Amplitudes and hidden symmetries in N=2 Chern-Simons Matter theory

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Based on

K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227** **(BCFW)**

K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672** **(Dual Superconformal symmetry)**

K.I, S.Jain, P.Nayak, T.Sharma, V.Umesh, **arXiv : 1712.nnppq** **(Yangian)**

References:

K.I, S.Jain, S.Mazumdar, S.Minwalla, V.Umesh, S.Yokoyama, **arXiv: 1505.06571**, JHEP 1510 (2015) 176.

S.Jain, M.Mandlik, S.Minwalla, T.Takimi, S.Wadia, S.Yokoyama, **arXiv: 1404.6373,** JHEP 1504 (2015) 129.

Part I

Introduction

N=2 Chern-Simons matter theory

- General renormalizable $\mathcal{N}=2$ theory with one fundamental multiplet

$$\mathcal{S}^{L}_{\mathcal{N}=2} = \int d^3x \Big[-\frac{\kappa}{4\pi}\epsilon^{\mu\nu\rho}\mathrm{Tr}\left(A_\mu\partial_\nu A_\rho - \frac{2i}{3}A_\mu A_\nu A_\rho\right) + \bar{\psi} i \not{\mathcal{D}}\psi - \mathcal{D}^\mu\bar{\phi}\mathcal{D}_\mu\phi + \frac{4\pi^2}{\kappa^2}(\bar{\phi}\phi)^3 + \frac{4\pi}{\kappa}(\bar{\phi}\phi)(\bar{\psi}\psi) + \frac{2\pi}{\kappa}(\bar{\psi}\phi)(\bar{\phi}\psi)\Big]$$

- The theory exhibits a **strong-weak self duality** under the duality map

$$\kappa' = -\kappa \ , \ \ N' = |\kappa| - N + 1 \ , \ \ \lambda' = \lambda - \mathrm{Sgn}(\lambda)$$

- **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** $2 \to 2$ scattering amplitudes to all orders in the 't Hooft coupling.

- In the (**non-anyonic**) symmetric, anti-symmetric and adjoint channels of scattering the **amplitude is tree-level exact to all orders in λ**.

- In the **(anyonic)** singlet channel the coupling dependence is **extremely simple**.

2→2 scattering amplitude to all orders in λ

- Tree level super amplitude

$$T_{\text{tree}} = \frac{4\pi}{\kappa} \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta(\sum_{i=1}^{4} p_i) \delta^2 \left(\mathcal{Q}\right)$$

$$\delta^2(Q) = \sum_{i<j=1}^{n} \langle ij \rangle \eta_i \eta_j$$

- **All loop** super amplitude

$$T^{\text{non-anyonic}}_{\text{all-loop}} = T_{\text{tree}}$$

$$T^{\text{anyonic}}_{\text{all-loop}} = N \frac{\sin(\pi\lambda)}{\pi\lambda} T_{\text{tree}}$$

$$S^{\text{non-anyonic}} = I + i\, T^{\text{non-anyonic}}_{\text{all-loop}}$$

$$S^{\text{anyonic}} = \cos\left(\pi\lambda\right) I + i\, T^{\text{anyonic}}_{\text{all-loop}}$$

- Passes all consistency checks: **Unitarity and Duality**

Motivation

- Why is the 2 → 2 particle scattering in the non-anyonic channels **tree level exact**? and why does it have a very **simple coupling dependence in anyonic channel**?
- Maybe some **powerful symmetry** that protects the amplitude from renormalization.
- Is it possible to compute **all loop m → n scattering amplitudes** in the N=2 theory at least in the planar limit?
- Does the **non-renormalization** results of the 2 → 2 scattering continue to persist for arbitrary higher point amplitudes?
- What are the **generalization of the crossing rules** for the anyonic channels in an arbitrary m →n scattering.
- These computations would **test the duality** in regions un-probed by large N perturbation theory yet.

What we do

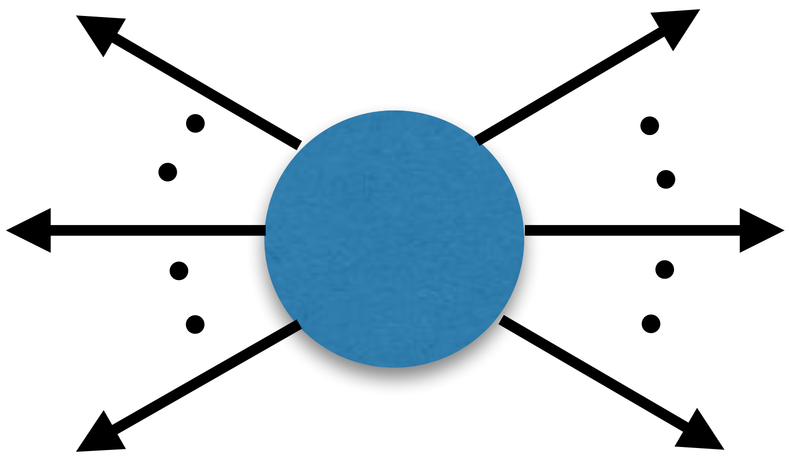
- As a first step towards the all loop m→n scattering, is it possible to write down **arbitrary m → n tree level amplitudes** ?
- We are able to achieve this via **BCFW recursions** **K.I, Jain, Nayak, Umesh**
- As a first step towards thinking about higher point loop amplitudes we identify a **hidden symmetry** in the 2 → 2 amplitude computed earlier that might explain the non-renormalization.
- This symmetry is known as **dual superconformal symmetry**.

  K.I, Jain, Majumdar, Nayak, Neogi, Sharma, Sinha, Umesh
- The superconformal symmetry and dual superconformal symmetry together generate an infinite dimensional symmetry known as the **Yangian**.

