# Stars and their evolution

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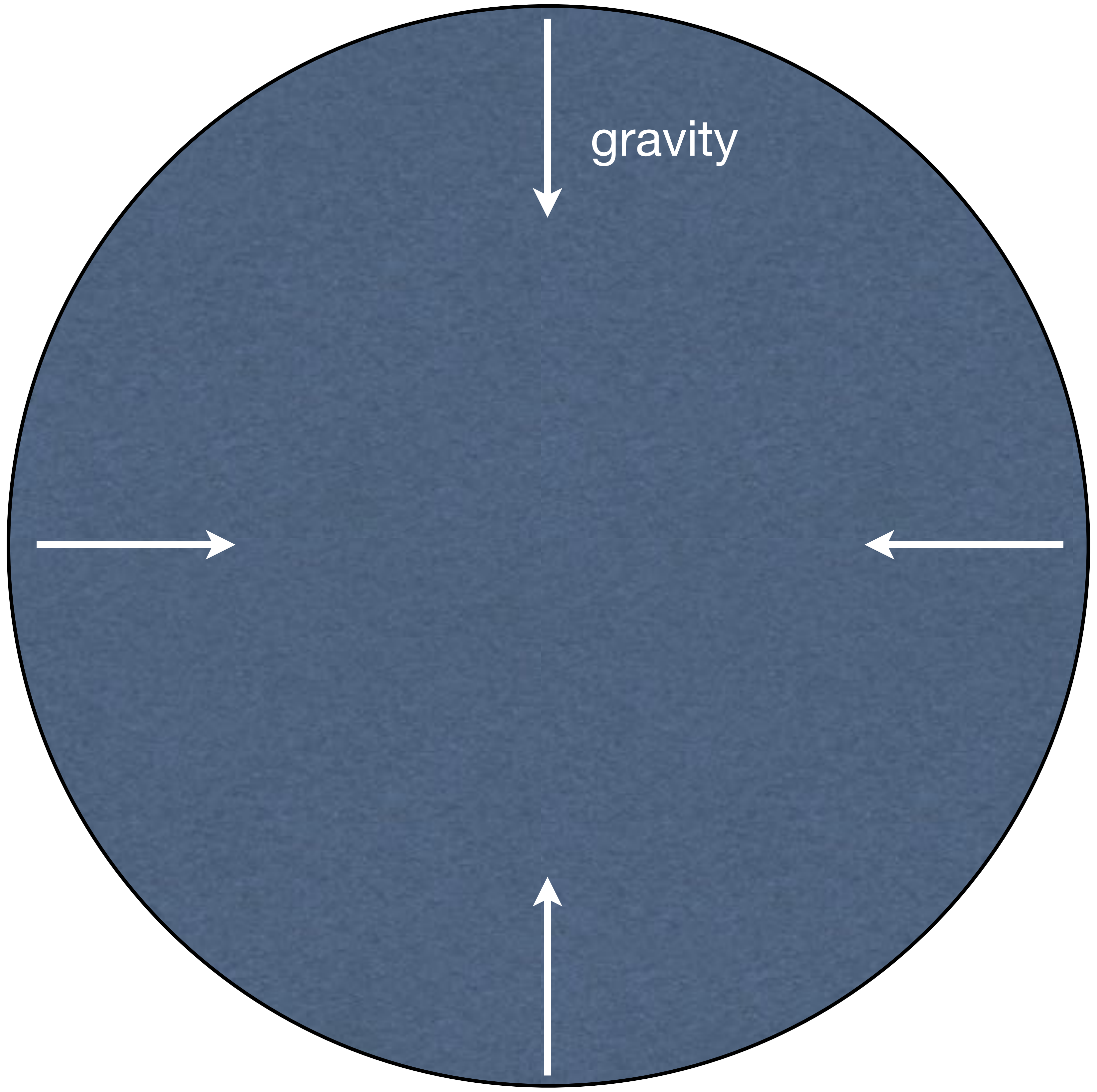
The Starry Sky
image by Mindaugas Vitkus

