Introduction to Holographic QCD

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Outline

1. Motivation:

   AdS/CFT correspondence: Foundations and applications

2. Transport properties (Quark-gluon plasma)

3. Chiral symmetry breaking

4. Mesons (Comparison to lattice gauge theory)

5. Applications to deep inelastic scattering

6. Entanglement Entropy

THE BIG PICTURE

Two current challenges in theoretical physics

Two current challenges in theoretical physics

Challenge Nr. 1:

Find a unified theory of all known interactions:

Electromagnetism, Weak force, Strong force

$\Leftrightarrow$ Gravity

Challenge: Quantization of gravity

Challenge Nr. 2: Strongly coupled systems

Describe observables and processes
in systems with a given interaction

Challenge: Beyond perturbation theory

Example: Yang-Mills theory

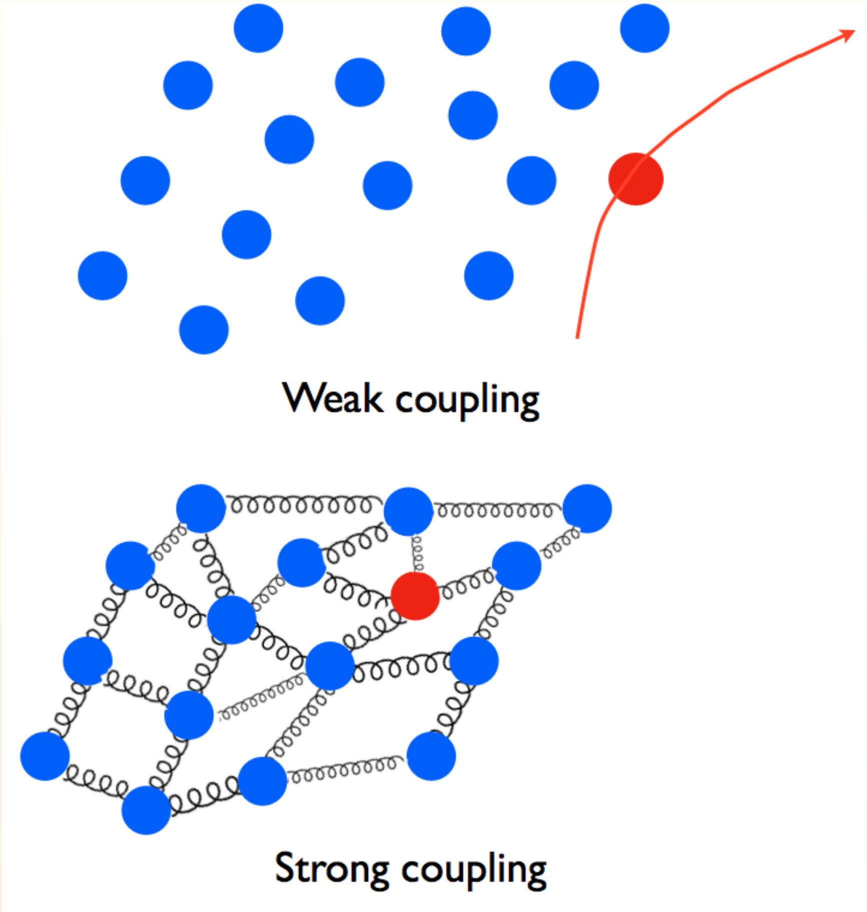
$$\mathcal{L} = -\frac{1}{2g^2}\,\mathrm{Tr}\,F^{\mu\nu}F_{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

Challenge in theoretical physics: Strong coupling

$g \ll 1$

$g \geq 1$



$g \geq 1$: Application of perturbation theory not possible

Recent development:

Unified theory of fundamental interactions and description of strongly coupled systems are much more closely related than we thought!

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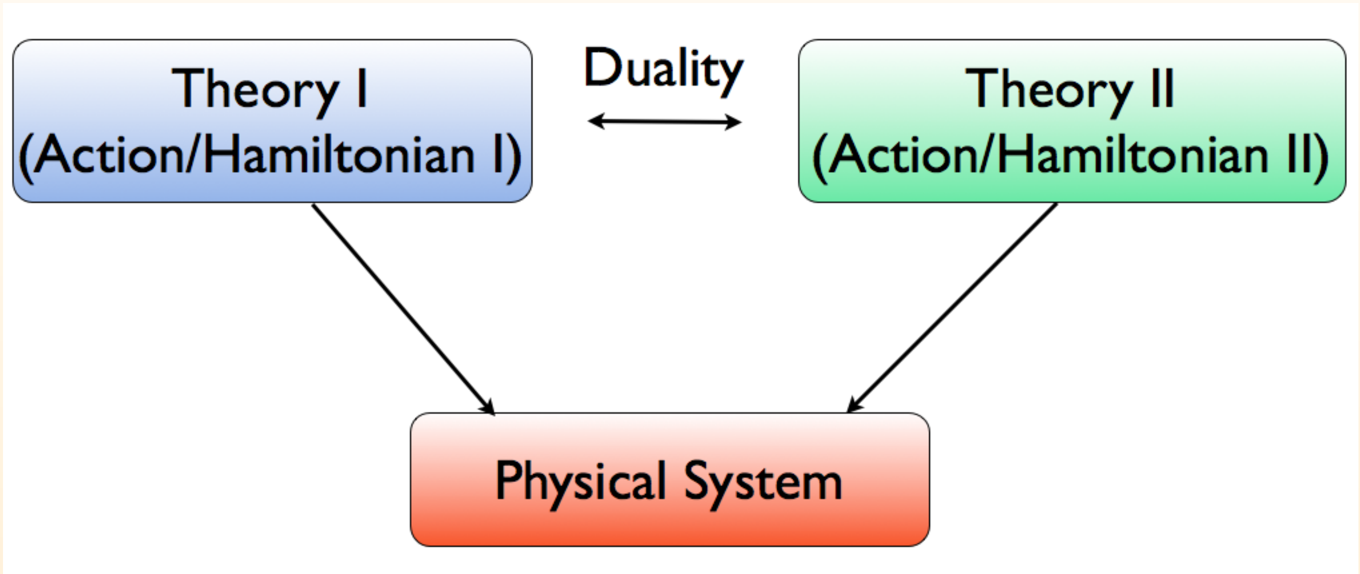
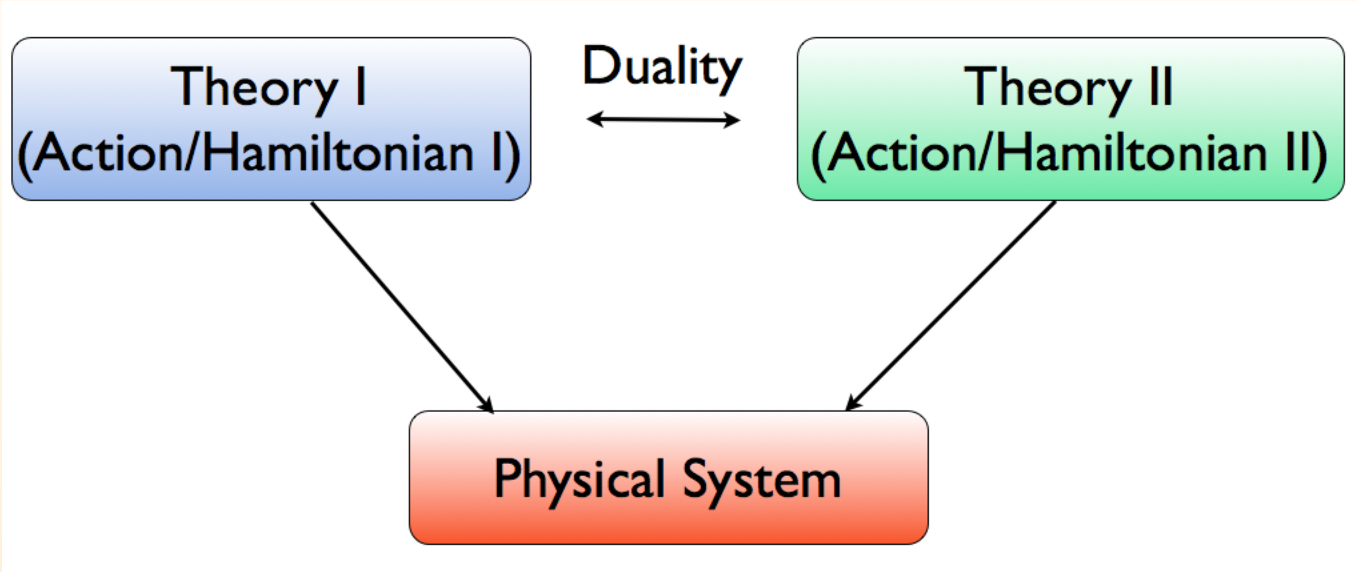
Gauge/gravity duality

Duality:


Theory I
(Action/Hamiltonian I)
Duality
Theory II
(Action/Hamiltonian II)
Physical System

Duality:



Gauge/Gravity Duality:

A theory without gravity is dual to a gravity theory.

Gauge/gravity duality

- Conjecture which follows from a low-energy limit of string theory

- Duality:

  Quantum field theory at strong coupling

  $\Leftrightarrow$ Theory of gravitation at weak coupling

- Holography:

  Quantum field theory in four dimensions

  $\Leftrightarrow$ Gravitational theory in five dimensions

Best understood example: AdS/CFT correspondence

AdS: Anti-de Sitter space, CFT: Conformal field theory

Anti-de Sitter Space

Hyperbolic space of constant negative curvature, has a boundary

Embedding of (Euclidean) AdS$_{d+1}$ into Mink$_{d+2}$:

$$-X_0^2 + X_1^2 + X_2^2 + \cdots + X_{d+1}^2 = -L^2$$

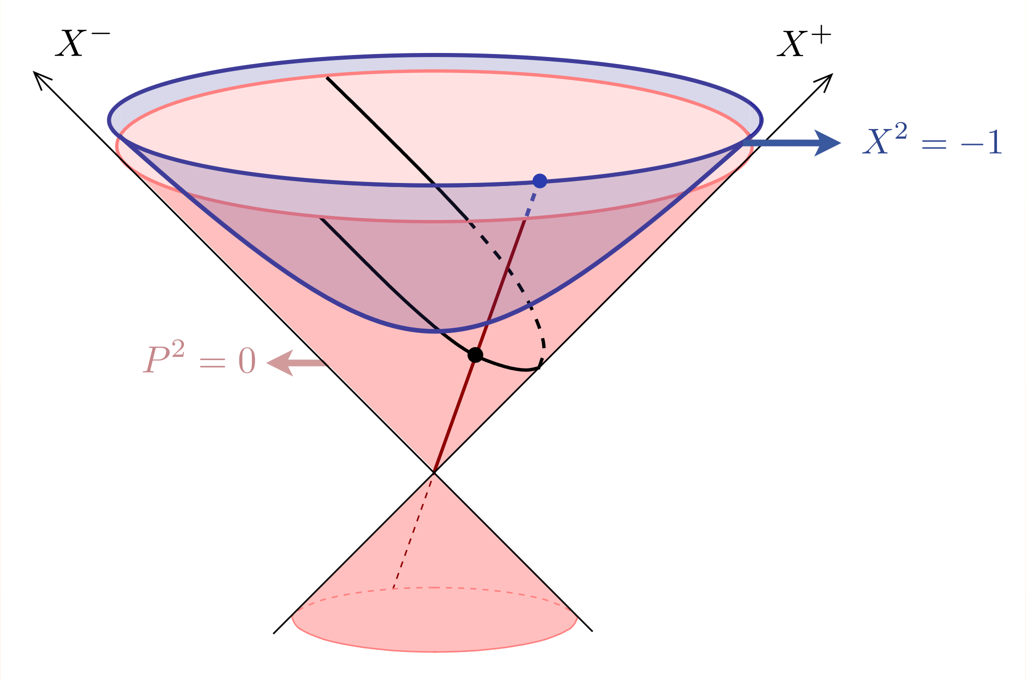
Isometries of Euclidean AdS$_{d+1}$:

$$SO(d+1,1)$$

Metric on Poincaré patch:

$ds^2 = e^{2r/L} dx_\mu dx^\mu + dr^2$ or

$ds^2 = \frac{L^2}{z^2}(dx_\mu dx^\mu + dz^2)$



Source: Costa, Penedones, Poland, Ryshkov 1109.6321

Quantum field theory

Quantum field theory

in which the fields transform covariantly under conformal transformations

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Conformal coordinate transformations:

Preserve angles locally: $dx'_\mu dx'^\mu = \Omega^2(x) dx_\mu dx^\mu$

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J.E., Osborn '97

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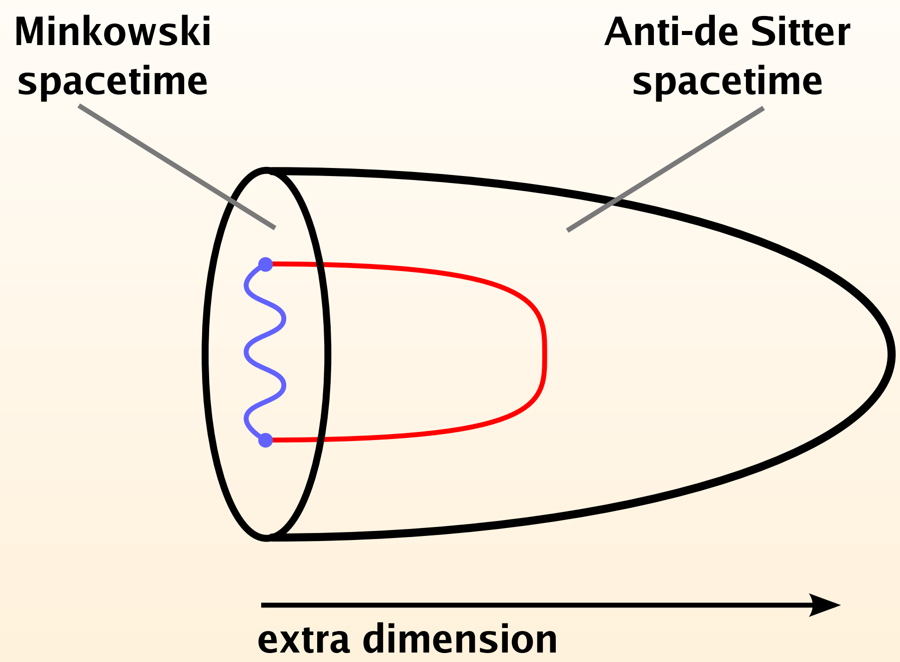
In AdS/CFT correspondence: Conformal field theory in 3+1 dimensions:
$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory (global symmetry $SO(4,2) \times SU(4)$)

AdS/CFT correspondence

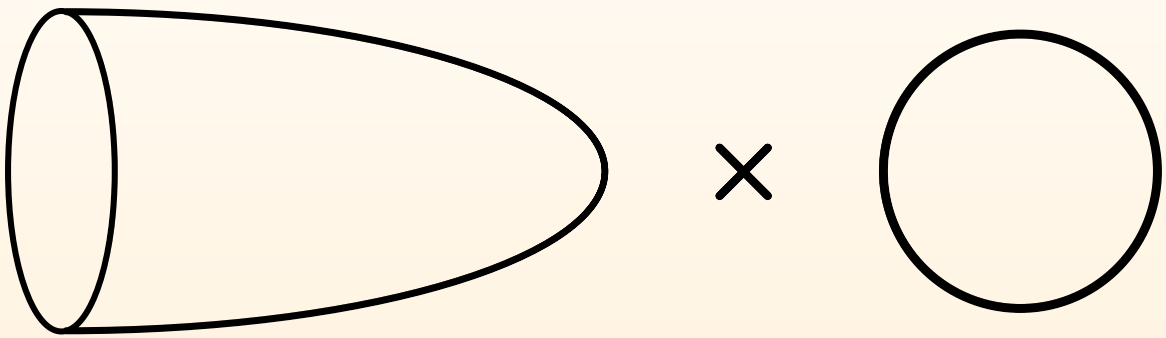


'Dictionary' Gauge invariant field theory operators
$\Leftrightarrow$ Classical fields in gravity theory

Symmetry properties coincide ($SO(4,2) \times SO(6)$)

Test: (e.g.) Calculation of correlation functions

String theory origin $\Rightarrow$ Ten dimensions



$$AdS_5 \times S^5$$

Symmetries of field theory and geometry coincide: $SO(4,2) \times SO(6)$

Internal manifold determines field content

Generalized AdS/CFT Correspondence: Gauge/gravity duality

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Generalization of AdS/CFT to quantum field theories of experimental relevance?

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Prototype candidate: Low-energy QCD

Generalized AdS/CFT Correspondence: Gauge/gravity duality

Generalization of AdS/CFT to quantum field theories of experimental relevance?

Prototype candidate: Low-energy QCD

- SU(3) gauge theory with matter (gluons and quarks)
- Strongly coupled at low energies $\Rightarrow$ mesons, baryons
- Beta function negative

Generalizations:

1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ Renormalization Group flows

   $\Rightarrow$ Finite temperature, finite density

2. More degrees of freedom are added (Example: quarks)

3. Large N limit continues to apply

Top-down vs. bottom-up

- Top-down: Begin with string theory in ten dimensions

  $\Rightarrow$ Lagrangian of dual field theory known, few parameters

Top-down vs. bottom-up

- Top-down: Begin with string theory in ten dimensions

  $\Rightarrow$ Lagrangian of dual field theory known, few parameters

- Bottom-up: Just work with deformations of AdS, ignore internal space

  $\Rightarrow$

  Calculations simpler, however only global symmetries of dual field theory known

  Universality

Top-down approach

Use 10-dimensional (super)gravity actions obtained from string theory

to describe

Dual degrees of freedom in strongly coupled quantum field theory

Quantum Theory of Gravity and Unification of Interactions:

Give up locality at very short distances

Natural cutoff: String length

$$l_s \sim \frac{1}{M_{Planck}},$$

Open strings: Gauge interactions Closed strings: Gravity

Higher oscillation modes may be excited $\Rightarrow$ Particles

Quantization:

Supersymmetric string theory is well-defined in $9 + 1$ dimensions

(no tachyons, no anomalies)

Supersymmetry: Bosons $\Leftrightarrow$ Fermions

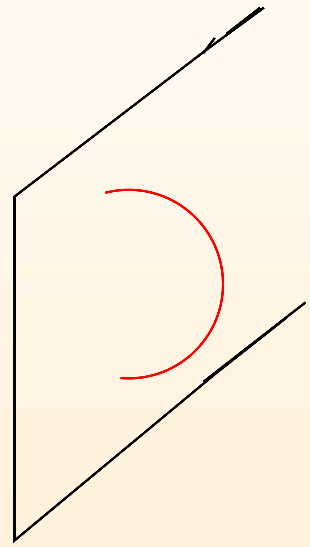
What is the meaning of the extra dimensions?