  K.I, Jain, Nayak, Sharma, Umesh, to appear
- If this is true for all higher point amplitudes, This suggests that the theory we are dealing with may be **integrable!**

Part II

All tree level amplitudes



- K.I, S.Jain, P.Nayak, V.Umesh, **arXiv :1710.04227**

BCFW recursions in 2+1 dimensions

- Recursion relations enable to construct **n point tree level scattering amplitudes from lower point tree level amplitudes.**

Britto, Cachazo, Feng, Witten

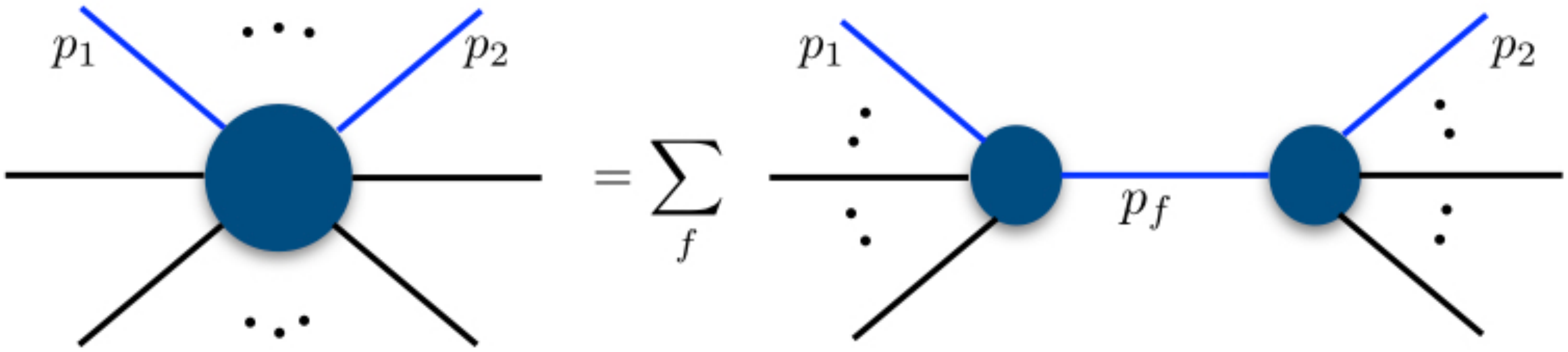
- Central idea: **Dixon**
  - Tree level amplitudes are **continuously deformable** analytic functions of momenta.
  - Only type of singularities that can appear at tree level are **simple poles.**
  - One can **reconstruct amplitudes** for generic scattering kinematics knowing its behavior in **singular kinematics**.
  - In these singular regions **amplitudes factorize** into causally disconnected amplitudes with fewer legs, connected by an i**ntermediate onshell state**.
- We will focus on situation where the external particles are massless.

BCFW recursions in 2+1 dimensions

- Promote the amplitude into a one complex parameter family of amplitudes

$$A(z) = A(p_1, \ldots p_i(z), p_{i+1}, \ldots p_l(z), \ldots, p_{2n})$$

- The necessary and sufficient conditions are:
  - The momentum deformation should preserve on-shell conditions and momentum conservation.
  - The amplitude should be asymptotically well behaved under the deformation.



- A higher point amplitude factorizes into lower point amplitudes!

Preserving the onshell conditions

- In 3d the momentum shift is non-linear in z

$$\begin{pmatrix} \hat\lambda_1 \\ \hat\lambda_2 \end{pmatrix} = R \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix}, \qquad R = \begin{pmatrix} \frac{z+z^{-1}}{2} & -\frac{z-z^{-1}}{2\ i} \\ \frac{z-z^{-1}}{2\ i} & \frac{z+z^{-1}}{2} \end{pmatrix}$$

$$p_i \to \frac{p_{ij}}{2} + qz^2 + \tilde q z^{-2} \qquad q^{\alpha\beta} = \frac{1}{4}(\lambda_2 + i\lambda_1)^\alpha(\lambda_2 + i\lambda_1)^\beta$$

$$p_j \to \frac{p_{ij}}{2} - qz^2 - \tilde q z^{-2} \qquad \tilde q^{\alpha\beta} = \frac{1}{4}(\lambda_2 - i\lambda_1)^\alpha(\lambda_2 - i\lambda_1)^\beta\ .$$

- The momentum deformations **preserve the onshell conditions**

$$p_i^2 = 0\ , p_j^2 = 0$$

$$q.\tilde q = -\frac{1}{4}p_i.p_j\ ,\ \ q + \tilde q = \frac{1}{2}(p_i - p_j)\ , q.p_{ij} = 0\ ,\ \ \tilde q.p_{ij} = 0$$

Gang, Huang, Koh, Lee, Lipstein

Asymptotic behavior

- Onshell susy methods, encode the **component amplitudes into a superamplitude**.
- **Susy ward identities** relate various component amplitudes and reduce the number of independent amplitudes.
- Susy also ensures that **if the independent component amplitudes are well behaved then the entire superamplitude is well behaved**.
- Using two independent methods we showed that the superamplitude is well behaved
  - **Background field expansion.** **Arkani-Hamed, Kaplan**
  - **Explicit Feynman diagram computation** of component amplitudes.
- The recursion formula then follows from **Cauchy residue theorem**.

The recursion formula for an arbitrary 2n point amplitude

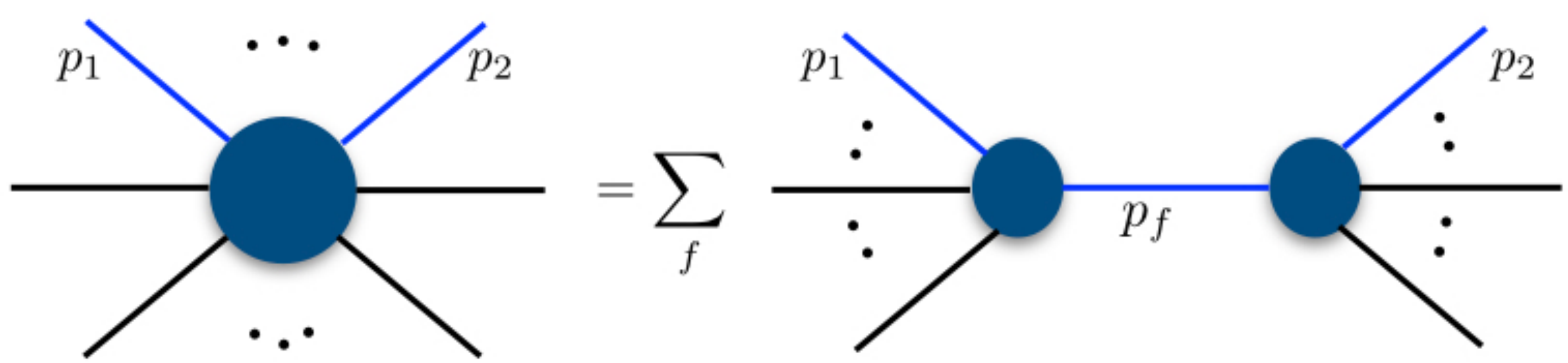
- Write a contour integral representation for the amplitude

$$\frac{1}{2\pi i}\oint_{C_{z=1}} \frac{dz}{z-1} A(z)$$

- Deform the contour to z→∞, If A(z) has no poles, the integral vanishes

$$A(z=1) = -\sum_{\text{poles}:z^i} \text{Res}_{z=z^i} \frac{A(z)}{z-1}$$

- remember that all the deformed momenta satisfy the onshell conditions!

