Mechanism Design

ICTS Course on Dynamics of Complex Systems

May 23, 2019

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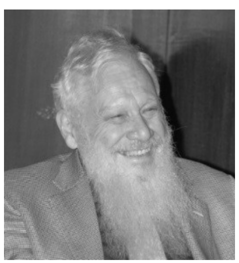
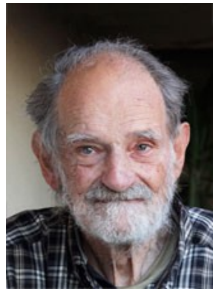
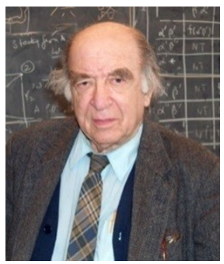
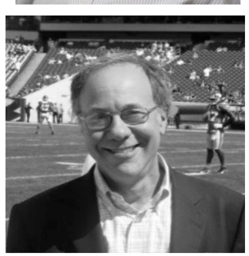
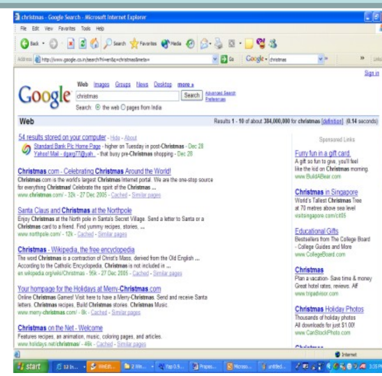
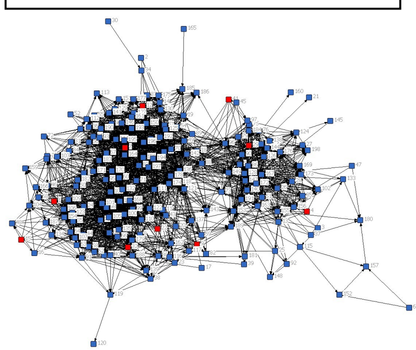
GTMD Meets Computer Science

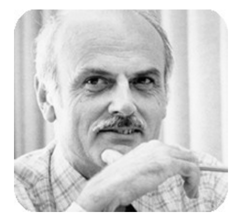
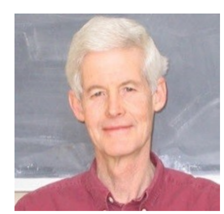


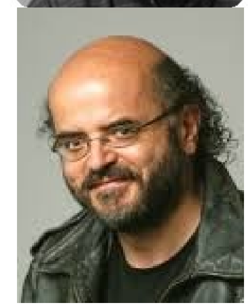


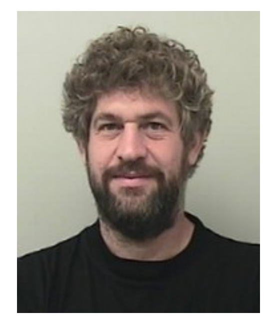





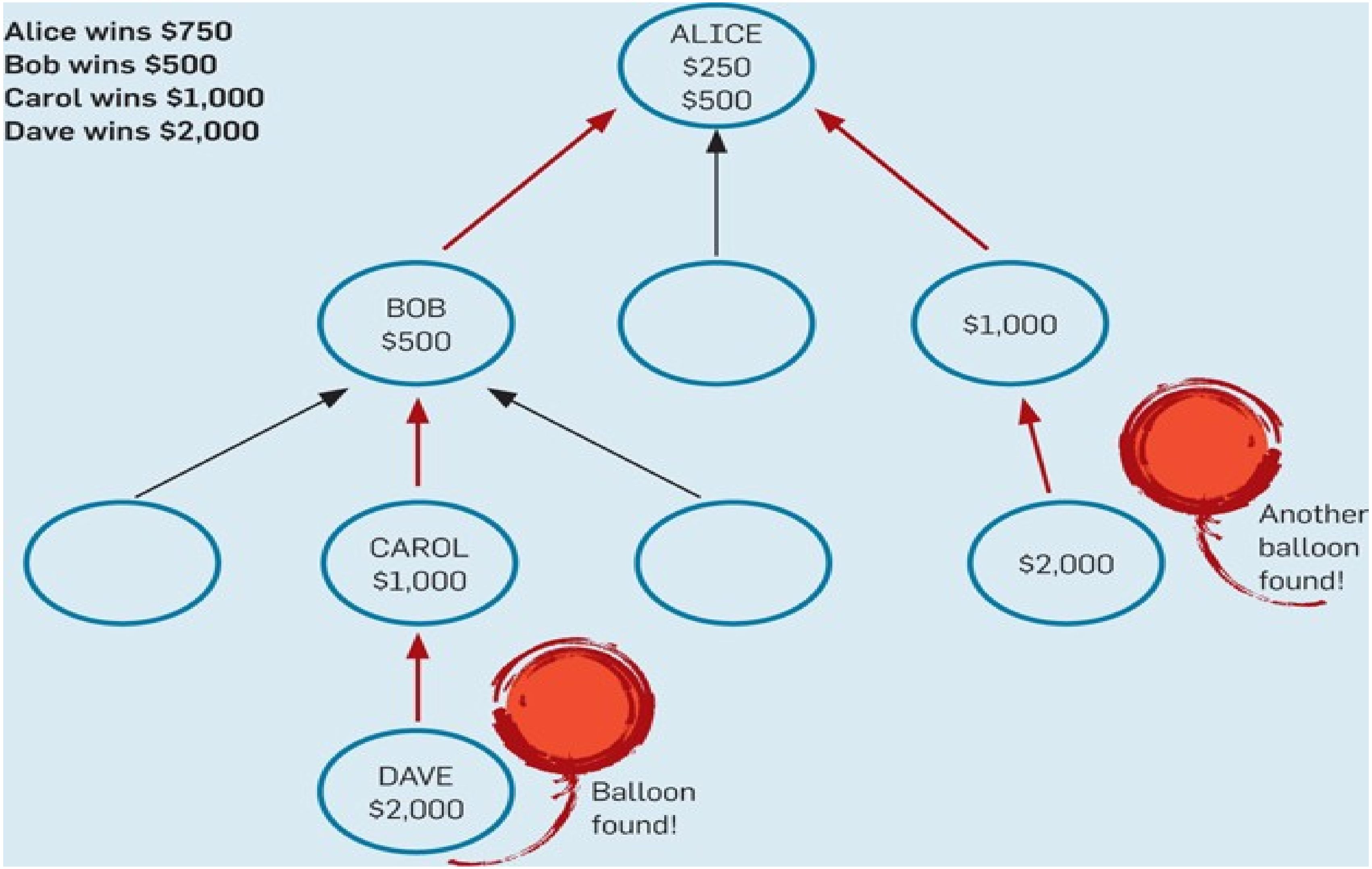


Geospatial Exploration

Mechanism Design Meets Computer Science, Communications of the ACM, Aug-2010

Image Courtesy: CACM, Aug-2010

Geospatial Exploration (contd.)



