10/12/14

Notes on State Dependent Operators

1) Based on
arXiv: 1310.6335, 1310.6334, 1211.6767

This is based on my work on the information paradox. However, some aspects of our solution are very interesting and appear to have broader significance on how quantum mechanics ties together with gravity.

2) We have checked
AdS $\leftrightarrow$ CFT
in 10,000 ways. However I want to ask the following fundamental question

"Are geometric bulk quantities given by Hermitian observables, or as functions of correlation functions"

a) Entanglement entropy: RT, as stated, is in the latter class.

b) Black hole info par: interior operators
c) Nato argument; version of counting argument
d) Our resolution; state dependence
e) Preserving linearity
f) relationship to ER = EPR

3) State Dependence of R.T. Formula

Consider the left side. within canonical gravity, the Area of the minimal surface is a Hermitian observable.

a) There are several ways to see this

$\hat{g}$ is an operator.

Given the wave-function on g, the minimal area surface is a well-defined functional of the metric

$$\hat{A}[g]$$

so $\hat{A}$ is an operator

b) Another way. Think of the set of all classical solutions of gravity. On each of these A_{min} is a well-def fn.

So A_{min} is a good fn on phase-space

By the usual quantization methods, this lifts to an operator on the Hilbert space.

On the other hand, S_{ent} is not a linear operator. Proof is very simple.

Consider $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. and a basis of states

$$|V_A\rangle \otimes |V_B\rangle$$

$\hat{S}_{EE} (|V_a\rangle \otimes |V_B\rangle) = 0;$ $\Rightarrow \hat{S}_{EE} |\psi\rangle = 0, \forall |\psi\rangle$

3.5)

To make this precise, consider the following state in the CFT.

$$|\psi\rangle = \alpha |E_1\rangle + \beta |E_2\rangle$$

$$|\alpha|^2 + |\beta|^2 = 1.$$

where E_1, E_2 are very different.

Now the geometric expected answer is as follows. with prob. $|\alpha|^2$ we get area A_1; with prob. $|\beta|^2$ we get area A_2.

How does one get this from R.T.

In fact

$$S_{ent}\left(\alpha |E_1\rangle + \beta |E_2\rangle\right)$$

$$\simeq |\alpha|^2 S_{ent}(|E_1\rangle) + |\beta|^2 S_{ent}(|E_2\rangle)$$

so we do have

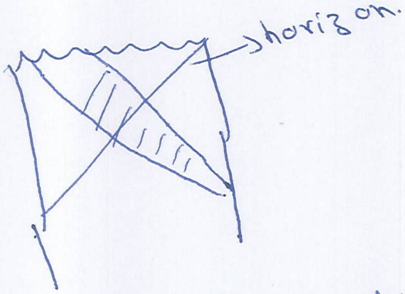
$$\langle A \rangle = S_{ent}$$

But conceptually, this is also a little unsatisfying because S_{ent} is not the expectation value of a Hermitian operator.

Besides, how do we get the prob. distribution for A^* from R.T?

4) I will now describe a similar setup in a geometry involving black-holes, which shows that certain bulk observables need to be state-dependent.

Consider a black hole geometry



Introduce tortoise coordinates r_*, which go to $-\infty$ at the horizon and Kruskal coordinates

$$U = e^{(-r_* + t)\frac{2\pi}{\beta}}, \quad V = e^{(r_* - t)\frac{2\pi}{\beta}}.$$

So the horizon is at $V = 0$.