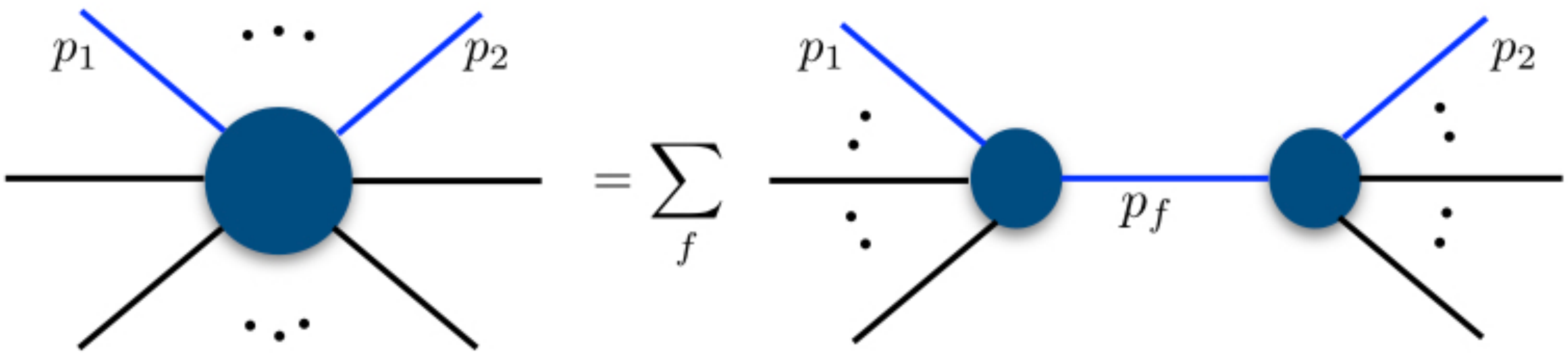


$$A(z=1) = -\sum_f \sum_{\text{poles}:z^i_f} \text{Res}_{z=z^i_f} \frac{1}{z-1} \frac{A_L(p_1 \ldots p_i(z), \ldots p_n) A_R(p_{n+1} \ldots p_j(z), \ldots p_{2n})}{\hat{p}_f^2(z)}$$

- We have used the fact that at **tree level the only possible singularities** are **simple poles**!

The recursion formula for arbitrary 2n point superamplitude

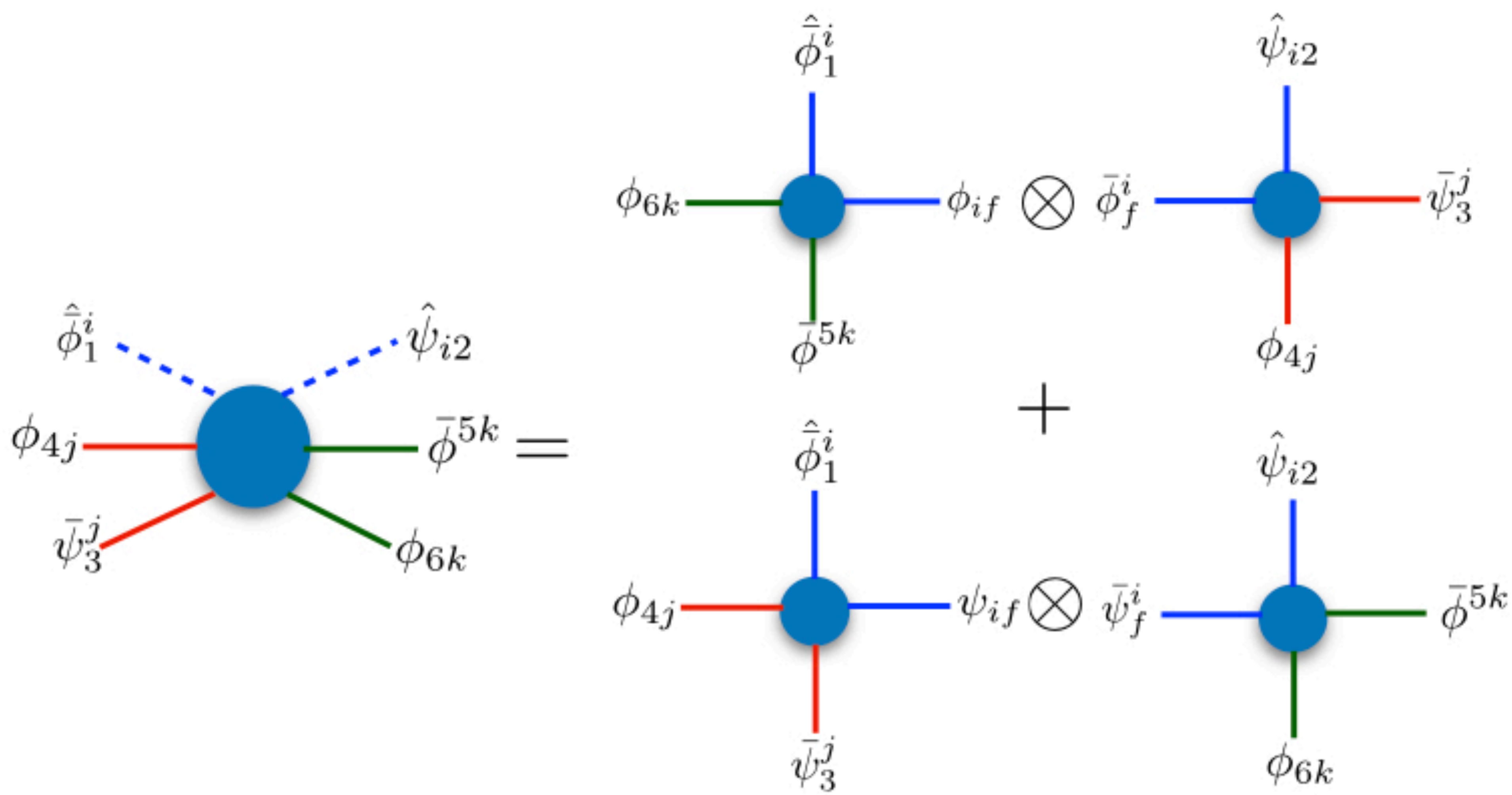
$$A_{2n}(z=1) = \sum_f \int \frac{d\theta}{p_f^2}\left(z_{a;f}\,\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2} A_L(z_{a;f},\theta)A_R(z_{a;f},i\theta) + (z_{a;f} \leftrightarrow z_{b;f})\right)$$



