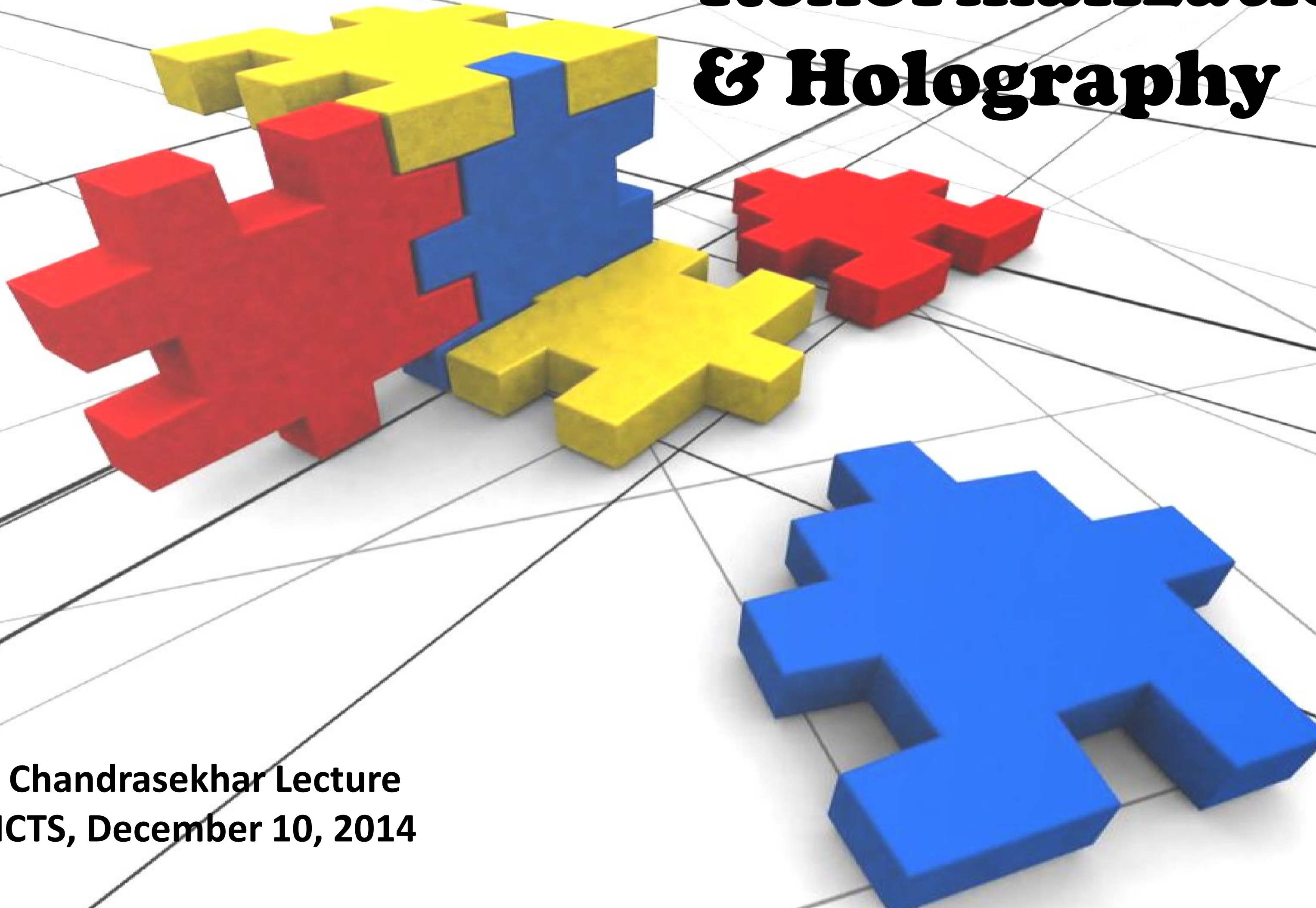
New Dialogues: Entanglement, Renormalization & Holography

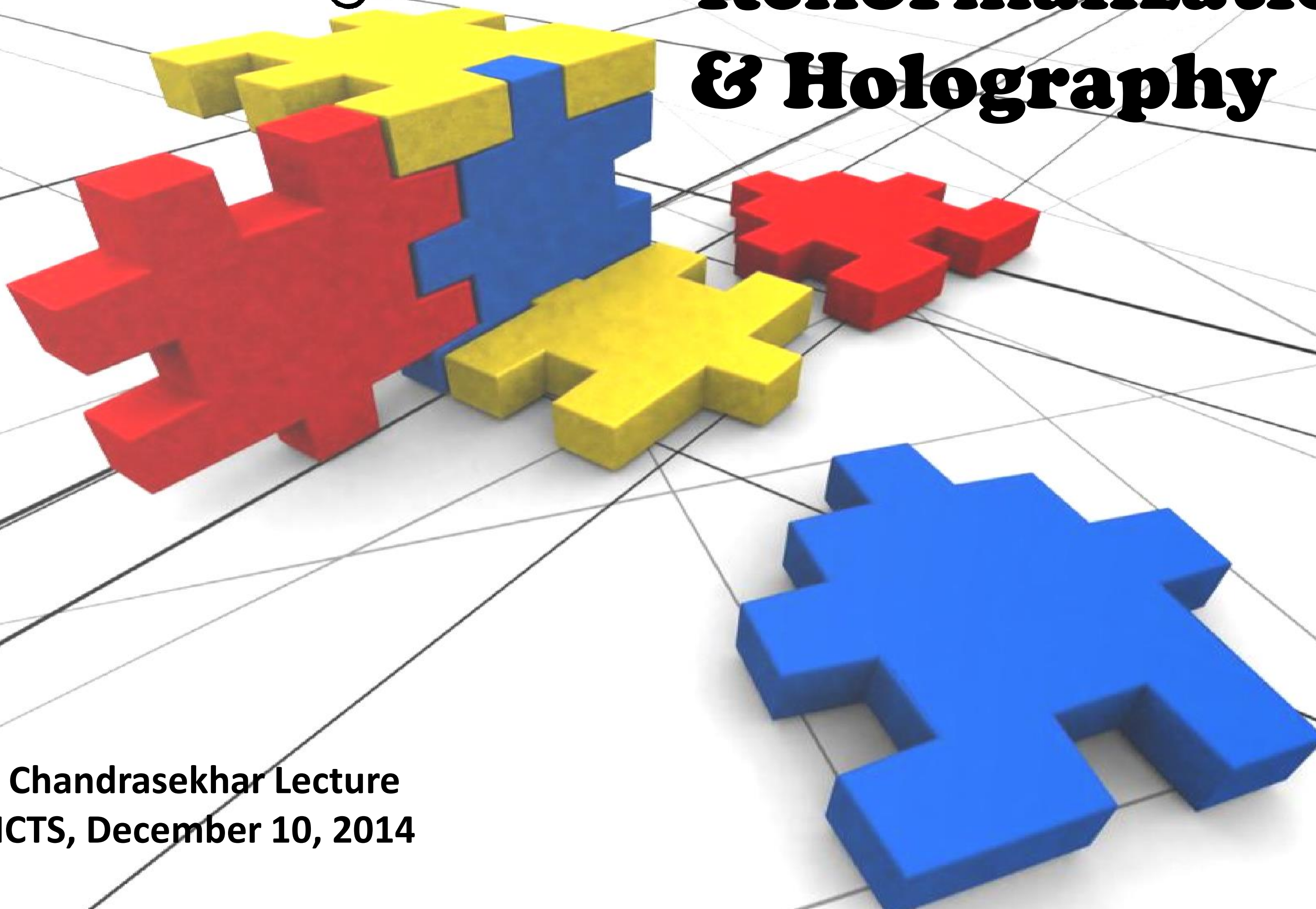


Chandrasekhar Lecture
ICTS, December 10, 2014

~~A New Alliance:~~

~~Connections:~~

~~New Dialogues:~~ A New Synthesis: Entanglement, Renormalization & Holography



Chandrasekhar Lecture
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New Collisions

In Physics : Entanglement, Renormalization & Holography

Chandrasekhar Lecture
ICTS, December 10, 2014

New Collisions

In Physics : ☼♏❍♋●♓⌘☝♓ॐ, ☝□●□♑◻♋□♒■ & ☜■⧫♋■♑❍♏

Chandrasekhar Lecture
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New Collisions

In Physics : Entanglement, Renormalization & Holography

Chandrasekhar Lecture
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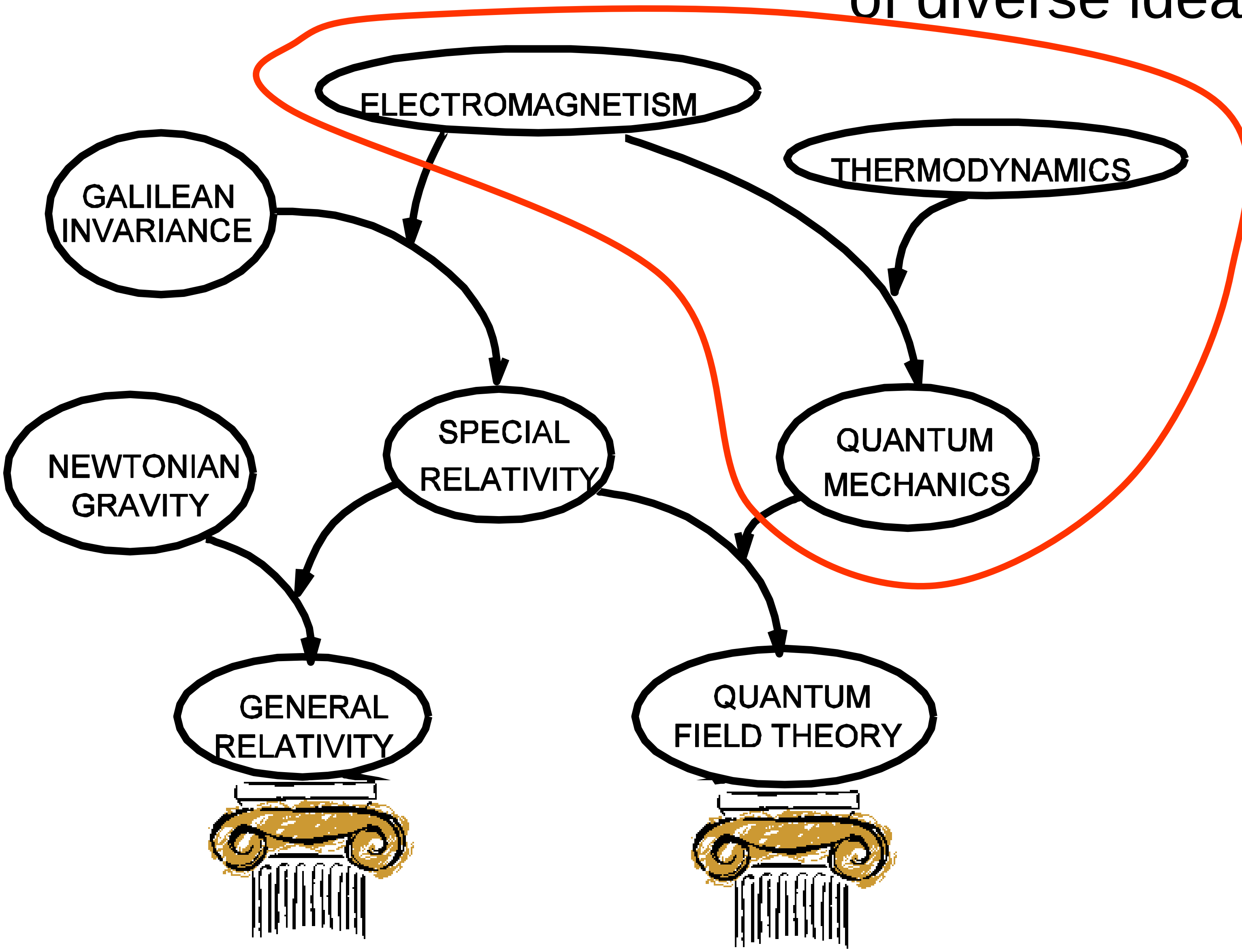
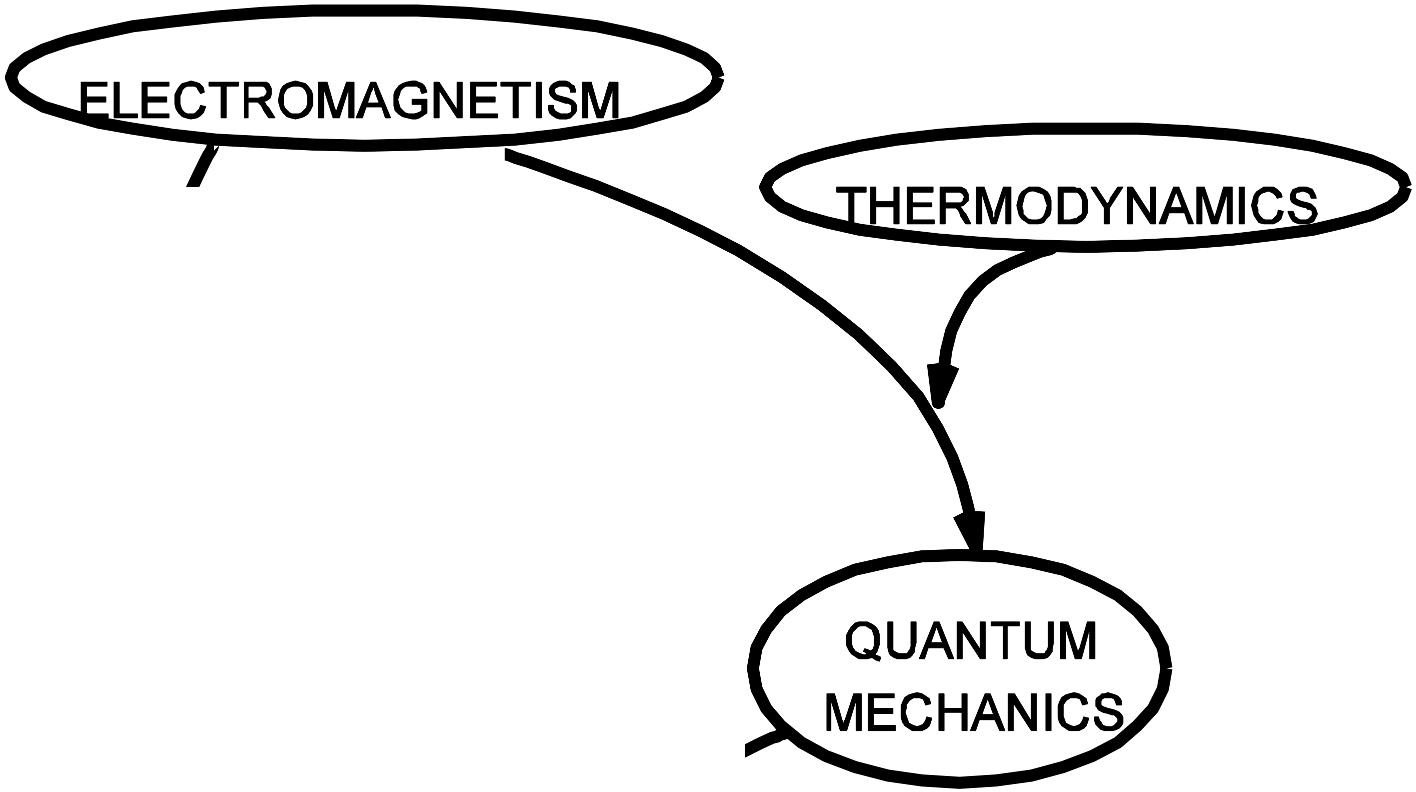
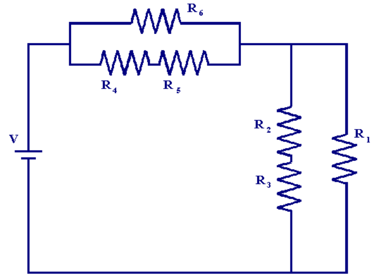
Physics of the 20th century:

A grand synthesis of diverse ideas!




ELECTROMAGNETISM
THERMODYNAMICS
QUANTUM MECHANICS



ELECTROMAGNETISM

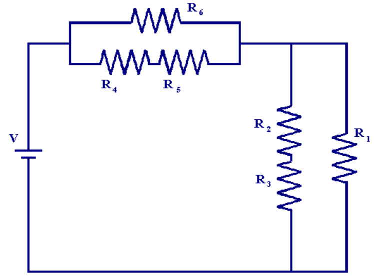
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

THERMODYNAMICS

QUANTUM MECHANICS



The Laws of Thermodynamics

0. Two bodies in thermal equilibrium are at same T
1. Energy can never be created or destroyed.

$$\boxed{\Delta U = T\,\Delta S - P\,\Delta V}$$

2. The total entropy of the UNIVERSE (= system plus surroundings) MUST INCREASE in every spontaneous process.

$$\boxed{\Delta S_{TOTAL} = \Delta S_{system} + \Delta S_{surroundings} > 0}$$

3. The entropy (S) of a pure, perfectly crystalline compound at T = 0 K is ZERO. (no disorder)

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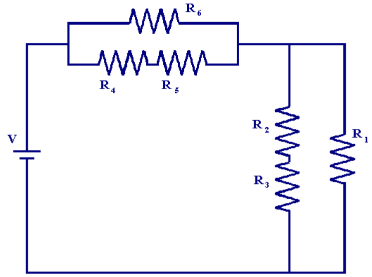

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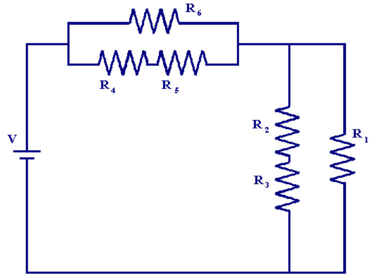

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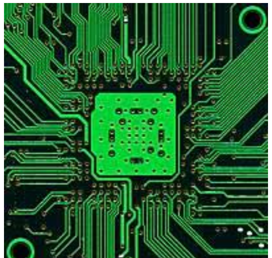
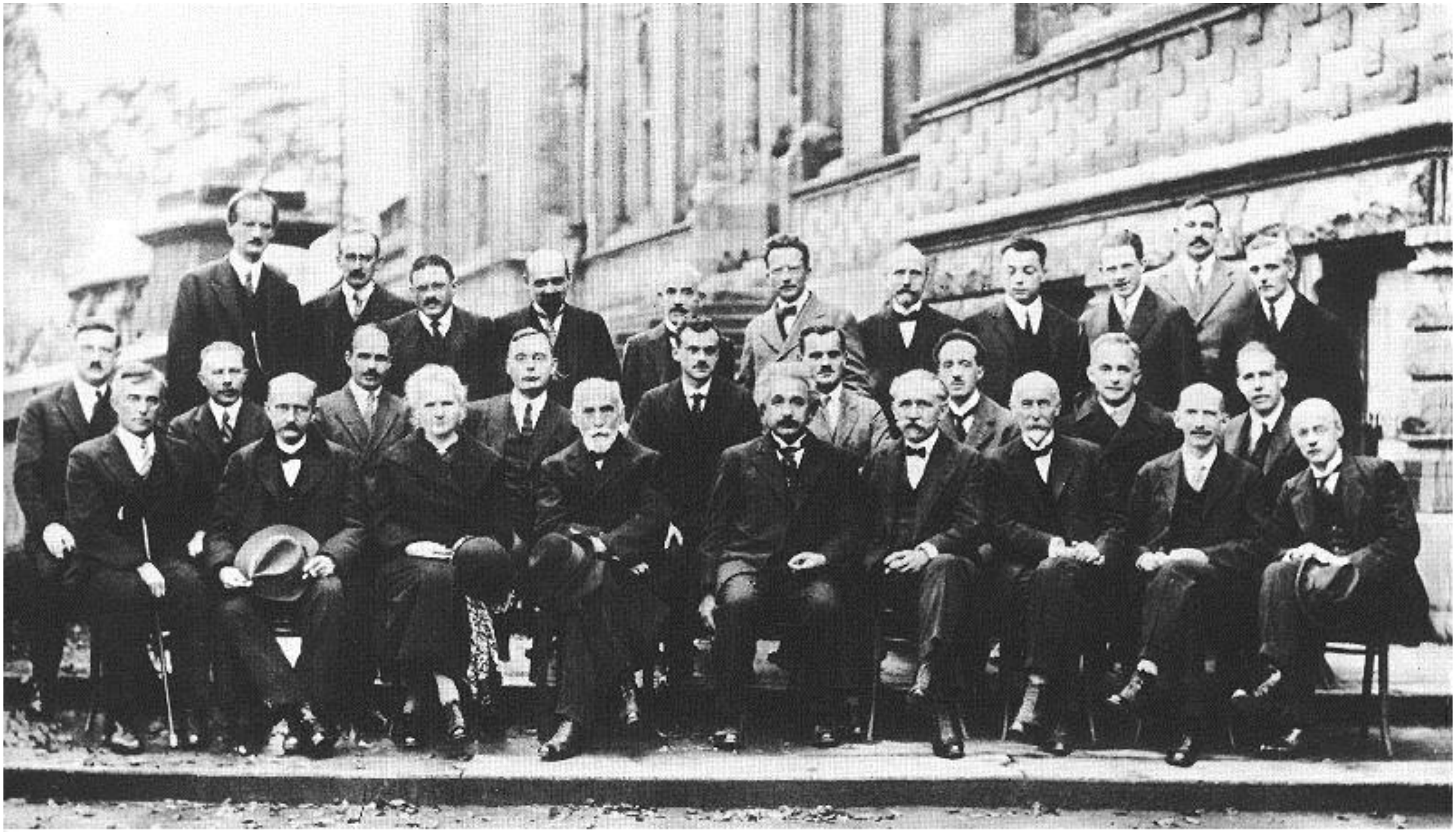
QUANTUM MECHANICS