Now near the horizon, the fields start behaving like two-d fields and the expansion is

Region I:

$$\phi = \int \frac{d\omega}{\sqrt{\omega}}\, a_\omega e^{i\omega(r_* + t)} + b_\omega e^{i\omega(r_* - t)} + h.c.$$

$$= \int \frac{d\omega}{\sqrt{\omega}}\, a_\omega U^{i\omega\beta/2\pi} + b_\omega V^{i\omega\beta/2\pi} + h.c.$$

Region II:

$$\phi = \int \frac{d\omega}{\sqrt{\omega}}\, a_\omega U^{i\omega\beta/2\pi} + c_\omega V^{-i\omega\beta/2\pi} + h.c.$$

5) Let us take two points close to the horizon. and consider

$$\lim_{U_1 \to U_2,\, V_1 \to V_2} \partial_V \phi(U_1, V_1)\, \partial_V \phi(U_2, V_2)$$

On very general grounds, since the physics becomes effectively 2d, and the short distance behaviour of the correlator is universal, we see that we must have

$$G = \frac{1}{(V_1 - V_2)^2}$$

This requires

$$\langle c_\omega b_\omega \rangle = \frac{1}{1 - e^{-\beta\omega}}$$

To see this consider
$\langle \phi_1 \phi_2 \rangle$.

$$= \int \frac{d\omega}{\omega} \left(\langle b_\omega c_\omega \rangle \frac{V_1^{i\omega\beta/2\pi}}{V_2^{i\omega\beta/2\pi}} + \langle b_\omega^{+} c_\omega^{+} \rangle \frac{V_2^{i\omega\beta/2\pi}}{V_1^{i\omega\beta/2\pi}} \right.$$

In the situation where
$\langle c_\omega b_\omega \rangle = \frac{1}{1 - e^{-\beta\omega}}$.

This becomes

$$\int \frac{d\omega}{\omega} \frac{1}{1 - e^{-\beta\omega}} \cdot \left(\frac{V_1}{V_2}\right)^{i\omega\beta/2\pi}$$

which has poles at $\beta\omega = 2\pi n$; picking up the poles gives us $\ln\left(1 - \frac{V_1}{V_2}\right)$!

6) Where are these c_ω operators in the CFT?

It seems possible to prove that these c_ω cannot exist as ordinary operators in the CFT.

7) Basically the proof goes as follows. Using the c_ω, one can construct the number operator as measured by the infalling observer.

$$N_a \propto (b_\omega^\dagger - e^{-\beta\omega/2} c_\omega)(b_\omega - e^{-\beta\omega/2} c_\omega^\dagger) + (c_\omega^\dagger - e^{-\beta\omega/2} b_\omega)(c_\omega - e^{-\beta\omega/2} b_\omega^\dagger).$$

and consider P, which projects onto states where $N_a \neq 0$.

Now we seem to have the following physical conditions. Energy eigenstates should have smooth horizons

$$\langle E | P | E \rangle \approx 0$$

but Schwarzchild number eigenstates should not

$$\langle N | P | N \rangle = O[1].$$

but, even restricting to N-eigenstates in some range

$$|E\rangle = |F_a\rangle + \frac{1}{N} |R_a\rangle.$$

where $|F_a\rangle$ is a linear combination of N-eigenstates in some range.

8)

However, one can show that for any projector, if we take two states

$$\langle u_1 | u_2 \rangle \simeq 1 - \varepsilon$$

then

$$\langle u_1 | P | u_1 \rangle - \langle u_2 | P | u_2 \rangle = O(\varepsilon)$$

Therefore

$$\frac{1}{D} \sum \langle E_a | P | E_a \rangle \simeq \frac{1}{D} \sum \langle F_a | P | F_a \rangle$$

since the F_a are almost orthonormal

$$\frac{1}{D} \sum \langle F_a | P | F_a \rangle \simeq \frac{1}{D} \sum \langle N_i | P | N_i \rangle$$

$$\simeq \mathrm{Tr}[1].$$

So we have a contradiction

9) ~~Anoth~~

<u>Another</u> <u>argument</u> [Depending on time]

Notice that

$$[H, c_\omega] = \omega c_\omega$$

opposite of usual sign

$$\Rightarrow \quad \mathrm{tr}(e^{-\beta H} c_\omega^\dagger c_\omega) = \frac{e^{\beta \omega}}{1 - e^{\beta \omega}} < 0 \,?!$$

