Near Barrier Reactions – many-body quantum dynamics in action

Mahananda Dasgupta

Department of Nuclear Physics

Australian National University

Canberra

E-mail: Mahananda.Dasgupta@anu.edu.au


> Click here to find out where canberra is
Canberra

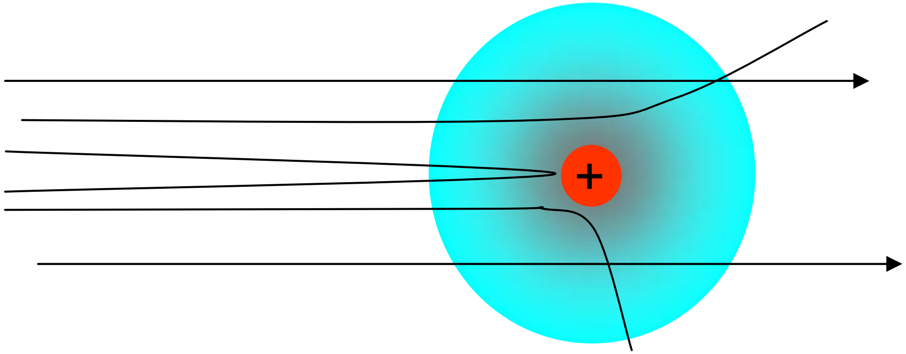

A
U
N

"The plan"

- Introduction and basic principles
- Nuclear structure effects on fusion
- Structure effects in reactions of weakly bound nuclei – mechanisms and time-scales
- Structure effects in evolution following capture

Geiger and Rutherford

George Gamow

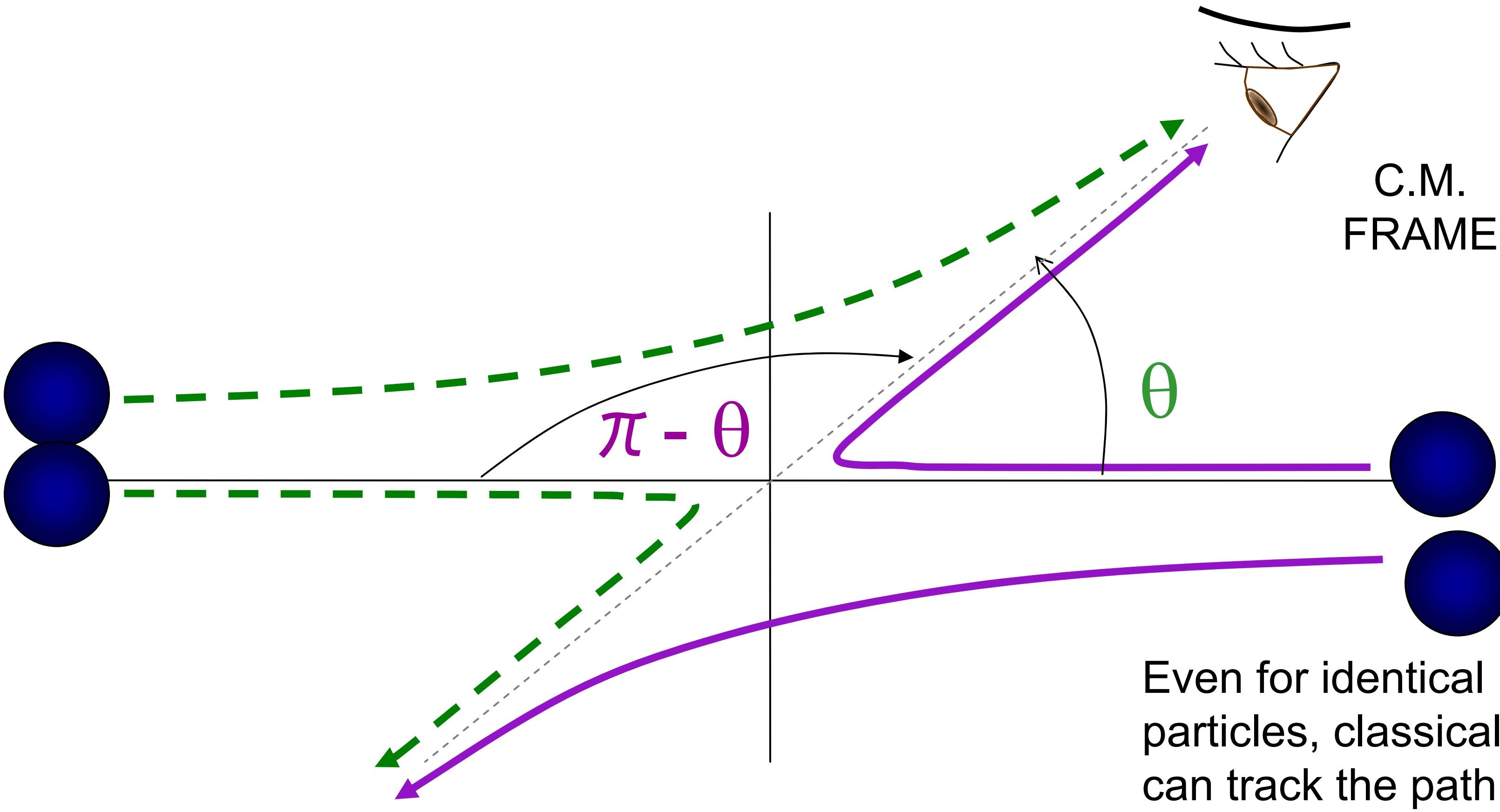


2011 - Centenary Rutherford's publication - discovery of the atomic nucleus

1928 - Tunnelling of alpha-particles (α-radioactivity)

First application of newly proposed Quantum theory

Scattering of identical nuclei - classical



Even for identical particles, classically we can track the path and know which particle scattered by θ or π-θ

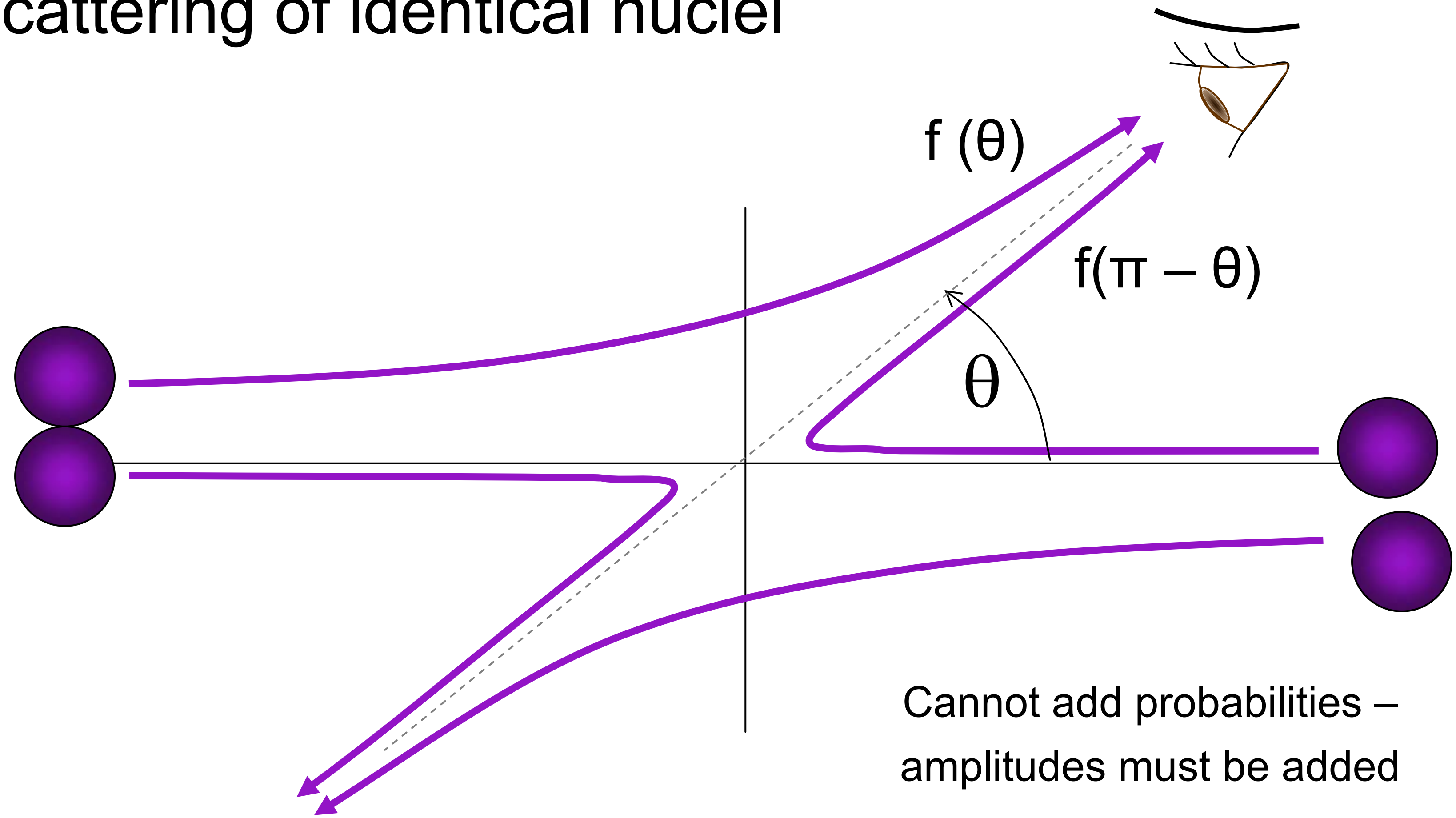
Probability of particle from left scattered by θ $= |f(\theta)|^2$

Prob. of particle from right scattered by $(\pi - \theta) = |f(\pi - \theta)|^2$

Detected events at θ proportional to sum the probabilities: $|f(\theta)|^2 + |f(\pi - \theta)|^2$

Detected at π/2 (in terms of f(θ))?