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V\right]\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$



1927 Solvay Conference on Quantum Mechanics



A. PICCARD E. HENRIOT P. EHRENFEST Ed. HERZEN Th. DE DONDER E. SCHRODINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN
P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

in the middle of the “collision”

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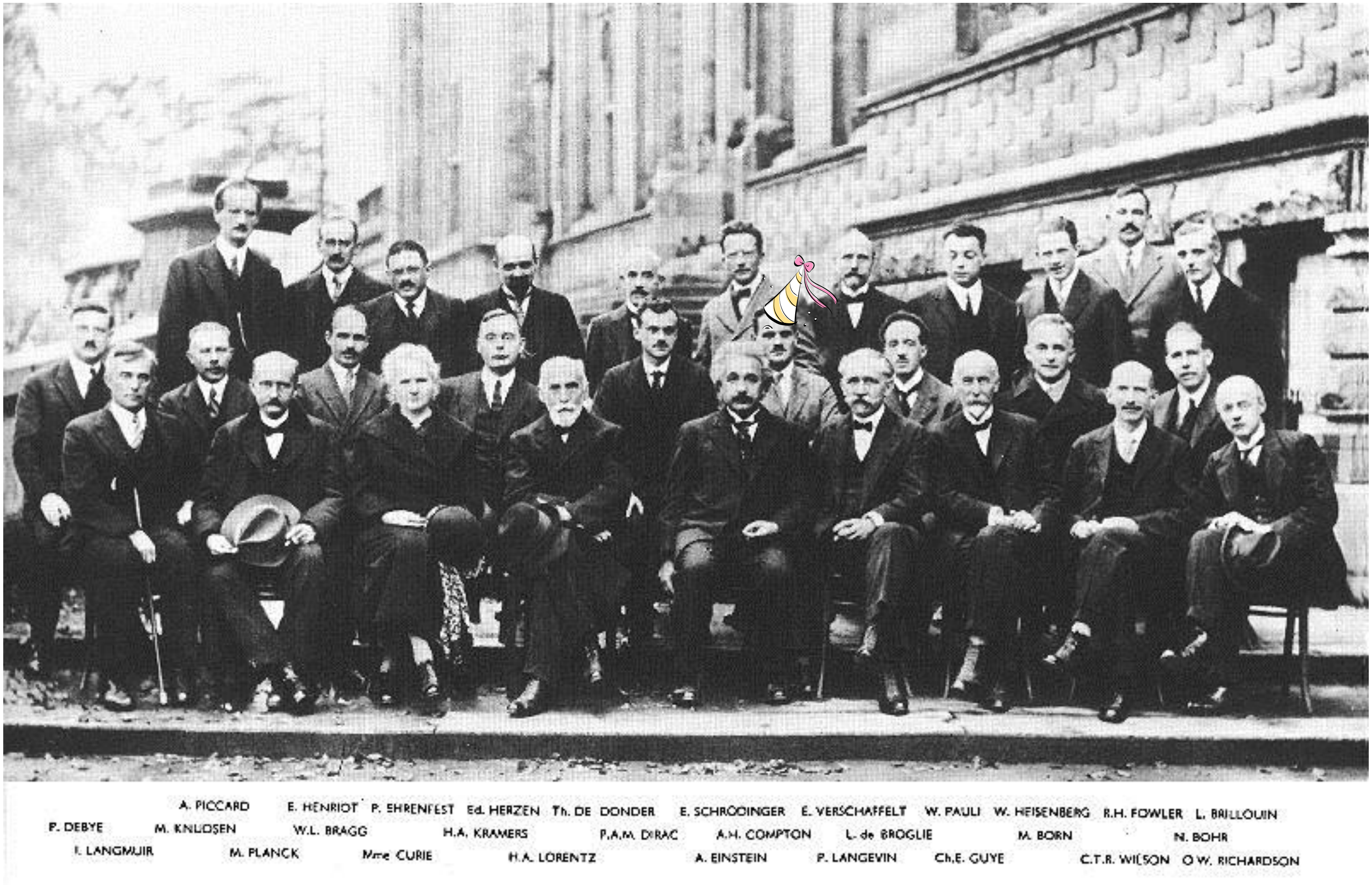
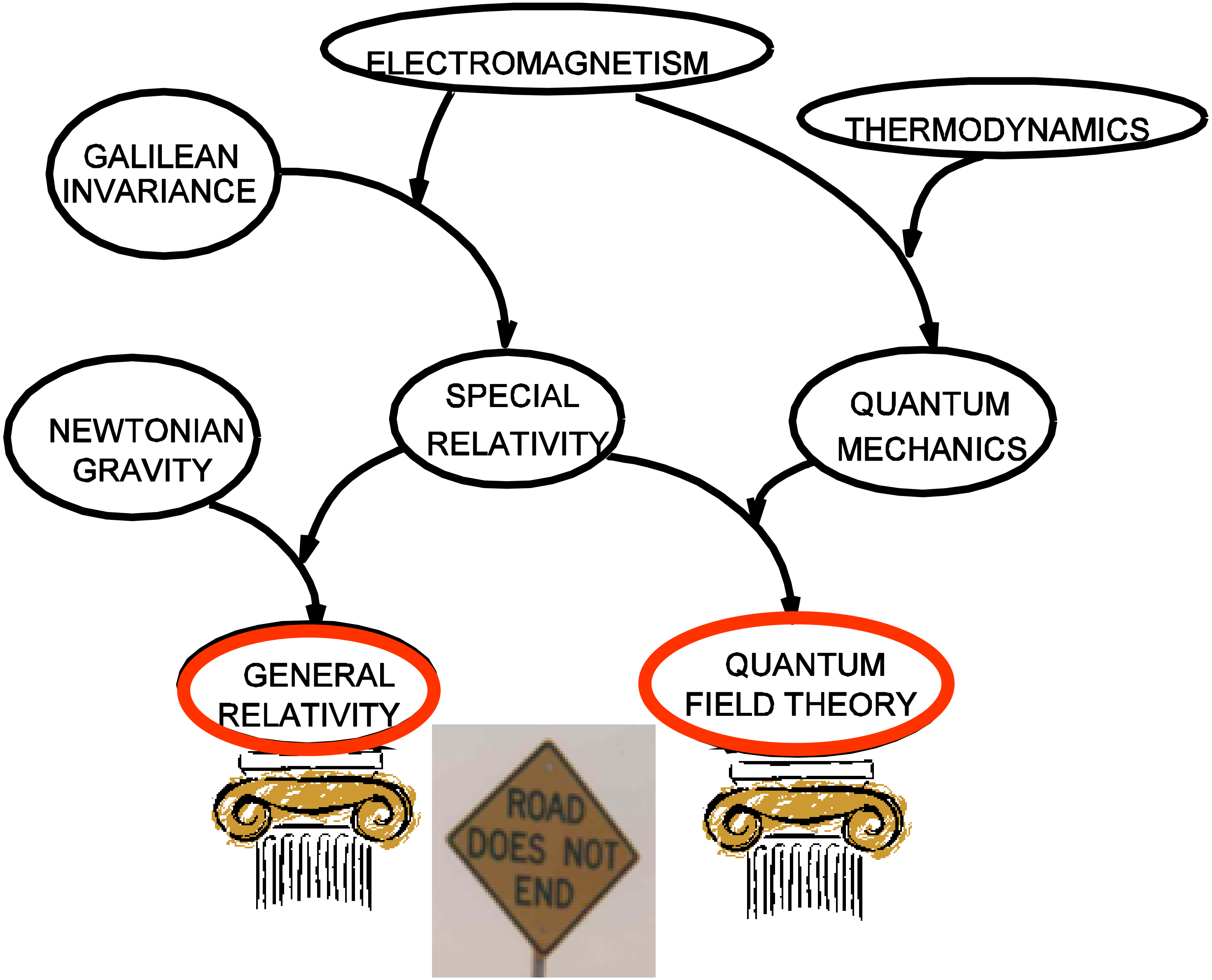


in the middle of the "collision":

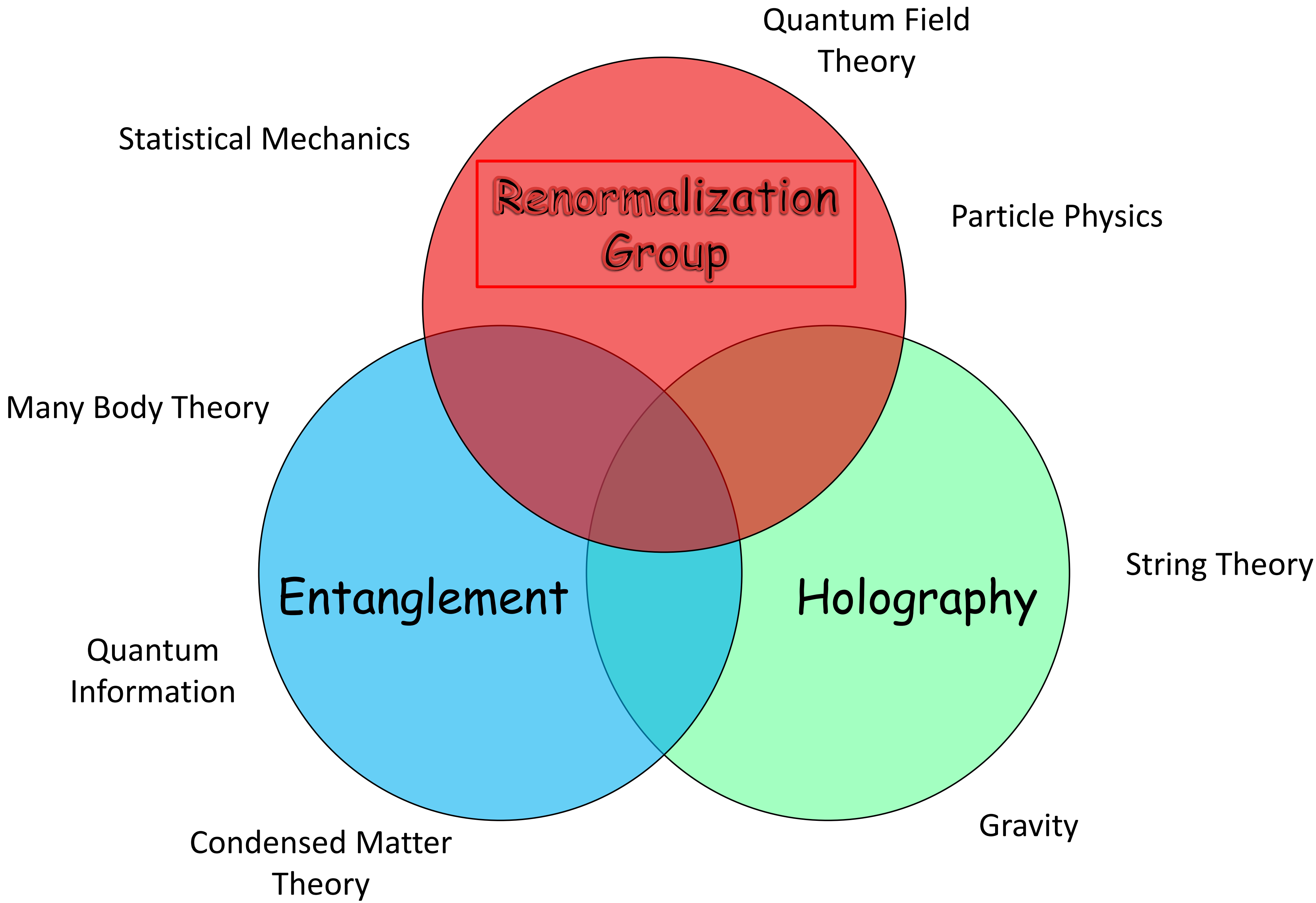
Do these people look happy?

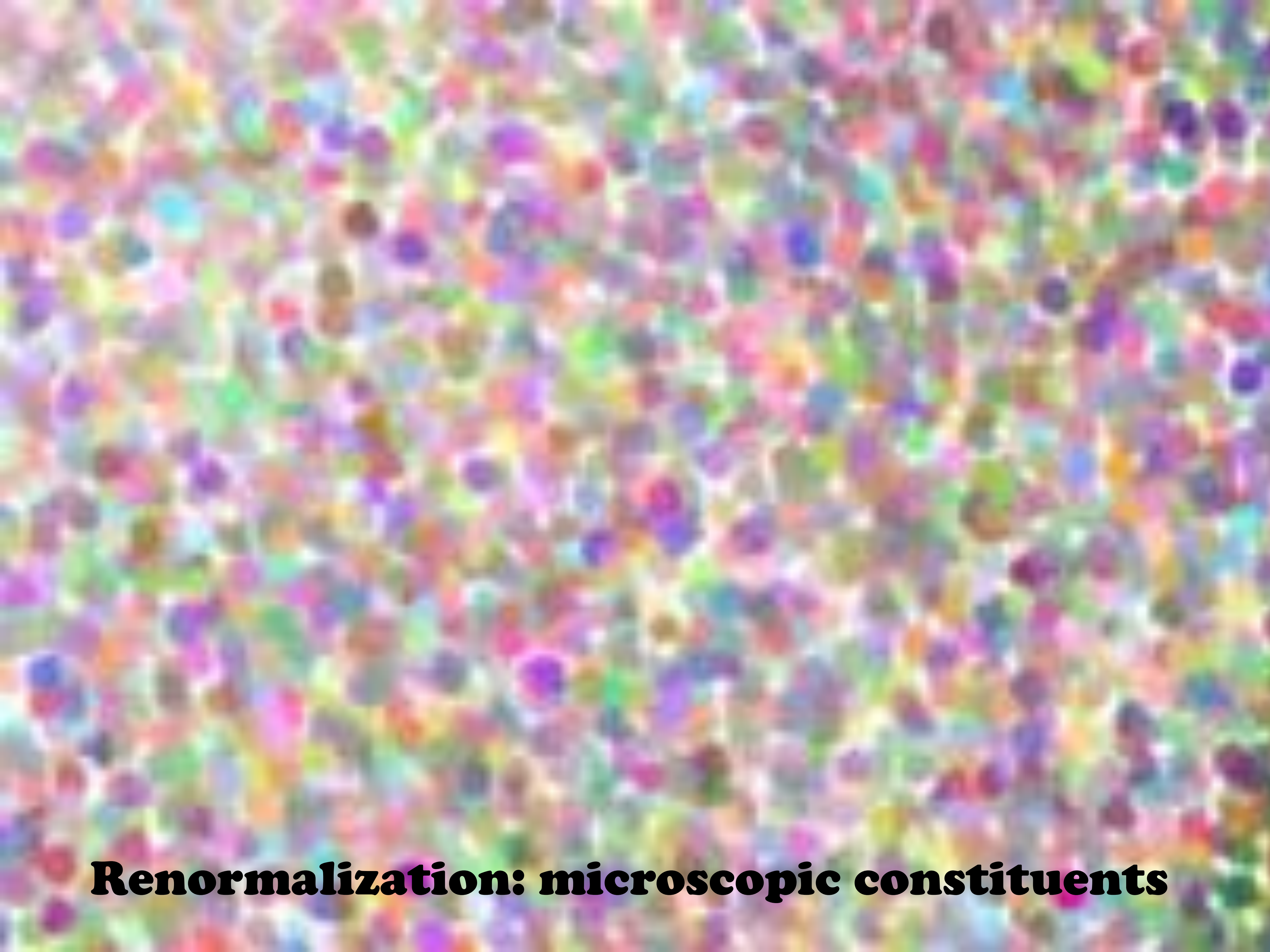
Physics of the 20th century:

A grand synthesis and progression of ideas!

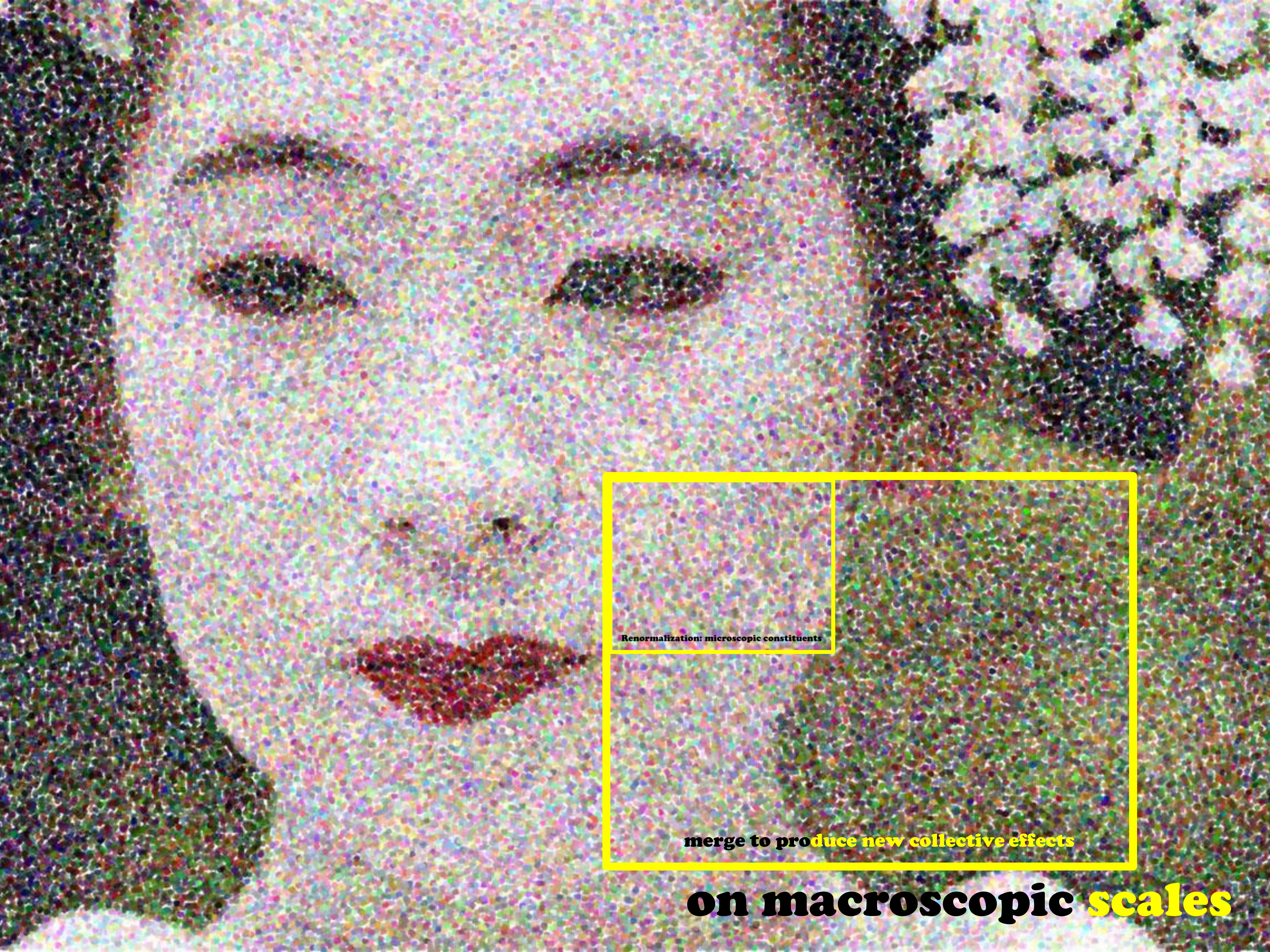


New Collisions in Theoretical Physics:




Renormalization: microscopic constituents


Renormalization: microscopic constituents
merge to produce new collective effects



Renormalization: microscopic constituents
merge to produce new collective effects
on macroscopic scales

Renormalization Group

Wikipedia:

"mathematical apparatus that allows systematic investigation of the changes of a physical system as viewed at different *distance scales*"

➡ **Universality**: physics at long distances is largely insensitive to details of physics at short distances

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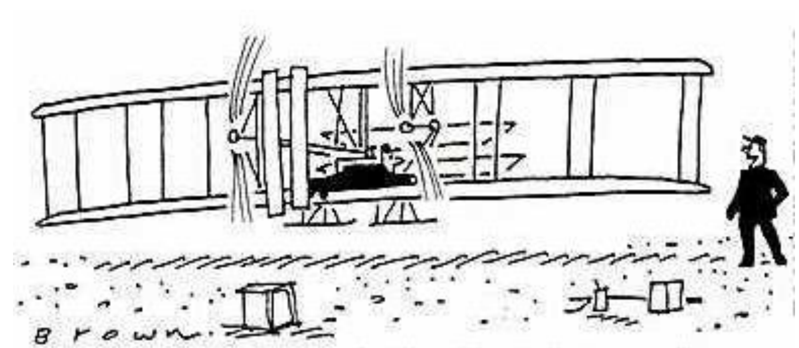
Why Physics Works!

