Flux periodicity in higher order topological superconductors

Sumathi Rao
Harish-chandra Research Institute, Allahabad
EDYTOP, ICTS, Bangalore
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Work in progress with

Suman Jyoti De (HRI) and

Udit Khanna (Tel Aviv University and Weizmann Institute)

Plan of the talk

- Brief introduction to higher order topological insulators and superconductors
- Flux periodicity of superconducting rings
- Our work and results

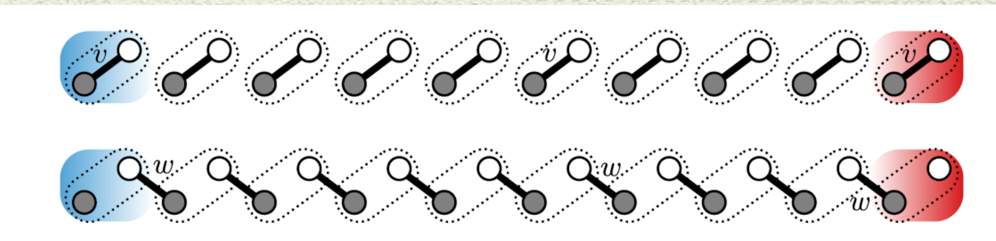
What are topological insulators?

- They are insulators which have some property to which an integer can be assigned which depends only on global properties and cannot be destroyed by impurities or disorder - topological protection

- At boundaries between different topological phases, can have gapless edge or surface states

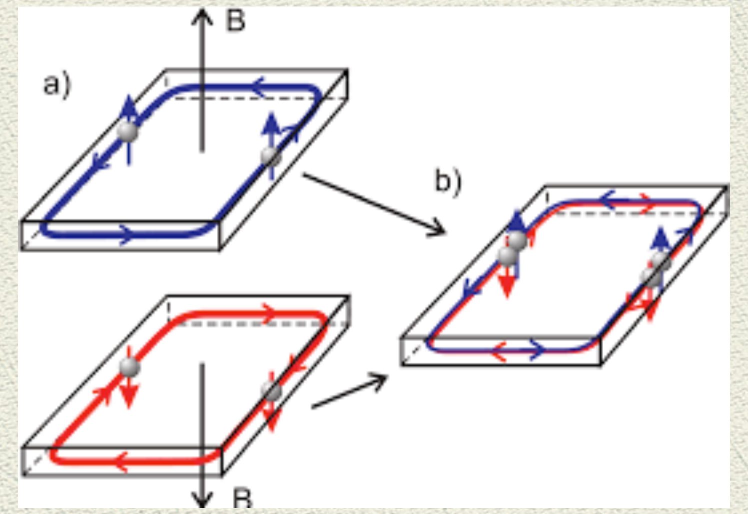
- Standard picture for topological insulators - gapped d-dimensional bulk and gapless d-1 dimensional edge states

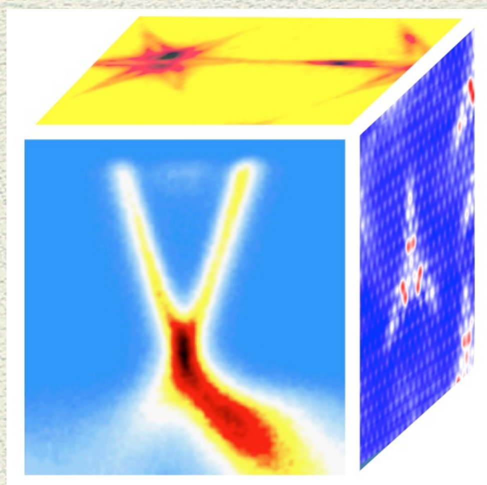
- Simplest example is the Su-Schrieffer- Heeger model in one dimension (d=1)



- Two distinct phases, one of which has edge states (d-1=1-1=0 dimensional modes) and one of which does not (non-topological phase)

- Quantum Hall effect and quantum spin Hall effect in two dimensions (d=2) with (d-1 =1) dimensional edge states

- 



- Topological insulators in three dimensions (d=3) with 3-1 = 2 dimensional gapless surface states

Higher order topological insulators

- d- dimensional gapped bulk with (d-n) dimensional gapless edges - n^{th} order topological insulator

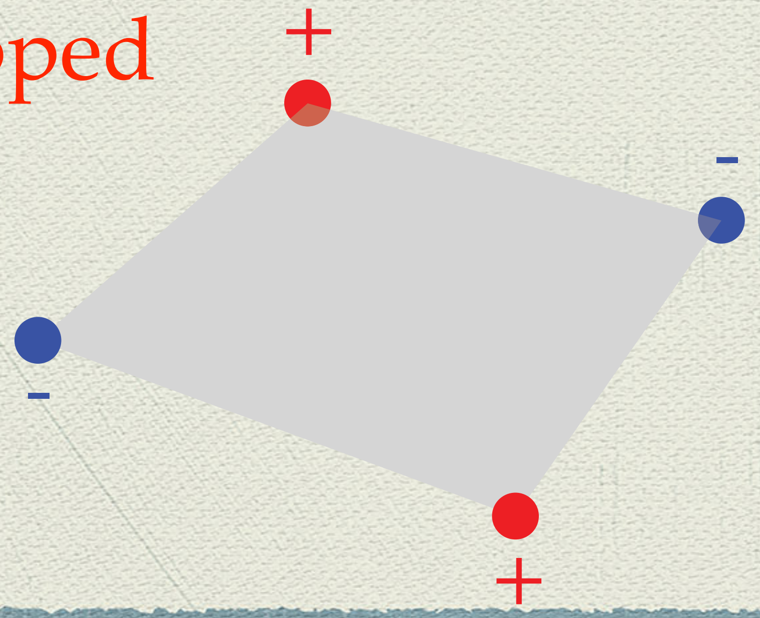
Benalcazar, Bernevig, Hughes, Science, 2017; PRB, 2017

Schindler *et al*, Science Advances, 2018

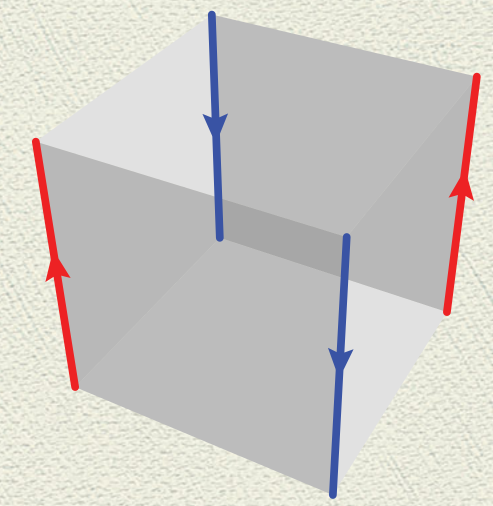
Langbehn *et al*, PRL, 2017

Zhida *et al*, PRL, 2017

- Examples - two dimensional lattices with zero dimensional gapless corner states - edge states gapped



- Three dimensional lattices with hinge states



- Here surfaces are gapped, only states localised on the hinges are gapless

- So even if band structure of material appears to be topologically trivial at first order, it can still have non-trivial topology

- In a certain sense, the bulk is gapped, and edges are also gapped, but are themselves topological, so that they have edge states - 2nd order topology

- Needs crystalline symmetries in the bulk to exist

- Recent claim that Bismuth is a higher order topological insulator
- Not surface of the crystal, but the hinges of the crystal host topologically protected conducting states