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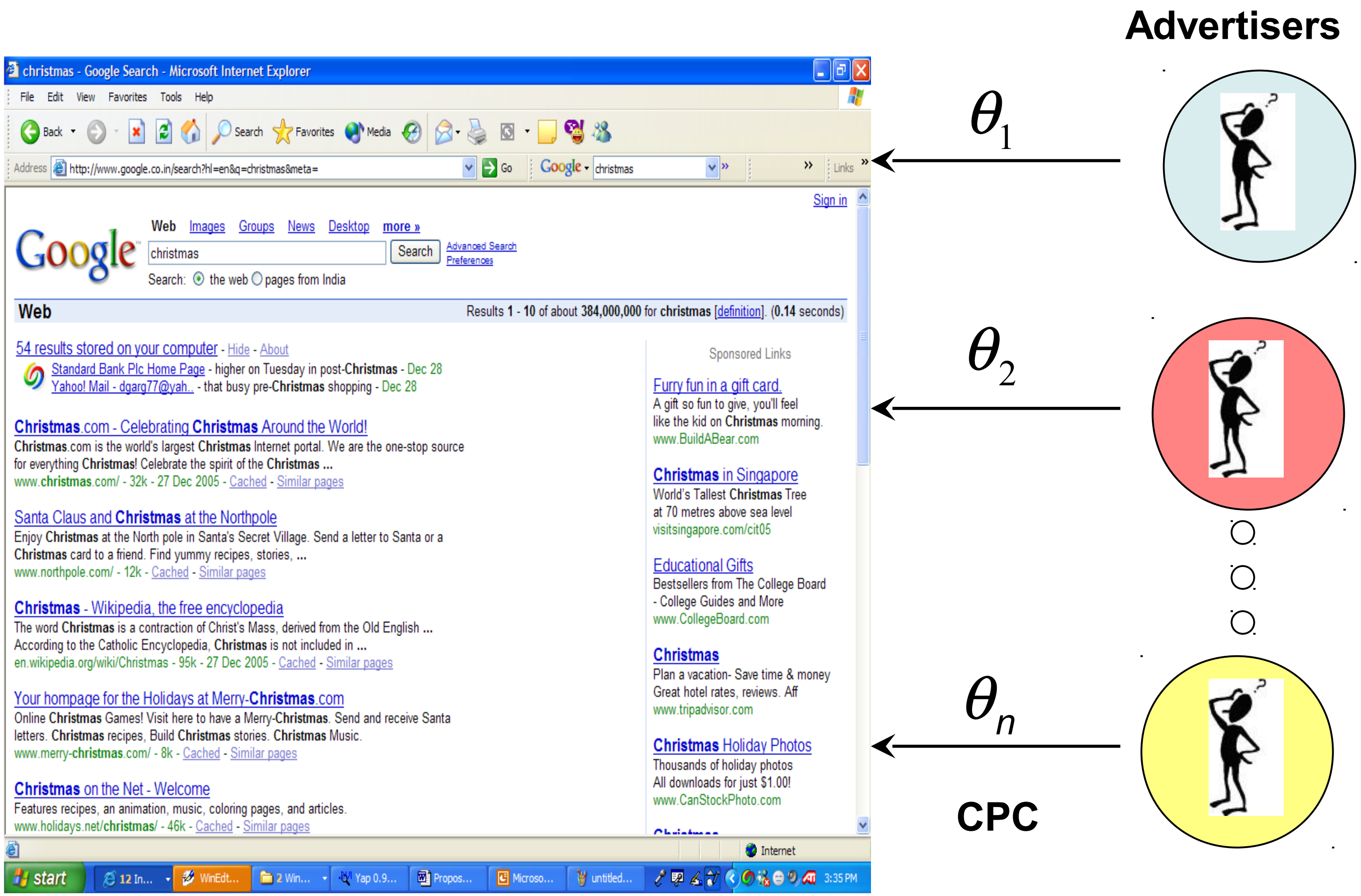
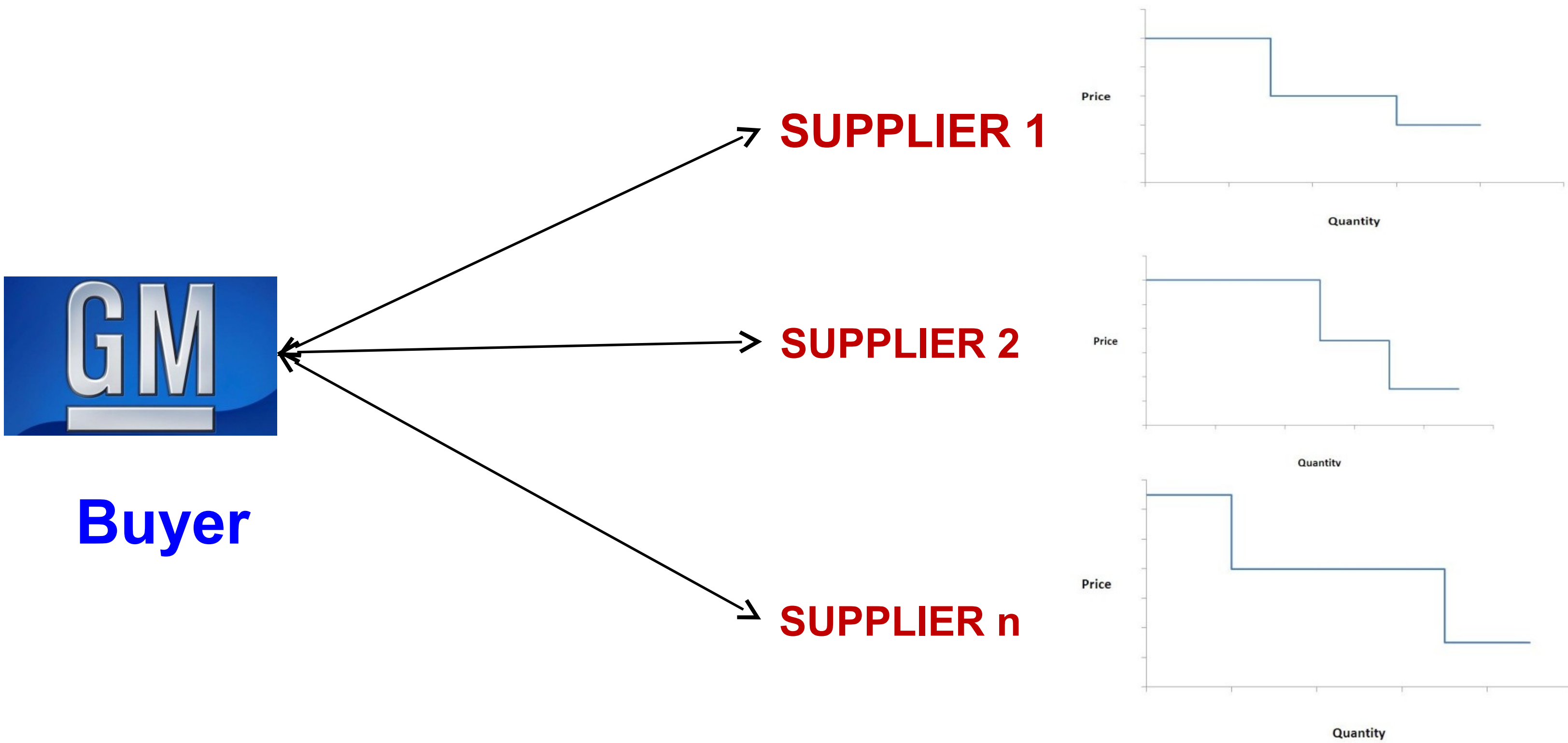
Sponsored Search Auction



Design an auction that maximizes social utility

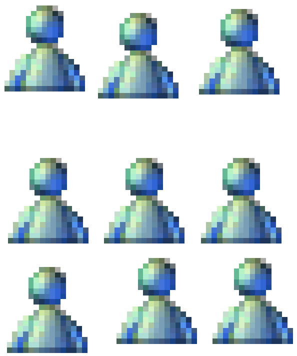
Procurement Auctions



Supply (cost) Curves

Budget Constraints, Lead Time Constraints, Learning by Suppliers, Learning by Buyer, Logistics constraints, Combinatorial Auctions, Cost Minimization, Multiple Attributes

Matching Market



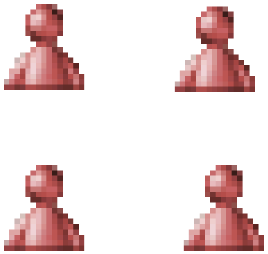
Students

Employees

Doctors

Farmers

Matching Market



Colleges

Companies

Hospitals

Consumers

Design a matching market that maximizes the social welfare and induces honest behavior

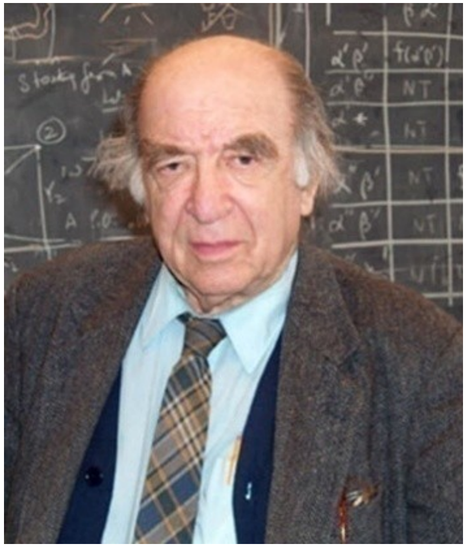
The Case for GTMD

Many modern problems involve strategic agents which can derail the algorithms and solutions in different ways

How do we realize social goals in the presence of self-interested agents?
How do we make agents behave honestly?

