INTERNATIONAL CENTRE for THEORETICAL SCIENCES

TATA INSTITUTE OF FUNDAMENTAL RESEARCH

BBH Source Modeling Using Numerical Relativity: Status & Challenges

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THE FUTURE OF GRAVITATIONAL-WAVE ASTRONOMY
International Center for Theoretical Sciences
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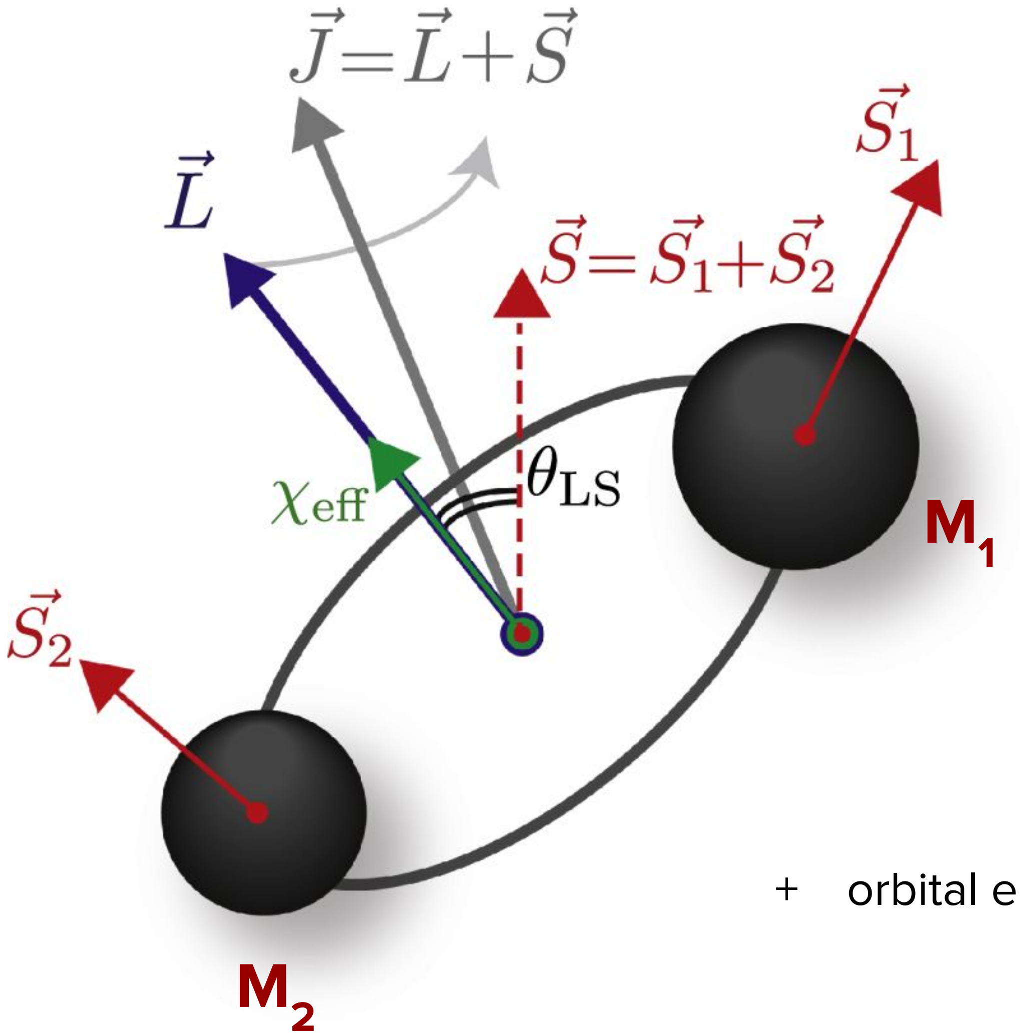
1. Source modeling



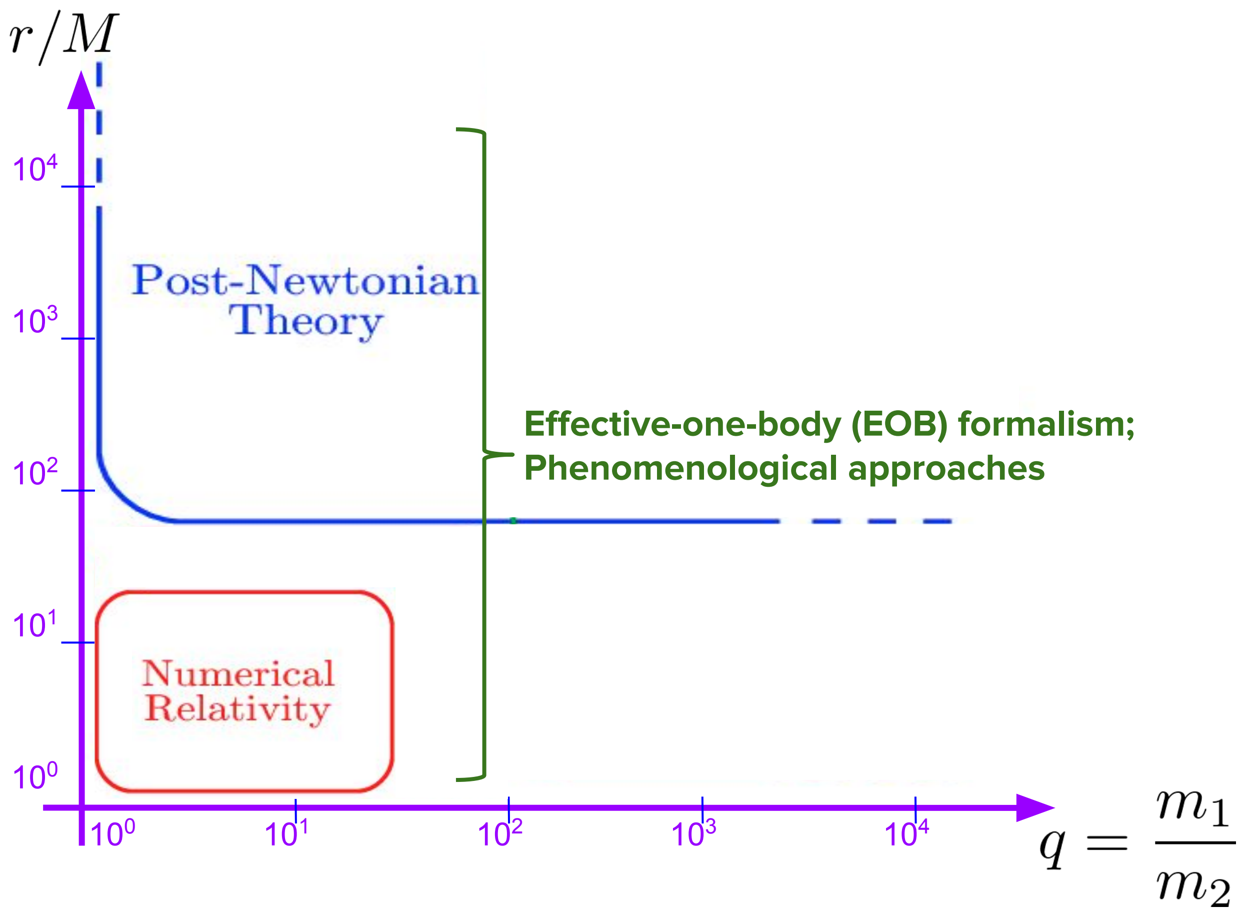
\+ orbital eccentricity ...

Figure credit: Rodriguez et al 2016 ApJL 832 L2

Current status of GW source modeling:

a. PN theory (brief)
b. NR
c. EOB formalism
d. Phenomenological models
e. Numerical Surrogates

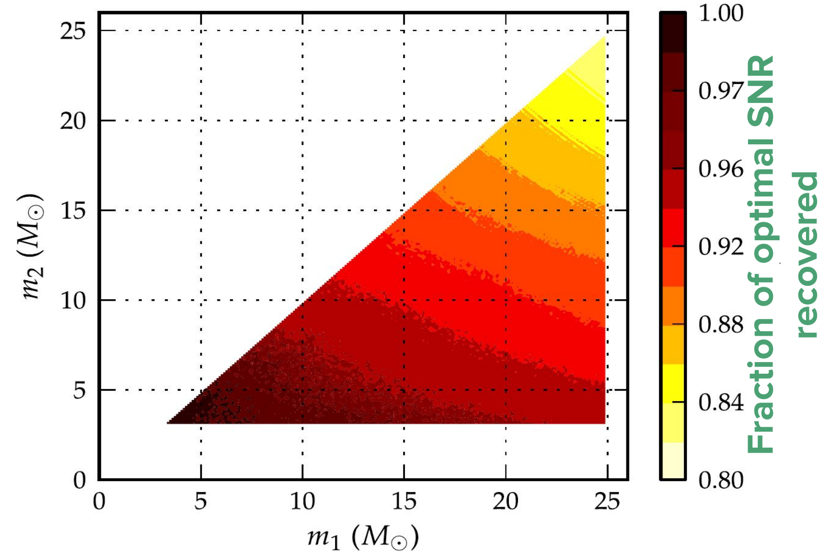
1. Source modeling

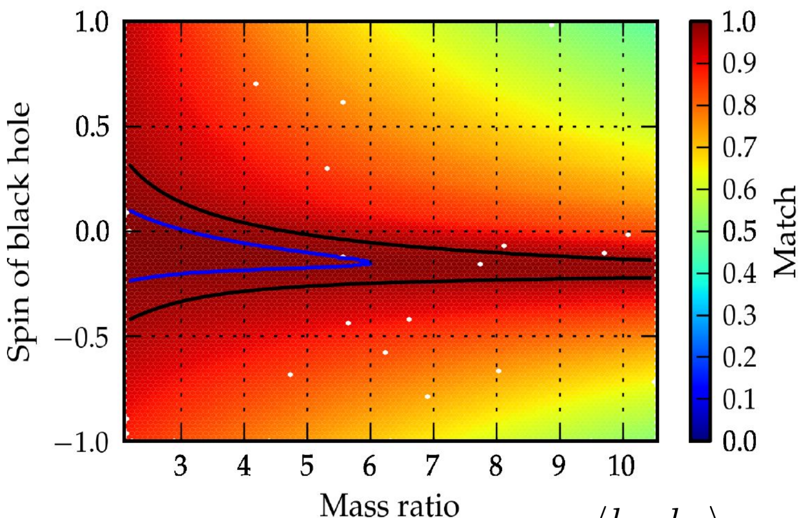


1a. Source modeling: Post Newtonian theory

- Sufficient for detecting BNS & low-mass BBHs with LIGO. Insufficient for LIGO binaries with masses $\gtrsim 12 M_\odot$
- Approximation gets worse with spins
- If heavy BHs (10 - 60 $M_\odot$) *dominate LIGO detection rates*, PN alone is not insufficient for LIGO detection (& PE)

(See this morning's panel)



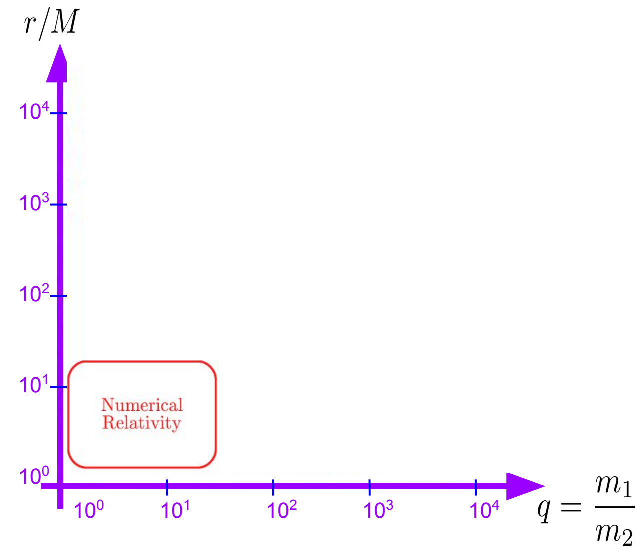


$$\frac{\langle h_1, h_2 \rangle}{|h_1||h_2|}$$

Buonanno & Sathyaprakash (2014), 1410.7832; PK et al (2013), 1310.7949; PK et al (2015), 1507.00103;

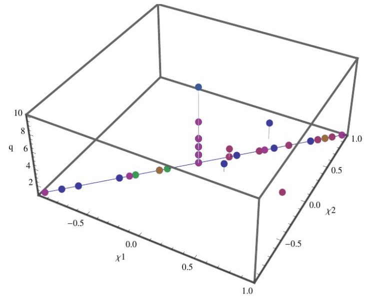
1b. Source modeling: Numerical Relativity

- Direct numerical evolutions of fully-nonlinear Einstein's equations

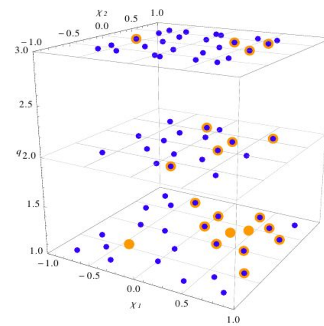


1b. Source modeling: Numerical Relativity

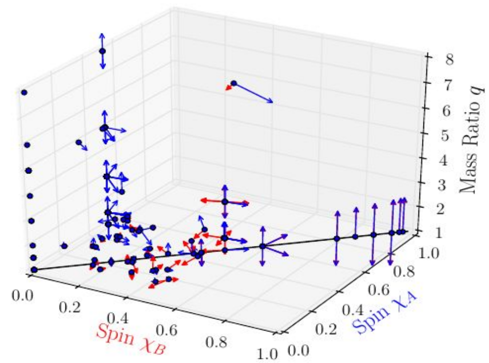
- Current catalogs



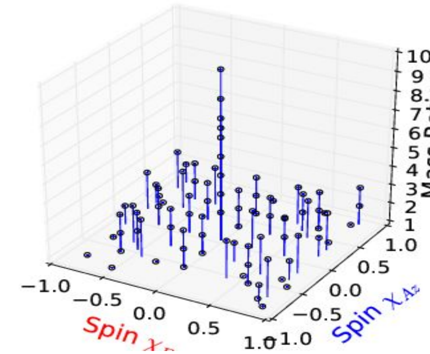
NINJA-2 Catalog (2012)



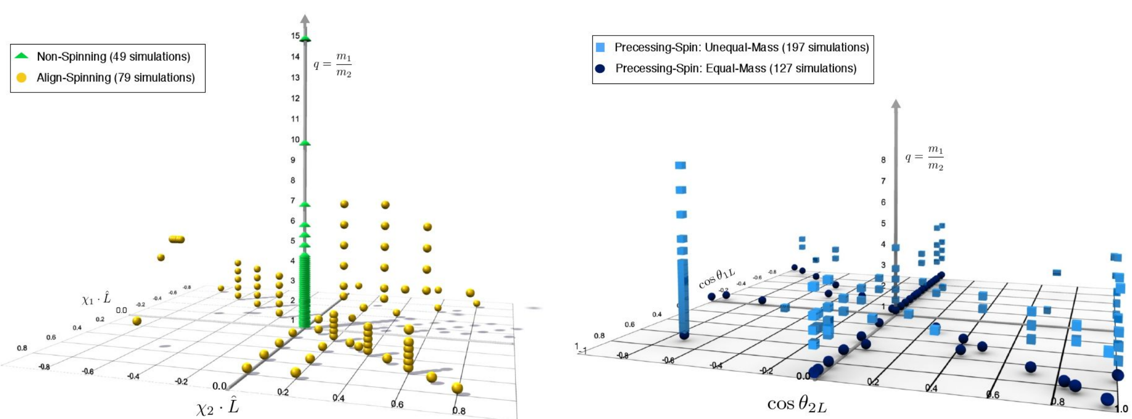
NRAR Catalog (2014)



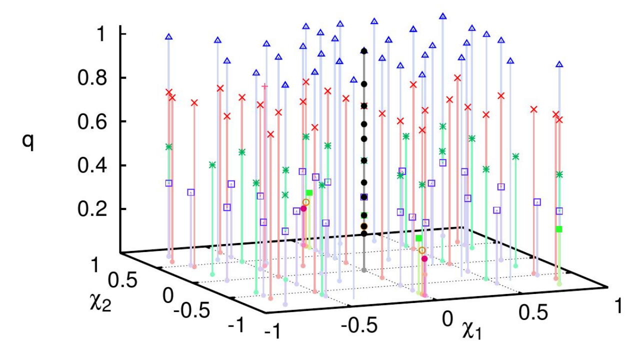
SXS Public Catalog (2013)



Chu, Fong, PK et al (SXS) (2016)



Jani et al (GATech) (2016)