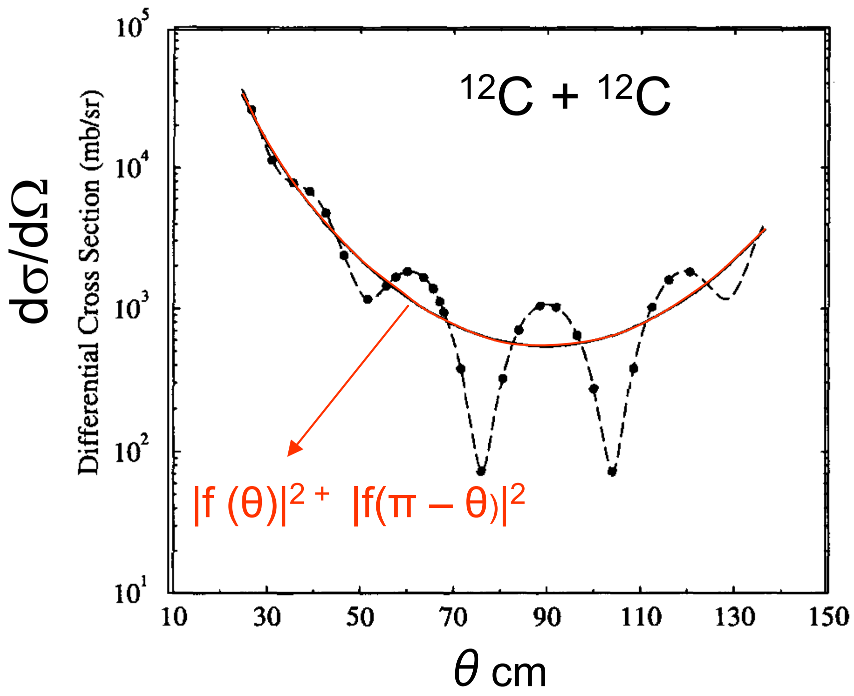
Scattering of identical nuclei



Cannot add probabilities – amplitudes must be added

Detected: $|f(\theta) + f(\pi - \theta)|^2$ (zero spin case)

$$|f(\theta)|^2 + |f(\pi-\theta)|^2 + \underbrace{f^*(\theta)\, f(\pi-\theta) + f(\theta)\, f^*(\pi-\theta)}_{\text{Interference}}$$

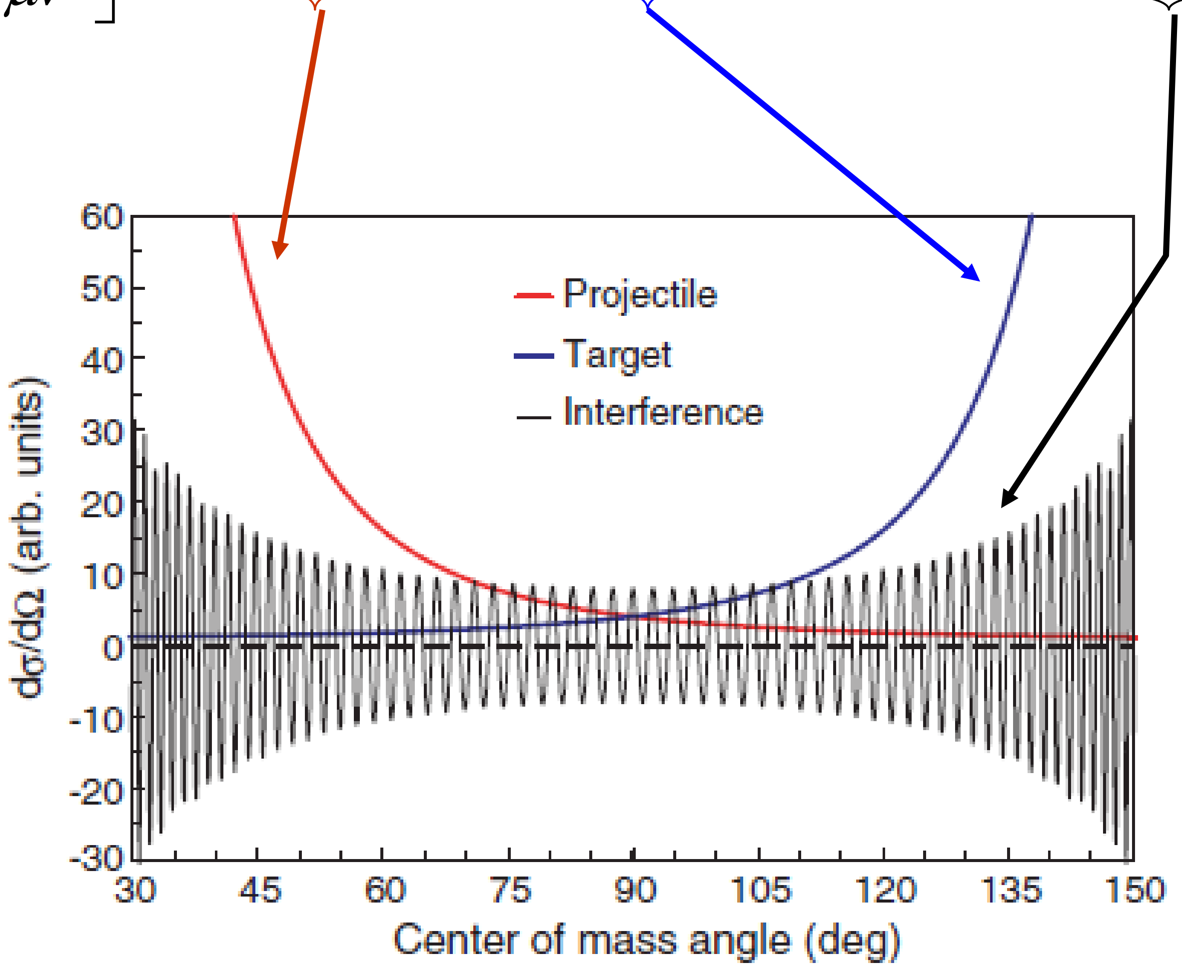


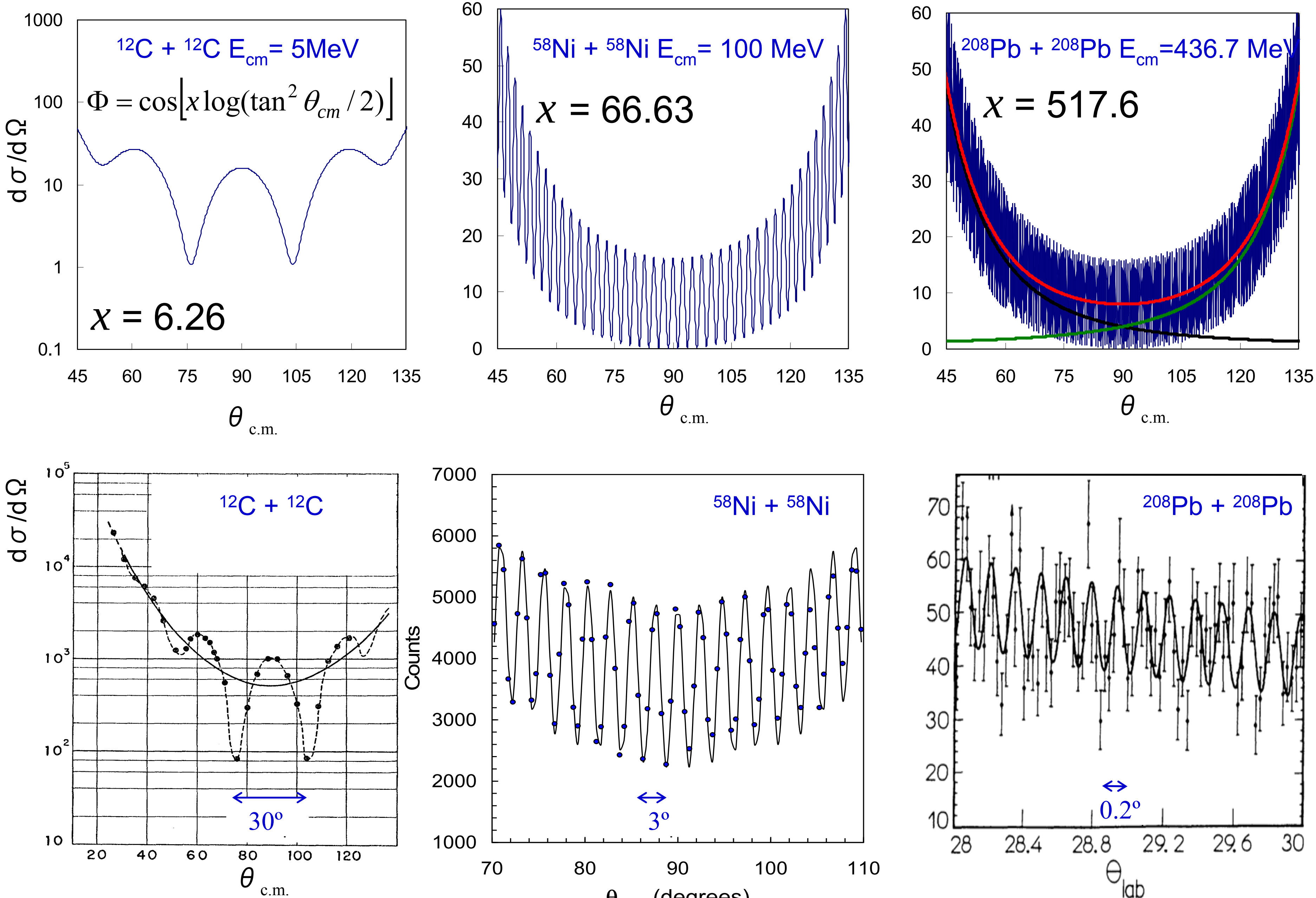
$|f(\theta)|2 + |f(\pi - \theta)|^2$ fails to explain data