Game theory and mechanism design have principled answers to these challenges and are key to filling an important gap

Leonid Hurwicz



Eric Maskin

Roger Myerson

2007 Nobel Prize In Economics for Game Theory and Mechanism Design

What is Mechanism Design?

Induces a game among the players such that in some equilibrium of the game, a desired social choice function is implemented

Reverse Engineering of Games

Example 1
Cake Cutting Problem

Mother
Social Planner
Mechanism Designer

Kid 1
Rational and Intelligent

Kid 2
Rational and Intelligent

Example 2
Baby's Mother Problem

Tenali Rama
(Birbal)
(King Soloman)
Mechanism Designer

Mother 1
Rational and Intelligent

Baby

Mother 2
Rational and Intelligent

Example 3: Vickrey Auction Game

1  40

2  50

3  60

4  80

Winner = 4

Payment = 60

Bidders: $N = \{1,2, \ldots, n\}$ ({1,2,3,4})

Valuations = $\{V_1, V_2, \ldots, V_n\}$ (40,50,60,80)

Strategy Sets (Bids)

$S_1 = S_2 = \ldots = S_n =$ [0, infty)

Allocation: Highest Bidder (4)

Payment: Highest non-winning bid (60)

Utilities: $U_i = x_i (V_i - P_i)$ (0,0,0,20)

Valuation minus payment for winner; zero for losers

Vickrey Auction is Truthful (DSIC)

V_1  b_1 V_2  b_2

There are two cases:

$V_1 >= b_2$; $V_1 < b_2$

Case 1: $V_1 >= b_2$

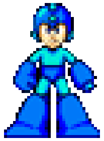
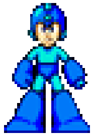
(1.1) $b_1 >= b_2$: Bidder 1 wins; $U_1 = V_1 - b_2 >= 0$

(1.2) $b_1 < b_2$: Bidder 1 loses; $U_1 = 0$;

If bidder 1 is truthful, $b_1 = V_1 >= b_2$; $U_1 = V_1 - b_2 >= 0$

Thus bidding truthfully is better whatever b_2

Vickrey Auction is Truthful (DSIC) (contd.)

V_1  b_1 V_2  b_2

Case 2: $V_1 < b_2$

(2.1) $b_1 >= b_2$: Bidder 1 wins; $U_1 = V_1 - b_2 < 0$

(2.2) $b_1 < b_2$: Bidder 1 loses; $U_1 = 0$

If bidder 1 is truthful, $b_1 = V_1 < b_2$; loses; $U_1 = 0$

<u>Thus bidding truthfully is better whatever b_2</u>

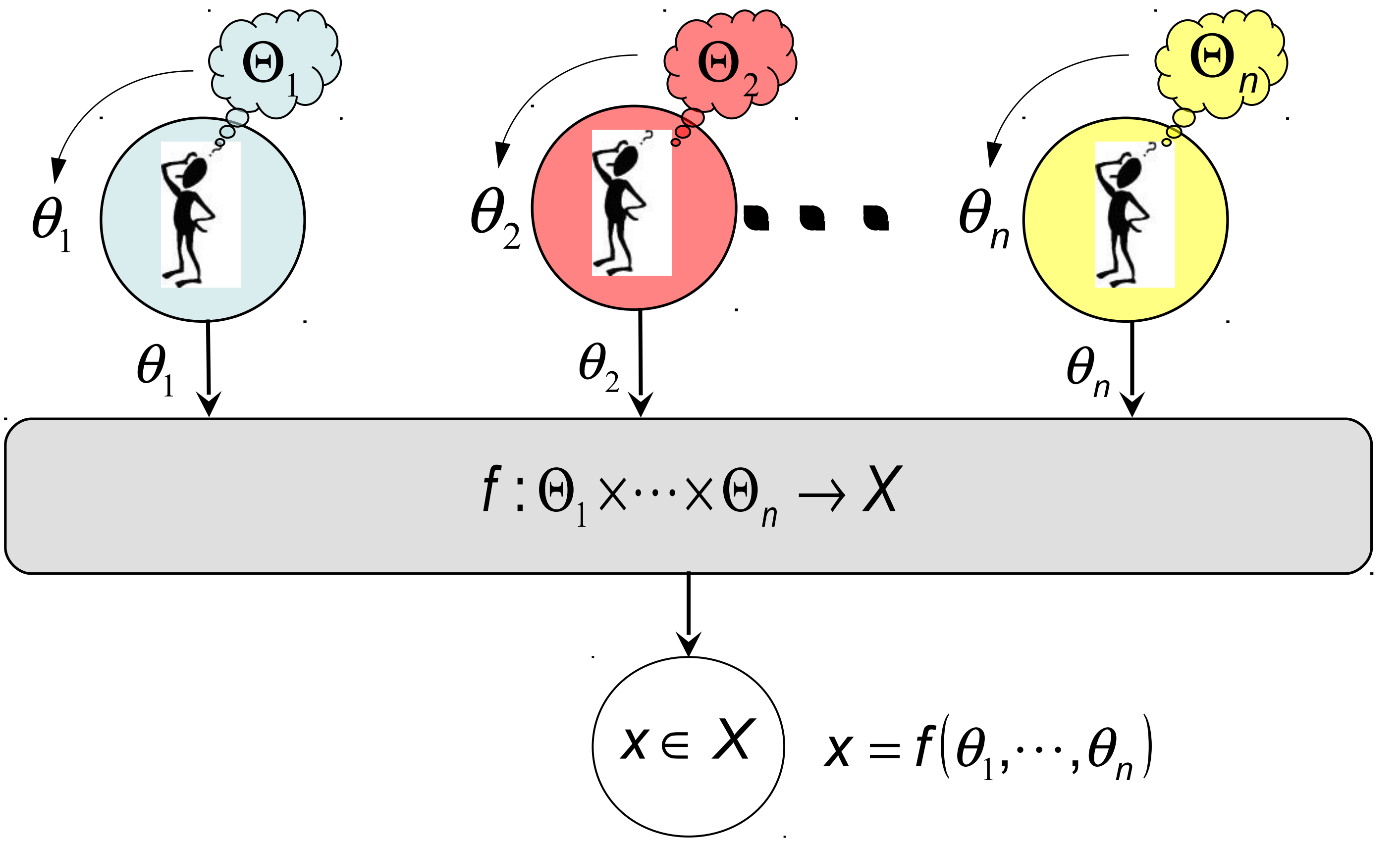
The Mechanism Design Problem

- n agents who need to make a collective choice from outcome set X
- Each agent i privately observes a signal θ_i which determines $i's$ preferences over the set X
- Signal θ_i is known as agent $i's$ type.
- The set of agent $i's$ possible types is denoted by Θ_i
- The agents types, $\theta = (\theta_1, \cdots, \theta_n)$ are drawn according to a probability distribution function $\Phi(.)$
- Each agent is rational, intelligent, and tries to maximize its utility function

$$u_i : X \times \Theta_i \rightarrow \Re$$

- $\Phi(.), \Theta_1, \cdots, \Theta_n, u_1(.), \cdots, u_n(.)$ are common knowledge among the agents

Social Choice Function (SCF)



