GAMES ON NETWORKS

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May 18th 2019

Summer Research Program on

Dynamics of Complex Systems (DCS2019)

"Under what conditions will cooperation emerge in a world of egoists without central authority?"

–*Robert Axelrod*

* *R.* Axelrod, *"The evolution of cooperation"* (Basic Books, 1984).

EXAMPLE #1:
EXTREME ALTRUISM IN *DICTYOSTELIUM DISCOIDEUM*

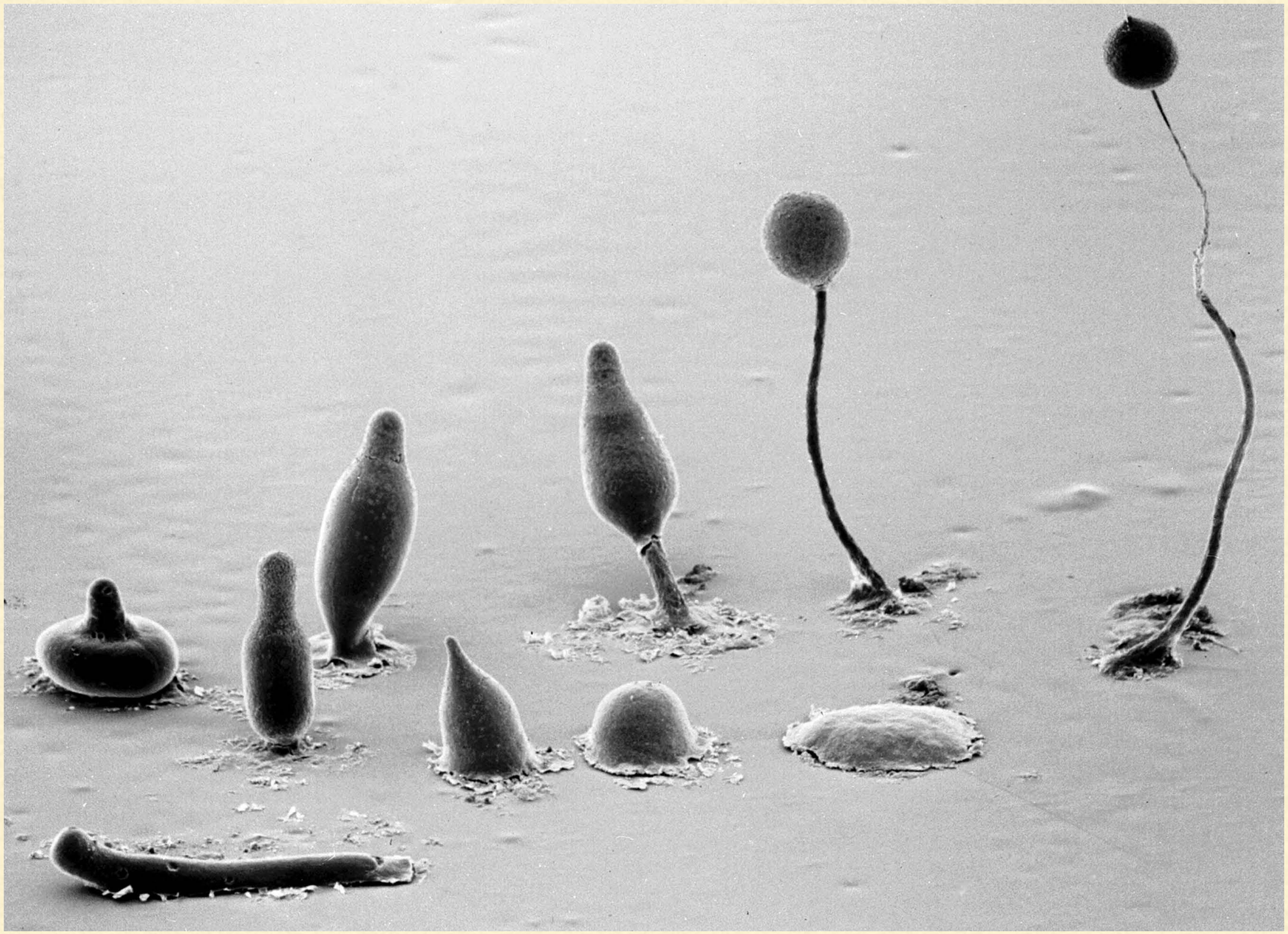


Photo: *M. J. Grimson & R. L. Blanton. Biological Sciences Electron Microscopy Laboratory, Texas Tech University*

EXAMPLE #2:
ANT BRIDGES

Source: https://www.quantamagazine.org/the-simple-algorithm-that-ants-use-to-build-bridges-20180226/

WHY COOPERATE?

- Cooperation is an organizational mechanism that is observed over a range of scales in the natural world.
- But… why would any individual agent decide to cooperate in a situation where it would be more personally beneficial to act otherwise?
- **Game theory** provides a theoretical framework for the understanding of the evolution of cooperative strategies in systems of interacting "rational" agents.

Photo: Alamsyah Rauf

COORDINATION-CENTRIC DILEMMAS

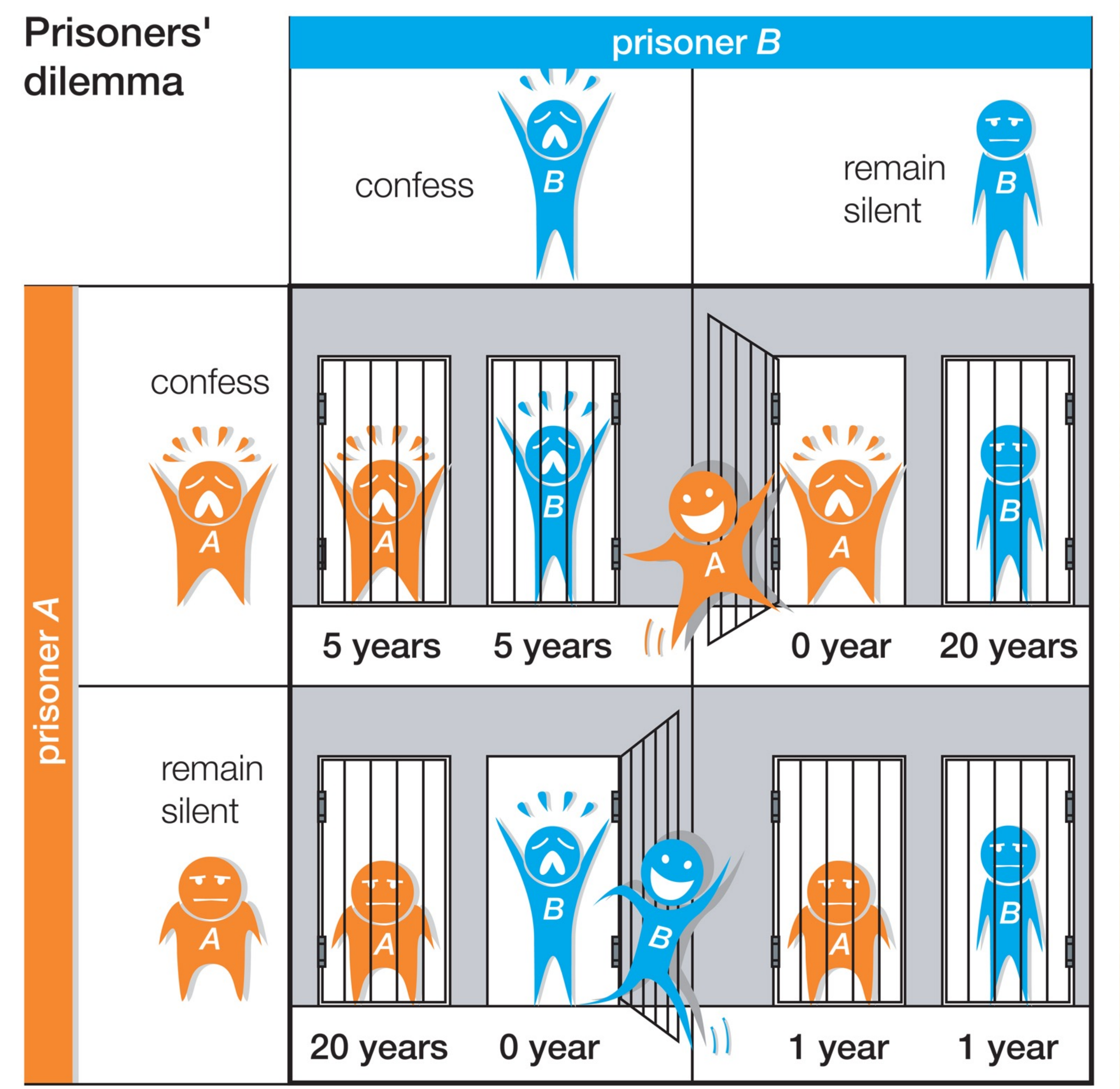
The Unscrupulous Diner's Dilemma

A group of friends make an agreement: they will divide the check evenly at the next restaurant that they go to.

One may order a cheap or an expensive item. Can you see where a <u>dilemma</u> may arise?

