Classical double copy for the Gravitational Binary

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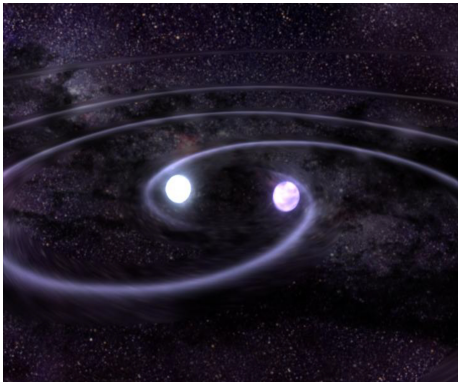
BalaFest
ICTS-TIFR, Bengaluru

The Gravitational two-body problem



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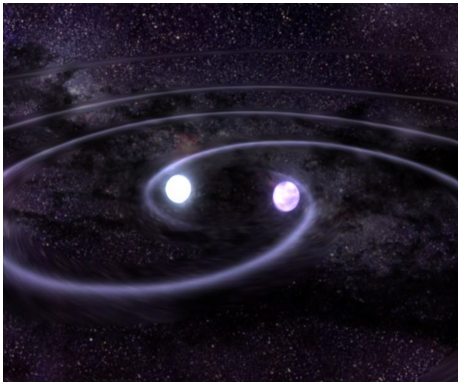

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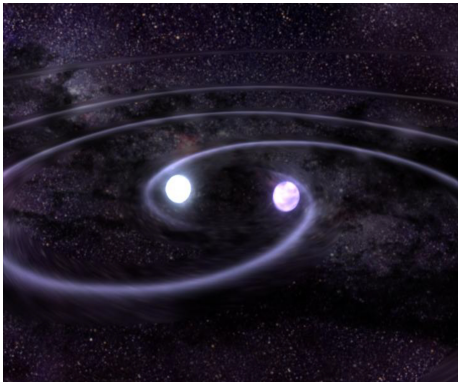

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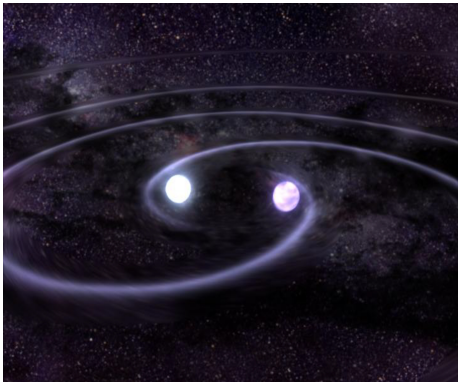

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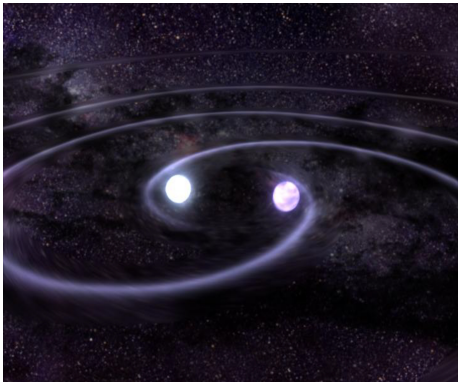

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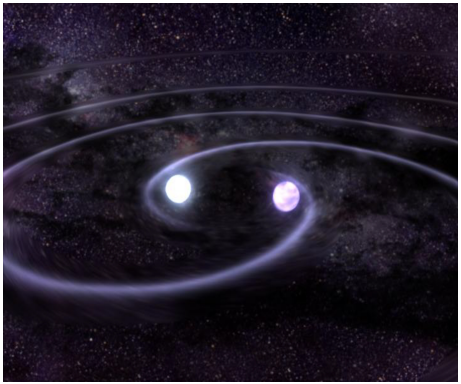

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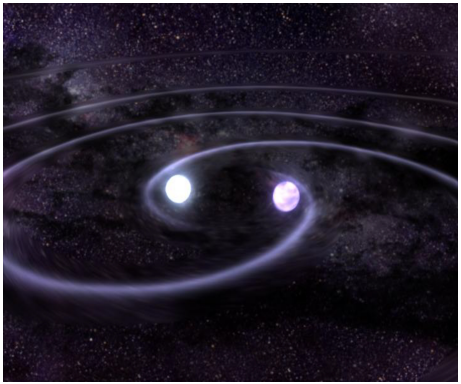

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- Declare victory?

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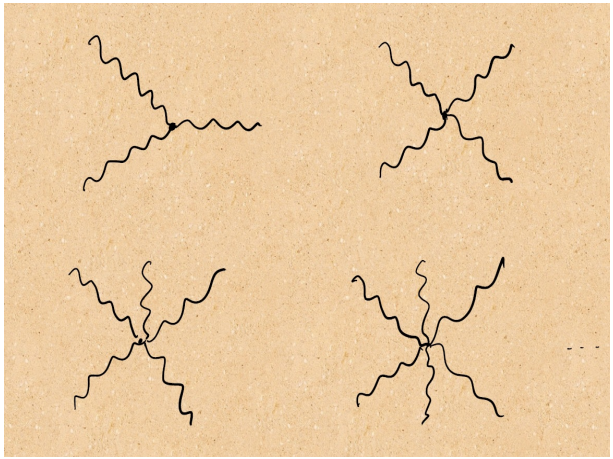
- Run into perturbative mess!