Two Fundamental Problems in Designing a Mechanism

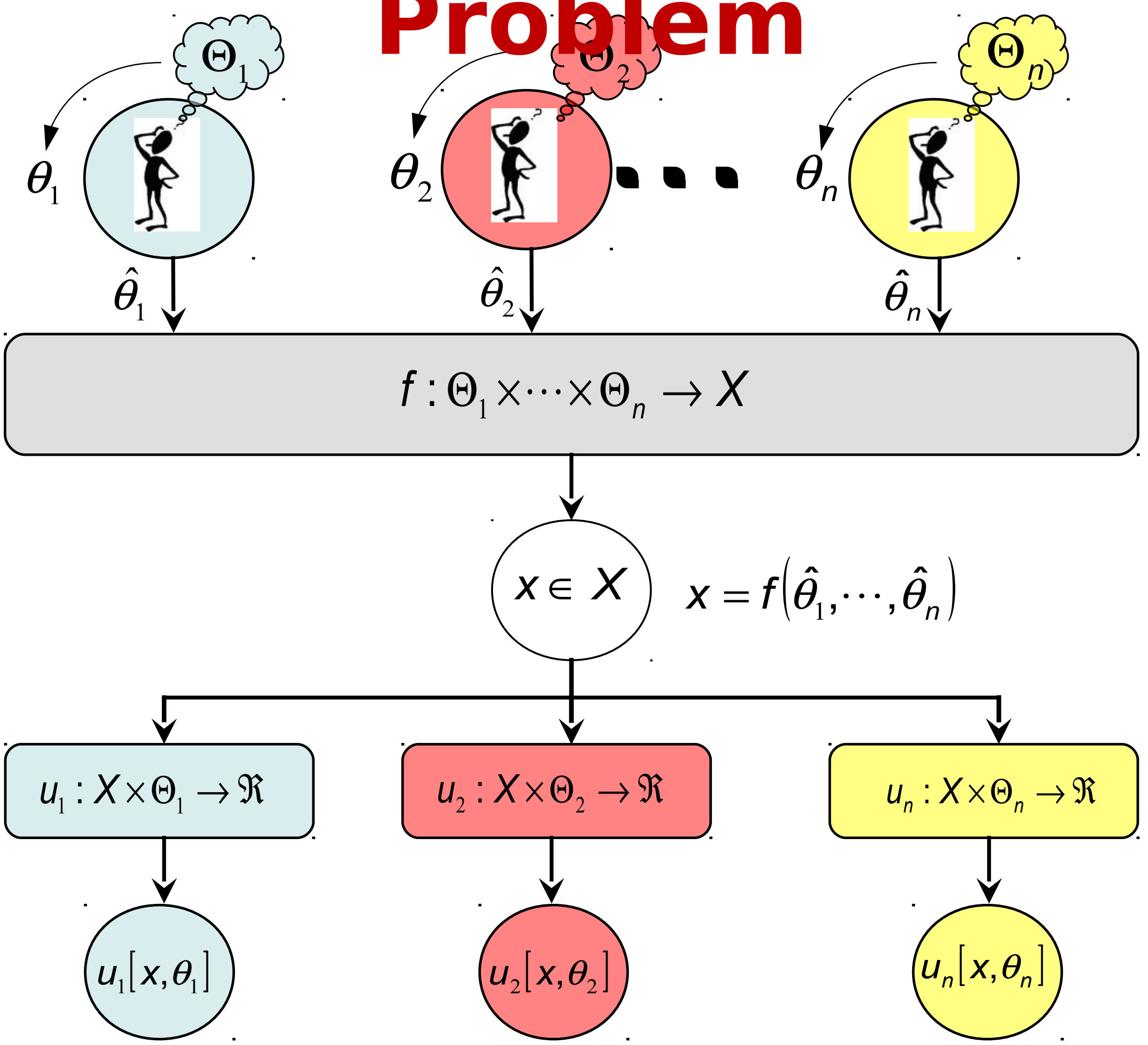
- **Preference Aggregation Problem**

  For a given type profile $\theta = (\theta_1, \cdots, \theta_n)$ of the agents, what outcome $x \in X$ should be chosen ?

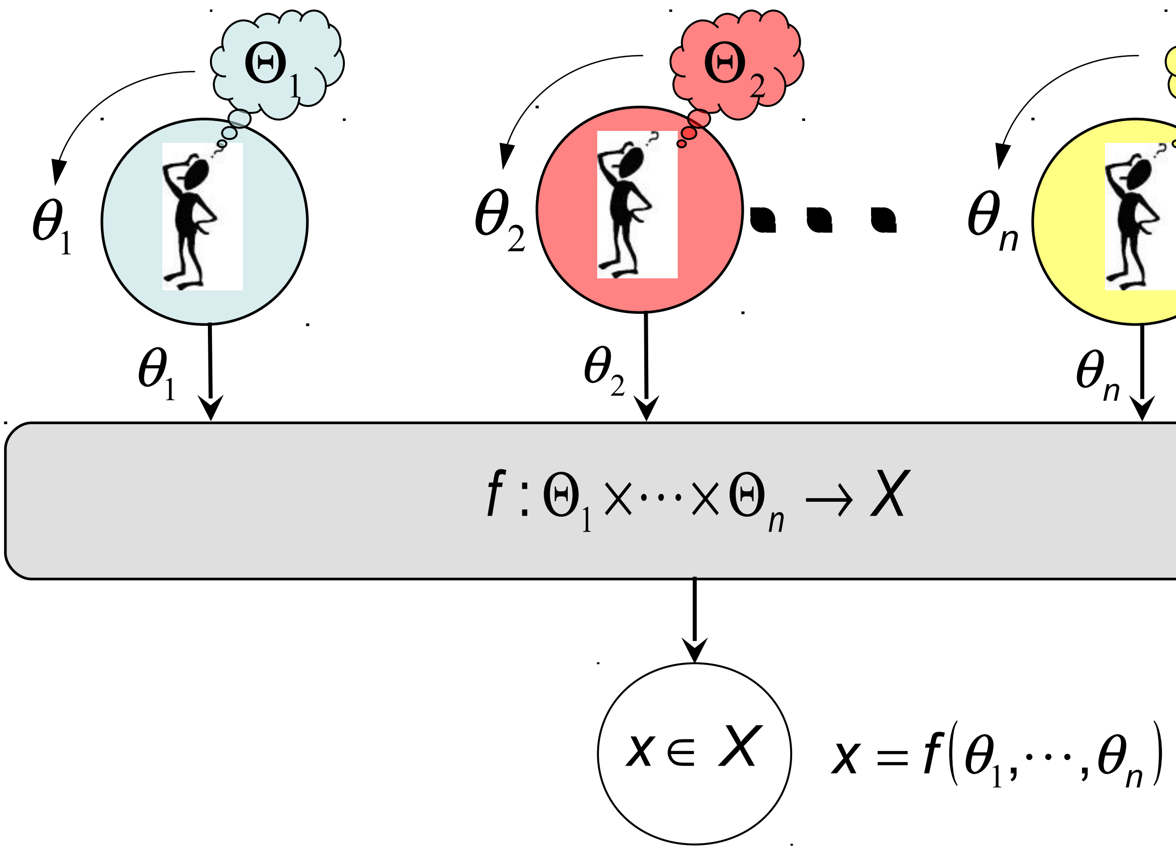
- **Information Revelation (Elicitation) Problem**

  How do we elicit the true type θ_i of each agent i, which is his private information ?