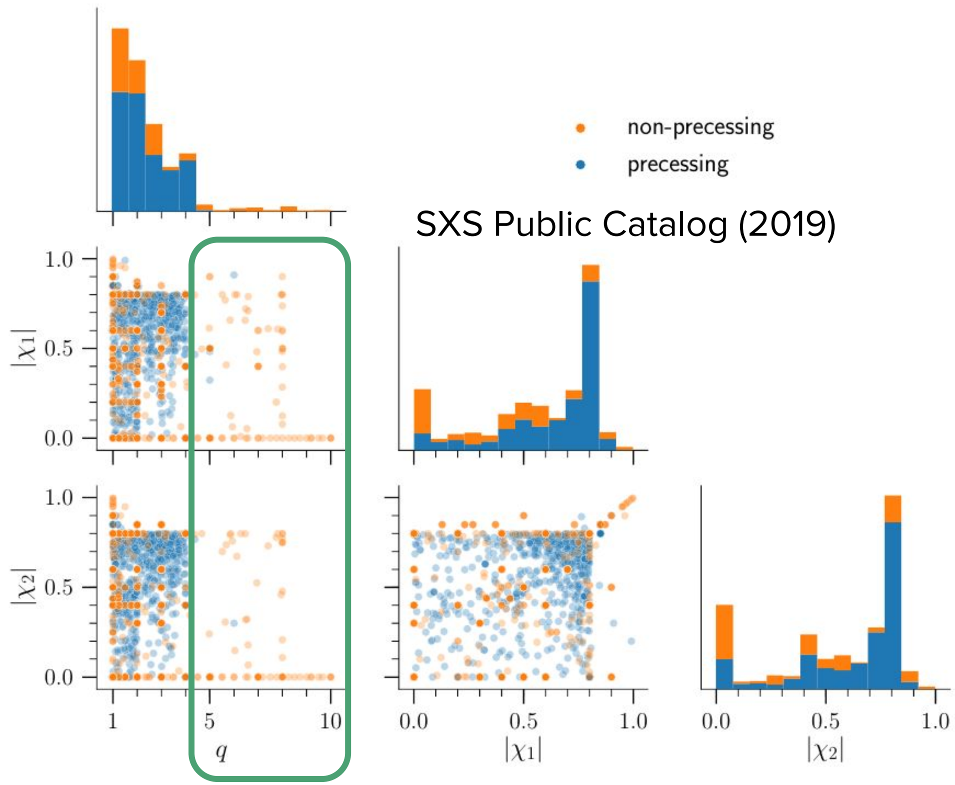
Healy et al (RIT) (2017)

1b. Source modeling: Numerical Relativity

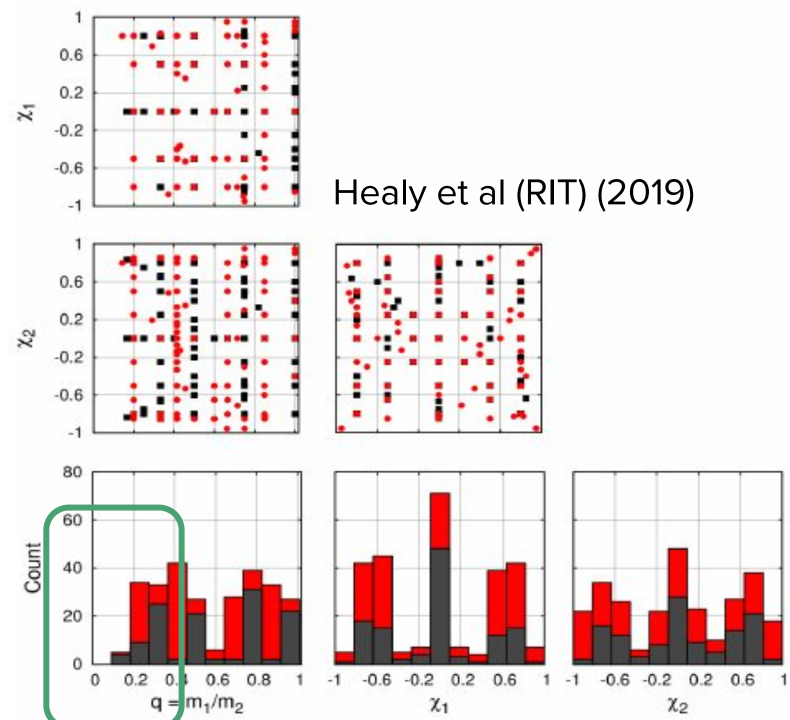
- Current catalogs

Catalog	Started	Updating?	Simulations	m_1/m_2 range	$\lvert\chi_1\rvert$ range	$\lvert\chi_2\rvert$ range	Precessing?	Median N_{cyc}	Public?
NINJA [97, 114]	2008	✗	63	1–10	0–0.95	0–0.95	✗	15	✗
NRAR [119]	2013	✗	25	1–10	0–0.8	0–0.6	✓	24	✗
Georgia Tech [121]	2016	✓	452	1–15	0–0.8	0–0.8	✓	4	✓
RIT (2017) [122]	2017	✓	126	1–6	0–0.85	0–0.85	✓	16	✓
RIT (2019) [123]	2017	✓	320	1–6	0–0.95	0–0.95	✓	19	✓
NCSA (2019) [124]	2019	✗	89	1–10	0	0	✗	20	✗
SXS (2018)	2013	✓	337	1–10	0–0.995	0–0.995	✓	23	✓
SXS (2019)	2013	✓	2018	1–10	0–0.998	0–0.998	✓	39	✓

Boyle et al (2019);



SXS Public Catalog (2019)



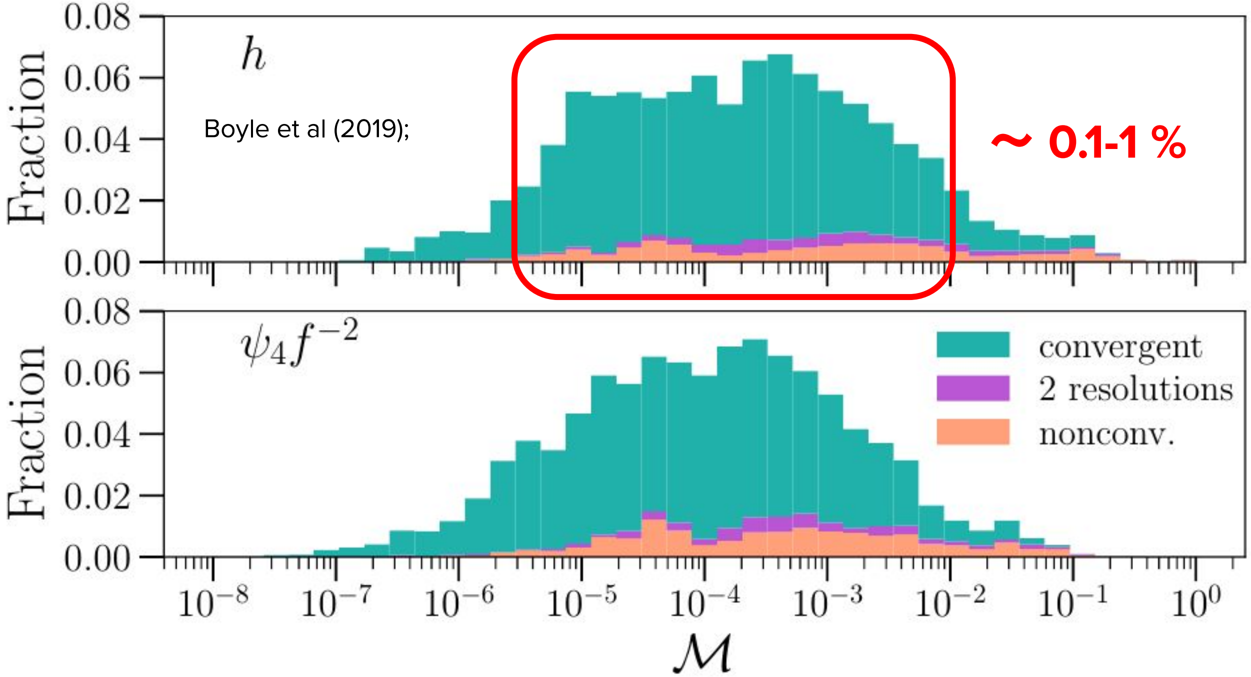
Healy et al (RIT) (2019)

1b. Source modeling: Numerical Relativity

- Current catalogs: Accuracy

Catalog	Started	Updating?	Simulations	m_1/m_2 range	$\|\chi_1\|$ range	$\|\chi_2\|$ range	Precessing?	Median N_{cyc}	Public?
NINJA [97, 114]	2008	✗	63	1–10	0–0.95	0–0.95	✗	15	✗
NRAR [119]	2013	✗	25	1–10	0–0.8	0–0.6	✓	24	✗
Georgia Tech [121]	2016	✓	452	1–15	0–0.8	0–0.8	✓	4	✓
RIT (2017) [122]	2017	✓	126	1–6	0–0.85	0–0.85	✓	16	✓
RIT (2019) [123]	2017	✓	320	1–6	0–0.95	0–0.95	✓	19	✓
NCSA (2019) [124]	2019	✗	89	1–10	0	0	✗	20	✗
SXS (2018)	2013	✓	337	1–10	0–0.995	0–0.995	✓	23	✓
SXS (2019)	2013	✓	2018	1–10	0–0.998	0–0.998	✓	39	✓

Boyle et al (2019);

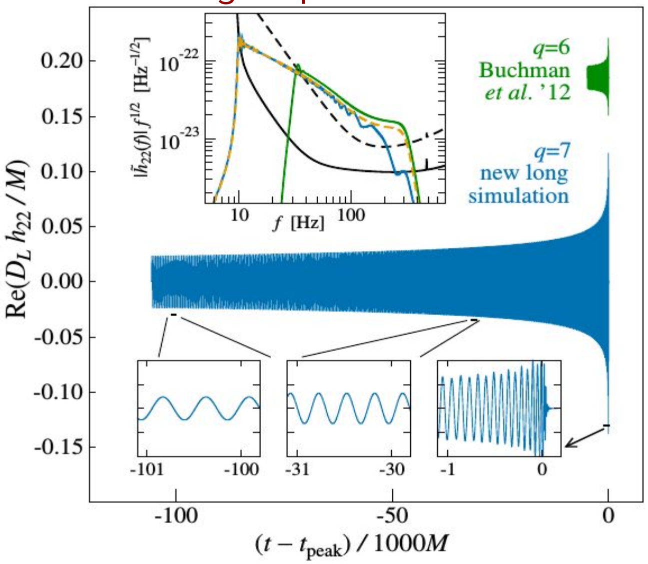


1b. Source modeling: Numerical Relativity

- Frontiers

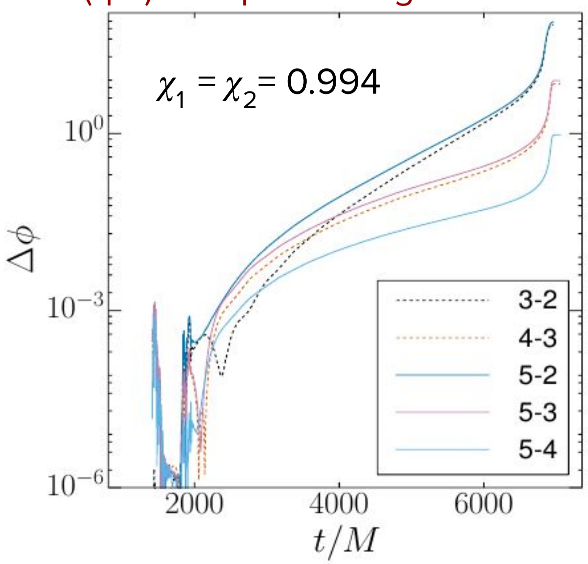
➢ *Critical for GW models of binary mergers*

Length up to ∽170 orbits



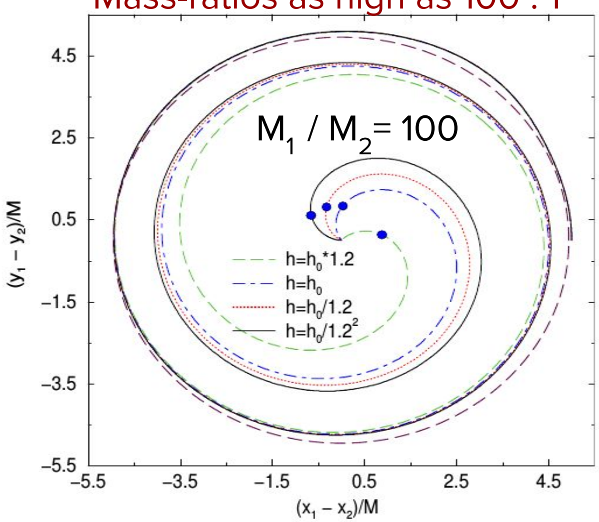
Szilagyi et al (2015)

(q=1) BH spins as high as 0.994



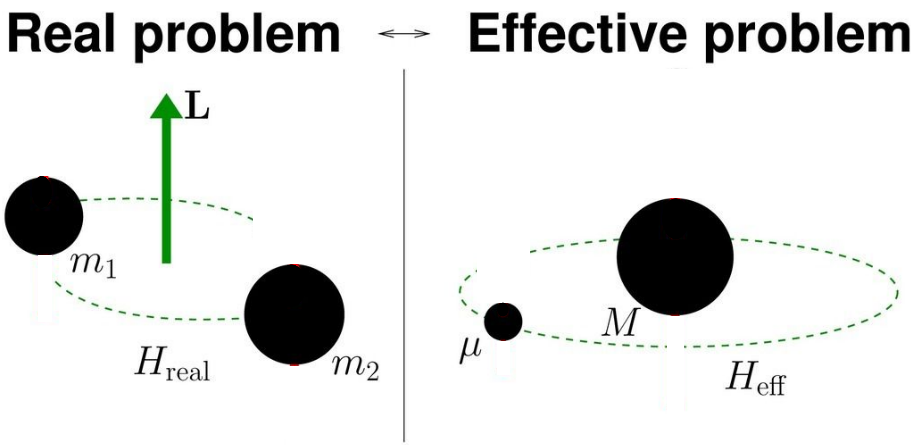
Scheel et al (2014)

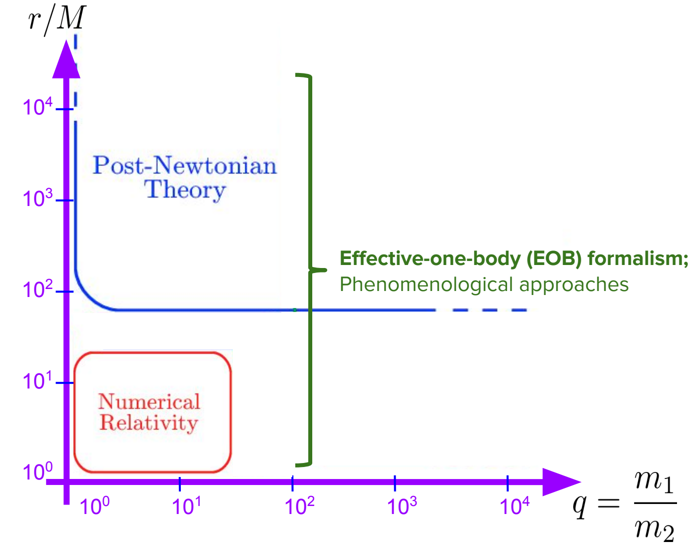
Mass-ratios as high as 100 : 1



Lousto & Zlochower (2011)

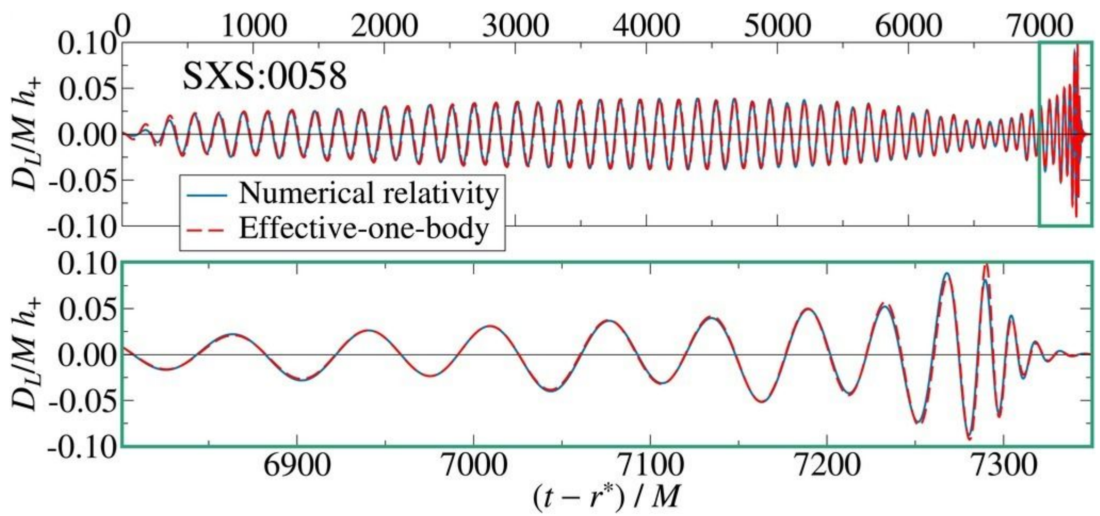
1c. Source modeling: Effective one body (EOB)





