Gravitational-wave source modeling using analytical methods

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Gravitational waveforms

first term of the multipole expansion of the form

$$h_{ij}^{\mathrm{rad}}(\boldsymbol{X},T) = \frac{4G}{c^4 R}\sum_{\ell=2}^{+\infty}\frac{1}{c^{\ell-2}\ell!}\Bigg\{ \underbrace{N_{L-2}}_{N_{i_1}\dots N_{i_{\ell-2}}} \underbrace{U_{ijL-2}}_{i_1\dots i_\ell} - \frac{2\ell}{c(2\ell+1)} N_{aL-2}\epsilon_{ab(i}V_{j)bL-2}\Bigg\}^{\mathrm{TT}}(T-R/c)$$

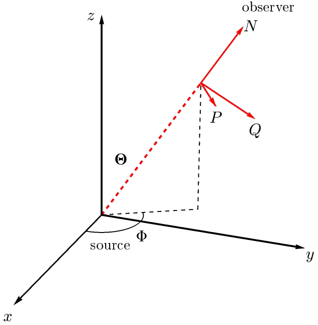
Generalizes the quadrupole formula

[Einstein (1918)]

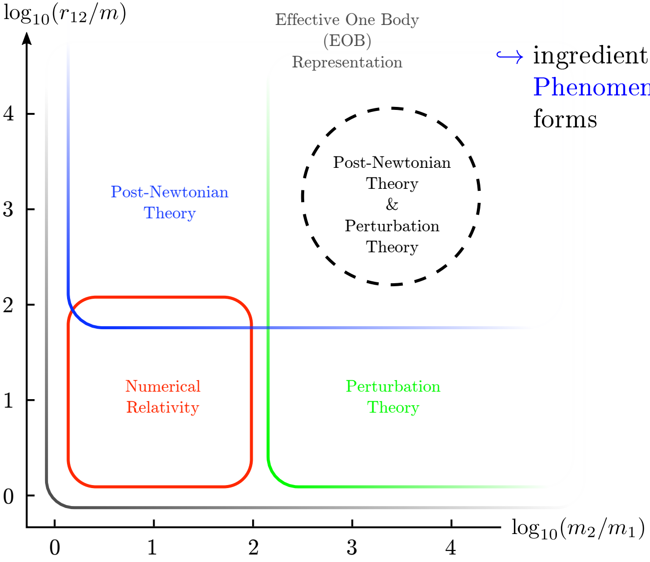
$$h_{ij}^{\mathrm{rad}}(\boldsymbol{X},T) = \frac{2G}{c^4 R}U_{ij}^{\mathrm{TT}}(T-R/c)$$

with $U_{ij} = I_{ij}^{(2)} + \mathcal{O}(G) = Q_{ij}^{(2)} + \mathcal{O}\left(\frac{1}{c^2}\right)$