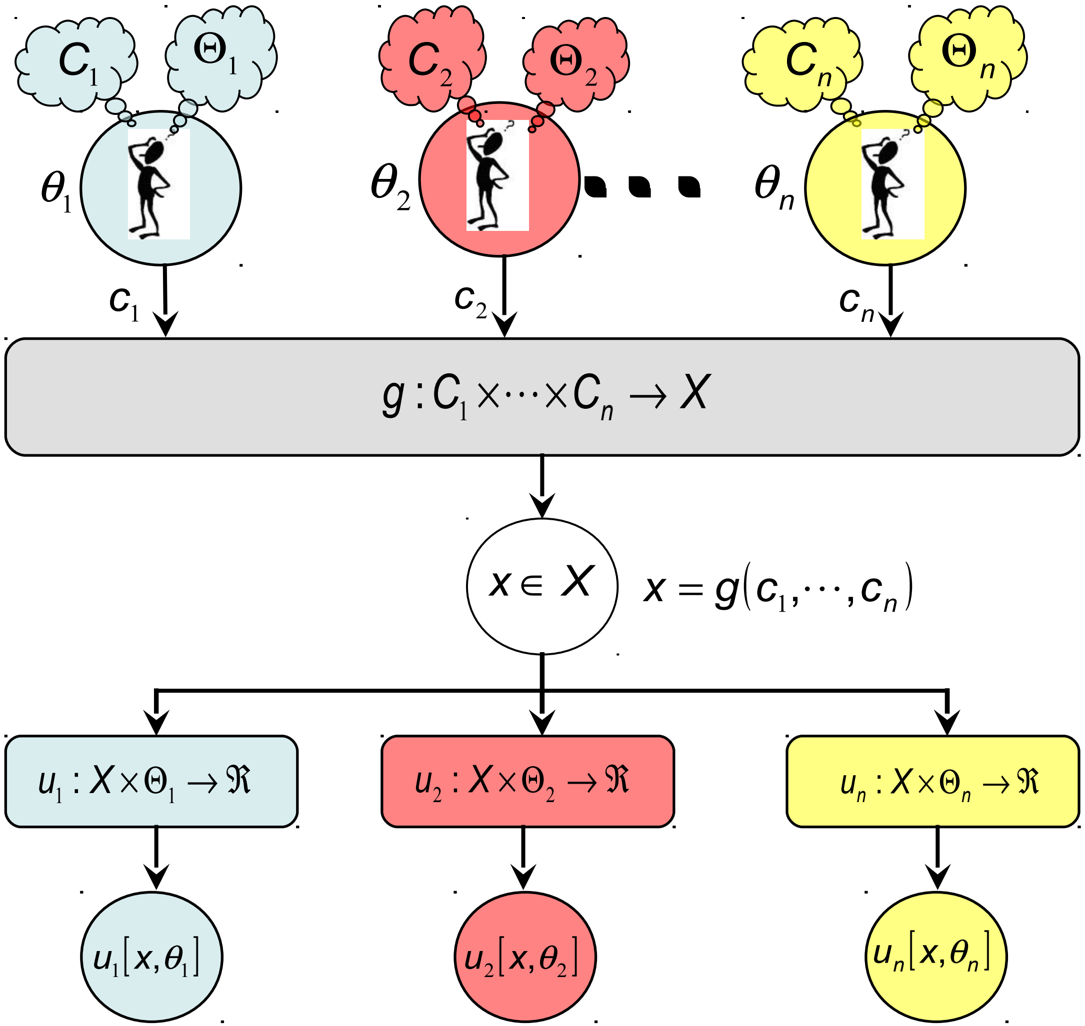
Information Elicitation Problem



Preference Aggregation Problem (SCF)



Indirect Mechanism



Social Choice Function and Mechanism

θ_1 . . . θ_n

Outcome Set

$f(\theta_1, \ldots, \theta_n) \in X$

$x = (y_1(\theta), \ldots, y_n(\theta), t_1(\theta), \ldots, t_n(\theta))$

S_1 . . . S_n

Outcome Set

$g(s_1(.), \ldots, s_n()) \in X$

$(S_1, \ldots, S_n, g(.))$

A mechanism induces a Bayesian game and is designed to implement a social choice function in an equilibrium of the game

Equilibrium of Induced Bayesian Game

⇨ Dominant Strategy Equilibrium (DSE)

A pure strategy profile $\left(s_1^d(.),\cdots s_n^d(.)\right)$ is said to be dominant strategy equilibrium if

$$u_i(g(s_i^d(\theta_i),s_{-i}(\theta_{-i})),\theta_i) \geq u_i(g(s_i(\theta_i),s_{-i}(\theta_{-i})),\theta_i)$$

$$\forall i \in N, \theta_i \in \Theta_i, s_i \in S_i, s_{-i} \in S_{-i}$$

⇨ Bayesian Nash Equilibrium (BNE)

A pure strategy profile $\left(s_1^*(.),\cdots s_n^*(.)\right)$ is said to be Bayesian Nash equilibrium

$$E_{\theta_{(-i)}}[u_i(g(s_i^*(\theta_i),s_{-i}^*(\theta_{-i})),\theta_i) \mid \theta_i] \geq E_{\theta_{(-i)}}[u_i(g(s_i(\theta_i),s_{-i}^*(\theta_{-i})),\theta_i) \mid \theta_i]$$

$$\forall i \in N, \theta_i \in \Theta_i, s_i \in S_i$$

⇨ Observation

Dominant Strategy-equilibrium $\Longrightarrow$ Bayesian Nash- equilibrium

Implementing an SCF

⇨ Dominant Strategy Implementation

We say that mechanism $M = \left(g(.), (C_i)_{i \in N}\right)$ implements SCF $f : \Theta \to X$ in dominant strategy equilibrium if

$$g\left(s_1^d(\theta_1), \cdots s_n^d(\theta_n)\right) = f(\theta_1, \cdots, \theta_n) \qquad \forall (\theta_1, \cdots, \theta_n)$$

⇨ Bayesian Nash Implementation

We say that mechanism $M = \left(g(.), (C_i)_{i \in N}\right)$ implements SCF $f : \Theta \to X$ in Bayesian Nash equilibrium if

$$g\left(s_1^*(\theta_1), \cdots s_n^*(\theta_n)\right) = f(\theta_1, \cdots, \theta_n) \qquad \forall (\theta_1, \cdots, \theta_n)$$

⇨ Observation

Dominant Strategy-implementation $\Longrightarrow$ Bayesian Nash- implementation

Andreu Mas Colell, Michael D. Whinston, and Jerry R. Green, "**Microeconomic Theory**", Oxford University Press, New York, 1995.

Properties of an SCF

- **Ex Post Efficiency**

  For no profile of agents' type $\theta = (\theta_1, \cdots, \theta_n)$ does there exist an $x \in X$ such that $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i) \;\forall i$ and $u_i(x, \theta_i) \geq u_i(f(\theta), \theta_i)$ for some i

- **Dominant Strategy Incentive Compatibility (DSIC)**

  If the direct revelation mechanism $D = \left(f(.), (\Theta_i)_{i \in N}\right)$ has a dominant strategy equilibrium $(s_1^d(.), \cdots s_n^d(.))$ in which

$$s_i^d(\theta_i) = \theta_i, \forall \theta_i \in \Theta_i, i \in N$$

- **Bayesian Incentive Compatibility (BIC)**

  If the direct revelation mechanism $D = \left(f(.), (\Theta_i)_{i \in N}\right)$ has a Bayesian Nash equilibrium $(s_1^*(.), \cdots s_n^*(.))$ in which

$$s_i^*(\theta_i) = \theta_i, \forall \theta_i \in \Theta_i, i \in N$$

Implementing an SCF

⇨ Dominant Strategy Implementation

We say that mechanism $M = \left(g(.), (C_i)_{i \in N}\right)$ implements SCF $f : \Theta \to X$ in dominant strategy equilibrium if

$$g\left(s_1^d(\theta_1), \cdots s_n^d(\theta_n)\right) = f(\theta_1, \cdots, \theta_n) \qquad \forall (\theta_1, \cdots, \theta_n)$$

⇨ Bayesian Nash Implementation

We say that mechanism $M = \left(g(.), (C_i)_{i \in N}\right)$ implements SCF $f : \Theta \to X$ in Bayesian Nash equilibrium if

$$g\left(s_1^*(\theta_1), \cdots s_n^*(\theta_n)\right) = f(\theta_1, \cdots, \theta_n) \qquad \forall (\theta_1, \cdots, \theta_n)$$

⇨ Observation

Dominant Strategy-implementation $\Longrightarrow$ Bayesian Nash- implementation

PROPERTIES OF SOCIAL CHOICE FUNCTIONS

DSIC (Dominant Strategy Incentive Compatibility)
Reporting Truth is always good

BIC (Bayesian Nash Incentive Compatibility)
Reporting truth is good whenever others also report truth

AE (Allocative Efficiency)
Allocate items to those who value them most

BB (Budget Balance)
Payments balance receipts and No losses are incurred

Non-Dictatorship
No single agent is favoured all the time

Individual Rationality
Players participate voluntarily since they do not incur losses