NOT Our World:



12 seconds! The heck with loop quantum gravity. We'll have to try a string theory approach.

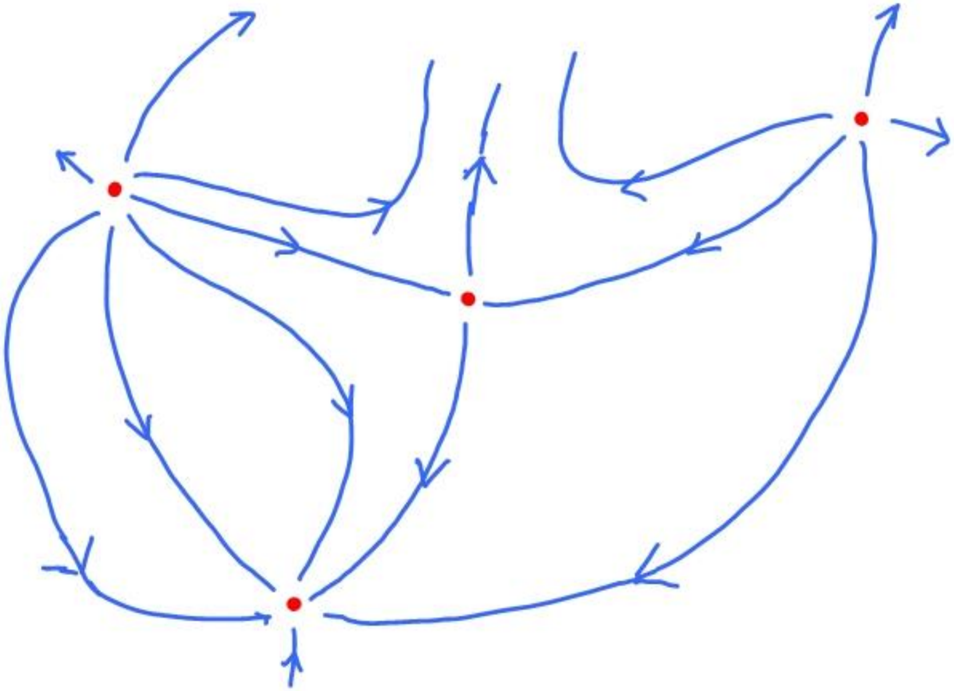
Renormalization Group Flows

Wikipedia:

"mathematical apparatus that allows systematic investigation of the changes of a physical system as viewed at different *distance scales*"

Effects of short-distance physics is absorbed in values of a few parameters of an effective theory

RG Flows describe how parameters of a quantum field theory change as more and more degrees of freedom at different scales are methodically "integrated out"



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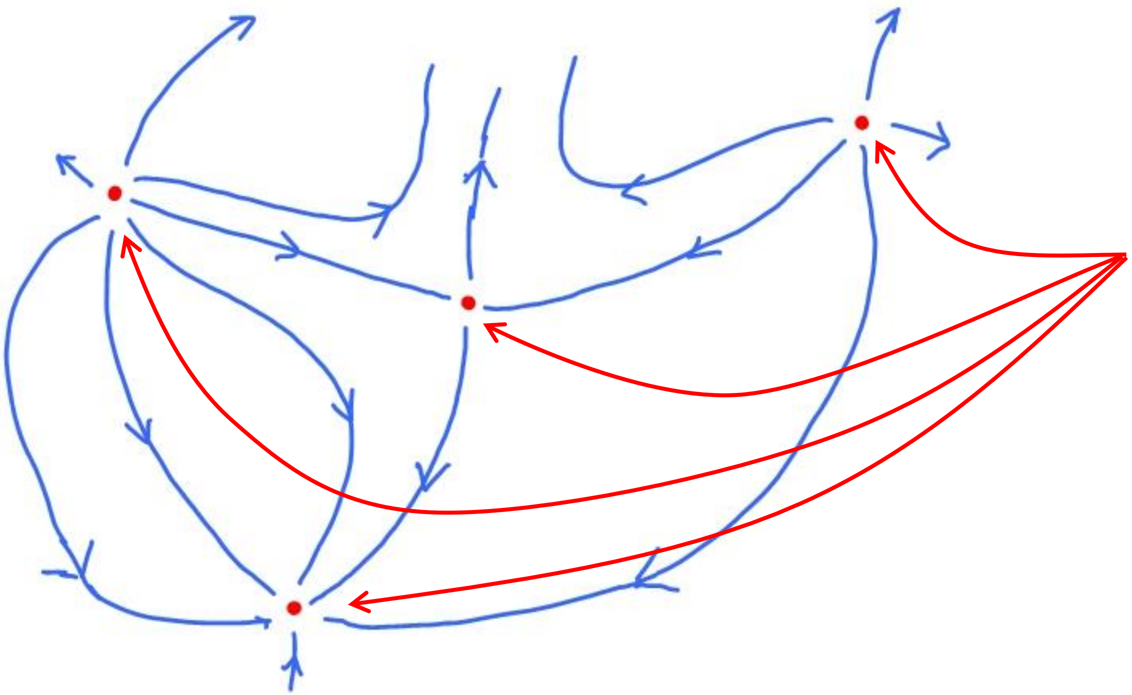

fixed points: flow is stationary

"critical phenomena"

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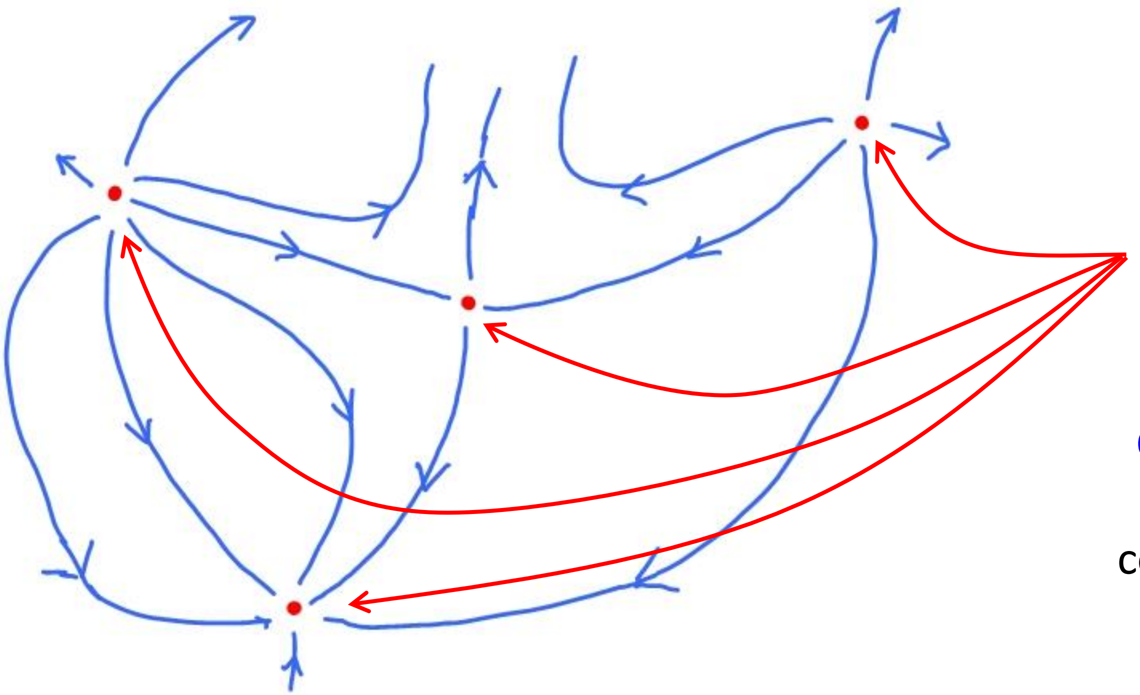

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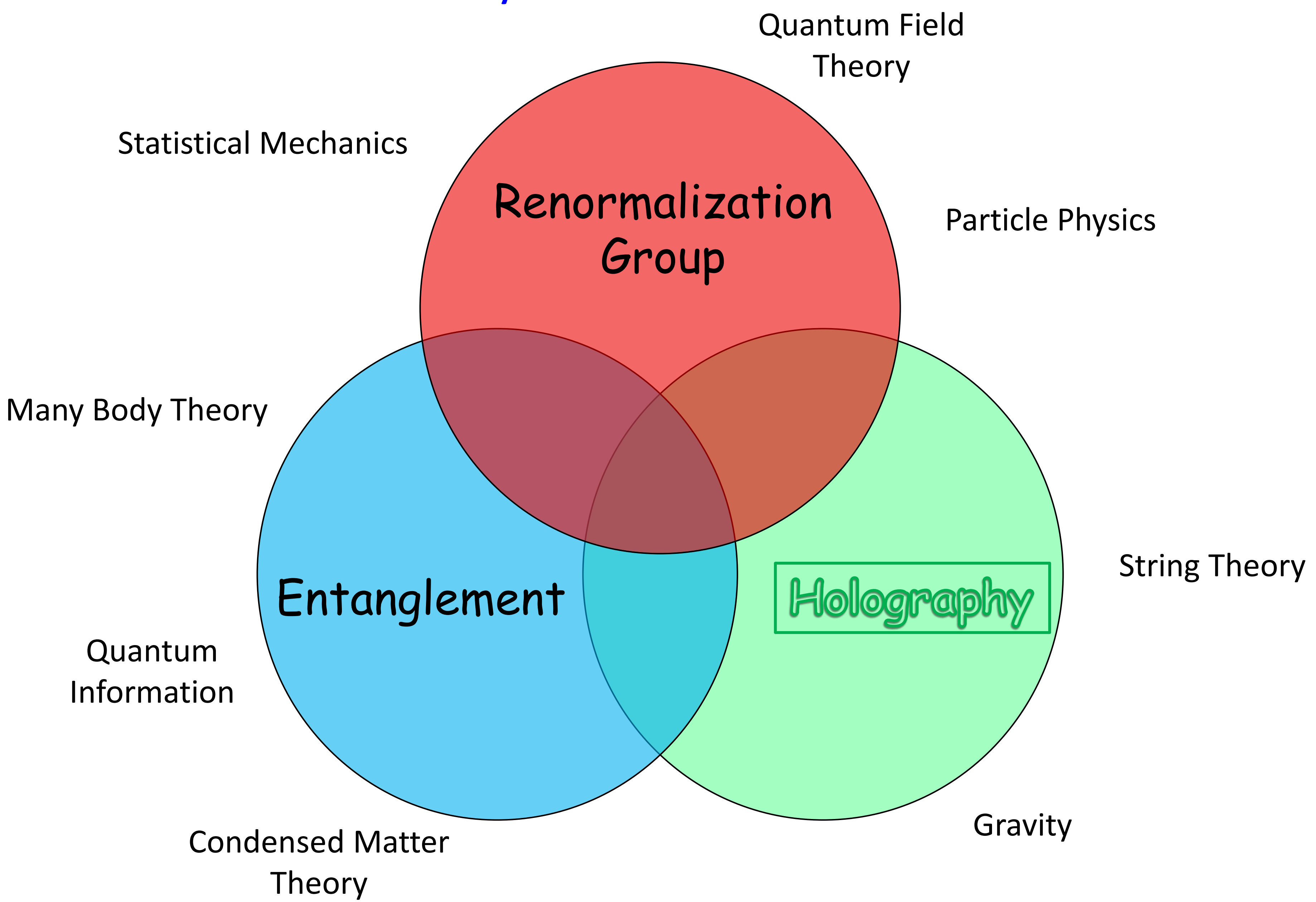
“critical phenomena”

Conformal Field Theory (CFT):
QFT with few dimensionless couplings; scale invariant theory

New Collisions in Theoretical Physics:



Holography

Holography

- idea in quantum gravity has its origins in trying to understand paradoxes related to the realization that black holes are thermal systems emitting (almost) blackbody radiation

- Bekenstein and Hawking: black hole horizons carry entropy!!

thermodynamics

relativity

geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\ \frac{\mathcal{A}}{4G}$$

quantum

gravity

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- quantum gravity provides a fundamental scale

$$\ell_P^2 = 8\pi\, G\;\hbar/c^3$$

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$$S_{BH} = 2\pi \, \frac{\mathcal{A}}{\ell_P^2} \; k_B$$

- ℓ_P **is really really small:** $\ell_P = 10^{-35}\ m$ (LHC: $\sim 10^{-20}\ m$)

- **get really big entropies!!**

big bag of hot gas: $S(sun) \simeq 10^{58}\ k_B$

black hole: $S(1\,M_\odot) \simeq 10^{77}\ k_B$

Holography

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$$S_{BH} = 2\pi \, \frac{\mathcal{A}}{\ell_P^2} \quad k_B$$

- k_B reminds us that entropy is associated with "heat"

$\longrightarrow$ black hole thermodynamics

- statistical mechanics also says: $S = -Tr\left[\rho_A \, \log \rho_A\right]$

$\longrightarrow$ black hole microstates

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$\longrightarrow$ black hole microstates

$\longrightarrow$ "holography" suggests that the horizon is a surface that encodes information about microstates but also that dynamics and evolution of black hole can be described in terms of an effective theory living on the horizon

('t Hooft; Susskind)

AdS/CFT Correspondence:

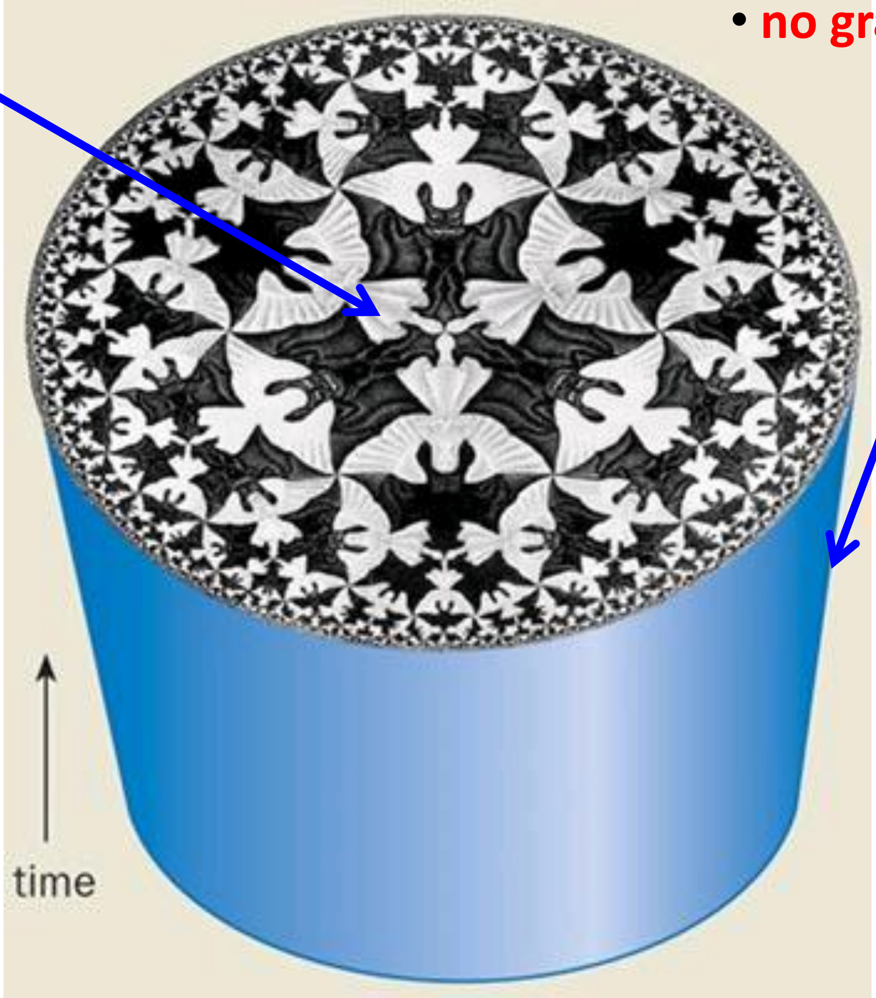
Bulk:

- quantum gravity
- negative cosmological constant
- **d+1** spacetime dimensions

Boundary:

- quantum field theory
- no scale (at quantum level)
- **d** spacetime dimensions
- **no gravity!**

anti-de Sitter space



conformal field theory

(Maldacena '97)

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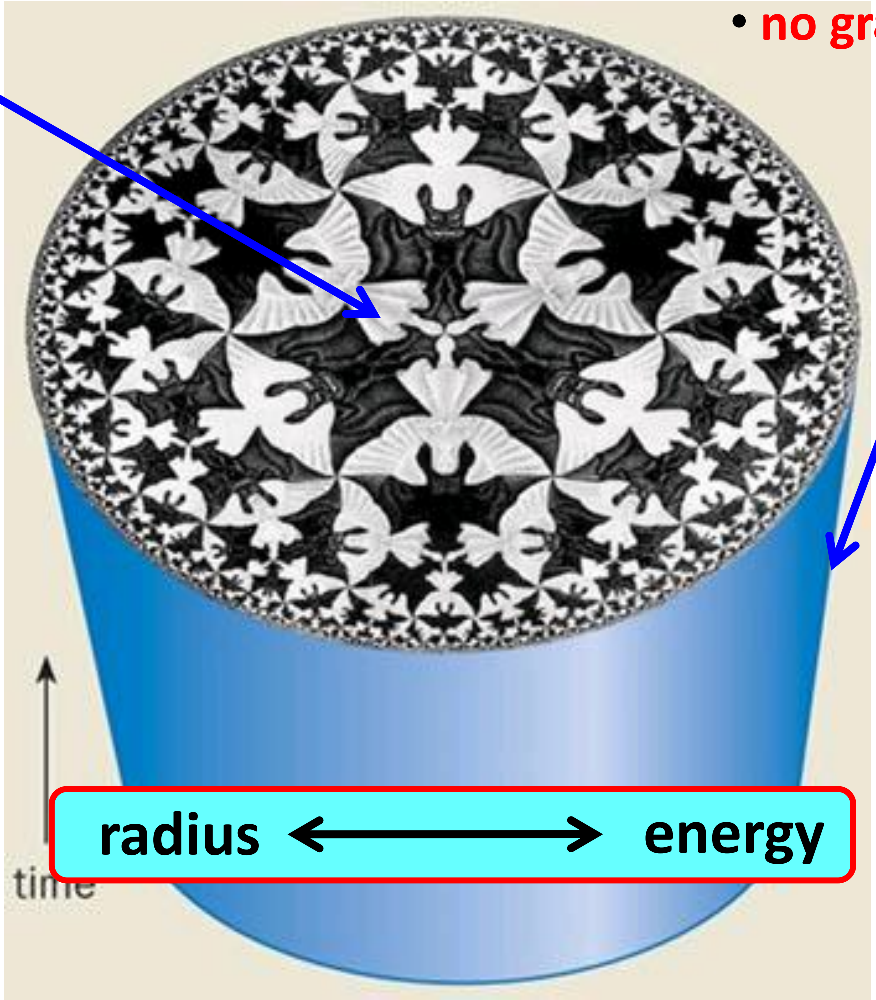