$$h_+ = h_{PP} = -h_{QQ} \qquad h_\times = h_{PQ}$$



Approximation methods for GW generation



$\hookrightarrow$ ingredients for EOB and Phenomenological waveforms

1. Post-Newtonian radiation of isolated systems

2. Post-Newtonian evolution of compact binaries

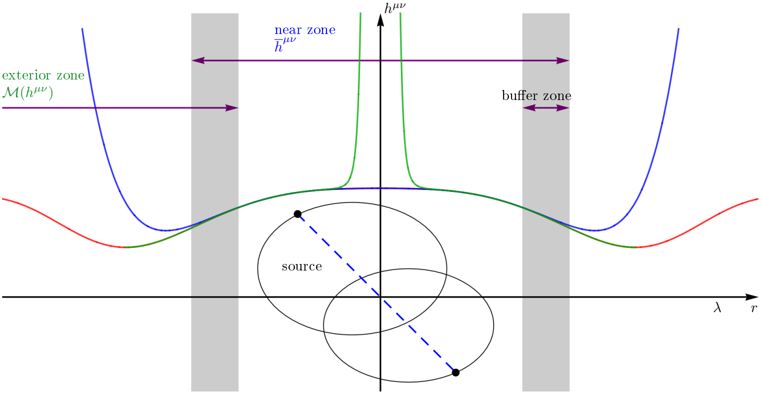
3. Lagrangian approach

4. Hamiltonian approach

5. Gravitational self-force calculations

Validity of the post-Newtonian regime

- Small velocities: $\max v \ll c$
- Size of matter source $D \ll \lambda$



PN iteration in harmonic coordinates

Harmonic gauge equations

$$\partial_\nu h^{\mu\nu} = 0 \qquad \text{(Gauge Cond on shell)}$$

$$\Box h^{\mu\nu} = \frac{16\pi G}{c^4}\tau^{\mu\nu} \equiv \frac{16\pi G}{c^4}|g|T^{\mu\nu} + \Lambda^{\mu\nu}(\partial h, \partial h) \qquad \text{(Relaxed EE)}$$

$h^{\mu\nu}$ searched in the form $\displaystyle\sum_{m\geq m_0(\mu,\nu)} c^{-m} h^{\mu\nu}_{[m]}$

- Assume that previous orders $h^{\mu\nu}_{[m']}$ are known
- Solution for $h^{\mu\nu}_{[m]}$

$$h^{\mu\nu}_{[m]} = 16\pi G\left\{\Box_{\mathrm{R}}^{-1}\left[\tau^{\mu\nu}(\overline{h}^{\alpha\beta})\right] + \sum_{\ell\geq 0}\overline{\partial_L\left(\frac{R_L^{\mu\nu}(t-r/c) - R_L^{\mu\nu}(t+r/c)}{r}\right)}\right\}_{[m-4]}$$

finite part of $\Box_{\mathrm{R}}^{-1}(r/r_0)^B[\;]$ $\qquad R_L^{\mu\nu} = R^{\mu\nu}_{i_1\ldots i_\ell}[\mathcal{M}(h^{\alpha\beta}), r_0]$

- Go to the next order

Blanchet-Damour-Iyer formalism

Outside the source:

$$\Box h^{\mu\nu} = \Lambda^{\mu\nu} \quad \text{(REE)} \qquad \partial_\nu h^{\mu\nu} = 0 \quad \text{(Gauge)} \qquad \text{with } h^{\mu\nu} = \sum_{n=1}^{+\infty} G^n h^{\mu\nu}_{(n)}$$

- Find the most general solution at linear order, with $\Lambda^{\mu\nu} \longrightarrow 0$
  - No-incoming wave solution of (REE): $h^{\mu\nu}_{(1)} = \sum_{\ell\geq 0} \overbrace{\partial_{i_1\ldots i_\ell}}^{\frac{\partial}{\partial x^{i_1}}\cdots\frac{\partial}{\partial x^{i_\ell}}} \left(\frac{\mathcal{H}^{\mu\nu}_{i_1\ldots i_\ell}(t-\frac{r}{c})}{r}\right)$
  - retarded solution of (REE) + (Gauge Cond):

$$h^{\mu\nu}_{(1)} = h^{\mu\nu}_{(1)}[\underbrace{I_L, J_L,}_{\text{source moments}} \quad \underbrace{W_L,\ X_L, Y_L, Z_L}_{\text{gauge moments}}]$$

- Solution for $h^{\mu\nu}_{(n)}$

$$h^{\mu\nu}_{(n)} = \underbrace{\mathrm{FP}\Box^{-1}_{\mathrm{R}}}_{\text{finite-part regularized retarded integral}} \Lambda^{\mu\nu}_{(n)} + (\text{homogeneous outgoing sol.})^{\mu\nu}$$

- Go to next order

Radiative moments

Link between the various multipole moment sets

$$I_L, J_L, \ldots \xrightarrow{\text{link S} \to \text{Can}} M_L, S_L \xrightarrow{\text{link Can} \to \text{R}} U_L, V_L$$

source $\to$ $I_L, J_L, \ldots$: integral over $\overline{\tau}^{\mu\nu}$ obtained by asymptotic matching

link S $\to$ Can

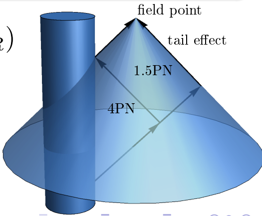
link Can $\to$ R: comparison between $h^{\mu\nu}_{\text{ext}}$ and h^{TT}_{ij} + coord. transf.