Photo: N. S. Glance and B. A. Huberman, Scientific American ***270****(3) 76-81 (1994)*

PRISONER'S DILEMMA (PD)

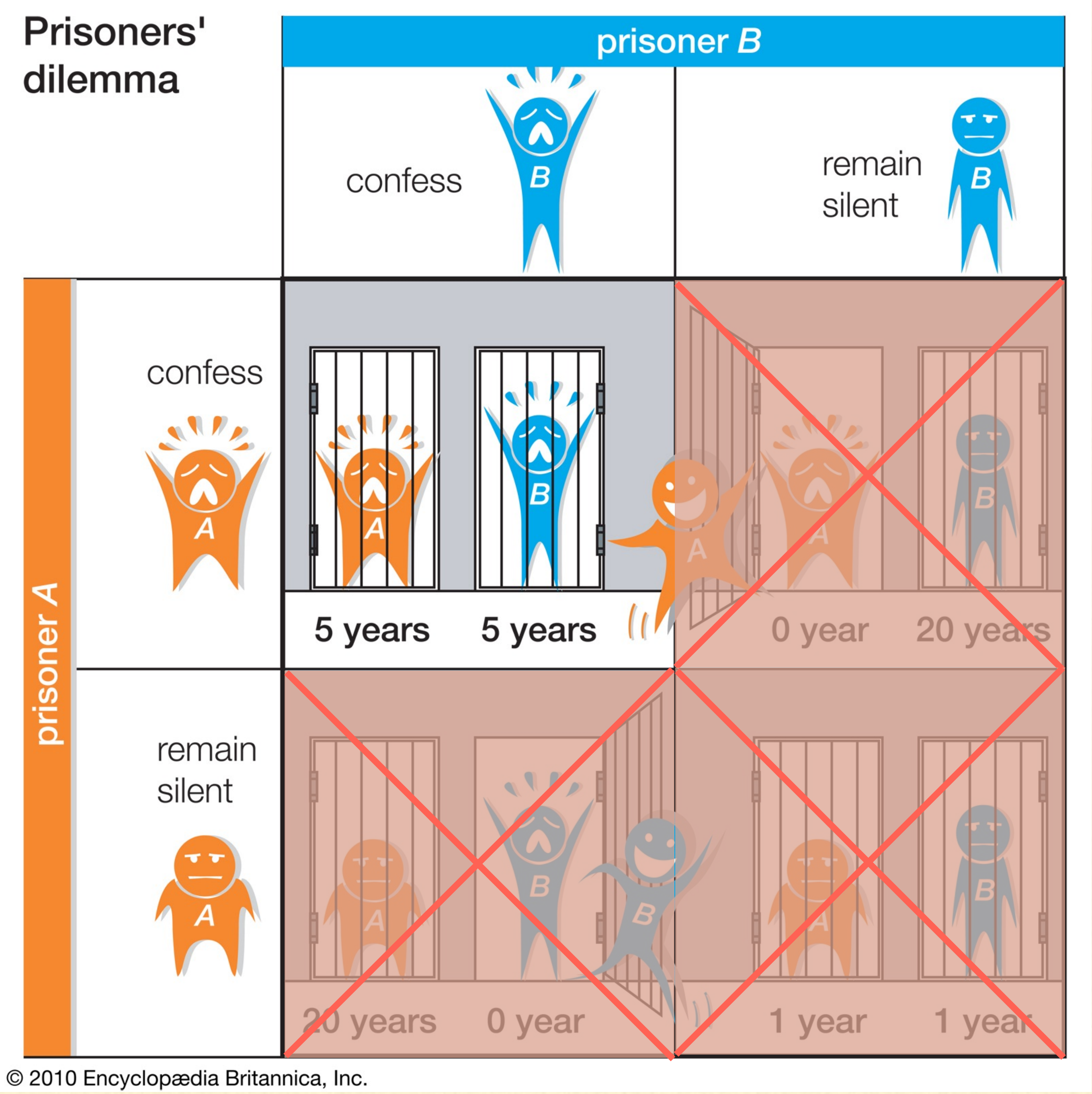


A pair of players are presented with a set of outcomes, depending on their choice of actions: **cooperate** or **defect**.

Depending on the pair of decisions, the players receive "payoffs". If both players cooperate, they each receive a higher payoff than if they both defect. But if one defects while the other cooperates, they receive the highest possible payoff.

If you were one of the two players, what do you think would be the best course of action?

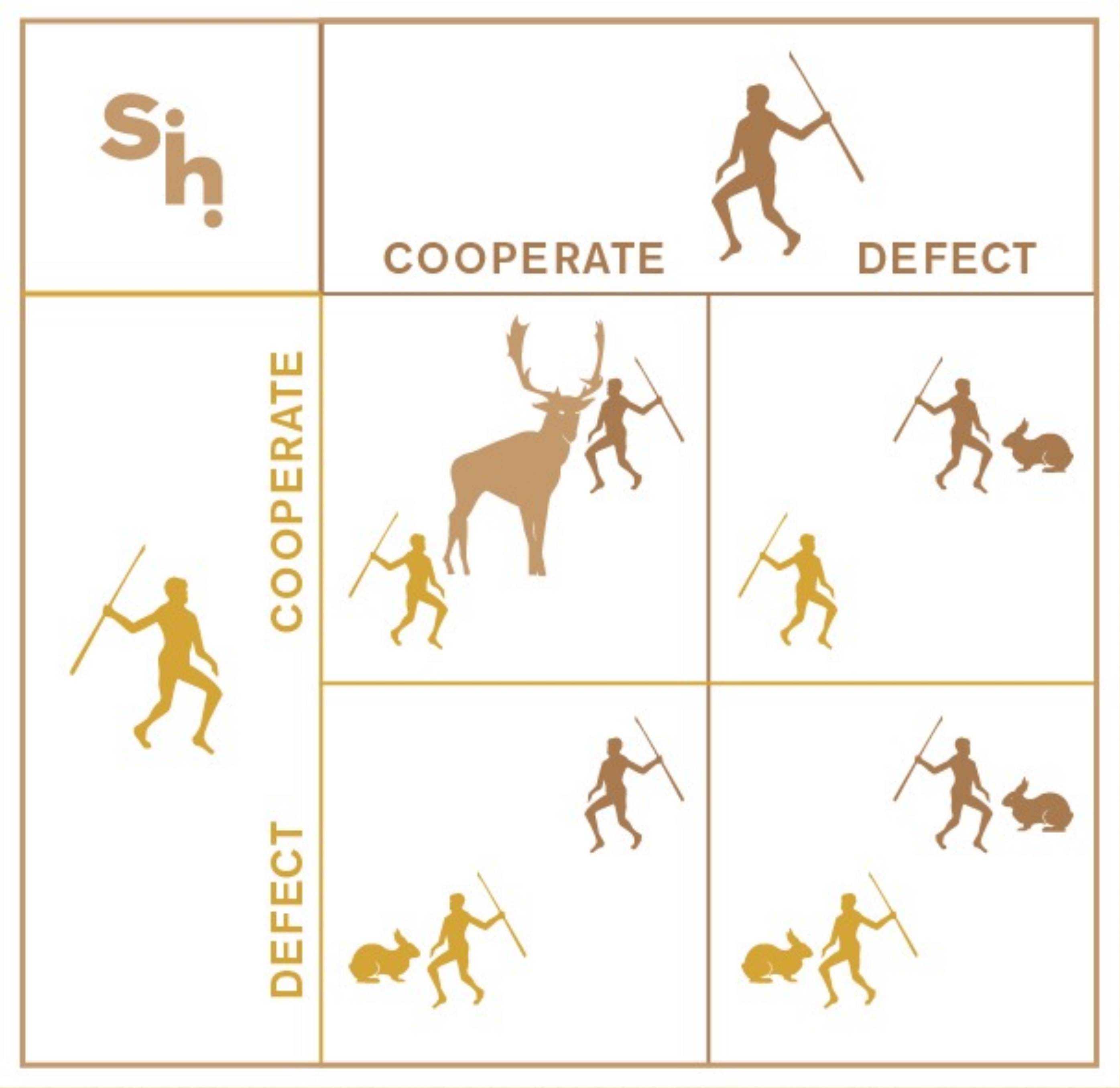
PRISONER'S DILEMMA (PD)



Players are assumed to be "rational": they purely wish to maximise their own payoff, and will utilise any strategy that they believe would lead to such an outcome.

The "dilemma" of the PD game is that although defection appears to be the "rational" choice for each player (leading to mutual defection as the rational outcome), they would have done better if they both chose to cooperate instead.

STAG HUNT



Two individuals would like to hunt a stag, but would be satisfied if they caught a hare. The two actions available to them are to cooperate (try to hunt the stag) or defect (catch a hare).

The agents can only catch the stag if they work together (i.e. both cooperate). If they aim to catch the stag alone they will be left empty-handed. If they try to catch a hare, they can do so regardless of what the other agent does.

What is the "best" strategy (if any) for an agent playing this game?