# **Stars**:
Self-gravitating globes of gas

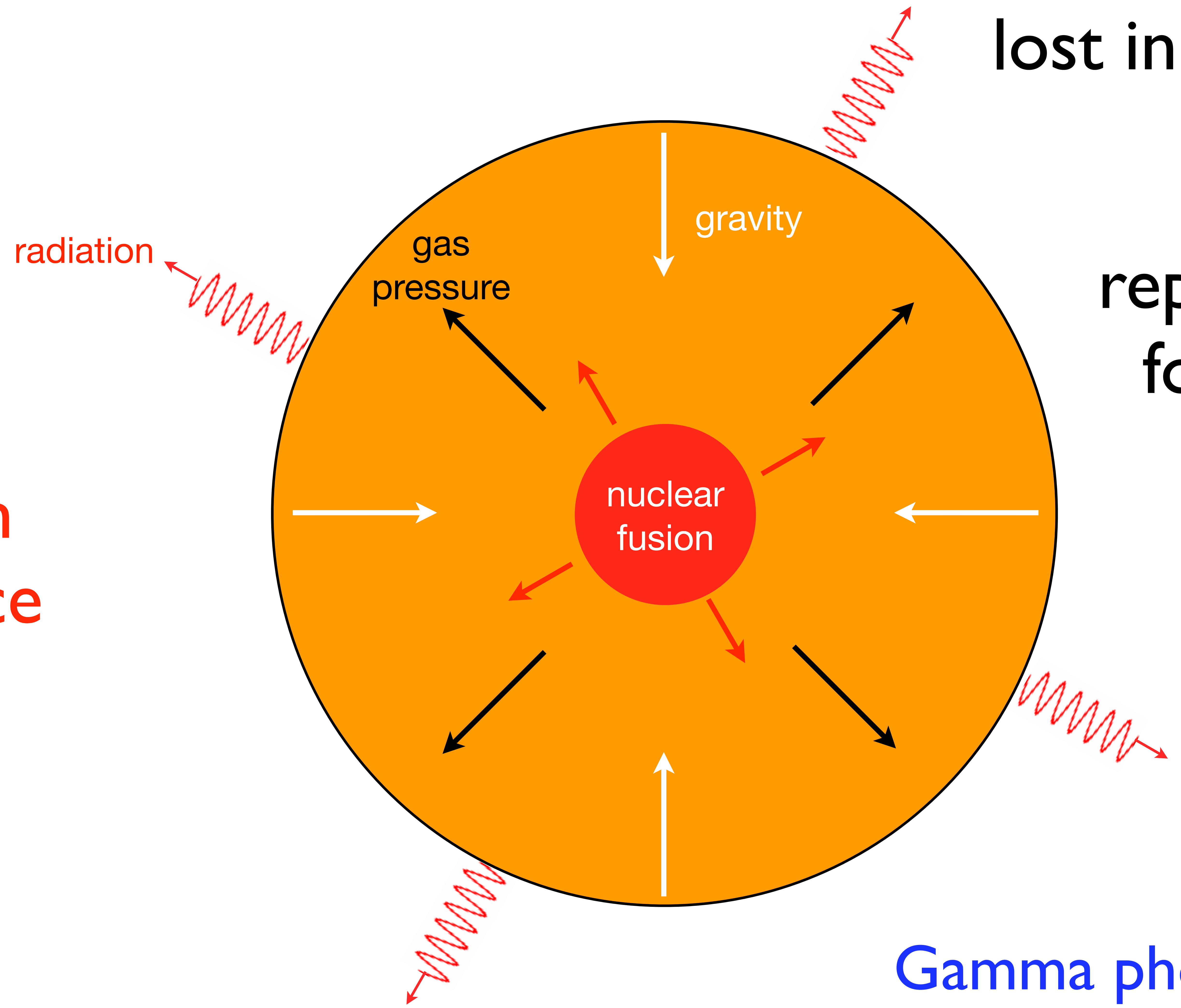


**Stars**:

Self-gravitating globes of gas supported by thermal pressure

Thermal energy is lost in radiation

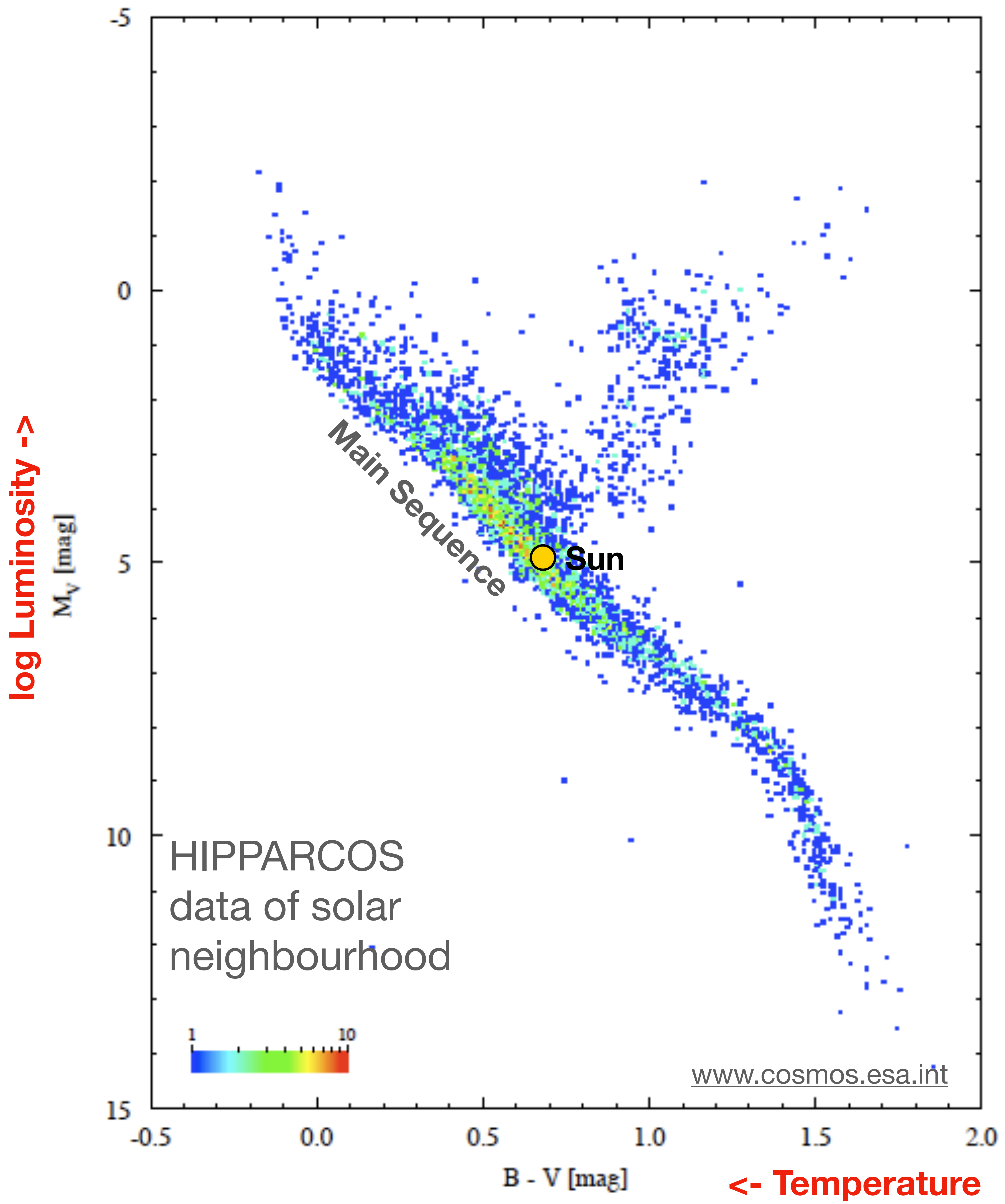
Need replenishment for stability

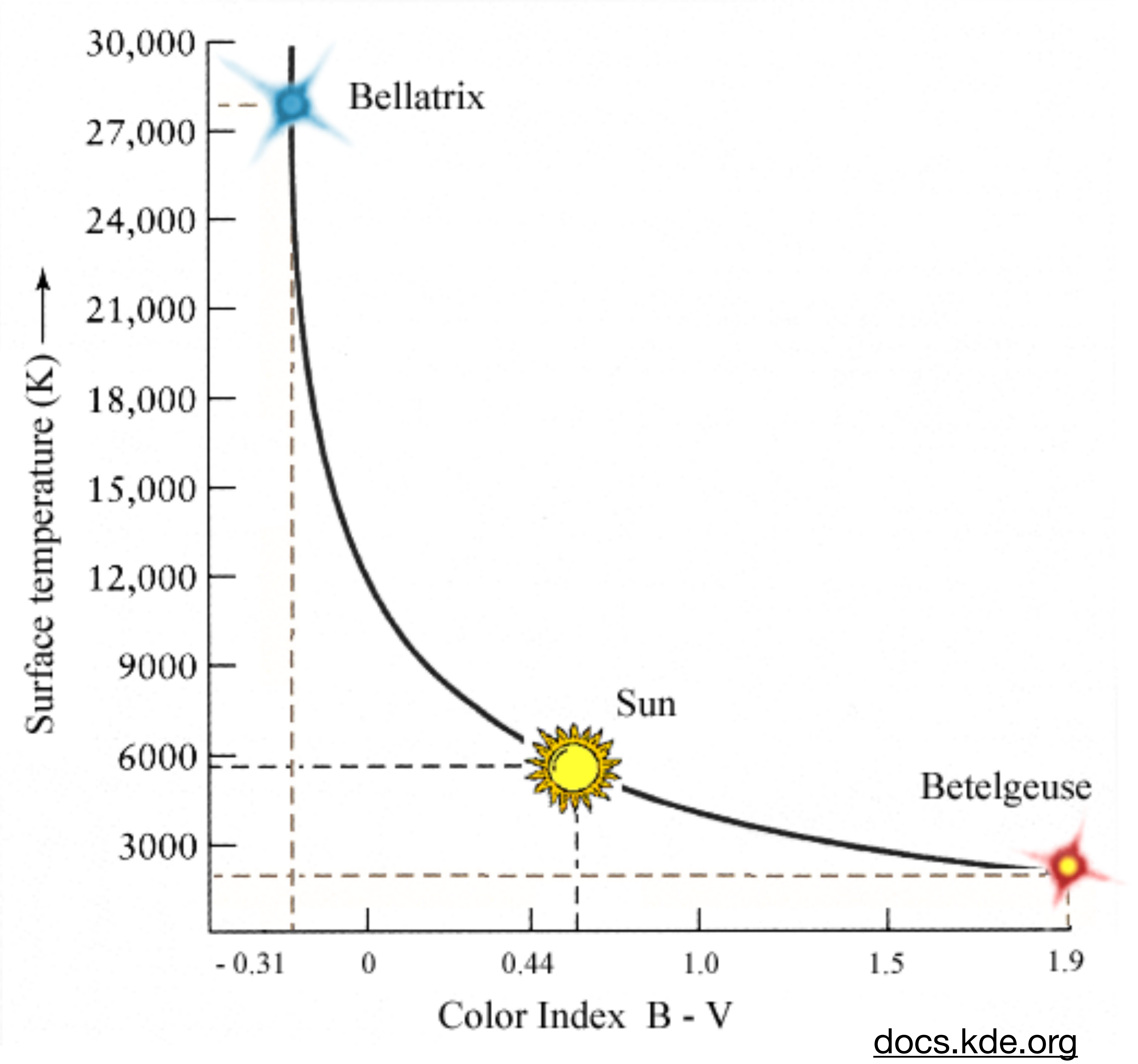



Nuclear fusion provides balance

Gamma photons are degraded by matter interaction to optical light

# Hertzsprung-Russell Diagram



# Stellar Structure: Hydrostatic Equilibrium

Spherically symmetric:

$$-\frac{dP}{dr} = \frac{G(M + 4\pi r^3 P/c^2)(\rho + P/c^2)}{r^2(1 - 2GM/rc^2)}$$