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Holographic RG flows:

(Freedman, Gubser, Pilch & Warner)

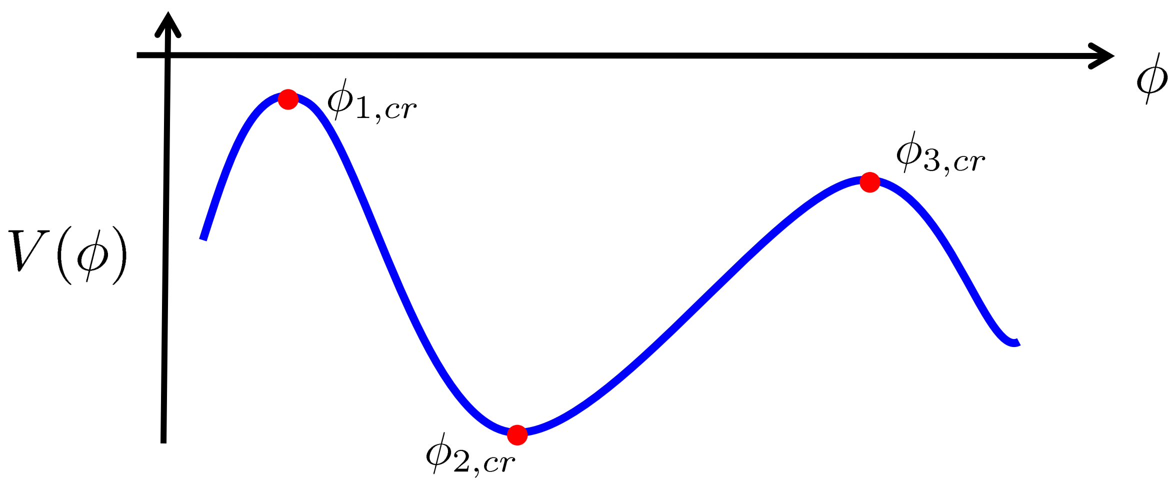
(Girardello, Petrini, Porrati & Zaffaroni)

$$I = \frac{1}{2\ell_P^{d-1}} \int d^{d+1}x\,\sqrt{-g}\left[R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

• imagine potential has stationary points giving negative cosmological constant

$$\longrightarrow \quad V(\phi_{i,cr}) = 2\,\Lambda_i \quad < 0$$

• $\phi = \phi_{i,cr}$ $\longrightarrow$ the gravity solution is AdS space – dual to a particular CFT



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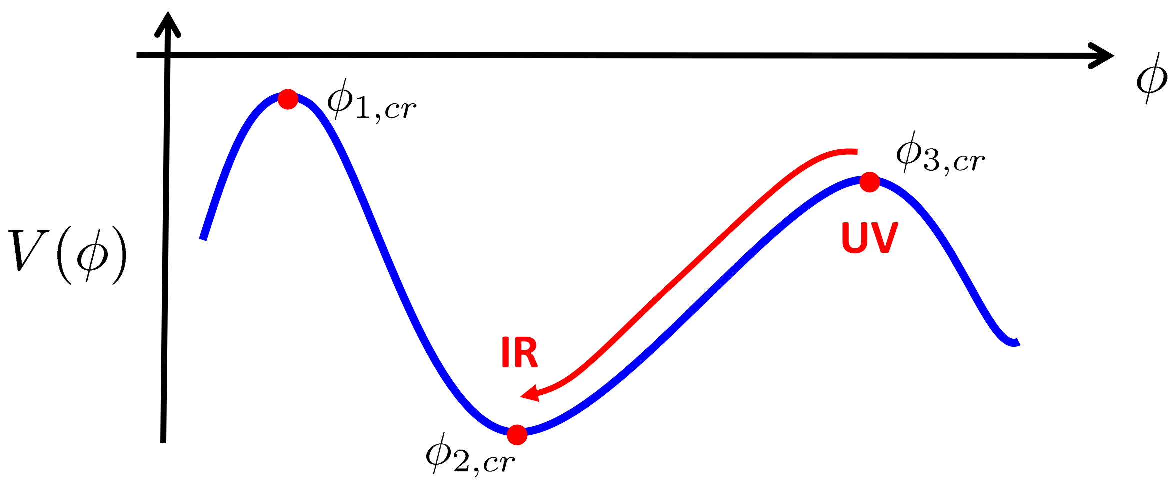
- Holographic RG flows are solutions starting at one stationary point at large radius and ending at another at small radius – connects two CFTs between high and low energies



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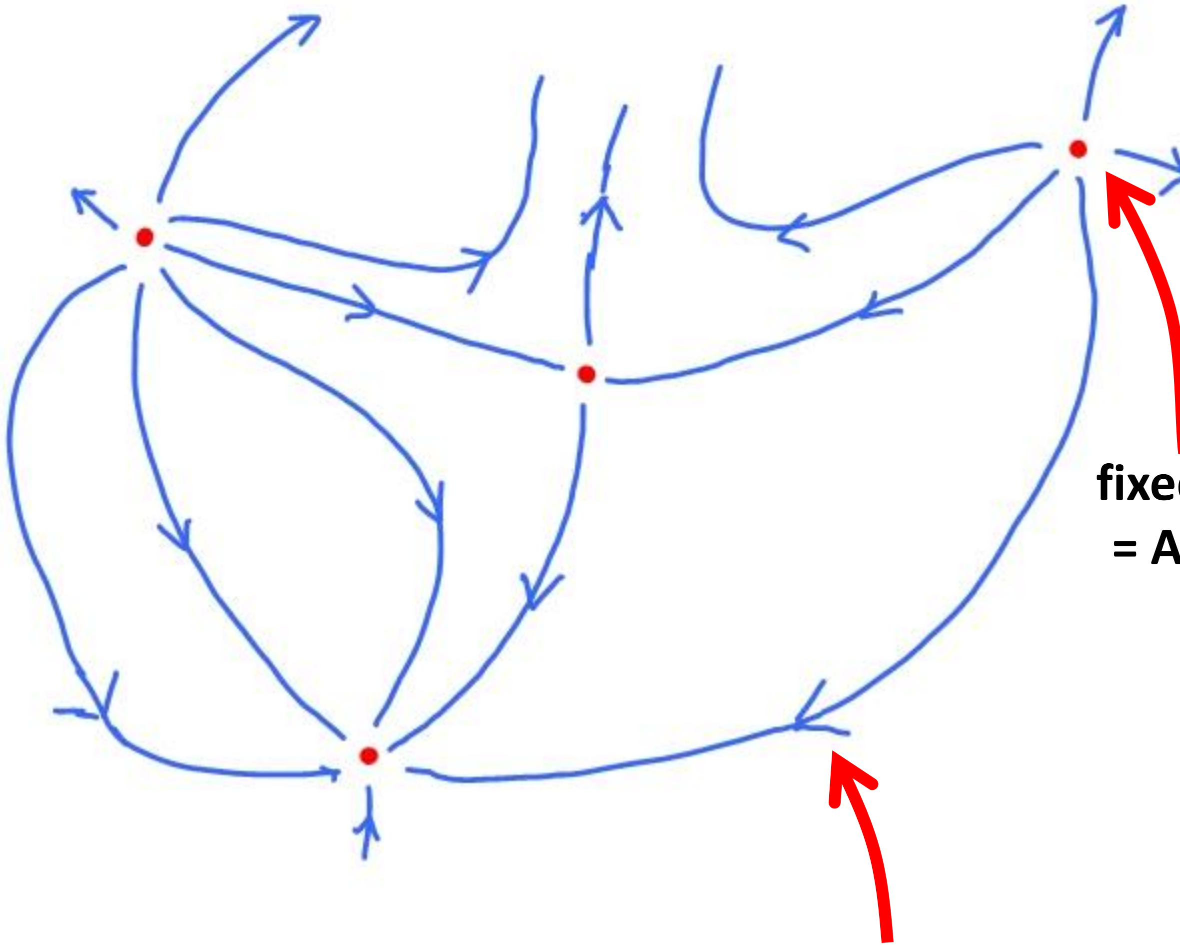
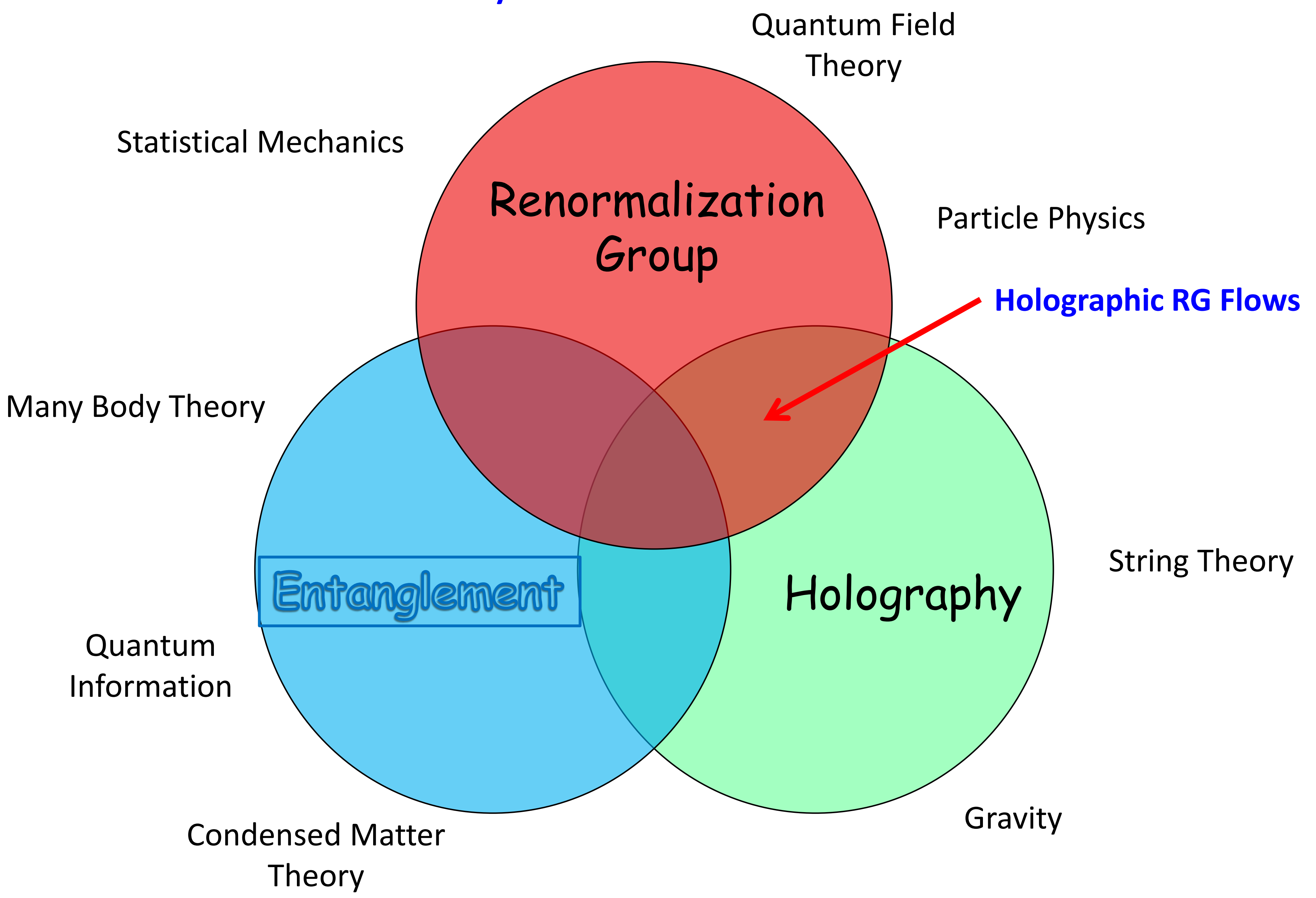


fixed points (CFT's)
= AdS spacetimes

RG flows = geometries interpolating
between asymptotic AdS regions

New Collisions in Theoretical Physics:



Quantum Entanglement

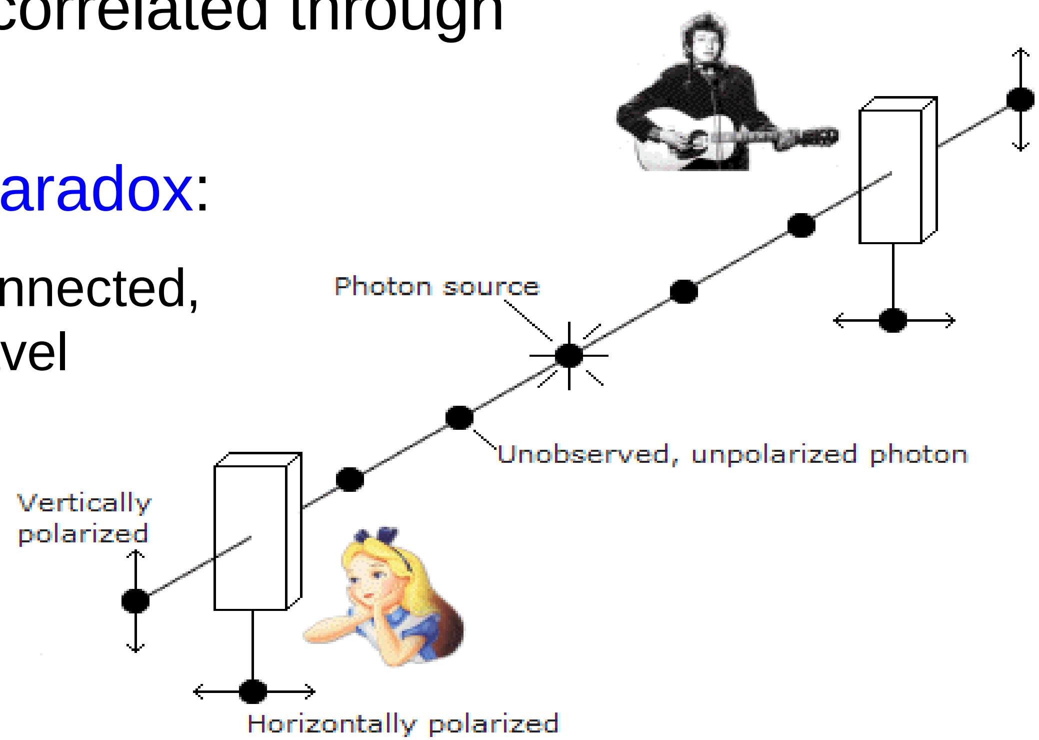
• different subsystems are correlated through global state of full system

Einstein-Podolsky-Rosen Paradox:

• properties of pair of photons connected, no matter how far apart they travel

"*spukhafte Fernwirkung*" = spooky action at a distance

$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\,\Big)$$



Quantum Information: entanglement becomes a resource for (ultra)fast computations and (ultra)secure communications

Condensed Matter: key to "exotic" phases and phenomena, e.g., quantum Hall fluids, unconventional superconductors, quantum spin fluids, ...

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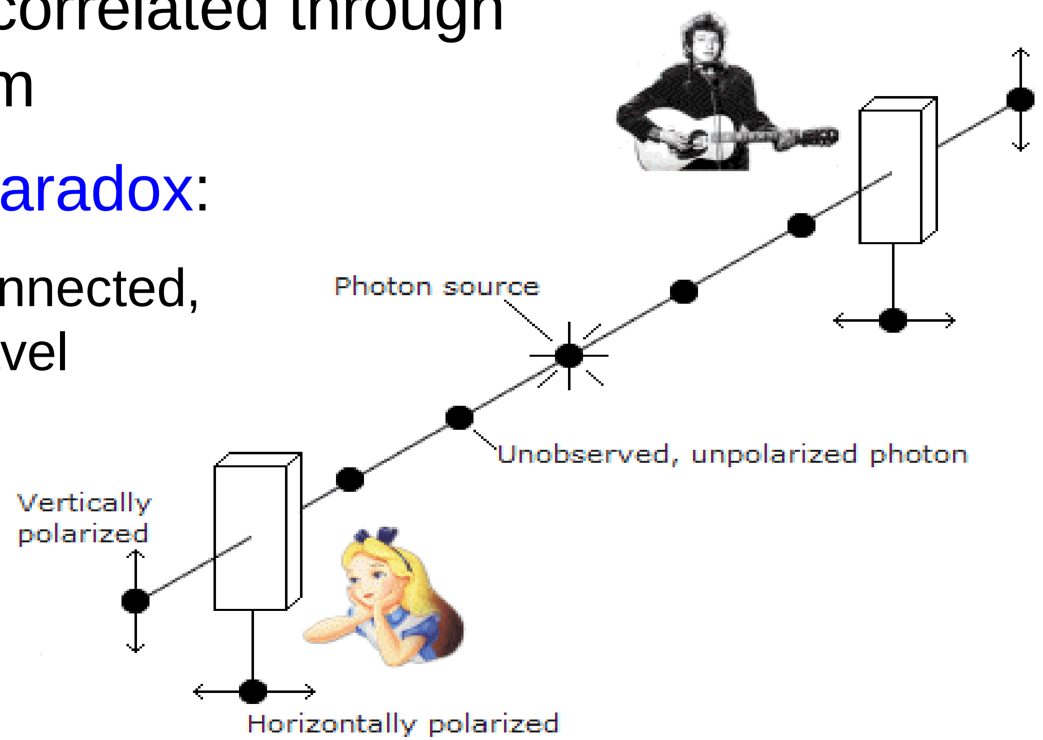

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compare: $$|\psi'\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle + |\downarrow\downarrow\rangle\Big)$$

$$= \frac{1}{2}\Big(|\uparrow\rangle + |\downarrow\rangle\Big) \otimes \Big(|\uparrow\rangle + |\downarrow\rangle\Big)$$ ➡ **No Entanglement!!**

$$|\psi''\rangle = \frac{1}{2}\Big(|\uparrow\uparrow\rangle + |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle - |\downarrow\downarrow\rangle\Big)$$ ➡ **Entangled!!**

Entanglement Entropy:

- general diagnostic: divide quantum system into two parts and use entropy as measure of correlations between subsystems
- <u>procedure:</u>
  - divide system into two subsystems, eg, A and B
  - trace over degrees of freedom in subsystem B
  - remaining dof in A are described by a density matrix ρ_A
  - calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \, \log \rho_A\right]$

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$$|\psi\rangle = \frac{1}{\sqrt{2}}\Big(\,|\!\uparrow\downarrow\rangle + |\!\downarrow\uparrow\rangle\Big) \longrightarrow \rho = \mathrm{Tr}_2\,(|\psi\rangle\langle\psi|) = \frac{1}{2}\,(|\!\downarrow\rangle\langle\downarrow\!| + |\!\uparrow\rangle\langle\uparrow\!|)$$

$$\longrightarrow S_{EE} = \log 2$$

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$$= \frac{1}{2}\Big(\,|\!\uparrow\rangle + |\!\downarrow\rangle\Big)\otimes\Big(\,|\!\uparrow\rangle + |\!\downarrow\rangle\Big) \longrightarrow S_{EE} = 0$$