Q.M.: The paths of the two ^{12}C nuclei cannot, even in principle, be distinguished i.e. we cannot track paths, unlike in the classical case

Bromley et al., Phys. Rev. 123, 878 (1961)

$$\frac{d\sigma}{d\Omega} = \left[\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 2\mu v^2}\right]^2 [\sin^{-4}(\theta_{cm}/2) + \cos^{-4}(\theta_{cm}/2) + 2\Phi \sin^{-2}(\theta_{cm}/2)\cos^2(\theta_{cm}/2)]$$

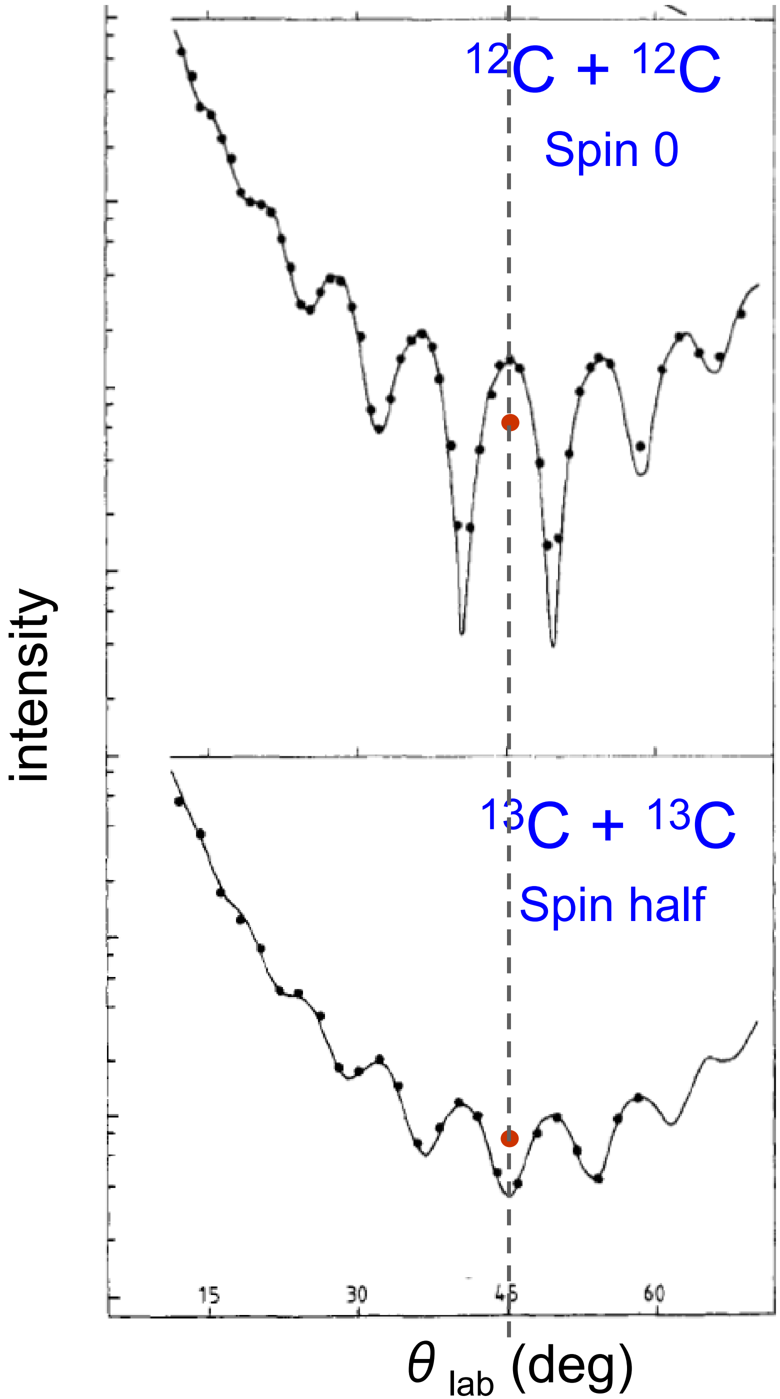




Bromley et al., Phys. Rev. 123, 878 (1961)

Hinde et al., PRC 76, 014617 (2007)

Villari et al., PRL 71, 2551 (1993)



^{12}C nucleus – ground state spin 0

(Scattering of Bosons)

$$= 4\,|\,f(\pi/2)\,|^2 \qquad \text{at } \theta = \pi/2$$

Distinguishable particles: $2\,|\,f(\pi/2)\,|^2$

^{13}C nucleus – ground state spin 1/2

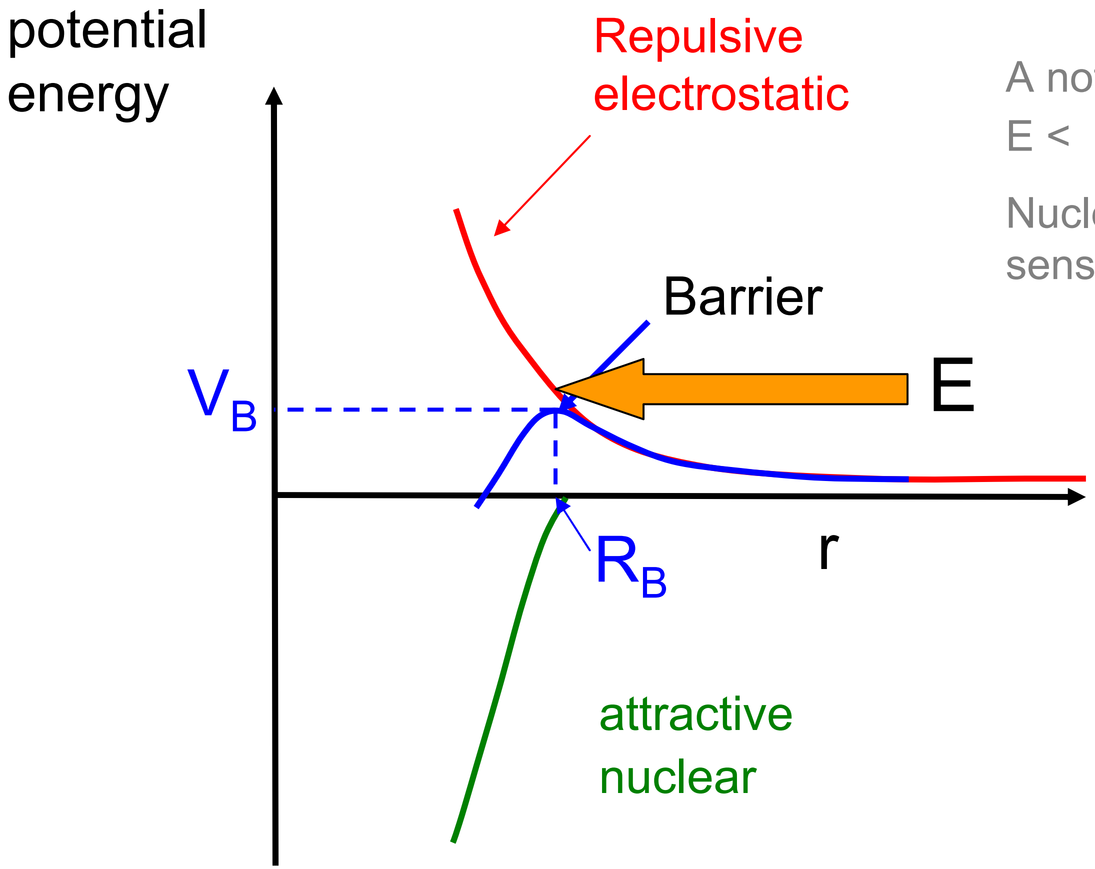
(Scattering of Fermions)

$$= \frac{3}{4}\left|f(\theta) - f(\pi-\theta)\right|^2 + \frac{1}{4}\left|f(\theta) + f(\pi-\theta)\right|^2$$