POSSIBILITIES AND IMPOSSIBILITIES - 1

Gibbard-Satterthwaite Theorem

When the preference structure is rich, a social choice function is DSIC iff it is dictatorial

Groves Theorem

In the quasi-linear environment, there exist social choice functions which are both AE and DSIC

The dAGVA Theorem

In the quasi-linear environment, there exist social choice functions which are AE, BB, and BIC

POSSIBILITIES AND IMPOSSIBILITIES -2

Green- Laffont Theorem

When the preference structure is rich, a social choice function cannot be DSIC and BB and AE

Myerson-Satterthwaite Theorem

In the quasi-linear environment, there cannot exist a social choice function that is BIC and BB and AE and IR

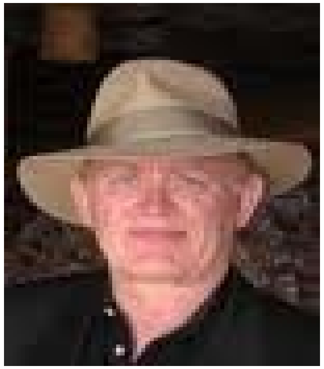
Myerson’s Optimal Mechanisms

Optimal mechanisms are possible subject to IIR and BIC (sometimes even DSIC)

Vickrey-Clarke-Groves (VCG) Mechanisms

Vickrey

Clarke



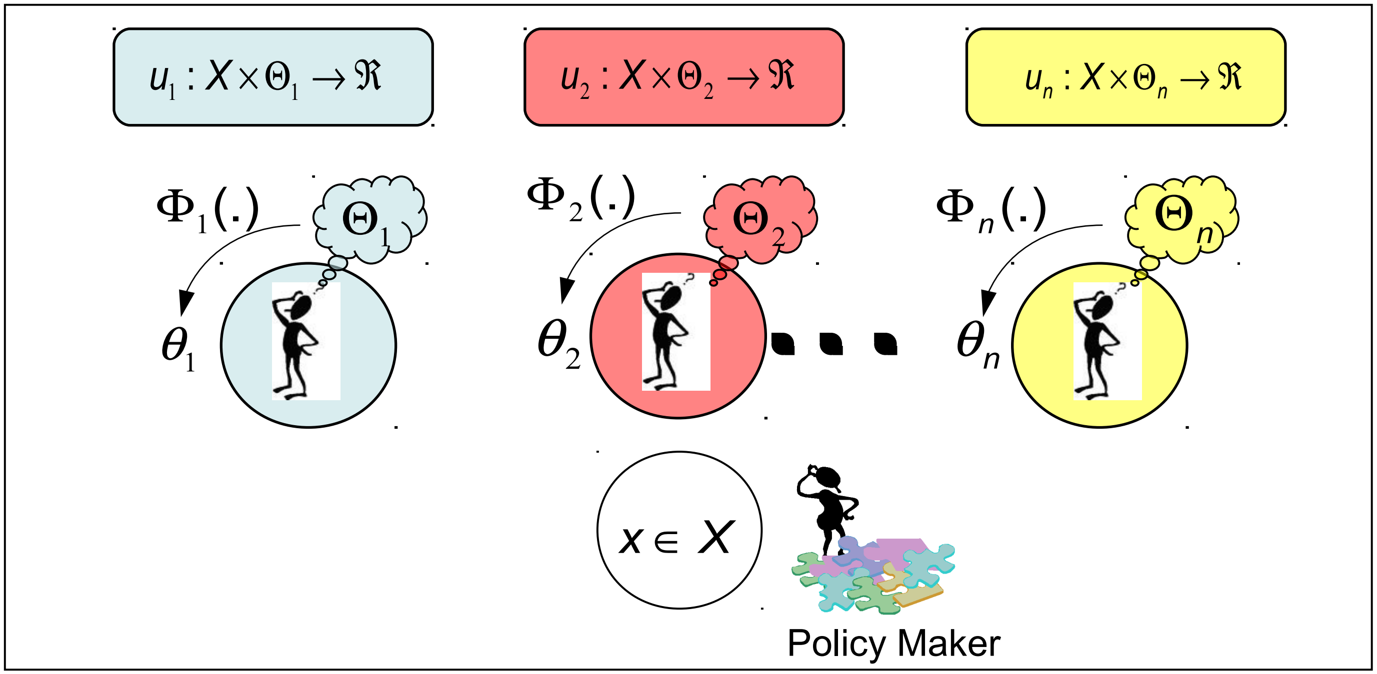
Groves

Only mechanisms under a quasi-linear setting satisfying
Allocative Efficiency
Dominant Strategy Incentive Compatibility

Quasi-Linear Environment

$$u_1(x,\theta_1) = v_1(k,\theta_1) + t_1$$

Valuation function of agent 1



$$X = \left\{ (k, t_1, \cdots, t_n) \mid k \in K, t_i \in \Re \ \forall i = 1, \cdots, n, \sum_i t_i \le 0 \right\}$$

project choice

Monetary transfer to agent 1

Properties of an SCF in Quasi-Linear Environment

- **Ex Post Efficiency**
- **Dominant Strategy Incentive Compatibility (DSIC)**
- **Bayesian Incentive Compatibility (BIC)**
- **Allocative Efficiency (AE)**

SCF $f(.) = (k(.), t_1(.), \cdots, t_n(.))$ is AE if for each $\theta \in \Theta$, $k(\theta)$ satisfies

$$k(\theta) \in \underset{k \in K}{\text{argmax}} \sum_{i=1}^{n} v_i(k, \theta_i)$$

- **Budget Balance (BB)**

SCF $f(.) = (k(.), t_1(.), \cdots, t_n(.))$ is BB if for each $\theta \in \Theta$, we have

$$\sum_{i=1}^{n} t_i(\theta) = 0$$

- **Lemma 1**

An SCF $f(.) = (k(.), t_1(.), \cdots, t_n(.))$ is ex post efficient in quasi-linear environment iff it is AE + BB

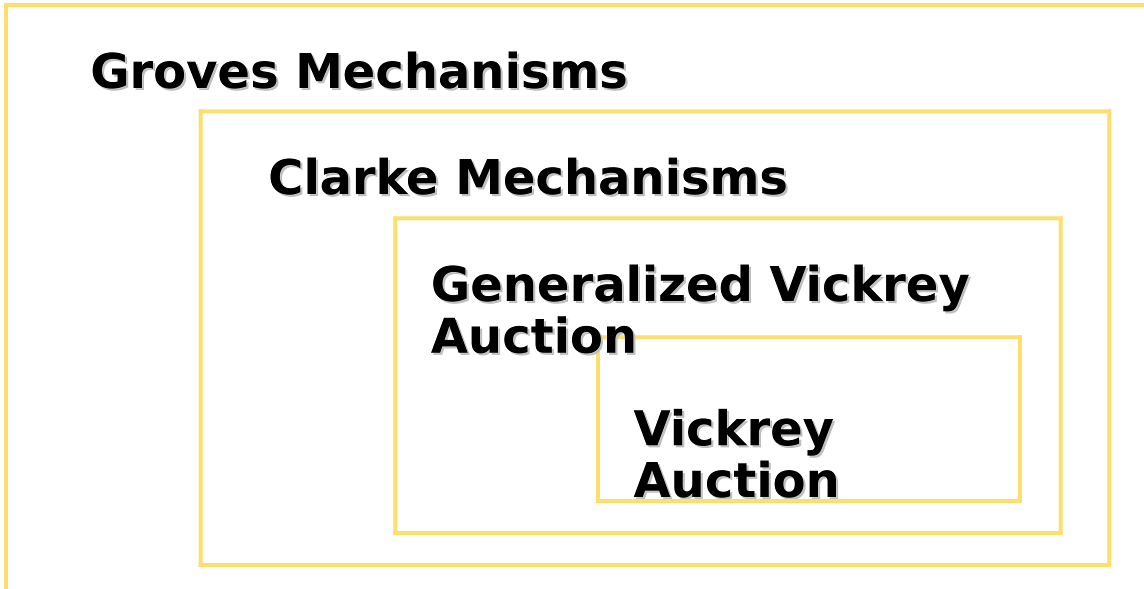
Groves Mecahnism: A Dominant Strategy Incentive Compatible Mechanism