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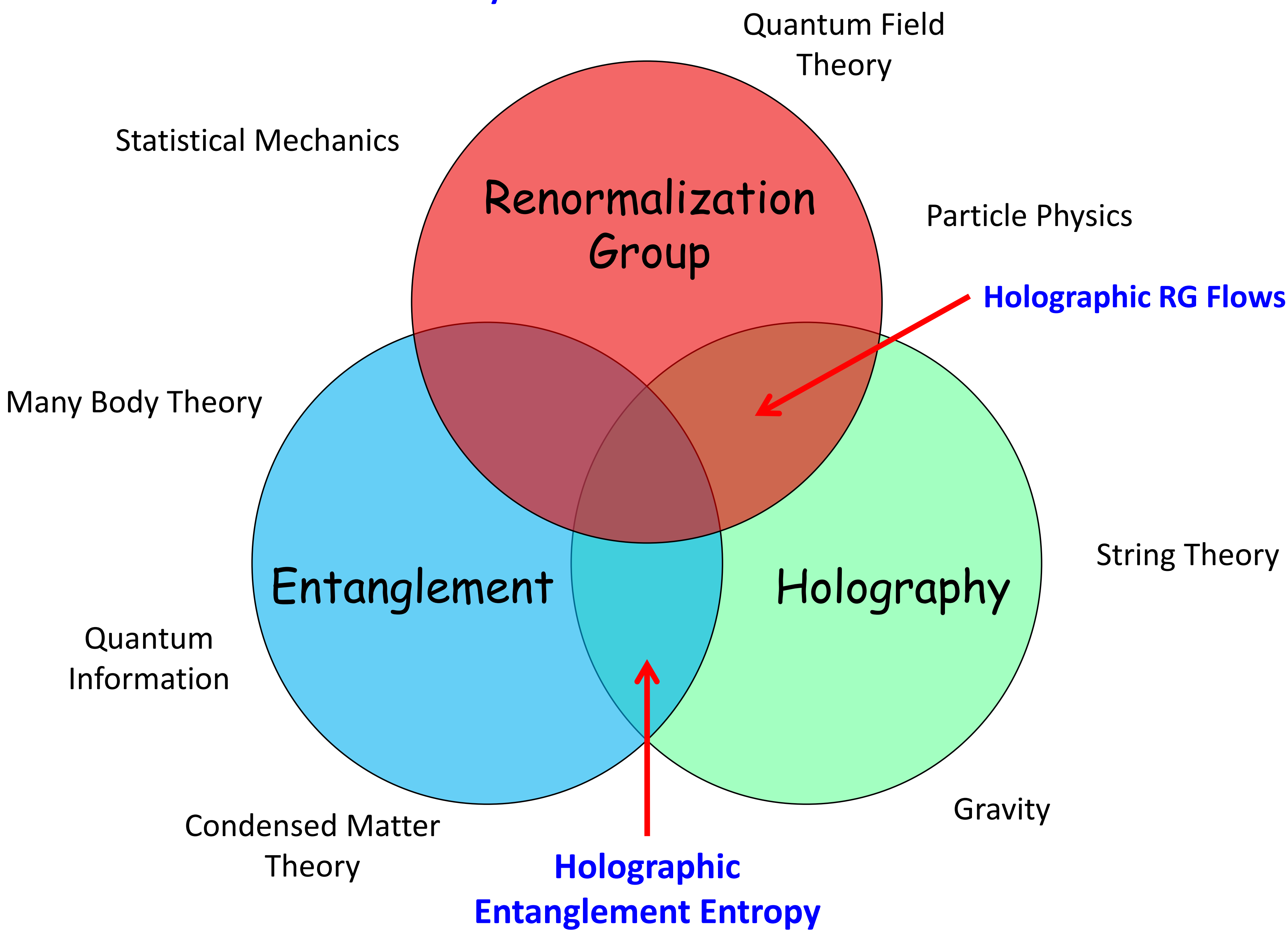
New Collisions in Theoretical Physics:



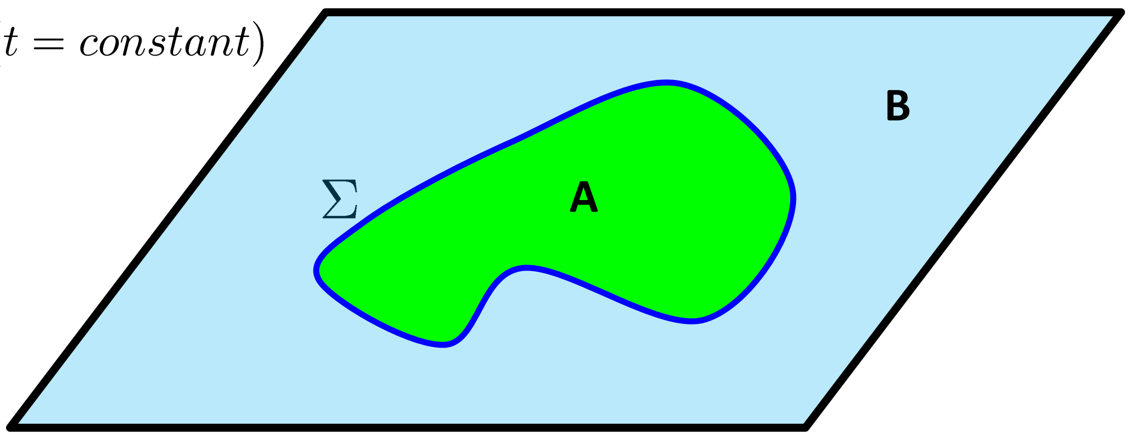
Entanglement Entropy 2:

- in the context of holographic entanglement entropy, S_{EE} is applied in the context of quantum field theory

- in QFT, typically introduce a (smooth) boundary or entangling surface Σ which divides the space into two separate regions
- integrate out degrees of freedom in "outside" region
- remaining dof are described by a density matrix ρ_A

⟶ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \log \rho_A\right]$

$(t = constant)$



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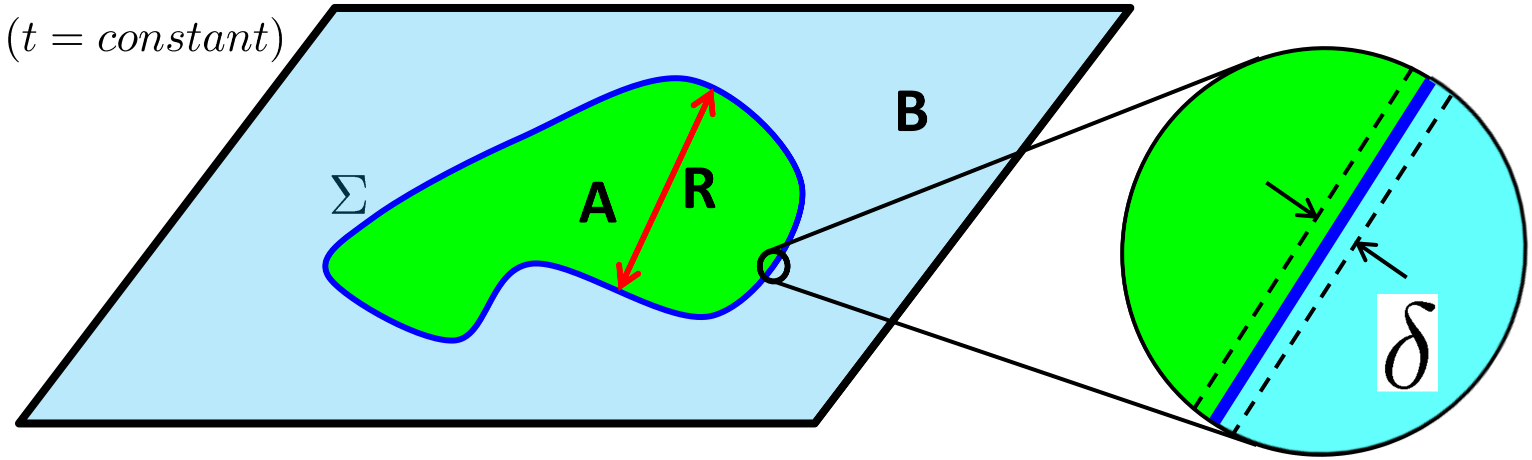
- remaining dof are described by a density matrix ρ_A

→ calculate von Neumann entropy: $S_{EE} = -Tr\left[\rho_A \, \log \rho_A\right]$



- result is UV divergent!
- must regulate calculation: δ = short-distance cut-off

$$S \; = \; c_0 \frac{R^{d-2}}{\delta^{d-2}} \; + \; c_2 \frac{R^{d-4}}{\delta^{d-4}} \; + \; \cdots$$

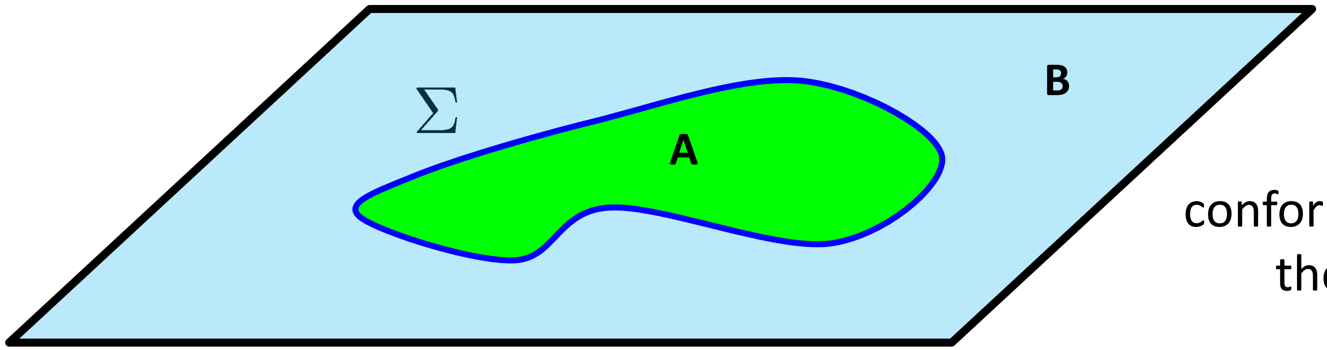
d = spacetime dimension

- careful analysis reveals geometric structure, eg, $S = \tilde{c}_0 \, \dfrac{\mathcal{A}_\Sigma}{\delta^{d-2}} + \cdots$

(Sorkin `85; Bombelli, Koul, Lee & Sorkin `86)

Holographic Entanglement Entropy:

(Ryu & Takayanagi)



conformal field theory

$$S(A) \;=\; ??$$

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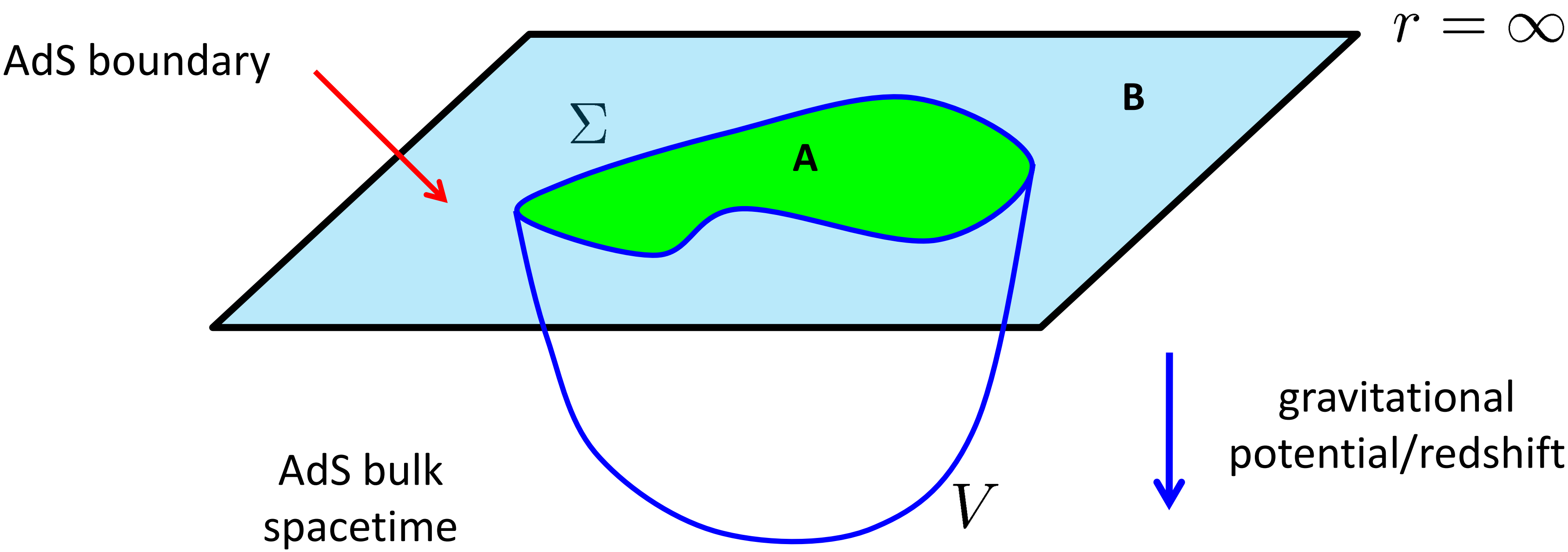

$$S(A) = \operatorname*{ext}_{\partial V = \Sigma} \frac{A_V}{4G_N}$$

looks like
BH entropy!

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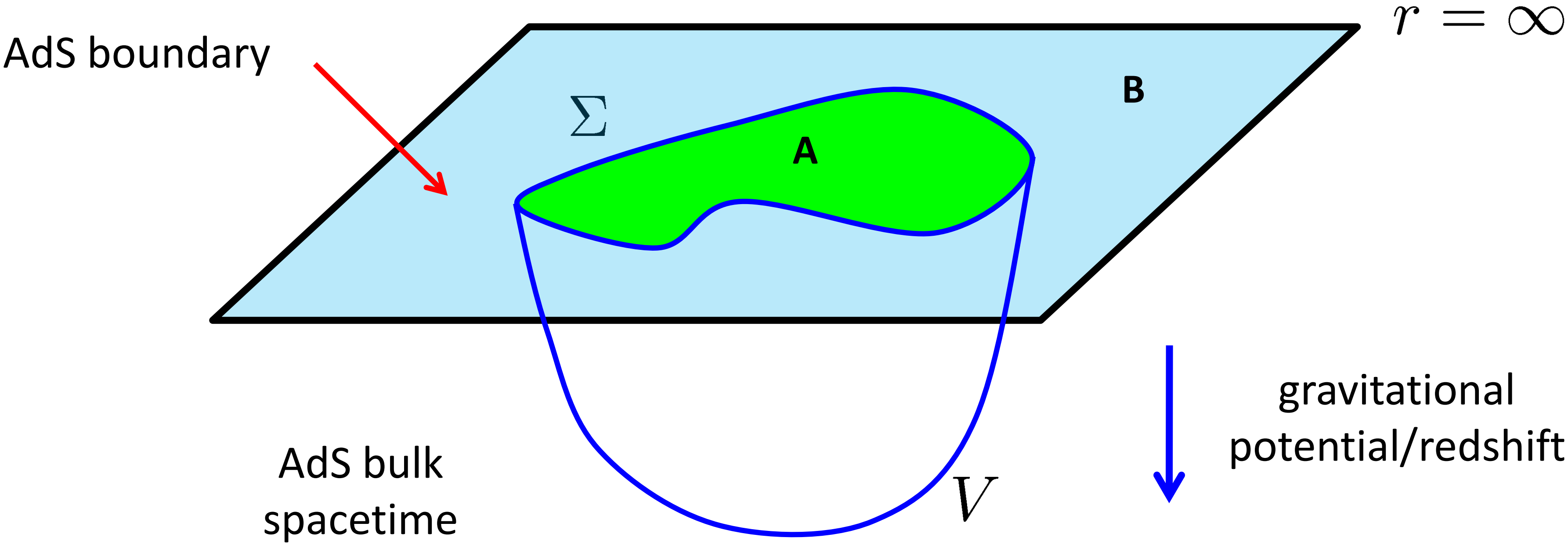

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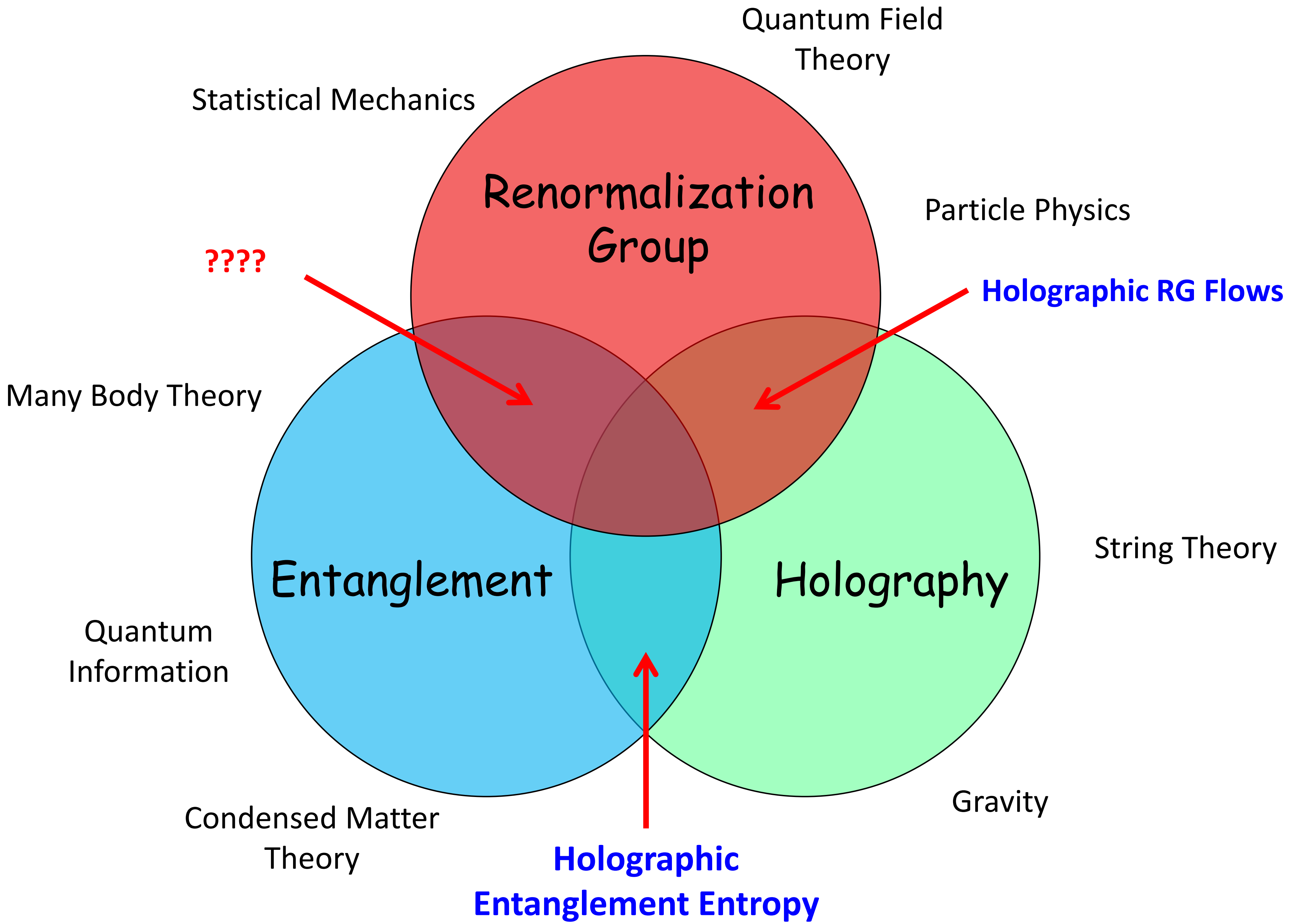