10)

Our construction

Before summarizing our construction, let me briefly mention how the b_ω operators are realized. We can associate

$$b_\omega \rightarrow \frac{O_\omega}{\sqrt{G_\omega}}$$

where O_ω is the mode of a G.F.F. that satisfies

$$\langle O(x_1) \ldots O(x_n)\rangle = \langle O(x_1) O(x_2)\rangle \ldots \langle O(x_{n-1}) O(x_n)\rangle + \text{permutations} + \frac{1}{N}.$$

We define $\tilde{O}_\omega$ as:

$$\tilde{O}_\omega \, O_{\omega_1} \ldots O_{\omega_p} |\Psi\rangle.$$

$$= e^{-\beta\omega/2} O_{\omega_1} \ldots O_{\omega_p} O_\omega^\dagger |\Psi\rangle.$$

For equilibrium states $|\Psi\rangle$.

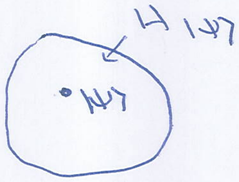
In the neighbourhood of an equilibrium state, this gives rise to linear =ns that clearly have a solution.

This solves all of the AMPS paradoxes and leads to a smooth horizon

11) <u>Properties of our construction</u>

The construction is obtained by taking a ~~space~~ state $|\psi\rangle$, and constructing a space

$H_{|\psi\rangle}$



On this space, $\tilde{O}$ acts linearly by construction. So it describes a given black hole and small excitations about it.

12)
However, there is no globally defined linear operator that obeys these conditions. This is guaranteed by M-P's arguments, but even in our case we see that equilibrium is <u>not</u> a linear notion.

Adding e^N eq. states can give a non-eq state.

Adding a few non-eq. states can give an eq. state.

(3)

Linearity for "small" superpositions

We have to make sure that, within EFT, the observer cannot detect any violation of Q.M

The test of any such violations is always to entangle the CFT with another system.

Consider a pointer with $(M+1)$-states and M equilibrium orthogonal states of the CFT

Now consider joint state

$$|\psi\rangle = \sum_{i=1}^{M} |i\rangle_{ptr} |\psi_i\rangle_{CFT}\, \underset{\substack{\uparrow \\ \text{coefficient}}}{\alpha_i} + |M+1\rangle_{ptr} \left(\sum \beta_i |\psi_i\rangle_{CFT}\right)$$

a) We would like that if we act with a unitary $\tilde{U}$ behind the horizon, then $\tilde{U}$ does not act on the ptr at all.

So

$$\tilde{U}|\psi\rangle = \sum_{i=1}^{M} \alpha_i |i\rangle \otimes \tilde{U} |\psi_i\rangle_{CFT} + |M+1\rangle\, \tilde{U} \left(\sum \beta_i |\psi_i\rangle_{CFT}\right)$$

b) We would like this unitary not to lead to superluminal communication. So density matrix of the ptr should be unchanged.

Initially

$$\rho_{PTR} = \sum |\alpha_i|^2 |i\rangle\langle i| + \alpha_i \beta_i^* |i\rangle\langle M+1| + \alpha_i^* \beta_i |M+1\rangle\langle i| + \left(\sum |\beta_i|^2\right) |M+1\rangle\langle M+1|$$

If the density matrix is unchanged we need:

denoting $|\chi\rangle = \tilde{U}\left(\sum \beta_i |\psi_i\rangle_{CFT}\right)$.

that

$$\langle \chi | \tilde{U} |\psi_i\rangle_{CFT} = \beta_i^* \quad ; \quad \langle \chi | \chi \rangle = \sum |\beta_i|^2$$

and this forces

$$|\chi\rangle = \sum_{i=1}^{M} \beta_i \tilde{U} |\psi_i\rangle_{CFT}.$$

14) There is no problem in ensuring the linear action of our operators on small sums of non-equilibrium states

$$\tilde{O}_\omega \left(\sum A_i |\psi_i\rangle\right) = \sum A_i O_\omega^\dagger |\psi_i\rangle e^{-\beta\omega/2}$$

for equilibrium states $|\psi_i\rangle$.