1. Let $f(.) = (k(.), I_0(.), I_1(.), \ldots, I_n(.))$ be allocatively efficient.
2. Let the payments be :

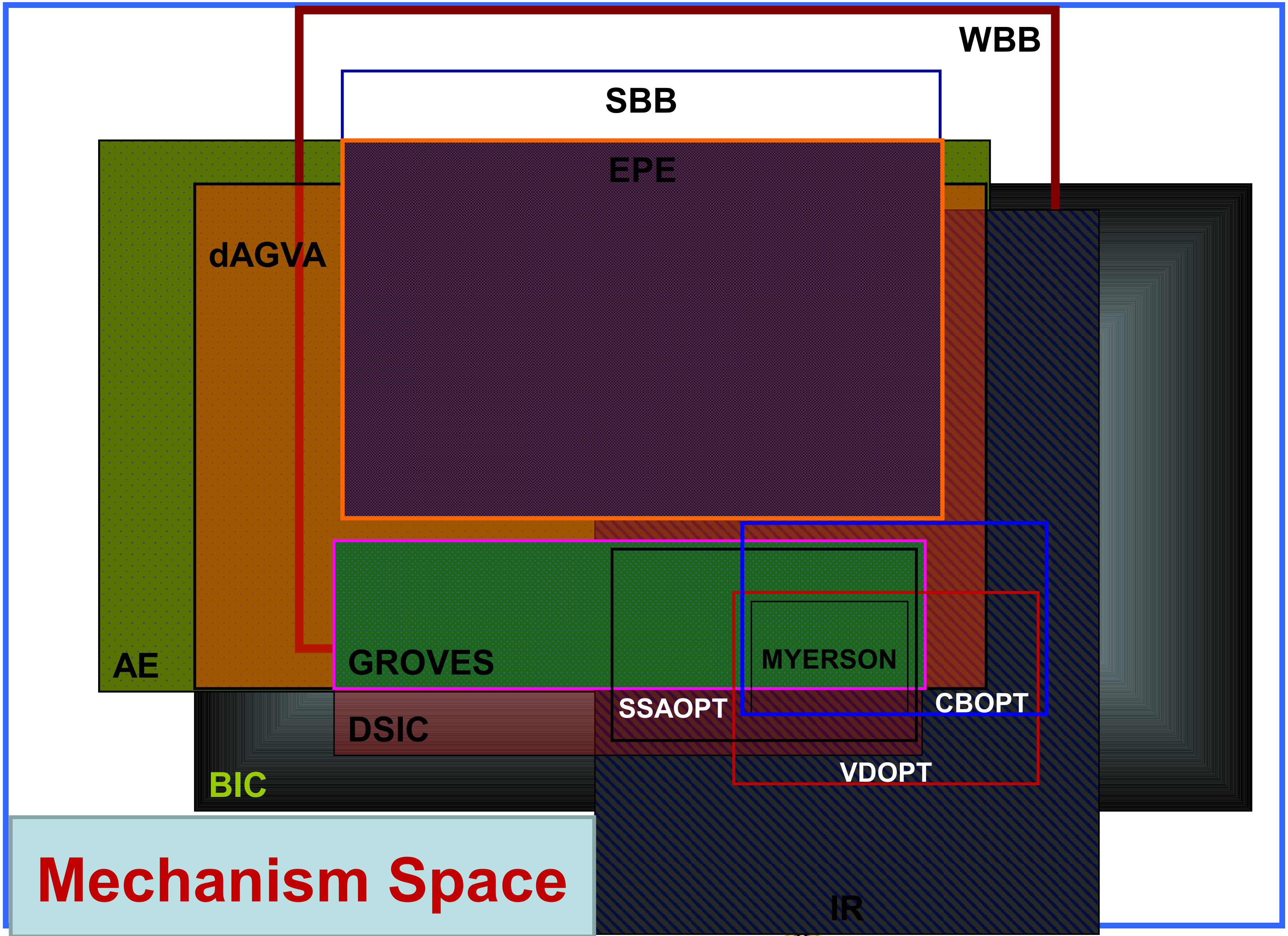
$$I_i(\theta) = \alpha_i(\theta_{-i}) - \sum_{j \neq i} b_j\left(\mu_i^*(\theta), \sigma_i^*(\theta)\right)$$

$$\forall \quad \theta \quad \in \quad \Theta$$

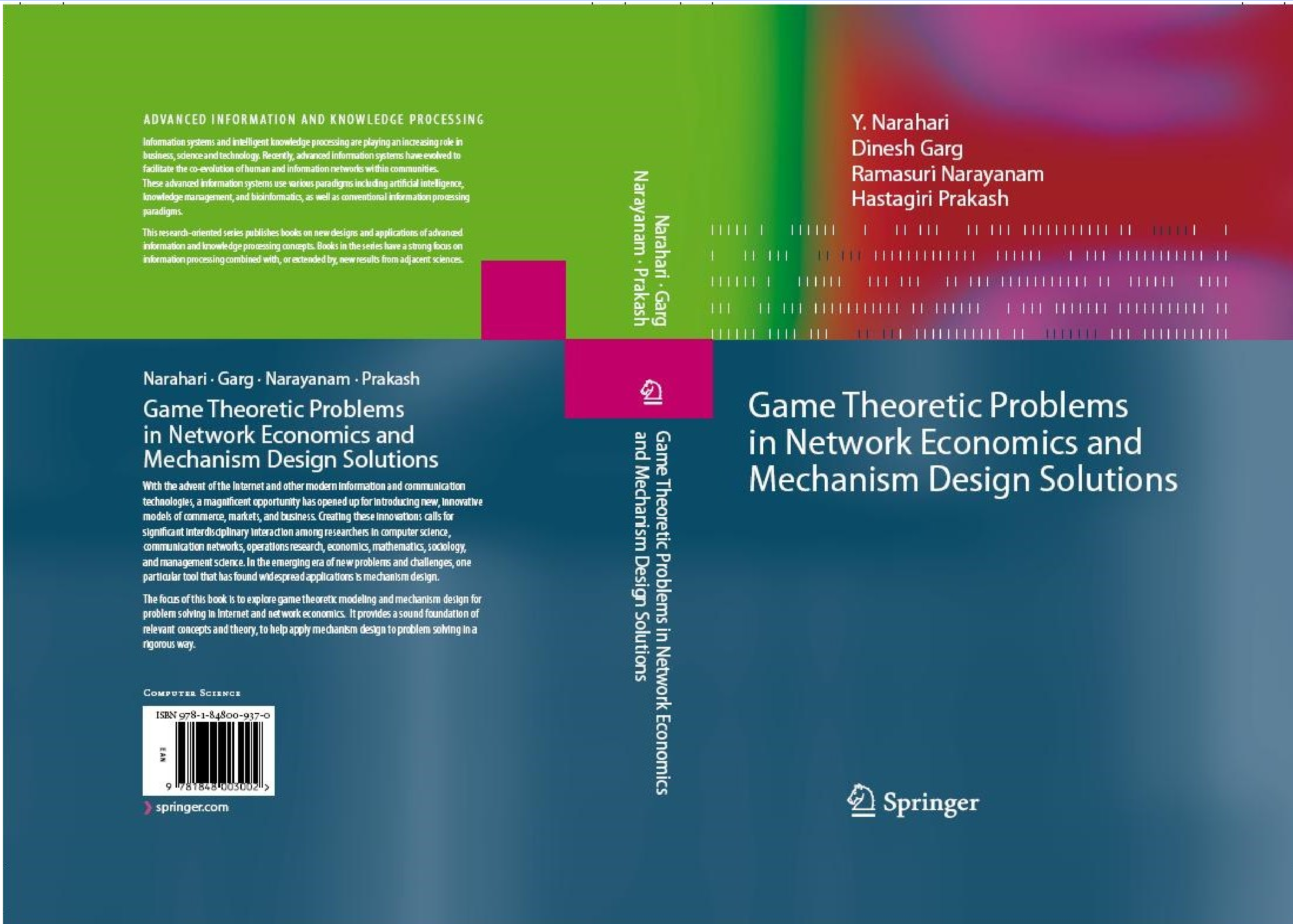
VCG Mechanisms (Vickrey-Clarke-Groves)