- $z_{a;f}\,,\, z_{b;f}$ are zeroes of $p_f^2(z)=0$
- The formula can be recursively applied to write down any **higher point superamplitude in terms of products of the four point superamplitude**.

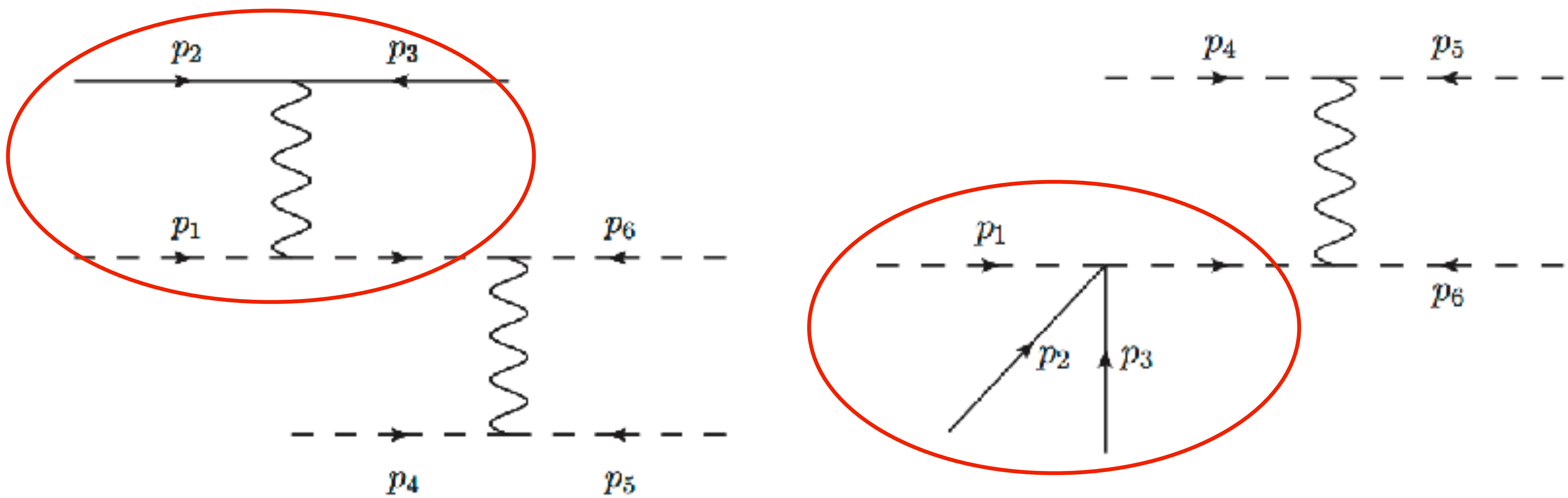
Eg: Six point amplitude as product of four point amplitudes

$$
\begin{aligned}
&\langle\bar\phi_1\psi_2\bar\psi_3\phi_4\bar\phi_5\phi_6\rangle = \\
&\left(z_{a;f}\,\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\,\langle\hat{\bar\phi}_1\hat\phi_f\bar\phi_5\phi_6\rangle_{z_{a;f}}\,\langle\hat{\bar\phi}_{(-f)}\hat\psi_2\bar\psi_3\phi_4\rangle_{z_{a;f}} + (z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\bigg|_{p_f=p_{234}} \\
&+\left(z_{a;f}\,\frac{z_{b;f}^2-1}{z_{a;f}^2-z_{b;f}^2}\,\langle\hat{\bar\phi}_1\hat\psi_f\bar\psi_3\phi_4\rangle_{z_{a;f}}\,\langle\hat{\bar\psi}_{(-f)}\hat\psi_2\bar\phi_5\phi_6\rangle_{z_{a;f}} + (z_{a;f}\leftrightarrow z_{b;f})\right)\frac{i}{p_f^2}\bigg|_{p_f=p_{256}}
\end{aligned}
$$



Eg: Six point amplitude: Asymptotic behavior

- The **Asymptotic behavior involves very precise cancellations of divergences** in the Feynman diagram approach.
- For eg, the process $\langle \bar{\psi}_1 \phi_2 \bar{\phi}_3 \psi_4 \bar{\phi}_5 \phi_6 \rangle$ gets contribution from 15 diagrams.
- 5 of them are well behaved, the remaining 10 are **individually divergent**, However the **divergences cancel pair wise**.
- Typical cancellations are between



$$\sim -\frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z}) \ , \ \sim \frac{8\pi^2 iz}{\kappa^2}\frac{\langle q3\rangle\langle 45\rangle\langle 56\rangle\langle 46\rangle}{p_{45}^2 p_{123}^2} + \mathcal{O}(\frac{1}{z})$$

Recursion relations for non-supersymmetric theories!

- BCFW does not apply to the **non-susy CS coupled to fermions/bosons** since the amplitudes **do not have good asymptotic behavior**.

- It is possible to extract the recursion relations for non-susy fermionic/bosonic CS matter theories from the N=2 results!! Eg:

    - At **tree level,** the Feynman diagrams for an **all fermion amplitude are same** for susy/non-susy theory.

    - **Susy ward identity**: The four point super amplitude is completely specified by one function, choose it to be the four fermion amplitude.