1. Compactification

2. D-Branes

D-Branes

D-branes are embedded in ten-dimensional space (Hypersurfaces)



D3-Branes: (3+1)-dimensional hypersurfaces

open strings may end on D-branes $\Leftrightarrow$ dynamics

D-Branes

In low-energy limit (strings pointlike) $\Rightarrow$

Open Strings $\Leftrightarrow$ Field theory (Gauge theory) degrees of freedom on the brane

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Open Strings $\Leftrightarrow$ Field theory (Gauge theory) degrees of freedom on the brane

Second interpretation of D-branes:

Solitonic solutions of ten-dimensional supergravity

heavy objects which curve the space around them

Elementary excitations: closed strings

AdS/CFT correspondence

Map:

Four-dimensional quantum field theory

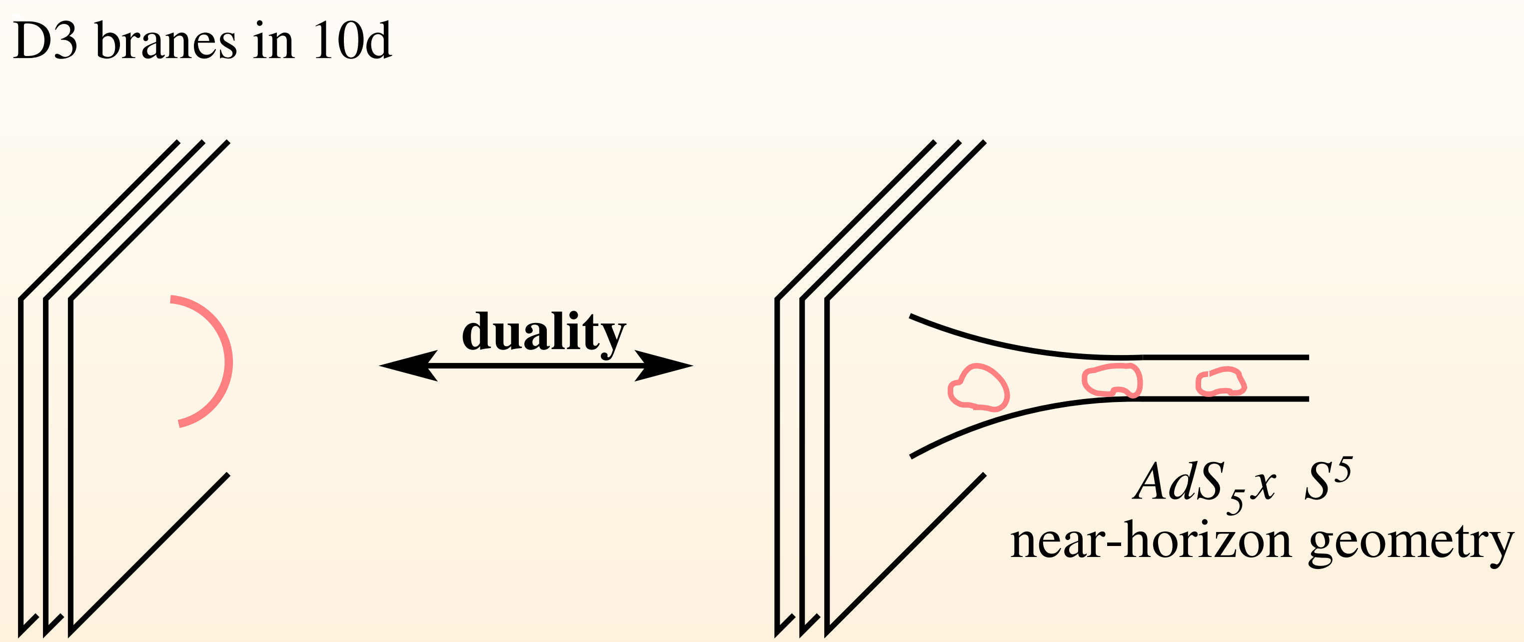
$\Leftrightarrow 5+5$-dimensional gravity theory!

arises from identifying the two different interpretations of D-branes

D3 Branes $\Rightarrow$

$\mathcal{N}=4$ Super Yang-Mills theory is dual to string theory on $AdS_5 \times S^5$

String theory origin of the AdS/CFT correspondence

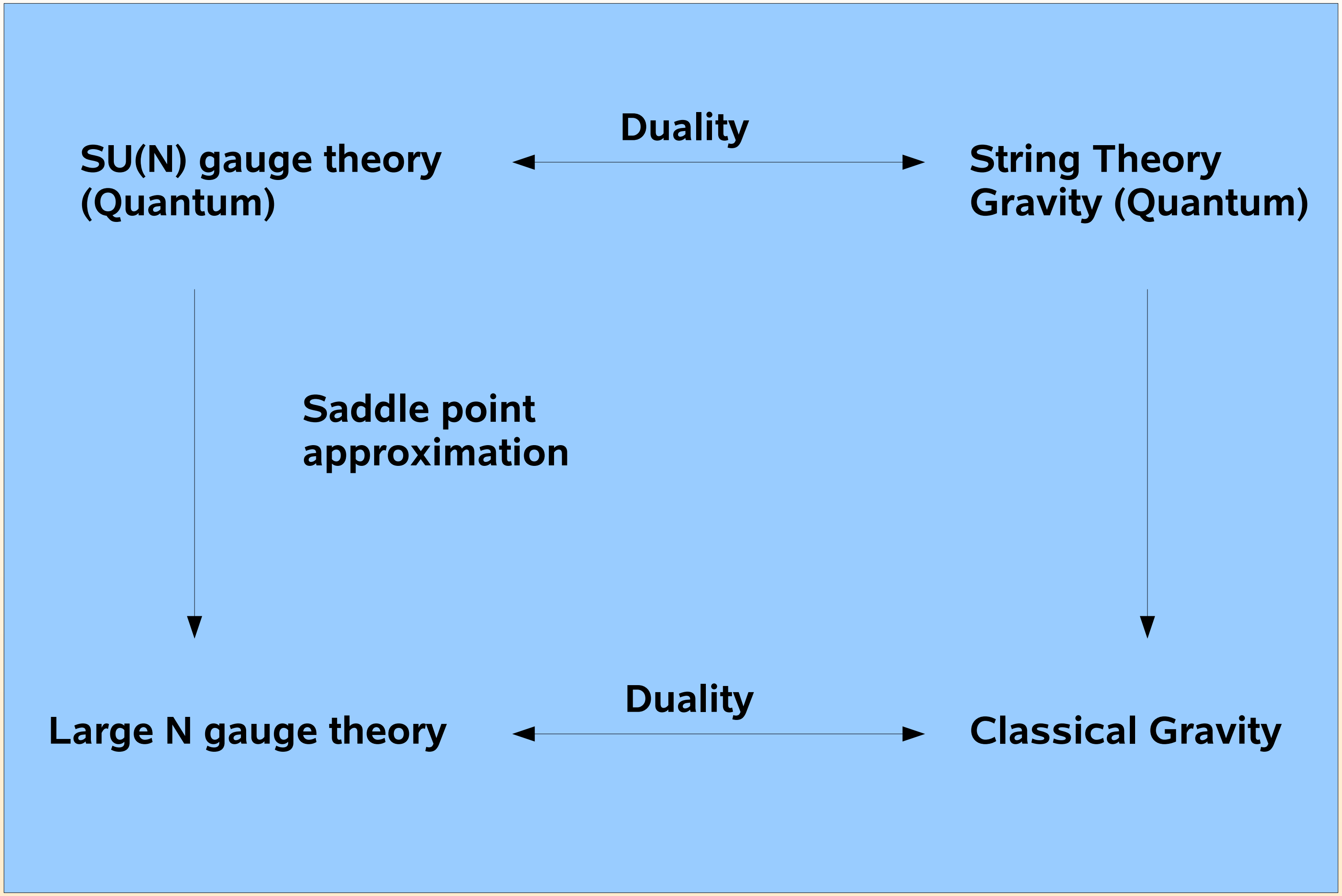


⇓ Low energy limit

Supersymmetric $SU(N)$ gauge theory in four dimensions $(N \to \infty)$

Supergravity on the space $AdS_5 \times S^5$

AdS/CFT correspondence (Maldacena 1997)



AdS/CFT correspondence

- Field-operator correspondence:

$$\langle e^{\int d^d x \, \phi_0(\vec{x}) \mathcal{O}(\vec{x})} \rangle_{CFT} = Z_{sugra} \Big|_{\phi(0,\vec{x}) = \phi_0(\vec{x})}$$

Generating functional for correlation functions of particular composite operators in the quantum field theory

coincides with

Classical tree diagram generating functional in supergravity

AdS/CFT correspondence

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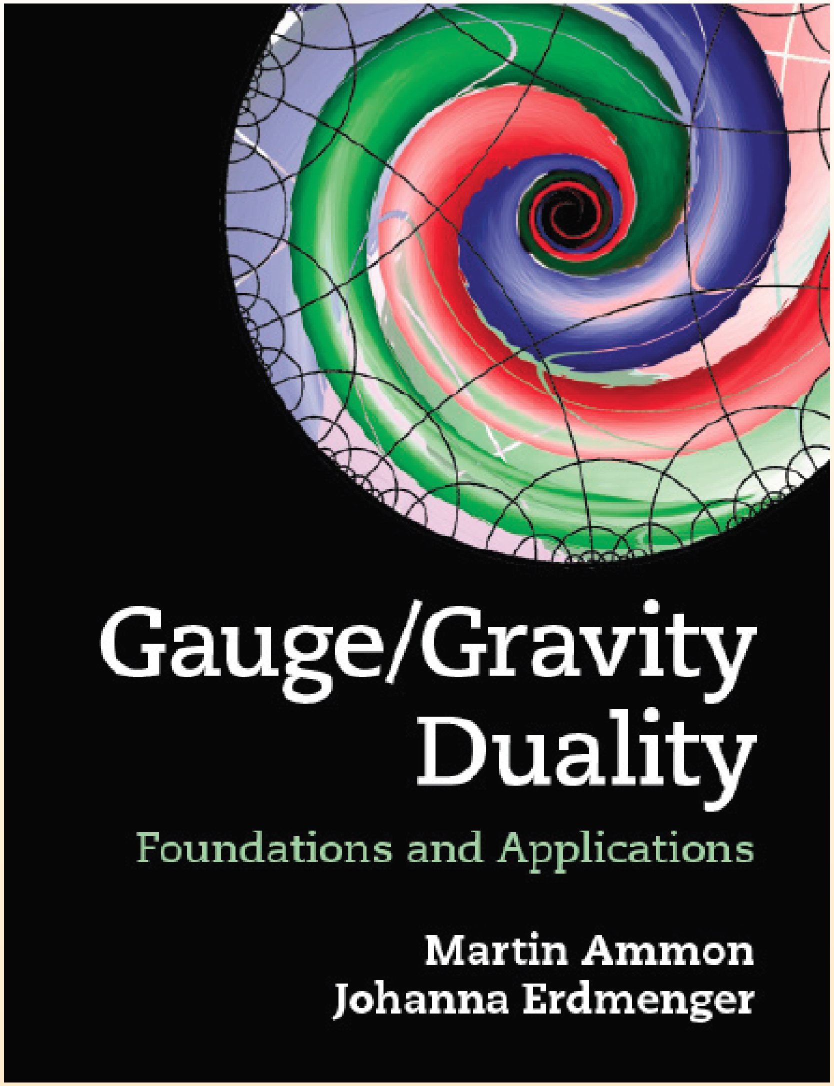
- **Dictionary:** field theory operators $\Leftrightarrow$ supergravity fields

$$\mathcal{O}_\Delta \leftrightarrow \phi_m \ , \qquad \Delta = \frac{d}{2} + \sqrt{\frac{d^2}{4} + L^2 m^2}$$

Book on gauge/gravity duality



Bottom-up approach

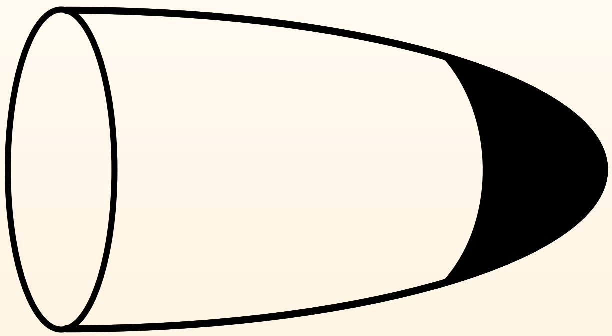
Strongly coupled quantum field theories

(difficult to solve)

are mapped to

Weakly coupled gravity theories (easy to solve)

Finite temperature field theory is dual to black hole embedded in AdS space



Motivation: In equilibrium, finite temperature given by periodic imaginary time (Matsubara formalism)

Euclidean signature black hole also requires compactified imaginary time for regularity at horizon!

Black hole metric (embedding in AdS):

$$ds^2 = \frac{L^2}{z^2}\left(f(z)d\tau^2 + d\vec{x}^2 + \frac{1}{f(z)}dz^2\right), \qquad f(z) = 1 - \frac{z^4}{z_h^4}.$$

Hawking temperature

$$T_h = 1/(\pi z_h)$$

is identified with temperature of dual field theory

Black holes

- Gravity objects
  with remarkable quantum properties

- Very massive objects

- Large mass $\Leftrightarrow$ Strong curvature

- Once matter or light passes the Schwarzschild radius, it is trapped inside the black hole and cannot escape any more. $\Rightarrow$ Horizon

- Hawking temperature: Bekenstein entropy

$$S = \frac{A_H}{4G}$$

Black holes

- In AdS/CFT:

  Black hole is a particular solution of the equations of motion

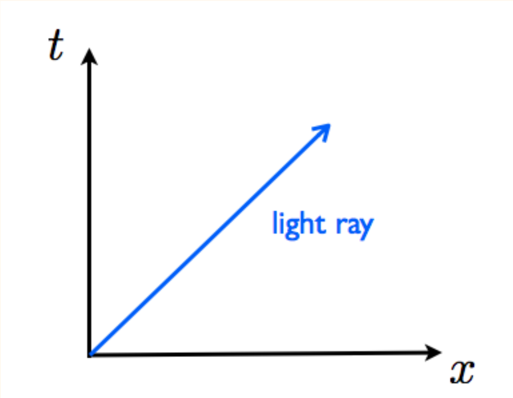
  of an abstract gravitational theory in Anti-de Sitter space

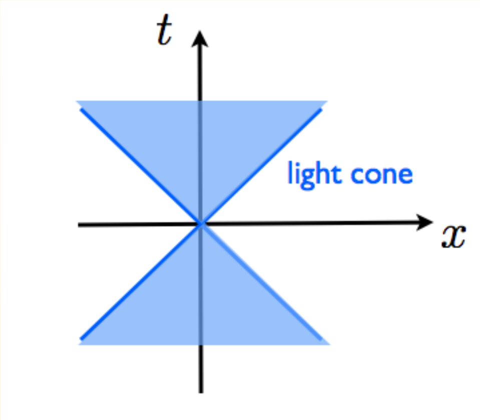
- Information content of black hole $\Leftrightarrow$ Quantum information theory

Causal structure of space-time

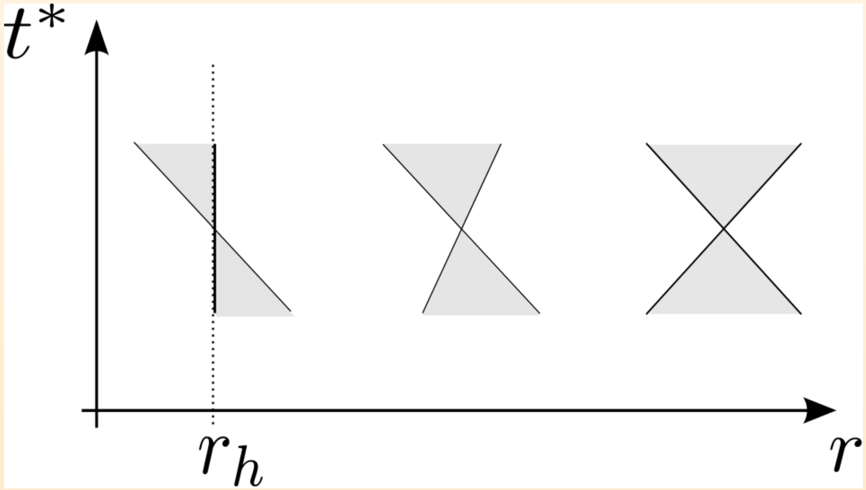
$x = ct$ Set $c = 1 \Rightarrow x = t$

Flat space:



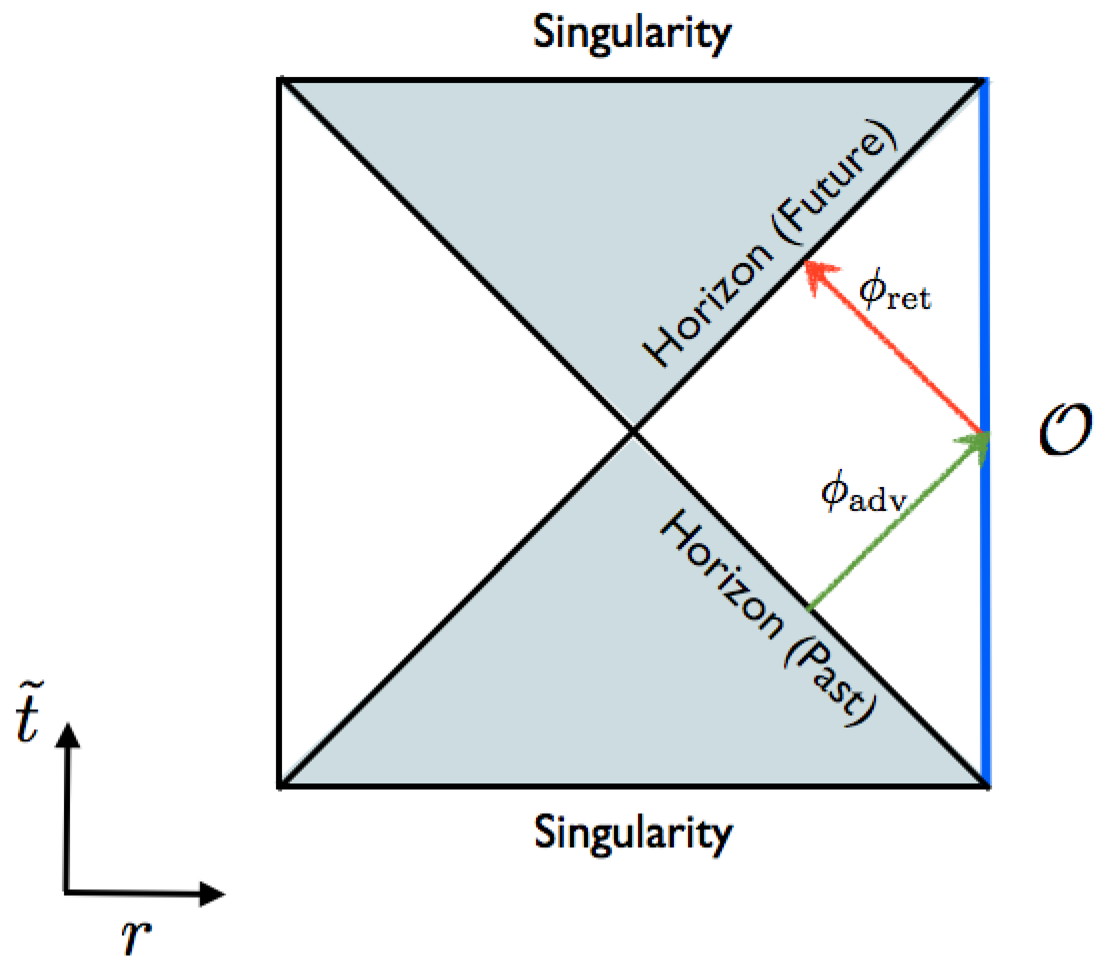


Black hole:



Retarded Green's Functions in Strongly Coupled Systems

Anti-de Sitter
black hole



Retarded Green's function: $G^R_{\mathcal{O}_A\mathcal{O}_B} = \left.\frac{\delta\langle\mathcal{O}_A\rangle}{\delta\phi_{B(0)}}\right|_{\delta\phi=0} = \frac{\delta\phi_{A(1)}}{\delta\phi_{B(0)}}$

subject to infalling boundary condition at horizon

Retarded Green's function and quasinormal modes

Complex energy eigenvalues of fluctuations about gravity background

Example: Fluctuations of scalar field $\Phi(x,z) = \int d^4k e^{-i\omega t + \vec{k}\cdot\vec{x}} \Phi(\omega, \vec{k}, z)$

Insert into equation of scalar fluctuation in AdS black hole background:

$$(\Box - m^2 L^2)\Phi(k,z) = 0$$

$$4u^3 \partial_u \left(\frac{f}{u} \partial_u \Phi(u,k) \right) + \frac{u}{(\pi T)^2 f} \left(\omega^2 - |\vec{k}|^2 f \right) \Phi(u,k) - m^2 L^2 \Phi(u,k) = 0\,.$$

$u = z^2/z_h^2$

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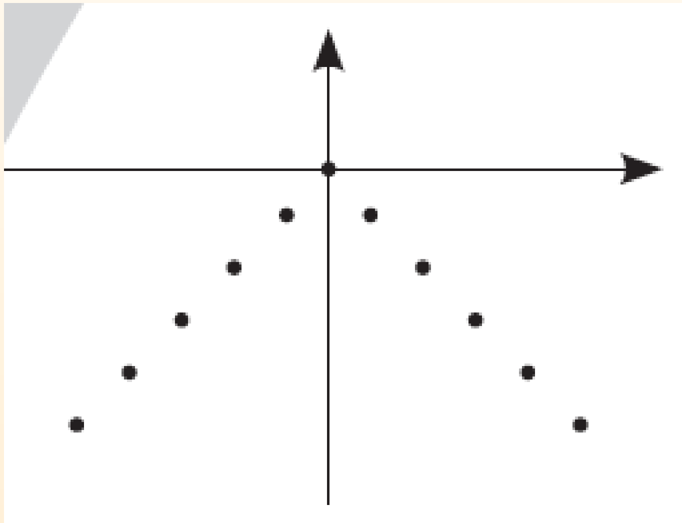
Asymptotic solution near boundary:

$$\Phi(u,k) \sim \phi_{(0)}(k) u^{(d-\Delta)/2}(1 + \mathcal{O}(u)) + \phi_{(1)}(k) u^{\Delta/2}(1 + \mathcal{O}(u))$$

Retarded Green's function and spectral function

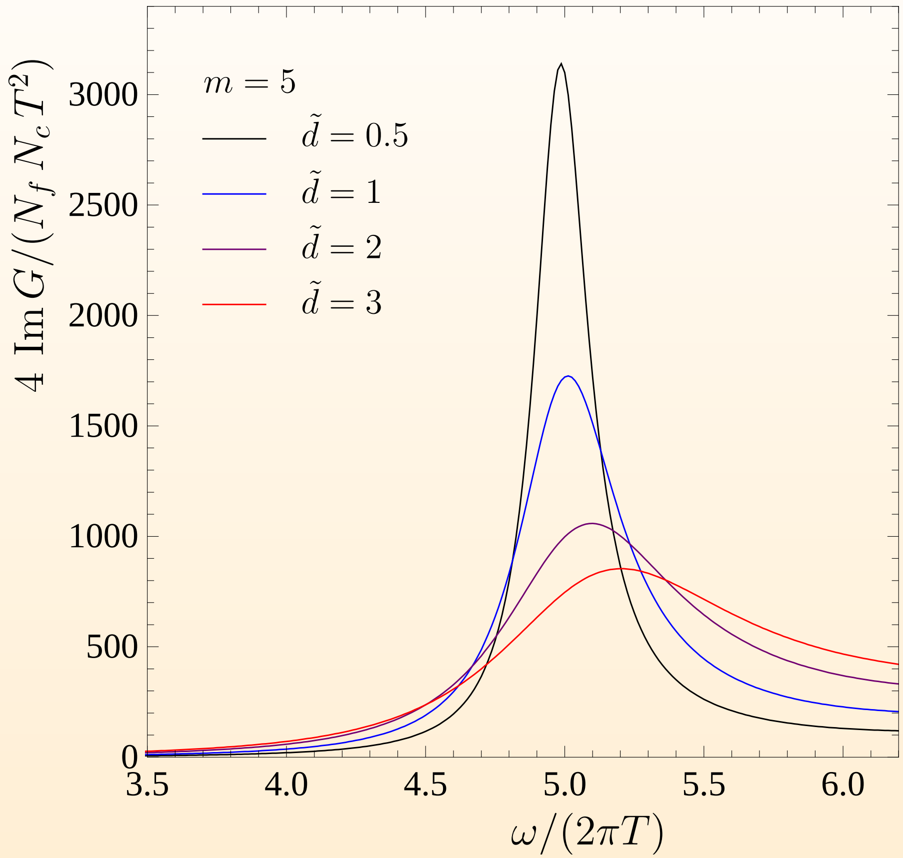
ω (which may be complex) determine poles of retarded Green's function

(location, decay width)



Spectral function: $\mathcal{R}(\omega, \vec{k}) = -2\mathrm{Im}G^R(\omega, \vec{k})$

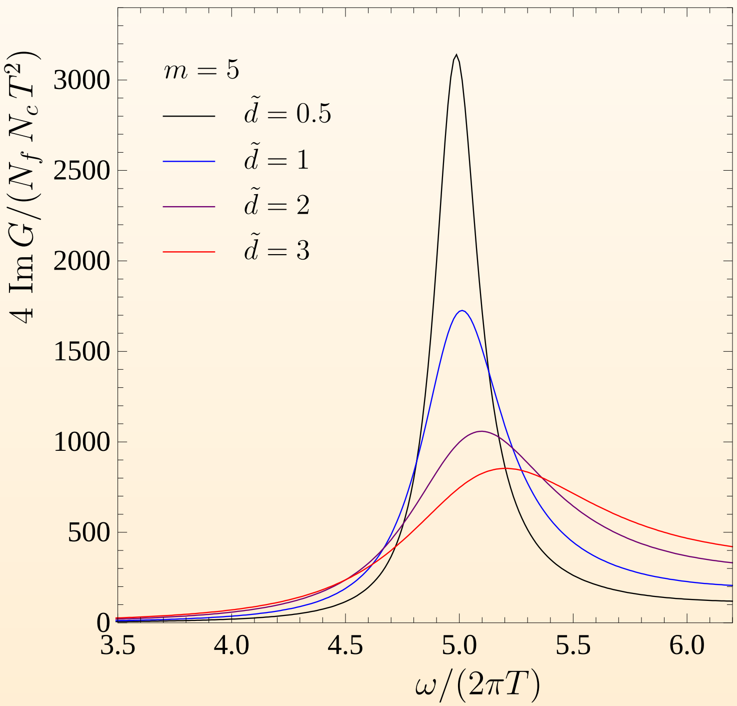
Example for spectral function: ρ meson in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



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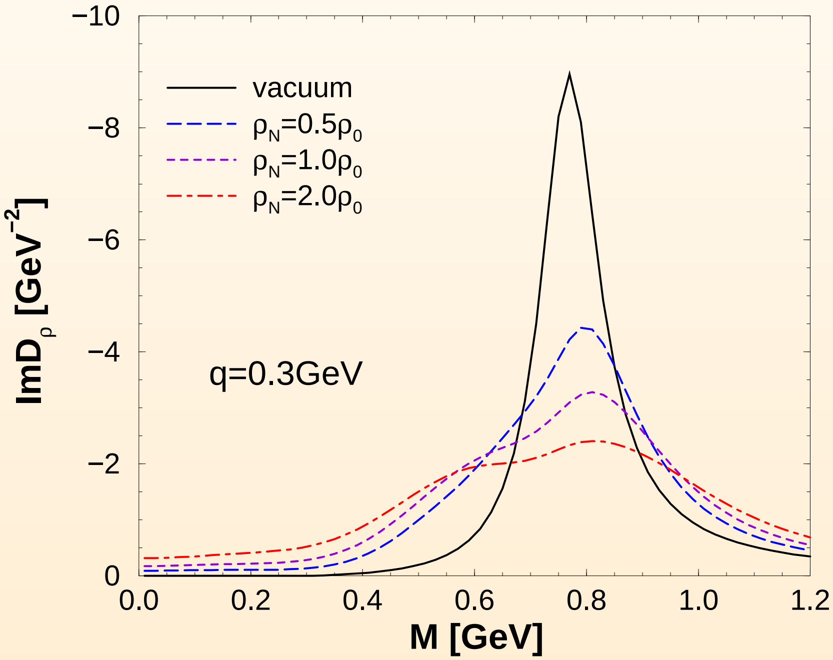

Field theory (Rapp, Wambach 2000)

First example for applications: Shear viscosity

Shear viscosity over entropy density

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

Kovtun, Son, Starinets PRL 2004

- Universal lower bound (does not depend on details of theory) at least to leading order
- Bound satisfied by the most strongly coupled systems ($g \to \infty$)
- Experimentally confirmed for quark-gluon plasma at RHIC accelerator
- Also relevant for electrons in solid?

Shear viscosity

- Hydrodynamics: Long wavelength, low-frequency fluctuations in fluids
- Expand physical quantities in derivatives of the fluid velocity: $\vec{v}$, $\nabla\vec{v}$, $\nabla\nabla\vec{v}\ldots$
- Relativistically: Four-velocity $u^\mu = (u^0, u^1, u^2, u^3)$, $u^\mu u_\mu = 1$

  $u^0 = 1/\sqrt{1-\vec{v}^2}$, $\vec{u} = \vec{v}/\sqrt{1-\vec{v}^2}$
- Consider energy-momentum tensor $T_{\mu\nu}$

  Contains information about energy density, energy and momentum flux
- Hydrodynamic expansion to first order in derivatives:

$$T_{\mu\nu}(x) = T^{(0)}_{\mu\nu}(x) + T^{(1)}_{\mu\nu}(x) + \ldots$$

$$T^{(0)}_{\mu\nu}(x) = (\epsilon + P)u_\mu u_\nu - P g_{\mu\nu}\,, \;\; T^{(1)}_{\mu\nu} = \eta\left(\partial_\mu u_\nu + \partial_\nu u_\mu - \tfrac{2}{3} g_{\mu\nu}\partial_\lambda u^\lambda\right) + \zeta g_{\mu\nu}\partial_\lambda u^\lambda$$

η shear viscosity, ζ bulk viscosity

Holographic calculation of shear viscosity

- Energy-momentum tensor $T_{\mu\nu}$ dual to graviton $g^{\mu\nu}$
- Calculate correlation function $\langle T_{xy}(x_1)T_{xy}(x_2)\rangle$ from propagation through black hole space
- Shear viscosity is obtained from Kubo formula:

$$\eta = -\lim \frac{1}{\omega}\,\mathrm{Im}G^R_{xy,xy}(\omega)$$

- Shear viscosity $\eta = \pi N^2 T^3/8$, entropy density $s = \pi^2 N^2 T^3/2$

$$\frac{\eta}{s} = \frac{1}{4\pi}\frac{\hbar}{k_B}$$

(Note: Quantum critical system: $\tau = \hbar/(k_B T)$)

Charged fluids: Vorticity

J.E., Haack, Kaminski, Yarom 0809.2488; JHEP 2009

Action of $\mathcal{N}=2$, $d=5$ Supergravity:
From compactification of $d=11$ supergravity on a Calabi-Yau manifold

$$S = -\frac{1}{16\pi G_5}\int\left[\sqrt{-g}\left(R+12-\frac{1}{4}F^2\right)-\frac{1}{2\sqrt{3}}A\wedge F\wedge F\right]d^5x$$

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Chern-Simons term leads to axial anomaly for boundary field theory:

$$\partial_\mu J^\mu_{(A)} = \frac{1}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

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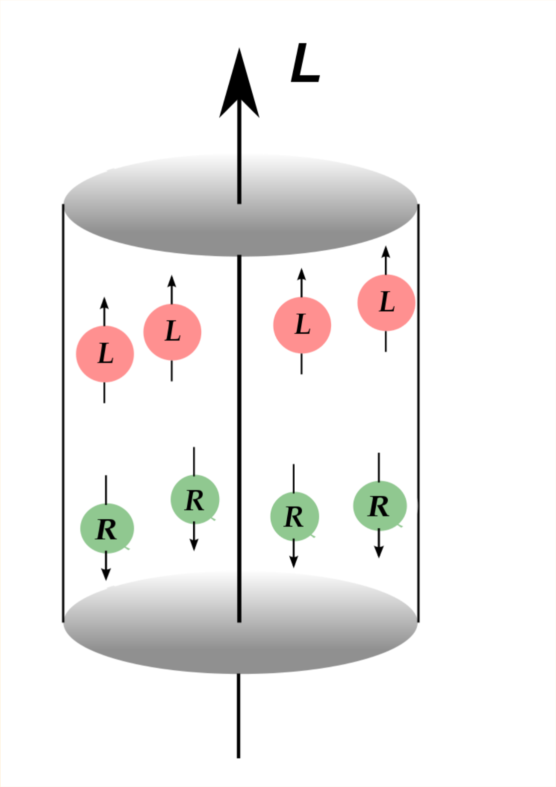
Contribution to relativistic hydrodynamics, proportional to angular momentum:

$J_\mu = \rho u_\mu + \xi \omega_\mu, \quad \omega_\mu = \frac{1}{2} \epsilon_{\mu\nu\sigma\rho} u^\nu \partial^\sigma u^\rho$, in fluid rest frame $\vec{J} = \frac{1}{2} \xi \nabla \times \vec{v}$

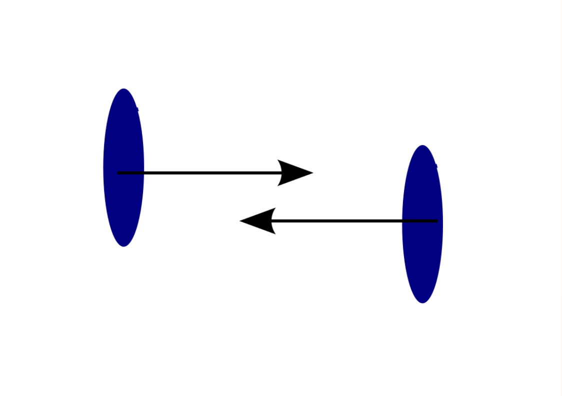
Chiral vortical effect

Chiral separation: In a volume of rotating quark matter, quarks of opposite chirality move in opposite directions. (Son, Surowka 2009)

Chiral vortical effect



Non-central
heavy ion collision



Anomaly induces topological charge $Q_5 \Rightarrow$ Axial chemical potential $\mu_5 \leftrightarrow \Delta Q_5$
associated to the difference in number of left- and right-handed fermions

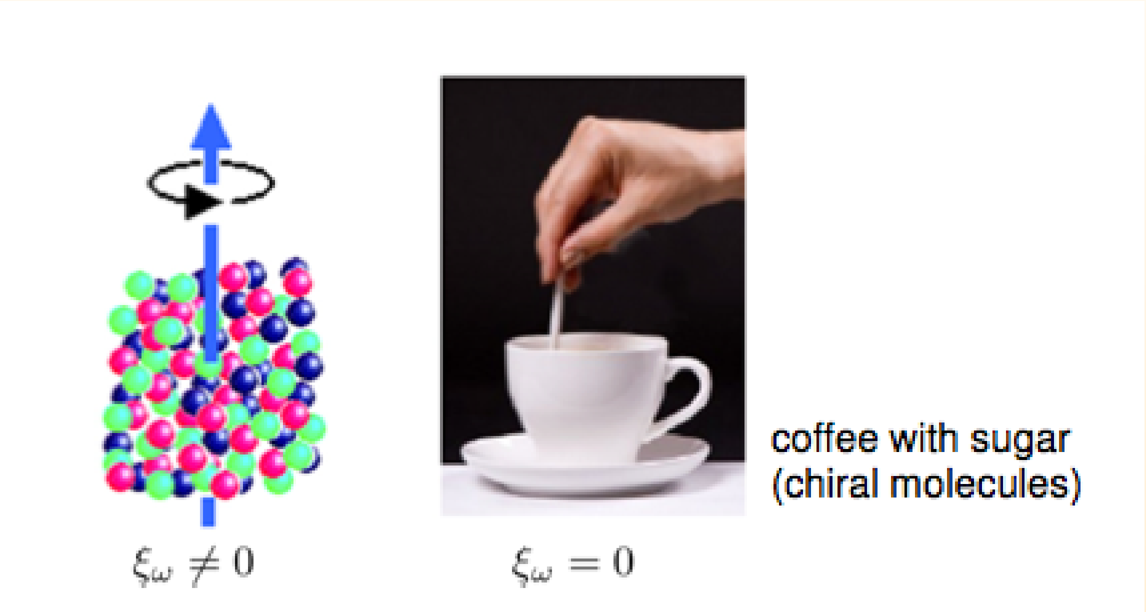
Proposal for experimental confirmation (Oz, Keren-Zur 2010):
Enhanced production of spin-excited hadrons along rotation axis

Chiral vortical effect

Chiral separation: Relativistic quantum effect

Chiral vortical effect

Chiral separation: Relativistic quantum effect



Non-equilibrium in holography

Modelled by time-dependent solutions in gravity

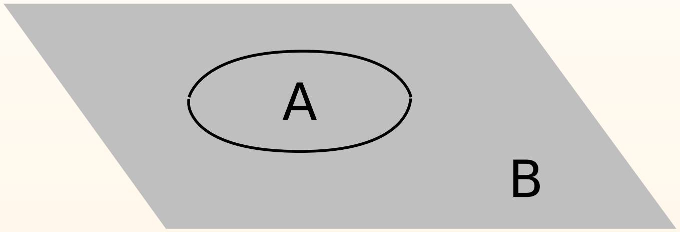
Shock waves, collapsing matter shells

Application to formation of quark-gluon plasma

Very short hydrodynamization time

Chesler+Yaffe, Romatschke, Ecker, van der Schee, ...