15)

Big pointers and relation to ER=EPR

Now lets couple the CFT to another "big" system: eg. another copy of the CFT.

Now lets say we take

$$|\psi\rangle_{ent} = \sum_{i=1}^{\dim(CFT)} |i\rangle|\psi_i\rangle \, ;$$

full system is in eq. and $|\psi_i\rangle$ are eq states.

Now if we try and impose

$$\tilde{O}_\omega A |\psi\rangle_{ent} = \sum_{i=1}^{\dim(CFT)} |i\rangle\left(\tilde{O}_\omega A|\psi_i\rangle\right).$$

and also

$$\tilde{O}_\omega A |\psi\rangle_{ent} = e^{-\beta\omega/2} A\, O_\omega^{+} |\psi\rangle_{ent}.$$

then we find that we must have

$$\tilde{O} A |\psi_i\rangle = A\, O_\omega^{+} |\psi_i\rangle, \ \forall i$$

there is certainly no linear operator that can satisfy all these equations.

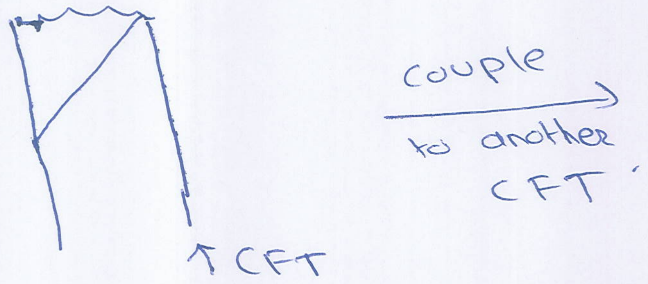
However there is a simple linear operator that acts on the big system, which satisfies this. This is obtained by solving

$$\tilde{O}_\omega A |\psi\rangle_{ent} = e^{-\beta\omega/2} A\, O_\omega^{+} |\psi\rangle_{ent}$$

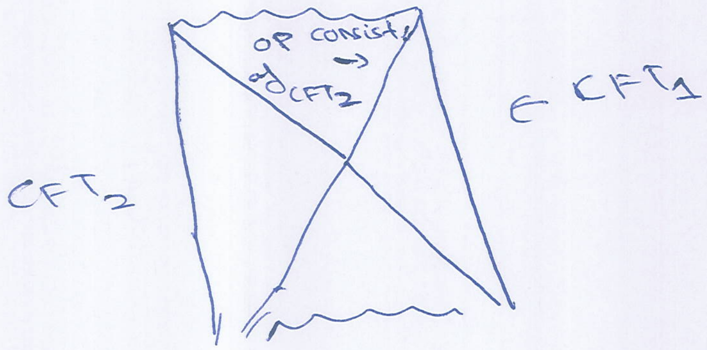
in the big H-space.

16) what is the geometric picture?

we have a B.H.



we find the <u>interior</u> <u>operators</u> in the first B.H. now comprise operators from the other CFT.



A <u>wormhole</u> has opened up. So this construction realizes ER = EPR in a precise sense.

Conclusion

Interesting story about emergent Q.M.

a) We see that the interior ops of a B.H. are realized as state-dependent operators.

b) This suggests the interior geometry emerges as a function of correlation functions.

c) Nevertheless Q.M. must be valid for observers in E.F.T.

d) Our construction passes several simple checks. Predicts the ER = EPR behaviour.

e) Suggests that Q.M. itself is emergent, in a sense. For the bulk theory. Very interesting direction.

f) May be wrong. In that case, will make a surprising prediction for black-holes.