Image: http://www.christopherxjjensen.com/research/projects/online-cooperative-resource/evolutionary-games-infographics/

SNOWDRIFT (OR "CHICKEN")

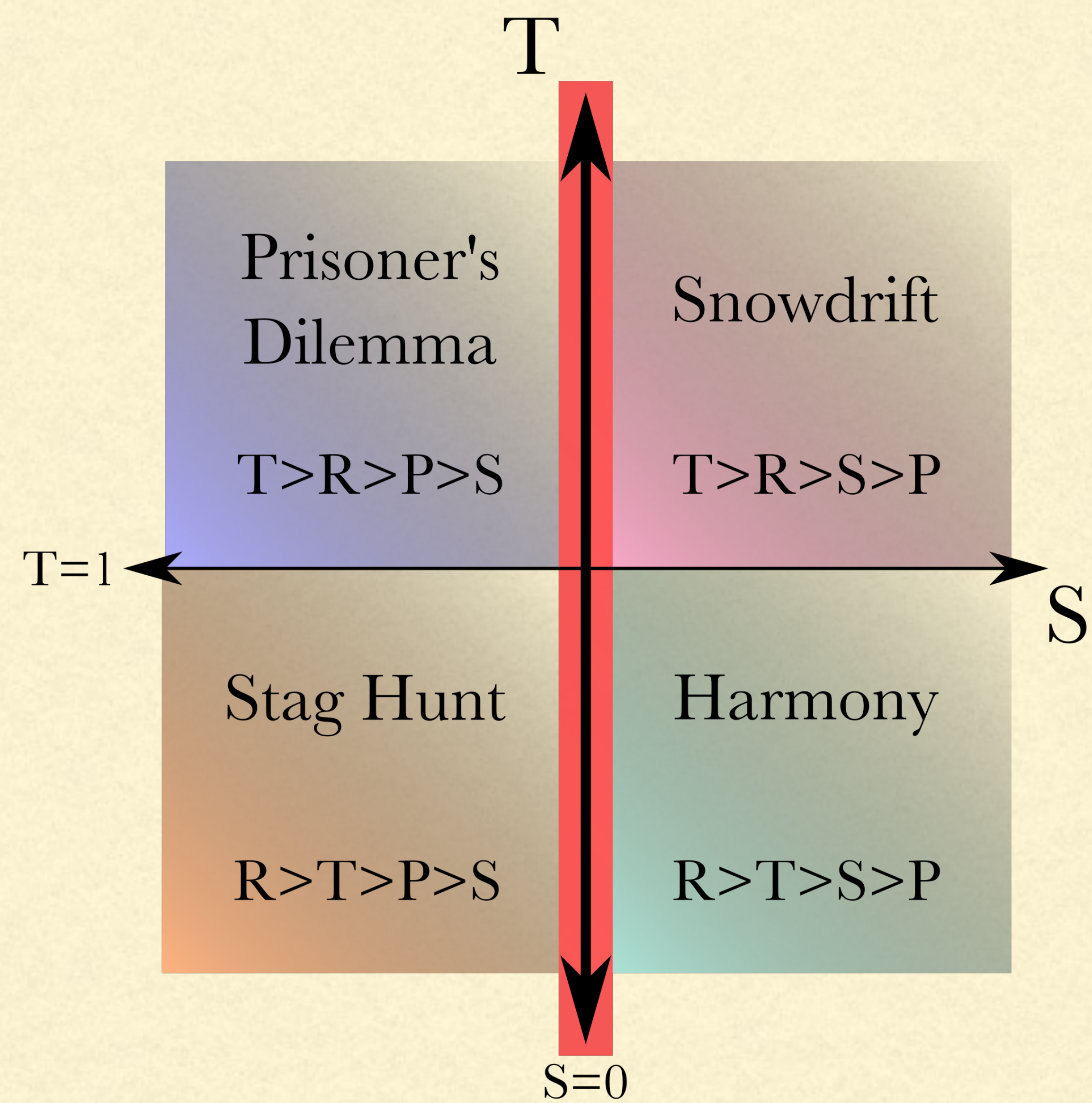
- One of the most well-known anti-coordination games.
- Players receive the lowest payoff if they both defect. If they both cooperate, they receive a small payoff. However, if one defects while the other cooperates, they receive the highest payoff.

Image: "*Rebel Without a Cause*", dir. Nicholas Ray (Warner Bros. 1955)

CANONICAL PAYOFF MATRIX FOR SYMMETRIC TWO PLAYER COOPERATIVE GAMES

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

T: Temptation (to defect while the other cooperates)
R: Reward (for mutual cooperation)
P: Punishment (for mutual defection)
S: Sucker's payoff (for cooperating while the other defects)



Following Nowak et al (1992), we can choose R=1 and P=0 w.l.o.g. (as R > P for all four games)

Image: S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

DOMINANT STRATEGIES AND NASH EQUILIBRIA

- A dominant strategy of an agent is one that yields a preferable outcome regardless of the strategy of the opponent.
- John Forbes Nash Jr. [left] proposed a solution of non-cooperative games in which agents know the (equilibrium) strategies of their opponents.
- Let (a_1, a_2) correspond to a pair of actions of the two players.
- The state (a_1, a_2) is said to be a **Nash equilibrium** if both players find that they receive a worse payoff upon unilaterally changing their action.
- If both players have the same dominant strategy, a unique Nash equilibrium exists.

Photo: https://www.nature.com/articles/522420a

PRISONER'S DILEMMA (T>R>P>S)

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

Player 2

Player 1 defects — P > S (Defection is preferable)

Player 1 cooperates — T > R (Defection is preferable)

Defection is the dominant strategy

Player 2 defects | Player 2 cooperates

Player 1

P > S (Defection is preferable)

T > R (Defection is preferable)

Defection is the dominant strategy

The Nash equilibrium is mutual defection

STAG HUNT (R>T>P>S)

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

Player 2

Player 1 defects: P > S (Defection is preferable)

Player 1 cooperates: R > T (Cooperation is preferable)

There is **no** dominant strategy

Player 1

Player 2 defects: P > S (Defection is preferable)

Player 2 cooperates: R > T (Cooperation is preferable)

There is **no** dominant strategy

The Nash equilibria are mutual defection and mutual cooperation

SNOWDRIFT (T>R>S>P)

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

Player 2

Player 1 defects — S > P (Cooperation is preferable)

Player 1 cooperates — T > R (Defection is preferable)

There is **no** dominant strategy

Player 2 defects

Player 2 cooperates

Player 1

S > P (Cooperation is preferable)

T > R (Defection is preferable)

There is **no** dominant strategy

There are two Nash equilibria: players choose opposing actions

HARMONY (R>T>S>P)

	Defect	Cooperate
Defect	P, P	T, S
Cooperate	S, T	R, R

Player 2

Player 1 defects S > P (Cooperation is preferable)

Player 1 cooperates R > T (Cooperation is preferable)

Cooperation is the <u>dominant strategy</u>

Player 2 defects

Player 2 cooperates

Player 1

S > P (Cooperation is preferable)

R > T (Cooperation is preferable)

Cooperation is the <u>dominant strategy</u>

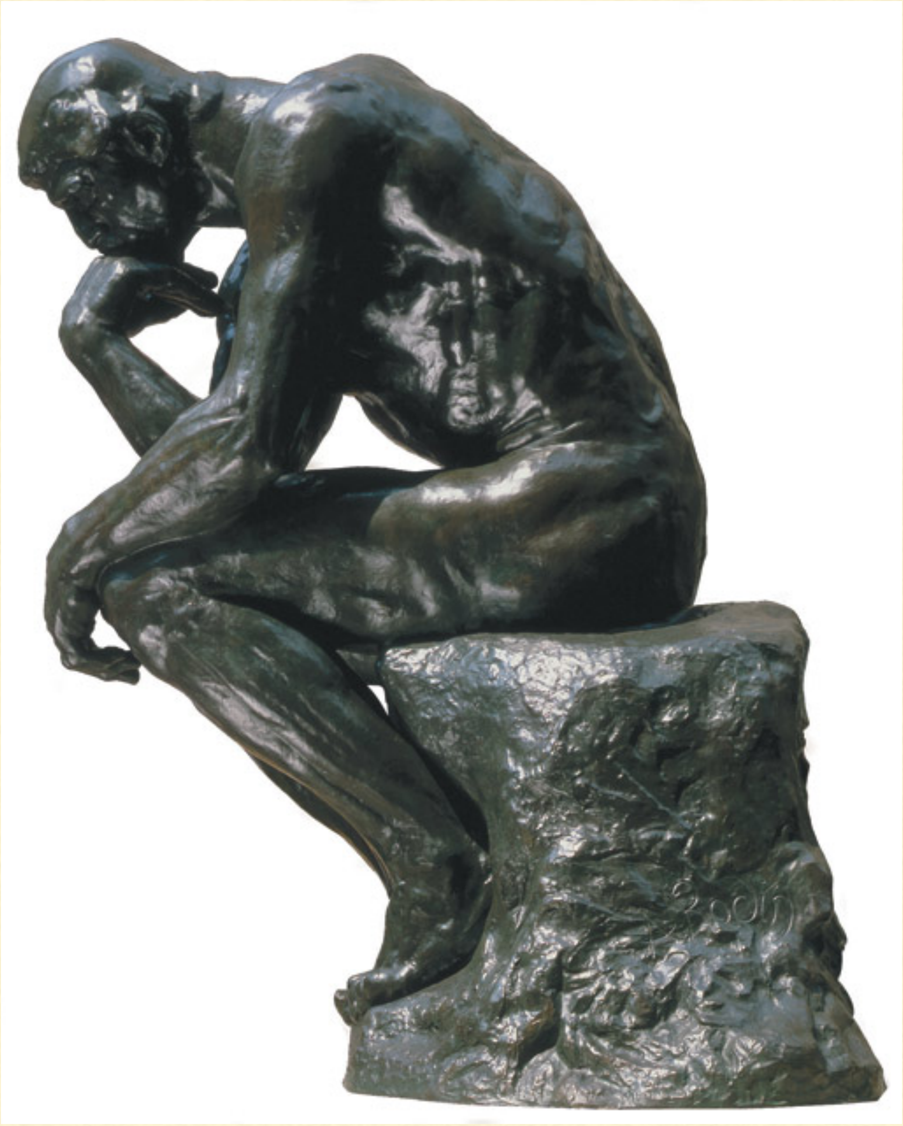
The Nash equilibrium is mutual cooperation

ARE AGENTS REALLY RATIONAL?

- “*It is certainly not by chance that a central recurrent theme in the history of game theory is how to define rationality. In fact, any working definition of rationality is a negative definition, not telling us what rational agents do, but rather what they do not.*” *

- While game theory predicts that interactions under the conditions of the Prisoner’s Dilemma will lead to defection, this is **not** what is commonly observed in experiments or in society.

- A refinement introduced to address this is the concept of “bounded rationality”: agents have cognitive limitations and/or may not be able perfectly process all available information.