Entanglement entropy



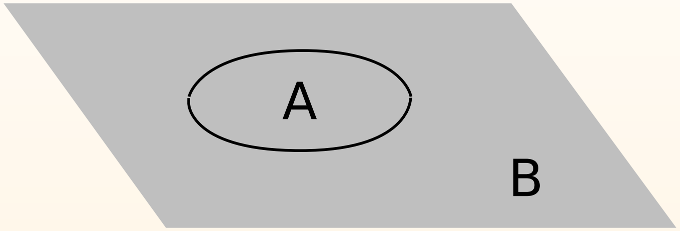
Von Neumann entropy $S_N = -\mathrm{Tr}\rho \ln \rho$

Reduced density matrix $\rho_A = \mathrm{Tr}_B \rho_{\mathrm{tot}}$

Entanglement entropy

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

Entanglement entropy



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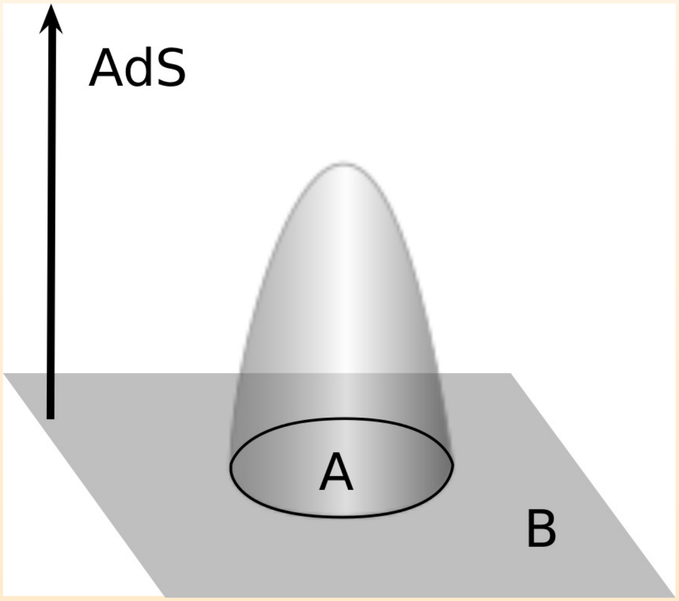
Entanglement entropy

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

Ryu-Takayanagi 2006:



$$S_A = \frac{\mathrm{Area}\gamma_A}{4G_N}$$

γ_A: Minimal area with $\partial A = \partial \gamma_A$

Entanglement entropy: Quantum mechanics

Density matrix $\rho = \sum_n p_n |\Psi_n\rangle\langle\Psi_n|$

Von Neumann entropy $S_{\mathrm{v}N} = -\mathrm{Tr}(\rho \ln \rho)$

Maximised when ρ diagonal with equal entries,
vanishes for pure states where $\rho^2 = \rho$

Entanglement entropy: Quantum mechanics

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Von Neumann entropy $S_{vN} = -\mathrm{Tr}(\rho \ln \rho)$

Maximised when ρ diagonal with equal entries,
vanishes for pure states where $\rho^2 = \rho$

Consider product Hilbert space $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$

Reduced density matrix

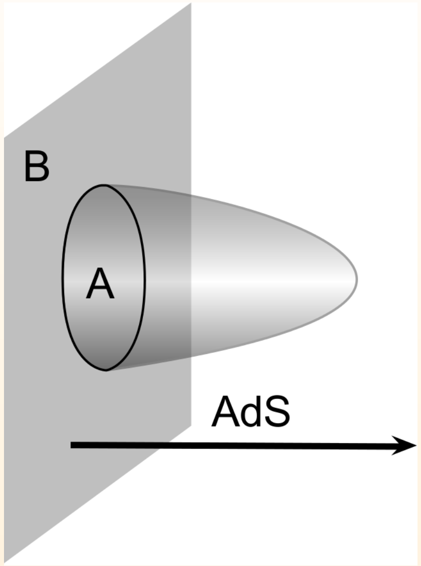
$$\rho_A = \mathrm{Tr}_B \rho_{\mathrm{tot}}$$

Entanglement entropy

$$S_A = -\mathrm{Tr}_A \rho_A \ln \rho_A$$

Analogy to black hole entropy
('Lost information' hidden in B)

Entanglement entropy: Gauge/gravity duality



Ryu-Takayanagi 2006:

$$S_A = \frac{\mathrm{Area}\gamma_A}{4G_N}$$

γ_A: Minimal area bulk surface with $\partial A = \partial \gamma_A$

Satisfies strong subadditivity

$$S_A + S_B \geq S_{A\cup B} + S_{A\cap B}$$

Entangement entropy: Examples

Conformal field theory in 1+1 dimensions (Cardy, Calabrese):

$$S = \frac{c}{3} \ln(\ell\Lambda)$$

Reproduced by Ryu-Takayanagi result

$\Lambda \propto 1/\epsilon$, ϵ boundary cut-off in radial direction

$c = 3L/(2G_3)$

Finite temperature (at small ℓ):

$$S(\ell) = \frac{c}{3} \ln\left(\frac{1}{\pi\epsilon T} \sinh(2\pi\ell T)\right)$$

J.E., Miekley 1709.07016

Analytic expression in closed form for strip region:

$$S_{EE} = \frac{L^{d-1}\left(\tilde{\ell}/\epsilon\right)^{d-2}}{2(d-2)G_N} + \frac{\sqrt{\pi}L^{d-1}}{4(d-1)G_N}\frac{\tilde{\ell}^{d-2}}{z_\star^{d-2}}\sum_{\Delta m=0}^{\frac{2(d-1)}{\chi}-1}\frac{(1/2)_{\Delta m}}{\Delta m!}\frac{\Gamma\left(\frac{d}{\chi}a^{\mathrm{EE}}_{-1/2}\right)}{\Gamma\left(\frac{d}{\chi}a^{\mathrm{EE}}_{0}\right)}\left(\frac{z_\star}{z_h}\right)^{\Delta m d} \quad (5.5\mathrm{b})$$

$$\times_{\frac{3d-2}{\chi}+1}F_{\frac{3d-2}{\chi}}\left(1, a^{\mathrm{EE}}_{-\frac{1}{2}}, \ldots, a^{\mathrm{EE}}_{\frac{d}{\chi}-\frac{3}{2}}, b^{\mathrm{EE}}_{\frac{1}{2}}, \ldots, b^{\mathrm{EE}}_{\frac{2(d-1)}{\chi}-\frac{1}{2}}; a^{\mathrm{EE}}_{0}, \ldots, a^{\mathrm{EE}}_{\frac{d}{\chi}-1}, b^{\mathrm{EE}}_{1}, \ldots, b^{\mathrm{EE}}_{\frac{2(d-1)}{\chi}}; \left(\frac{z_\star}{z_h}\right)^{\frac{2(d-1)d}{\chi}}\right)$$

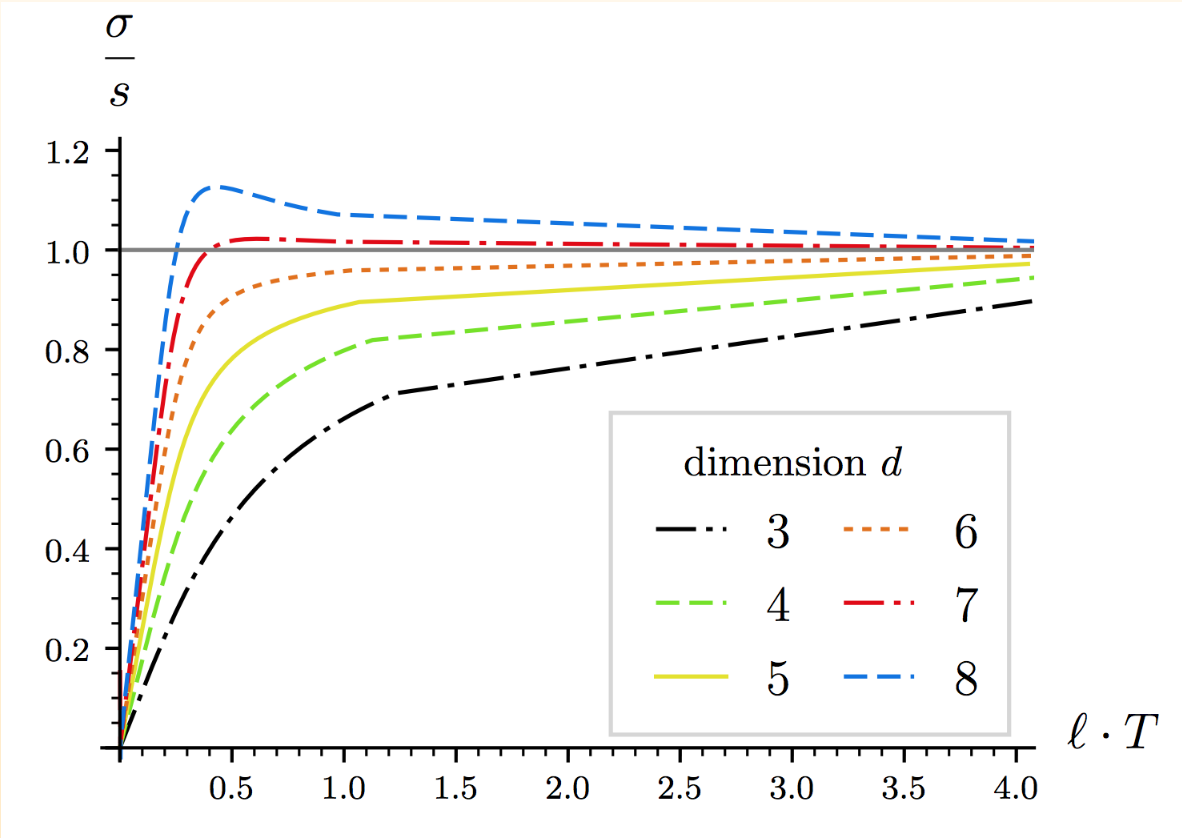
z_*: Turning point of minimal surface

Given implicity in terms of strip width ℓ

J.E., Miekley 1709.07016

Entanglement density

$$\sigma = \frac{S(T) - S(T=0)}{vol(A)}$$



Non-monotonic behaviour signals violation of area theorem

Generalized AdS/CFT Correspondence: Gauge/gravity duality

Generalization of AdS/CFT to quantum field theories of experimental relevance?

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Generalizations:

1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ Renormalization Group flows

2. More degrees of freedom are added (Example: quarks)

Generalized AdS/CFT Correspondence: Gauge/gravity duality

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Generalizations:

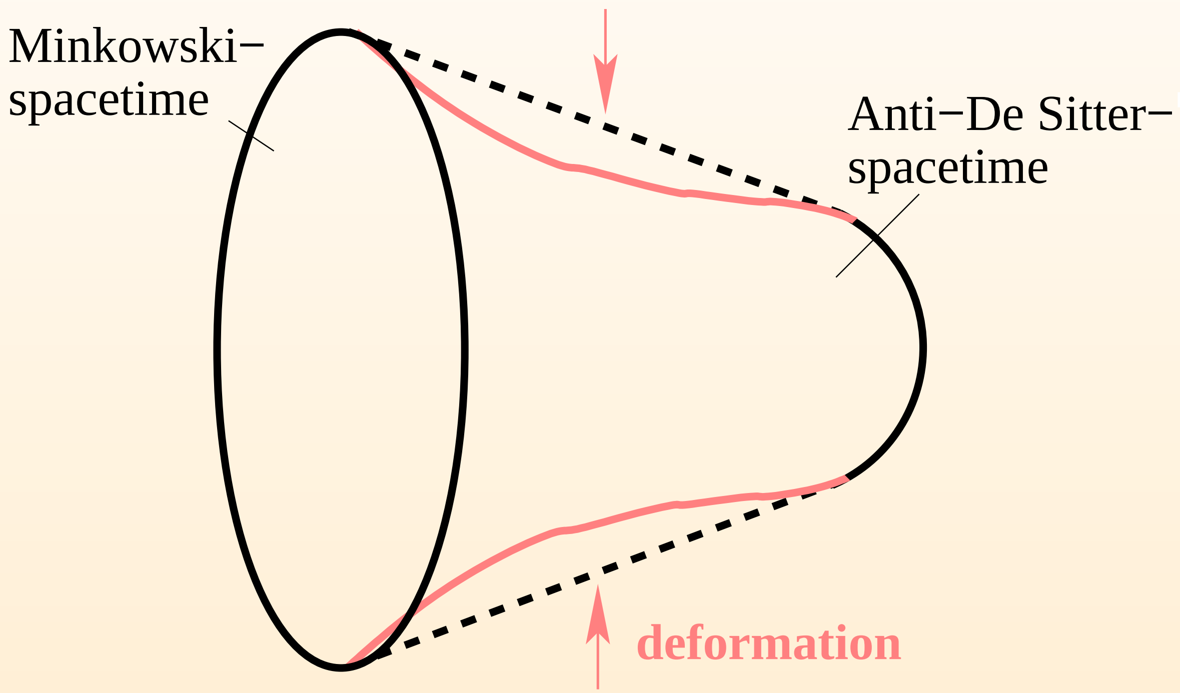
1. Symmetry requirements are relaxed in a controlled way

   $\Rightarrow$ Renormalization Group flows

2. More degrees of freedom are added (Example: quarks)

3. Large N limit continues to apply

Deformations of AdS_5 and S^5

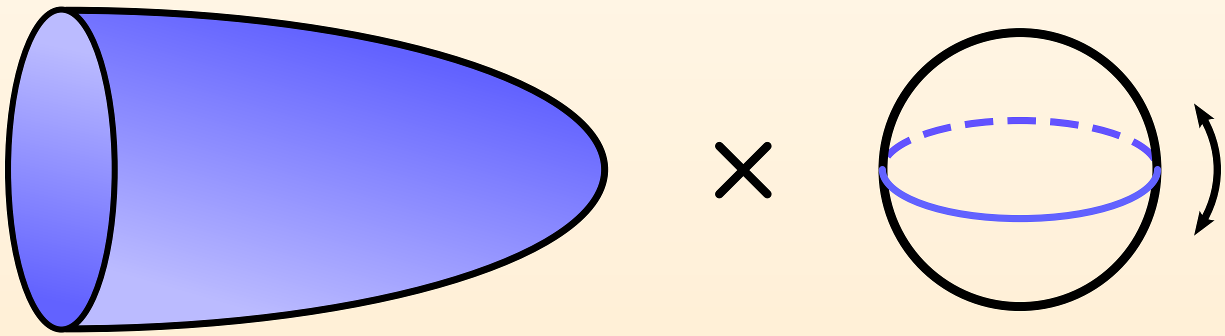


Fifth Dimension $\Leftrightarrow$ Energy scale

Supersymmetry breaking by deforming the sphere S^5

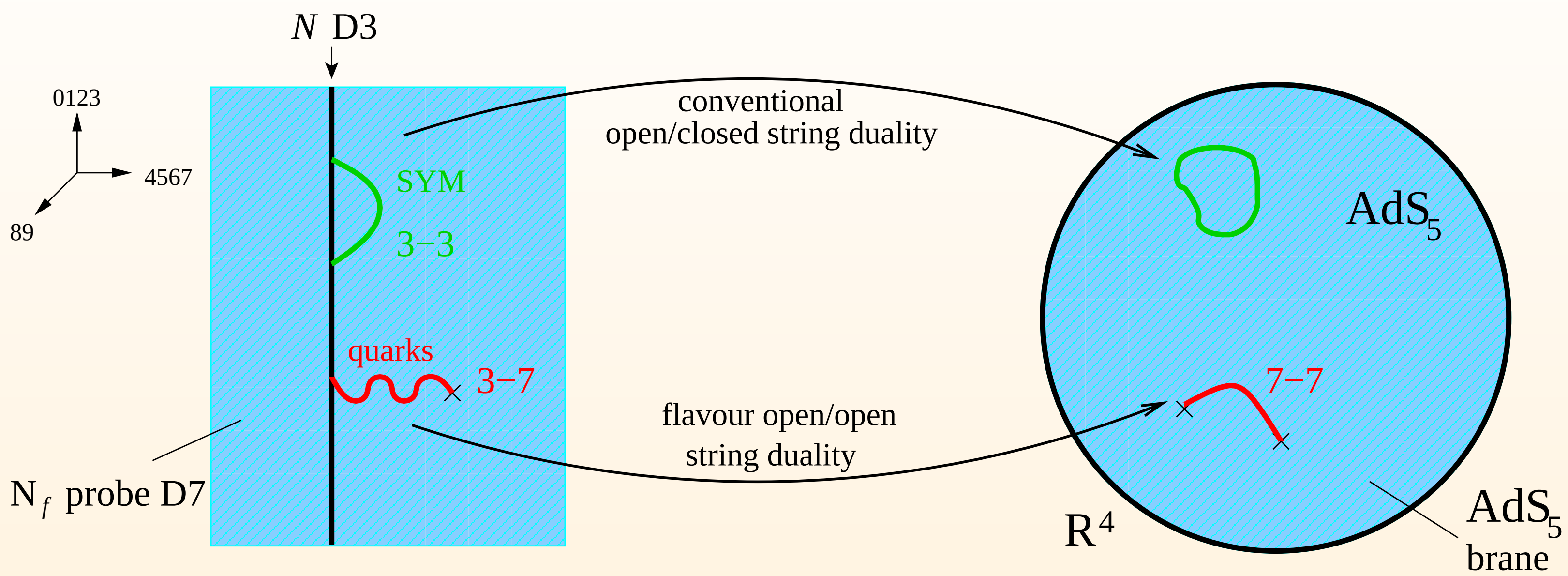
Add D7-Branes (eight-dimensional surfaces) to ten-dimensional space