Tolman-Oppenheimer-Volkoff (TOV) equation [full GR]

$$-\frac{dP}{dr} = \frac{GM\rho}{r^2}$$

Newtonian Gravity [for small values of $M/R$ ]

$$M(r) = \int_0^r 4\pi r^2 \rho(r) dr$$

Supplement these with Equation of State to solve for equilibrium structure

# Virial Theorem

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$ (hydrostatic equilibrium)

Multiply both sides by $4\pi r^3$ and integrate over the full configuration: $r = 0 \to R$

$$4\pi R^3 P(R) - \int_0^R 4\pi r^2 \cdot 3P dr = \int_0^R 4\pi r^2 dr \left[-\frac{GM(r)\rho(r)}{r}\right]$$

RHS = Total gravitational energy of the configuration $E_g$ ( < 0 )

and since $P = (\gamma - 1)u_{\rm th}$ , where $u_{\rm th}$ is the Thermal (kinetic) energy density,

the 2nd term in LHS = $3(\gamma - 1)E_{\rm th}$ , $E_{\rm th}$ being the total thermal (kinetic) energy

Hence $$E_g + 3(\gamma - 1)E_{\rm th} = 4\pi R^3 P(R)$$ : **Virial Theorem**

must be obeyed by all systems in hydrostatic equilibrium

For $\gamma = 5/3$ and $P(R) = 0$ : $E_g + 2E_{\rm th} = 0$

Note: $E_{\rm tot} = E_g + E_{\rm th} = E_g/2 = -E_{\rm th}$

# A Rough Guide to Stellar Structure

$$\frac{dP}{dr} = -\frac{GM(r)\rho(r)}{r^2}$$

stellar radius $R$

central pressure $P_\mathrm{c}$

total mass $M$

Using a linear approximation: $\dfrac{0 - P_\mathrm{c}}{R} = -\dfrac{GM}{R^2} \cdot \dfrac{M}{\frac{4\pi}{3}R^3}$

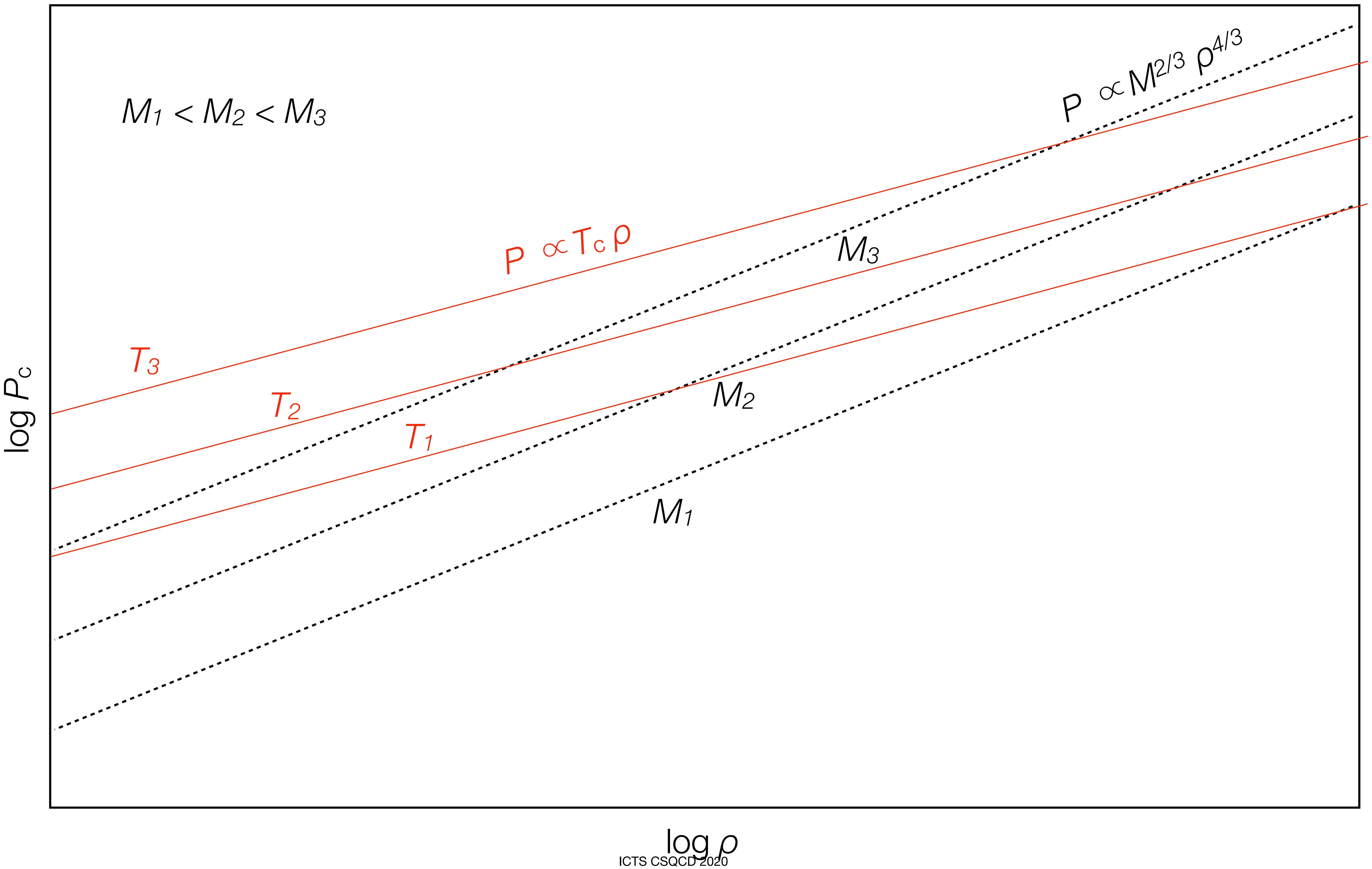
$$P_\mathrm{c} = \frac{3}{4\pi}\frac{GM^2}{R^4} = \left(\frac{4\pi}{3}\right)^{1/3} GM^{2/3}\rho^{4/3}$$