$U_L, V_L \to$ waveform

Link Can $\longrightarrow$ R, e.g., for U_L

$$U_L[I, J, \ldots] = U_L^{\text{inst}}[M, S] + U_L^{\text{tail}}[M, S] + U_L^{\text{tail-tail}}[M, S] + U_L^{\text{mem}}[M, S] + \ldots$$

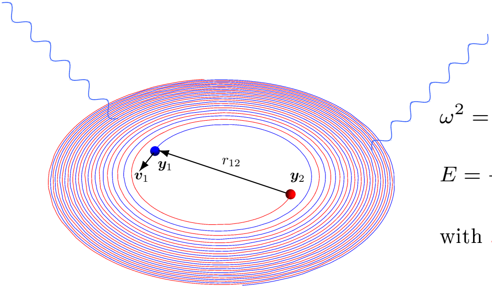
- instantaneous terms: function of $\partial_t^k M_L(T_R)$, $\partial_t^k S_L(T_R)$
- tail terms: depend weakly on the source past history
- memory terms: depend strongly on the source past history



Case of compact binaries

- Leading order flux $= \dfrac{G}{5c^5}\left[\dddot{Q}_{ij}(T_{\text{ret}})\dddot{Q}_{ij}(T_{\text{ret}}) + \mathcal{O}\left(\dfrac{1}{c^2}\right)\right]$ → Newtonian order

balance equations for E, $\boldsymbol{J}$ $\Rightarrow$
- $e \searrow 0$ for isolated binaries
- E and $r_{12} \searrow$ at a rate $\sim \epsilon^{5/2}$ $=$ 2.5PN order



$$\omega^2 = \frac{Gm}{r_{12}^3}\left[1 + \left(\frac{Gm}{r_{12}c^2}\right)(...) + ... + \left(\frac{Gm}{r_{12}c^2}\right)^4(...)\right]$$

$$E = -\frac{\mu c^2 x}{2}\left[1 + x(...) + ... + x^4(...)\right]$$

with $x = \left(\dfrac{Gm\omega}{c^3}\right)^{2/3}$

state of the art

- For circular orbits: $E = E(x)$, $\mathcal{F} = \mathcal{F}(x)$, $h_{+,\times} = h_{+,\times}(x)$ gauge invariant

Possible approaches

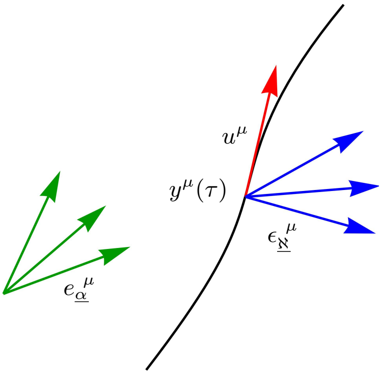
Equations of motion of the sources:

- PN Iteration Scheme in Harmonic coordinates:
  - French flavored: effective $T^{\mu\nu}_{\rm eff}$ + dim reg + asymptotic matching
    $\hookrightarrow$ Blanchet, Damour, Iyer, Le Tiec, Marsat, Bohé, Bernard, Marchand...
  - QFT flavored: effective action + dim reg + zero-bin subtraction
    $\hookrightarrow$ Goldberger, Rothstein, Porto, Foffa, Sturani, Kol, Levi, Galley...
  - American flavored: perfect fluid + splitting of volume integrals
    $\hookrightarrow$ Will, Wiseman, Kidder, Pati...
- Method à la Einstein-Infeld-Hoffmann (strong-field region avoidance)
  $\hookrightarrow$ Futamase, Itho, Asada
- Reduced ADM Hamiltonian approach: ADM
  $\hookrightarrow$ Schäfer, Jaranowski, Damour, Steinhoff, Hergt, Hartung, ...