Auguste Rodin, Le Penseur *(1880).*

* *G. Szabó & G. Fáth,* Physics Reports ***446****, 97-216 (2007).*

THE ITERATED PRISONER'S DILEMMA

- The limitations of classical game theory can be seen most clearly when considering the *Iterated Prisoner's Dilemma* (IPD)*. Here, the agents choose their **action** (either cooperate [C] or defect [D]) at each step, based on their choice of **strategy**, and are assigned payoffs depending on the collective set of actions

- The strategy could be deterministic or probabilistic, and may incorporate memory of previous actions.

- If the IPD game is played X times, and it is *known* that the game will be played X times, the only rational strategy for a player (arrived at via induction) is to "always defect". This is the so-called "backward induction paradox".

- It is alternatively argued that agents use "inductive reasoning" to dynamically process information regarding their environment, and update their strategies based on how they perform.

* *R.* Axelrod, *"The evolution of cooperation"* (Basic Books, 1984).

AXELROD'S TOURNAMENT

- In 1980, Robert Axelrod organised a tournament to find the "best" possible strategy for the IPD.
- Numerous programs, each of which encoded a particular strategy, competed against each other and themselves.
- The strategy determined the choice of action (C or D), based on the previous timeline of actions. The actions could also be random.

Photo: http://skytteprize.statsvet.uu.se/PrizeWinners.aspx

AXELROD'S TOURNAMENT

PAYOFF (AGENT 1)	S	T	T	S	T	S	P	R	S
Agent 1: "Random"	C	D	D	C	D	C	D	C	C
Agent 2: "Random"	D	C	C	D	C	D	D	C	D
PAYOFF (AGENT 2)	T	S	S	T	S	T	P	R	T

- The winning strategy was "Tit for tat" developed by Anatol Rapoport, where a player simply mimics the opponent's action in the next round.
- It was subsequently shown that a superior strategy is "Win-stay lose-shift", where a player switches action only if unsuccessful.

EMERGENCE OF COOPERATION ON NETWORKS

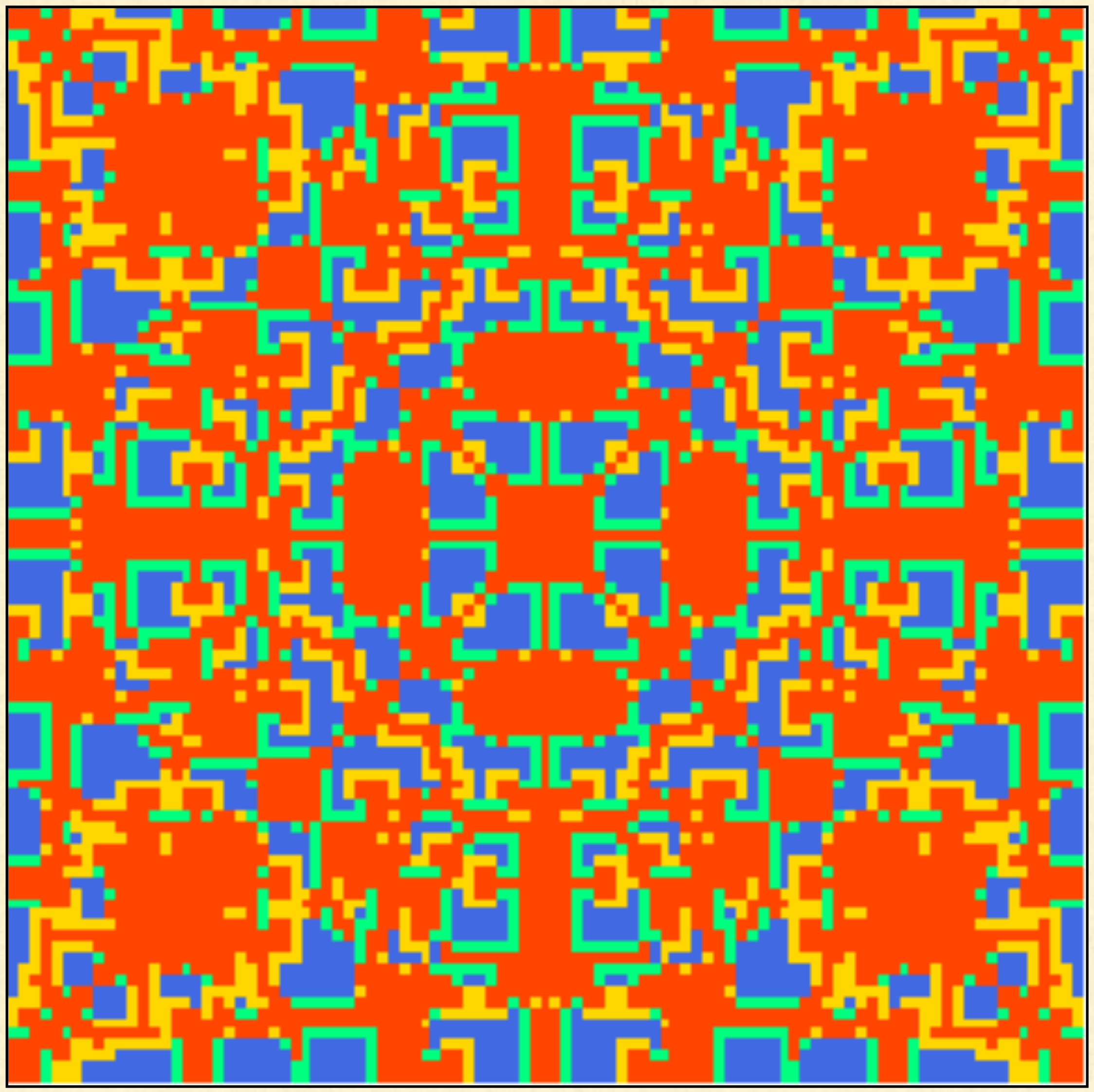
- In the case of *well-mixed populations*, in which any two agents have an equal likelihood of interacting, natural selection tends to favour defectors.*

- However, this argument doesn't necessarily hold for interactions over social networks. In this case, cooperation can emerge through **network reciprocity**, i.e. cooperators survive by forming (possibly dynamic) clusters on the network.

* J. Maynard Smith, "*Evolution and the Theory of Games*" (1982).

AN EVOLUTIONARY KALEIDOSCOPE

N=99x99 agents, T=1.9



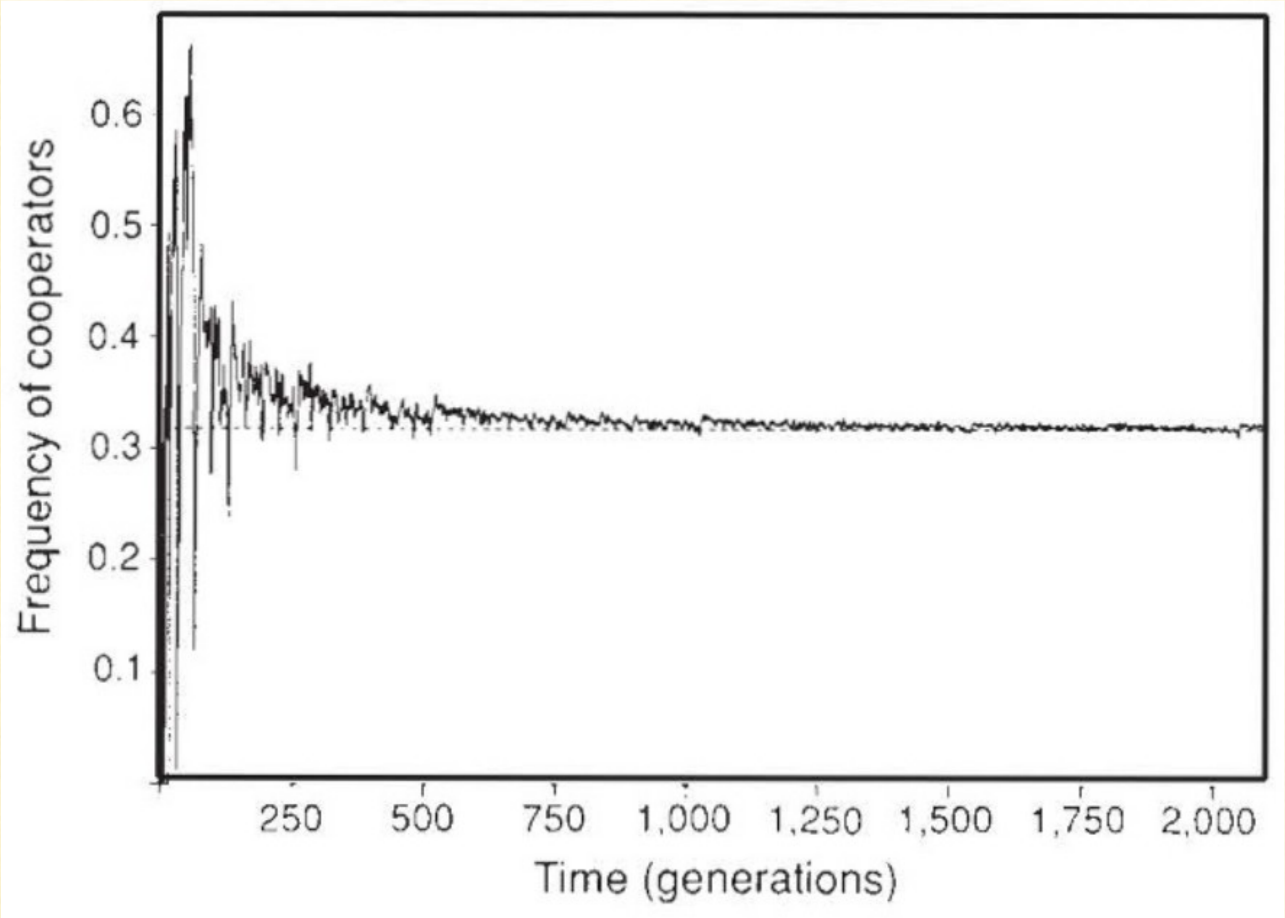
Blue C->C, **Red** D->D, **Yellow** C->D, **Green** D->C

- Nowak and May* demonstrated that when the IPD is played on a square lattice, rapidly evolving spatial patterns arise if agents employ *unconditional imitation.*
- This is a deterministic strategy in which each agent plays the game with the 8 agents in its neighbourhood (as well as itself) and, in the next step, copies the action of the most successful neighbour.