(“gravitational pressure” $P_\mathrm{grav}$)

For equilibrium, this pressure needs to be matched by the Equation of State

e.g. Thermal Pressure: $P = \dfrac{kT}{\mu m_\mathrm{p}}\rho$

# Thermal pressure support



# Main Sequence

Hydrogen burning star $T_{\rm c} \approx T_{\rm H}$

$$P_{\rm c} \approx GM^{2/3}\rho^{4/3} = \frac{kT_{\rm H}}{\mu m_{\rm p}}\rho \quad : \quad \rho \propto M^{-2} \ ; \ R \propto M \text{ and } T_{\rm c} \propto \frac{M}{R}$$

Luminosity $L =$ Radiative Energy Content / Radiation Escape Time

Radiation Escape Time = $\left(\frac{R}{l}\right)^2 \cdot \frac{l}{c}$ where $l =$ mean free path $= \frac{1}{n\sigma} = \frac{1}{\rho\kappa}$

opacity

Hence $L \approx \frac{aT^4R^3}{(R^2/lc)} \approx aclT^4R$ In Main Sequence: $L \propto lR \propto lM$

High mass stars: opacity: Thomson scattering $\kappa =$constant; $l \propto \frac{R^3}{M}$ : $L_{\rm MS} \propto M^3$

Low mass stars: $l \propto \frac{T^{3.5}}{\rho^2}$ : $L_{\rm MS} \propto M^5$

*On average* 
$$\boxed{\begin{array}{l} L_{\rm MS} \propto M^4 \\ t_{\rm MS} \propto M^{-3} \end{array}}$$