Schindler *et al*, Nat. Phys., 2018

Higher order topological superconductors

- Generalise to Majorana modes at corners of two dimensional topological superconductors and chiral hinge Majorana modes at edges of three dimensional topological superconductors

- Start with a toy model - Hamiltonian in terms of Majoranas similar to Kitaev model written in terms of Majoranas

`Usual’ topological superconductors

Brief introduction to `usual' topological superconductors and Majorana modes

- Simplest case is to study a toy model proposed by Kitaev

$$H = -\mu \sum_{x=1}^{N} c_x^\dagger c_x - \frac{1}{2} \sum_{x=1}^{N} (t c_x^\dagger c_{x+1} + \Delta c_x c_{x+1} + h.c.)$$

- Here μ is the chemical potential, t is hopping and Δ is the pairing term

- Can rewrite model as follows

$$c_x = \frac{1}{2}(\gamma_{A,x} + i\gamma_{B,x})$$

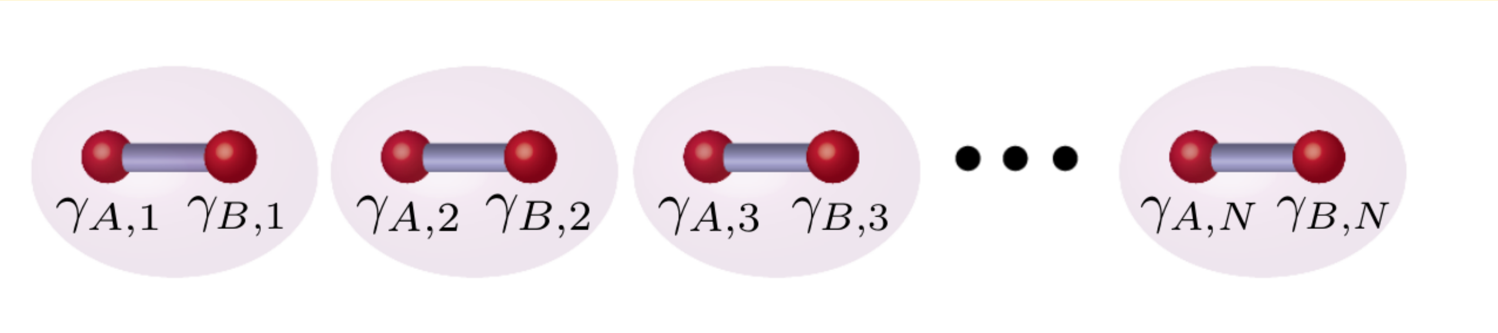
$$c_x^\dagger = \frac{1}{2}(\gamma_{A,x} - i\gamma_{B,x})$$

-

- Here $\gamma_{A,x}$ and $\gamma_{B,x}$ are Majorana modes
- They anti-commute like fermions, and are hermitian $\gamma_{A,x} = \gamma_{A,x}^\dagger$ and satisfy $\gamma_{A,x}^2 = \gamma_{B,x}^2 = 1$ whereas the fermion operators satisfy $c_x^2 = 0$
- Pairs of Majoranas form normal fermions

- One limit : When $\mu < 0$ and $t = \Delta = 0$
- Only bonds between Majoranas at same x

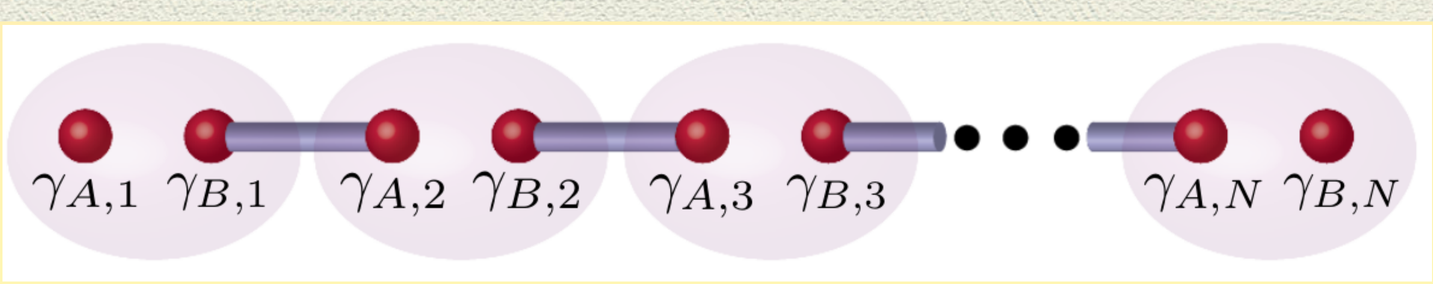
$$H = -\frac{\mu}{2}\sum_{x=1}^{N} c_x^\dagger c_x = -\frac{\mu}{2}\sum_{x=1}^{N}(1 + i\gamma_{A,x}\gamma_{B,x})$$

- Ground state is unique and ends of the chain do not play any special role
- 

- Second limit : When $t = \Delta \neq 0$ and $\mu = 0$

$$H = -\frac{t}{2}\sum_{x=1}^{N}(c_x^\dagger c_{x+1} + c_x c_{x+1} + h.c.)$$

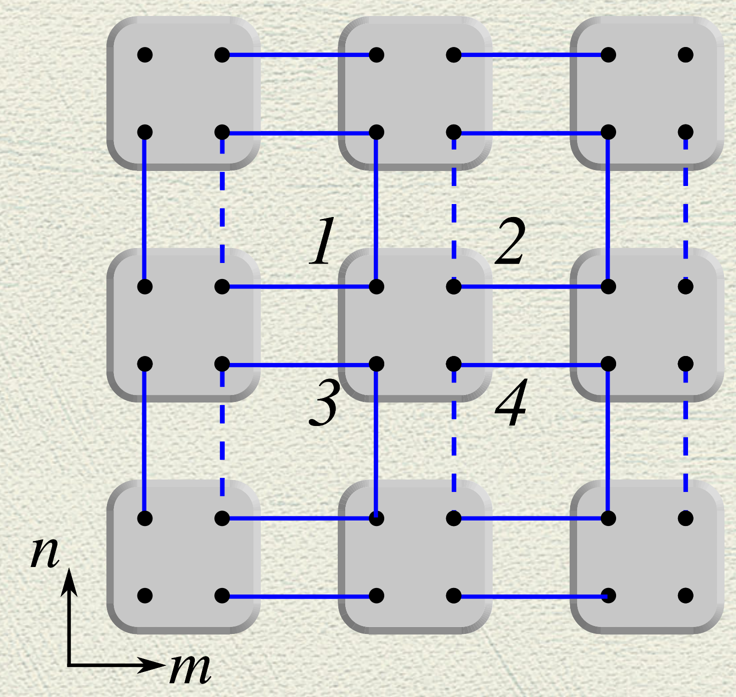
$$= -i\frac{t}{2}\sum_{x=1}^{N-1}\gamma_{B,x}\gamma_{A,x+1}$$



- Here bonds are between Majoranas at adjacent sites, but unpaired Majoranas $\gamma_{A,1}$ and $\gamma_{B,N}$ at the two ends

- Can make a non-local fermion from these two Majoranas - double degeneracy depending on whether or not the state is occupied
- Main point - when the model is written in terms of Majoranas, very easy to identify the end states