- **Allocatively efficient, individual rational, and dominant strategy incentive compatible with quasi-linear utilities.**



Mechanism Space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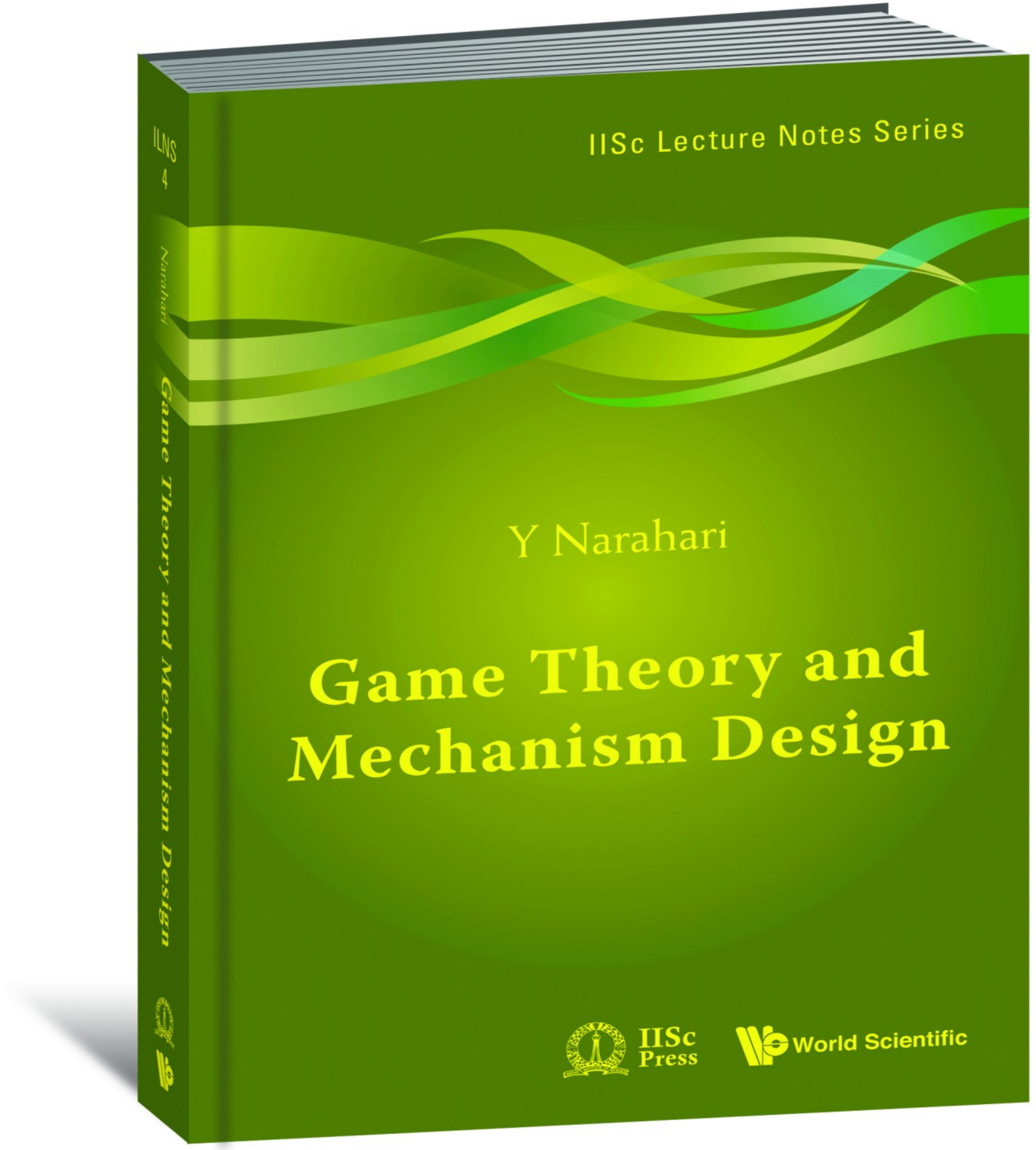
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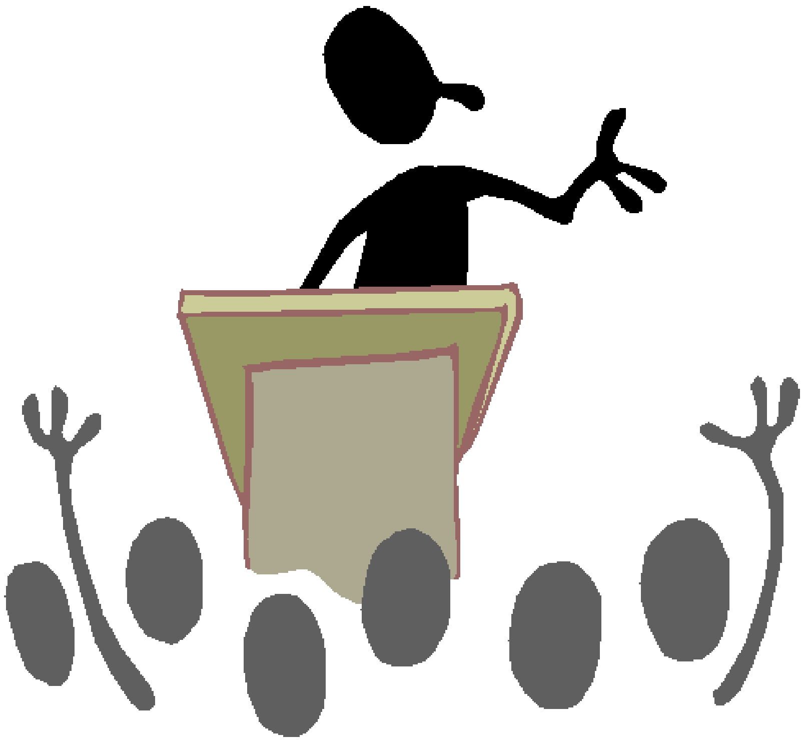
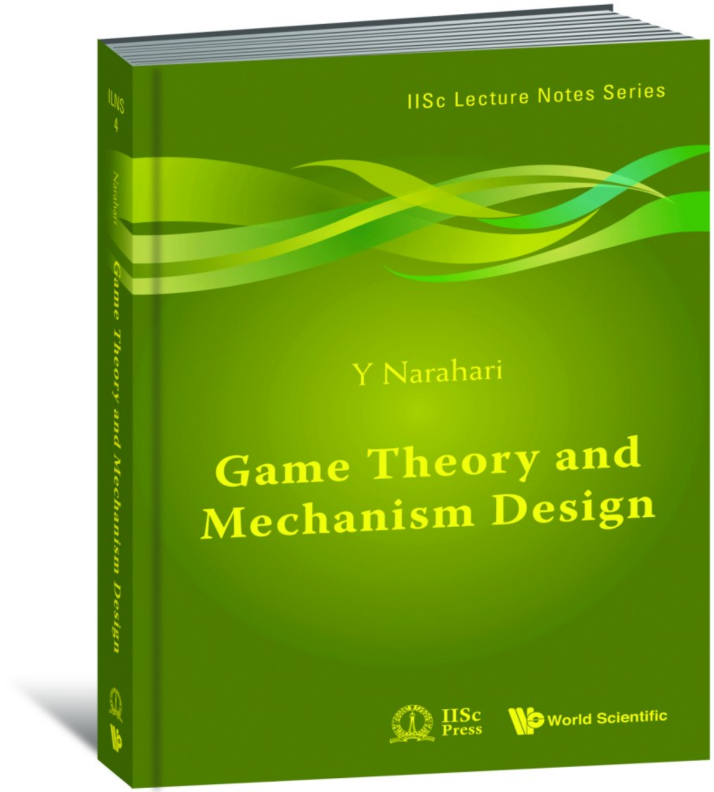
http://www.gametheory.net
A rich source of material on game theory and game theory courses

http://lcm.csa.iisc.ernet.in/hari
Course material and several survey articles can be downloaded

Questions …





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