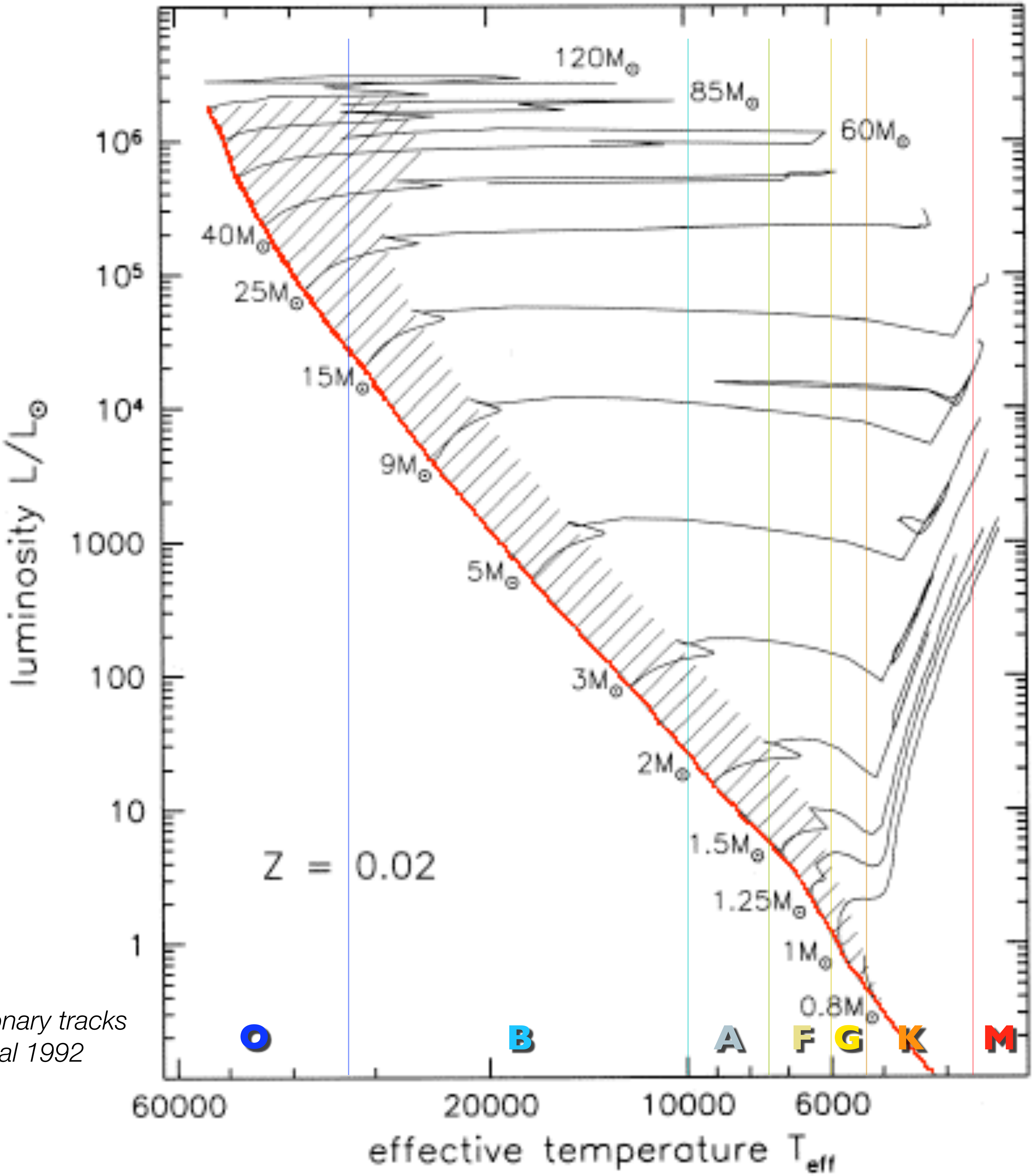
# Theoretical H-R diagram

On Main Sequence

$$L \propto M^4\,;\; R \propto M$$

$$\Rightarrow T_{\rm eff}^4 \propto \frac{L}{R^2} \propto M^2$$

$$\Rightarrow L_{\rm MS} \propto T_{\rm eff}^8 \sim T_{\rm col}^8$$

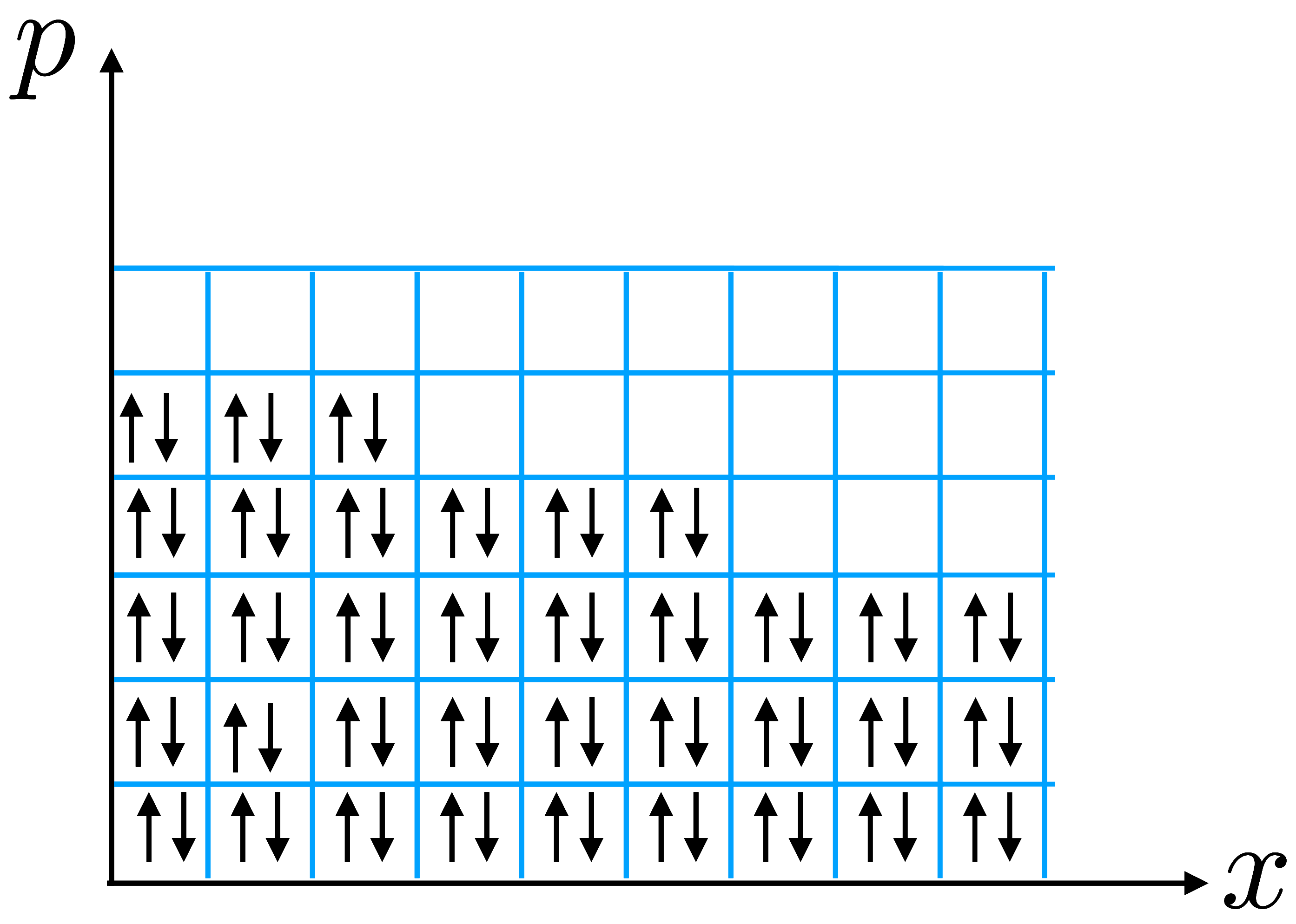



*Stellar evolutionary tracks by Schaller et al 1992*

# Degeneracy pressure

Electrons are elementary particles with half-integral spin: $\hbar/2$

Such particles obey Fermi-Dirac statistics, leading to Pauli's exclusion principle:




Phase space (6-dim): position and momentum

Divided into quantum cells of volume $h^3$