- Several free parameters in radiative & conservative dynamics;

  ⇒ Calibrated to NR

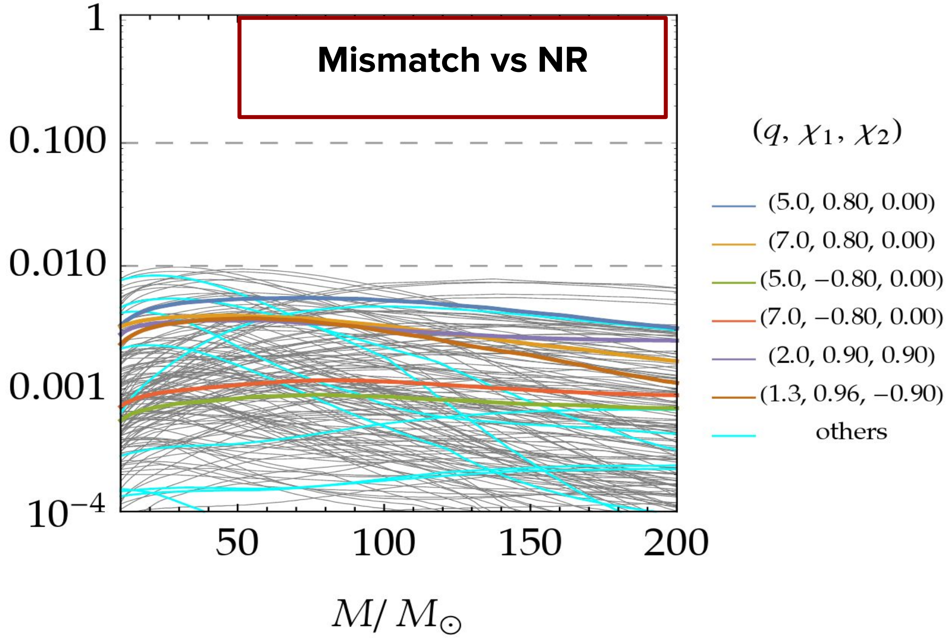


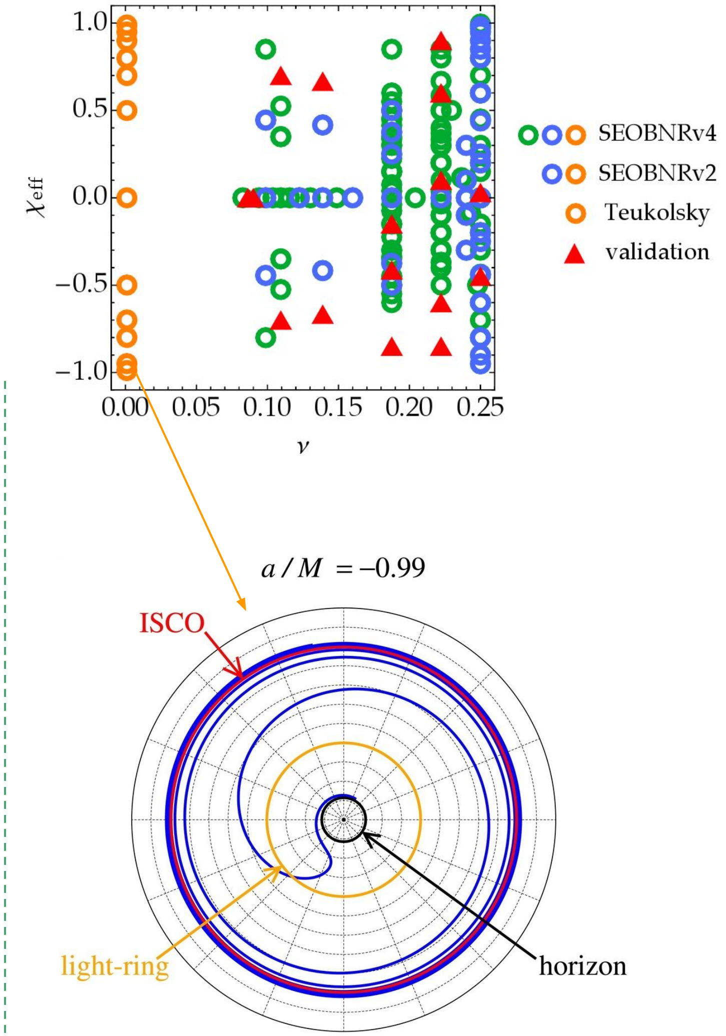
Buonanno et al (1998), gr-qc/9811091; Pan et al (2011), 1106.1021; Image credit: A. Taracchini/AEI

1c. Source modeling: Effective one body (EOB)

- Cutting-edge: EOBNRv4
- Calibrated to 141 NR + 10 numerical Teukolsky waveforms
- Agreement with NR to *better than 1%*

$$\text{Mismatch}(h_1, h_2) = 1 - \frac{\langle h_1, h_2 \rangle}{|h_1||h_2|}$$

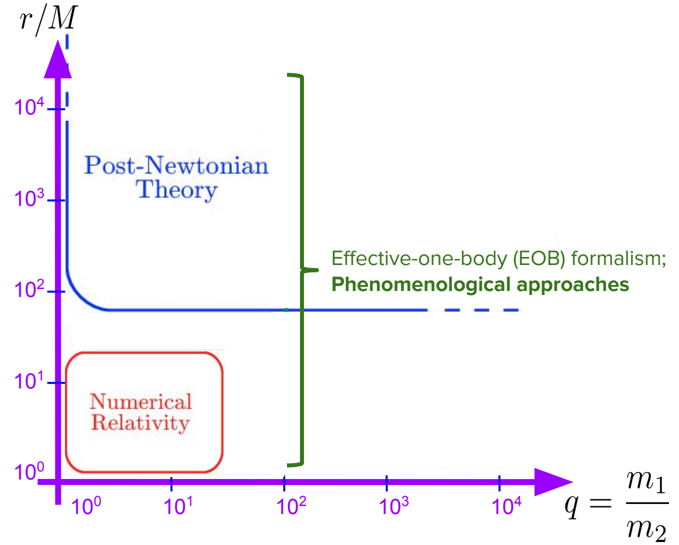


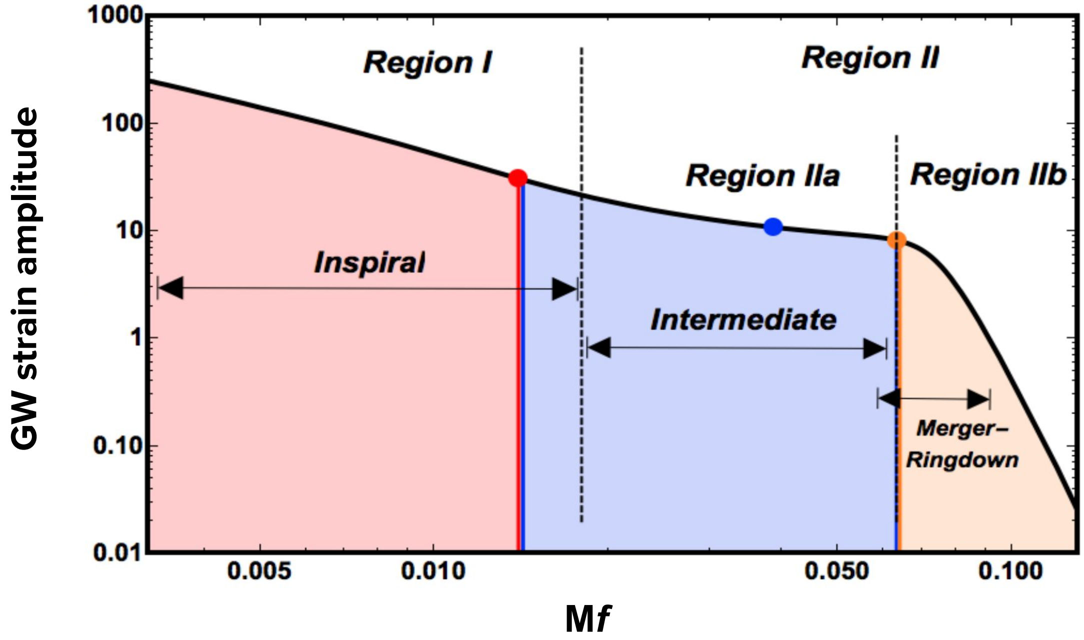


Bohe, ..,PK et al (2016), 1611.03703; Image credit: A. Taracchini/AEI

1d. Source modeling: IMRPhenom

- Guided by need to reduce computational cost of waveform generation
- PN-inspired ansatz for GW amp/phase, and NR (+EOB hybrids) used to calibrate the ansatz

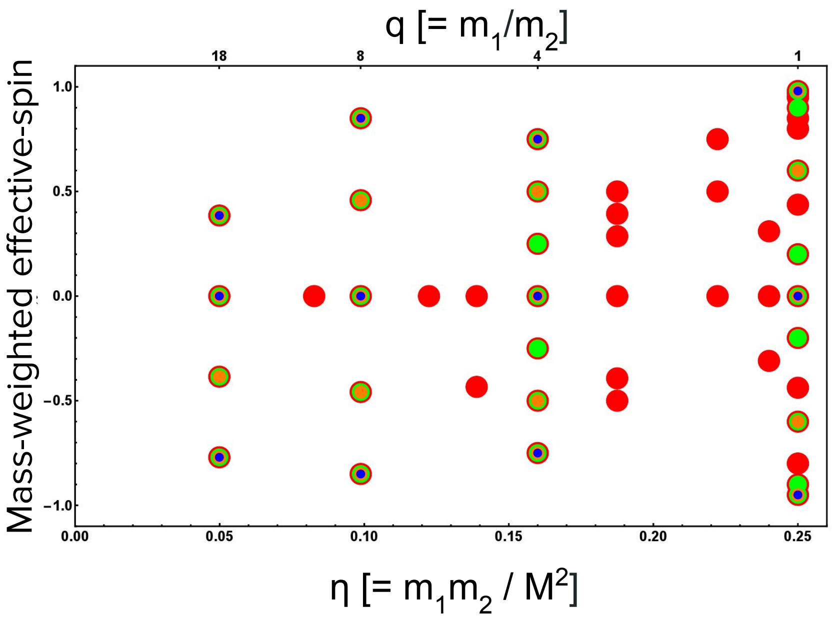


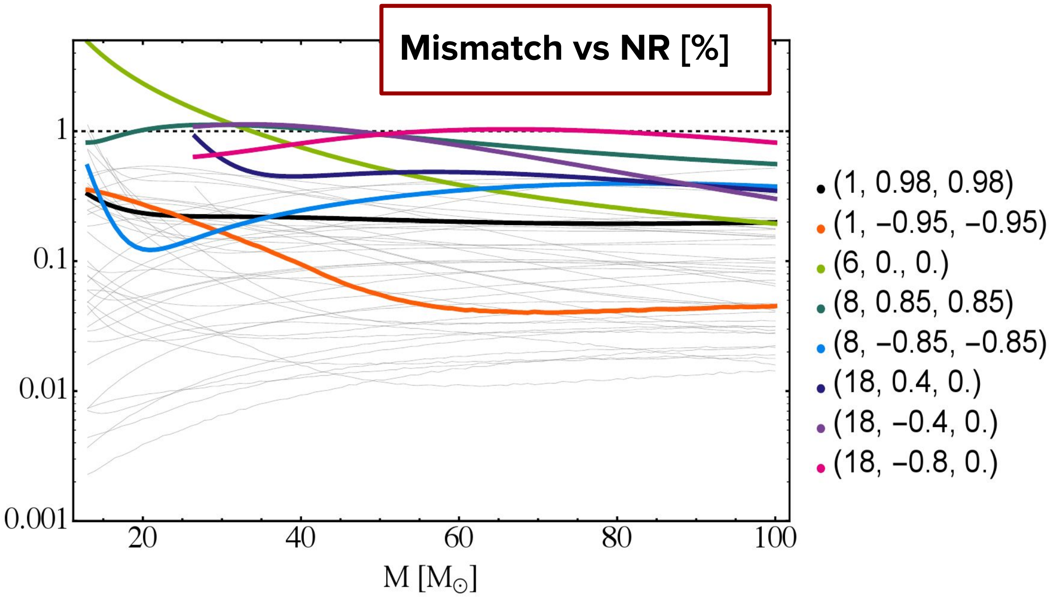


Khan et al (2015), 1508.07253; Santamaria et al (2010), 1005.3306; Ajith et al (2007), 0704.3764/0710.2335; Ajith et al (2009), 0909.2867;

1d. Source modeling: IMRPhenom

- Guided by need to reduce computational cost of waveform generation
- PN-inspired ansatz for GW amp/phase, and NR (+EOB hybrids) used to calibrate the ansatz
- State of art - PhenomD (version 4)





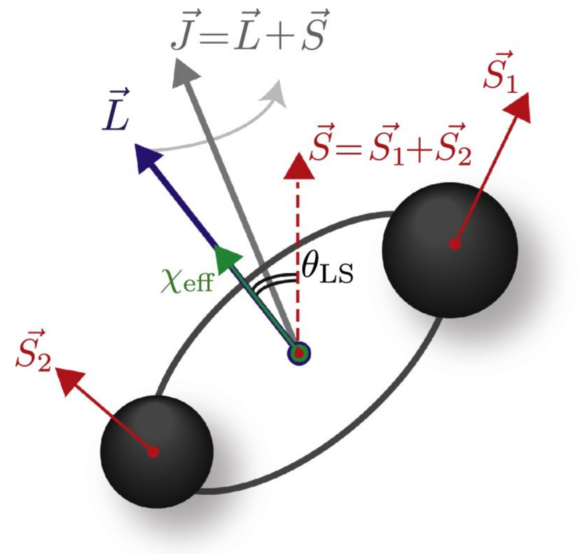
Khan et al (2015), 1508.07253; Santamaria et al (2010), 1005.3306; Ajith et al (2007), 0704.3764/0710.2335; Ajith et al (2009), 0909.2867;

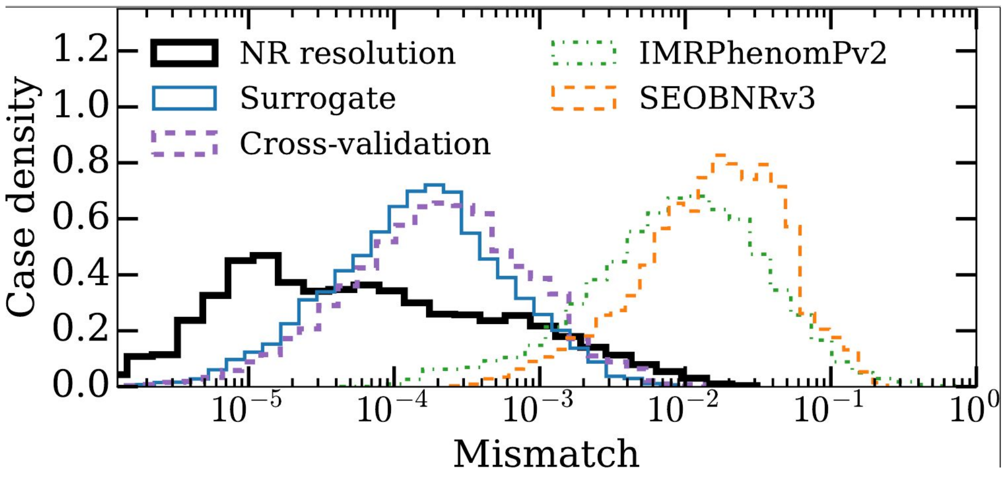
1e. Source modeling: NR surrogate model

- Time-domain surrogates have been built for NR waveforms directly. Used 744 NR simulations:
    - Full-precession; l <= 4 modes
    - **1 ≤ q ≤ 2; spin magnitudes < 0.8**
    - Length ≈ 4500M