* M. A. Nowak & R. M. May, *Nature* **359**, 829-829 (1992).

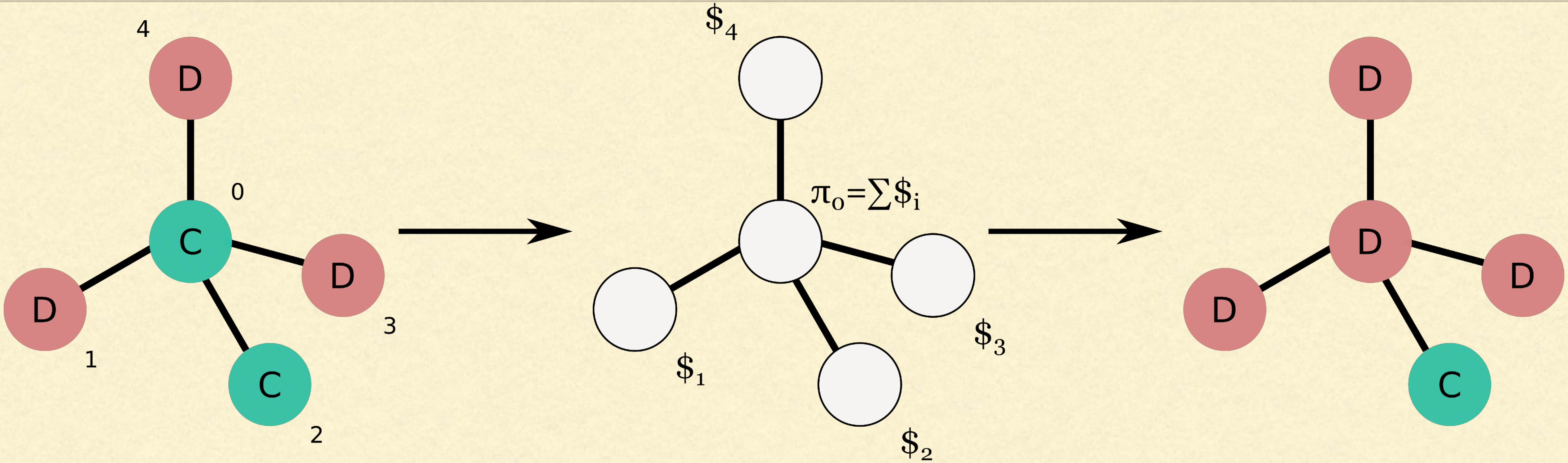
AN EVOLUTIONARY KALEIDOSCOPE

- It was found that the fraction of cooperators fluctuates around a non-zero value, regardless of the initial fraction of cooperators, or the starting configuration.
- Similar results can be obtained even if agents interact with four neighbours (or six neighbours in a hexagonal lattice), and also regardless of whether self-interactions are included or excluded.
- Individual cells do not have memory, or elaborate strategies



M. A. Nowak & R. M. May, *Nature* **359**, 829-829 (1992).

THE DYNAMICS: HOW AGENTS UPDATE THEIR ACTIONS ON A NETWORK



Agents play with all their neighbours

Agents accumulate payoffs from each interaction

Agents update their actions

Once agent *i* finds its accumulated payoff (π_i), it views the accumulated payoffs of each of its neighbours, and makes a decision deterministically or probabilistically. This process occurs simultaneously for all agents, i.e. this is a **synchronous** update.

A PROBABILISTIC UPDATING STRATEGY

- In certain contexts although agents may have information regarding the payoffs of their neighbours, they may not necessarily copy their actions. In such settings agents may copy the action of another with some probability.

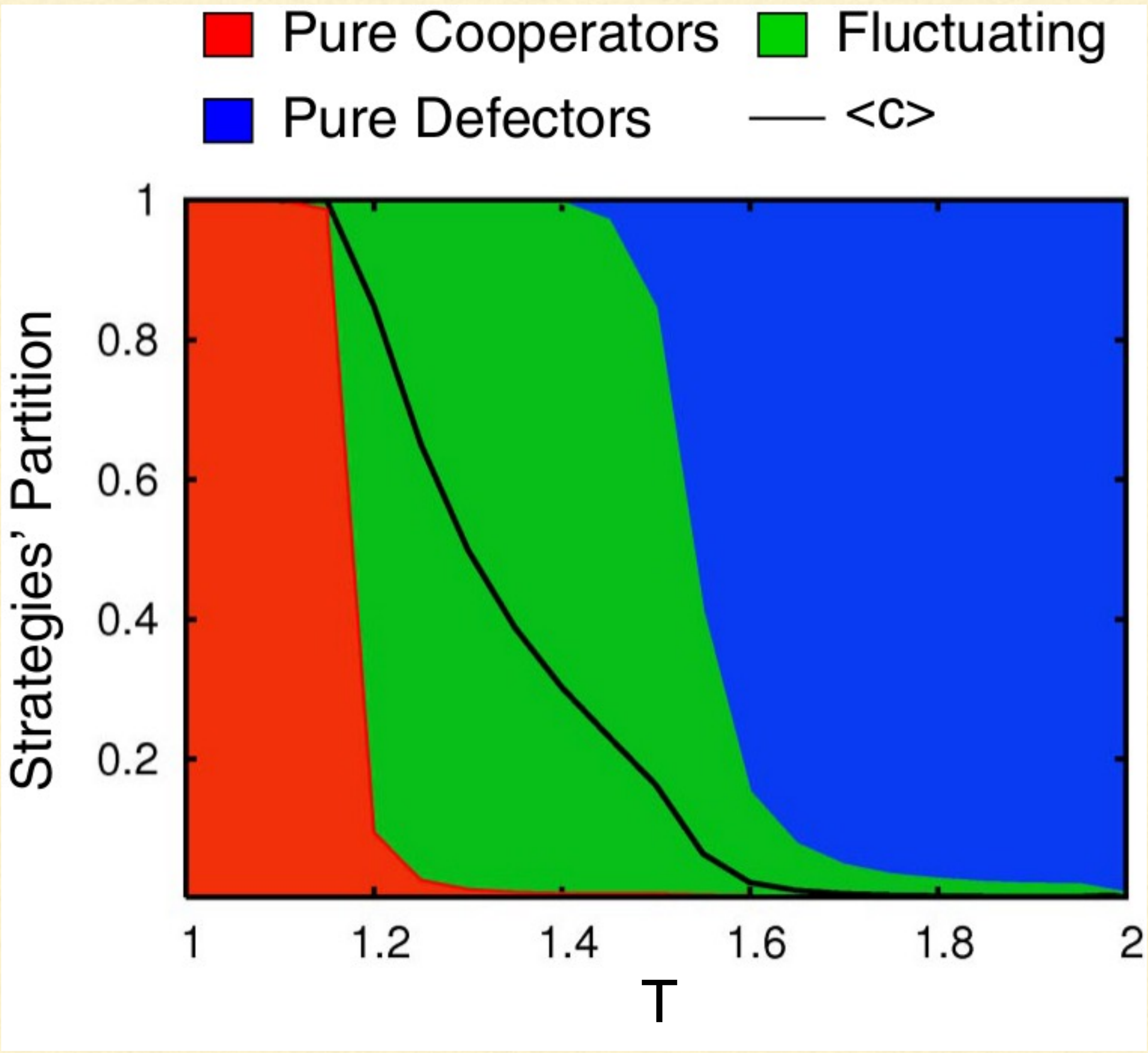
- A commonly used probabilistic strategy* is as follows:

Each agent i on a network compares its payoff π_i with that of a randomly chosen neighbour j (π_j). If $\pi_i \geq \pi_j$, the agent repeats its action in the next step, otherwise it copies the action of j with a probability proportional to the difference between their respective payoffs, and dependent on the temptation T and their degrees ($k_{i,j}$), namely

$$\Pi_{i \to j} = \frac{\pi_j - \pi_i}{T \max(k_i, k_j)}$$

* F. C. Santos & J. M. Pacheco, *Phys. Rev. Lett.* **95**, 098104 (2005).