    - Use this information recursively in the BCFW formula!

- An arbitrary higher point tree level amplitude in the fermionic CS matter theory can be entirely written in terms of **4 fermion amplitude**.

Recursion relations for non-supersymmetric theories!

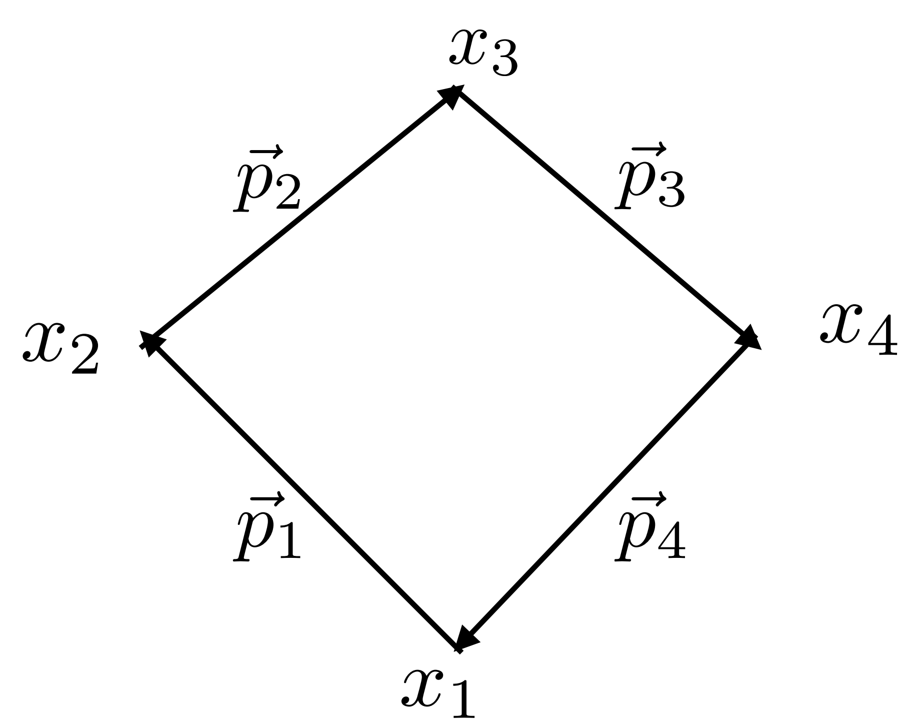
$$
\begin{aligned}
&\langle \bar\psi_1 \psi_2 \bar\psi_3 \psi_4 \bar\psi_5 \psi_6 \rangle = \\
&\Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \Bigg[-\frac{z_{a;f}^2 + 1}{2 z_{a;f}} + i\, \frac{z_{a;f}^2 - 1}{2 z_{a;f}}\, \frac{\langle \hat 1 4 \rangle}{i \langle \hat f 4 \rangle} \frac{\langle \hat f 6 \rangle}{\langle \hat 2 6 \rangle} \Bigg] \\
&\qquad \times \langle \hat{\bar\psi}_1 \hat\psi_f \bar\psi_3 \psi_4 \rangle \langle \hat{\bar\psi}_{(-f)} \hat\psi_2 \bar\psi_5 \psi_6 \rangle_{z_{a;f}} \\
&+ (z_{a;f} \leftrightarrow z_{b;f}) \Bigg) \frac{i}{p_f^2} \Bigg|_{p_f = p_{234}} \\
&- \Bigg(z_{a;f} \frac{z_{b;f}^2 - 1}{z_{a;f}^2 - z_{b;f}^2} \Bigg[-\frac{z_{a;f}^2 + 1}{2 z_{a;f}} + i\, \frac{z_{a;f}^2 - 1}{2 z_{a;f}}\, \frac{\langle \hat 1 6 \rangle}{i \langle \hat f 6 \rangle} \frac{\langle \hat f 4 \rangle}{\langle \hat 2 4 \rangle} \Bigg] \\
&\qquad \times \langle \hat{\bar\psi}_1 \hat\psi_f \bar\psi_5 \psi_6 \rangle \langle \hat{\bar\psi}_{(-f)} \hat\psi_2 \bar\psi_3 \psi_4 \rangle_{z_{a;f}} \\
&+ (z_{a;f} \leftrightarrow z_{b;f}) \Bigg) \frac{i}{p_f^2} \Bigg|_{p_f = p_{256}}
\end{aligned}
$$

Main highlights

- We obtained **BCFW recursion relations for arbitrary $m \to n$ tree level scattering amplitudes** in N=2 Chern-Simons matter theory.
- We were also able to extract the **recursions for non-supersymmetric Chern-Simons theory coupled to fundamental fermions**.
- Similar exercise can also be done for the bosonic theory.
- We saw an explicit example of the recursions for a **six point amplitude as a product of four point amplitudes**.
- The recursions can be iteratively applied to write **all higher point amplitudes in terms of products of four point amplitudes**.

Part III

Hidden symmetry: Dual superconformal invariance



- K.I, S.Jain, S.Majumdar, P.Nayak, T.Neogi, T.Sharma, R.Sinha, V.Umesh, **arXiv : 1711.02672**

Dual variables

- The dual variables realize momentum conservation linearly in the x variables

$$x_{i,i+1}^{\alpha\beta} = x_i^{\alpha\beta} - x_{i+1}^{\alpha\beta} = p_i^{\alpha\beta} = \lambda_i^{\alpha} \lambda_i^{\beta}$$
$$\theta_{i,i+1}^{\alpha} = \theta_i^{\alpha} - \theta_{i+1}^{\alpha} = q_i^{\alpha} = \lambda_i^{\alpha} \eta_i$$

- momentum and supermomentum conservation imply

$$P^{\alpha\beta} = \sum_i p_i^{\alpha\beta} = x_{n+1}^{\alpha\beta} - x_1^{\alpha\beta} = 0,$$
$$\mathcal{Q}^{\alpha} = \sum_i q_i^{\alpha} = \theta_{n+1}^{\alpha} - \theta_1^{\alpha} = 0.$$

- The **four point super amplitude in dual space**

$$\mathcal{A}_4 = \frac{\langle 12 \rangle}{\langle 23 \rangle} \delta(\sum_{i=1}^{4} p_i) \delta^2(\mathcal{Q}) \xrightarrow[\text{dual space}]{} \mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}} \delta^{(3)}(x_1 - x_5) \delta^{(2)}(\theta_1 - \theta_5)$$

- Goal is to show that this is **invariant under the superconformal symmetry in the dual variables**.