Back to higher order topological superconductors

- 

Wang, Lin, Hughes, 2018

$$H = -2it\sum_{m,n}[\gamma^2_{m,n}\gamma^1_{m+1,n} + \gamma^4_{m,n}\gamma^3_{m+1,n} - \gamma^2_{m,n}\gamma^4_{m,n+1} + \gamma^1_{m,n}\gamma^3_{m,n+1}]$$

- Each unit cell has 4 Majoranas
- Hoppings chosen so that one Majorana is left out at each corner - corner Majorana modes
- Pi flux in each plaquette because of change in sign in one hopping

- Pi flux needed to ensure quantisation of the quadrupole moment, which is the essential physics of higher order topological insulators in two dimensions

- Edges are like Kitaev chains with Majoranas at the end (but with a common edge Majorana at corner, not two with one each coming from the x and y edges)

- Here Majorana corner modes by construction
- Choice of grouping the Majoranas to make two complex fermions decides how Hamiltonian splits into normal state band and superconducting pairing gaps.
- So try to choose groupings that give gapless normal bands and pairing terms describing intrinsic superconducting tendency

$$H = \int d\mathbf{k}\ \psi_k^\dagger \mathcal{H}(\mathbf{k}) \psi_k \quad \text{with} \quad \psi_k = (c_k, c_{-k}^\dagger)^T \quad \text{and}$$

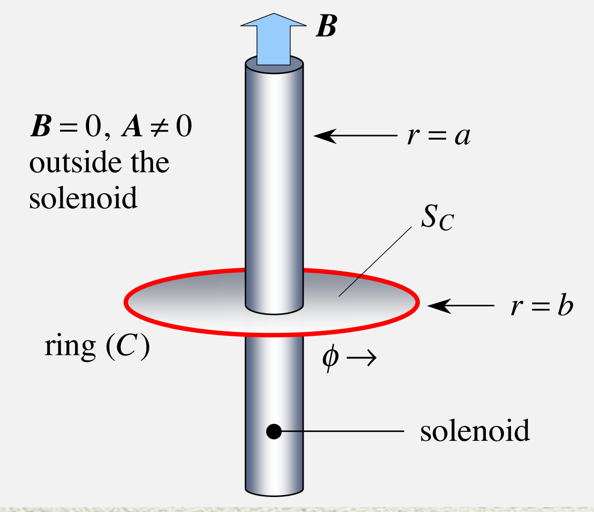
$$\mathcal{H}(\mathbf{k}) = t(\cos k_x \sigma_x \tau_z + \cos k_y 1 + \sin k_x \sigma_x \tau_y + \sin k_y \sigma_x \tau_x)$$

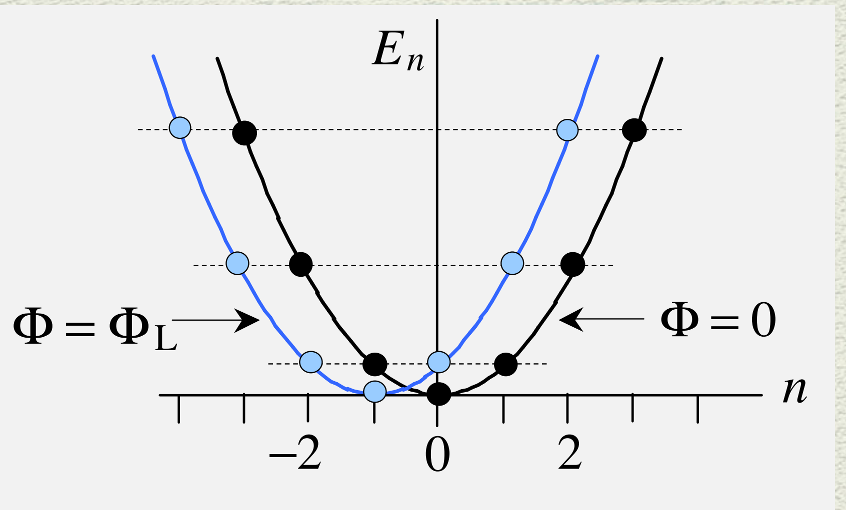
- Can check that this Hamiltonian is the same as the earlier one - has corner Majorana modes by construction

- Can show that the model continues to have corner modes even after perturbations are added provided particle-hole symmetry and some mirror symmetries are respected

Flux periodicity in superconductors

Aharonov-Bohm periodicity in metal rings





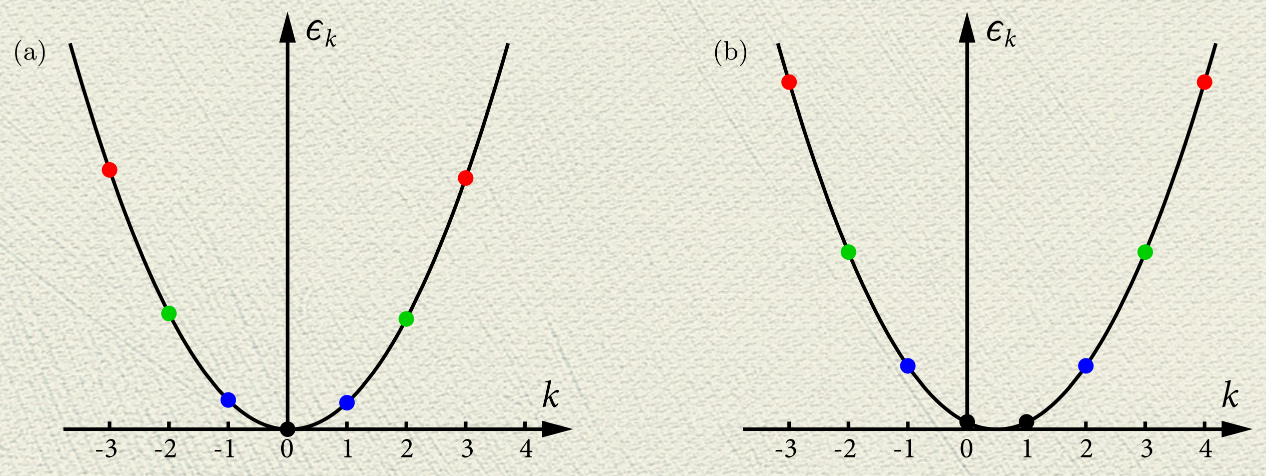
- Current through thin metal ring oscillates with periodicity $\phi_L = \phi_0 = hc/e$
- From $E_n = \dfrac{\hbar^2}{2mb^2}(n + \phi/\phi_0)^2$ and $I = -\dfrac{\partial E_n}{\partial \phi}$

Periodicity in superconducting ring

- But flux periodicity in superconducting rings was seen to be $hc/2e$
- Naively factor of $2e$ due to Cooper pairs carrying twice the charge
- Not correct, because Cooper pairs not tightly bound and not clear whether they traverse the ring separately - need better argument

- Two classes of superconducting wave-functions

- Pairing of electrons with momenta k and -k leading to condensate with q = 0 for $\phi = 0$ and with momenta k and -k+1 leading to condensate with q=1 for $\phi = \phi_0/2$



- Byers and Yang, Onsager

- But this simple picture not always true

- s-wave superconducting rings with diameter smaller than coherence length shows hc/e periodicity