Multipolar post-Minkowskian formalisms in harmonic coordinates

- Regularization based approach: finite part reg + asymptotic matching
  $\hookrightarrow$ Blanchet, Damour, Iyer (see also early works by Thorne)
- Integral splitting: perfect fluid + splitting of volume integrals
  $\hookrightarrow$ Will, Wiseman (in the line of Epstein, Wagoner)
- QFT flavored: effective action + dim reg + zero-bin subtraction
  $\hookrightarrow$ Goldberger, Ross

Effective action for extended bodies



$\epsilon_{\underline{\aleph}} = \Lambda_{\underline{\aleph}}{}^{\underline{\alpha}} e_{\underline{\alpha}}$

- $e_{\underline{\alpha}}{}^\mu$: field tetrad
- $e_{\underline{\aleph}}{}^\mu$: tetrad attached to the body
- $\Lambda_{\underline{\aleph}}{}^{\underline{\alpha}}$: Lorentz matrices (6 d.o.f.)

$$S = \int \mathrm{d}\tau\, L[u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}, R_{\mu\nu\rho\sigma}, \nabla_\lambda R_{\mu\nu\rho\sigma}, \ldots]$$
$$= S_{\text{grav}} + S_{\text{point part.}} + S_{\text{non-minimal}}$$

Skeleton stress-energy tensor

Description of the dynamics in terms of:

- Worldline density: $n(x) = \int \mathrm{d}\lambda \frac{\delta^4(x^\mu - y^\mu)}{\sqrt{-g}}$
- Effective momenta: $p_\mu = \frac{\partial L}{c\partial u^\mu} \qquad S_{\mu\nu} = 2\frac{\partial L}{\partial \Omega^{\mu\nu}}$
- Effective quadrupole moment: $J^{\mu\nu\rho\sigma} = -\frac{6}{c^2}\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}}$
- Effective octupole moment: $J^{\lambda\mu\nu\rho\sigma} = -\frac{12}{c^2}\frac{\partial L}{\partial \nabla_\lambda R_{\mu\nu\rho\sigma}}$

[Bailey, Israel (1975); Dixon (70's); Steinhoff, Pueztfeld (2009); Marsat (2015)]

$$
\begin{aligned}
T^{\mu\nu} = & \, n\left[p^{(\mu}u^{\nu)}c + \frac{c^2}{3}R^{(\mu}{}_{\lambda\rho\sigma}J^{\nu)\lambda\rho\sigma} + \frac{c^2}{6}\nabla^\lambda R^{(\mu}{}_{\tau\rho\sigma}J_\lambda{}^{\nu)\tau\rho\sigma} + \frac{c^2}{12}\nabla^{(\mu}R_{\lambda\tau\rho\sigma}J^{\nu)\lambda\tau\rho\sigma}\right] \\
& + \nabla_\rho\left\{n\left[u^{(\mu}c\,S^{\nu)\rho} - \frac{c^2}{6}R^{(\mu}{}_{\tau\lambda\sigma}J^{|\rho|\nu)\tau\lambda\sigma} - \frac{c^2}{3}R^{(\mu}{}_{\tau\lambda\sigma}J^{\nu)\rho\tau\lambda\sigma} + \frac{c^2}{3}R^{\rho}{}_{\tau\lambda\sigma}J^{(\mu\nu)\tau\lambda\sigma}\right]\right\} \\
& - \frac{2c^2}{3}\nabla_\rho\nabla_\sigma\left\{nJ^{\rho(\mu\nu)\sigma}\right\} + \frac{c^2}{3}\nabla_\lambda\nabla_\rho\nabla_\sigma\left\{nJ^{\sigma\rho(\mu\nu)\lambda}\right\} + \ldots
\end{aligned}
$$

Description of the system dynamics

GR action for point particles

$$S_{\rm grav} + S_{\rm point\ part.} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\Big[g^{\mu\nu}(\Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\rho} - \Gamma^\rho_{\mu\nu}\Gamma^\lambda_{\rho\lambda}) - \underbrace{\frac{1}{2}g_{\mu\nu}\Gamma^\mu\Gamma^\nu}_{\text{gauge fixing term}}\Big]$$

$$-\sum_A c^2\int \mathrm{d}t\Big[m_A\sqrt{-(g_{\mu\nu})_A v_A^\mu v_A^\nu/c^2} + \frac{1}{2}S^A_{\mu\nu}\Omega^{\mu\nu}_A\Big]$$