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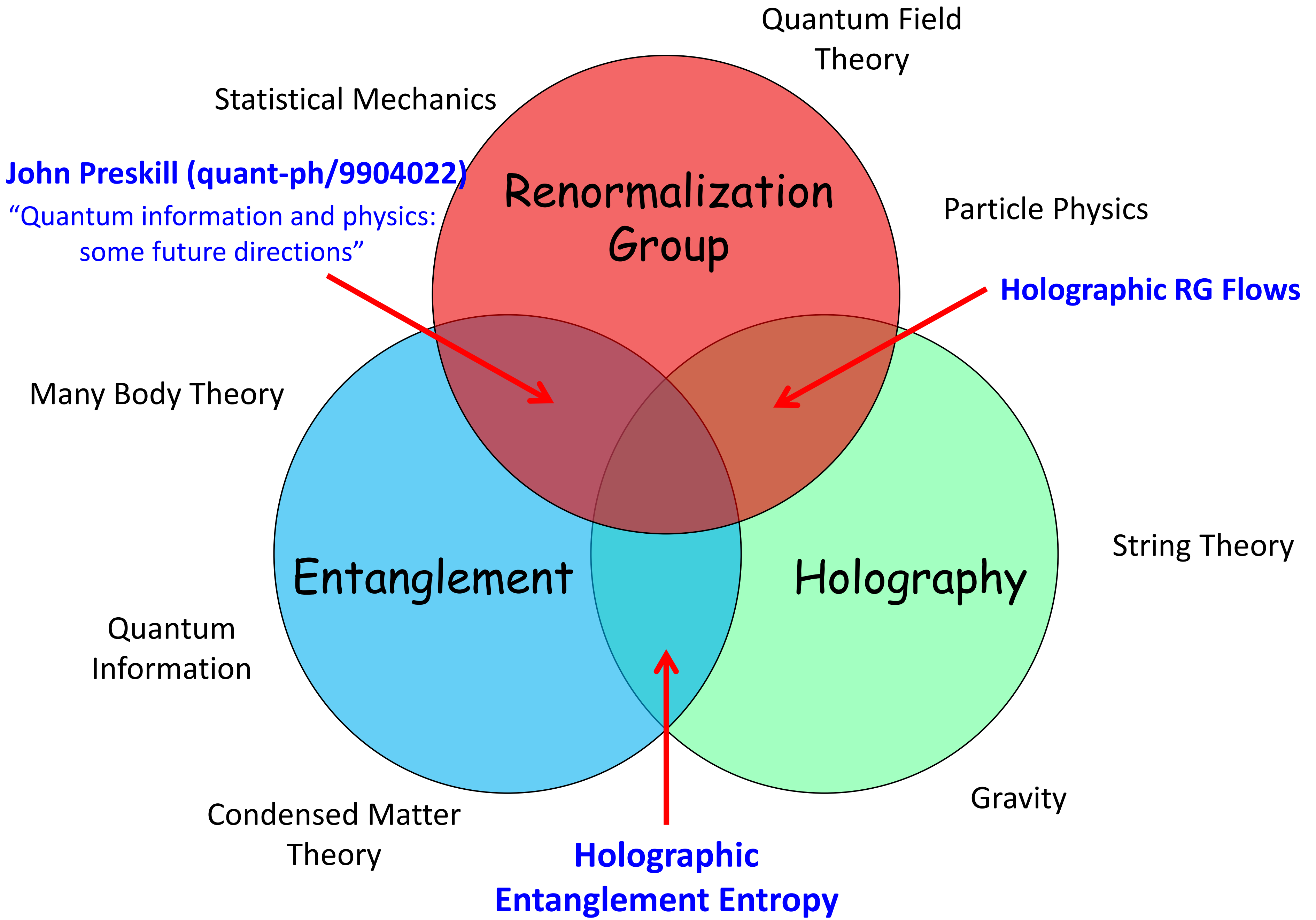


Zamolodchikov c-theorem (1986):

• renormalization-group flows can seen as one-parameter motion:

$$\frac{d}{dt} \equiv -\beta^i(g)\,\frac{\partial}{\partial g^i} \qquad (\, t \text{ = scale; } \beta^i = \partial_t g^i \,)$$

in the space of (renormalized) coupling constants $\{g^i,\ i = 1, 2, 3, \cdots\}$ with beta-functions as “velocities”

• for unitary, Lorentz-invariant, renormalizable QFT’s in two dimensions, there exists a positive-definite real function of the coupling constants : $C(g)$

1. monotonically decreasing along RG flows: $\frac{d}{dt}C(g) \leq 0$

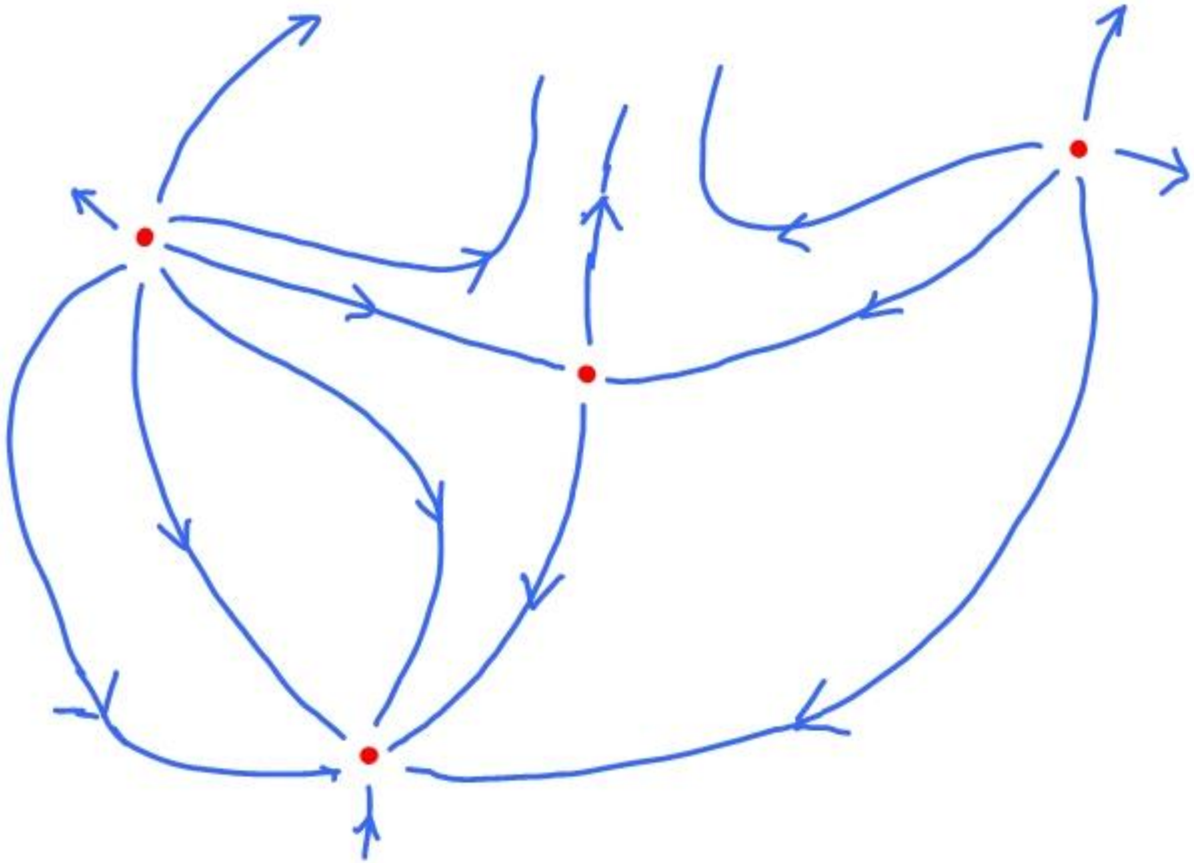
2. “stationary” at fixed points : $g^i = (g^*)^i$

$$\beta^i(g^*) = 0 \longleftrightarrow \frac{\partial}{\partial g^i}C(g) = 0$$

3. at fixed points, it equals central charge of corresponding CFT

$$C(g^*) = c$$

with Zamolodchikov's framework:



BECOMES

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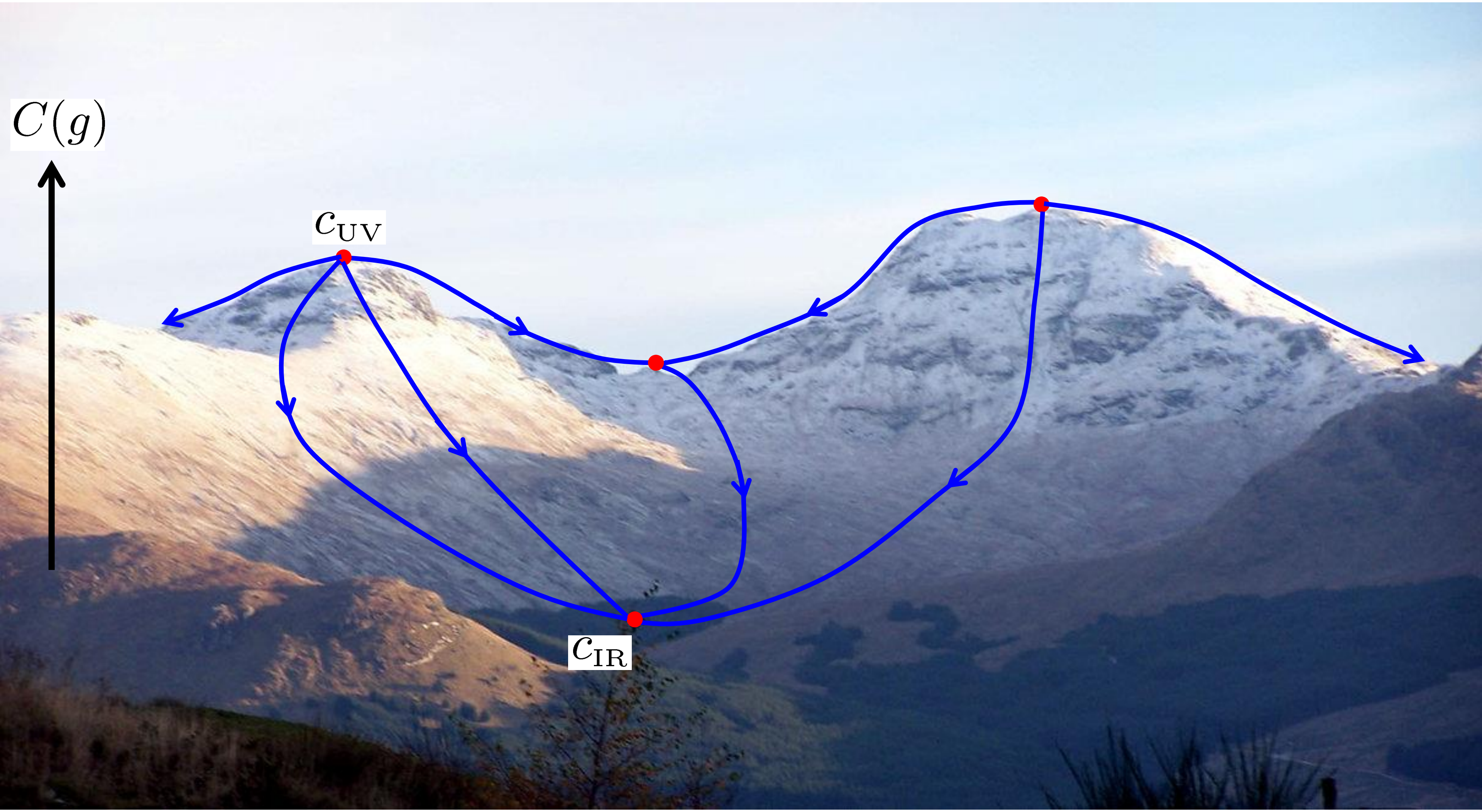


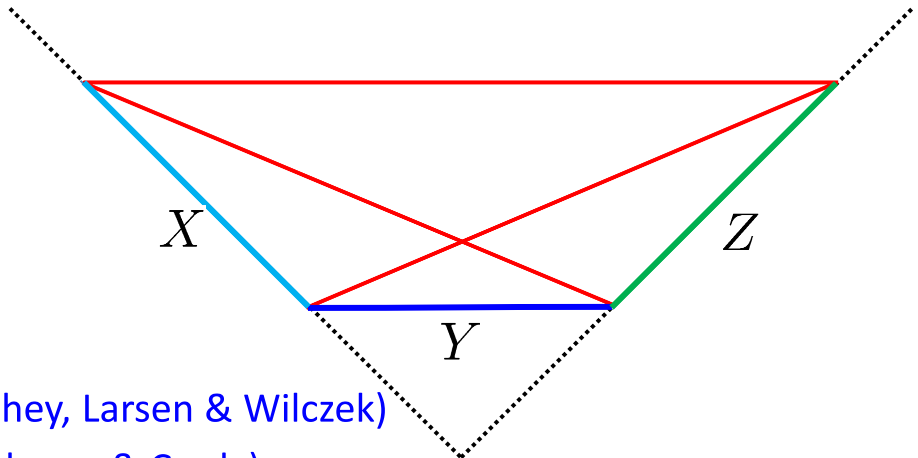
Consequence for any RG flow in d=2: $c_{\mathrm{UV}} > c_{\mathrm{IR}}$

(Casini & Huerta '04)

RG flows Meet Entanglement:

- c-theorem for d=2 RG flows can be established using unitarity, Lorentz invariance and **strong subaddivity inequality**:

$$S(X\cup Y\cup Z)-S(X\cup Y)-S(Y\cup Z)+S(Y)\leq 0$$



- define: $C(\ell)=3\,\ell\;\partial_\ell S(\ell)$

$$\longrightarrow \quad \partial_\ell C(\ell)\leq 0$$

- for d=2 CFT: $S=\frac{c}{3}\log(\ell/\delta)+a_0$ (Holzhey, Larsen & Wilczek) (Calabrese & Cardy)

$$\longrightarrow \quad C_{\rm CFT}(\ell)=c$$

- hence it follows that: $\boxed{c_{\rm UV}>c_{\rm IR}}$

C-theorems in higher dimensions??

d=2: $$\langle\, T_\mu{}^\mu \,\rangle = -\frac{c}{12}\, R$$

d=4: $$\langle\, T_\mu{}^\mu \,\rangle = \frac{c}{16\pi^2}\, I_4 - \frac{a}{16\pi^2} E_4 - \frac{a'}{16\pi^2}\, \nabla^2 R$$

where $I_4 = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}$ and $E_4 = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2$

- in 4 dimensions, have three central charges: $c,\ a,\ a'$
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a-theorem: proposed by Cardy (1988)

- numerous nontrivial examples, eg, perturbative fixed points (Osborn '89), SUSY gauge theories (Anselmi et al '98; Intriligator & Wecht '03)
- holographic field theories with Einstein gravity dual (Freedman et al '99; Giradello et al '98)
- progress stalled; no proof found; . . . .

- **past few years have seen a resurgence of interest and rapid progress**

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past three years have seen a resurgence of interest and remarkable progress:

- RG flows in generalized holographic models with higher curvatures (RM & Sinha '10)

→ found new holographic c-theorem: $[a_d^*]_{UV} \geq [a_d^*]_{IR}$

$$a_d^* = \frac{\pi^{(d-2)/2}\, L^{d-1}}{8\Gamma(d/2)\, G_N f_\infty^{(d-1)/2}} \left(1 - \frac{2(d-1)}{d-3}\lambda f_\infty - \frac{3(d-1)}{d-5}\mu f_\infty^2\right)$$

where $\alpha^2 - f_\infty + \lambda f_\infty^2 + \mu f_\infty^3 = 0$

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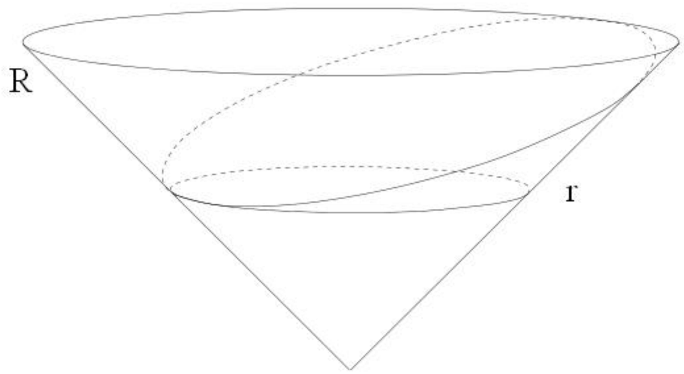

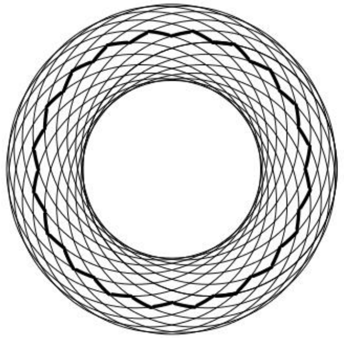

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- **d=4 a-theorem proved!** dilaton effective actions (Komargodski & Schwimmer '11)

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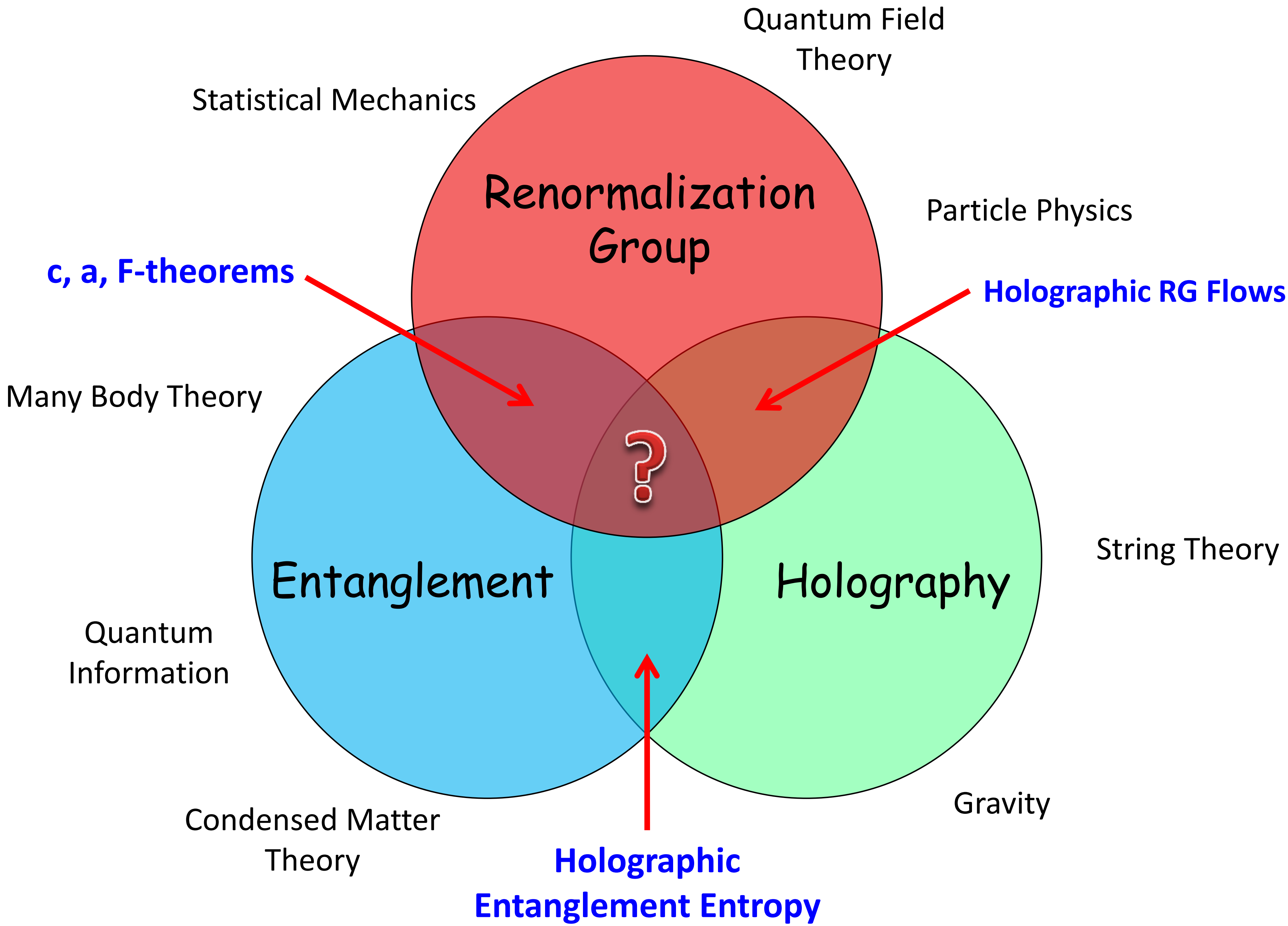
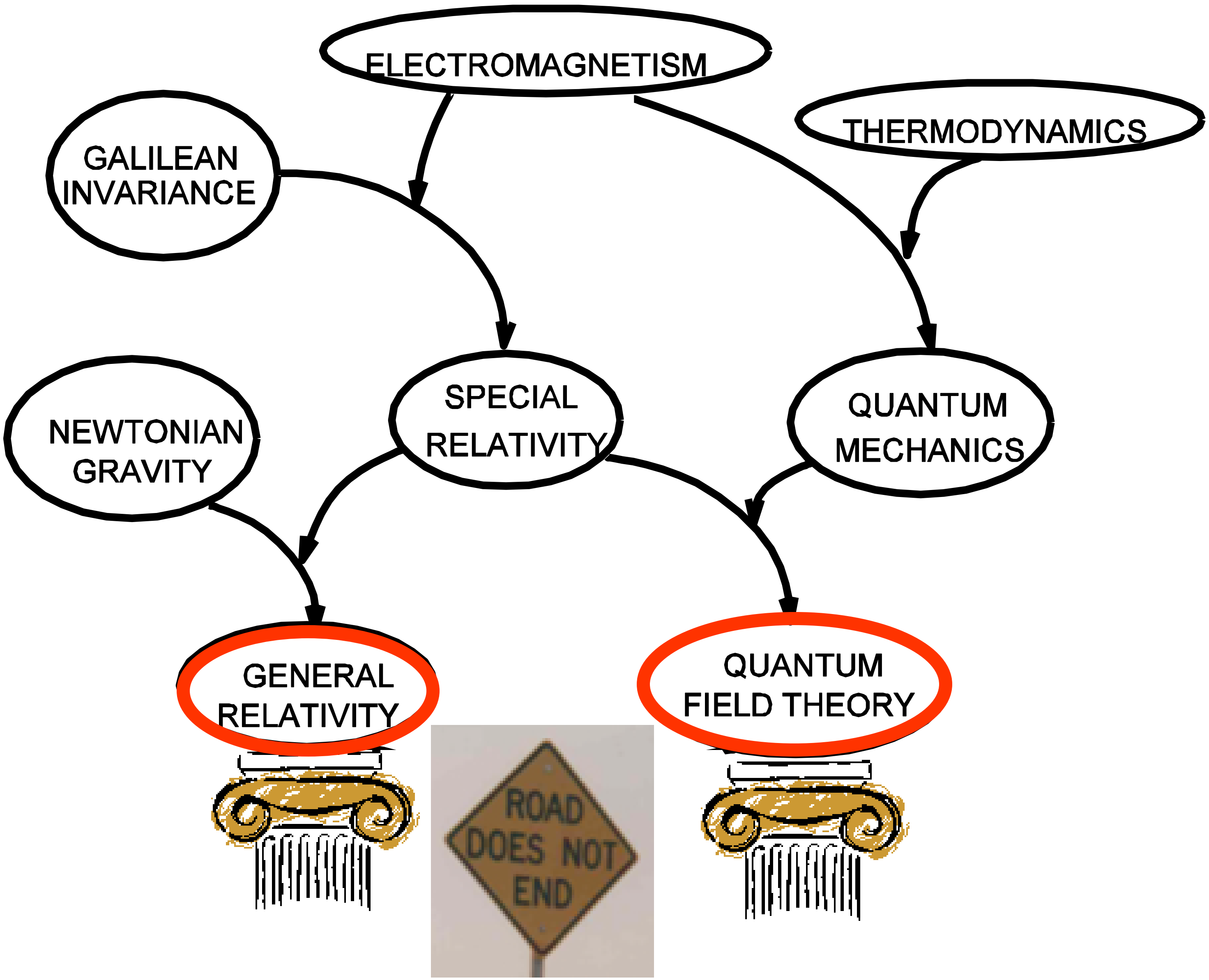