Each cell may contain at most 2 electrons of opposite spin

*The denser the material gets, the higher the momentum rises*

# Degeneracy pressure

Let number of electrons per unit volume: $n_e$

Momentum up to which states are filled: $p_{\mathrm{F}}$

No. of phase space cells: $$\frac{4\pi}{3}\frac{p_{\mathrm{F}}^3}{h^3} = \frac{n_e}{2}$$

Thus $p_{\mathrm{F}} = \left(\frac{3}{8\pi}\right)^{1/3} h n_e^{1/3}$

Pressure = momentum flux

$P = n_e\, v\, p$

$$v = p/m_e \rightarrow P \propto n_e^{\frac{5}{3}} \quad (v \ll c)$$

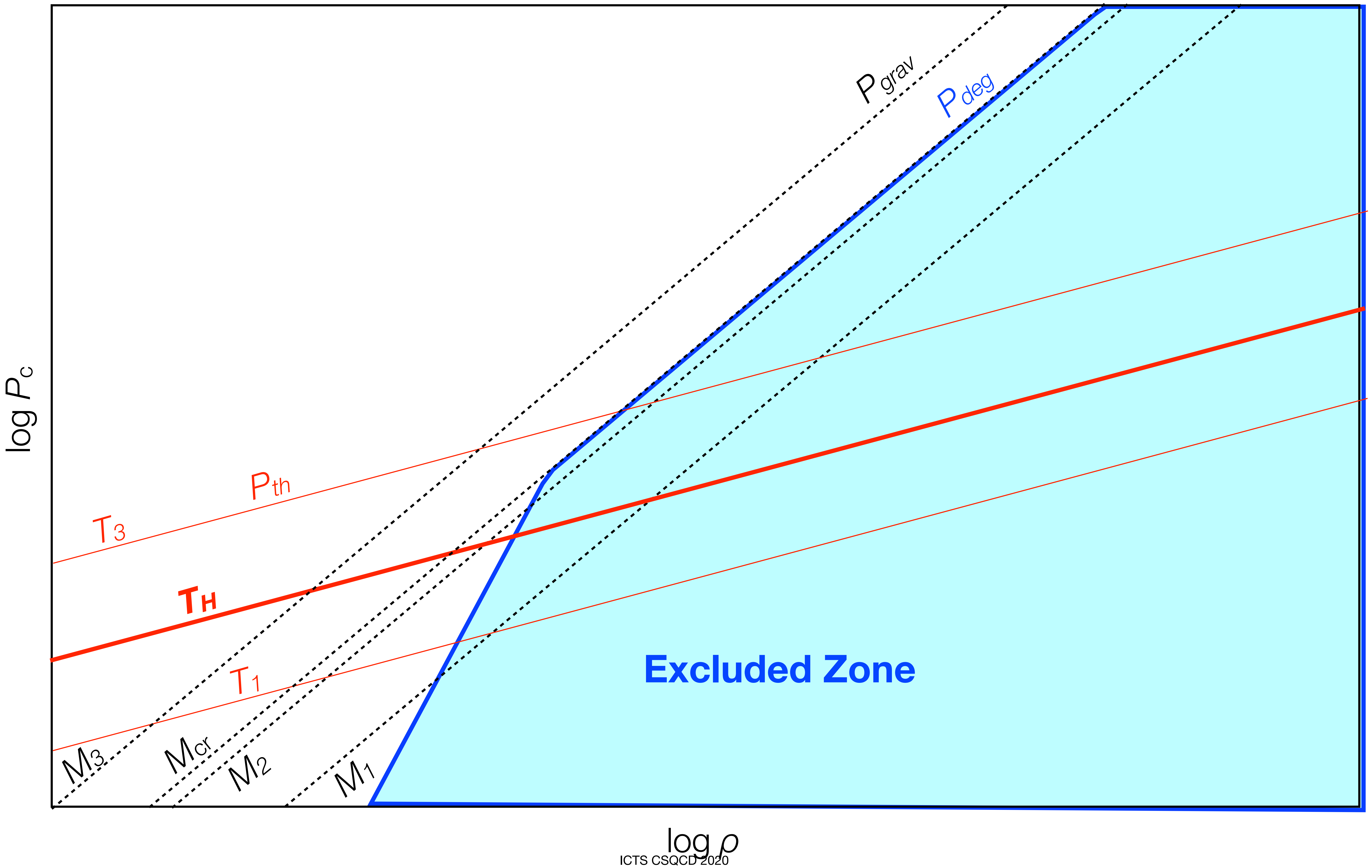
$$P \propto n_e^{\frac{4}{3}} \quad (v \rightarrow c)$$