Two non-minimal pieces

$$L_{\rm tides} = \sum_{\ell\geq 2}\frac{1}{\ell!}\Big\{\frac{1}{2}\mu^{(\ell)}(G_L(\tau))^2 + \frac{1}{2c^2}\frac{\ell}{\ell+1}\sigma^{(\ell)}(H_L(\tau))^2 + \frac{1}{2c^2}\mu'^{(\ell)}(\dot{G}_L(\tau))^2 + \ldots\Big\}$$

[for a relativistic def. of Love numbers, see Damour, Nagar (2009); Binnington, Poisson (2009)]

$$L_{\rm spin\ int} = \sum_{n=1}^{+\infty}\frac{\kappa_{GS^{2n}}}{(2n)!m^{2n-1}}D_{\mu_{2n}}\ldots D_{\mu_3}\left(\frac{G_{\mu_1\mu_2}}{\sqrt{-u^2}}\right)S^{\mu_1}\ldots S^{\mu_{2n-1}}S^{\mu_{2n}}$$

mass-like tidal moments

current-like tidal moments

$$+\sum_{n=1}^{+\infty}\frac{\kappa_{HS^{2n+1}}}{(2n+1)!m^{2n}}D_{\mu_{2n+1}}\ldots D_{\mu_3}\left(\frac{H_{\mu_1\mu_2}}{\sqrt{-u^2}}\right)S^{\mu_1}\ldots S^{\mu_{2n}}S^{\mu_{2n+1}}$$

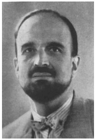
[Levi, Steinhoff (2015)]

Dixon moments J obtained through $T^{\mu\nu}_{\text{pole-dipole}} + T^{\mu\nu}_{\text{spin int}} = T^{\mu\nu}_{\text{skeleton}}$

Fokker action

metric perturbation

- Start from the action $S[h^{\alpha\beta}, \boldsymbol{y_A}]$
- Compute $h^{\alpha\beta}_{\text{(FE)}}$ given by $\left(\dfrac{\delta S}{\delta h^{\alpha\beta}}\right) = 0$ (REE)
- Build a Fokker action: $S_{\text{Fokker}} = S[h^{\alpha\beta}_{\text{(FE)}}[\boldsymbol{y_A}], \boldsymbol{y_B}]$



Credit: Huygens ING

Adriaan Fokker

$$\left(\frac{\delta S}{\delta y^i_A}\right)_{h=h_{\text{(FE)}}[\boldsymbol{y}_A]} = 0 \quad \text{and} \quad h_{\text{(FE)}}[\boldsymbol{y}_A] \text{ built with } \Box^{-1}_{\text{sym.}}$$

$$\Leftrightarrow \quad \frac{\delta S_{\text{Fokker}}}{\delta y^i_A} = 0$$

Computation by means of Feynman diagrams possible:

$$Z[\rho, j^i, s^{ij}] = \mathrm{e}^{\frac{1}{\epsilon}S_{\text{eff}}} = \int \mathcal{D}\phi \mathcal{D}A_i \mathcal{D}\sigma_{ij} \mathrm{e}^{\frac{1}{\epsilon}\int \mathrm{d}^4x \frac{c^3}{4\pi G}\left[\frac{1}{2}\phi\Box\phi + \rho\phi + \mathcal{O}\left(\frac{8\pi G}{c^2}\phi\right) + (\text{sim. for } A_i, \sigma_{ij})\right]}$$

has a classical limit given by the saddle point method: $\mathrm{e}^{\frac{1}{\epsilon}S_{\text{Fokker}}}$

see e.g., [Damour, Jaranowski (2017)] for discussion

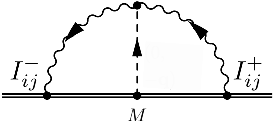
Treatment of the divergences I

Most important recent result: 4PN dynamics recently obtained by 3 groups

ADM: Damour, Jaranowski, Schäfer (2014)
Fokker: Bernard, Blanchet, Bohé, F, Marchand, Marsat (2018a,b)
Diagrammatic: Foffa, Sturani (2019), Foffa, Porto, Rothstein, Sturani

Two types divergences at the 4PN order:

- Due to the effective point-particle modeling: "UV" divergences
  $\hookleftarrow$ unphysical: must cancel each other
- Due to the expansion of the retardation in the near zone: "IR" divergences
  $\hookleftarrow$ must be eliminated by contributions of radiative sector d.o.f.