Superconformal algebra in dual space

- The N=2 superconformal algebra in **dual space** is generated by

$$\{P_{\alpha\beta},\ M_{\alpha\beta},\ D,\ K_{\alpha\beta},\ R,\ Q_\alpha,\ \bar{Q}_\alpha,\ S_\alpha,\ \bar{S}_\alpha\}$$

$$P_{\alpha\beta} = \sum_{i=1}^{n} \frac{\partial}{\partial x_i^{\alpha\beta}}, \quad D = -\sum_{i=1}^{n}\left(x_i^{\alpha\beta}\frac{\partial}{\partial x_i^{\alpha\beta}} + \frac{1}{2}\theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}\right),$$

$$Q_\alpha = \sum_{i=1}^{n}\frac{\partial}{\partial \theta_i^\alpha}, \quad \bar{Q}_\alpha = \sum_{i=1}^{n}\theta_i^\beta \frac{\partial}{\partial x_i^{\beta\alpha}},$$

$$M_{\alpha\beta} = \sum_{i=1}^{n}\left(x_{i\alpha}{}^{\gamma}\frac{\partial}{\partial x_i^{\gamma\beta}} + \frac{1}{2}\theta_{i\alpha}\frac{\partial}{\partial \theta_i^\beta}\right), \quad R = \sum_{i=1}^{n}\theta_i^\alpha \frac{\partial}{\partial \theta_i^\alpha}$$

- The remaining generators can be expressed using the inversion operator

$$I\left[x_i^{\alpha\beta}\right] = \frac{x_i^{\alpha\beta}}{x_i^2}, \quad I\left[\theta_i^\alpha\right] = \frac{x_i^{\alpha\beta}\theta_{i\beta}}{x_i^2}$$

$$K_{\alpha\beta} = I P_{\alpha\beta} I, \quad S_\alpha = I Q_\alpha I, \quad \bar{S}_\alpha = I\bar{Q}_\alpha I.$$

Dual superconformal invariance N=2 vs ABJM

- Note that the **delta functions for N=2 transform under the inversion** as

$$I\left[\delta^{(3)}(x_1-x_5)\delta^{(2)}(\theta_1-\theta_5)\right] = x_1^4\,\delta^{(3)}(x_1-x_5)\delta^{(2)}(\theta_1-\theta_5)$$

- Whereas in the **N=4 and ABJM case, the corresponding delta function is invariant under the inversion**!

$$A^{(4)}_{ABJM} = \frac{1}{\sqrt{x^2_{1,3}x^2_{2,4}}}\delta^{(3)}(x_1-x_5)\delta^{(6)}(\theta_1-\theta_5)$$

$$\tilde{K}^{\alpha\beta}\mathcal{A}^{(4)}_{ABJM} = \left(K^{\alpha\beta}+\frac{1}{2}\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)}_{ABJM} = 0\ ,\ \ \Delta_i = \{1,1,1,1\}$$

Gang, Huang, Koh, Lee, Lipstein

- So **it was expected that the superamplitude in the N=2 theory would not have any dual superconformal invariance** at all.

- However, in the N=2 case, **dual superconformal invariance, still works but the weights become non-homogeneous.**

Dual superconformal invariance of the superamplitude

- The four point amplitude in the N=2 theory is

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- In this form the **translation, Lorentz invariance and supersymmetry invariance of the amplitude is manifest.**

- The amplitude is just a function of the square differences in the x variable.

- Under Dilatations it transforms as a eigenfunction of weight 4.

- Under R symmetry it transforms as eigenfunction of weight 2.

- To show the dual superconformal invariance it is sufficient to show the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

Dual superconformal invariance of the superamplitude

$$K_{\alpha\beta}\left[\mathcal{A}_4\right] = IP_{\alpha\beta}I\left[\mathcal{A}_4\right]$$
$$= I\sum_{i=1}^{4}\partial_{i\alpha\beta}\left[x_1^4\sqrt{\frac{x_2^2x_4^2}{x_1^2x_3^2}}\;\mathcal{A}_4\right]$$
$$= I\left[-\frac{1}{2}x_1^4\sqrt{\frac{x_2^2x_4^2}{x_1^2x_3^2}}\left(3\frac{x_1^{\alpha\beta}}{x_1^2}+\frac{x_2^{\alpha\beta}}{x_2^2}+\frac{x_4^{\alpha\beta}}{x_4^2}-\frac{x_3^{\alpha\beta}}{x_3^2}\right)\mathcal{A}_4\right]$$
$$= -\frac{1}{2}\left(3x_1^{\alpha\beta}+x_2^{\alpha\beta}+x_4^{\alpha\beta}-x_3^{\alpha\beta}\right)\mathcal{A}_4$$
$$= -\frac{1}{2}\left(\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}_4 \qquad \text{w/} \quad \{\Delta_j\}=\{3,1,-1,1\}$$

- So the invariance under $K_{\alpha\beta}, S_\alpha, \bar{S}_\alpha$

$$\tilde{K}^{\alpha\beta}\mathcal{A}^{(4)} = \left(K^{\alpha\beta}+\frac{1}{2}\sum_{j=1}^{4}\Delta_j x_j^{\alpha\beta}\right)\mathcal{A}^{(4)} = 0 \qquad \tilde{\bar{S}}_\alpha\left[\mathcal{A}_4\right] = \left(\bar{S}_\alpha+\frac{1}{2}\left(\sum_{j=1}^{4}\Delta_j\theta_{j\alpha}\right)\right)\left[\mathcal{A}_4\right]=0$$

$$S_\alpha\left[\mathcal{A}_4\right] = IQ_\alpha I\left[\mathcal{A}_4\right] = IQ_\alpha\left[x_1^4\sqrt{\frac{x_2^2x_4^2}{x_1^2x_3^2}}\;\mathcal{A}_4\right] = 0.$$

Dual superconformal invariance at all loops

- We showed that the function A4 is dual superconformal invariant!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

- The **tree level superamplitude is dual superconformal invariant**.

$$T_{tree} = \frac{4\pi}{\kappa}\mathcal{A}_4$$

- The **all loop results** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** are **also dual superconformal invariant.**

$$T_{sym}^{all\ loop} = T_{Asym}^{all\ loop} = T_{Adj}^{all\ loop} = T_{tree}$$

$$T_{sing}^{all\ loop} = N\frac{\sin\pi\lambda}{\pi\lambda}T_{tree}$$

- Now that we know this symmetry exists, **can we reverse the argument and do an S matrix bootstrap to fix the general structure of the amplitude**?