	0	1	2	3	4	5	6	7	8	9
N D3	X	X	X	X						
1,2 D7	X	X	X	X	X	X	X	X		



Quarks: Low energy limit of open strings between D3- and D7-Branes

Quarks (fundamental fields) from brane probes



$N \to \infty$ (standard Maldacena limit), N_f small (probe approximation)

duality acts twice:

$\mathcal{N} = 4$ SU(N) Super Yang-Mills theory coupled to $\mathcal{N} = 2$ fundamental hypermultiplet	$\longleftrightarrow$	IIB supergravity on $AdS_5 \times S^5$ + Probe brane DBI on $AdS_5 \times S^3$

Karch, Katz 2002

Quarks are introduced into AdS/CFT via the addition of brane probes

$N_f \ll N_c, \quad (N_f = 1$ on our case)

Brane probes: open strings between D3 branes and brane probe correspond to fundamental degrees of freedom in the field theory

Field theory described:

$$\mathcal{L} = \frac{1}{4} Tr\, F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \bar{\psi} \not{D} \psi$$

gauge group $SU(N)$

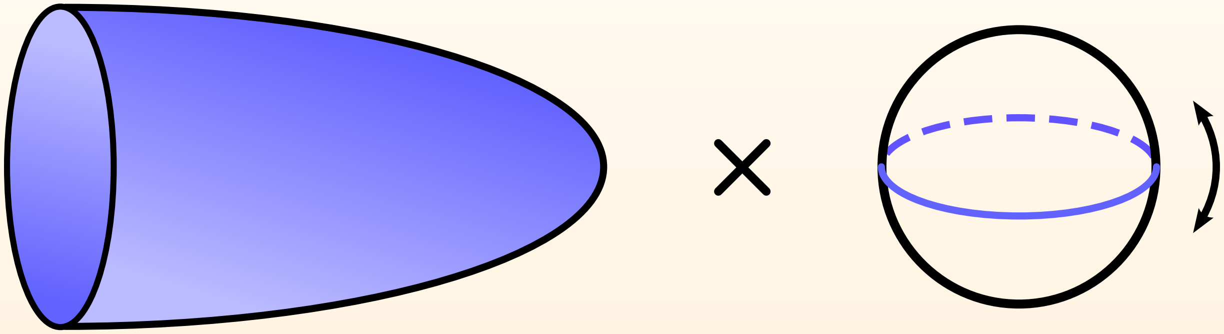
$U(1)_A$ symmetry: two Dirac fermions ψ_L, ψ_R; $\quad \psi = \psi_L + \psi_R$

$$\psi_L \to e^{i\alpha} \psi_L \,, \psi_R \to e^{-i\alpha} \psi_R$$

chiral symmetry broken by condensate $\langle \bar{\psi} \psi \rangle$

Application: Quarks in the AdS/CFT correspondence

Add D7-Branes (eight-dimensional surfaces) to ten-dimensional space



Meson masses from fluctuations of the surface (D7-Brane):

π meson (pseudoscalar):
Energy eigenvalues of D7-brane coordinate fluctuations

ρ meson (vector meson):
Energy eigenvalues of gauge field fluctuations on D7-brane

Combine the deformation of the supergravity metric
with the addition of brane probes:

Dual gravity description of chiral symmetry breaking and Goldstone bosons

J. Babington, J. E., N. Evans, Z. Guralnik and I. Kirsch,

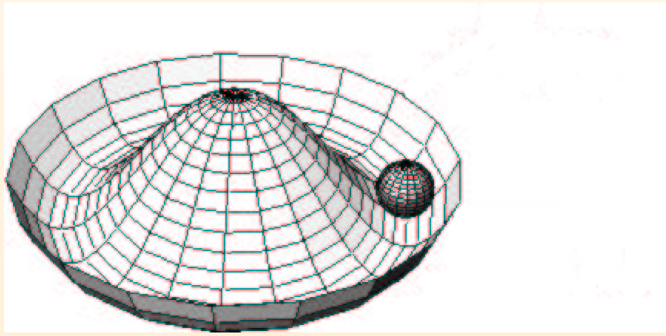
“Chiral symmetry breaking and pions in non-SUSY gauge/gravity duals”

Phys. Rev. D 69 (2004) 066007 [arXiv:hep-th/0306018].

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Gravitational realization of

Spontaneous chiral symmetry breaking



New ground state given by quark condensate $\langle\bar{\psi}\psi\rangle$

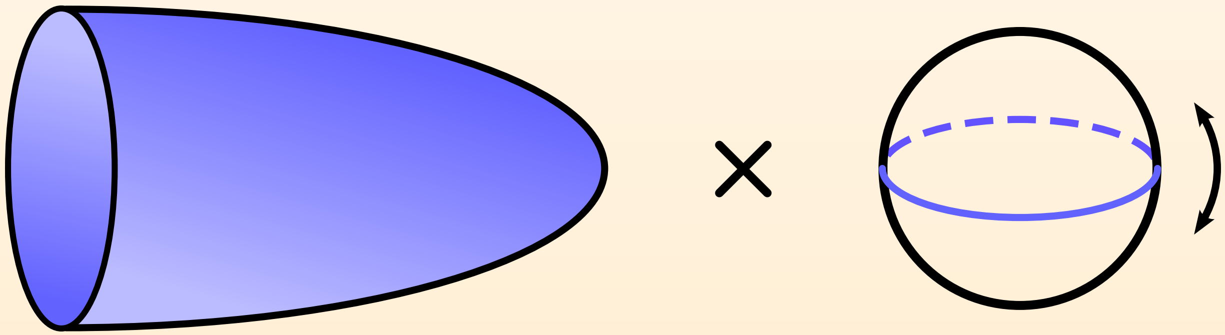
Spontaneous symmetry breaking $\rightarrow$ Goldstone bosons (Mesons)

Light mesons

Babington, J.E., Evans, Guralnik, Kirsch PRD 2004

Meson masses obtained from fluctuations of hypersurface probe D7-brane

in a confining non-supersymmetric ten-dimensional gravity background



π pseudoscalar meson mass: From fluctuations of D-brane

ρ vector meson mass: From fluctuations of gauge field on D-brane

D7 brane probe in gravity backgrounds dual to

confining gauge theories without supersymmetry.

Example:

Constable-Myers background (particular deformation of $AdS_5 \times S^5$ metric)

- The deformation introduces a new scale into the metric.
- In UV limit, geometry returns to $AdS_5 \times S^5$ with D7 probe wrapping $AdS_5 \times S^3$.

1. Start from Dirac-Born-Infeld action for a D7-brane embedded in deformed background

$$S_{\mathrm{D7}} \sim \int d^8\xi \, \sqrt{-\det(P[G] + 2\pi\alpha' F)}$$

2. Derive equations of motion for transverse scalars (w_5, w_6)

3. Solve equations of motion numerically using shooting techniques

   Solution determines embedding of D7-brane (e.g. $w_5 = 0, w_6 = w_6(\rho)$)

4. Meson spectrum:

   Consider fluctuations δw_5, δw_6 around a background solution obtained in 3.

   Solve equations of motion linearized in $\delta w_5, \delta w_6$

Asymptotic behaviour of supergravity solutions

UV asymptotic behaviour of solutions to equation of motion:

$$w_6 \propto m\, e^{-r} + c\, e^{-3r}$$

Identification of the coefficients as in the standard AdS/CFT correspondence:

m quark mass, $c = \langle \bar{q} q \rangle$ quark condensate

Here:

$m \neq 0$: explicit breaking of $U(1)_A$ symmetry

$c \neq 0$: spontaneous breaking of $U(1)_A$ symmetry

The Constable-Myers deformation

$\mathcal{N} = 4$ super Yang-Mills theory deformed by VEV for $tr\, F^{\mu\nu} F_{\mu\nu}$
(R-singlet operator with $D = 4$) $\quad \rightarrow$ non-supersymmetric QCD-like field theory

The Constable-Myers background is given by the metric

$$ds^2 = H^{-1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta/4} dx_4^2 + H^{1/2} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{(2-\delta)/4} \frac{w^4 - b^4}{w^4} \sum_{i=1}^{6} dw_i^2,$$

where

$$H = \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\delta} - 1 \qquad (\Delta^2 + \delta^2 = 10)$$

and the dilaton and four-form

$$e^{2\phi} = e^{2\phi_0} \left(\frac{w^4 + b^4}{w^4 - b^4} \right)^{\Delta}, \qquad C_{(4)} = -\frac{1}{4} H^{-1} dt \wedge dx \wedge dy \wedge dz$$

This background has a singularity at $w = b$

The Constable-Myers deformation

The Constable-Myers background is dual to a confining gauge theory

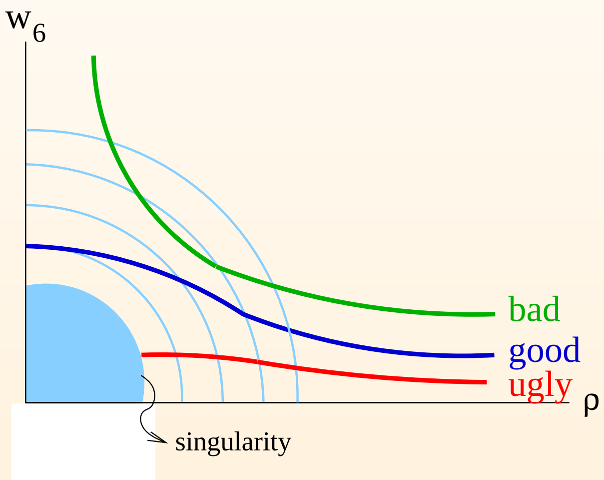
since the Wilson loop displays an area law

Gravity dual of Wilson loop:

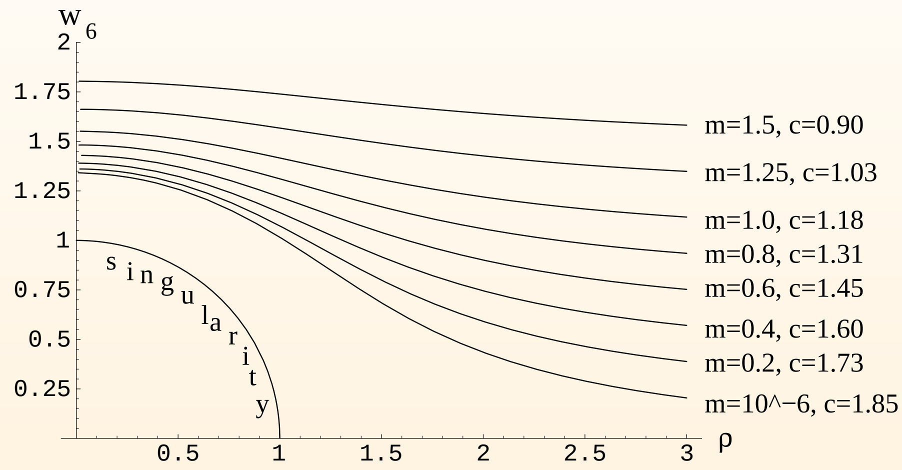
Minimal surface in dual geometry ending on the loop

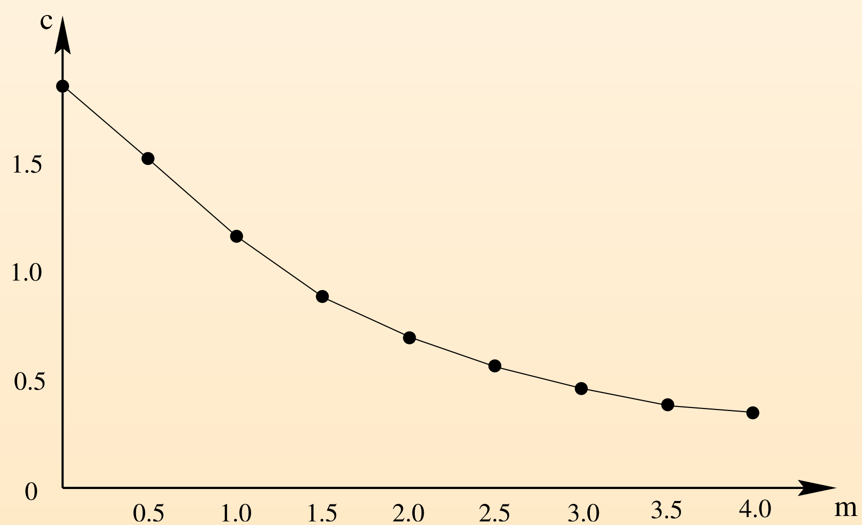
Chiral symmetry breaking

Solution of equation of motion for probe brane



Numerical Result:





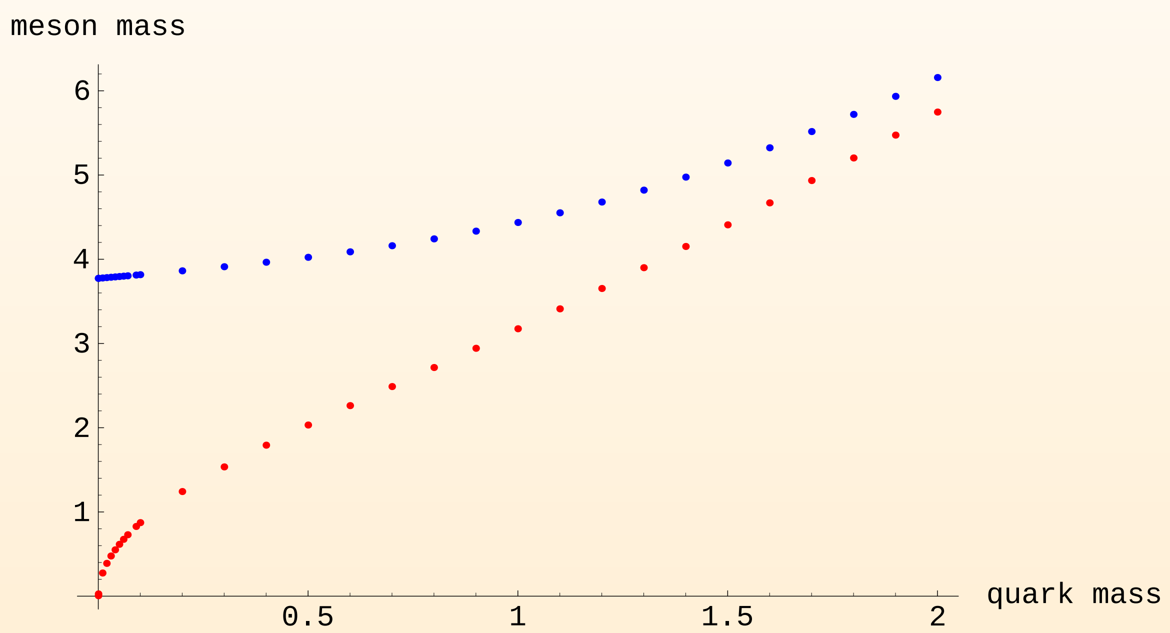
Result:

Screening effect: Regular solutions do not reach the singularity

Spontaneous breaking of $U(1)_A$ symmetry: For $m \to 0$ we have $c \equiv \langle \bar{\psi}\psi \rangle \neq 0$

Meson spectrum

From fluctuations of the probe brane

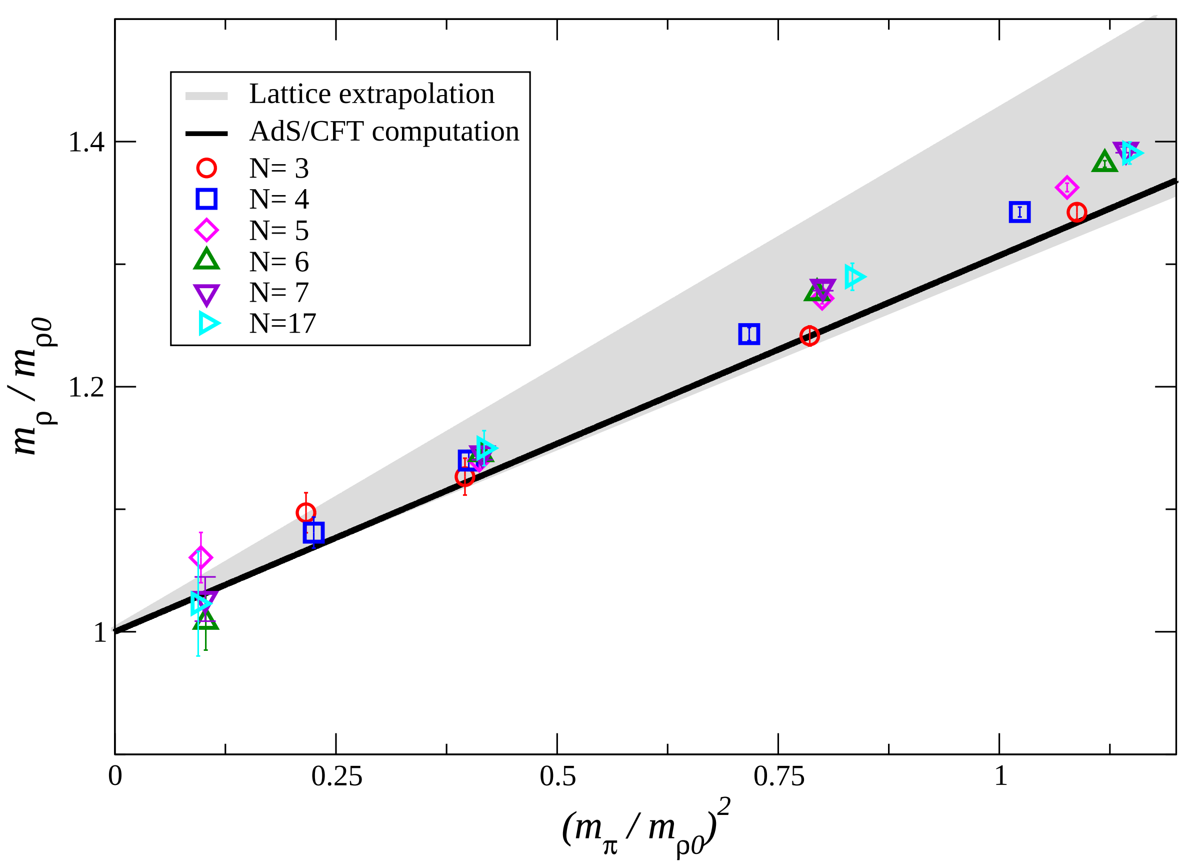


Goldstone boson (η')

Gell-Mann-Oakes-Renner relation: $M_{Meson} \propto \sqrt{m_{Quark}}$

Comparison to lattice gauge theory

Mass of ρ meson as function of π meson mass^2 (for $N \to \infty$)



Comparison to lattice gauge theory

Gauge/Gravity Duality: J.E., Evans, Kirsch, Threlfall '07, review EPJA

Lattice gauge theory: Lucini, Del Debbio, Bali, Panero et al '13

Result Gauge/Gravity Duality:

$$\frac{m_\rho(m_\pi)}{m_\rho(0)} = 1 + 0.307 \left(\frac{m_\pi}{m_\rho(0)}\right)^2$$

Result Lattice Gauge Theory (Bali, Bursa '08): Slope 0.341 ± 0.023

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Why is the agreement so good?