IPD ON ERDÖS-RÉNYI NETWORKS

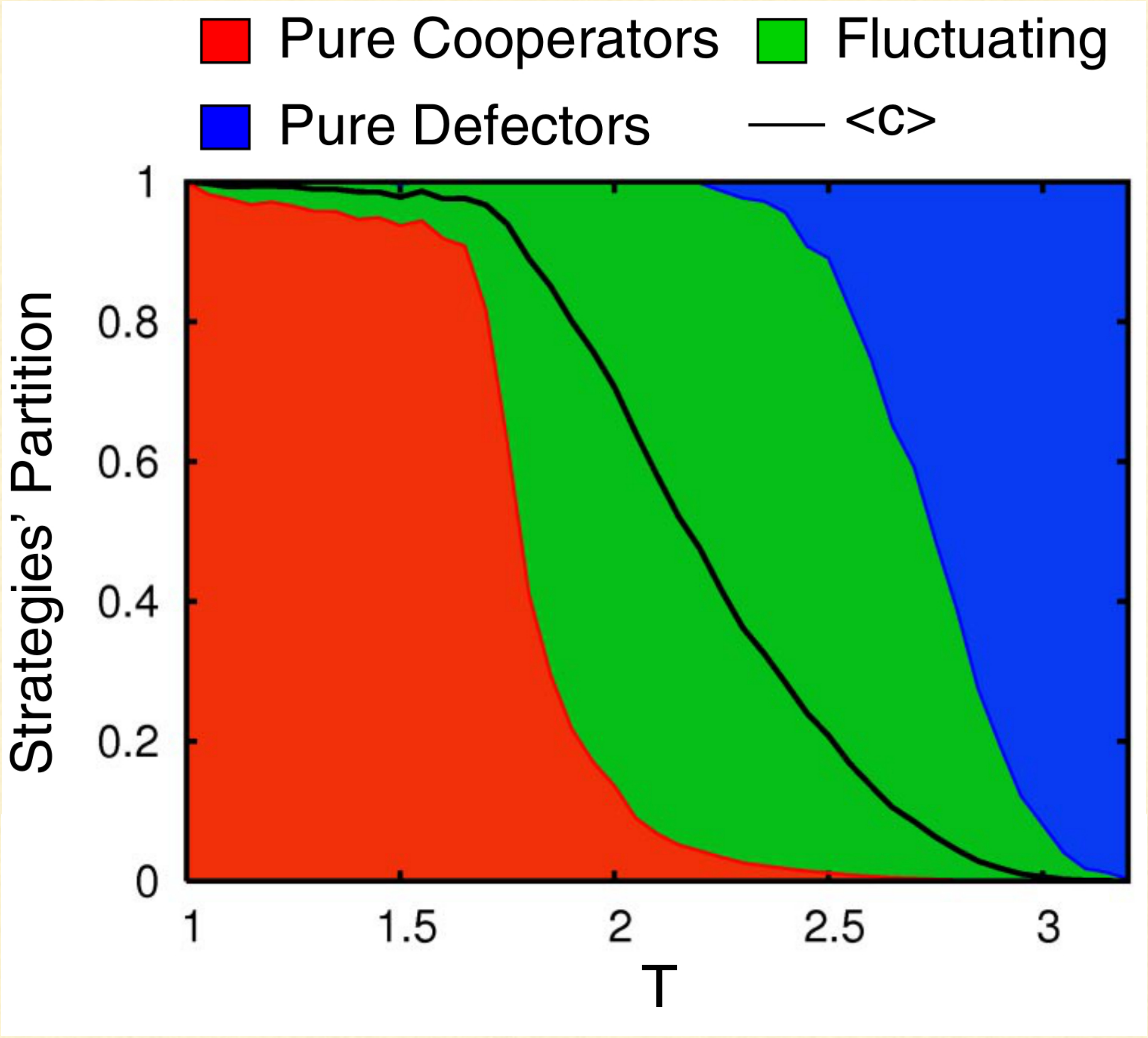


It was found that when agents playing an IPD on an Erdös-Rényi (ER) random network use this probabilistic strategy to update their choice of action, three different strategies emerge*.

The adjacent figure corresponds to the case N=4000 nodes, and average degree $\langle k \rangle = 4$.

* J. Gómez-Gardeñes et al., *Phys. Rev. Lett.* **98**,108103 (2007).

IPD ON SCALE-FREE NETWORKS

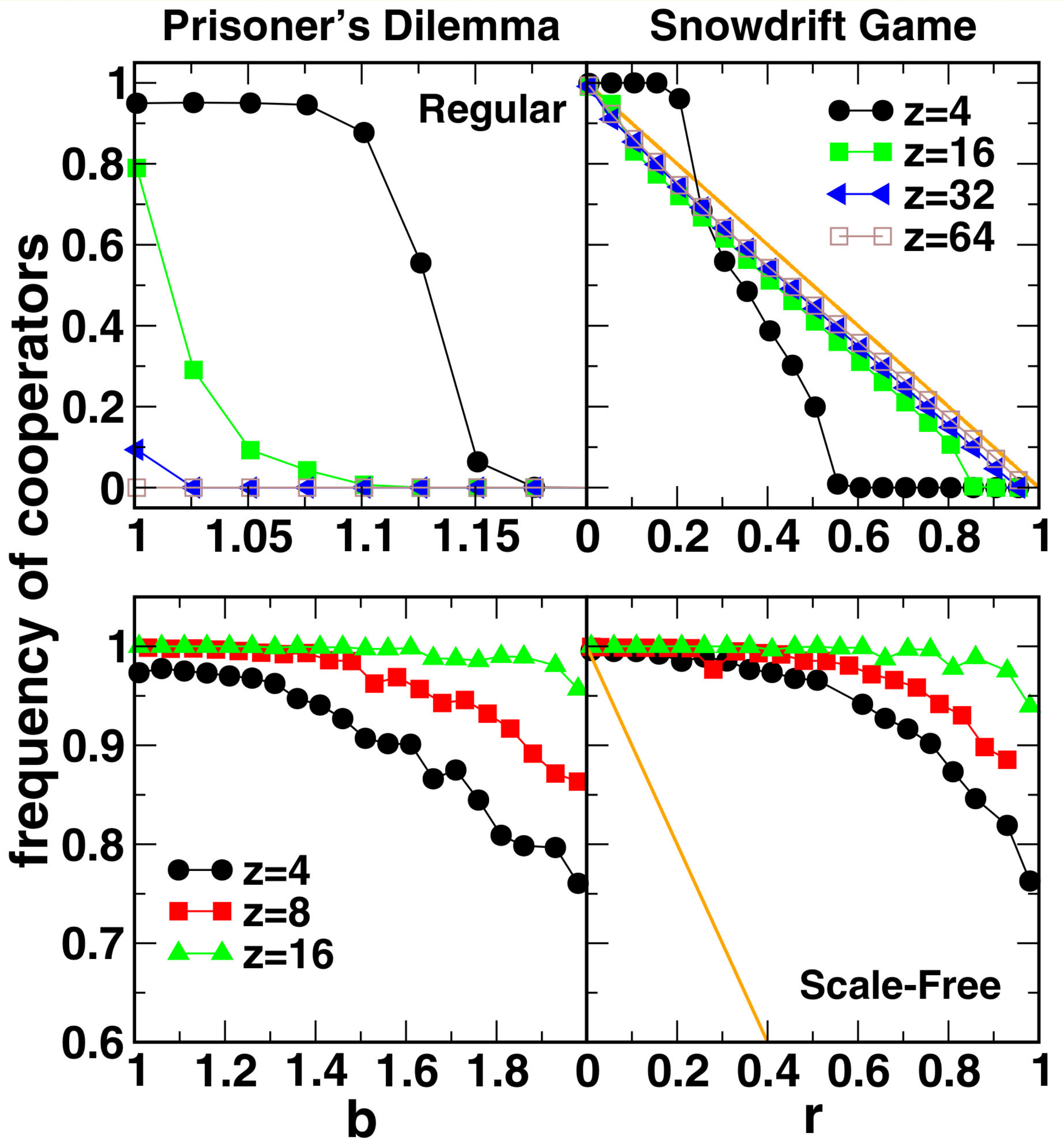


When the same rules were implemented for the case of Scale-Free (SF) random networks, the cooperation regime was enhanced*.

The adjacent figure corresponds to the case N=4000 nodes, and average degree $\langle k \rangle = 4$. The exponent of the SF network is -3.

* J. Gómez-Gardeñes et al., *Phys. Rev. Lett.* **98**,108103 (2007).

IPD ON SCALE-FREE NETWORKS



In fact, while an increase in degree is found to reduce the overall level of cooperation for the PD on ER networks, it *enhances* cooperation for the case of SF networks*.

The adjacent figure corresponds to the case N=10^4 nodes. For the PD, T=b(>1), R=1, P=S=0 and for the SD, T=β(>1), R=β-1/2, S=β-1, P=0, such that r=1/(2β-1). The average degree is z.

* F. C. Santos & J. M. Pacheco, *Phys. Rev. Lett.* 95, 098104 (2005)..

NOISY COMMUNICATION

One can also incorporate tunable external noise by employing the **Fermi rule*.** Here, each agent i randomly picks a neighbour j and copies its action with a probability $\Pi_{i\to j}$.

The probability is proportional to the Fermi distribution function

$$\Pi_{i\to j} = \frac{1}{1+\exp(-\beta(\pi_j - \pi_i))}$$

where β can be thought of as the inverse of temperature K, or "noise", in the decision making process and π_i & π_j are the payoffs of i and j respectively.