  Loder *et al*, 2007

- High temperature d -wave superconducting rings shows hc/e periodicity

  Loder *et al*, 2007

- Topological superconducting rings have hc/e periodicity

  Liu,Cole,Sau, 2019

Our work and results

Model and symmetries

- We start with a four band model defined by

$$\mathcal{H}(\mathbf{k}) = (b_x + \lambda\cos(k_x))\tau_z\sigma_x + \lambda\cos(k_y)1_\tau\sigma_y$$
$$+\Delta\sin(k_x)\tau_y\sigma_x + \Delta\sin(k_y)\tau_x\sigma_x,$$

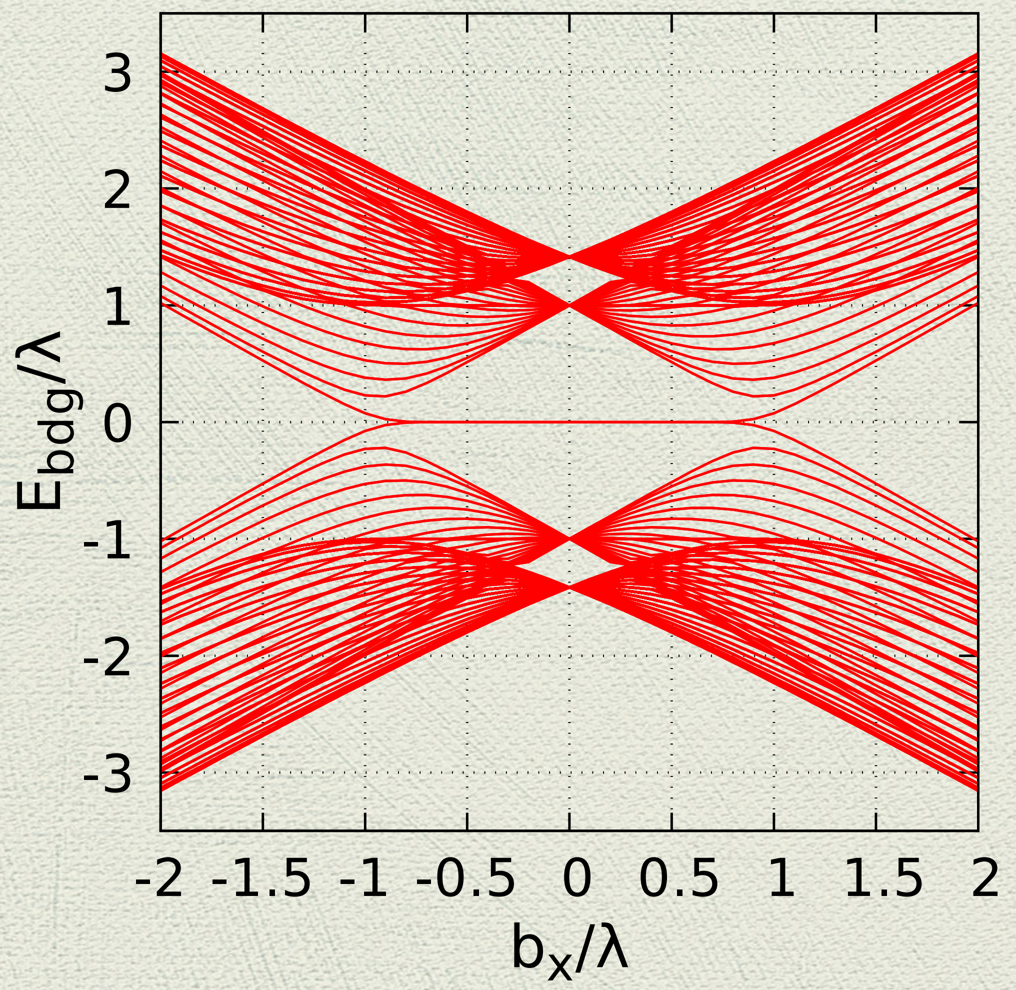
- σ, τ denote operators in spin/Nambu space

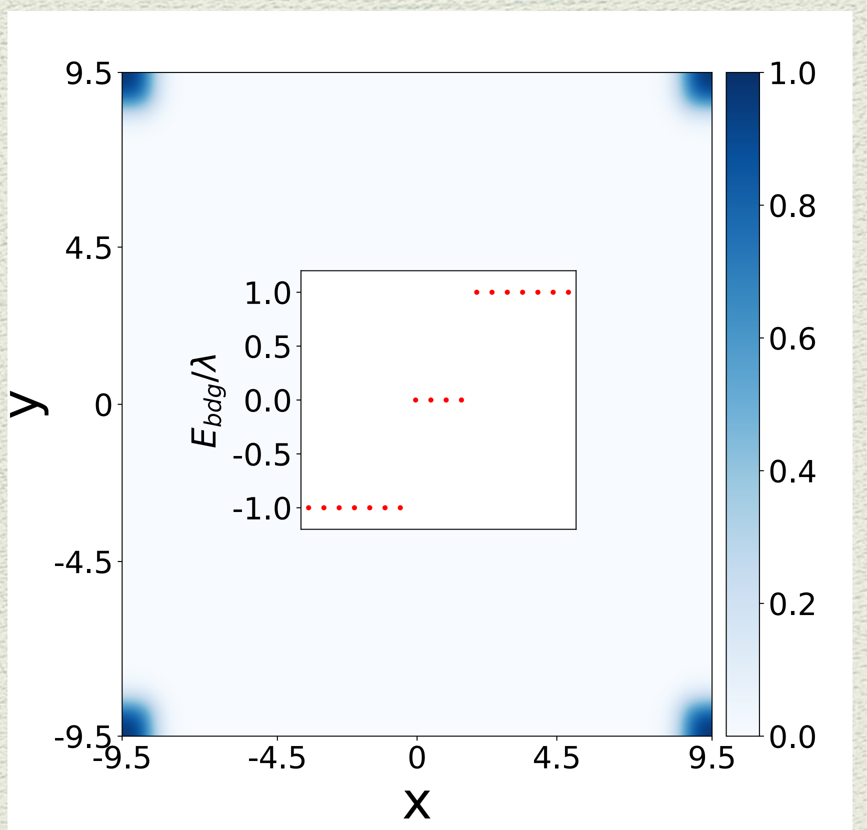
- Particle-hole symmetry $\tau_x\mathcal{H}^T(-k)\tau_x^{-1} = -\mathcal{H}(k)$
  and mirror symmetries $\mathcal{M}_{x,y}\mathcal{H}(\mathbf{k})\mathcal{M}_{x,y}^{-1} = \mathcal{H}(\hat{m}_{x,y}\mathbf{k})$
  Mirror symmetries anti-commute $\{\mathcal{M}_x, \mathcal{M}_y\} = 0$

- For $b_x = 0$ and $\lambda = \Delta = t$, this reduces to the earlier model. In fact, can be shown to be topological for $|b_x| < \lambda$
- Normal state of this model, $\Delta = 0$ corresponds to 2 dimensional Dirac metal with 4 mirror symmetric Dirac nodes
- Can then include pairing

- Non-commuting mirror symmetries which are needed to generate higher order topology, only possible for $p_x + ip_y$ superconductors since superconducting gap has to transform non-trivially in both directions.
- Will not work for other possible pairings such as s, d, p_x, p_y