Physics of the 20th century:

A grand synthesis and progression of ideas!



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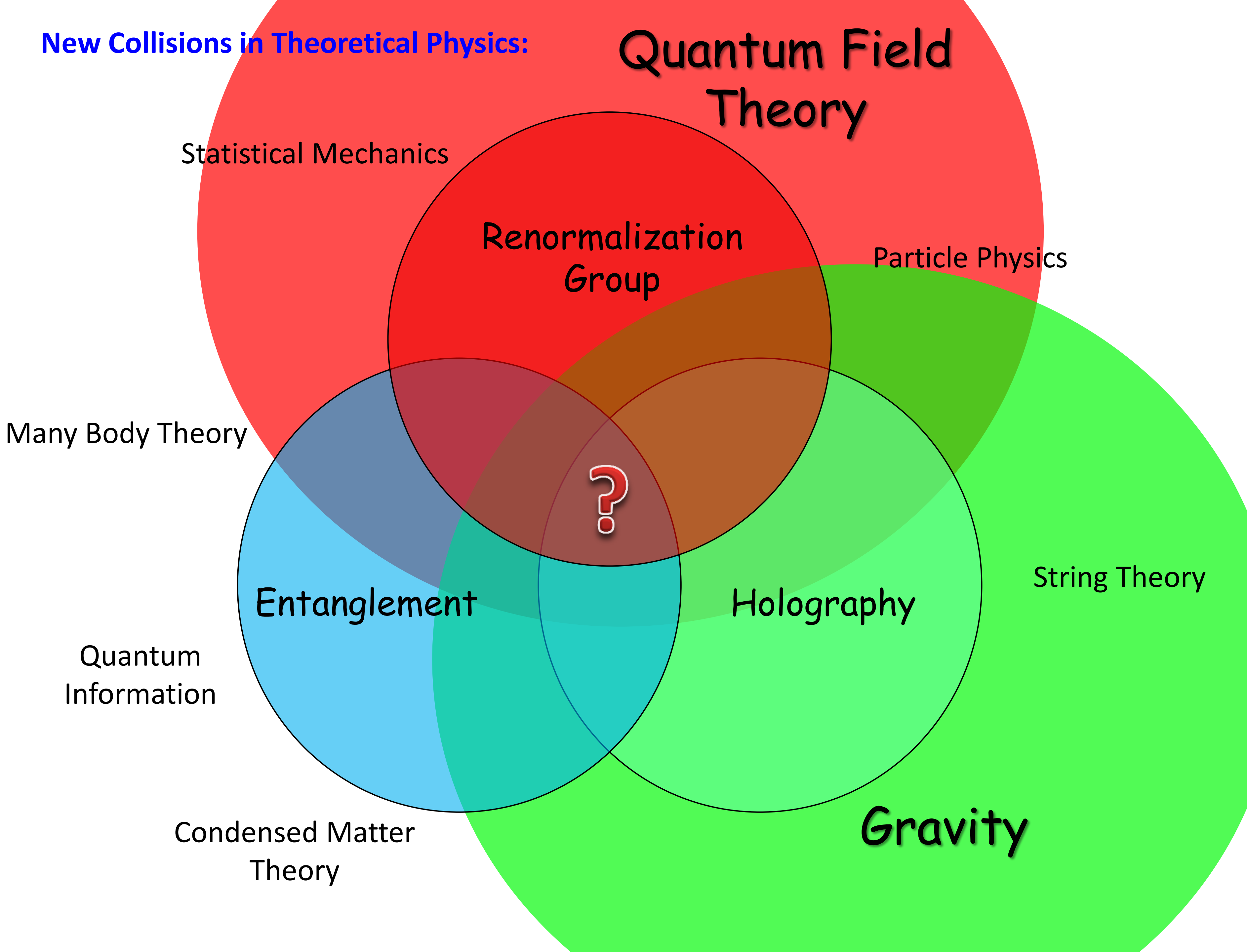

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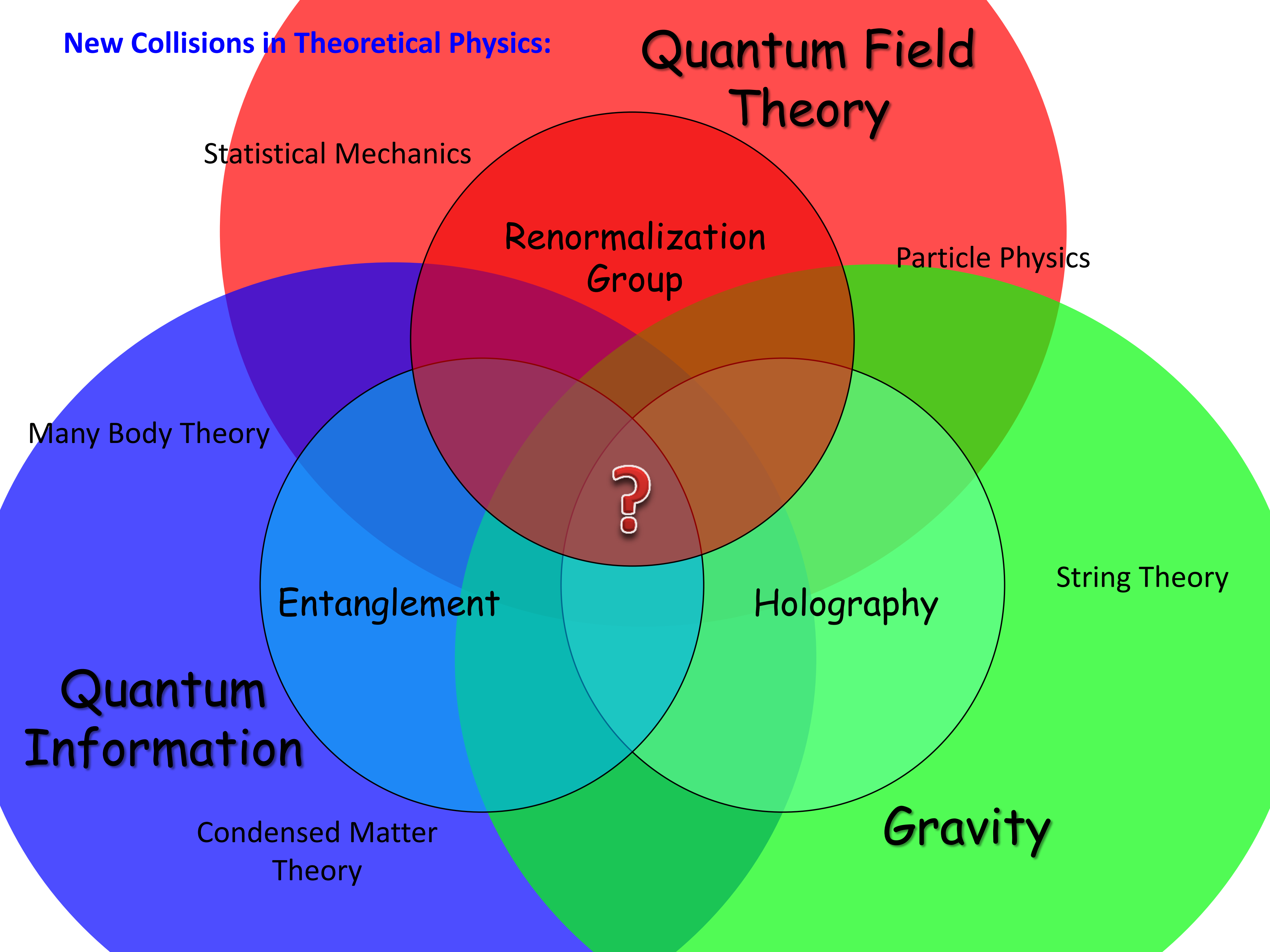

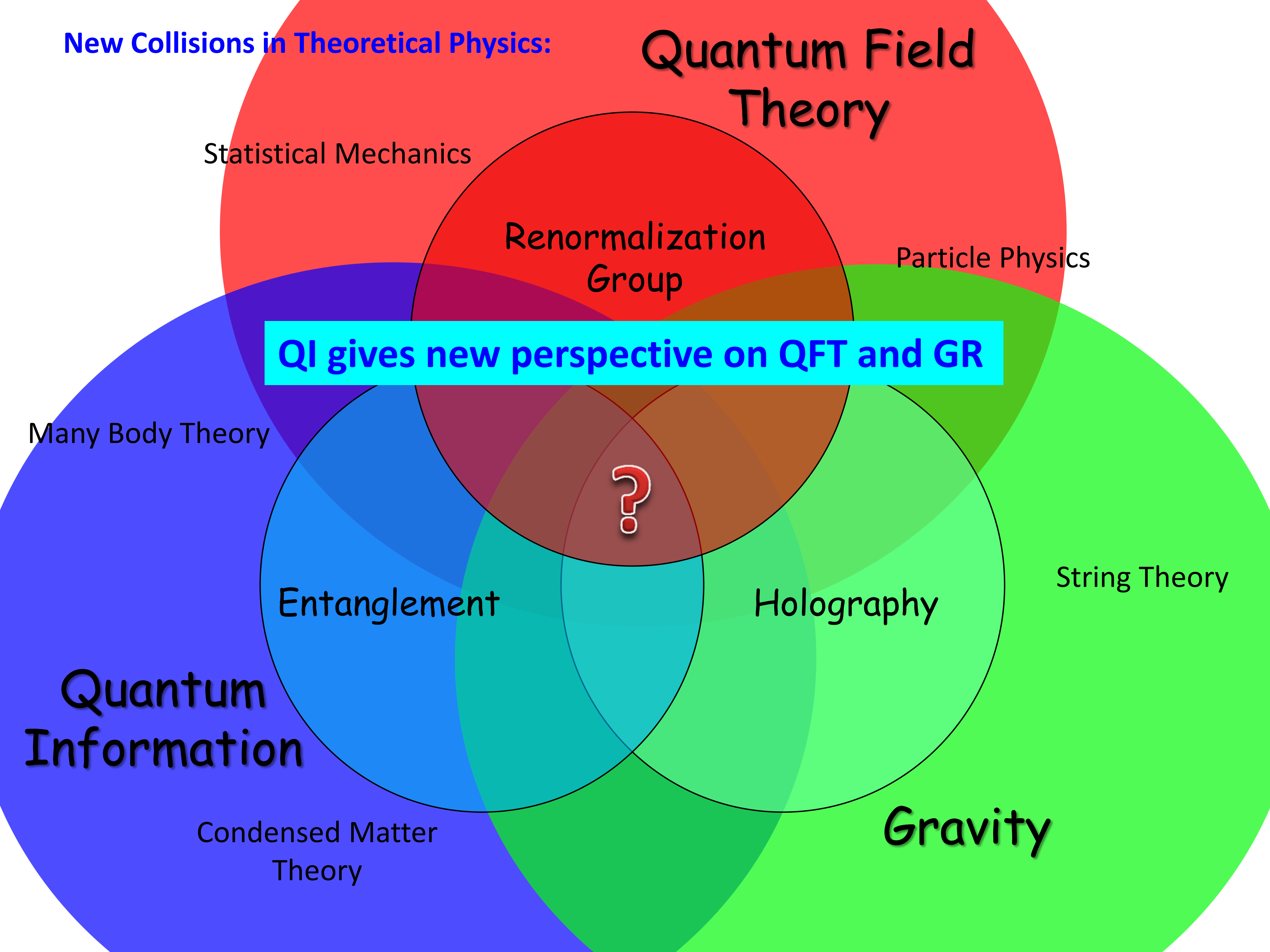
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Holography
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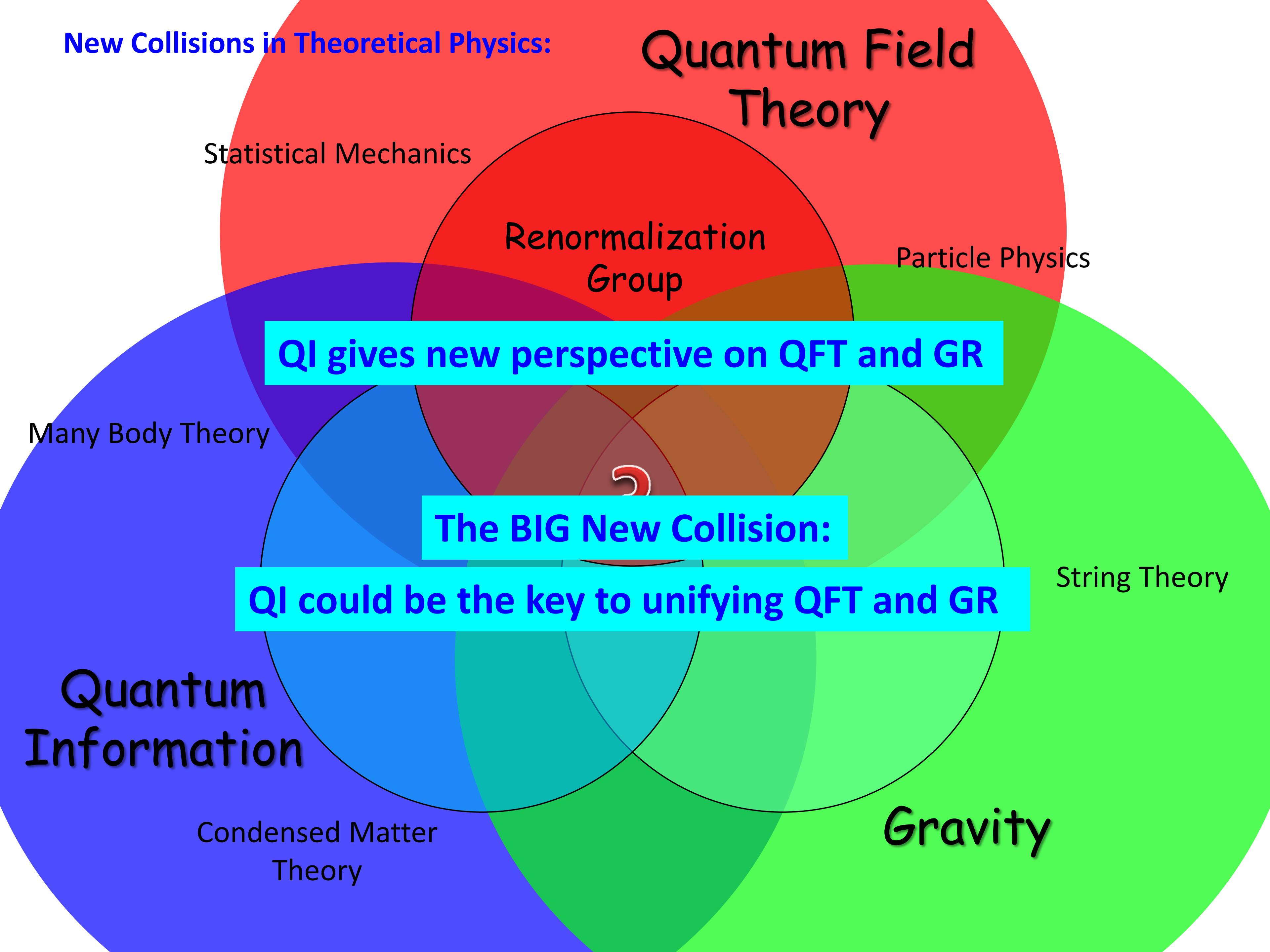
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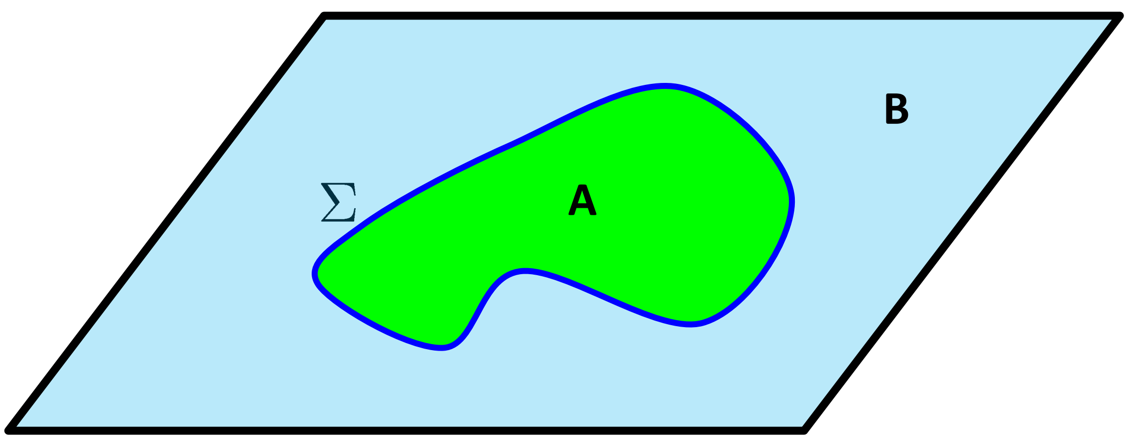
Spacetime and Entanglement

- interesting clues connecting spacetime and entanglement:

$\longrightarrow$ Bekenstein-Hawking formula encodes black hole entropy in spacetime geometry

$$S_{BH} = \frac{k_B\, c^3}{\hbar}\,\frac{\mathcal{A}}{4G}$$



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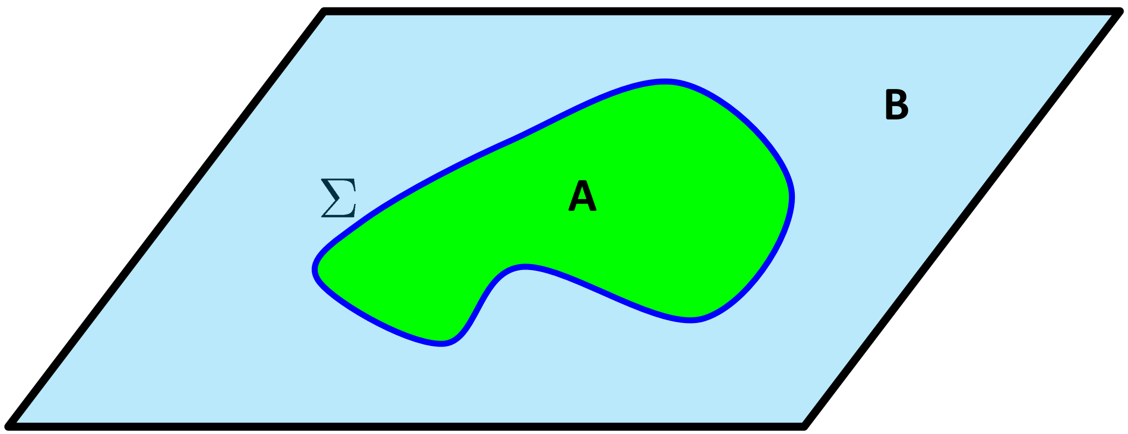
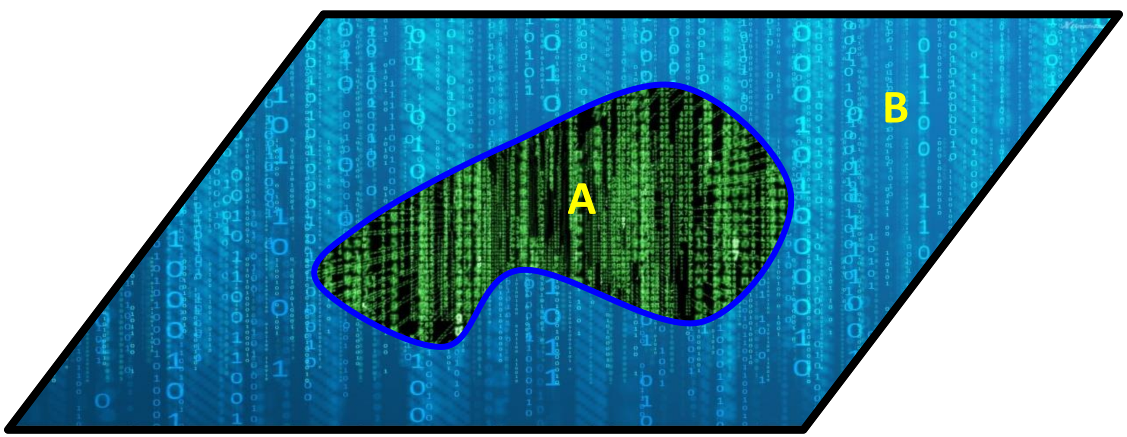
→ appearance of "area law" in entanglement entropy (Sorkin)

→ application of BH formula to holographic entanglement entropy



Spacetime = Entanglement

- black hole entropy is entanglement entropy (Sorkin, ….)
- connectivity of spacetime requires entanglement (van Raamsdonk)
- spacetime entanglement conjecture (Bianchi & RM)
- AdS spacetime as a tensor network (MERA) (Swingle, Vidal, ….)
- "ER = EPR" conjecture (Maldacena & Susskind)
- hole-ographic spacetime (Balasubramanian, Chowdhury, Czech, de Boer & Heller; RM, Rao & Sugishita; Czech Dong & Sully; ….)



Gravitation from Entanglement in AdS/CFT:

- relative entropy: $S(\rho_1|\rho_0) = \mathrm{tr}(\rho_1 \log \rho_1) - \mathrm{tr}(\rho_1 \log \rho_0)$
- let: $\rho_0 \; = \;$ reference state; $\rho_1 \; = \;$ perturbed state

$\longrightarrow$ "1st law" of entanglement entropy: $\delta S_A = \delta \langle H_A \rangle$

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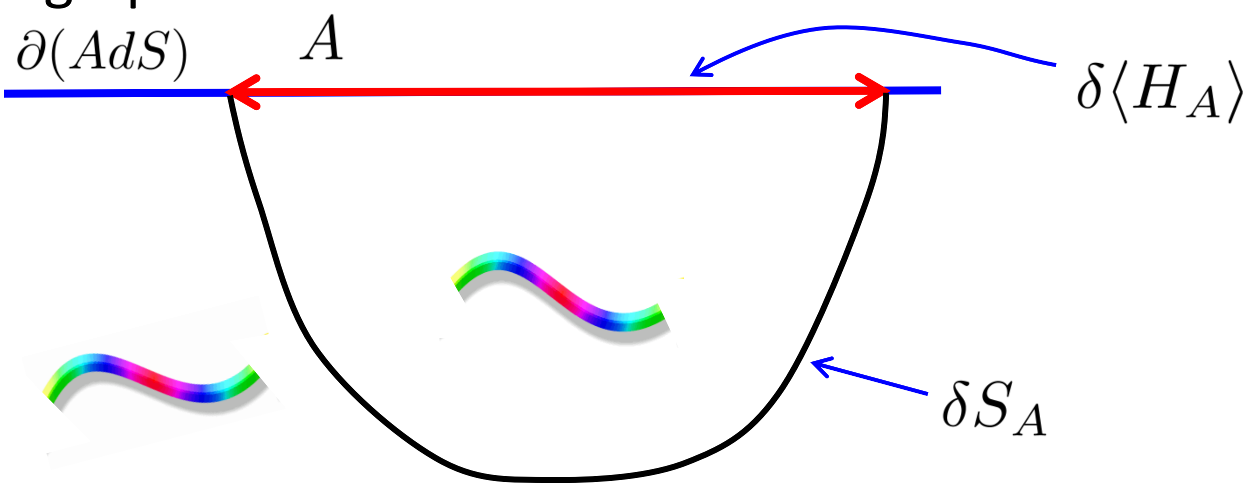

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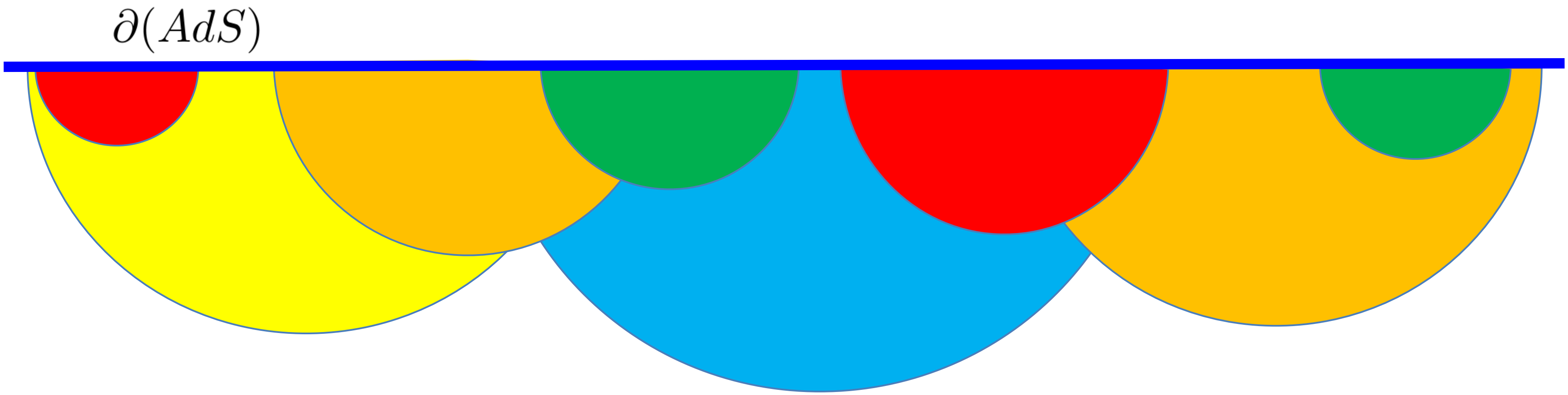

- apply 1st law for spheres of all sizes, positions and in all frames:

1st law of S_{EE} $\Longleftrightarrow$ **bulk geometry satisfies linearized Einstein eq's**

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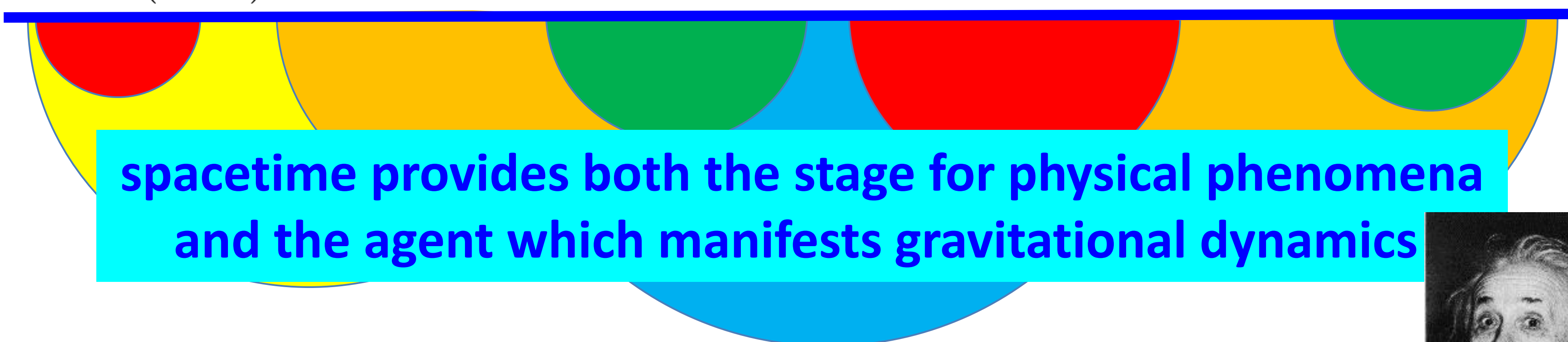
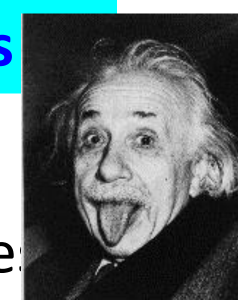


spacetime provides both the stage for physical phenomena and the agent which manifests gravitational dynamics



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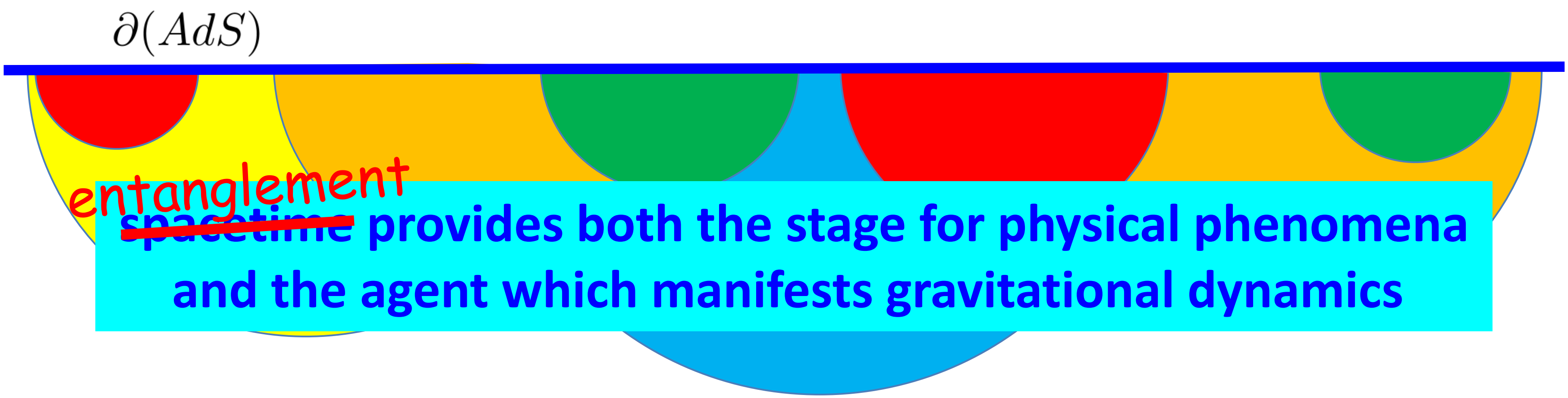

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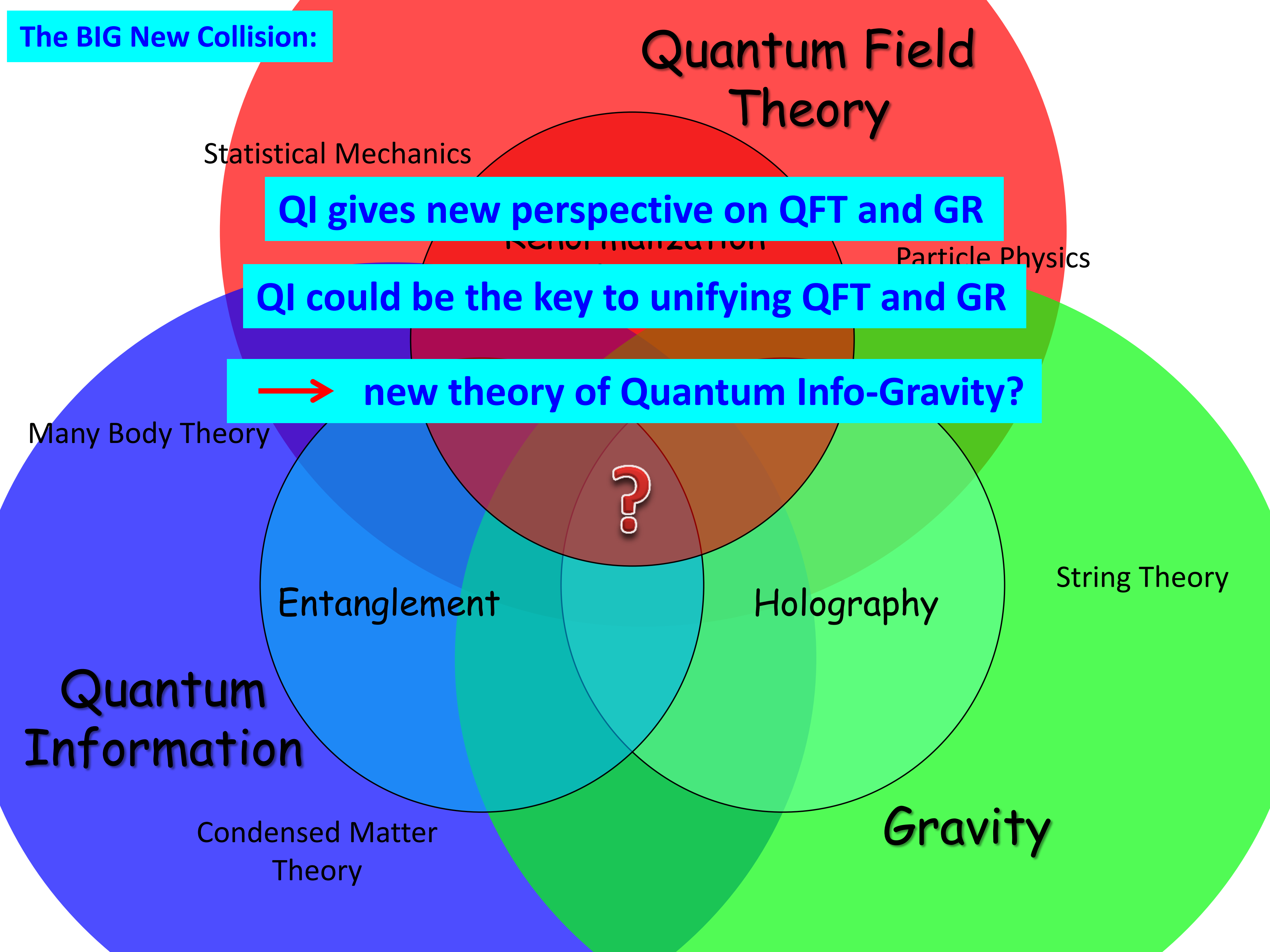
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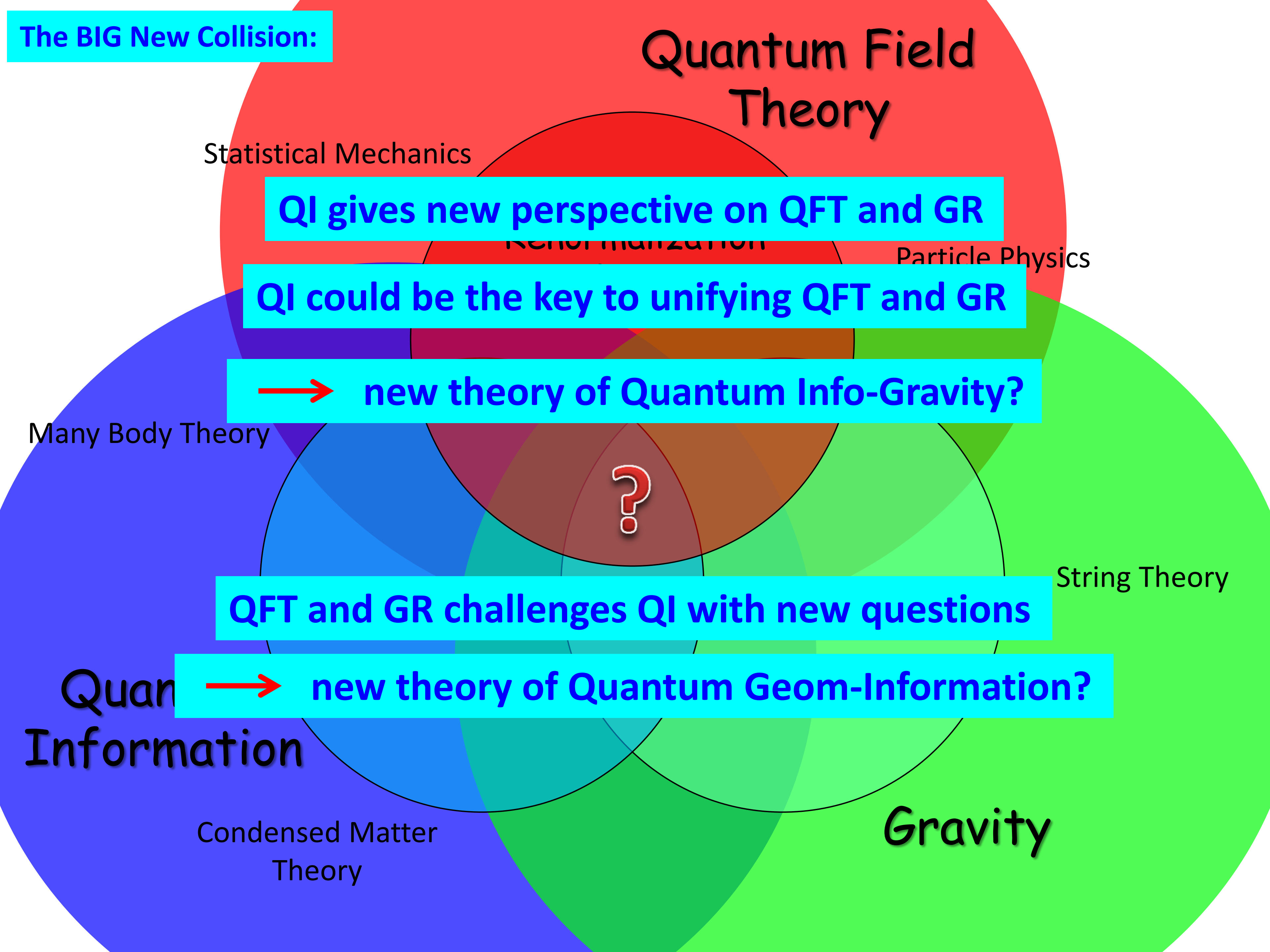

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QFT and GR challenges QI with new questions

→ new theory of Quantum Geom-Information?

The BIG New C

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