$$= |\,f(\pi/2)\,|^2 \qquad \text{at } \theta = \pi/2$$

Plattner and Sick, Eur. J. Phys. 2 (1981) 109

Attraction vs. repulsion – the nuclear balancing act



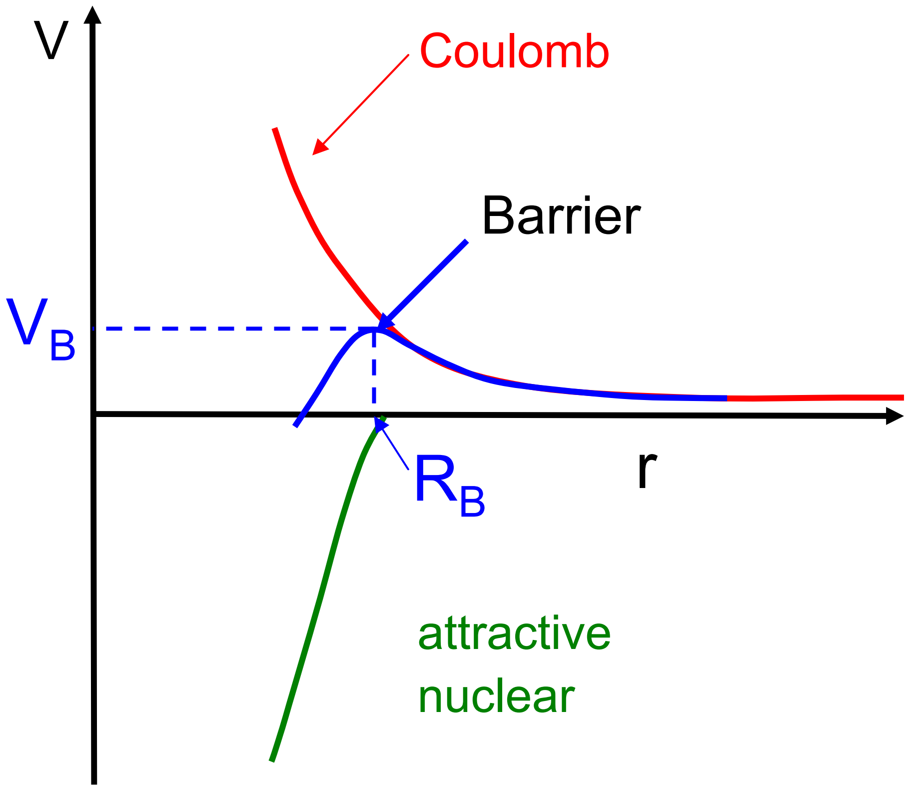
A note: In these lectures:

E < threshold for pion production

Nuclei made of nucleons – not sensitive to substructure

- Barrier – a result of attractive and repulsive potentials
- Alpha decay → alpha particles leaving the nucleus face a barrier
- Fission → barrier needs to be overcome as a nucleus splits into two nuclei

A home work problem – calculate $V(r) = V_{coulomb} + V_{nuclear} + V_{centrifugal}$



$$V_{Coulomb}(r) = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 r}$$

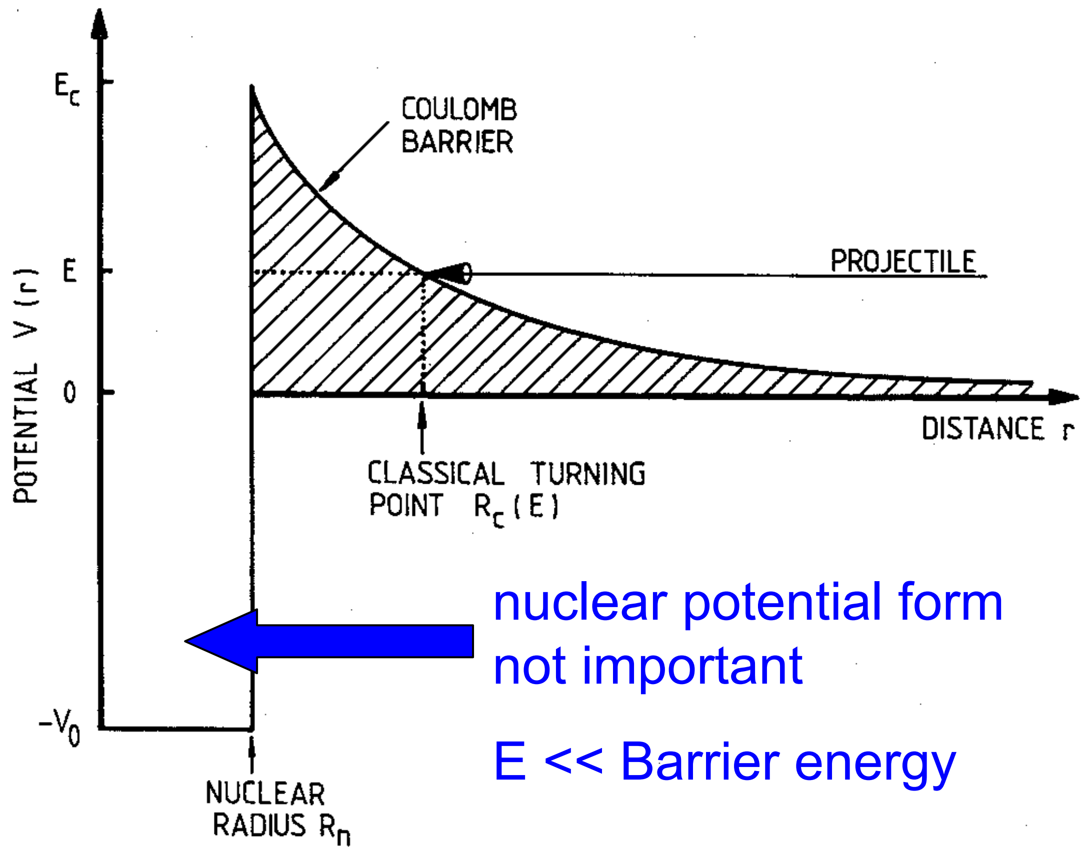
$$V_{nuclear}(r) = \frac{-V_0}{1 + \exp(\frac{r - R_0}{a})}$$

$$V_{centrifugal}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2}$$

^{16}O + ^{144}Sm: Calculate all three potentials (in MeV) and the total V as a function of r (from 5- 20 fm) for angular momentum values of 0, 50. Draw all on same graph. Use V_0 = 100 MeV, R_0=8.60 fm, a=0.75 fm.

V_B, R_B for angular momentum values of 0,50

- Nuclear Astrophysics (G. Martínez Pinedo) – charged particle reactions



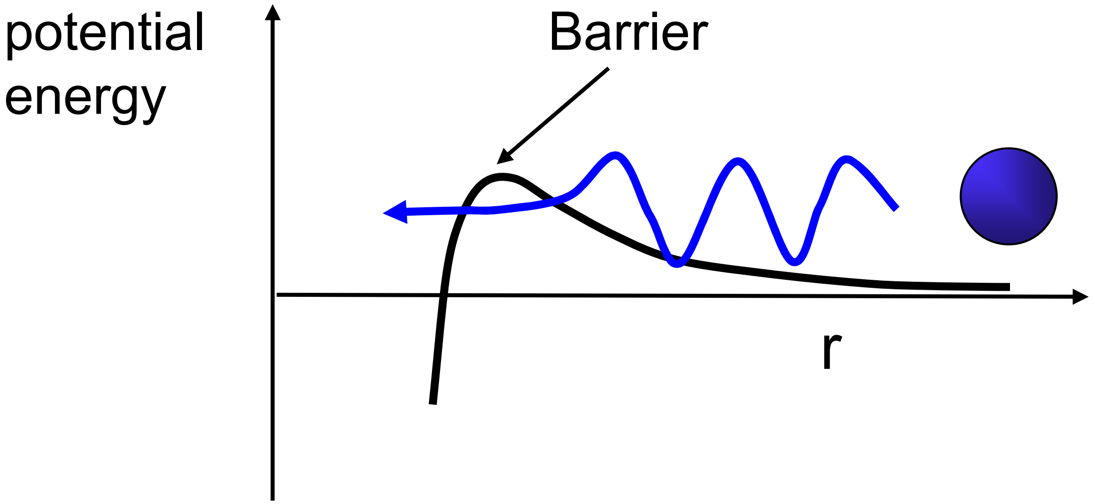
nuclear potential form not important

E << Barrier energy

- High energy reactions (T. Aumann) – E>> Barrier

  → not sensitive to barrier energy/shape

Near-barrier collisions – fascinating playground of many body quantum physics



- Near barrier – gentle collisions – explore many body aspects

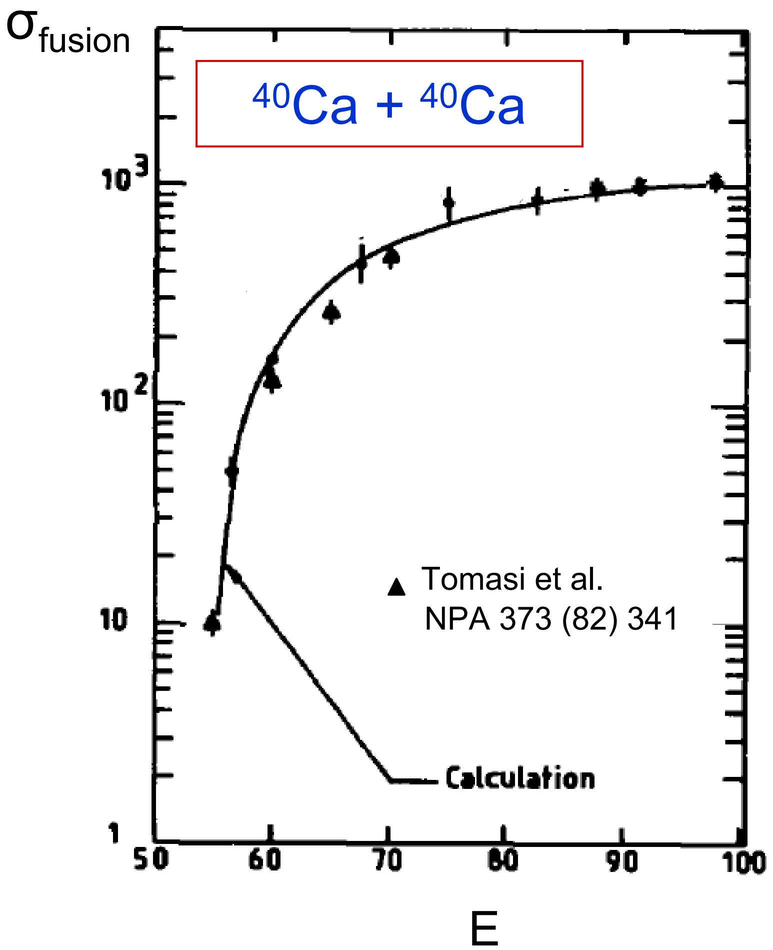
  reaction time scales ~ internal motion e.g. rotations, vibrations