Spectrum

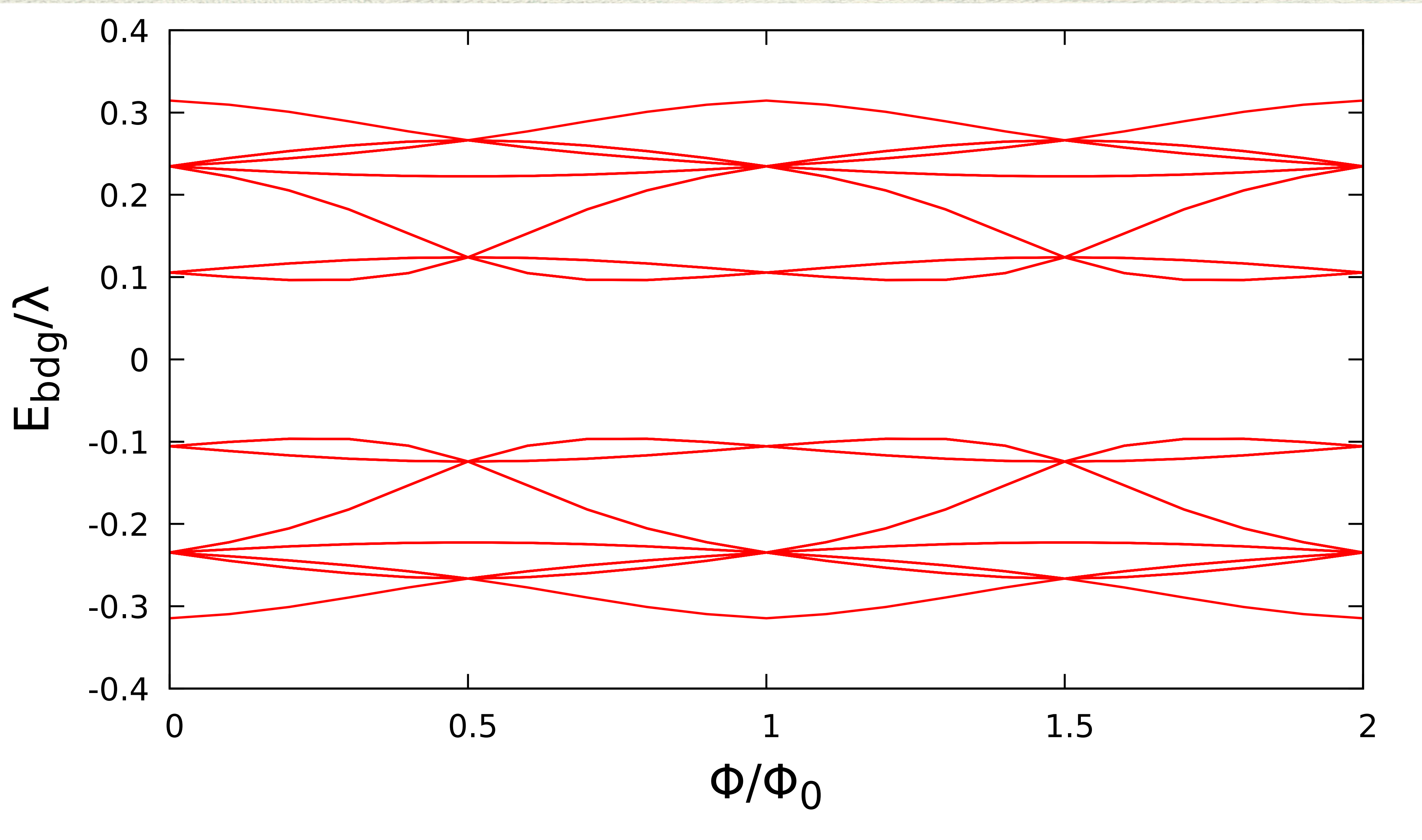




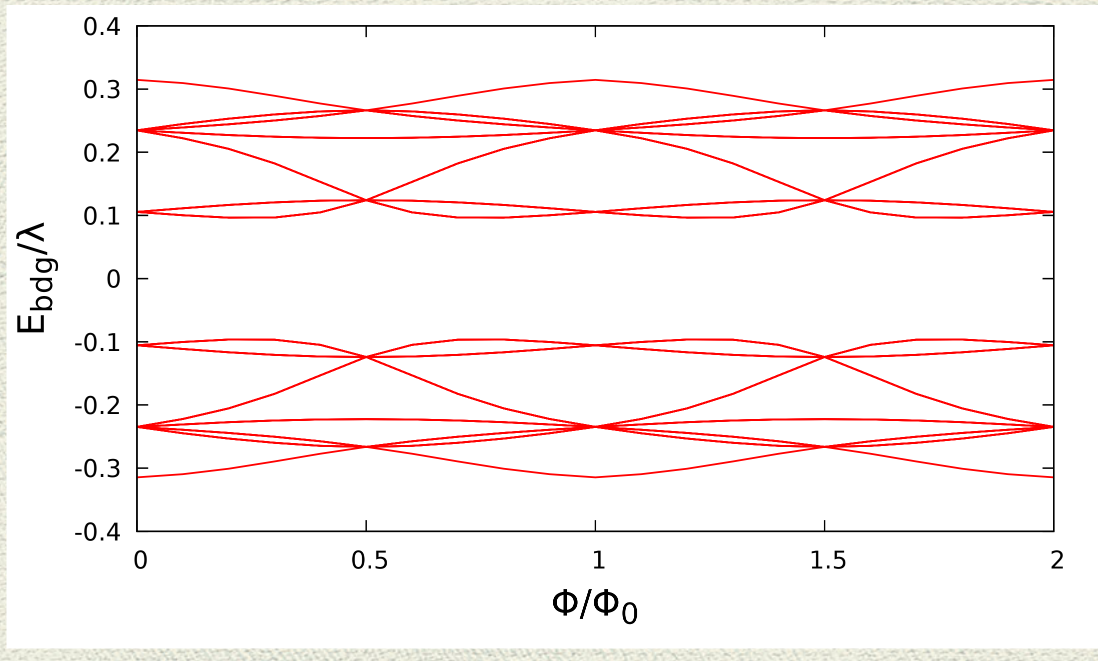
- For $b_x/\lambda < 1$ second order topological TSC_2 superconductor with four corner states