Credit: Foffa, Porto, Rothstein, Sturani (2019)

back reaction of the exterior-zone tail effect

Common tool: dimensional regularization

Treatment of the divergences II

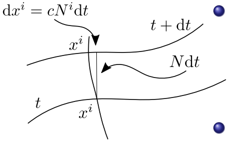
Three ways to proceed with this regularization:

- Fokker Lagrangian in harmonic coordinates:
  - UV div. of S_{local} eliminated by redefining $\boldsymbol{y}_{1,2} \to \boldsymbol{y}_{1,2} + \delta\boldsymbol{y}_{1,2}$
  - finite part regularization kept on the top of dim reg (asymptotic matching)
  - UV div. of the tail radiation reaction cancels the IR pole
- Diagrammatic approach:
  - UV div. of the local part eliminated by adding innocuous counter-terms
  - IR div. transformed into UV div. by:
    (i) tracking the pole nature of vanishing self terms: Zero-Bin subtraction
    (ii) adding UV divergent exterior-zone counter-terms
  - $L_{n\mathrm{PN}}^{\mathrm{UV}\ (\mathrm{IR\ near+self-ZB})} + L_{n\mathrm{PN}}^{\mathrm{UV}\ (\mathrm{exterior})} \to$ finite
- Reduced Hamiltonian in ADM gauge:
  - UV poles of H_{local} eliminated by adding an appropriate total derivative
  - reaction radiation treated in 3 dimensions
    $\to$ finite part produced by the related UV poles not controlled
  - unknown coefficient determined by matching with GSF results

ADM Hamiltonian approach

Assumptions:

- asymptotically flat space-time
- in appropriate asymptotic coordinates:



$$(ds)^2 = -N^2(cdt)^2 + (dx^i - N^i cdt)(dx^j - N^j cdt)\gamma_{ij}$$

- near ι^0: $g_{\mu\nu} - \eta_{\mu\nu} = \mathcal{O}(1/r) \qquad \partial_i g_{\mu\nu} = \mathcal{O}(1/r^2)$

Hamiltonian including surface boundary integrals

$$H[\gamma_{ij}, \pi_{ij}, N, N^i; q^a, \pi_a] = \int d^3x (N\mathcal{H} - cN^i \mathcal{H}_i) + \frac{c^4}{16\pi G} \int_{\iota^0} dS_i \partial_j (\gamma_{ij} - \delta_{ij}\delta^{kl}\gamma_{kl})$$

ADMTT gauge: $\gamma_{ij} = \left(1 + \frac{1}{8}\phi\right)^4 \delta_{ij} + h_{ij}^{\mathrm{TT}}, \quad \pi^{ij}\delta_{ij} = 0$

Constraints:

- $\mathcal{H} = 0 \quad \Leftrightarrow \quad \Delta\phi = \ldots$ elliptic equation
- $\mathcal{H}_i = 0 \quad \Leftrightarrow \quad \Delta \underbrace{\pi^i}_{\text{vector part of } \pi^{ij}} = \ldots$ 3 elliptic equations

Reduced Hamiltonian

Property of ADMTT gauge

Injecting the constraints in H does not affect the dynamics

$$H_{\text{red}}[h_{ij}^{\text{TT}}, \pi^{ij}_{\text{TT}}; q^a, \pi_a] = -\frac{c^4}{16\pi G}\int \mathrm{d}^3x\, \Delta\phi[h_{ij}^{\text{TT}}, \pi^{ij}_{\text{TT}}; q^a, \pi_a]$$