➢ **Valuable tool for waveform modeling**

➢ **Direct application of NR to PE (extracted HM from GW events*)**

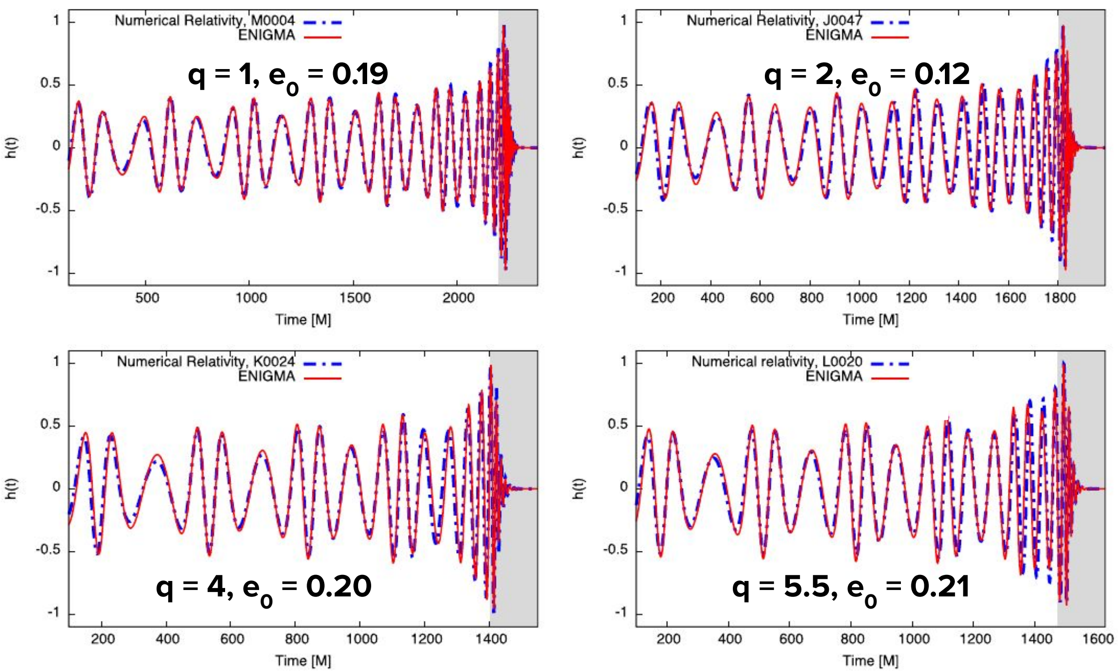


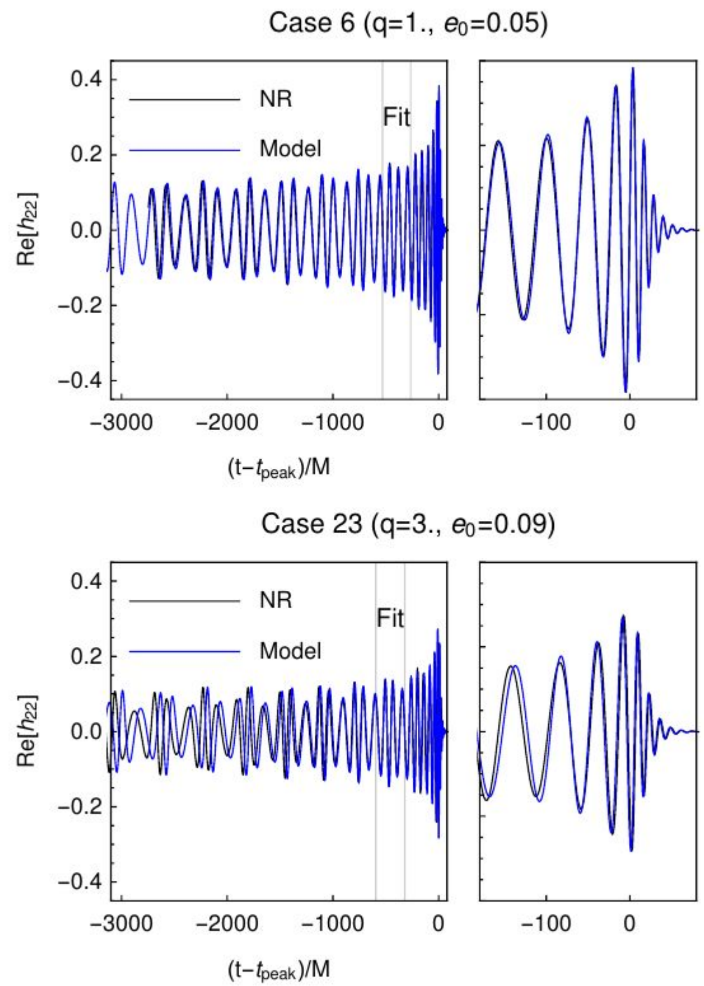


Blackman et al (2017), 1705.07089; Blackman et al (2017), 1701.00550;
*PK et al (2018), 1808.08004; [Figure credit: Rodriguez et al 2016 ApJL 832 L2]

1f. Source modeling: Eccentric

- First inspiral + plunge-merger models for eccentric orbits recently (2017-18):
- Mass ratios <= 6; **NO spins;**





Huerta, ..**PK**.. et al, Phys. Rev. D 97, 024031 (**2018**)

Hinder, Kidder, Pfeiffer, Phys. Rev. D 98, 044015 (**2018**)

1. Source modeling: Summing up current status

- In general:
  - Inspiral + merger-ringdown modeling benefits from NR.
  - ...

- BHB in quasi-circular orbits:
  - Models exist that capture spin+precession effects through inspiral + merger-ringdown;
  - Current EOB / Phenom models calibrated to NR waveforms:
    - densely to **q ~ 6**; and v. sparsely to q ~ 15;
    - BH **spin magnitudes up to ~0.8** (densely);
  - Model accuracy: approx **0.1 - 1%** mismatch vs NR (i.e. SNRs of 40-50).
  - NR accuracy: approx **0.01 - 1%** mismatch

- BHB in eccentric orbits:
  - Models exist that capture eccentric motion through inspiral + merger-ringdown;
  - Calibrated to NR waveforms with:
    - mass ratios up to **q ~ 5**;
    - **e_0 up to 0.2** (10 orbits before merger);
    - No spins!
  - Model accuracy: approx **0.1 - 3%** mismatch vs NR

Current open questions and future needs for BBH modeling:

a. LIGO Sources
b. LISA Sources (MBHB)

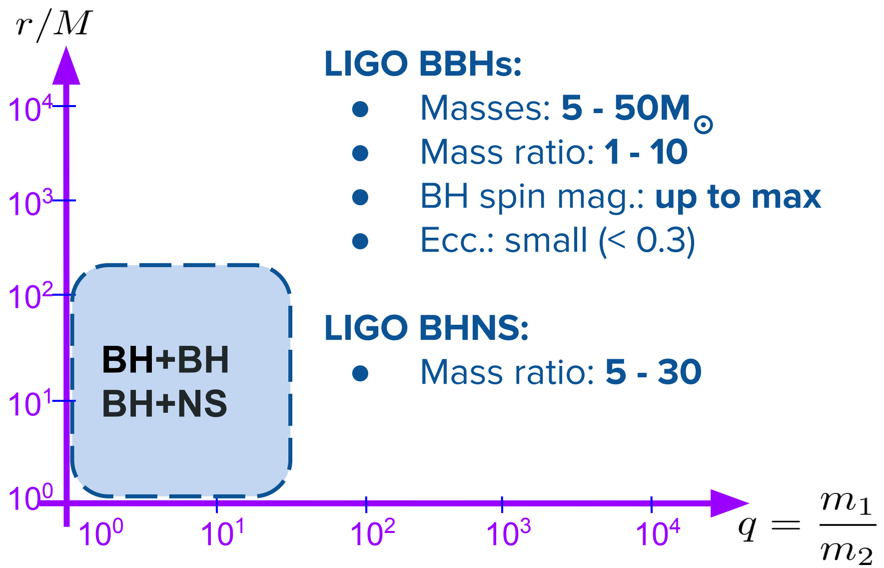
2. LIGO: BBH sources modeling needs

LIGO BBHs:

- Masses: **5 - 50M$_\odot$**
- Mass ratio: **1 - 10**
- BH spin mag.: **up to max**
- Ecc.: small (< 0.3)

LIGO BHNS:

- Mass ratio: **5 - 30**



Accuracy of present-day NR waveforms is sufficient. We still need:

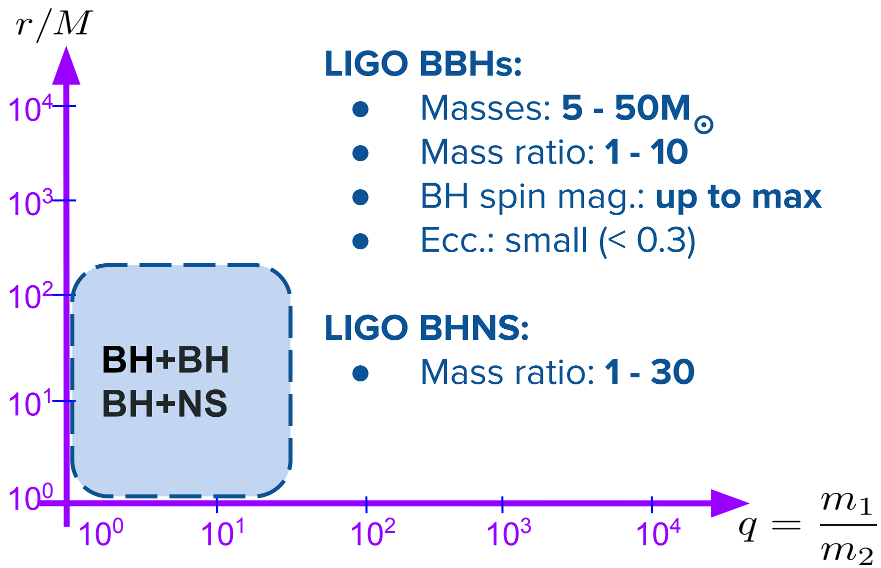
1. More model accuracy for high q (> 6)
   ⇒ more NR waveforms!
2. Subdominant harmonics for high q
   ⇒ more NR waveforms!
3. Spinning+eccentric models
   ⇒ need NR waveforms!

⇒ *(1) and (2) especially for BHNS..*

SNR	10	10^2
Mismatch (vs NR)	< 2 %	< 0.02%

*using criterion from Lindblom et al (2008), 0809.3844;

2. LIGO → LISA: BBH sources modeling needs



LIGO BBHs:

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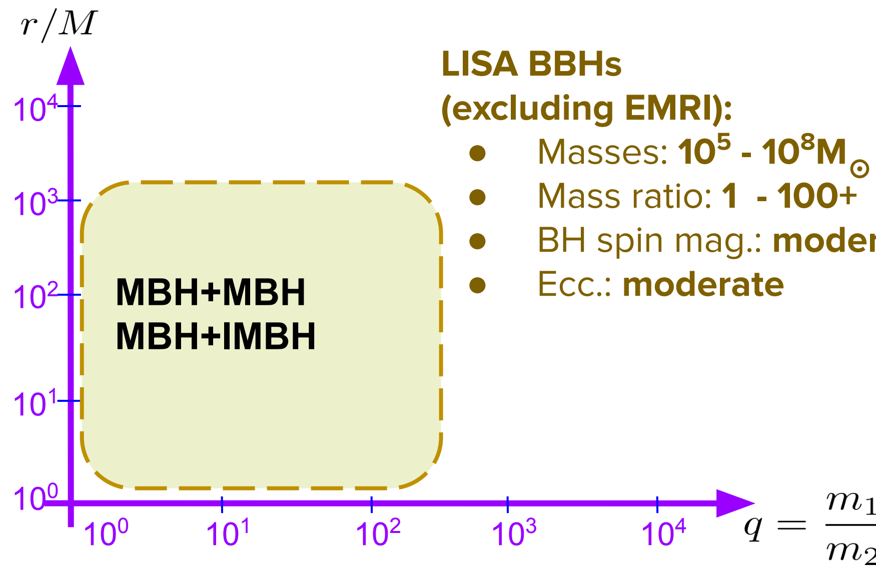
LIGO BHNS:

- Mass ratio: **1 - 30**



LISA BBHs (excluding EMRI):

- Masses: **10^5 - 10^8M$_\odot$**
- Mass ratio: **1 - 100+**
- BH spin mag.: **moderate**
- Ecc.: **moderate**

2. LIGO → LISA: BBH sources modeling needs

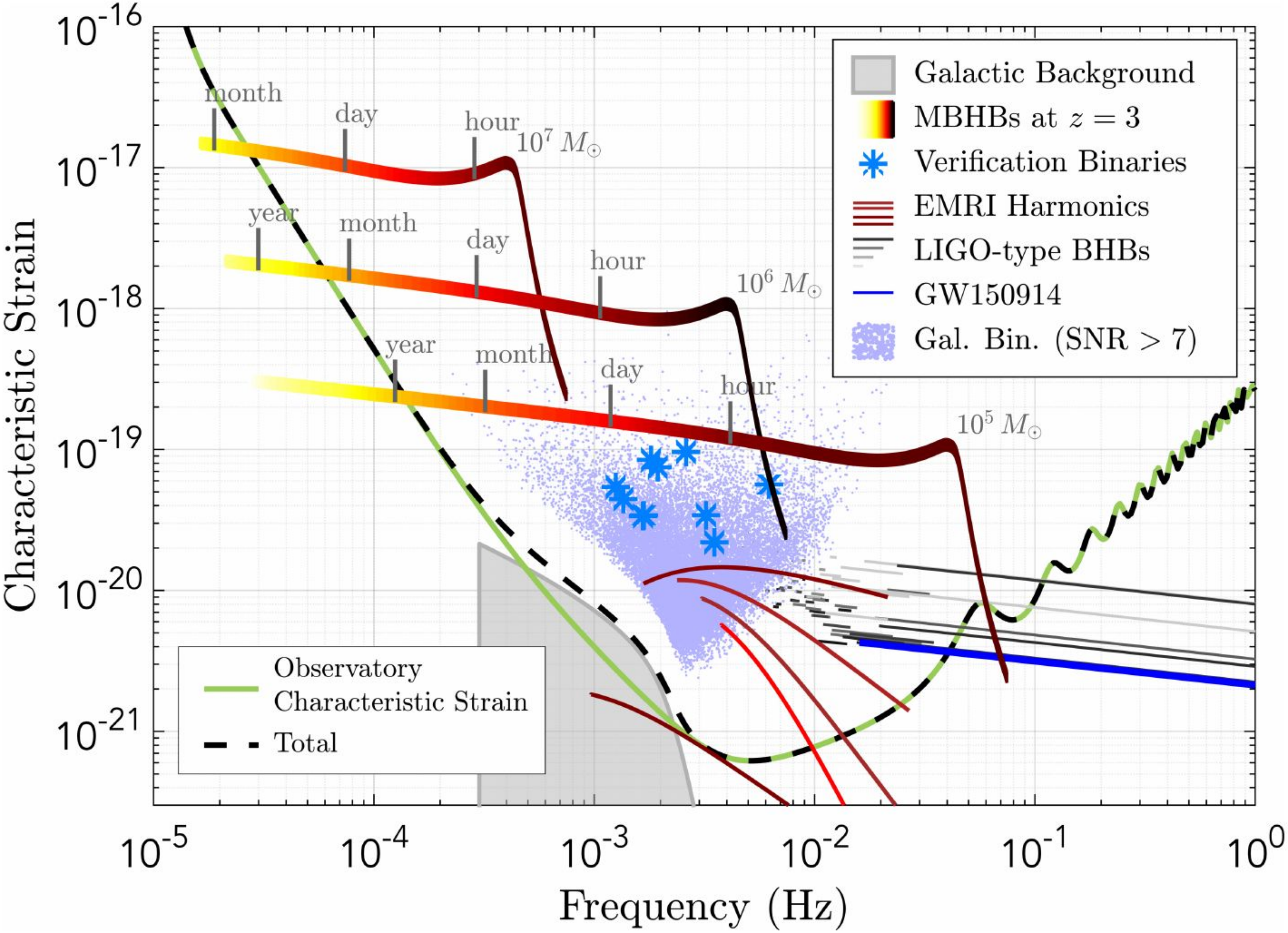
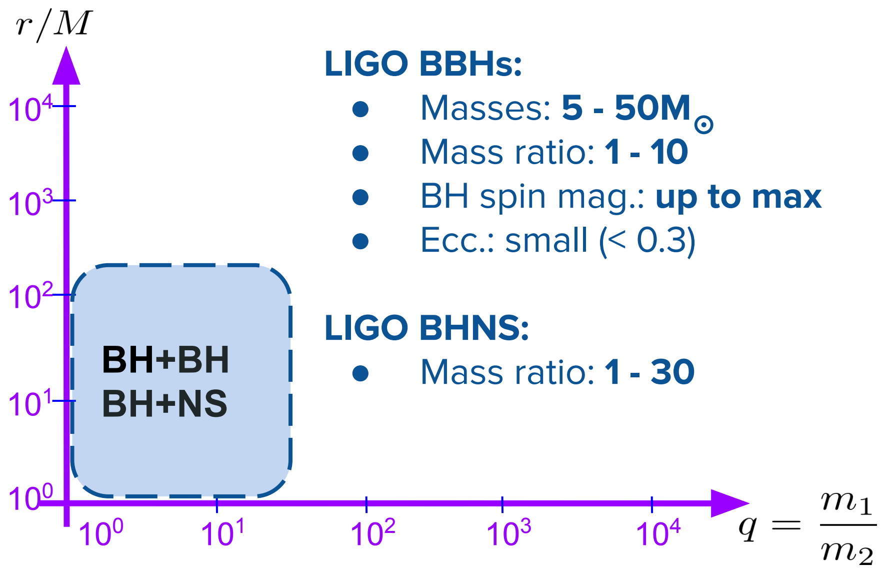


Figure 1 of LISA L3 document

2. LIGO → LISA: BBH sources modeling needs



LIGO BBHs:

- Masses: **5 - 50M$_\odot$**
- Mass ratio: **1 - 10**
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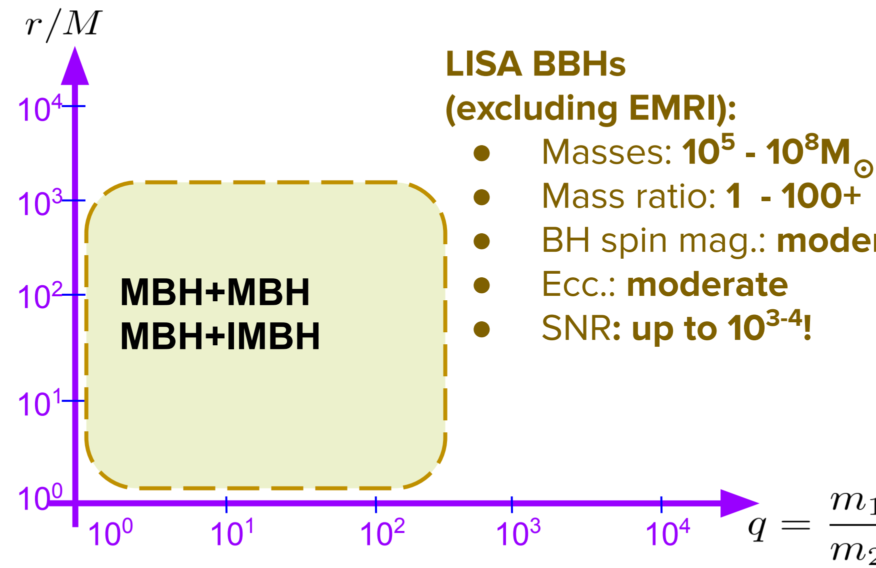
LIGO BHNS:

- Mass ratio: **1 - 30**



LISA BBHs (excluding EMRI):

- Masses: **10^5 - 10^8M$_\odot$**
- Mass ratio: **1 - 100+**
- BH spin mag.: **moderate**
- Ecc.: **moderate**
- SNR**: up to $10^{3\text{-}4}$!**

<u>Waveforms needed for:</u>

1. **Testing GR**
2. Astrophysical parameter estimation
3. Minimizing residuals after the hierarchical subtraction of loud signals
4. Detection (subset of over 2. and 3.)