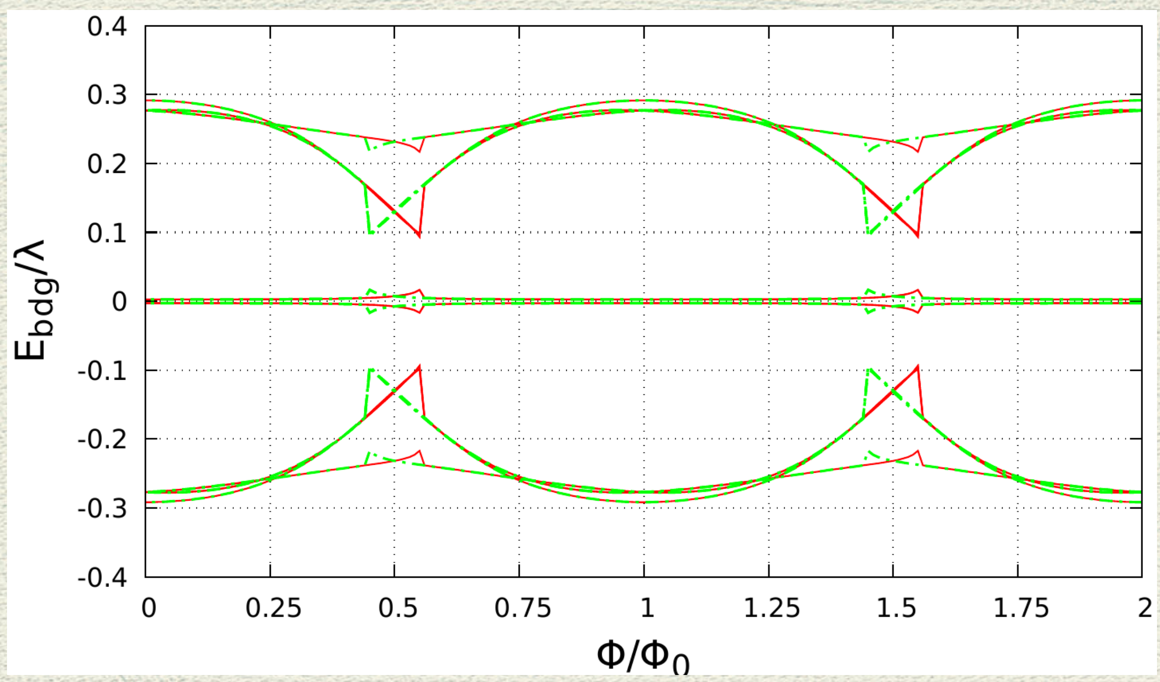
Vortex at the origin

- Introduce solenoid at origin of 2D sample, so that any closed loop encircling the origin encloses a flux ϕ
- First, we set $\Delta = 0$ - i.e. we turn off superconductivity
- Spectrum shows standard periodicity with respect to ϕ/ϕ_0


0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
E_{bdg}/λ
0
0.5
1
1.5
2
Φ/Φ_0

- $\Delta \neq 0$, constant Δ not even qualitatively correct in the presence of vortex
- Need to find self-consistent Δ by solving the BdG equations in the presence of the vortex self-consistently
- Close to $\phi = \phi_0/2$, find two self-consistent solutions (reminiscent of two classes of wave-functions)





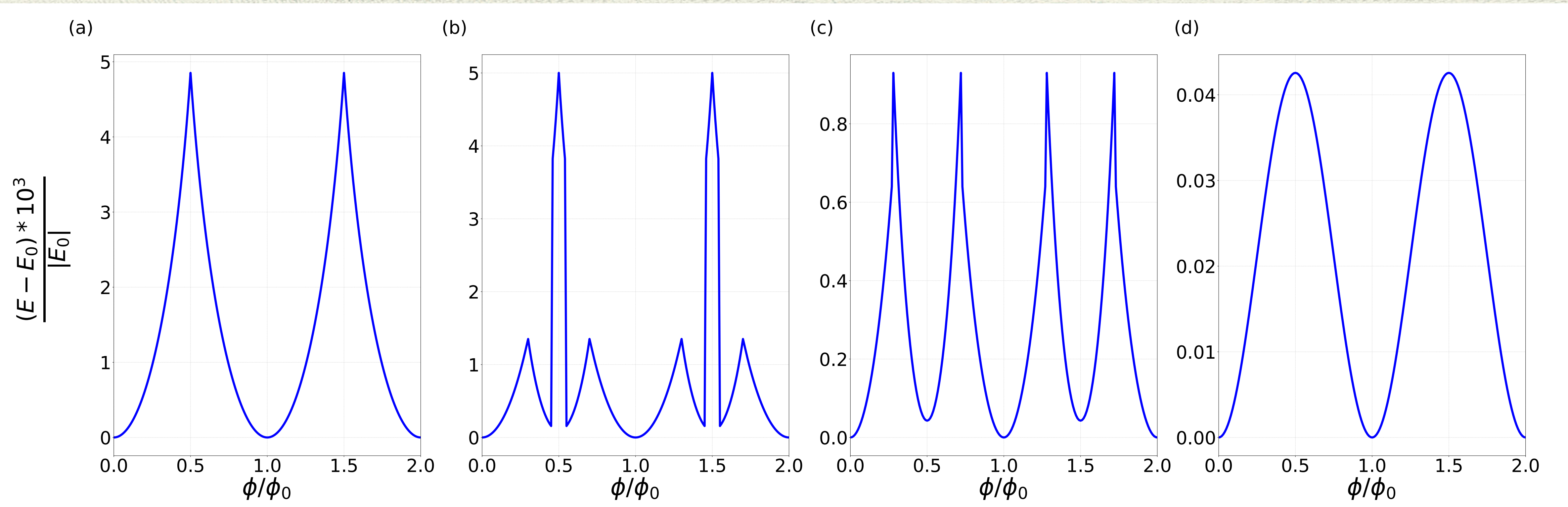
- Plot of lowest three self-consistent energy eigenvalues for $\Delta = 0$ and $\Delta \neq 0$

- Besides the zero energy states of the Majoranas, there are also two self-consistent states close to $\phi = \phi_0/2$