- Fundamental quantum mechanics problems – diverse areas

  nuclei isolated – “mini universe”- next lecture

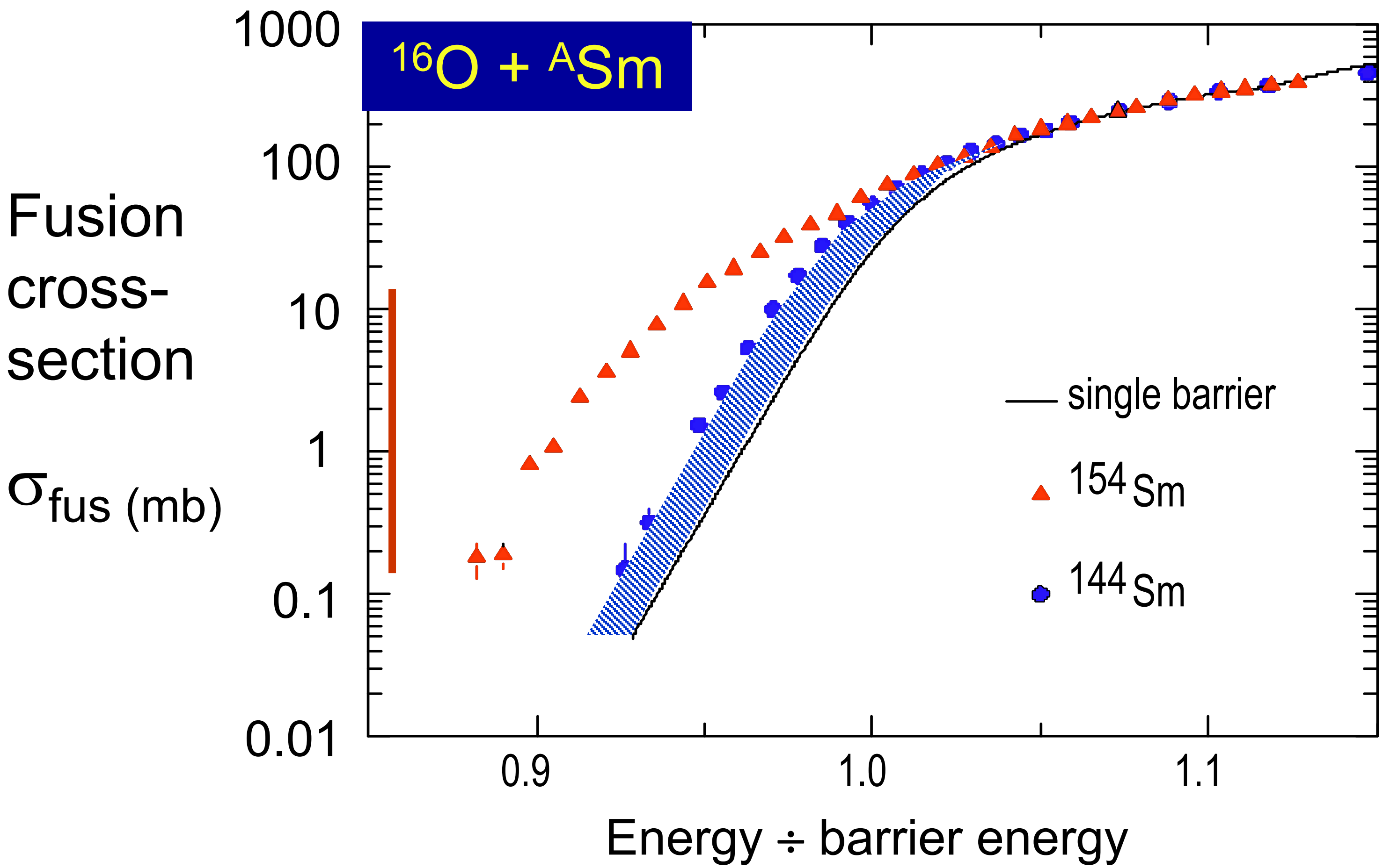
- Heavy element formation – near barrier energies

Nuclear fusion – the textbook treatment



Single barrier model works well for fusion of light nuclei

Fusion of heavy nuclei: experiment vs. expectations



Stockstad et al, PRL 41, 468 (1978)

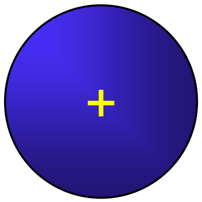
Wei et al., Phys. Rev. Lett., 67 (1991) 3368

Morton et al, PRL, 72 (1994)4074

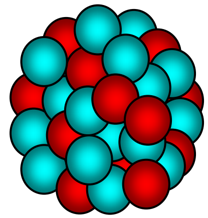
Factors of 10-100 mismatch - fundamental physics missing

inversion of fusion cross sections → potential – double valued!

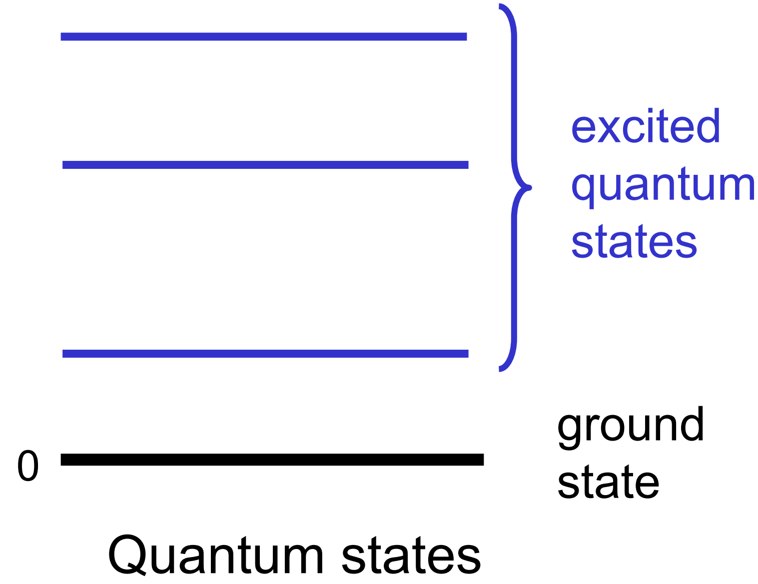
Balantekin, Koonin, Negele, PRC 28, 1565 (1983); K. Hagino lectures



Not an elementary particle



Many body quantum system



- Colliding nuclei in a superposition of quantum states

  (i.e. can't tell which state, until a measurement is made)

  → Dramatically alters reaction dynamics

- Contrast - high energy reactions (Aumann) - reaction dynamics and structure are less entangled → allows extraction of structure information

Combining structure and reactions

Details: K.Hagino's lectures; also revisit in next lecture

- Colliding nuclei in a superposition of intrinsic states:

→ set of intrinsic states

$$\psi(r,\xi) = \sum_n u_n(r)\, \varphi(\xi)$$

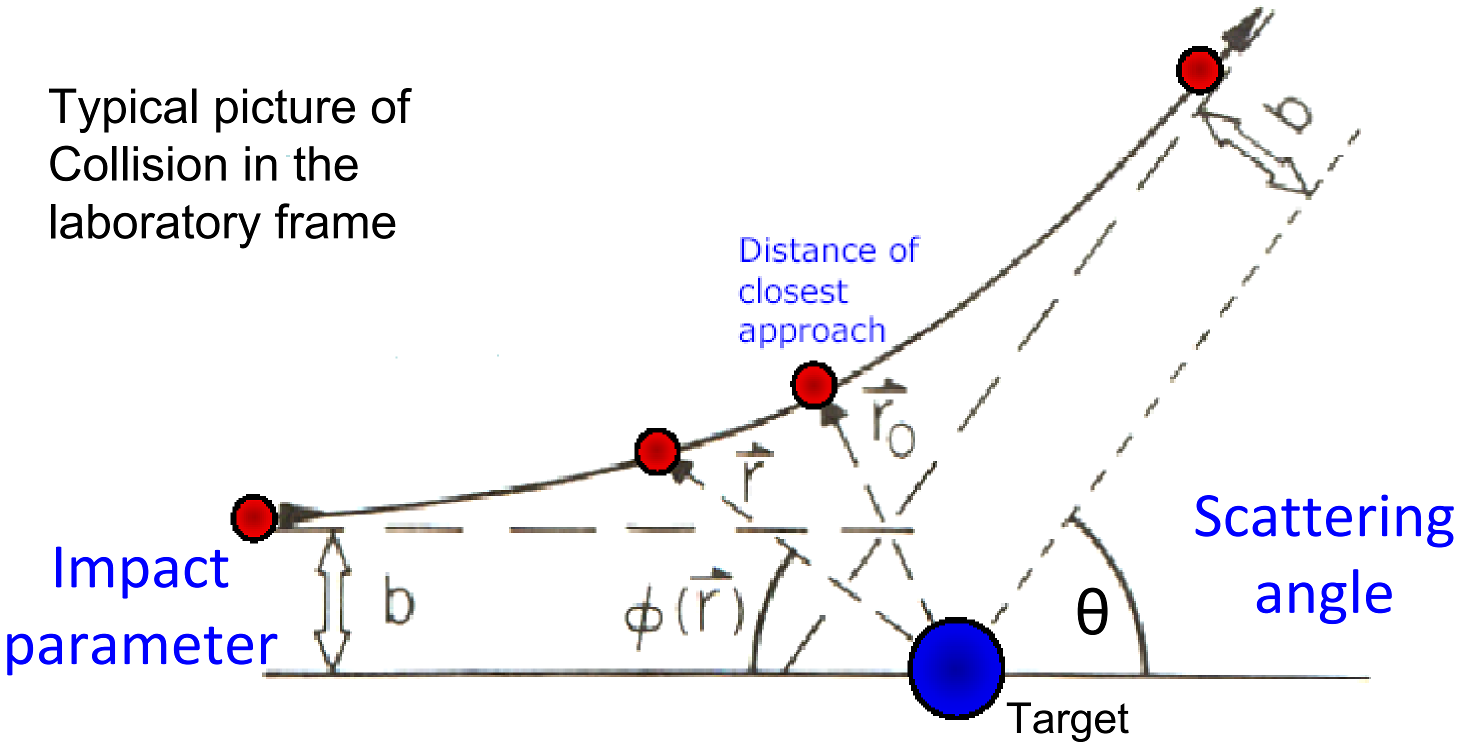
- Coupled reaction channels model

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2} + V(r) + H(\xi) + V_{coup}(r,\xi) \right] \psi(r,\xi) = E\, \psi(r,\xi)$$