Comparison to lattice gauge theory

D7 probe brane action expanded to quadratic order:

$$S = \tau_7 \mathrm{Vol}(S^3) \mathrm{Tr} \int \mathrm{d}^4 x \mathrm{d}\rho \, \rho^3 \left[\frac{1}{\rho^2 + |X|^2} |DX|^2 + \frac{\Delta m^2 L^2}{\rho^2} |X|^2 + (2\pi\alpha' F)^2 \right]$$

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Phenomenological model:

J.E., Evans, Scott 2014

Metric

$$ds^2 = \frac{L^2 d\rho^2}{\rho^2+|X|^2}+\frac{\rho^2+|X|^2}{L^2}\mathrm{d}x^2$$

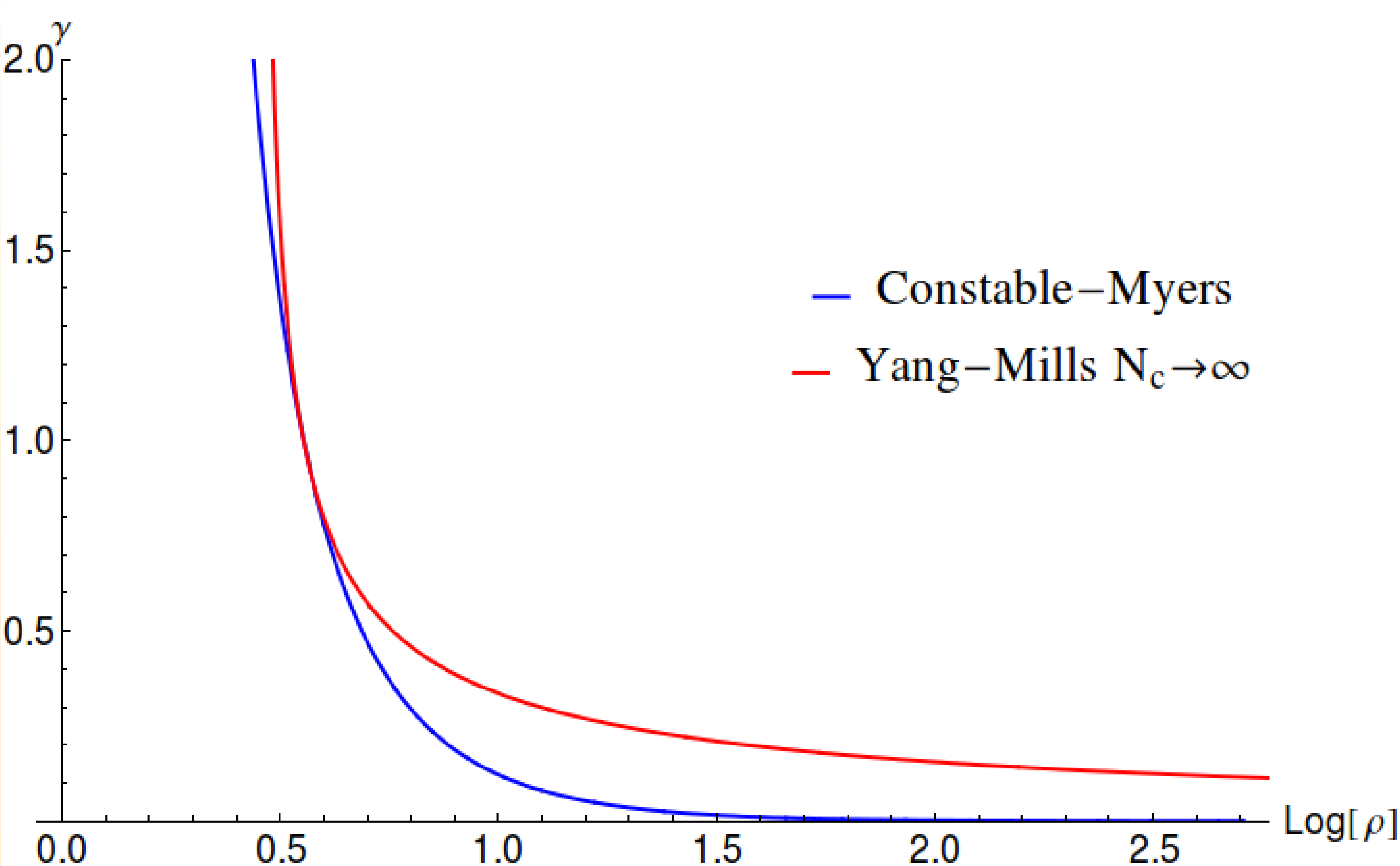
Fluctuations $X = l(\rho)e^{2i\pi^a T^a}$

Make contact with QCD by choosing

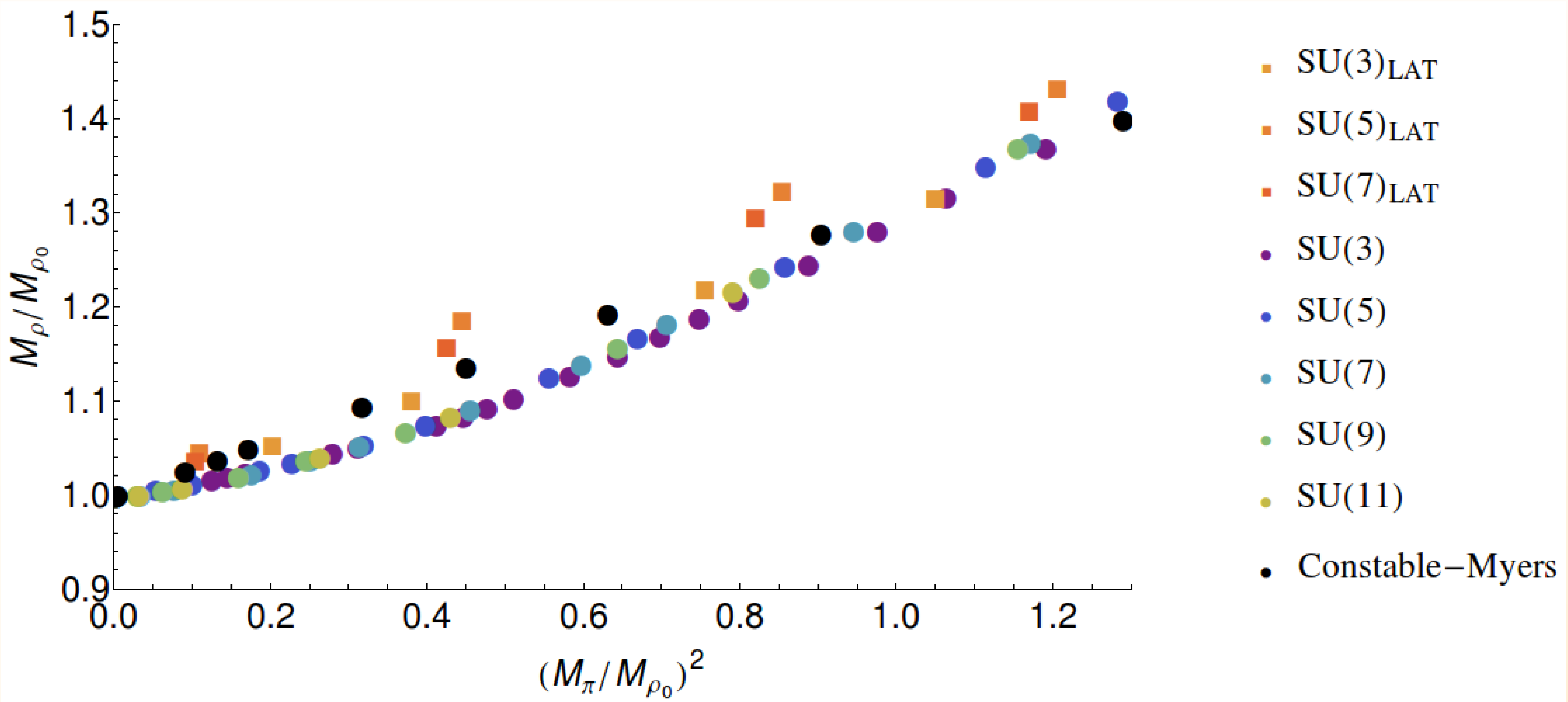
$$\Delta m^2 L^2 = -2\gamma = -\frac{3(N^2-1)}{2N\pi}\alpha$$

Anomalous dimension for gravity theory and for Yang-Mills

J.E., Evans, Scott



ρ meson vs. π meson mass^2



Sakai-Sugimoto model

$D4/D8/\bar{D}8$ brane model Sakai+Sugimoto 12/2004

(cf. work by A. Rebhan)

Chiral symmetry breaking $SU(3) \times SU(3) \rightarrow SU(3)$

$\Rightarrow$ More realistic

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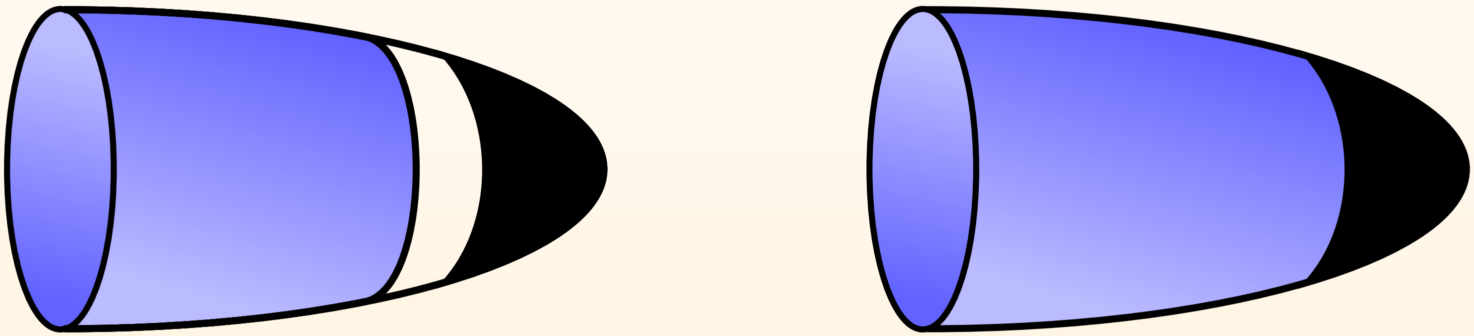
Energy scale is compactification scale of 5th dimension

Quark masses cannot be dialled

FInite Temperature: D7 brane embedding in black hole background



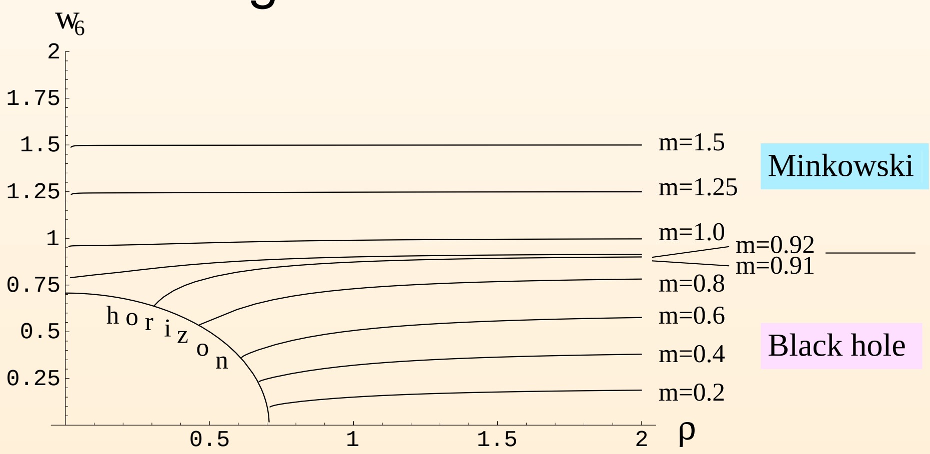
First order phase transition

Babington, J.E., Evans, Guralnik, Kirsch
Mateos, Myers, Thomson

D7 brane embedding in black hole background

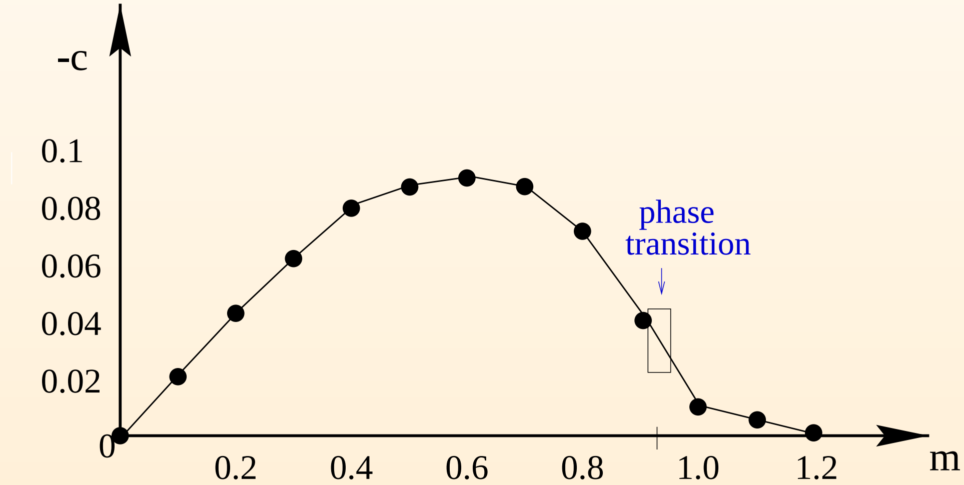
Babington, J.E., Evans, Guralnik, Kirsch 0306018

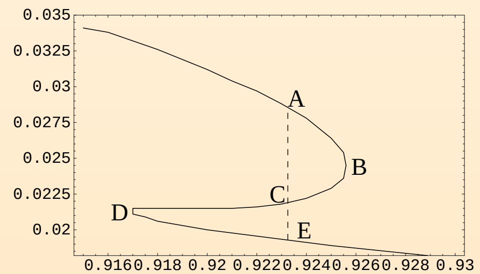
Embeddings



Phase transition at $m_c \approx 0.92$
(1st order)

Condensate $c \equiv \langle \bar{\psi}\psi \rangle$ vs. quark mass m
m in units of T





Kirsch 2004

Masses and decay widths of mesons - Spectral functions

Standard procedure in D3/D7: Mateos, Myers et al 2003

Meson masses calculated from linearized fluctuations of D7 embedding

Fluctuations: $\delta w(x, \rho) = f(\rho) e^{i(\vec{k} \cdot \vec{x} - \omega t)}$, $M^2 = -k^2$

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For black hole embeddings, ω develops negative imaginary part

$\Rightarrow$ damping $\Rightarrow$ decay width

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$\Rightarrow$ damping $\Rightarrow$ decay width

Make contact with hydrodynamics: Starinets, Kovtun ....

Spectral function determined by poles of retarded Green function

Quasinormal modes

Identify mesons with resonances in spectral function

Landsteiner, Hoyos, Montero

Hydrodynamics from AdS/CFT Son, Starinets et al

Transport processes in the quark-gluon plasma

Hydrodynamics from AdS/CFT

Son, Starinets et al

Transport processes in the quark-gluon plasma

Time dependence requires Minkowski signature AdS black hole

Infalling boundary condition required at black hole horizon

Hydrodynamics from AdS/CFT — Son, Starinets et al

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Spectral function from imaginary part of retarded Green function

$$G^R{}_{\mu\nu}(\omega, \boldsymbol{k}) = -i \int d^4x \; e^{i\,\vec{k}\vec{x}}\, \theta(x^0)\, \langle [J_\mu(\vec{x}), J_\nu(0)] \rangle$$

Correlator calculated from propagation through AdS black hole space

Here: J_μ: flavour current dual to gauge field on D7 brane

Spectral function $\Rightarrow$ Resonance spectrum (vector mesons)

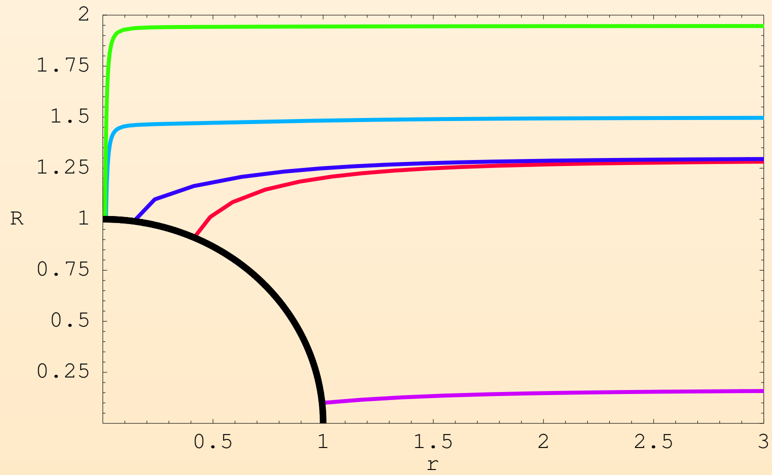
Finite $U(1)$ baryon density

Mateos, Myers, Matsuura et al

Baryon density n_B and $U(1)$ chemical potential μ
from VEV for gauge field time component:

$$\bar{A}_0(\rho) \sim \mu + \frac{\tilde{d}}{\rho^2}, \qquad \tilde{d} = \frac{2^{5/2}}{N_f \sqrt{\lambda} T^3} n_B$$

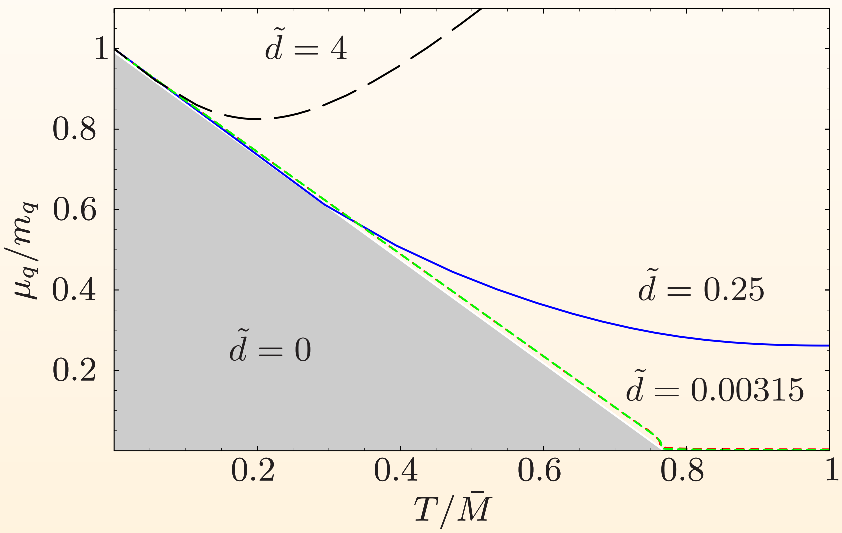
At finite baryon density, all embeddings are black hole embeddings



Phase diagram with finite $U(1)$ baryon density

Phase diagram:

grey region: $n_B = 0$
white region: $n_B \neq 0$



Sin, Yogendran et al; Mateos, Myers et al; Karch, O'Bannon; . . .