Constraining amplitudes from dual superconformal symmetry

- The **four point amplitude in momentum space** can be interpreted as a **four point correlator in dual space**, then dual conformal invariance fixes

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle$$
$$= \frac{1}{x_{12}^{\Delta_1+\Delta_2} x_{34}^{\Delta_3+\Delta_4}} \left(\frac{x_{24}}{x_{14}}\right)^{\Delta_1-\Delta_2} \left(\frac{x_{14}}{x_{13}}\right)^{\Delta_3-\Delta_4} f(u,v,\kappa,\lambda)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}.$$

- Since $x_{ij}^2 = p_i^2 = 0$, the correlator is understood in the limit

$$\left.\frac{u}{v}\right|_{onshell} = \left.\frac{x_{12}^2 x_{34}^2}{x_{14}^2 x_{23}^2}\right|_{onshell} = \left.\frac{p_1^2 p_3^2}{p_2^2 p_4^2}\right|_{onshell} = constant$$

- **If dual superconformal symmetry is exact it fixes the momentum (x) dependence completely***

$$f(u,v,\kappa,\lambda) = g(\kappa,\lambda)$$

Constraining amplitudes from dual superconformal symmetry

- In general the S matrix could get complicated functions with poles and branch cuts.

- If dual conformal invariance is an exact symmetry at loop level then no such behavior appears.

- Non trivial momentum dependence could still appear from $x_{i,j}^{\Delta}$ when Δ gets correction from loops.

- This can give rise to log dependence for instance, However these do not appear if there are no IR divergences.

- **If we assume that there are no IR divergences (none seen in the calculation), and that the dual conformal invariance is an exact symmetry, then the momentum dependence is fixed**.

4 point amplitude as a free field correlator in dual space

- Recall that $\{\Delta_1, \Delta_2, \Delta_3, \Delta_4\} = \frac{1}{2}\{4-1, 1, -1, 1\}$
- The factor of 4 is due to momentum+supermomentum conservation and can be removed. $I\left[\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)\right] = x_1^4\, \delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$
- With this identification the operator dimensions are

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$
$$\Delta_2 = \Delta_4 = \frac{1}{2}$$

- The four point correlator in dual space gets fixed to (cancellations in limiting sense)

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = g(\kappa, \lambda)\sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

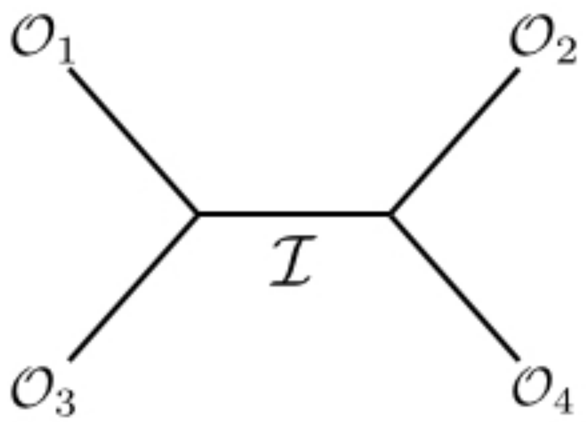
- Same as the amplitude without the delta functions!

$$\mathcal{A}_4 = \sqrt{\frac{x_{1,3}^2}{x_{2,4}^2}}\delta^{(3)}(x_1 - x_5)\delta^{(2)}(\theta_1 - \theta_5)$$

4 point amplitude as a free field correlator in dual space

- In a general CFT, in the **double light cone limit, only Identity operators are expected to contribute**!
- In the channel where $(\mathcal{O}_1, \mathcal{O}_3)$ and $(\mathcal{O}_2, \mathcal{O}_4)$ are brought together

$$\tilde{\Delta}_1 = \Delta_3 = -\frac{1}{2}$$
$$\Delta_2 = \Delta_4 = \frac{1}{2}$$



- The four point amplitude can be accounted for by an **identity exchange**.

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)\rangle = \langle \mathcal{O}_1(x_1)\mathcal{O}_3(x_3)\rangle\langle \mathcal{O}_2(x_2)\mathcal{O}_4(x_4)\rangle$$
$$= c_1 c_2 \sqrt{\frac{x_{13}^2}{x_{24}^2}}$$

- This suggests $g(\kappa, \lambda) = c_1 c_2$.
- It would be interesting to understand the CFT interpretation of these operators, and also to see what happens in the cross channel.

Part III
Summary

Summary

- We started with a goal of computing **arbitrary m→n tree level scattering amplitudes** in U(N) $\mathcal{N}=2$ Chern-Simons matter theories with fundamental matter.

- We achieved this via **BCFW recursion relations**, this enabled us to express arbitrary n point amplitudes as products of four point amplitudes!

- We saw an explicit example where the six point amplitude is expressed as a product of two four point amplitudes via two factorization channels.

- The non-susy amplitudes do not satisfy the BCFW requirements.

- However we were able to use the fact that the four point superamplitude in the $\mathcal{N}=2$ theory is specified by one function and that the tree level amplitudes are identical to the non-susy case to write recursions for the non-susy theory as well.

- We saw an explicit example for the six fermion amplitude in the fermion coupled Chern-Simons theory.

Summary

- We showed that the **all loop 2→2 scattering amplitude** computed in **K.I, Jain, Mazumdar, Minwalla, Umesh, Yokoyama** is **dual superconformal invariant**.
- Thus **dual superconformal symmetry is all loop exact**, at least for the 4 point amplitude.
- The presence of dual conformal symmetry then allows us to interpret the **amplitude in momentum space as a correlator in dual space**.
- We argued that **if dual conformal symmetry was an exact symmetry it fixed the momentum dependence of the amplitude completely**.
- We interpreted the four point amplitude in dual space as a free field correlator where the identity operator exchange accounted for it.
- However, **general principles such as unitarity, duality and dual conformal symmetry are insufficient to fix the overall coupling dependence**.