Structure of the nucleus strongly affects reaction dynamics

Also in atomic collisions – but coupling strengths, atomic structure such that → potential renormalization

And yet – we picture reactions classically?



Why?

Wavelength associated with a 100 MeV ^{32}S nucleus?

(non-relativistic ok)

(a) 0.5 fm → Large mass – short wavelengths (Rutherford expt λ ?)

(b) 50 fm

(c) 100 fm

Classical concept of trajectories requires wavelength of <u>relative motion</u> < interaction region

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{2\pi\hbar c}{\sqrt{2mc^2 E}}$$

So we can use some clever tricks for calculations

$$\hbar c = 197.3 \text{ MeV fm}$$

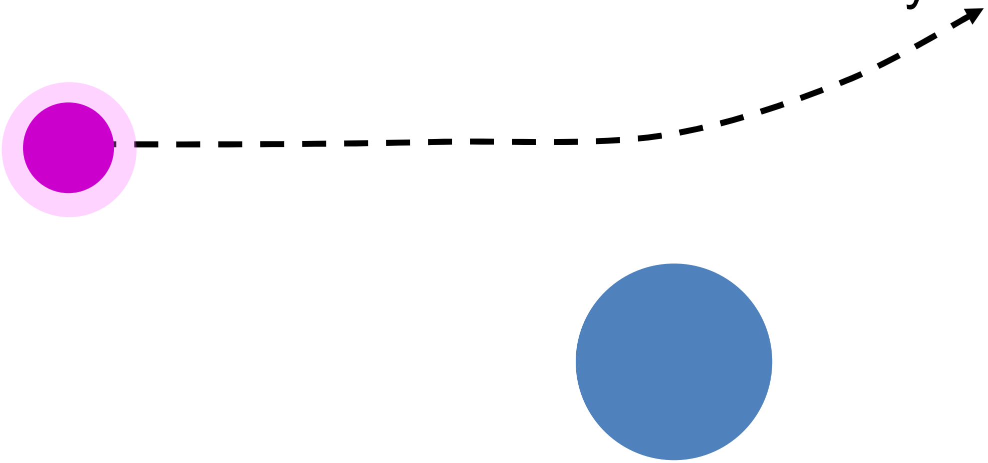
(~ 200)

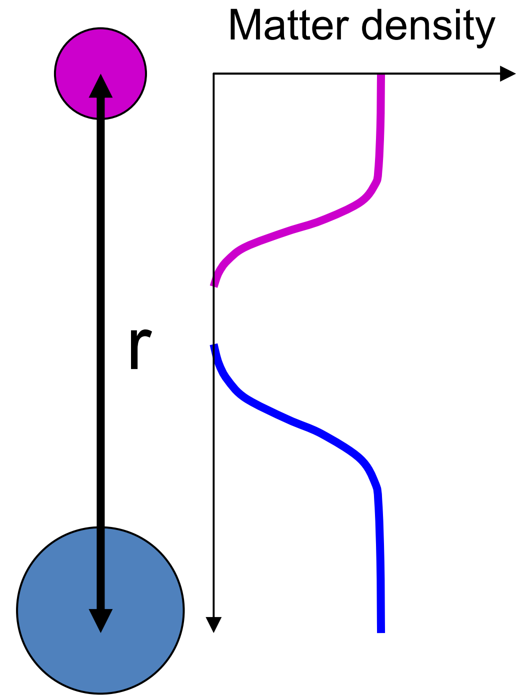
$$u = 931.5 \text{ MeV}$$

(~ 1000)

$$l = \frac{\mu v_\infty b}{\hbar} = \frac{\sqrt{2\mu c^2 E_{cm}}\; b}{\hbar c}$$

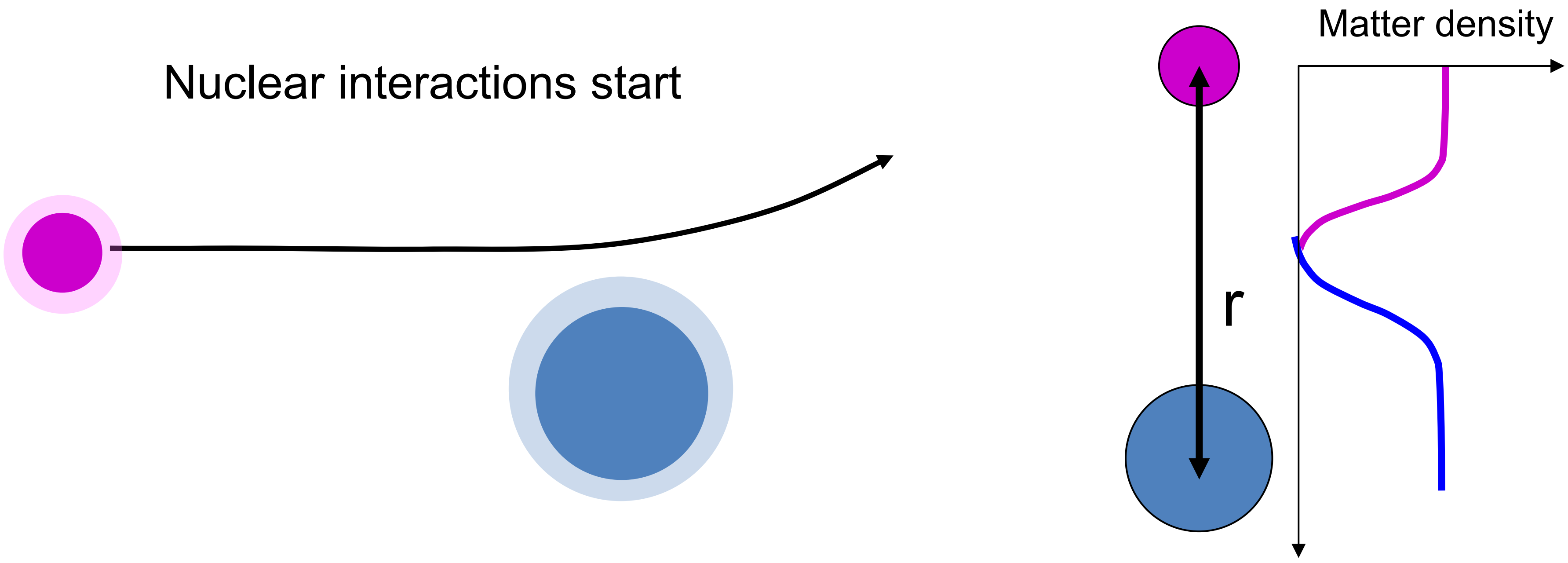
Interactions due to Coulomb field only





- Rutherford scattering (pure Coulomb) – Elastic scattering

- Inelastic scattering (Coulomb excitation) - colliding nuclei excited to higher energy levels due to (changing) Coulomb field between the two nuclei
  - only way nuclear/atomic properties enter – electromagnetic matrix elements between the initial and final state
  - one of the methods to experimentally determine B(E λ) values