- Total energy as function of ϕ/ϕ_0 for different values of

  $b_x/\lambda = 0, \quad 0.3, \quad 0.8, \quad 1.5(\text{no superconductivity})$

- Note (almost) degeneracy for $b_x/\lambda = 0.8$



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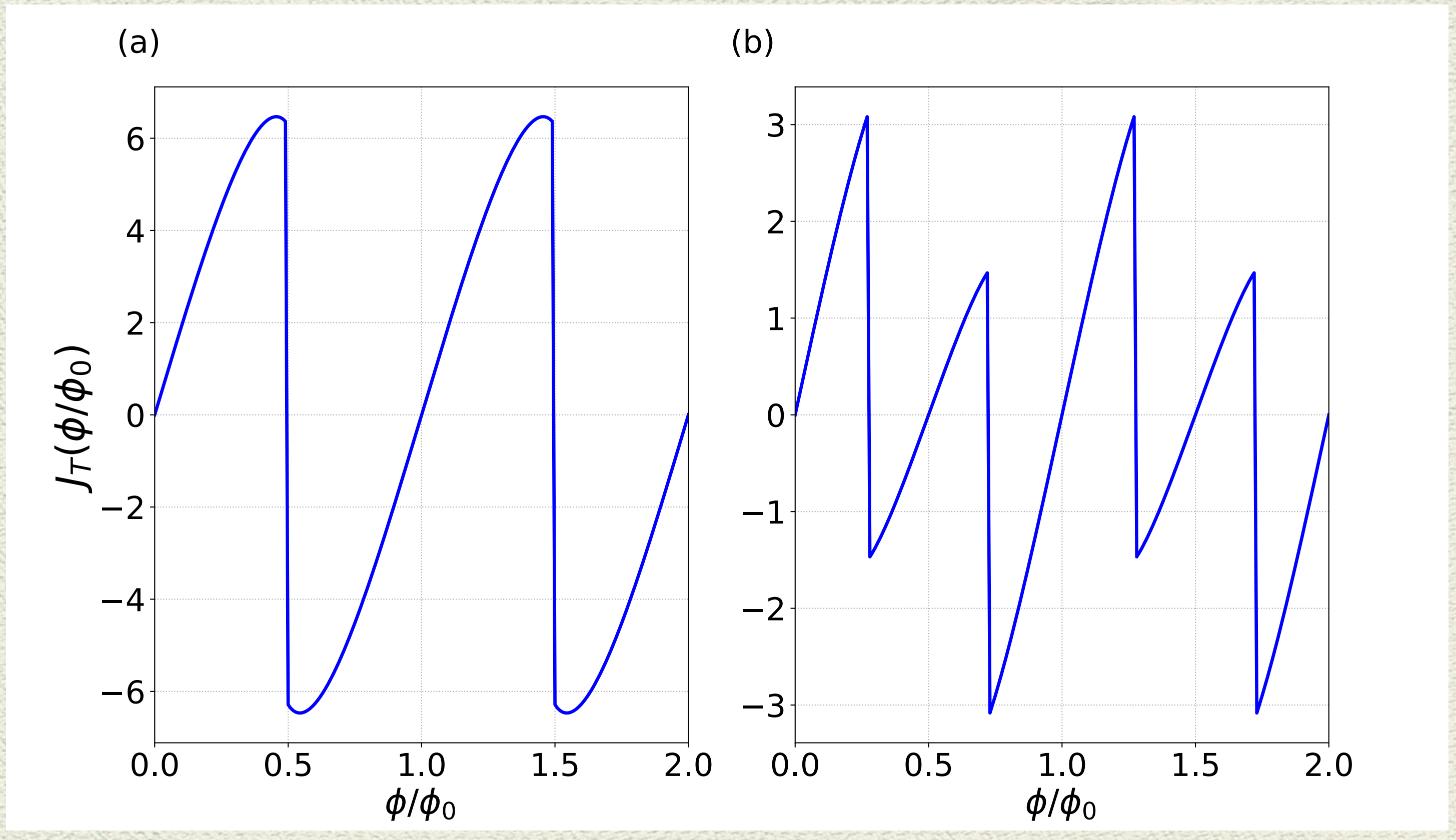
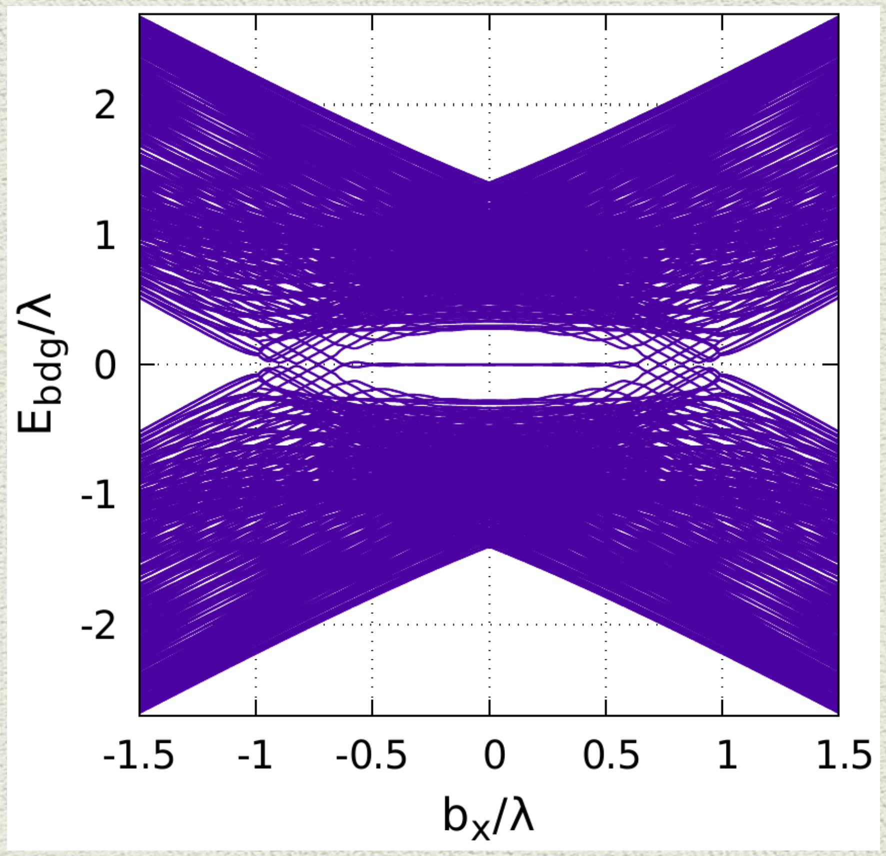
- Also computed total circulating current around the vortex for $b_x/\lambda = 0.0$ and 0.7