$\Rightarrow$ will set the strictest accuracy requirements.

Q. Are current NR waveforms sufficiently accurate for MBHBs?

SNR	10	10^2	10^3	10^4
Mismatch (vs NR)	< 2 %	< 0.02%	< 2x10^{-4} %	< 2x10^{-6} %

*using criterion from Lindblom et al (2008), 0809.3844;

2. LIGO → LISA: NR sources modeling needs

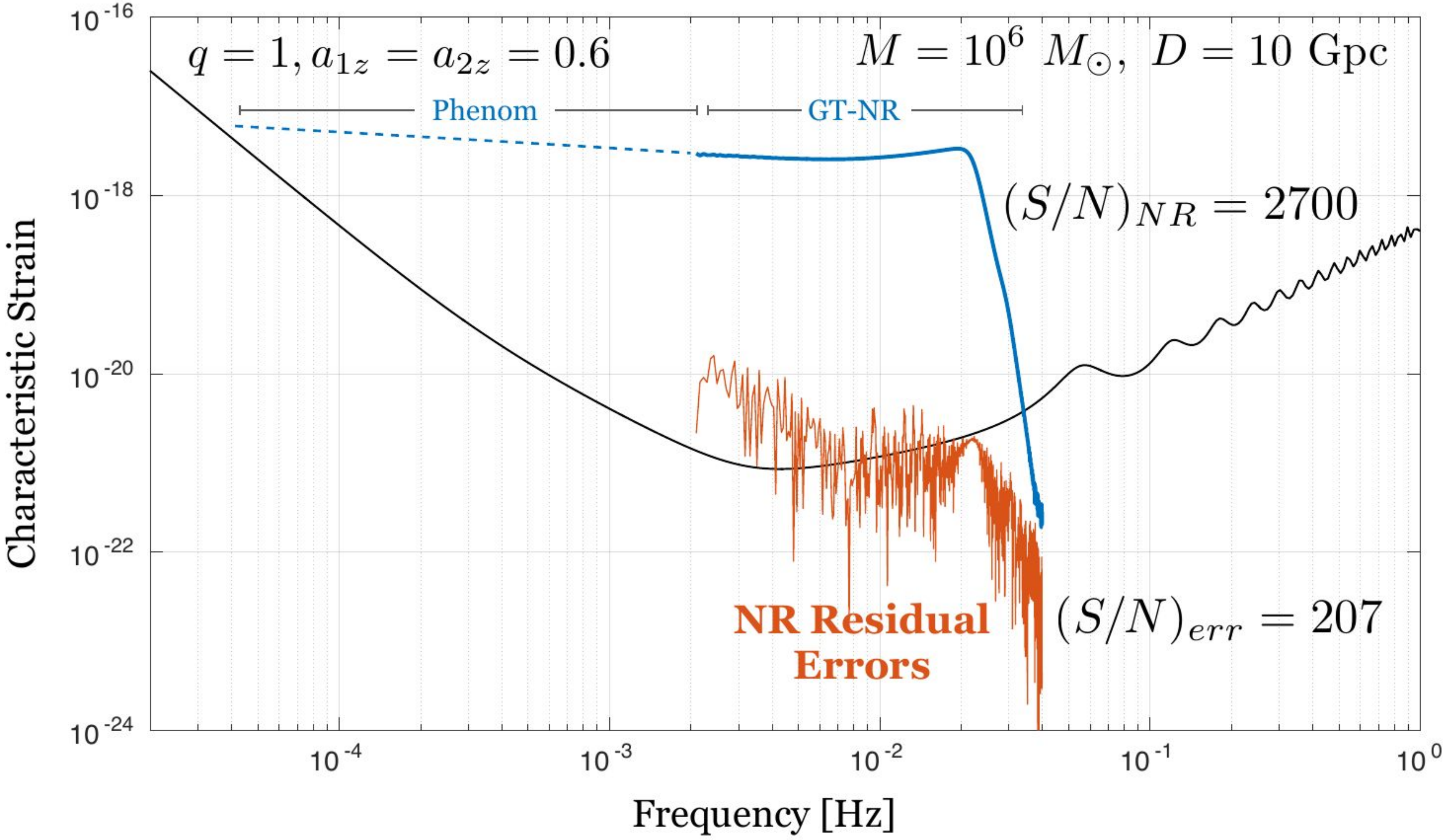
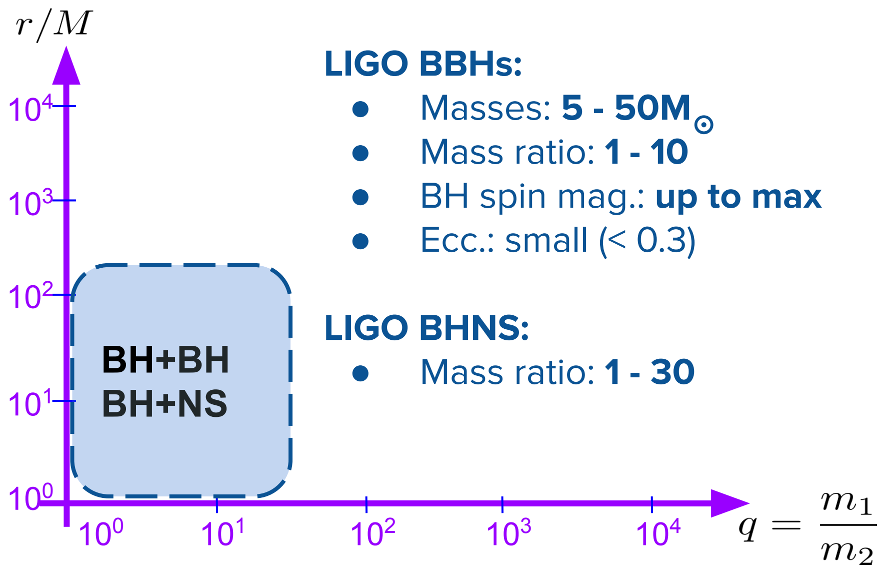
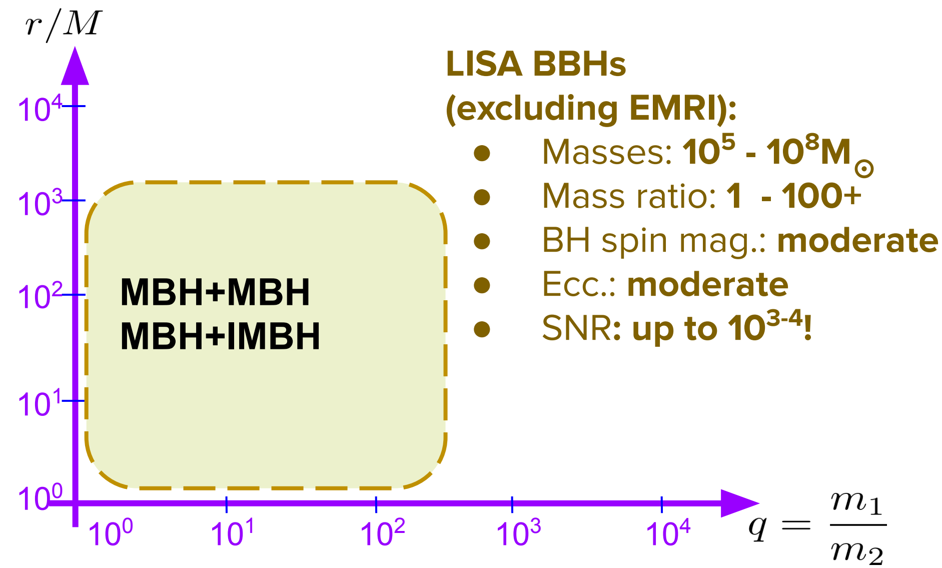


Figure credit: D. Shoemaker, APS April Meeting 2019

2. LIGO - LISA: BBH sources modeling needs





<u>Waveforms needed for:</u>

1. **Testing GR** $\Rightarrow$ will set the strictest accuracy requirements.
2. Astrophysical parameter estimation
3. Minimizing residuals after the hierarchical subtraction of loud signals
4. Detection (subset of over 2. and 3.)

Q. Are current NR waveforms sufficiently accurate for MBHBs?

A. <u>**NO!**</u> (We really need to quantify this soon!)

3. Summing up

- For LIGO observations of stellar-mass BHBs / BHNS:
  - Current models for quasi-circular BHBs work to SNRs ~50 and q < 8.
  - Need modeling of higher mass ratios. **More NR information needed.**
  - Need modeling of spinning eccentric binaries. **Little NR information available.**

- For LISA observations of MBHBs with q ~ O(10):
  - Current model (NR) waveforms work to SNRs of 40-50 (120 - 150).
  - Observations @ SNRs ~ 100 - 1000+!
  - **Need more accurate NR waveforms to improve models.**

- For MBHBs with q ~ O(100):
  - EOB incorporates information from test-particle limit. Will need further calibration/validation against NR for q = 10 - 100.
  - **No reliable NR information available!**

- IMBH + MBH (IMRIs) need generic binary models (eccentric w/ spins).
  No reliable models or NR information available!

- LISA can record SNRs ~ $O(10\text{-}10^4)$. Even NR waves are not accurate to such high SNRs, this will hinder tests of GR. ***Need overall better accuracy in all above approaches.***

Extras

LISA: Science

- Use ringdown from merging BBHs to test if post-merger BHs are as predicted by GR (measure more than 1 mode)
- Origin, growth and merger history of supermassive black holes across cosmic ages
- Study growth mechanisms of SMBHs from earliest quasars - including measuring dimensionless spin parameters to 0.1 and misalignment of spin to 10 degrees
- Observe EM counterparts to merging BBHs (requirement on localization)
- Test existence of Intermediate-Mass black holes (IMBH)
- Origins of stellar-mass black holes (multi-band GW astronomy!)