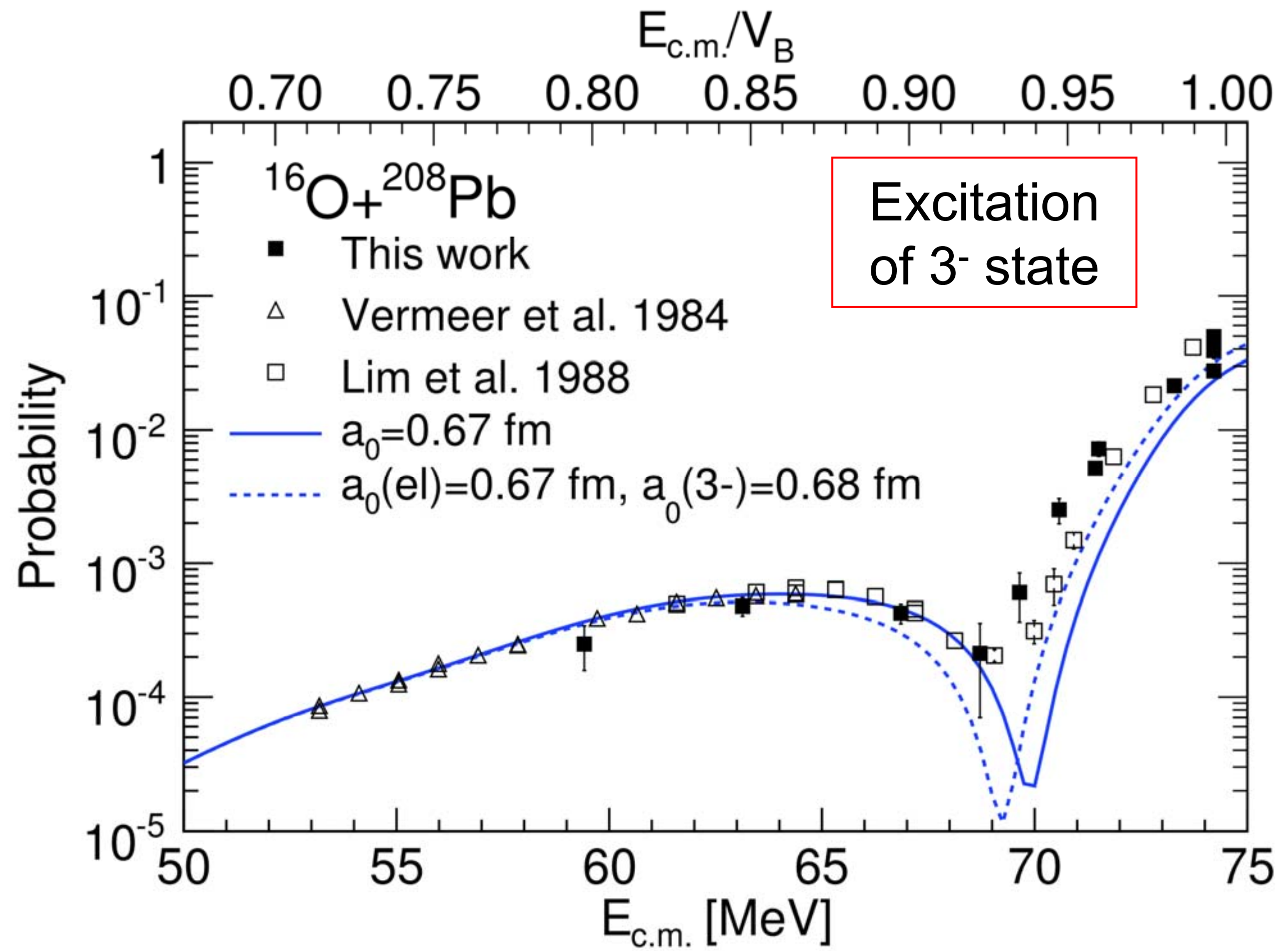


- Inelastic scattering (Coulomb and nuclear) - colliding nuclei excited to higher energy levels due to (changing) Coulomb and nuclear interactions (Theo. methods: Distorted Wave Born Approx (DWBA), Coupled channels)

- Coulomb-nuclear interference effects

Why?

Also discussions during the last lecture of T. Aumann

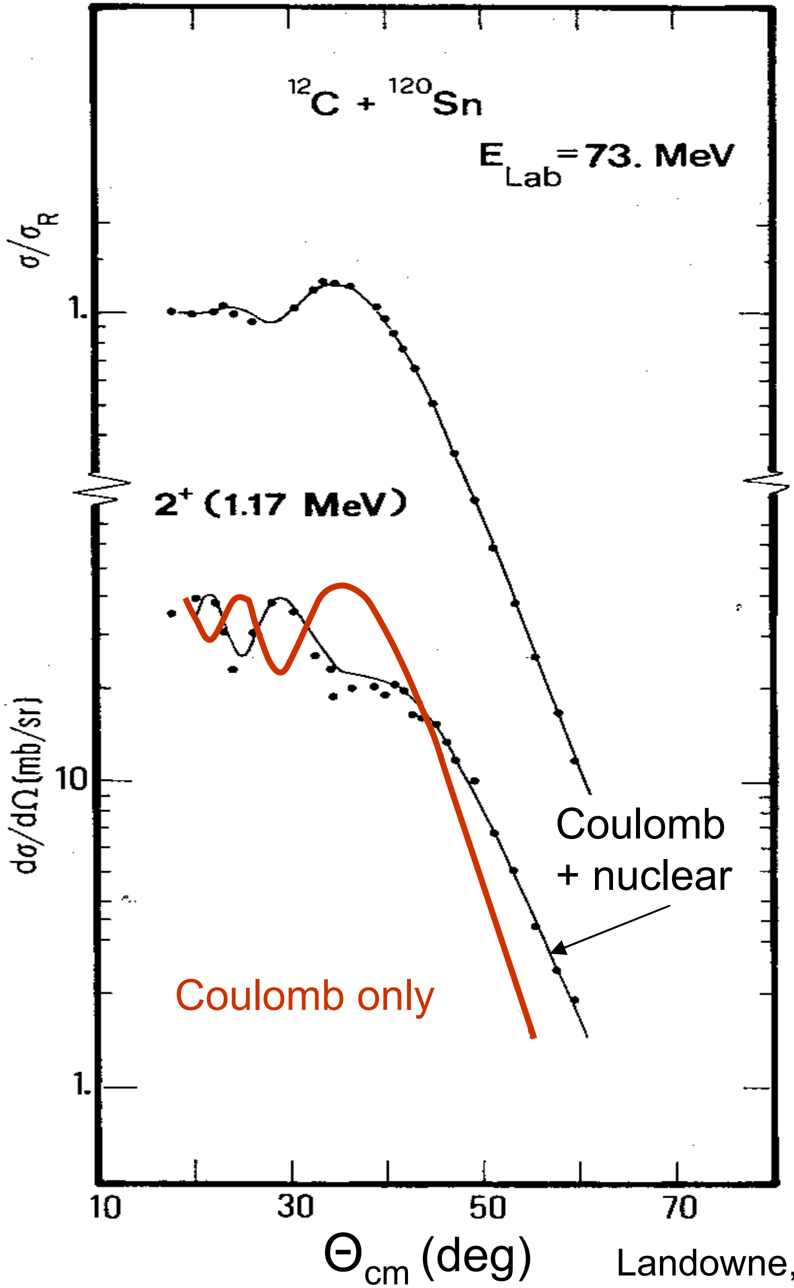


Evers et al., PRC 81, 014602 (2010)

Position of the minimum – determined by the relative phase between the nuclear and Coulomb scattering amplitudes

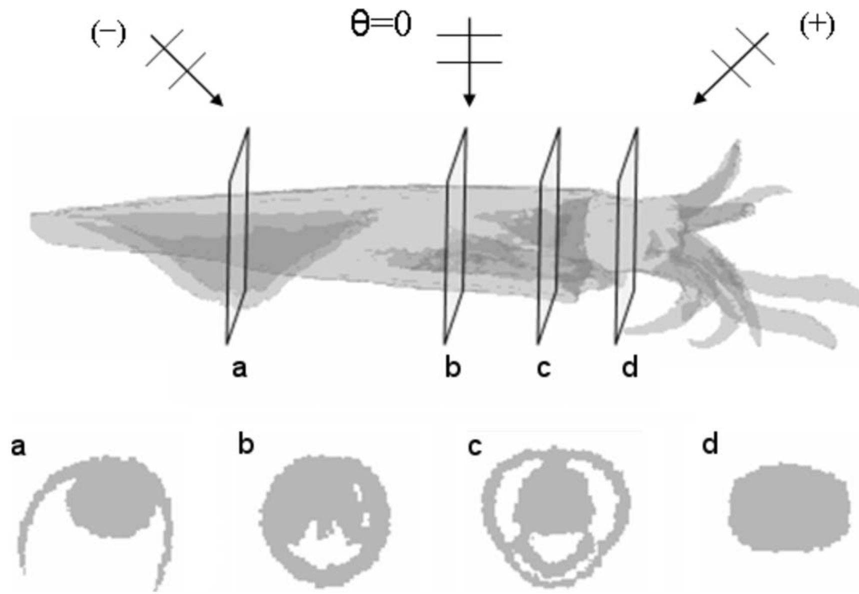
→ Sensitive measure of nuclear interactions

DWBA calculations – elastic, inelastic scattering in nuclear scattering



Landowne, Vitturi in treatise on heavy ion science

DWBA application in acoustic scattering of inhomogeneous objects:



Jones et al., J. Acoust. Soc. Am. 125 (2009)73

ICES Journal of Marine Science, 60: 625–635. 2003
doi:10.1016/S1054–3139(03)00063-8