Gauge field on D7: background + fluctuations

$$\hat{A}_\mu(\rho, \vec{x}) = \delta_\mu^0 \tilde{A}_0(\rho) + A_\mu(\vec{x}, \rho)$$

Insert this into equations of motion from DBI action of D7 brane

In Fourier space, use gauge invariant quantities

$$E_x = \omega A_x + q A_0, \qquad E_{y,z} = \omega A_{y,z} \qquad (q = 0)$$

Green functions

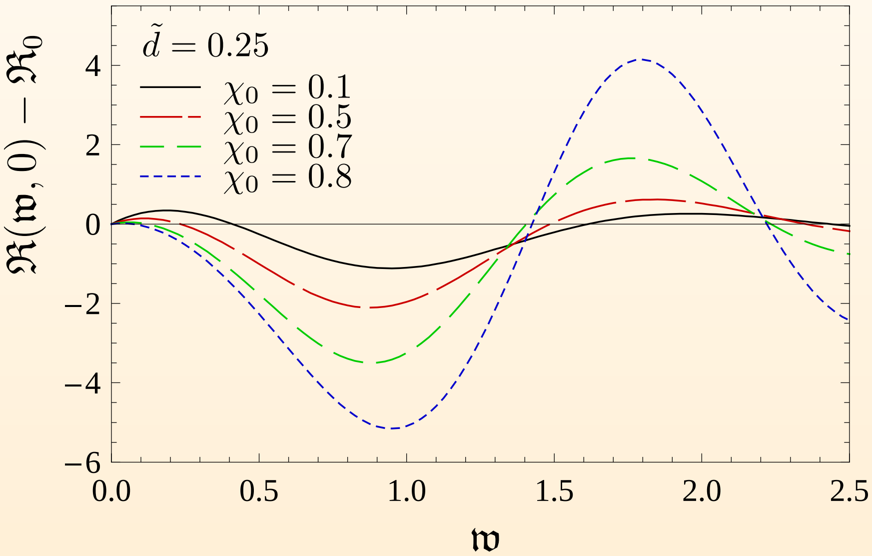
$$G^R = G^R_{xx} = G^R_{yy} = G^R_{zz} = \frac{N_f N_c T^2}{8} \lim_{\rho \to \infty} \left(\rho^3 \frac{\partial_\rho E(\rho)}{E(\rho)} \right)$$

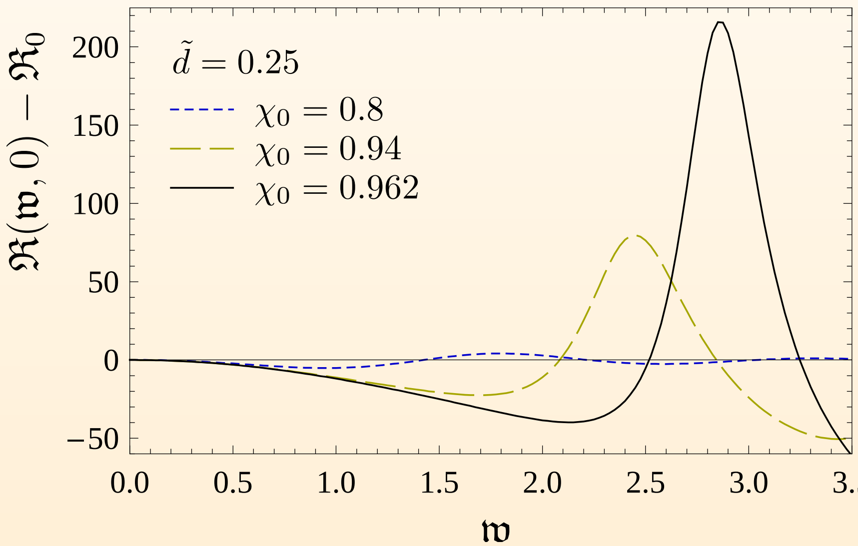
At vanishing density: Myers, Starinets, Thompson

Spectral function for vector mesons with $U(1)$ quark chemical potential

J.E., Kaminski, Rust 0710.0334

Spectral functions - temperature dominated regime

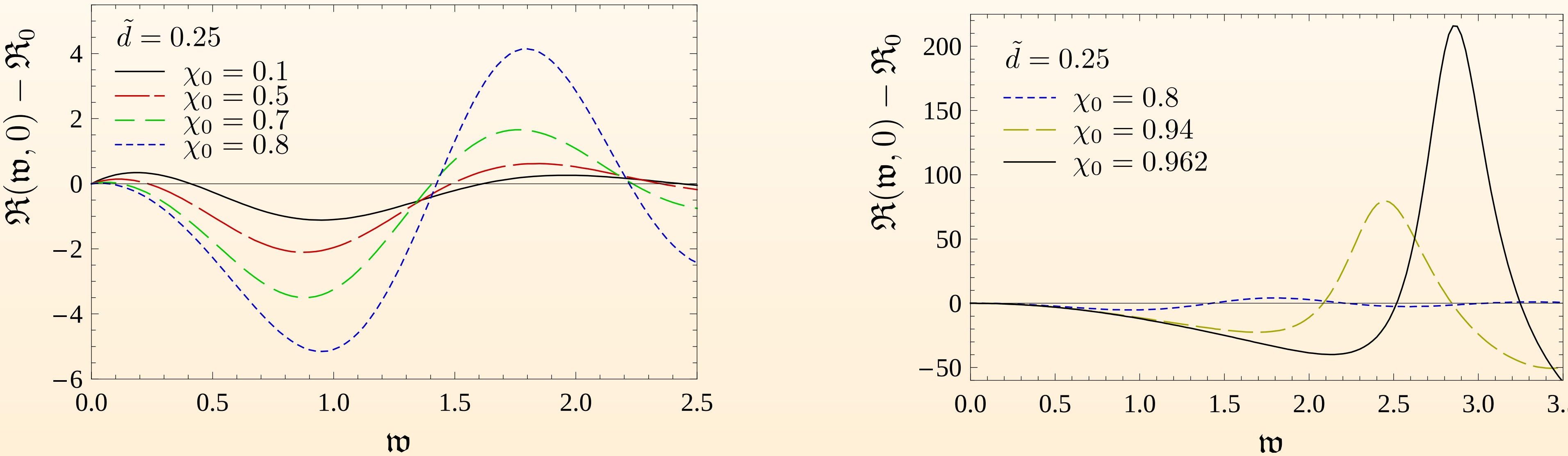




$(\mathfrak{R}_0 = N_f N_c T^2 \pi \mathfrak{w}^2)$

J.E., Kaminski, Rust 0710.0334

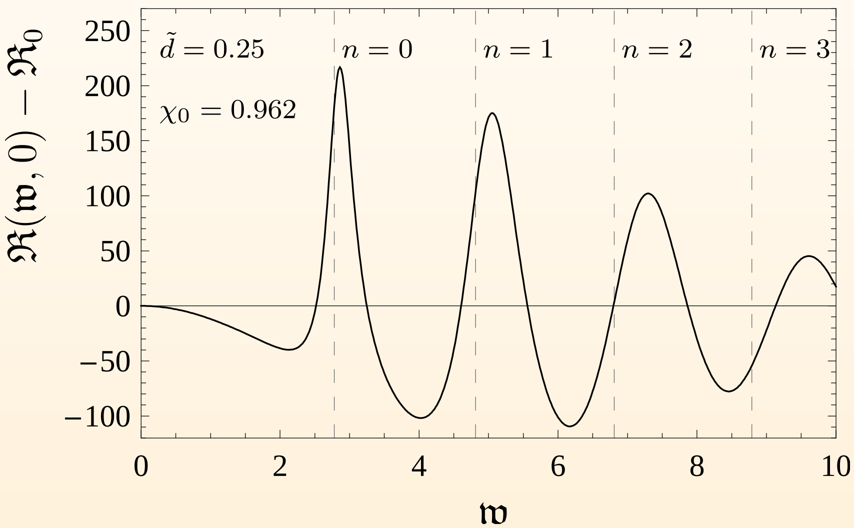
Spectral functions - temperature dominated regime

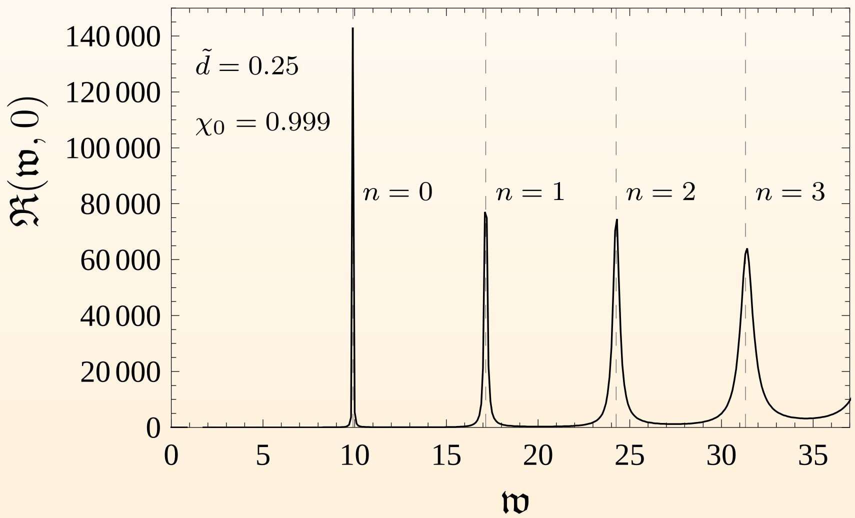


$(\mathfrak{R}_0 = N_f N_c T^2 \, \pi \mathfrak{w}^2)$

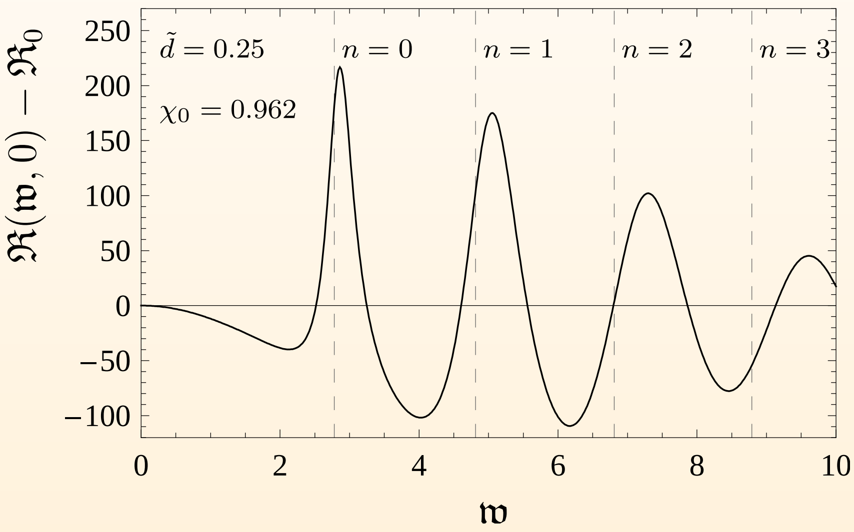
For increasing m/T, peaks first move to smaller, then to larger frequencies

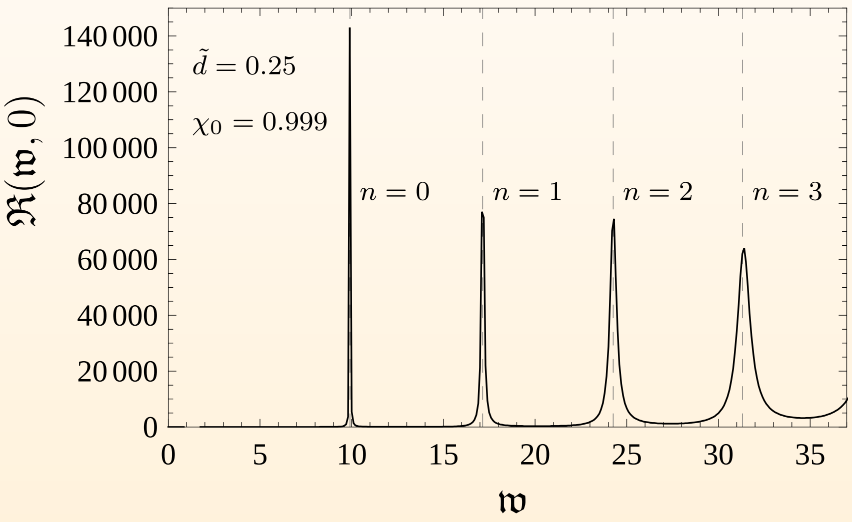
Spectral functions - potential-dominated regime





Spectral functions - potential-dominated regime

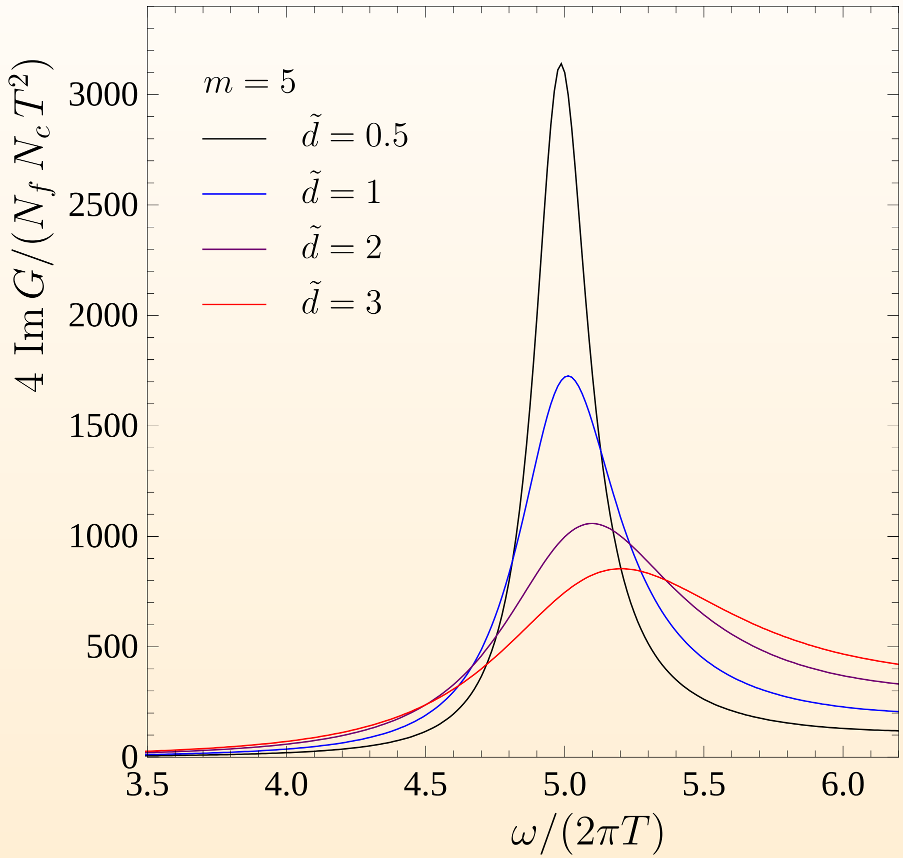




Agrees with supersymmetric meson spectrum

(Calculated analytically by Kruczenski, Mateos, Myers, Winters 0304032)

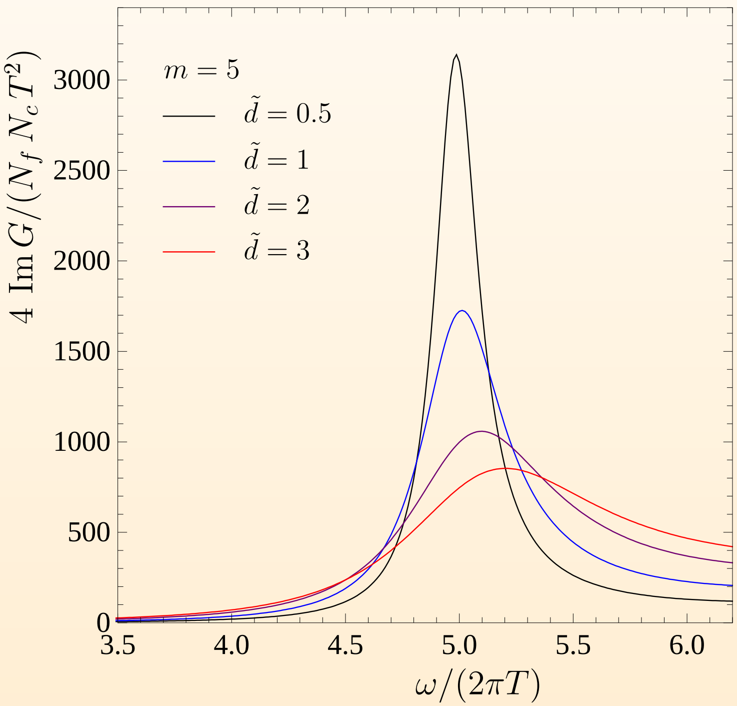
ρ vector meson spectral function in dense hadronic medium



AdS/CFT result (J.E., Kaminski, Kerner, Rust 2008)