LISA: MBHs

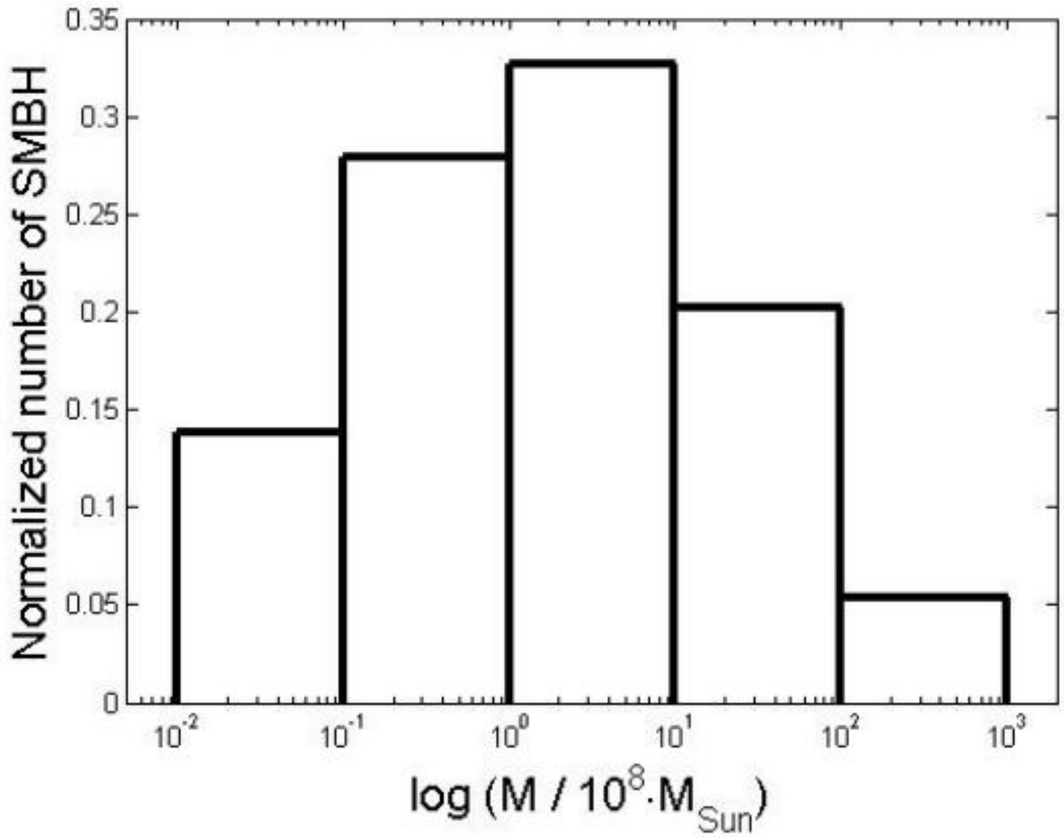


Figure: arXiv:1811.00808

LISA: MBHs

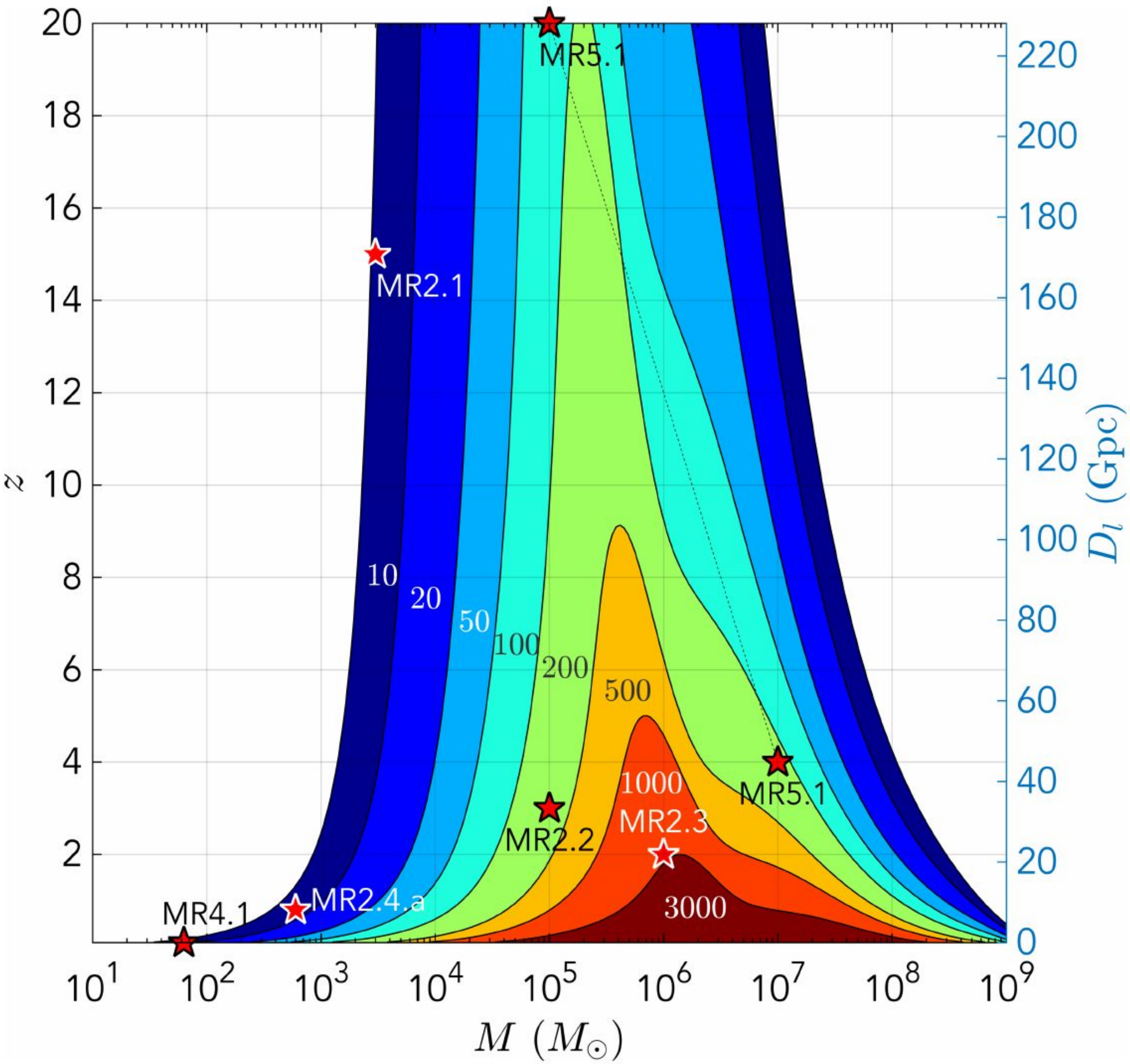
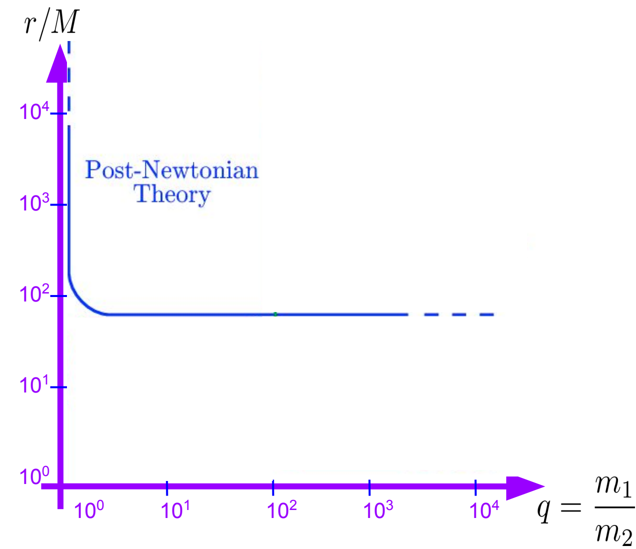


Figure 3 of LISA L3 document

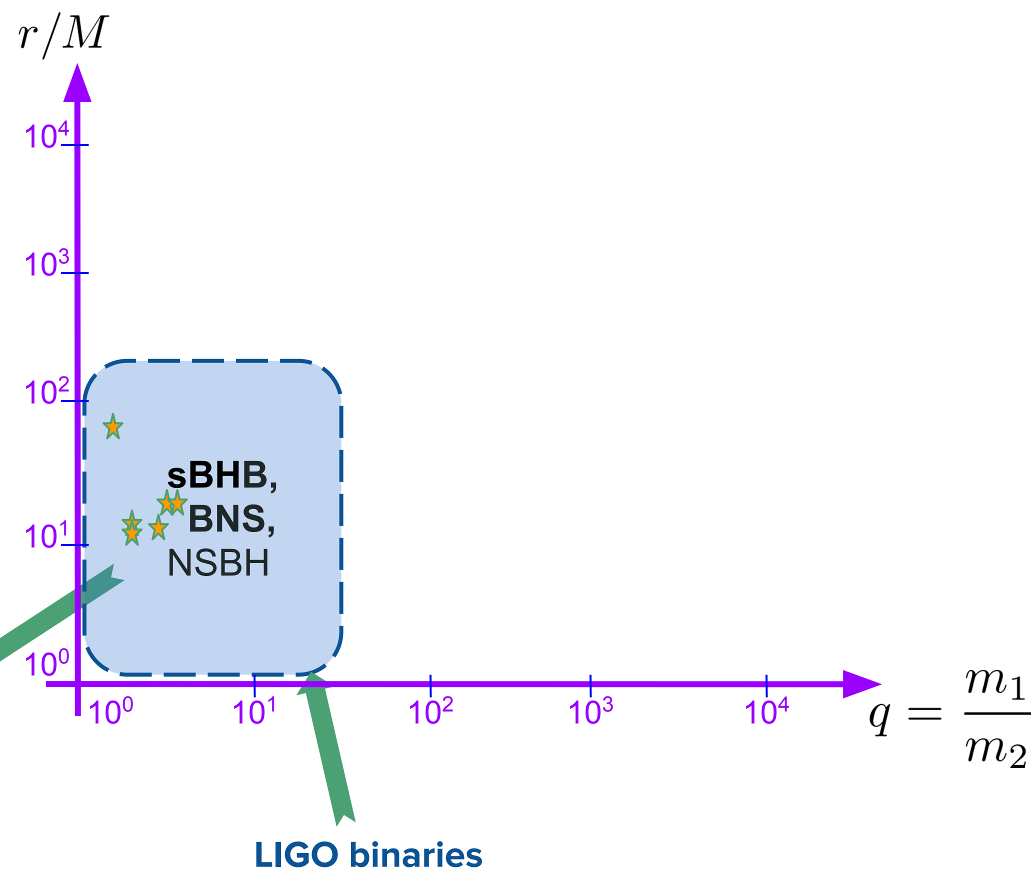
2a. Source modeling: Post Newtonian theory

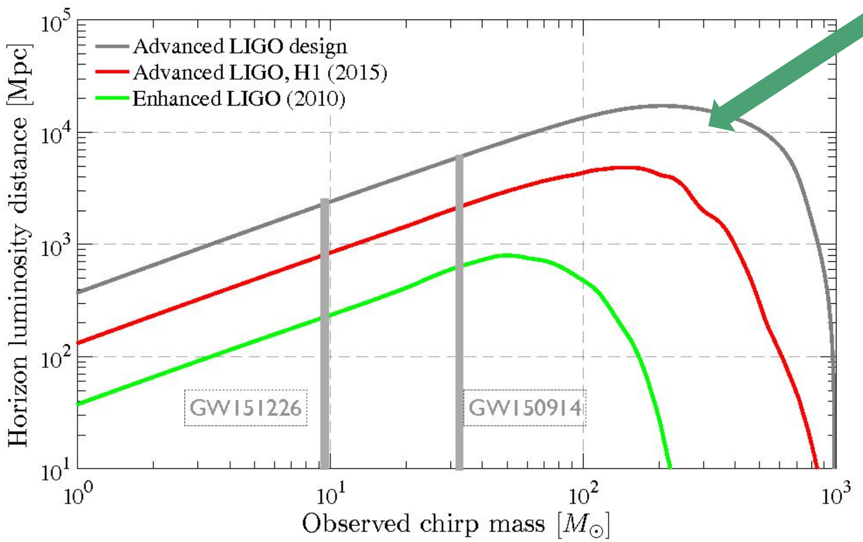
- Slow-motion weak-field approximation
- Perturbative expansions in orbital velocity (v/c)
- Equation of Motion:
  2PN EoM - [Ohta et al, '73],
  3.5PN EoM - [Iyer & Will, '93]
  4PN EoM - [Damour et al, '14; Bernard et al '15]
  1.5PN SO - [Barker et al '75]
  2PN SS - [Kidder et al, '93]
  4PN NNLO SS - [Hartung et al '11, Levi et al '11]
  3.5PN NNLO SO - [Hartung et al '11, Marsat et al '13]



1. LIGO Sources

- LIGO is sensitive to 10-10^3 Hz
- Binary mergers of stellar-collapse black holes and / or neutron stars
- More massive sources visible further away (up to cosmological distances)





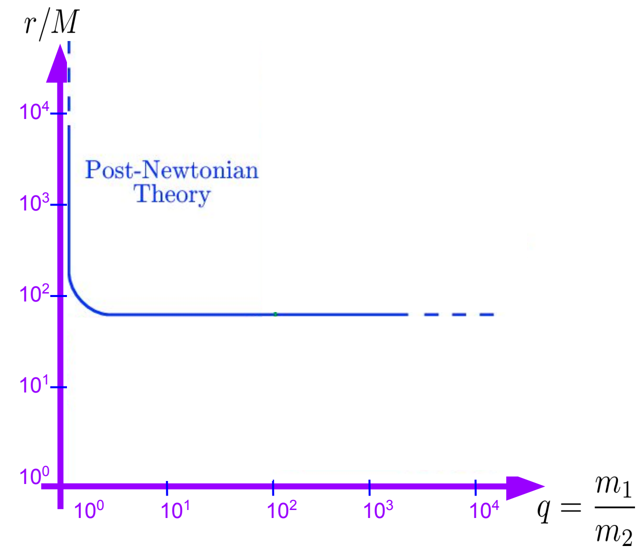
$$= M \times \eta^{3/5}$$

Figure: Colpi et al, 1610.05309

1a. Source modeling: Post Newtonian theory

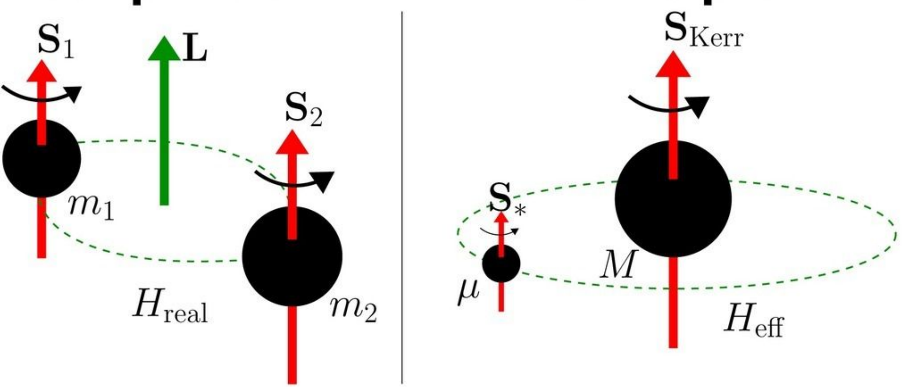
- Slow-motion weak-field approximation
- Perturbative expansions in orbital velocity (v/c)
- State of the art (circa '14):

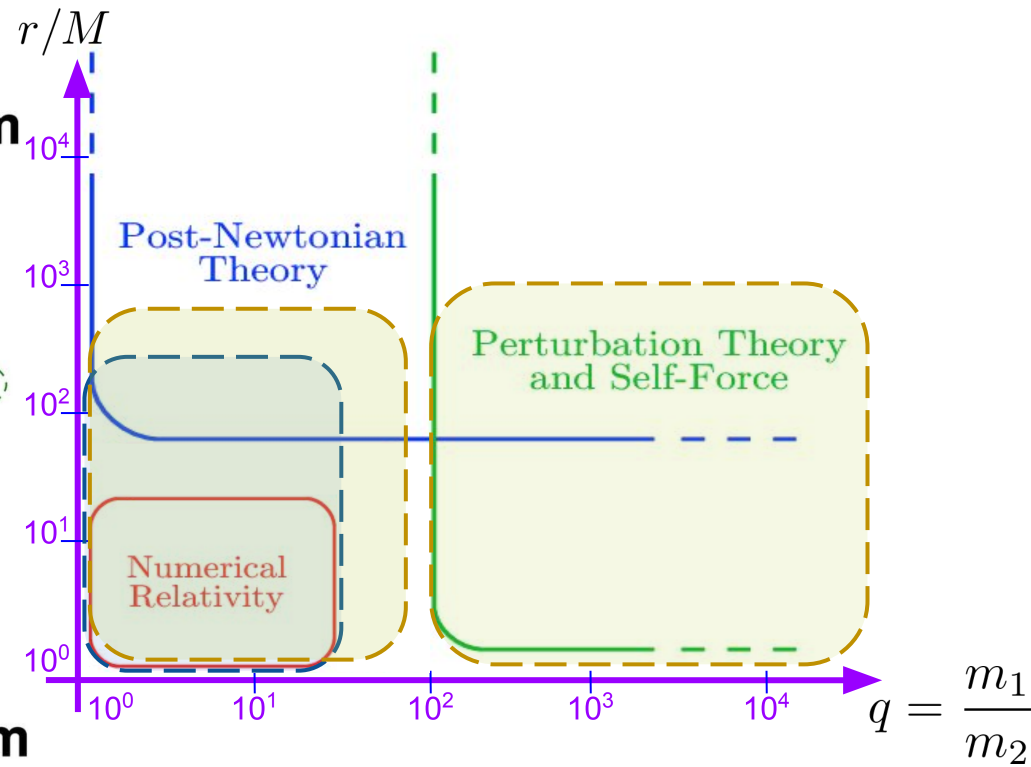
	No Spin	Spin-Linear	Spin-Squared
Conservative Dynamics	4PN[a] [121, 122, 133] [126, 158–164]	3.5PN [52, 54, 141] [140, 165–169]	3PN [52, 54, 138] [137, 170–172]
Energy Flux at Infinity	3.5PN [95, 173, 174]	4PN [175–178]	2PN [53, 54, 179–181]
RR Force	4.5PN [37, 93, 183–185]	4PN [186–188]	4.5PN [189]
Waveform Phase[c]	3.5PN [190]	4PN [175, 177, 178]	2PN [54, 179–181, 191]
Waveform Amplitude[e]	3PN[d] [194–197]	2PN [191, 198]	2PN [53, 54, 191, 198]



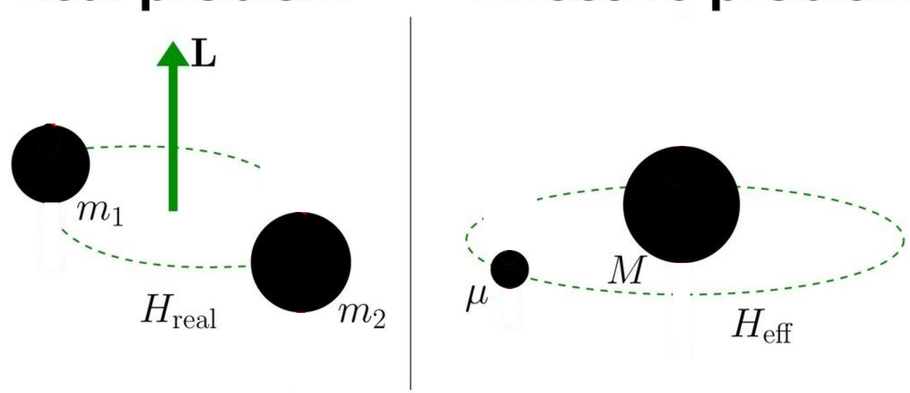
Buonanno & Sathyaprakash (2014), 1410.7832;

Real problem ↔ Effective problem



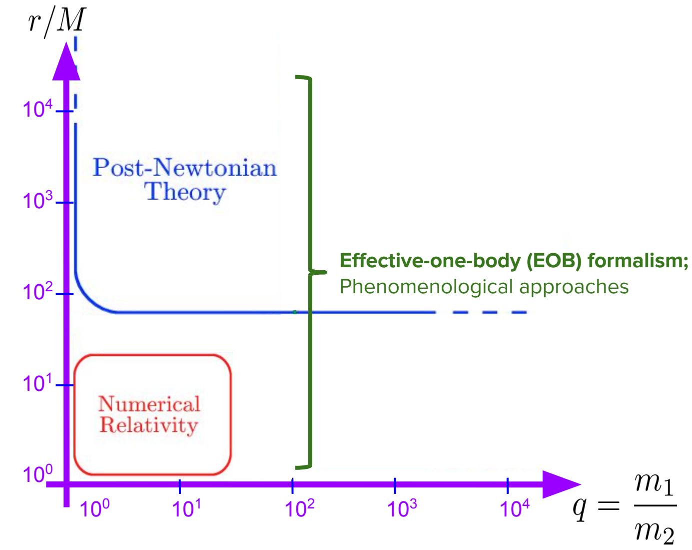


Real problem ↔ Effective problem



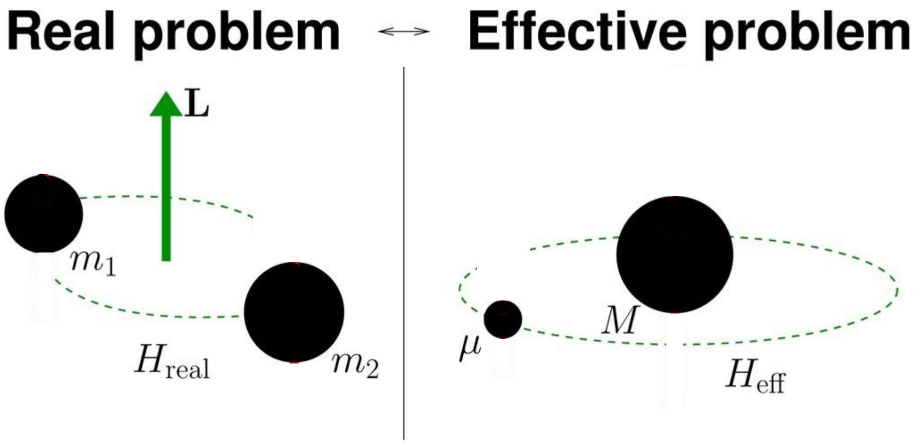
1c. Source modeling: Effective one body (EOB)

