Ecology, Epidemiology and Disease Modelling: Computational Lab problems

(Week 2: 15 - 19 May, 2017)

(Samit Bhattacharyya & Somdatta Sinha)

DAY-1:

A. Discrete Generation Population Dynamics Models:

1) Take the following models and work out the questions given below –

   a. **Logistic Model**: $N(t+1) = rN(t)\left(1 - \frac{N(t)}{K}\right) \quad K = 1$

   b. **Ricker Model**: $N(t+1) = N(t)e^{\left\{r\left(1-\frac{N(t)}{K}\right)\right\}} \quad K = 1$

   c. **Hassell Model**: $N(t+1) = \frac{rN(t)}{(1+N(t))^b} \quad b = 3, \; b = 6$

(i) Find the **Steady States** of each model, and plot the **Time series** (*N* versus *t*) and for different values of r to show what types of dynamics the model exhibits.

(ii) Plot **Return Maps** (*N*(t) versus *N*(t+1)) for at least 3 values of *r* at different dynamical regimes.

(iii) Write the logic/code/algorithm for the **Bifurcation Diagrams** on variation of the parameter $r \in (1, 4)$ for Model (a), and $r \in (0, 50)$ for Models (b and c), and plot them.

B. Continuous Generation Population Dynamics Models:

1. Find the **Steady States** of the following differential equation model and calculate the **Local Stability** of the steady states:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{K}\right), \quad K = 1$$

   Plot the **Time Series** for different values of *r*.

2. Find the **Steady States** of the following two-variable coupled differential equation model and calculate the **Local Stability** of the steady states:

$$\frac{dx}{dt} = g(x) - bxy,$$
$$\frac{dy}{dt} = cxy - dy$$

   For (a) $g(x) = ax$, (b) $g(x) = ax(1 - x/K)$, K=1

   Plot **Phase Portrait** (*x* versus *y*) of the two-variable system.

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DAY-2:

A. Generic Compartmental Models in Epidemiology:

1) The standard **SIR** and **SEIR** models are given below: Find their **Steady States** and calculate the **Local Stability** of the **Steady States**.

$$\frac{ds}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{ds}{dt} = \gamma I$$

$$\frac{ds}{dt} = -\beta S(I + \varepsilon E)$$
$$\frac{dE}{dt} = \beta S(I + \varepsilon E) - \kappa E$$
$$\frac{dI}{dt} = \kappa E - \gamma I$$
$$\frac{ds}{dt} = \gamma I$$

Plot **Time Series** and **Phase Portrait** for both models for the following parameter values.

Parameters	Values
Transmission rate (β)	1 per 3000 per day
Latency (k)	0.5 per day
Recovery (γ)	0.17 per day
Infectivity by exposed class (ε)	0.3
Initial condition ($S0, E0, I0$)	(995, 0, 5)

1. Simulate the following **Ross** and **MacDonald**

$$\frac{dI_h}{dt} = abmI_m S_h - \gamma I_h$$
$$\frac{dI_m}{dt} = acI_h S_m - \mu I_m$$

$$S_h + I_h = 1$$
$$S_m + I_m = 1$$

$$\frac{dI_h}{dt} = abmI_m S_h - \gamma I_h$$
$$\frac{dE_m}{dt} = acI_h S_m - acI_h(t-\tau)S_m(t-\tau)e^{-\mu\tau} - \mu E_m$$
$$\frac{dI_m}{dt} = acI_h(t-\tau)S_m(t-\tau)e^{-\mu\tau} - \mu I_m$$

Parameters	Range	Values for simulation
Biting rate (a)	0.01-0.5 / day	0.2
Infection probability (b)	0.2-0.5 /day	0.5
Mosquito: Human (m)	0.5-40	5
Recovery rate of human (γ)	0.005-0.05 / day	0.02
Infection probability of mosquito (c)	0.5	0.5
Mosquito mortality rate (μ)	0.05-0.5 /day	0.05
Delay (τ)	8-10 days	8