Gravitational Waves from Soft Theorem

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Goal: Summarize the results on gravitational waves from soft theorem

1. New viewpoint for older results (memory effect)

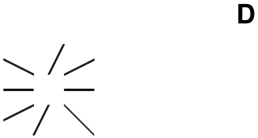
Strominger, $\cdots$

2. Some new results

Alok Laddha, A.S. arXiv:1806.01872

Biswajit Sahoo, A.S. arXiv:1808.03288

Consider an explosion in space



A bound system at rest breaks apart into fragments carrying four momenta $p_1, p_2, \cdots$ of masses $m_1, m_2, \cdots$

$$(p_a^0)^2 = \vec{p}_a^{\,2} + m_a^2, \quad a = 1, 2, \cdots, \quad \sum_a \vec{p}_a = 0$$

This process emits gravitational waves

Detector D placed far away detects $\quad h_{\mu\nu} \equiv (g_{\mu\nu} - \eta_{\mu\nu})/2$

Physical components: h_{ij}^{TT} (transverse, traceless)

Our focus will be on the late time tail of the radiation – the value of h_{ij}^{TT} at a large time u after the passage of the peak

It has the form

$$h_{ij}^{TT} = A_{ij}^{TT} + \frac{1}{u} B_{ij}^{TT} + \cdots, \quad \text{for large u}$$

A_{ij}: memory term

– a permanent change in the state of the detector after the passage of gravitational waves

Zeldovich, Polnarev; Braginsky, Grishchuk; Braginsky, Thorne; $\cdots$

– connected to the leading soft graviton theorem   Strominger; $\cdots$

B_{ij}: tail term

– connected to logarithmic terms in the subleading soft theorem

Laddha, A.S.

$$h_{ij}^{TT} = A_{ij}^{TT} + \frac{1}{u} B_{ij}^{TT} + \cdots$$

$$A^{\mu\nu} = -\frac{2\,G}{R} \sum_a \frac{p_a^\mu p_a^\nu}{p_a \cdot n}, \quad u \cdot v = -u^0 v^0 + \vec{u} \cdot \vec{v}$$

$$B^{\mu\nu} = \frac{2\,G^2}{R} \Bigg[2 \sum_{a,b} \frac{n \cdot p_b}{n \cdot p_a} p_a^\mu p_a^\nu$$

$$+ \sum_{\substack{a,b \\ b \neq a}} \frac{n_\rho p_a^{(\nu}}{p_a \cdot n} (p_a^{\mu)} p_b^\rho - p_b^{\mu)} p_a^\rho) \frac{p_b \cdot p_a}{\{(p_b \cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}} \{2(p_b \cdot p_a)^2 - 3 m_a^2 m_b^2\} \Bigg]$$

n=(1, n̂), n̂: unit vector towards the detector

R: distance to detector, G: Newton's constant

$$h_{ij} = A_{ij}^{TT} + \frac{1}{u} B_{ij}^{TT}, \quad \text{for large } u$$

$$A^{\mu\nu} = -\frac{2\,G}{R} \sum_{a} \frac{p_a^{\mu} p_a^{\nu}}{p_a \cdot n}$$

$$B^{\mu\nu} = \frac{2\,G^2}{R} \Bigg[2 \sum_{a,b} \frac{n \cdot p_b}{n \cdot p_a}\, p_a^{\mu} p_a^{\nu}$$

$$+ \sum_{\substack{a,b \\ b \neq a}} \frac{n_\rho p_a^{(\nu}}{p_a \cdot n} \left(p_a^{\mu)} p_b^{\rho} - p_b^{\mu)} p_a^{\rho}\right) \frac{p_b \cdot p_a}{\{(p_b \cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}} \left\{2(p_b \cdot p_a)^2 - 3 m_a^2 m_b^2\right\} \Bigg]$$

If a significant fraction of energy is carried away by radiation, then the sum over a,b includes integration over outgoing flux of radiation, regarded as a flux of massless particles.

$$h_{ij} = A^{TT}_{ij} + \frac{1}{u} B^{TT}_{ij}, \quad \text{for large } u$$

$$A^{\mu\nu} = -\frac{2G}{R} \sum_a \frac{p_a^\mu p_a^\nu}{p_a \cdot n}$$

$$B^{\mu\nu} = \frac{2G^2}{R} \Bigg[2 \sum_{a,b} \frac{n \cdot p_b}{n \cdot p_a} p_a^\mu p_a^\nu$$

$$+ \sum_{\substack{a,b \\ b\neq a}} \frac{n_\rho p_a^{(\nu}}{p_a \cdot n} (p_a^{\mu)} p_b^\rho - p_b^{\mu)} p_a^\rho) \frac{p_b \cdot p_a}{\{(p_b \cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}} \left\{ 2(p_b \cdot p_a)^2 - 3 m_a^2 m_b^2 \right\} \Bigg]$$

1. The result is a statement in classical GR, even though it was ‘derived’ using soft graviton theorem (a result for S-matrix).