Non canonical EOM for $(h_{ij}^{\text{TT}},\ \pi^{ij}_{\text{TT}})$:

$$\frac{c^3}{16\pi G}\partial_t \pi^{ij}_{\text{TT}} = -\delta^{\text{TT}ij}_{kl}\frac{\delta H_{\text{red}}}{\delta h_{kl}^{\text{TT}}} \qquad \frac{c^3}{16\pi G}\partial_t h_{ij}^{\text{TT}} = \delta^{\text{TT}ij}_{kl}\frac{\delta H_{\text{red}}}{\delta \pi^{kl}_{\text{TT}}}$$

Construction of the N-body Hamiltonian $(q^a \to y_A^i,\ \pi^a \to p_i^A)$:

- Combine evolution equations for h_{ij}^{TT}, π^{ij}_{TT} into $\Box h_{ij}^{\text{TT}} = \mathcal{O}\left(\frac{1}{c^4}\right)$ $\leftarrow h_{ij}^{\text{TT}} \sim$ 2PN
- $H_{\text{red}} \xrightarrow{\text{Legendre transform}} R[h_{ij}^{\text{TT}}, \partial_t h_{ij}^{\text{TT}}, y_A^i, p_i^A]$
- Perform a Fokker type reduction [Schäfer (1985), Jaranowski, Schäfer (1998, 2000)]

$$H_{\text{cons}}[y_A^i, p_i^A, \dot y_A^i, \dot p_i^A, \ldots] = R[y_A^i, p_i^A, h_{ij}^{\text{TT}}(y_A^k, p_k^A, \dot y_A^k, \dot p_k^A, \ldots), \partial_t h_{ij}^{\text{TT}}(y_A^k, p_k^A, \dot y_A^k, \dot p_k^A)]$$

Main features of the ADM approach

- Only one retarded field h_{ij}^{TT} that appears at high orders

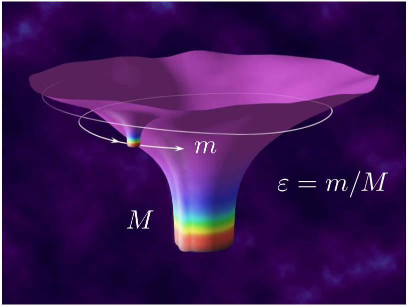
  Less problems with retardation and matching

- No UV poles nor logarithms at 3PN
  Elimination by adding a mere total derivative at 4PN
- E, $\boldsymbol{P}$, $\boldsymbol{J}$ immediate to compute
- Standard techniques to construct action-angle variables
- Poincaré invariance may be checked through the Poisson bracket algebra
- Spin can be added but delicate construction [Steinhoff, Schäfer (2009)]

  $\hookleftarrow$ formalism on a fixed curved background [Barausse, Racine, and Buonanno (2009)]
  better understood [Vines, Kunst, Steinhoff, Hinderer (2016), Witzany, Steinhoff, Lukes-Gerakopoulos (2019)]
- Hamiltonian formulation of perfect fluids in GR can be used

  [Holm (1987), Blanchet, Damour, Schäfer (1990)]

Gravitational self-force [adapted from Pound, Warburton, Miller, Wardell (Capra 19)]



Credit: NASA

Einstein field equations

$$G_{\mu\nu}[g] = \frac{8\pi G}{c^4} T^{\mu\nu} \text{(REE)} + \text{Gauge Cond}$$

- $g_{\mu\nu} = g^{(0)}_{\mu\nu} + \sum_{n=1}^{+\infty} \varepsilon^n h^{(n)}_{\mu\nu}$
- Assume the previous orders are known: $h^{(n' \leq n)}_{\mu\nu}[x, y^\lambda_{(0)} + \varepsilon y^\lambda_{(1)} + \ldots]$
- Solve for the world-line corrections $y^\mu_{(n)}(\tau)$ and $h^{(n)}_{\mu\nu}$

Form of the equations of motion in the background

$$\frac{D^2 y^\mu}{\mathrm{d}\tau^2} = \sum_{n=1}^{+\infty} \varepsilon^n \underbrace{F^\mu_{(n)}}_{n^{\text{th}} \text{ order self-force}}$$

Ongoing work: $n \leq 2 \Rightarrow$ 2^{nd} order GSF

Contribution to the GW phase at 2GSF order

For GW, relevance of the secular time scale: $\tau_s = GM/(\varepsilon c^3)$
$\hookleftarrow$ taken into account by multi-scale analysis: $t_{\text{orb}} = \phi_{\text{orb}}/\Omega_{\text{orb}}$, $t_{\text{slow}} = \tilde{t} = \varepsilon t$