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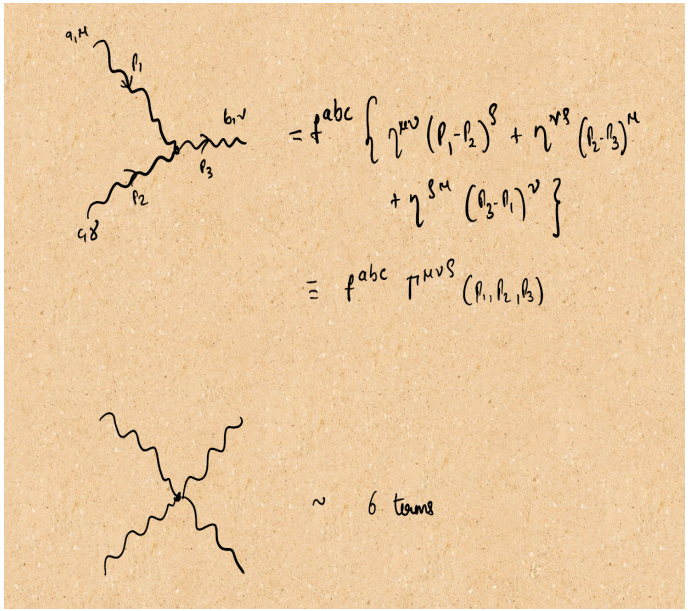

Genesis: String Theory (KLT) $\rightarrow$ Quantum Field Theory (BCJ)

- Bern, Carrasco and Johansson(BCJ) discovered an amazing set of relations between perturbative gravity scattering amplitudes, and the corresponding gauge theory scattering amplitudes.
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  Remarkably, the numerators also satisfy $n_i \pm n_j \pm n_k = 0$.

$$\textit{Color} \rightarrow \text{kinematics}$$

$$\mathcal{A}_{YM}^{\text{n-point}} = \sum_{\alpha} \frac{n_\alpha c_\alpha}{D_\alpha} \rightarrow \mathcal{A}_{grav}^{\text{n-point}} = \sum_{\alpha} \frac{n_\alpha n_\alpha}{D_\alpha}$$

- BCJ double copy relations proved for all tree level amplitudes. Highly non-trivial evidence for many loop level amplitudes.

A Scalar two-body problem

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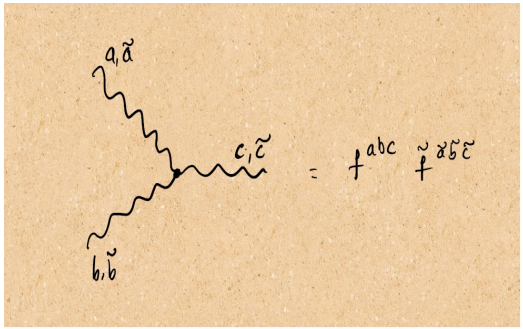

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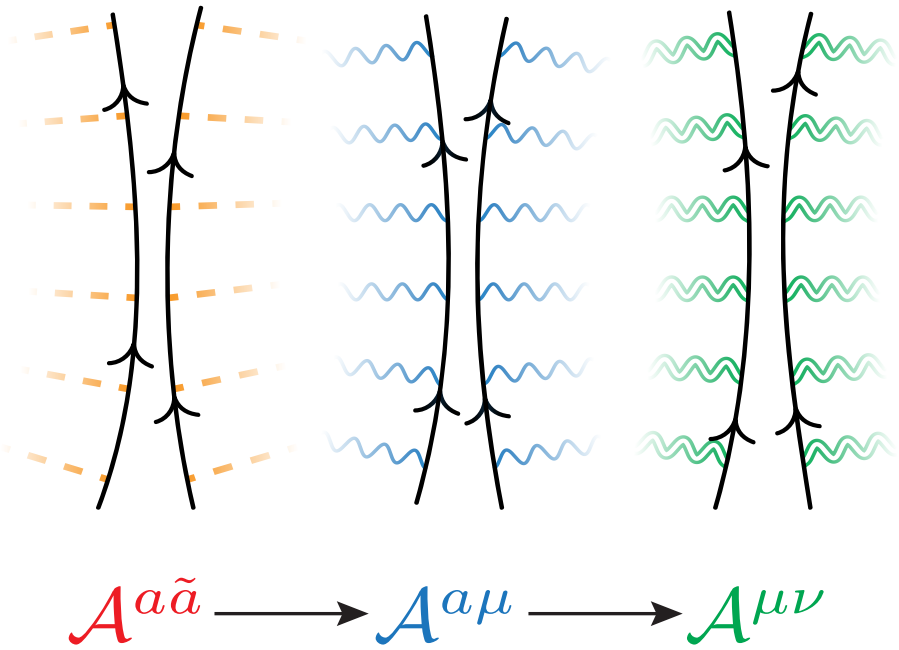
Bi-adjoint scalar	Scalars (Spin 0)	Trajectory, two kinds of color charge

The Classical Double Copy for radiation



Goldberger, Ridgway '16
Goldberger, SP, Thompson '17

Image credits: Rijan Maharjan

The Classical Double Copy

- Bound sources : Goldberger, Ridgway '17

The Classical Double Copy

- Bound sources : Goldberger, Ridgway '17
- Spinning sources: Goldberger, SP, Li '17; SP, Li '18

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Classical double copy from color-kinematics in the soft limit

- Athira PV, Manu '19

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- Does this work for curved spacetimes? (Ongoing work)

Thank You!