$$\Pi_{i\to j} = \frac{1}{2}$$

RANDOM

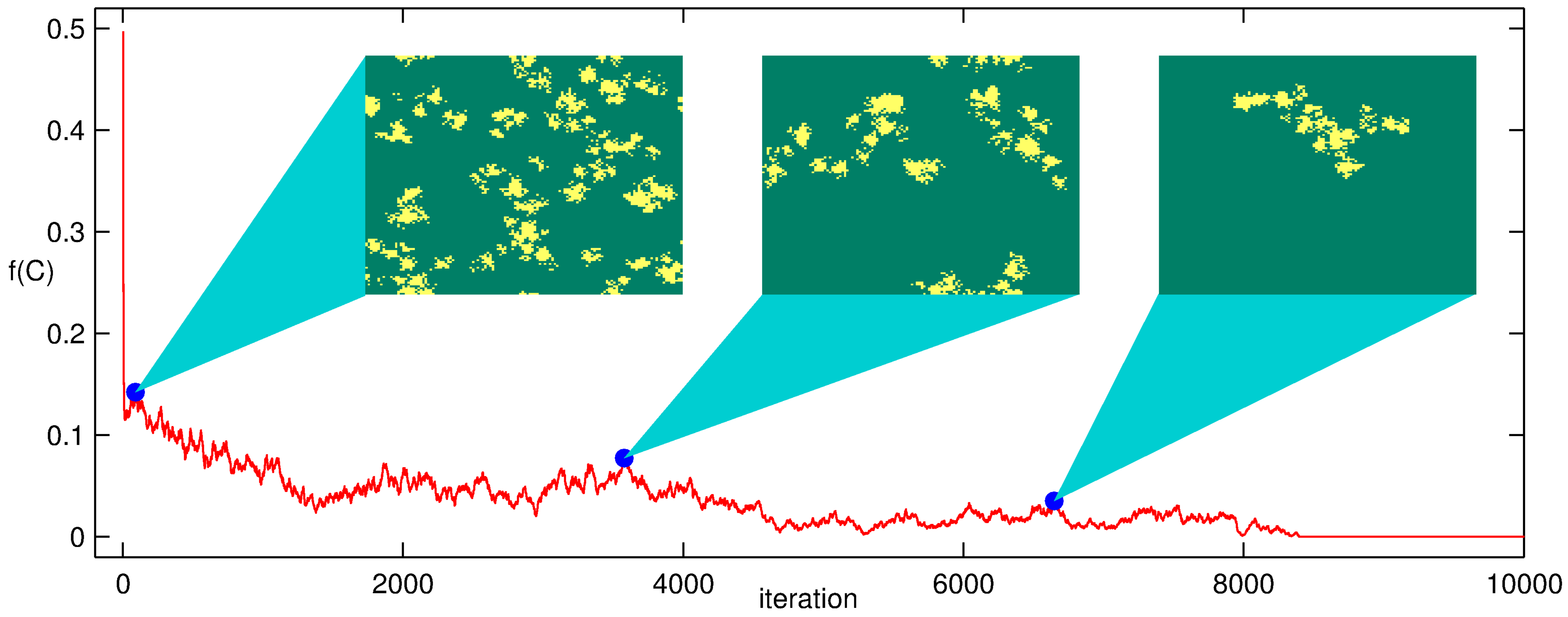
$$\Pi_{i\to j} = \begin{cases} 0, & \text{if } \pi_j < \pi_i \\ 1, & \text{if } \pi_j > \pi_i \end{cases}$$

DETERMINISTIC

$$\infty \longleftarrow K \longleftarrow 0$$

* G. Szabó & C. Toke, *Phys. Rev. E* 58, 69-73 (1998).

DYNAMICS ON A LATTICE

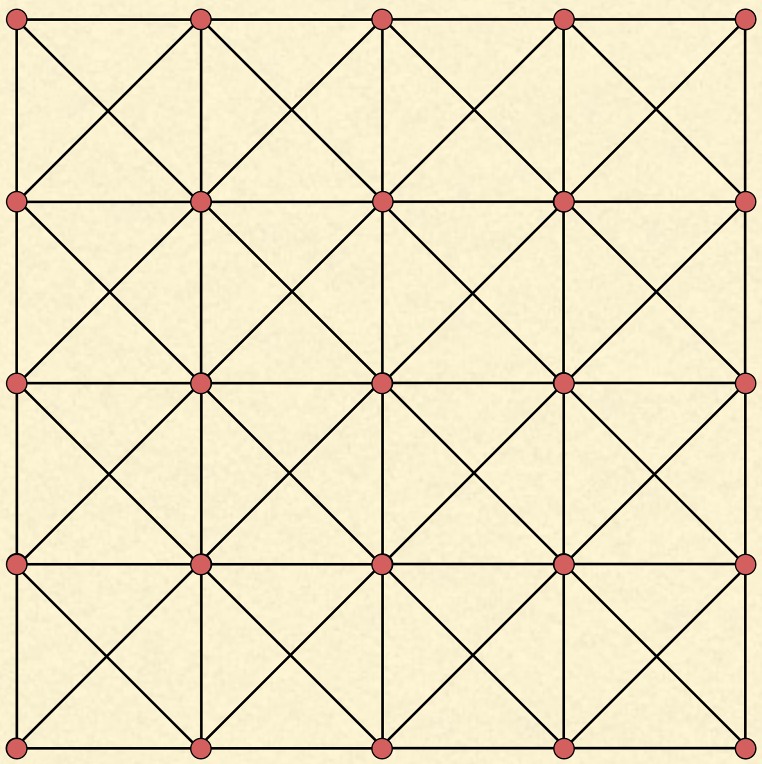


When these rules are applied to agents on a lattice that play IPD with their neighbours, we find two possible outcomes:

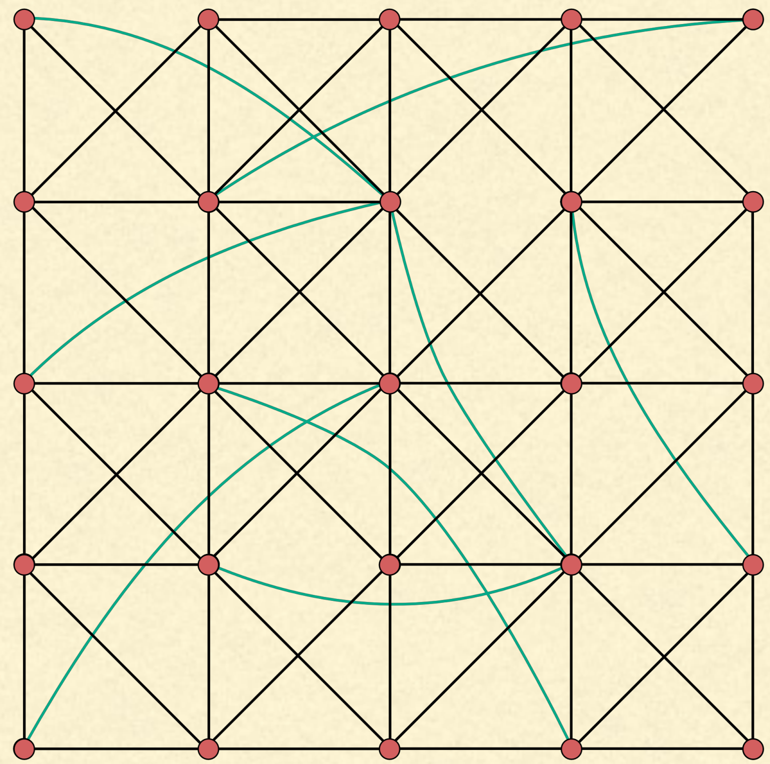
- all agents eventually defect (i.e. the fraction of cooperators, f(C)=0),
- the fraction of cooperators fluctuates around a non-zero value (0<f(C)<1).

* S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

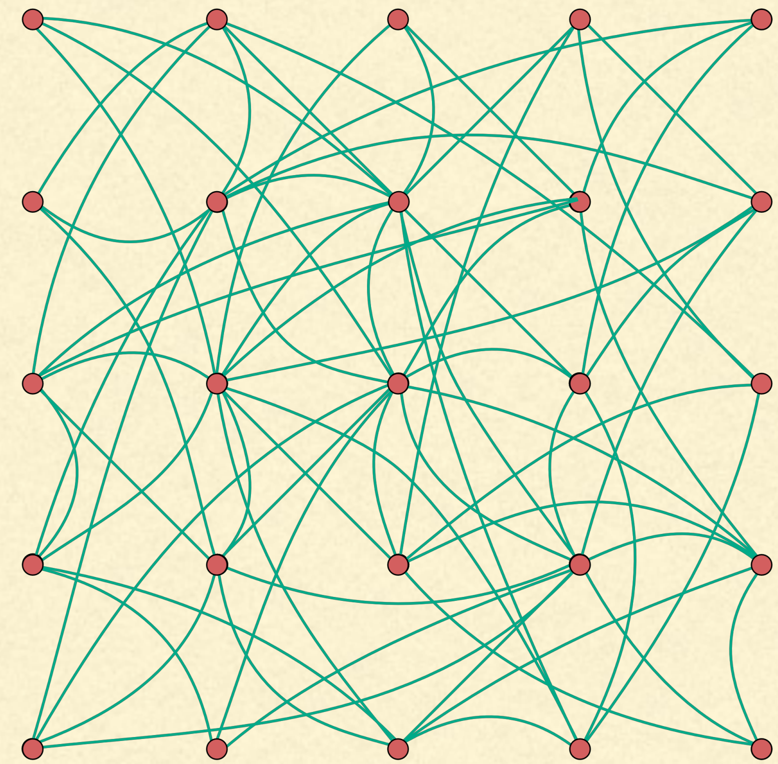
NETWORK TOPOLOGIES



Lattice [p=0]



Small world network [0<p<1]



Random network [p=1]