- General relativistic extension of 2-body to 1-body mapping of Newtonian problem



Buonanno et al (1998), gr-qc/9811091;

2c. Source modeling: Effective one body (EOB)



- General relativistic extension of 2-body to 1-body mapping of Newtonian problem

- Conservative dynamics:

$$ds^2_{\text{eff}} = -A(R_{\text{eff}})dt^2_{\text{eff}} + \frac{D(R_{\text{eff}})}{A(R_{\text{eff}})}dR^2_{\text{eff}} + C(R_{\text{eff}})R^2_{\text{eff}}\, d\Omega^2_{\text{eff}}$$

- Identify:
  - $m_1 + m_2 \rightarrow M$;
  - $m_1 m_2 / M \rightarrow \mu$;

- Require the effective spacetime reduce to Schwarzschild at first order leads to (2PN)

- Mapping energy levels between 2-body and EOB description gives

$$H^{2\,\text{body}}_{\text{PN}} \rightarrow H^{EOB}$$

$$A(R) = 1 - \frac{2M}{R} + 2\eta\left(\frac{M}{R}\right)^3; \;\; D(R) = 1 - 6\eta\left(\frac{M}{R}\right)^2$$

↓

Pade re-summed

Buonanno et al (1998), gr-qc/9811091; Image credit: A. Taracchini/AEI

2c. Source modeling: Effective one body (EOB)

- Radiative dynamics:

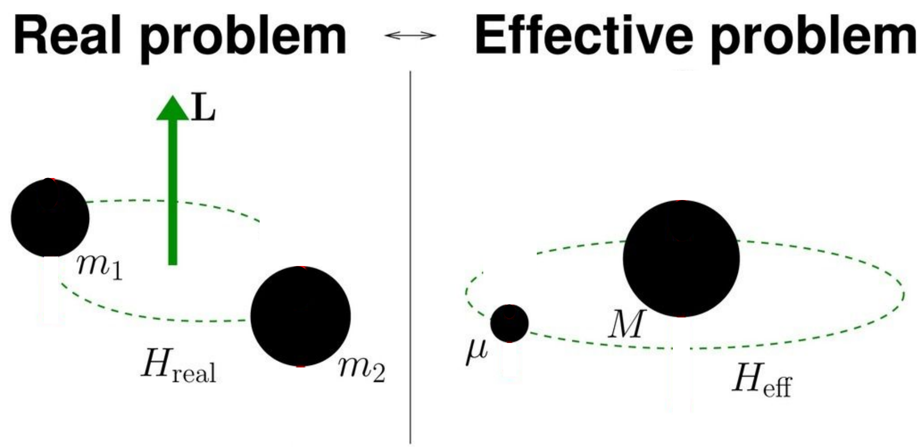
$$\frac{dE}{dt} \propto -\frac{\omega^2}{8\pi} \sum_{l,m} m^2 \left| \frac{\mathcal{R}}{M} h_{lm}(t) \right|^2$$

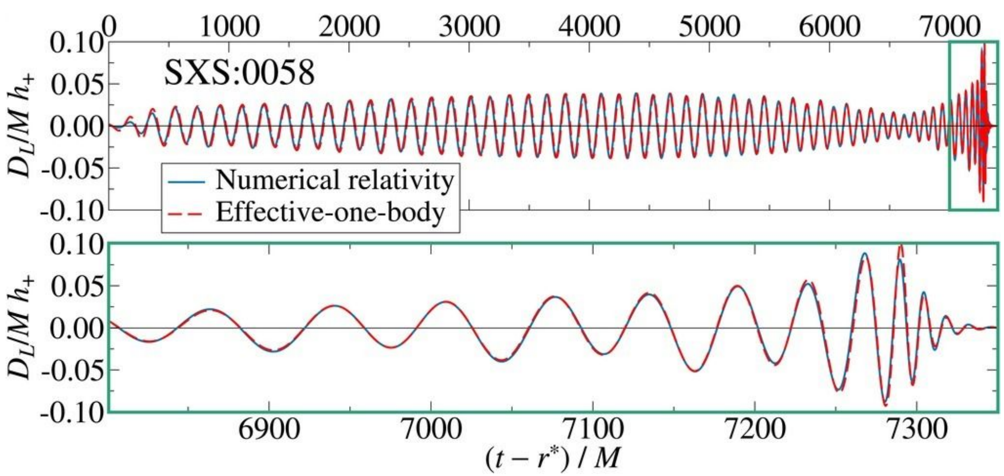
- Waveform multipoles are factorized:

$$h_{lm} = h^A_{lm}\, h^B_{lm}\, T_{lm} \mathrm{e}^{i\delta_{lm}}\, c_{lm}\, N_{lm}$$

where, all but the last factor are **re-summed**

- Several free parameters that are calibrated to NR

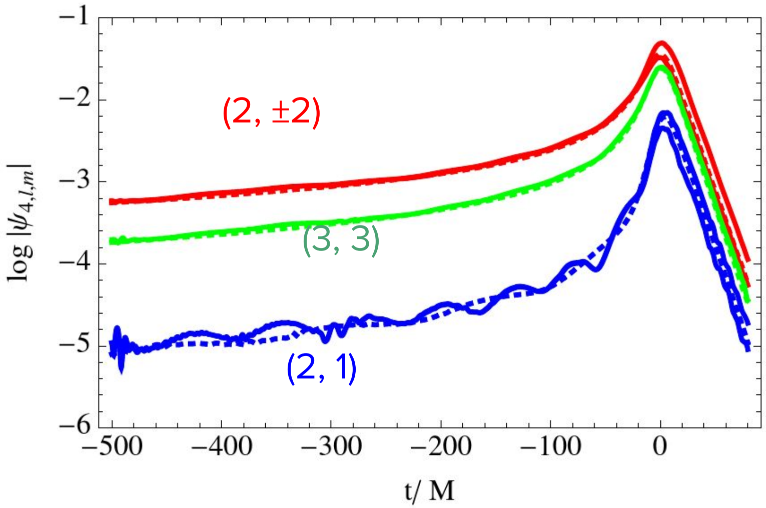


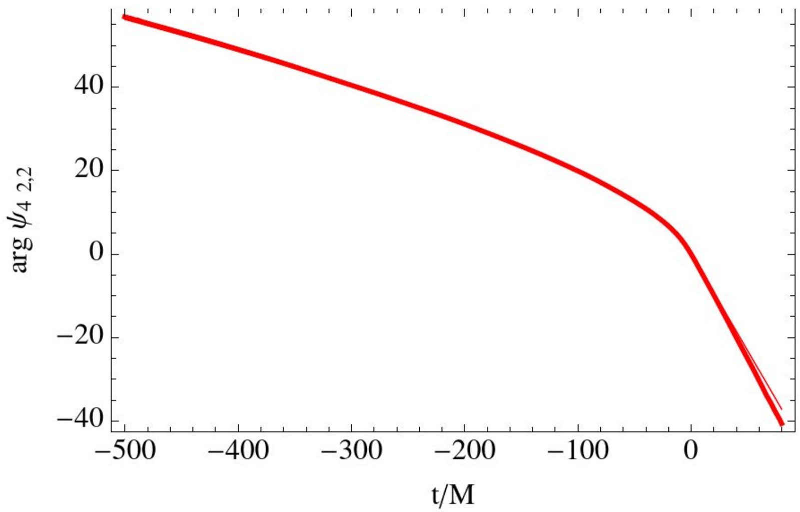


Buonanno et al (1998), gr-qc/9811091; Pan et al (2011), 1106.1021; Image credit: A. Taracchini/AEI

2e. Source modeling: Including spin-induced orbital precession

- So far source models included restricted spin description, with both BH spins (anti)parallel to orbital ang. momentum
- Dominant GW emission directions are ⊥ to the plane of the binary. In a coordinate system aligned with that direction (**QA**), most of the signal power resides in the (l = 2, |m| = 2) spin-weighted spherical harmonics
- ***Schmidt et al <u>identified</u> both GW mode amplitude (in 2010) and phasing (in 2012) of QA-frame waveforms for precessing binaries <u>with</u> equivalent non-precessing-binary waveforms!***
- Application - generic Phenom / EOB models developed by applying a time-dependent rotation to non-precessing binary waveforms.

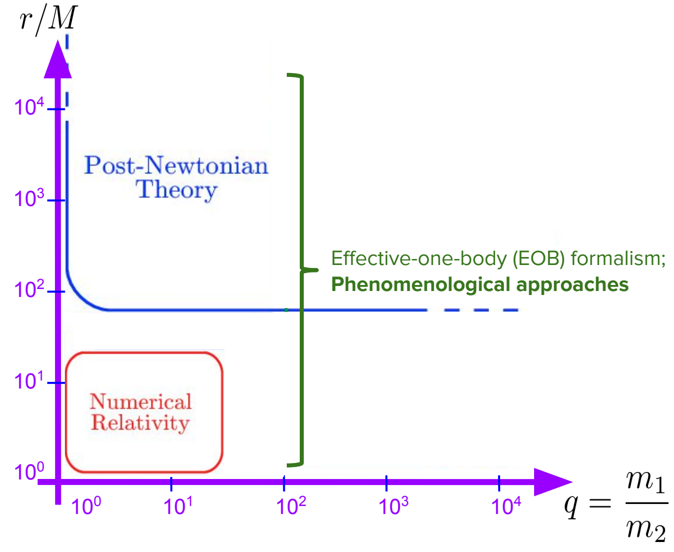




Schmidt et al (2010), 1012.2879; Boyle et al (2011), 1110.2965; Schmidt et al (2012), 1207.3088; Pekowsky et al (2013), 1304.3176;

1d. Source modeling: Phenom

- Guided by need to reduce computational cost of waveform generation, closed-form GW strain models in frequency-domain were developed
- PN-inspired ansatz is taken for amplitude/phase, and PN/EOB+NR hybrid waveforms are used to calibrate the ansatz

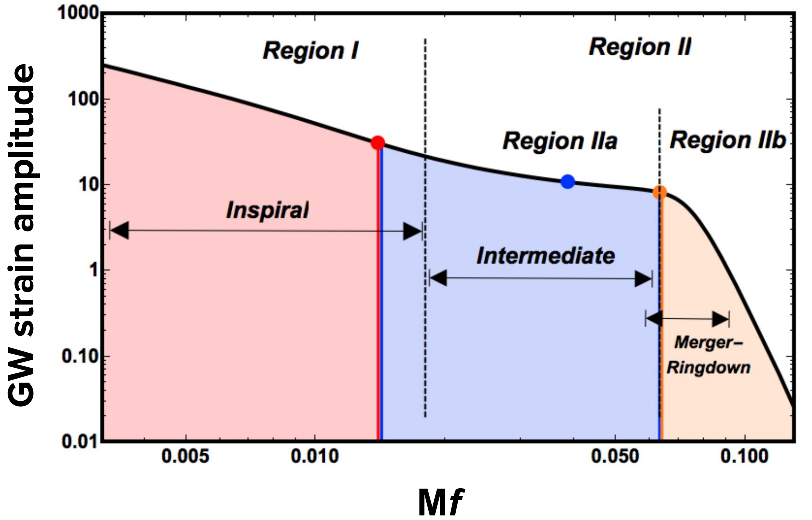


$$A_{\rm Int} = A_0 \left(\delta_0 + \delta_1 f + \delta_2 f^2 + \delta_3 f^3 + \delta_4 f^4\right)$$

$$\phi_{\rm Int} = \frac{1}{\eta}\left(\beta_0 + \beta_1 f + \beta_2 \,{\rm Log}(f) - \frac{\beta_3}{3} f^{-3}\right)$$

$$\frac{A_{\rm MR}}{A_0} = \gamma_1 \frac{\gamma_3 f_{\rm damp}}{(f - f_{\rm RD})^2 + (\gamma_3 f_{\rm damp})^2} e^{-\frac{\gamma_2 (f - f_{\rm RD})}{\gamma_3 f_{\rm damp}}}$$

$$\phi_{\rm MR} = \frac{1}{\eta}\left\{\alpha_0 + \alpha_1 f - \alpha_2 f^{-1} + \frac{4}{3}\alpha_3 f^{3/4} + \alpha_4 \tan^{-1}\left(\frac{f - \alpha_5 f_{\rm RD}}{f_{\rm damp}}\right)\right\}.$$

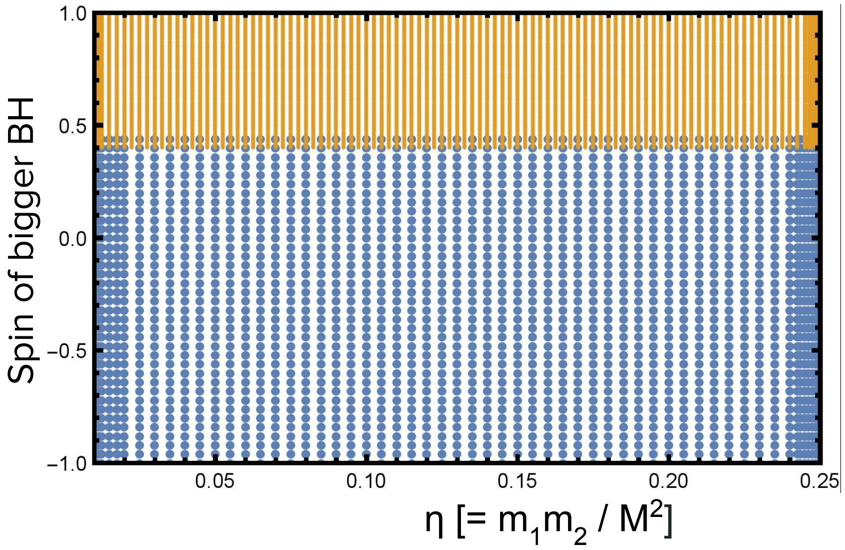


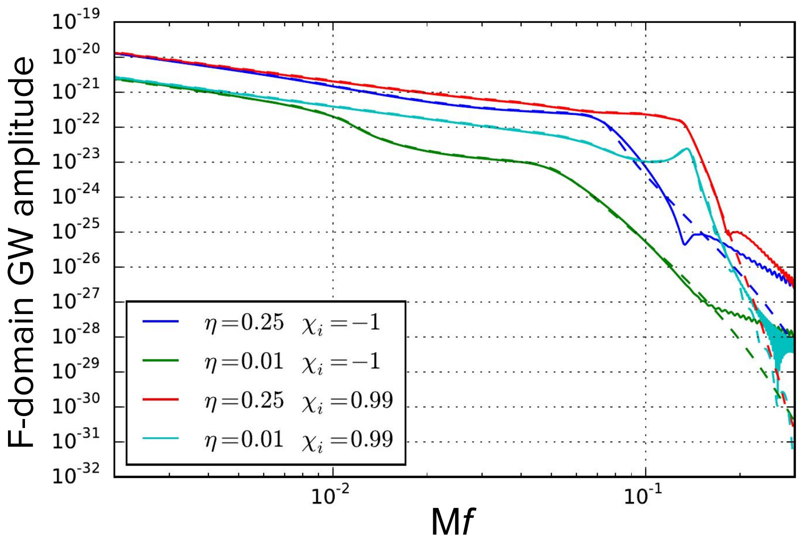
Khan et al (2015), 1508.07253; Santamaria et al (2010), 1005.3306; Ajith et al (2007), 0704.3764/0710.2335; Ajith et al (2009), 0909.2867;

3a. GW PE: Reduced-order modeling

- ROM is a technique to create surrogate models for computationally expensive waveform models:
  - Define a region of parameter space
  - Compute a basis for GW amplitude & phase
  - Compute projection coefficients for a dense set of training waveforms & interpolate them
  - Store spline-interpolation coefficients on disk

- Evaluation of ROM is straightforward:
  - Read in spline coefficients
  - Evaluate splines at required parameter values
  - Combine with basis vectors to generate GW templates





Purrer et al (2014), 1402.4146; Purrer et al (2015), 1512.02248;

3a. LIGO PE: Reduced-order modeling

- ROM is a technique to create surrogate models for computationally expensive waveform models:
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  - Store spline-interpolation coefficients on disk