2. The result is given completely in terms of the momenta of final state objects without knowing what caused the explosion or how the objects moved during the explosion

– consequence of soft graviton theorem

3. The contribution to $B^{\mu\nu}$ from massless particles vanishes.

A similar result exists for a general scattering process

– gives the gravitational wave-form $\tilde{h}_{ij}$ at low frequency in terms of momenta of incoming and outgoing particles.

$$\tilde{h}_{ij}^{TT}(\vec{x},\omega) = C_{ij}^{TT}$$

$$C^{\mu\nu} = \frac{2\,G}{i\,R}\sum_a \eta_a \frac{p_a^\mu p_a^\nu}{p_a\cdot k}\left\{1 - 2\,i\,G\,\ln\omega \sum_{b,\eta_b=-1} k\cdot p_b\right\}$$
$$-2\,\frac{G^2}{R}\,\ln\omega \sum_a \sum_{\substack{b\neq i\\ \eta_a\eta_b=1}} \frac{k_\rho p_a^{(\nu}}{p_a\cdot k}\,(p_a^{\mu)} p_b^\rho - p_b^{\mu)} p_a^\rho)$$
$$\times \frac{p_b\cdot p_a}{\{(p_b\cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}}\,\{2(p_b\cdot p_a)^2 - 3m_a^2 m_b^2\} + \text{finite}\,.$$

$k = -\omega\,(1,\hat{n})$, η_a: +1 if a is incoming, -1 if a is outgoing.

– Matches explicit results in special cases

Peters; Ciafaloni, Colferai, Veneziano; Addazi, Bianchi, Veneziano

We shall not attempt to explain these results from soft graviton theorem.

Instead, in the rest of the talk I shall try to explain how the various parts of $B^{\mu\nu}$ might arise in a classical GR analysis.

Laddha, A.S., work in progress

$$B^{\mu\nu} = \frac{2\,G^2}{R}\Bigg[2\sum_{a,b}\frac{n\cdot p_b}{n\cdot p_a}\,p_a^\mu p_a^\nu$$

$$+\sum_{\substack{a,b\\ b\neq a}}\frac{n_\rho p_a^{(\nu}}{p_a\cdot n}\,(p_a^{\mu)}p_b^\rho - p_b^{\mu)}p_a^\rho)\,\frac{p_b\cdot p_a}{\{(p_b\cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}}\,\{2(p_b\cdot p_a)^2 - 3m_a^2 m_b^2\}\Bigg]$$

$B^{TT}_{\mu\nu}$ gives the coefficient of 1/u term for transverse, traceless component of $h_{\mu\nu}$.

1. After the explosion, the final state objects exert long range gravitational force on each other causing them to accelerate

– they continue to radiate.

This leads to the second term in $B^{\mu\nu}$ proportional to:

$$\sum_{\substack{a,b\\ b\neq a}} \frac{n_\rho p_a^{(\nu}}{p_a\cdot n}\,(p_a^{\mu)} p_b^\rho - p_b^{\mu)} p_a^\rho)\,\frac{p_b\cdot p_a}{\{(p_b\cdot p_a)^2 - m_a^2 m_b^2\}^{3/2}}\,\{2(p_b\cdot p_a)^2 - 3m_a^2 m_b^2\}$$

Both massive and massless particles contribute to this.

2. The component of the gravitational wave associated with the memory effect will have to travel in the background geometry of the ‘hard’ particles / radiation before reaching the detector

This disperses the gravitational wave-form associated with the memory effect

Peters; Goldberger, Ross

– responsible for the first term in $B^{\mu\nu}$ proportional to

$$2\sum_{a,b}\frac{n\cdot p_b}{n\cdot p_a}\,p_a^{\mu}p_a^{\nu}$$

The task left for a direct GR proof of the result is to show that there are no other effects to this order.

The different origin of the two terms is also visible in the analysis based on soft theorem.

The first term comes from loop momenta $< 1/u$, while the second term comes from loop momenta $> 1/u$.

For some reason that we do not understand, for massless final state particles the two effects cancel each other

$\Rightarrow$ no contribution to $B^{\mu\nu}$ for binary black hole merger, but there can be appreciable contributions for supernovæ and other explosive events.