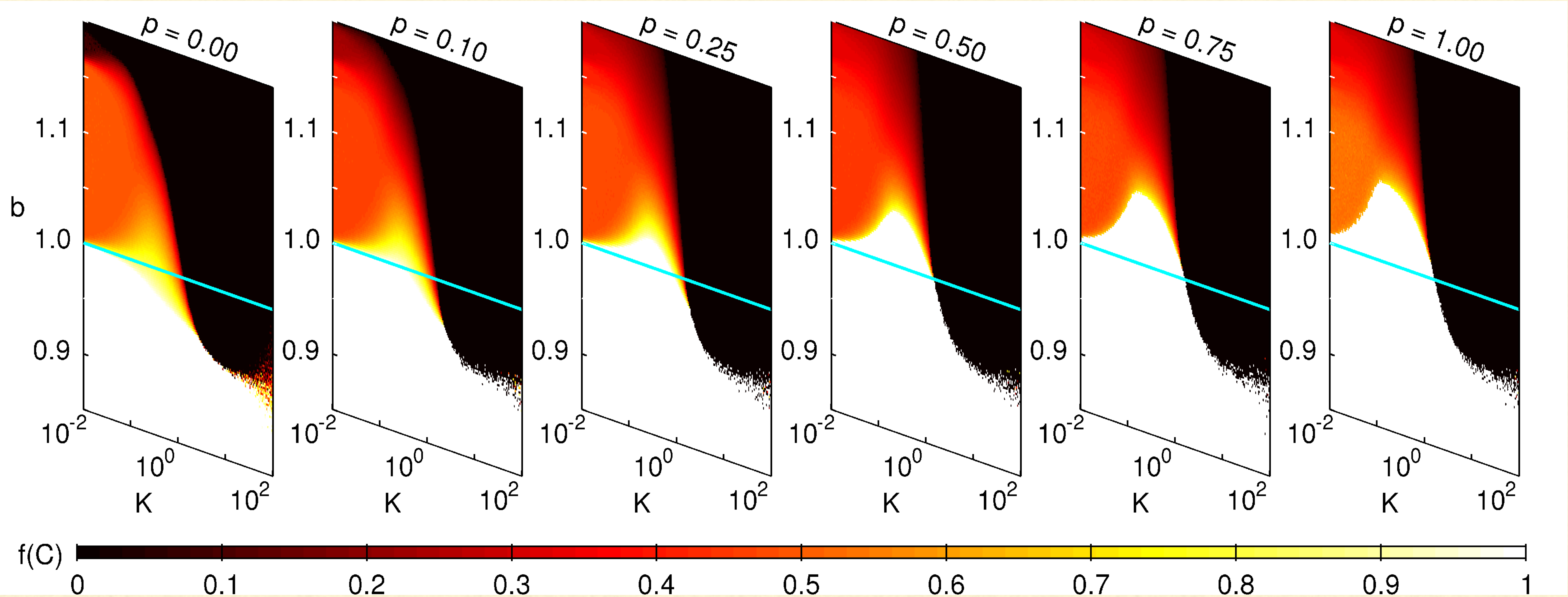
We interpolate between a lattice and a random network using the approach of Watts and Strogatz*: cycle through and rewire each link with probability p.

For the range $0<p<1$, we have a *small world* network structure.

* D. Watts & S. Strogatz, *Nature* 393, 440-442 (1998).

We examine the dependence of the fraction of cooperators f(C) on the Temptation (b), temperature (K) and rewiring probability (*p*).

Recall that when b<1, agents play the Stag Hunt, and when b>1 agents play the Prisoner's Dilemma. Mutual cooperation is <u>not</u> expected to be a stable outcome in the latter case.

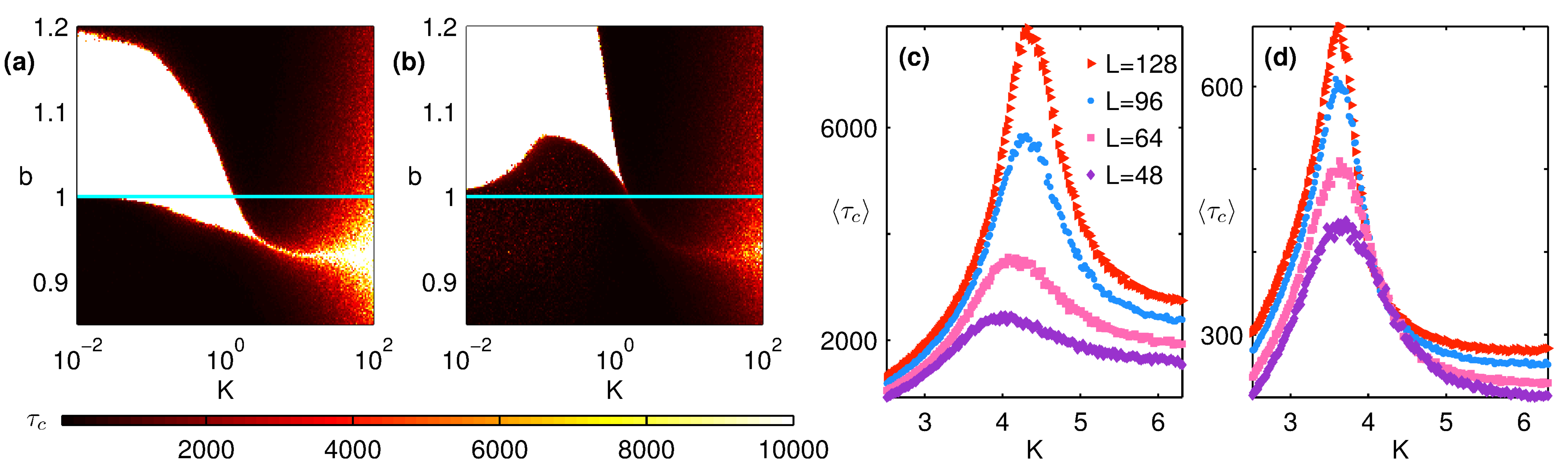


[data for N=128^2 , k_{avg} = 8]

However, as we approach *p*=1, a regime of pure cooperation emerges for T>1.

* S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

Next, we look at the convergence time τ_c at each point in parameter space (the number of iterations taken for the system to reach one of the two possible absorbing states).



For (a) *p*=0 & (b) *p*=1, we observe that the boundaries of fluctuation regime are characterised by a sharp change in the convergence time. Hence, the fluctuation regime is hence <u>qualitatively distinct</u> from the "noisy" regime, seen at high temperatures.

For (c) *p*=0 & (d) *p*=1, we find a divergence in convergence time across the all D-all C boundary on increasing the system size.

* S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018).

Does network structure enhance cooperation? The jury is still out…

PROCEEDINGS OF THE ROYAL SOCIETY B

Proc. R. Soc. B (2006) 273, 51–55
doi:10.1098/rspb.2005.3272
Published online 4 October 2005

Graph topology plays a determinant role in the evolution of cooperation

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We study the evolution of cooperation in communities described in terms of graphs, such that individuals occupy the vertices and engage in single rounds of the Prisoner's Dilemma with those individuals with whom they are connected through the edges of those graphs. We find an overwhelming dominance of cooperation whenever graphs are dynamically generated through the mechanisms of growth and preferential attachment. These mechanisms lead to the appearance of direct links between hubs, which constitute sufficient conditions to sustain cooperation. We show that cooperation dominates from large population sizes down to communities with nearly 100 individuals, even when extrinsic factors set a limit on the number of interactions that each individual may engage in.

Keywords: ev

Abbr

1. INTRODUCTION

Cooperation is an esser organisms have coope organisms throughout we know that animals offspring, and in group risk of predation. In sp the evolution of coop fundamental challenge Hammerstein 2003), t diverse as anthropol economics, psycholog physics, etc., who often (Maynard-Smith 198 mathematical framew (PD) as a metaphor unrelated individuals Nowak & May 1992 individuals are either accordingly whenever receive R upon mutua defection. A defector amount T and the exp that $T>R>P>S$ (Mes in a single round of the the opponent's decision unable to resist invasio under replicator dynam well-mixed population somewhat alleviated, h in a spatially structured such that individuals with their nearest

Physica A 412 (2014) 169–179

Contents lists available at ScienceDirect

Physica A

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ELSEVIER

Community structure inhibits cooperation in the spatial prisoner's dilemma

CrossMark

Jianshe Wu*, Yanqiao Hou, Licheng Jiao, Huijie Li

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tial prisoner's dilemma.
tween structure and cooperation.
on the other kinds of games.

RACT

solved issue across structure and the e is ubiquitous in nmunity structure r coefficient of a cates that smaller ation in statistical nmunity networks random networks nmunity structure

enging problem the emergence [5]. Many efforts that the average ing or collapsing the well known f cooperation in olute payoffs are e promotion for e networks with

PNAS

Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma

Carlos Gracia-Lázaro[a], Alfredo Ferrer[a], Gonzalo Ruiz[a], Alfonso Tarancón[a,b], José A. Cuesta[a,c], Angel Sánchez[a,c,1], and Yamir Moreno[a,b,1]

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Edited by Simon A. Levin, Princeton University, Prince

It is not fully understood why we cooperate w daily basis. In an increasingly global world, networks and relationships between individ more complex, different hypotheses have be explain the foundations of human cooperation to account for the true motivations that are b enon. In this context, population structure has foster cooperation in social dilemmas, but th this mechanism have yielded contradictory resu ally, the issue lacks a proper experimental tes We have performed the largest experiments to playing a spatial Prisoner's Dilemma on a latti network (1,229 subjects). We observed that th tion reached in both networks is the same, co level of cooperation of smaller networks or ur tions. We have also found that subjects respo tion that they observe in a reciprocal manner, b cooperate if, in the previous round, many of t themselves did so, which implies that human neighbors' payoffs when making their decisio but only their actions. Our results, which are recent theoretical predictions based on this b gest that population structure has little relevan promoter or inhibitor among humans.

evolutionary game dynamics | network reciprocity | conditional cooperation

The strong cooperative attitude of humans c of *Homo economicus* and poses an evolut (1, 2). This conundrum is because many of o be framed as Prisoner's Dilemmas (3–5) or Pu

SCIENTIFIC REPORTS

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SUBJECT AREAS:
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STATISTICAL PHYSICS
SOCIAL EVOLUTION
NONLINEAR PHENOMENA

Received
2 November 2014

Accepted
25 February 2015

Published
7 April 2015

Network Modularity is essential for evolution of cooperation under uncertainty

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Cooperative behavior, which pervades nature, can be significantly enhanced when agents interact in a structured rather than random way; however, the key structural factors that affect cooperation are not well understood. Moreover, the role structure plays with cooperation has largely been studied through observing overall cooperation rather than the underlying components that together shape cooperative behavior. In this paper we address these two problems by first applying evolutionary games to a wide range of networks, where agents play the Prisoner's Dilemma with a three-component stochastic strategy, and then analyzing agent-based simulation results using principal component analysis. With these methods we study the

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THANK YOU FOR YOUR COOPERATION!

photo: Reuters