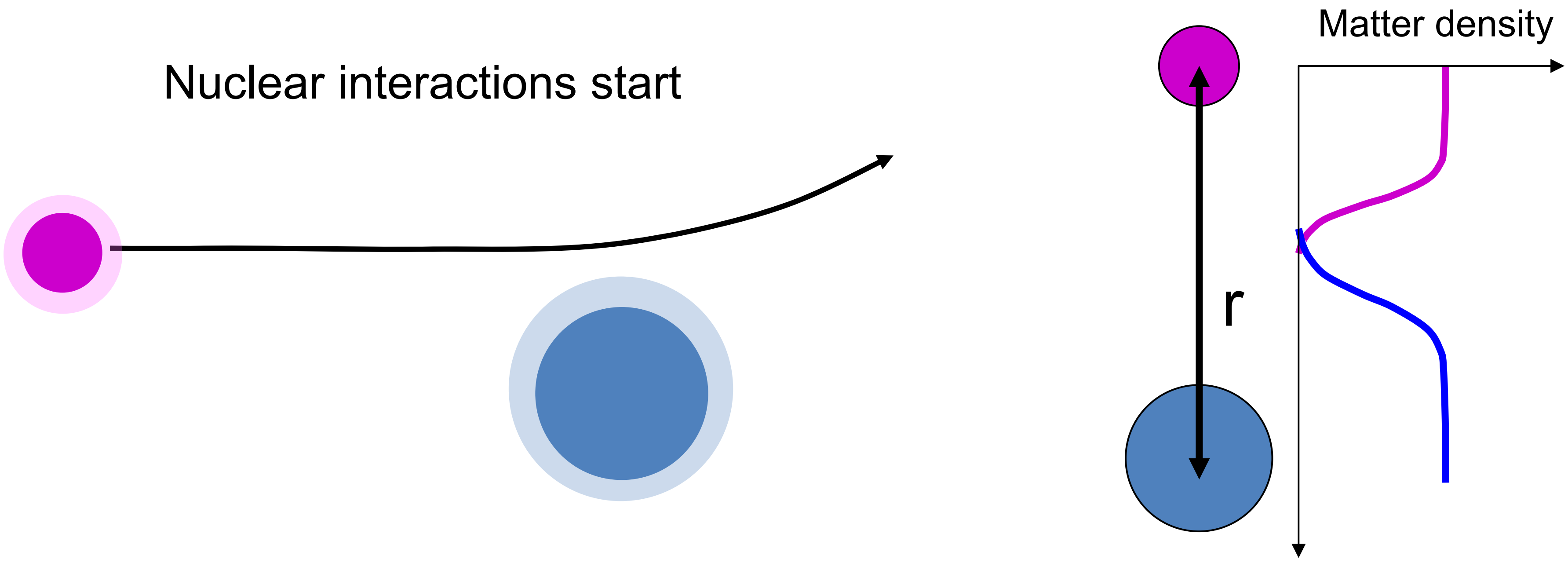
Available online at www.sciencedirect.com
SCIENCE DIRECT®

Validation of the stochastic distorted-wave Born approximation model with broad bandwidth total target strength measurements of Antarctic krill

David A. Demer and Stéphane G. Conti

Introduction

The United States of America's Antarctic Marine Living Resources Program (AMLR) uses multi-frequency echo-sounders and echo-integration to map the dispersion of Antarctic krill (*Euphausia superba*) over large areas and to estimate their abundance (Hewitt and Demer, 2000). The



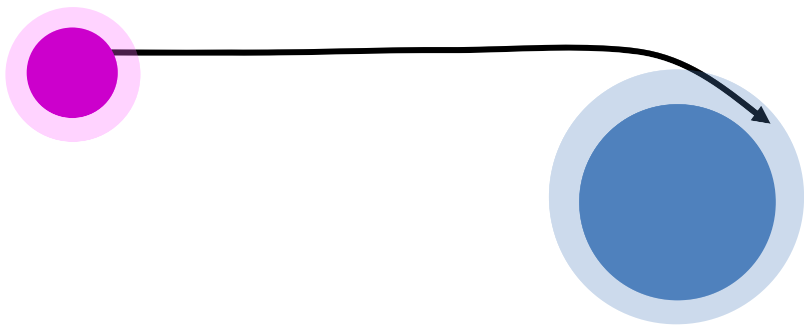
- Inelastic scattering (Coulomb and nuclear)

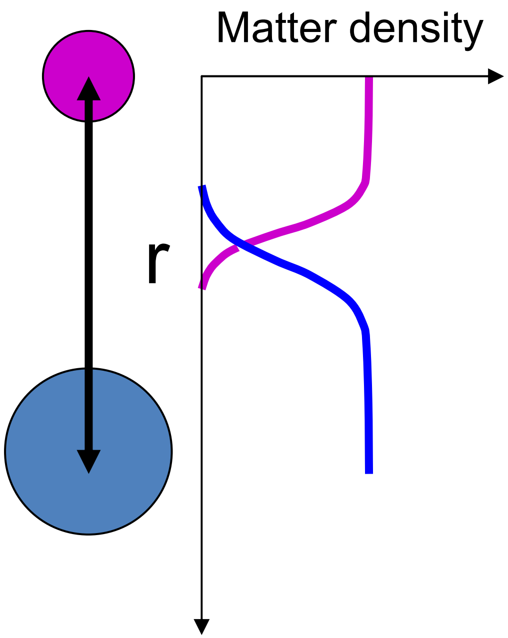
- transfer reactions –- if transfer process is weak and proceeds directly (i.e. not transfer following inelastic excitation) q.m. calculation method – distorted wave Born approximation (DWBA)

- Full calculation (Coupled channels)

DWBA: relative motion before and after non-elastic event are described by waves distorted by elastic scattering and absorption (not two plane waves as in Born Approximation)

Nuclear interactions stronger

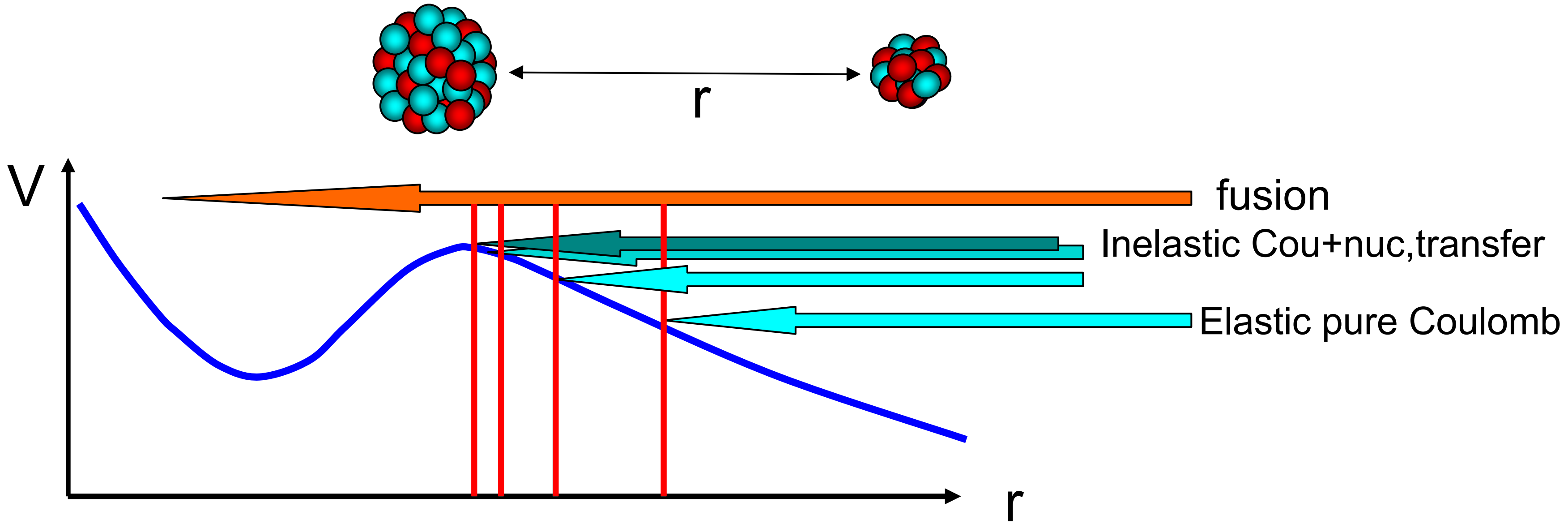




- Multi-nucleon transfer
- Deep inelastic reactions
- Nuclear fusion

Radial dependence of probabilities

- Mapping energy to radial separation



- Hierarchy of complexity
- Thus it is not a either/or situation, e.g. if energy is sufficient inelastic excitation, transfer all occur on the way to fusion
  → Coupled channels
- Quasi elastic: elastic + peripheral - probes “tail” of the nuclear potential

Washiyama et al. PRC73 (2006) 034697 ; Evers et al., PRC78 (2008)034614

Inelastic scattering, few nucleon transfer – good quantum theories

- Few-nucleon transfer
- Multi-nucleon transfer
- Deep inelastic reactions
- Nuclear fusion (Complete damping of kinetic energy – Compound nucleus formed)

Increasing complexity

Complete damping of K.E. → leads to fusion is not (yet) described quantum mechanically

imaginary potential or incoming wave boundary condition - like a "blackhole"

Main messages

- Colliding nuclei – many body quantum systems
- Heavy nuclei – structure and reaction dynamics entangled (coupled channels)
- Various classes of reactions - impact parameter concept

Additional material follows

For pure coulomb interaction

$$f(\theta) = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 2\mu v^2} \sin^{-2}(\theta_{cm}/2) \exp[(-i\alpha)\log\{(1-\cos\theta_{cm})/2\} + i\pi + 2i\eta_0]$$

Where $\alpha = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar v}$ $\exp(2i\eta_0) = \frac{\Gamma(1+i\alpha)}{\Gamma(1-i\alpha)}$

$$\frac{d\sigma}{d\Omega} = |f(\theta) + f(\pi-\theta)|^2$$

$\sin^{-4}\{(\pi - \theta_{cm})/2\}$

$$= \left[\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 2\mu v^2}\right]^2 [\sin^{-4}(\theta_{cm}/2) + \cos^{-4}(\theta_{cm}/2) + 2\Phi \sin^{-2}(\theta_{cm}/2)\cos^2(\theta_{cm}/2)]$$

where $\Phi = \cos\left[\frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 \hbar v} \log\{\tan^2(\theta_{cm}/2)\}\right]$

Derivation of f(θ) for coulomb scattering: See “The theory of atomic collisions”, 3rd edition, by Mott and Massey, page 55, note V= ZZ’ε^2/r, whilst we have used $Z_1 Z_2 e^2/4\pi\varepsilon_0 r$