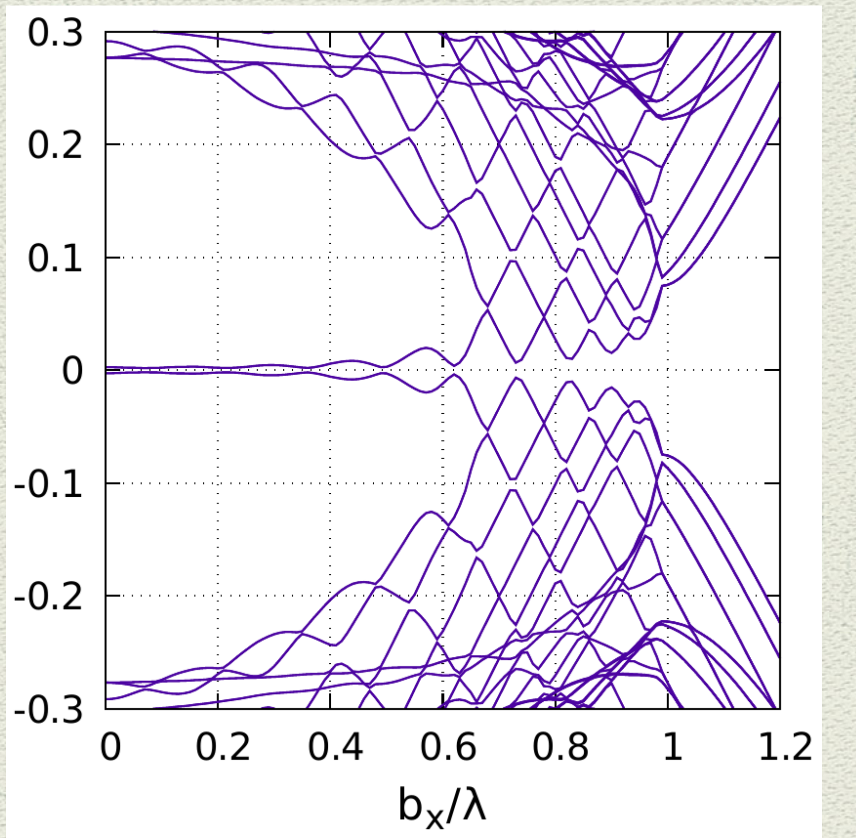


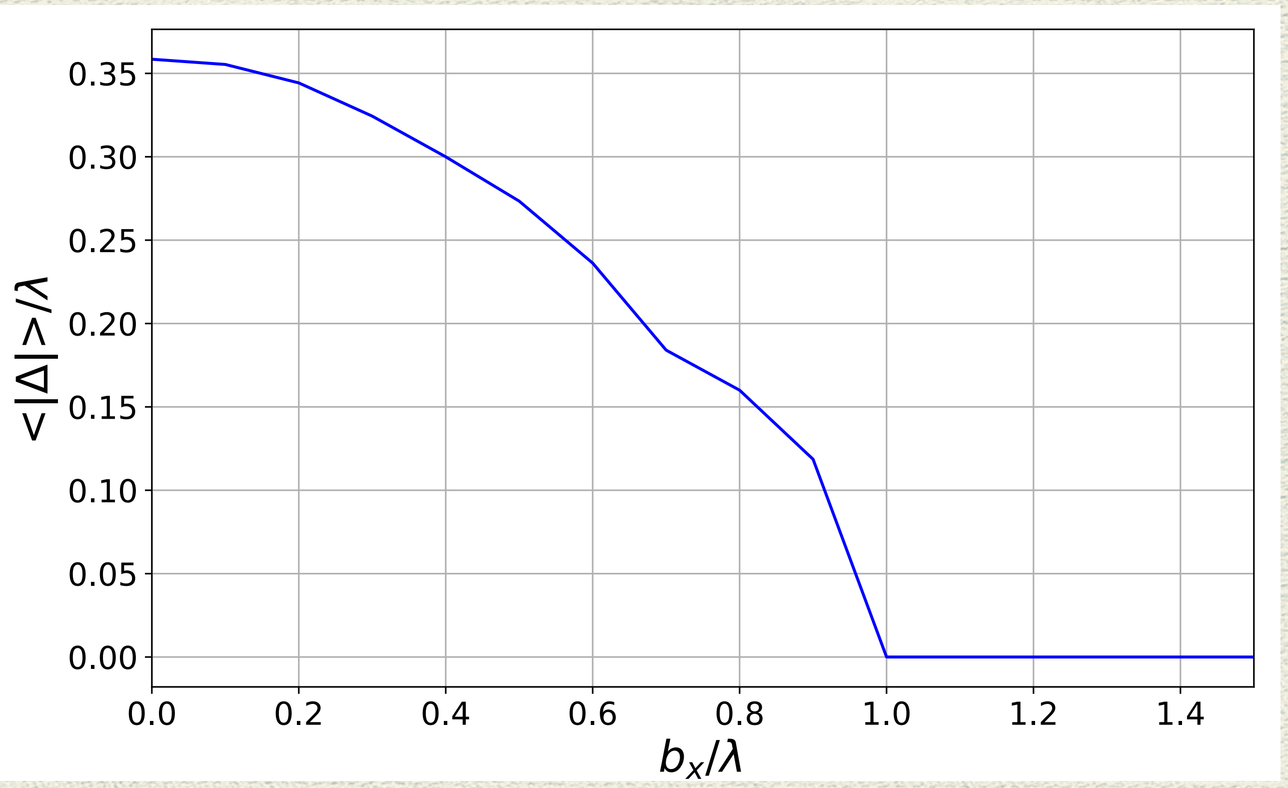
- Note change in `periodicity'. Strictly periodic only in ϕ_0 but roughly periodic in $\phi_0/2$

Why does periodicity change ?

- Self consistent spectrum in the absence of any vortex
- Shows crossover from TSC to normal superconductor







- Note that change to normal metal only around $b_x/\lambda \simeq 1$
- But topological superconductivity only survives till $b_x/\lambda \simeq 0.6$ because of mixing of the zero energy levels with higher energy levels

- So TSC_2 with ϕ_0 periodicity transitions to normal superconductor with $\phi_0/2$ periodicity to normal metal with ϕ_0 periodicity

- Issue that both topological superconductors and normal metals have ϕ_0 periodicity circumvented by separation via normal superconductor with $\phi_0/2$ periodicity

Tuning the transitions

- If we can tune between these phases, then the change in periodicity of the circulating current can be measured
- Need to change parameters in the Hamiltonian - so one possibility, couple to light

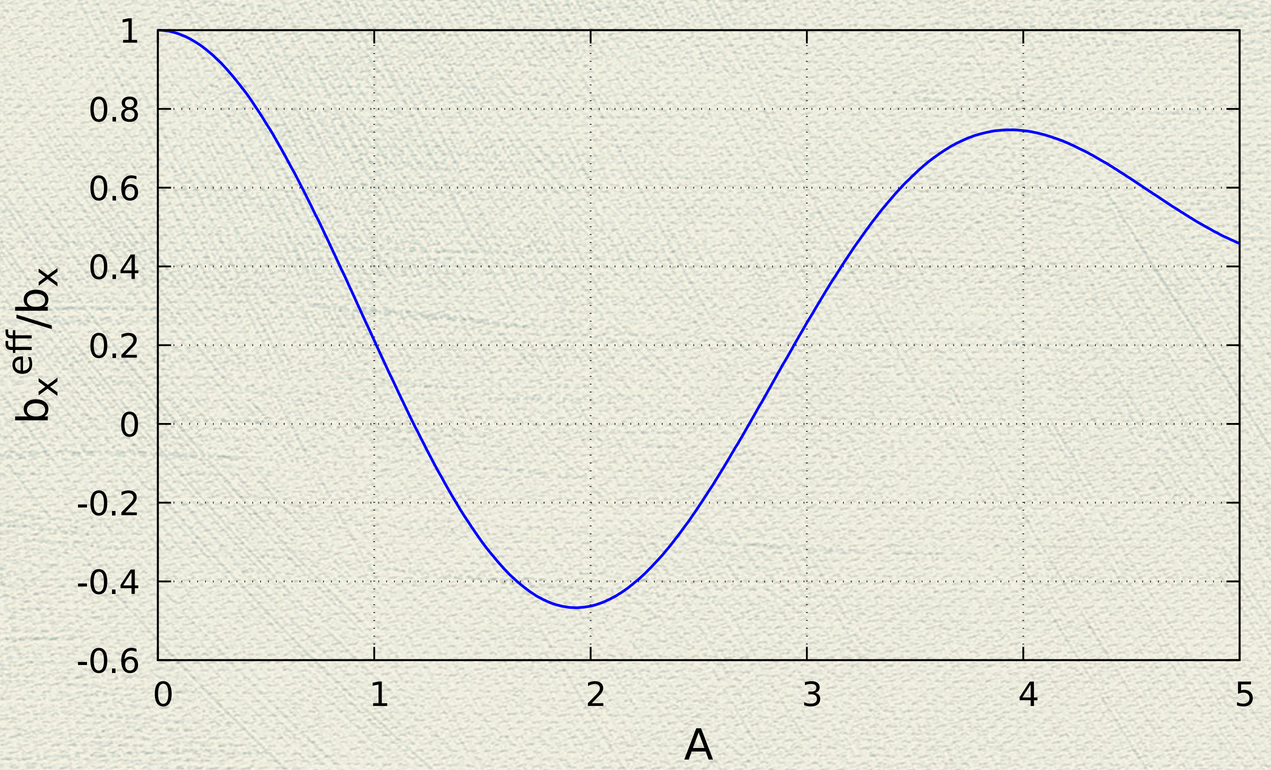
Floquet Hamiltonian

- Shine elliptically polarized light

$$A(t) = (A_x \cos(\omega t), A_y \cos(\omega t + \theta), 0), \quad \omega = 2\pi/T$$

- For high frequencies, $\omega \gg \lambda, \Delta$ obtain effective static Hamiltonian by expanding in powers of $1/\omega$

- Computation to $O(1/\omega^2)$ shows that b_x can be tuned by applying light



$$A_x = A_y = A$$
$$\lambda = \Delta$$
$$\theta = \pi/2$$

- So by shining light, can change periodicity of circulating current

Conclusion

- Studied corner Majorana modes in a model of a higher order topological superconductor
- Have introduced a vortex at the origin in a higher order topological superconductor
- Periodicity of energy levels and circulating current changes from ϕ_0 to $\phi_0/2$ as the superconductor transitions to a normal superconductor
- Change in periodicity can be tuned by coupling to light

Future

- 2D p+ip superconductors in first order topological superconductors - check whether self-consistent solutions lead to new results.

- Goal - to move the corner Majoranas for braiding. Since there are four Majoranas, it should be possible to get non-abelian statistics