[Pound (2015)]

GSF expansion of the phase

$$\phi = \frac{1}{\varepsilon} \underbrace{\phi_{(0)}}_{\text{adiabatic}} + \underbrace{\phi_{(1)}}_{\text{post-adiabatic}} + \mathcal{O}(\varepsilon)$$

- $\phi_{(0)}$ adiabatic: secular effect on τ_s scale due to $\langle F^\mu_{(1)} \rangle_{\text{orbit}}$
- $\phi_{(1)}$ post-adiabatic: needed for precision GR

Three contributions

- 1^{st} order, oscillatory dissipative self-force
- 1^{st} order conservative self-force
- 2^{nd} order orbit averaged self-force

Hierarchy of equations to be solved

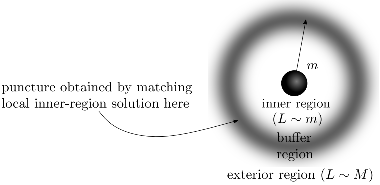
$$\Box h^{(1)} = T^{(1)} \qquad \Box h^{(2)} + G^{(2)}[h^{(1)}, h^{(1)}] = -T^{(2)}$$

2^{nd} order field equation

2^{nd} order field equation made tractable by:

- writing $\Box = -c^{-2}\partial_t^2 + \partial_{r^*}^2 + \ldots$ in terms of ϕ_{orb} and $\tilde{t}$:
  $\Box = \Box_\omega^{(0)} + \varepsilon \Box_\omega^{(1)}$
- obtaining the expression of the singular part of $h^{(2)}$: puncture $h^{(2)\text{P}}$

  [Pound (2012, 2017, 2014), Pound, Miller (2014), Warburton, Wardell (2014)]



Final equation: to be solved with non-incoming wave condition

$$\Box^{(0)} h_\omega^{\text{R}(2)} = -G_\omega^{(2)}[h^{(1)}, h^{(1)}] - \Box_\omega^{(0)} h^{(2)\text{P}} - \Box_\omega^{(1)} h^{(1)}$$

$G_\omega^{(2)}[h^{(1)}, h^{(1)}] = G_\omega^{(2)}[h^{\text{P}(1)}, h^{\text{P}(1)}] + 2G_\omega^{(2)}[h^{\text{R}(1)}, h^{\text{P}(1)}] + G_\omega^{(2)}[h^{\text{R}(1)}, h^{\text{R}(1)}]$

expressed as derivative w.r.t. parameters

asymptotic matching near $\mathcal{I}^+$

On-going work for quasi-circular orbits

Conclusion: where we are

Self-force calculations:

- Right-side of the 2^{nd} order field controlled for circular orbits
  $\hookleftarrow$ improvement to be done
- Long task list including generalization to generic orbits
- Overlap of validity domain allowed comparison with PN calculations
  $\hookleftarrow$ $E_{1\mathrm{GSF}}(x)$ matched with $E_{4\mathrm{PN\ ADM}}(x)$ to get the unknown coefficient

Post-Newtonian calculations:

- Equations of motion
  - point particles: 4PN order (Fokker, diagrammatic, ADM), static part at 5PN order (diagrammatic) [Foffa, Mastrolia, Sturani, Sturm, Torres Bobadilla]
  - spinning objects: 4PN order (diagrammatic) [completed by Levi, Steinhoff (2015)]
  - tidal effects: 2PN (Fokker) [completed by Bini, Damour, F]
- Flux and phase
  - point particles: 3.5PN order + strict 4.5PN order
  - spinning objects: SO 4PN, SS 3PN, SSS 3.5PN
  - tidal effects: 2PN [Henry, F (in preparation)]
  - tidal effects with spin: 1.5PN [Abdelsalhin, Gualtieri, Pani (2018)]
- Amplitudes
  - point particles: 3PN [Blanche, F, Iyer, Sinha (2008)]
  - spinning objects: 2PN [Buonanno, F, Hinderer (2013)]