- Evaluation of ROM is straightforward:
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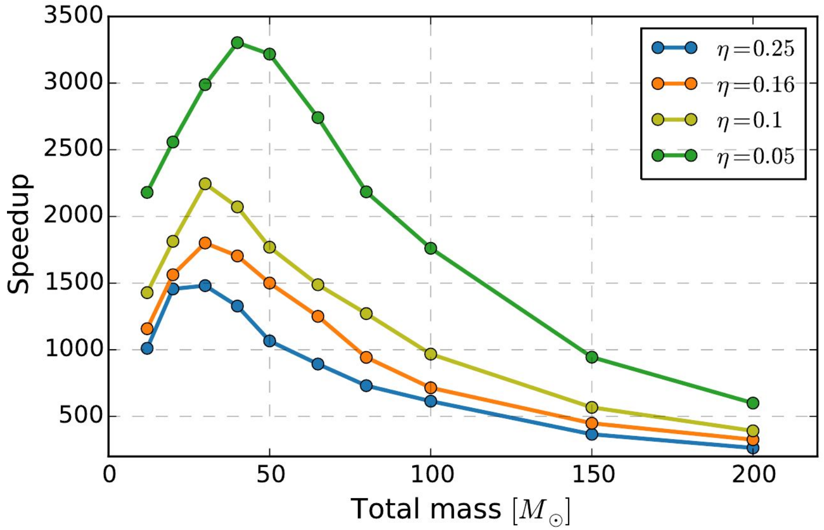
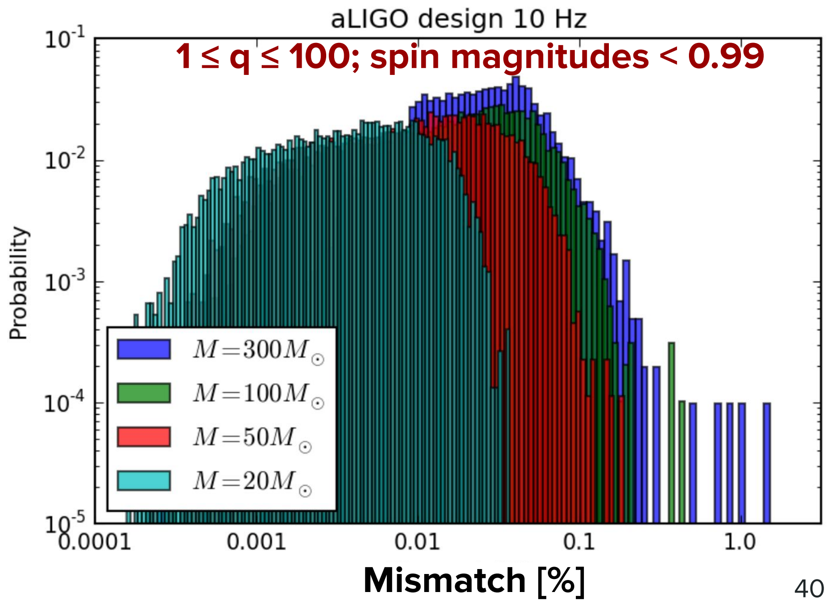
  - Evaluate splines at required parameter values
  - Combine with basis vectors to generate GW templates

- **Evaluation in polynomial time (esp. EOB)**
- **Marginal loss in accuracy**



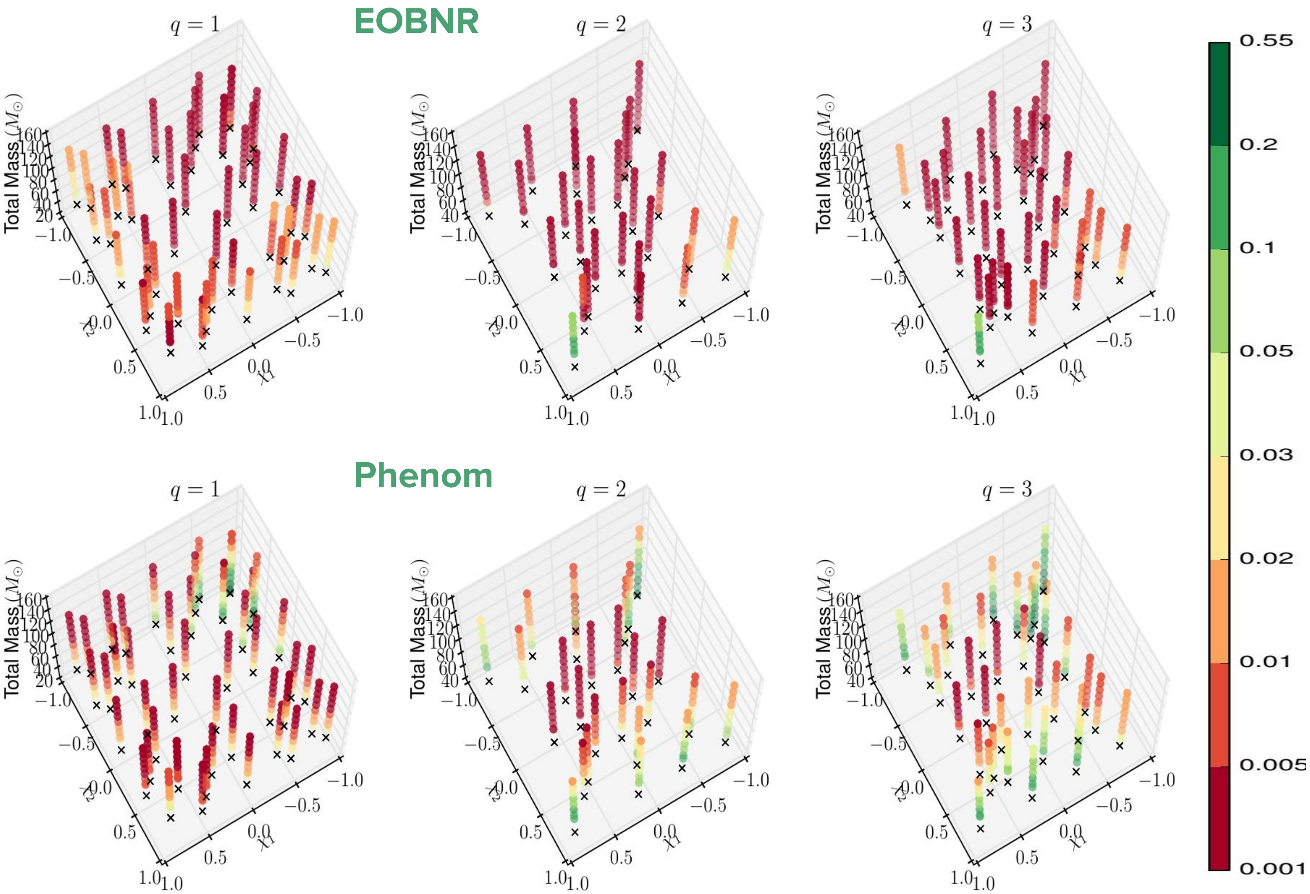


Purrer et al (2014), 1402.4146; Purrer et al (2015), 1512.02248;

3. GW PE: Model requirements

Model accuracy up to reqd. SNR:

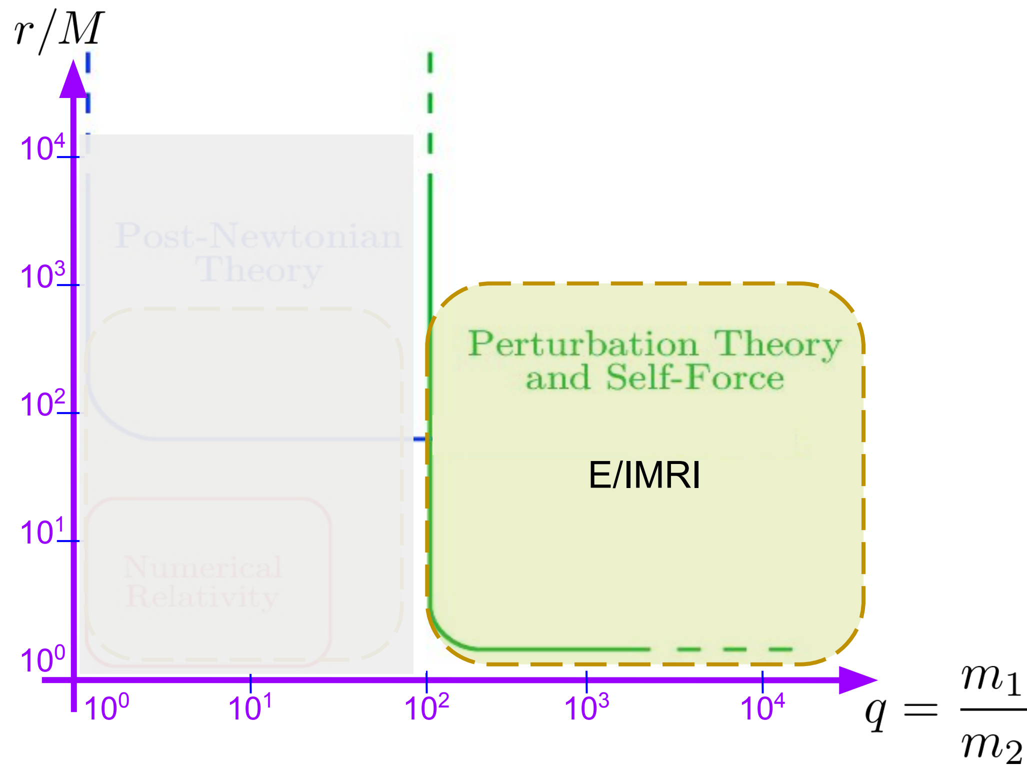
SNR	10	10^2	10^3	10^4
Mismatch (vs NR)	< 2 %	< 0.02%	< 2×10^{-4} %	< 2×10^{-6} %



*Lindblom et al (2008), 0809.3844; PK et al (2016), 1601.05396;

4. LISA Sources: EMRI, IMRI

- EMRIs will be seen by LISA at SNRs 20 - 100s, few every year!



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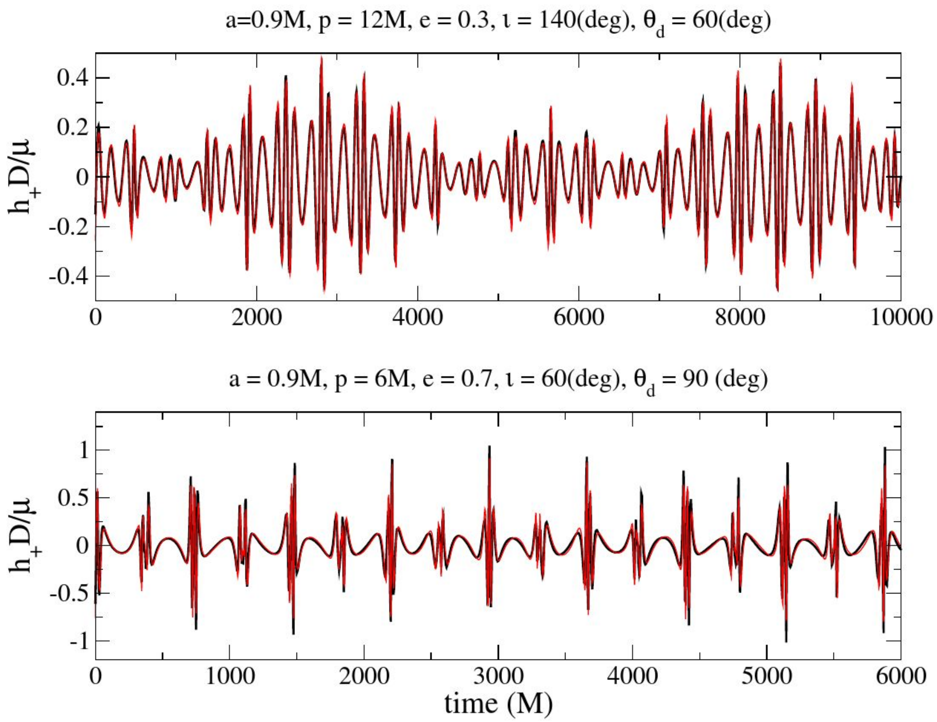
- Kludge waveforms are available for EMRIs:
    - Inexpensive and tested for PE (under idealized conditions)

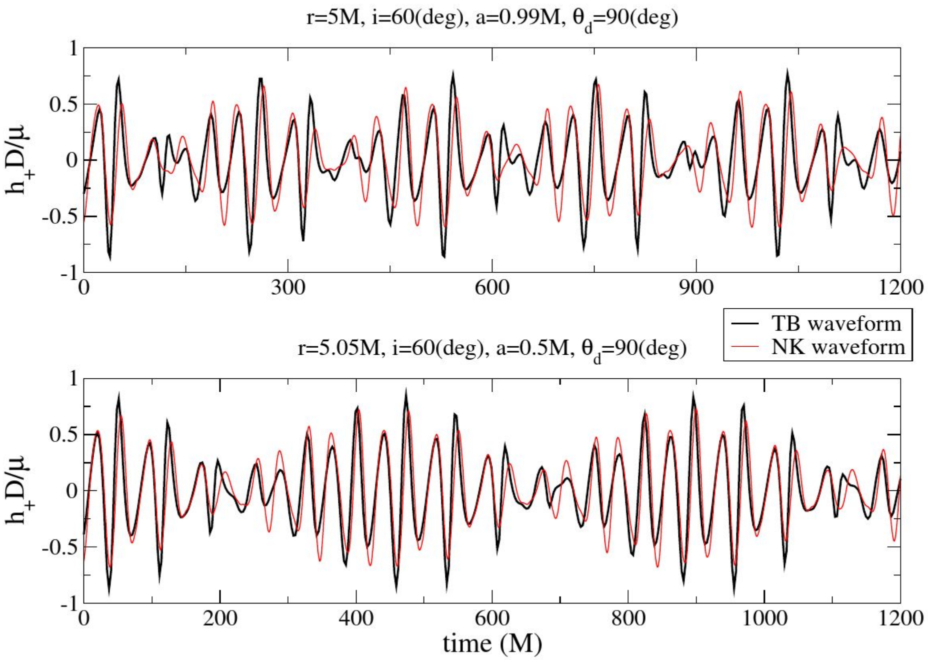
    - Agree with numerical Teukolsky codes



Babak et al (2006), gr-qc/0607007; Gair et al (2005), gr-qc/0510129; Barack et al (2004), gr-qc/0310125;

4. LISA Sources: EMRI, IMRI

- EMRIs will be seen by LISA at SNRs 20 - 100s, few every year!
- Kludge waveforms are available for EMRIs:
  - Inexpensive and tested for PE (under idealized conditions)
  - Agree with numerical Teukolsky codes ... up until r ~ 5M
  - Sufficient for detection and maybe even PE - e.g. $O(10^{-2})$ accuracy if not $O(10^{-4})$



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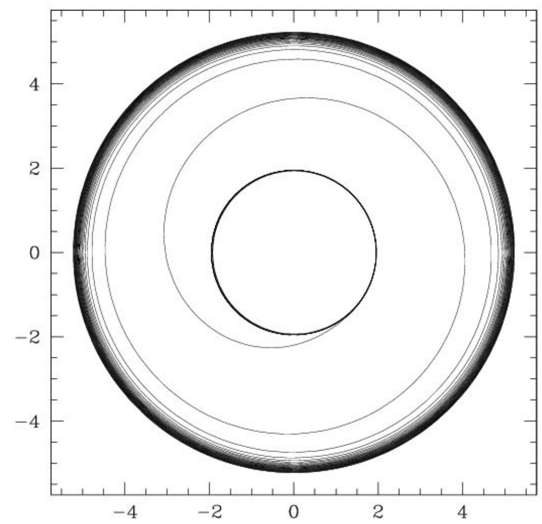
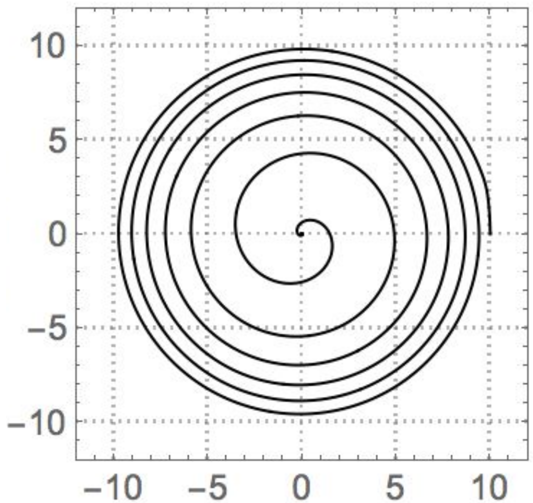
4. LISA Sources: EMRI, IMRI

- EMRIs will be seen by LISA at SNRs 20 - 100s, few every year!
- Kludge waveforms are available for EMRIs
- Need waveforms from self-force program to:
    - Calibrate *kludge* models, extend to IMRIs
    - Validate models and compare to observed signals
    - Test GR / no-hair theorem: for precision tests, models need to track GW emission to better than O(1) cycle over thousands





Babak et al (2006), gr-qc/0607007; Gair et al (2005), gr-qc/0510129; Barack et al (2004), gr-qc/0310125; Image credit: arXiv:1003.0485 & einsteintoolkit.org;