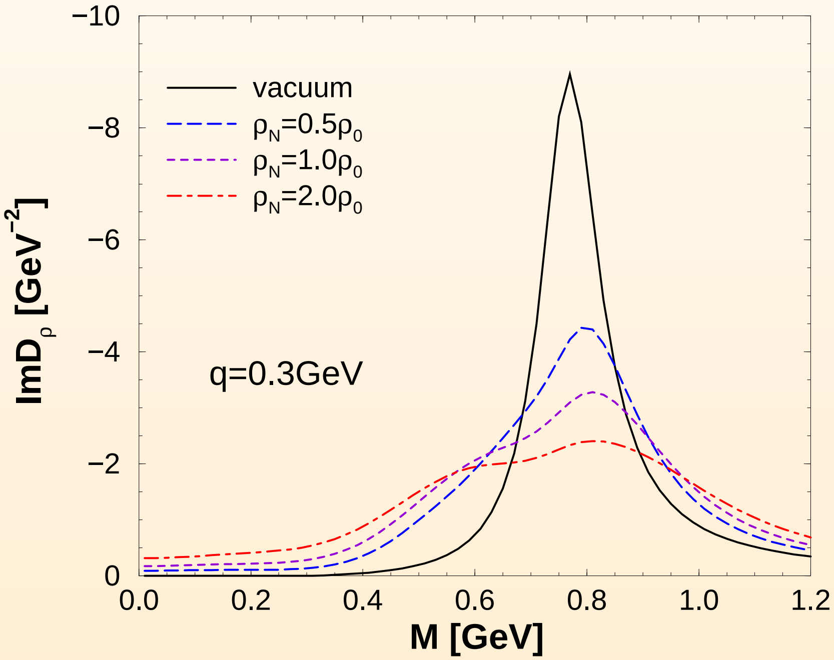
Spectral function at finite baryon density

ρ vector meson spectral function in dense hadronic medium



AdS/CFT result

(J.E., Kaminski, Kerner, Rust 2008)



Field theory (Rapp, Wambach 2000)

Brower, Polchinski, Strassler, Tan JHEP 0712 (2007) 005

Pomeron:

Coherent color-singlet excitation in high-energy hadronic scattering

At large s, small t, large N

it contributes the leading singularity in the angular momentum plane

Pomeron in AdS/CFT: (large N)

Calculation of field theory amplitude from string amplitude in ten-dimensional $AdS_5 \times S^5$ space with cut-off

Four-dimensional scattering given by coherent sum over scattering in the six transverse dimensions

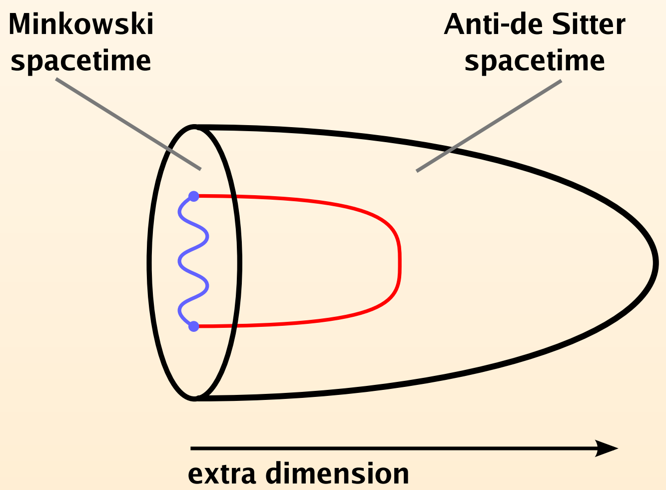
Hard scattering

Holographic encoding of gauge theory physics:

Low energy states at small r, high energy states at large r (near boundary)

Warped space:

$$ds^2 = \frac{r^2}{L^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{L^2}{r^2}dr^2 + L^2 ds_X^2$$



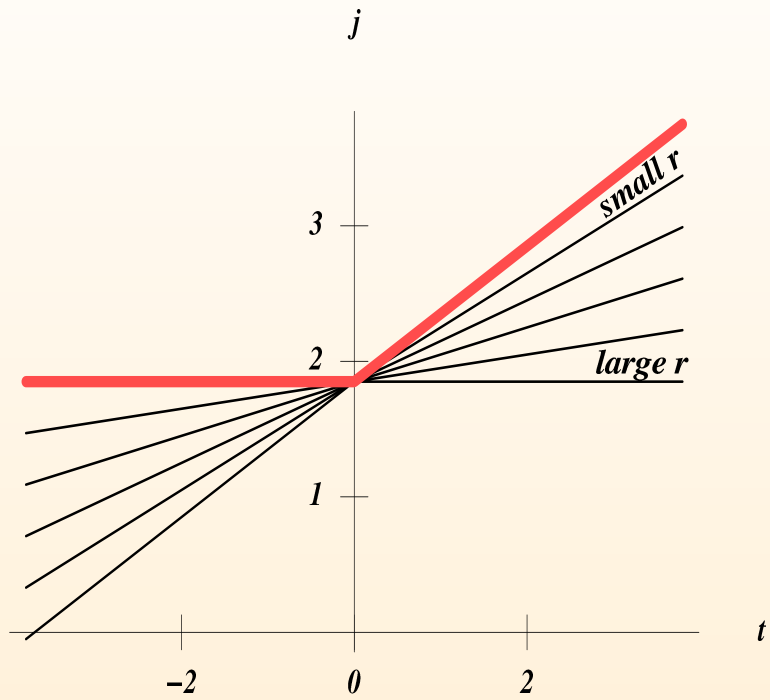
$$p_\mu = \frac{r}{L}\tilde{p}_\mu$$

$$\mathcal{A}(s,t) \propto s^{\alpha(t,r)}$$

p_μ conserved momentum, corresponding to invariance under translation of x^μ

$\tilde{p}_\mu$ momentum in local inertial coordinates for momenta localized at r

Pomeron in gauge/gravity duality



At large s, highest trajectory will dominate:

t positive: r small: soft (Regge) pomeron, properties determined by confining dynamics: glueball

t negative: r large: hard (BFKL) pomeron, two-gluon perturbative small object

Recent refinements

Ballon-Bayona, R. Quevedo, Costa 1704.08280

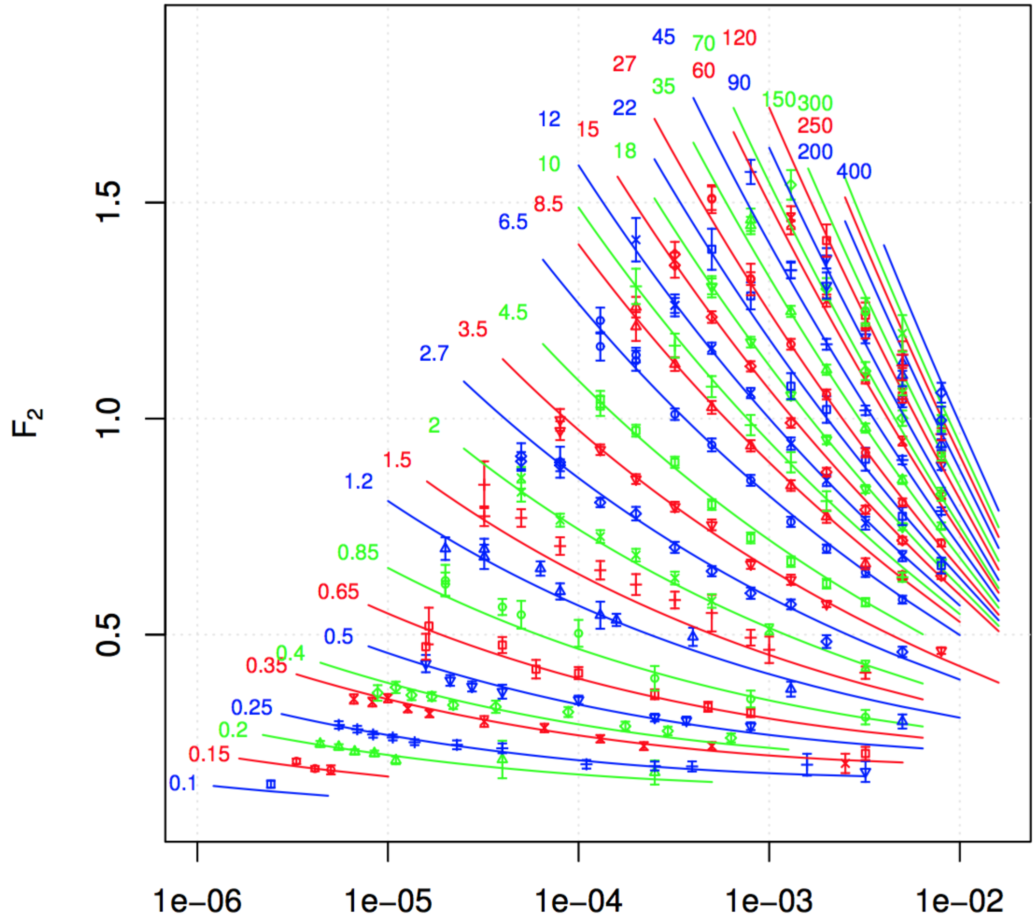
Exchange of higher-spin fields in the graviton Regge trajectory
dual to glueball states of twist two

First four pomeron trajectories are considered; fit to HERA data

$x < 0.01$

$0.1 < Q^2 < 400$ in GeV^2

$\chi^2_{\text{d. o. f.}} = 1.7$



Example: Froissart bound

High-energy behaviour of total cross-sections in two-particle scattering

Heisenberg 1952:

A target hadron is surrounded by a pion field with energy density $\propto e^{-m_\pi r}$

Inelastic processes will occur when the collision is close enough to locally yield enough energy to create a pion pair

$\Rightarrow$

$$\sigma \propto \frac{1}{m_\pi^2} \ln^2 \frac{E}{m_\pi}$$

Example: Froissart bound

Froissart 1961:

At high energies, the total cross-section for two-particle scattering (protons) has an upper bound

$$\sigma \propto \ln^2 \frac{s}{s_0}$$

s centre-of-mass energy, s_0 energy scale

General argument based on unitarity of S matrix and
analyticity properties of the scattering amplitude

Example: Froissart bound

Froissart 1961:

At high energies, the total cross-section for two-particle scattering (protons) has an upper bound

$$\sigma \propto \ln^2 \frac{s}{s_0}$$

s centre-of-mass energy, s_0 energy scale

General argument based on unitarity of S matrix and analyticity properties of the scattering amplitude

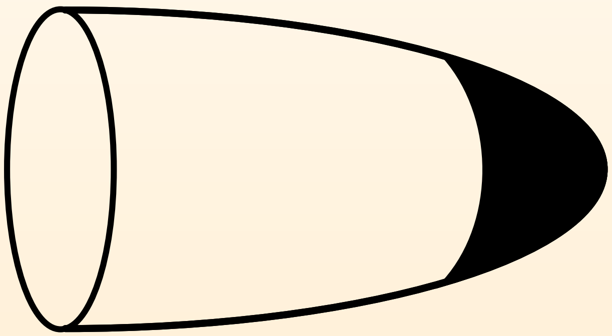
QCD considerations link the Froissart bound at high energies to the dynamics of ultra-soft gluons (strongly coupled)

Example: Froissart bound in gauge/gravity duality

Giddings Phys.Rev. D67 (2003) 126001; Kang, Nastase Phys.Rev. D72 (2005) 106003

AdS metric with IR cutoff ('hard wall'), point mass m is placed on this IR wall

This creates perturbations of the AdS space which may lead to the formation of a black hole in AdS space



Geometrical cross section of this black hole $\Leftrightarrow$
maximum possible scattering cross section in the field theory

$$\sigma \leq \sigma_{\rm BH} = \pi r_h^2 \propto \ln^2 \frac{E}{E_0}$$

Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

Subleading corrections $\propto -\ln(s/s_0)$ and $\propto \ln s/s_0 \ln \ln s/s_0$,
from higher curvature corrections

improve fits to cosmic ray and LHC data

Subleading corrections to Froissart bound from AdS black holes

Diez, Godbole, SInha, Phys.Lett. B746 (2015) 285

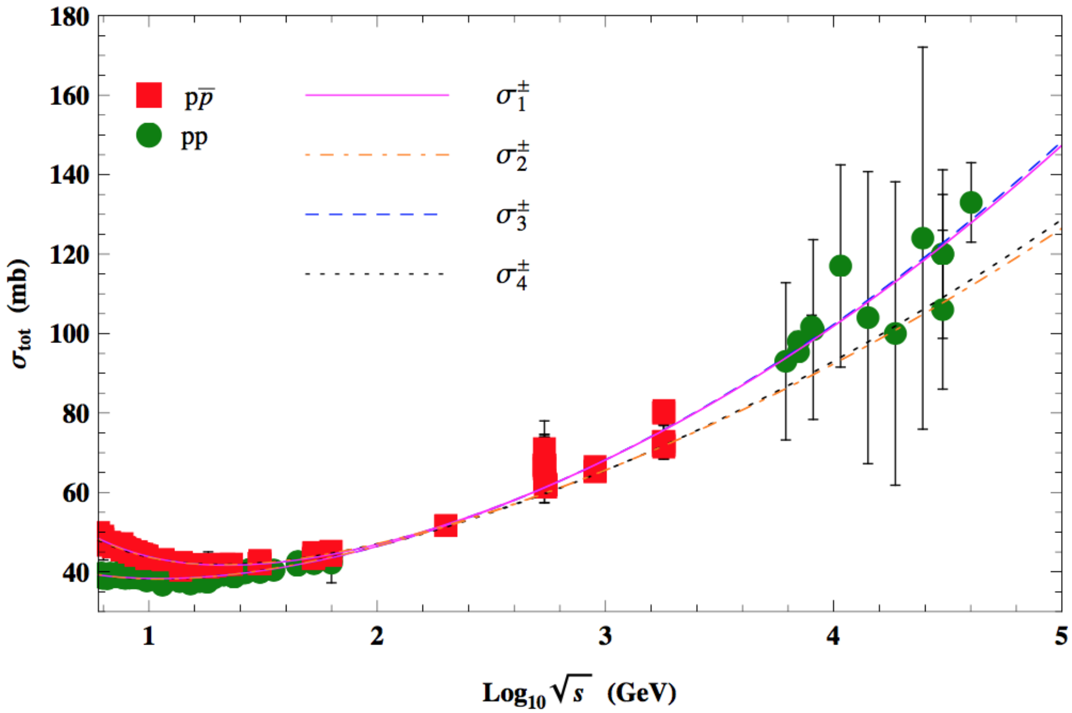
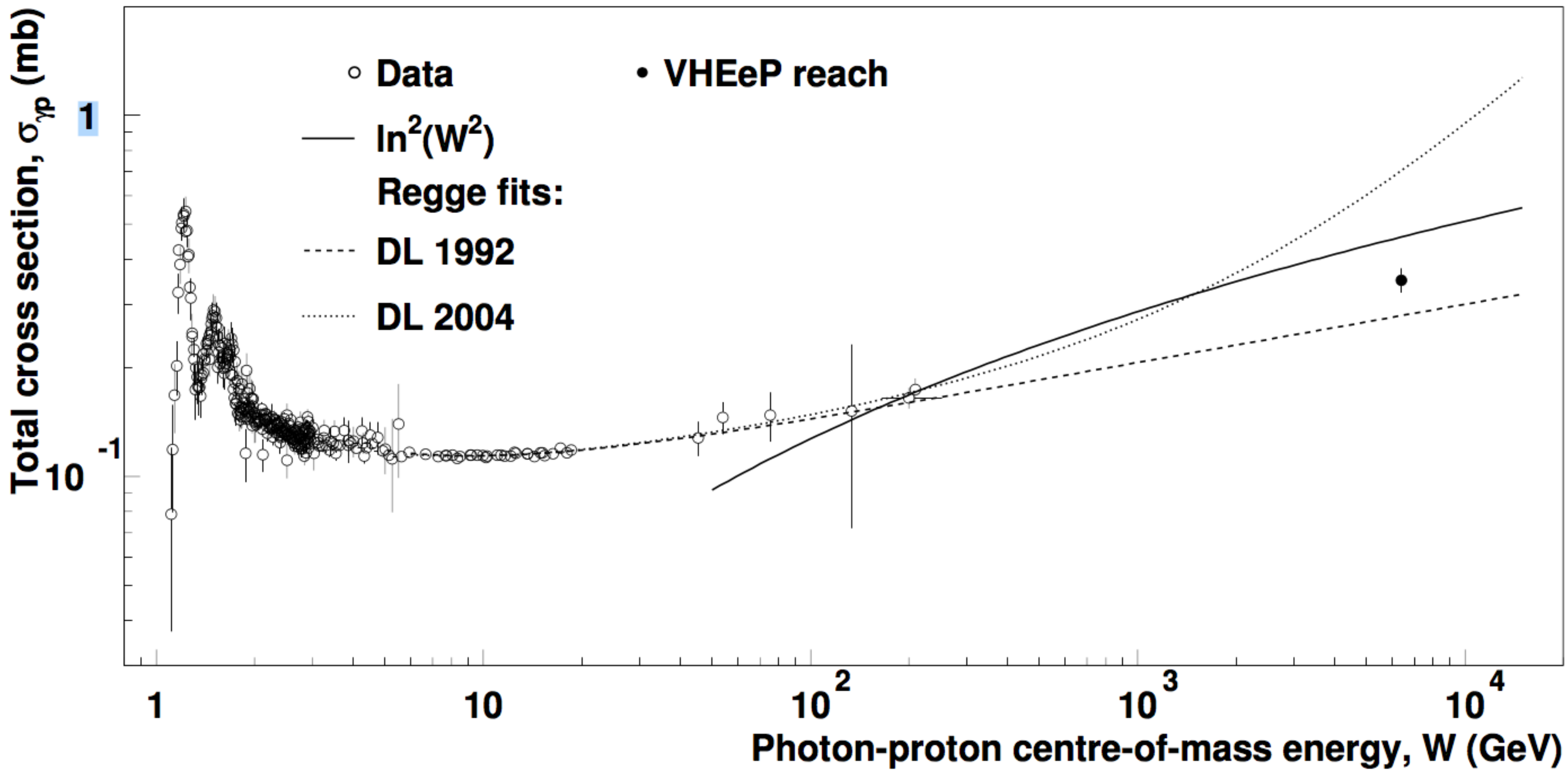


FIG. 1: (Colour online.) Fit results to experimental values of σ_{tot}^{pp} and $\sigma_{tot}^{p\bar{p}}$. The magenta solid, orange dot-dashed, blue dashed and black dotted curves are the (57)-(60) fits to the pp (green circles) and $p\bar{p}$ (red squares) data points, respectively. The data are from CDF, E710, E811, UA1, UA4, UA5 experiments [35–42]. The pp data points also include σ_{tot}^{pp} results from the LHC (at $\sqrt{s} = 7, 8$ TeV) [43–45] and cosmic-ray data [46].

Total γp cross section

Caldwell, Wing Eur.Phys.J. C76 (2016) 463



Conclusions and outlook

- Gauge/gravity duality :

  New duality between quantum field theory and gravity

- New approach for describing strongly coupled systems

- Application examples:

  Transport, hydrodynamics, non-equilibrium, mesons, glueballs, Wilson loops, deep inelastic scattering ...

- Much more to explore!