Yangian Symmetry

- The presence of the **superconformal and dual superconformal symmetries indicate a Yangian symmetry** in the amplitude.

- A Yangian algebra is an associative Hopf Algebra generated by

$$[J^A, J^B] = f_C^{AB} J^C \ , \ [J^A, Q^B] = f_C^{AB} Q^C$$

- J^A take values in a Lie group G, both J^A and Q^A are constrained to obey the Serre relations in addition to Jacobi Identity.

$$[Q^A,[Q^B,J^C]] + [Q^B,[Q^C,J^A]] + [Q^C,[Q^A,J^B]] = \frac{1}{24} f^{ADK} f^{BEL} f^{CFM} f_{KLM} \{J_D, J_E, J_F\}$$

$$[[Q^A,Q^B],[J^C,Q^D]] + [[Q^C,Q^D],[J^A,Q^B]] = \frac{1}{24}(f^{AGL} f^{BEM} f_K^{CD} + f^{CGL} f^{DEM} f_K^{AB}) \times f^{KFN} f_{LMN} \{J_G, J_E, J_F\}$$

- Under repeated commutations the Q's generate an infinite dimensional Symmetry algebra.

Yangian Symmetry in N=2 Chern-Simons theory

- The Yangian has a basis (labelled by levels)

$$\mathcal{J}_0^A = J^A \ , \ \mathcal{J}_1^A = Q^A \ , \ldots , \ \mathcal{J}_n^A$$

$$[\mathcal{J}_{(0)}^A, \mathcal{J}_{(0)}^B\} = f^{AB}{}_C \mathcal{J}_{(0)}^C \qquad [\mathcal{J}_{(0)}^A, \mathcal{J}_{(1)}^B\} = f_C^{AB} \mathcal{J}_{(1)}^C$$

- The generators $\mathcal{J}_n^A$ are "n local" operators. The infinite dimensional symmetry is generated by commutators of $\mathcal{J}_n^A$.
- For the N=2 theory the spacetime superconformal symmetry is $Osp(2|4)$

$$\mathcal{J}_{(0)}^A = \{p_{\alpha\beta}, m_{\alpha\beta}, d, k_{\alpha\beta}, r, q_\alpha, \bar{q}_\alpha, s_\alpha, \bar{s}_\alpha\}$$

- A general ansatz for the Level 1 generators **Kazakov et al.**

$$\mathcal{J}_{(1)}^A = \frac{1}{2} f^A_{BC} \sum_{j<k} \mathcal{J}_{j,(0)}^C \mathcal{J}_{k,(0)}^B + \sum_k v^l \mathcal{J}_{l,(0)}^A$$

- The dual generators K and S when restricted to onshell superspace map to Level 1 Yangian generators of the spacetime superconformal algebra.

$$K_{\alpha\beta}(\lambda_\alpha, \eta) \equiv p_{\alpha\beta}^{(1)}$$

$$\bar{S}_\alpha(\lambda_\alpha, \eta) \equiv \bar{q}_\alpha^{(1)}$$

- The remaining restricted dual generators have a trivial automorphism to Level 0 generators of the spacetime superconformal algebra.

K.I, Jain, Nayak, Sharma, Umesh, to appear

Yangian Symmetry in N=2 Chern-Simons theory

- Note that K, S commute!, so how does the infinite dimensional algebra appear?

$$K_{\alpha\beta}(\lambda_\alpha, \eta) \equiv p^{(1)}_{\alpha\beta}$$
$$\bar{S}_\alpha(\lambda_\alpha, \eta) \equiv \bar{q}^{(1)}_\alpha$$

- The remaining Level 1 generators are obtained by commuting K with the $\mathcal{J}^A_{(0)}$
- These remaining Level 1 (other than [K,S]) generate the Level 2 upon commutation and so on.
- The Yangian works exactly the same way as it did for N=4 SYM and ABJM.
- The Yangian invariance of the amplitude then boils down to the statement

$$\mathcal{J}^A_{(0)}\mathcal{A}_4 = 0\ ,\ \mathcal{J}^A_{(1)}\mathcal{A}_4 = 0\ ,\ \Longrightarrow\ \ \mathcal{Y}\mathcal{A}_4 = 0,$$

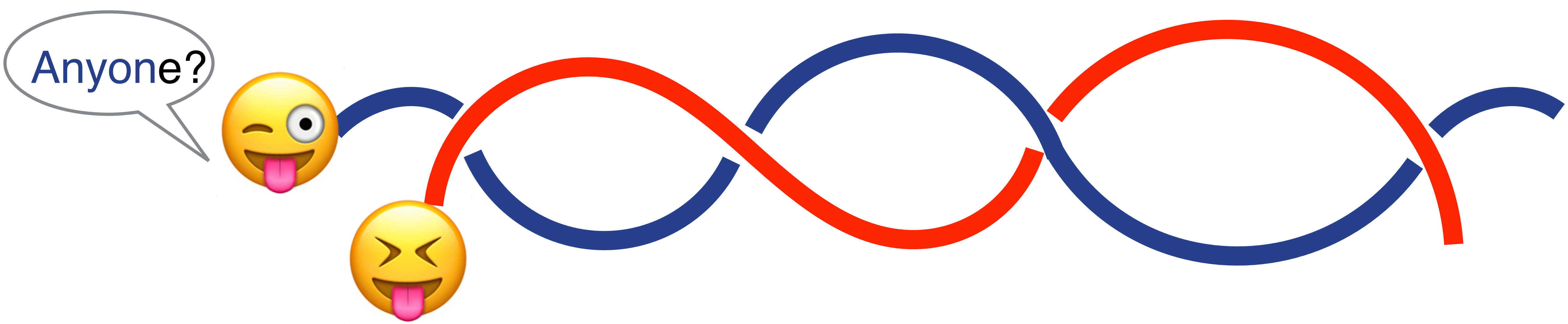
- Thus **superconformal and dual superconformal symmetries generate a Yangian symmetry!**

K.I, Jain, Nayak, Sharma, Umesh, to appear


We are happy together!

Thank you!!


So are you!
You are excluded!
And you!
You too!


Anyone?