$n_{\mathrm{e}} = \rho/(\mu_{\mathrm{e}} m_{\mathrm{p}})$

$P_{\mathrm{deg}} \propto \rho^{5/3}$ (non-relativistic)

$\propto \rho^{4/3}$ (relativistic)

# Stellar Equilibrium



# White Dwarfs

Configurations supported by Electron Degeneracy Pressure

$P_{\rm deg} = K_1 m_{\rm e}^{-1} (\rho/\mu_{\rm e} m_{\rm p})^{5/3}$ (non-relativistic)

$P_{\rm deg} = K_2 (\rho/\mu_{\rm e} m_{\rm p})^{4/3}$ (relativistic) : when $p_{\rm F} \gtrsim m_{\rm e} c$ ($\rho \gtrsim 10^6$ g cm$^{-3}$)

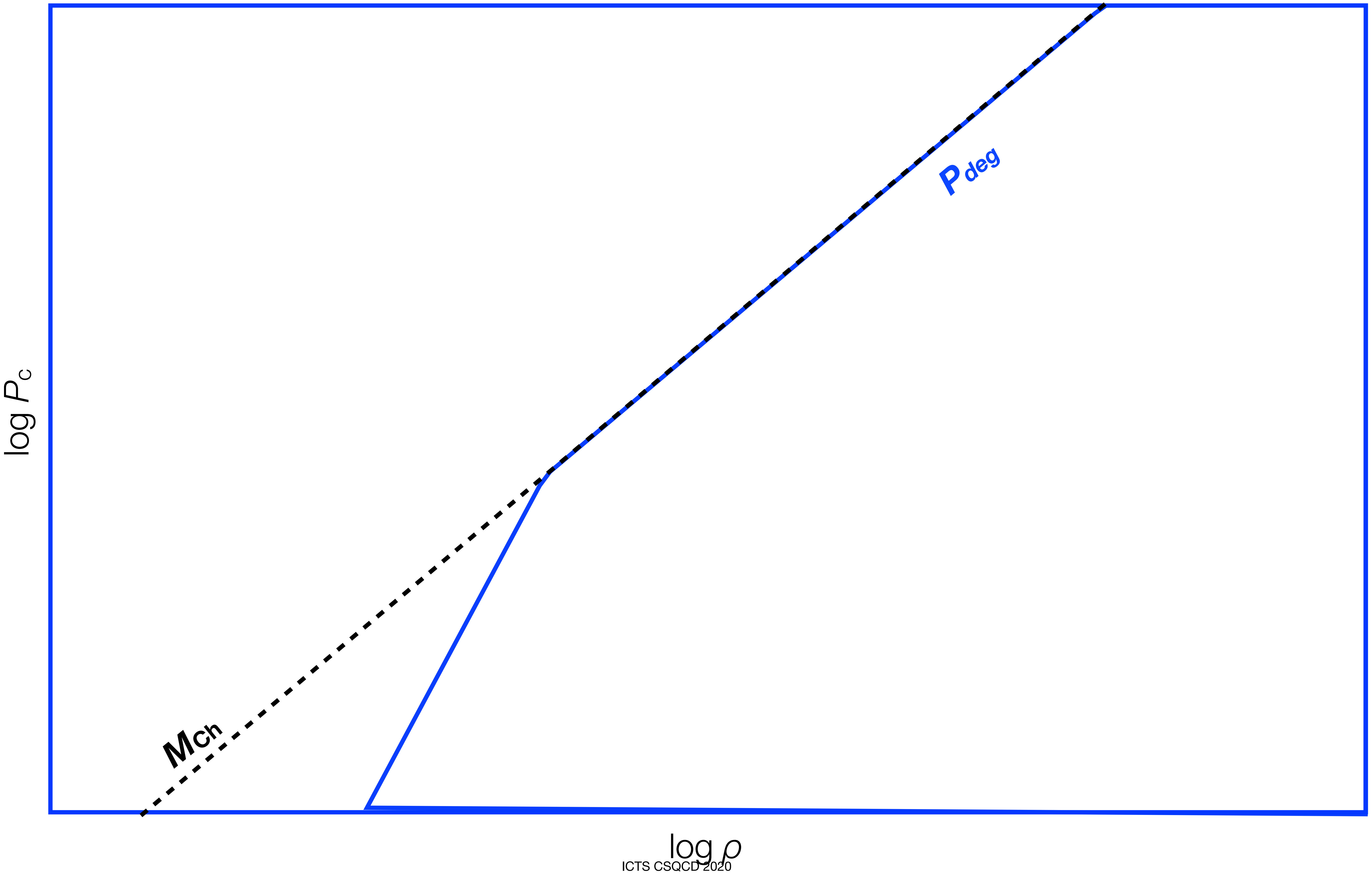
Equilibrium condition: $P_{\rm deg} \approx G M^{2/3} \rho^{4/3}$

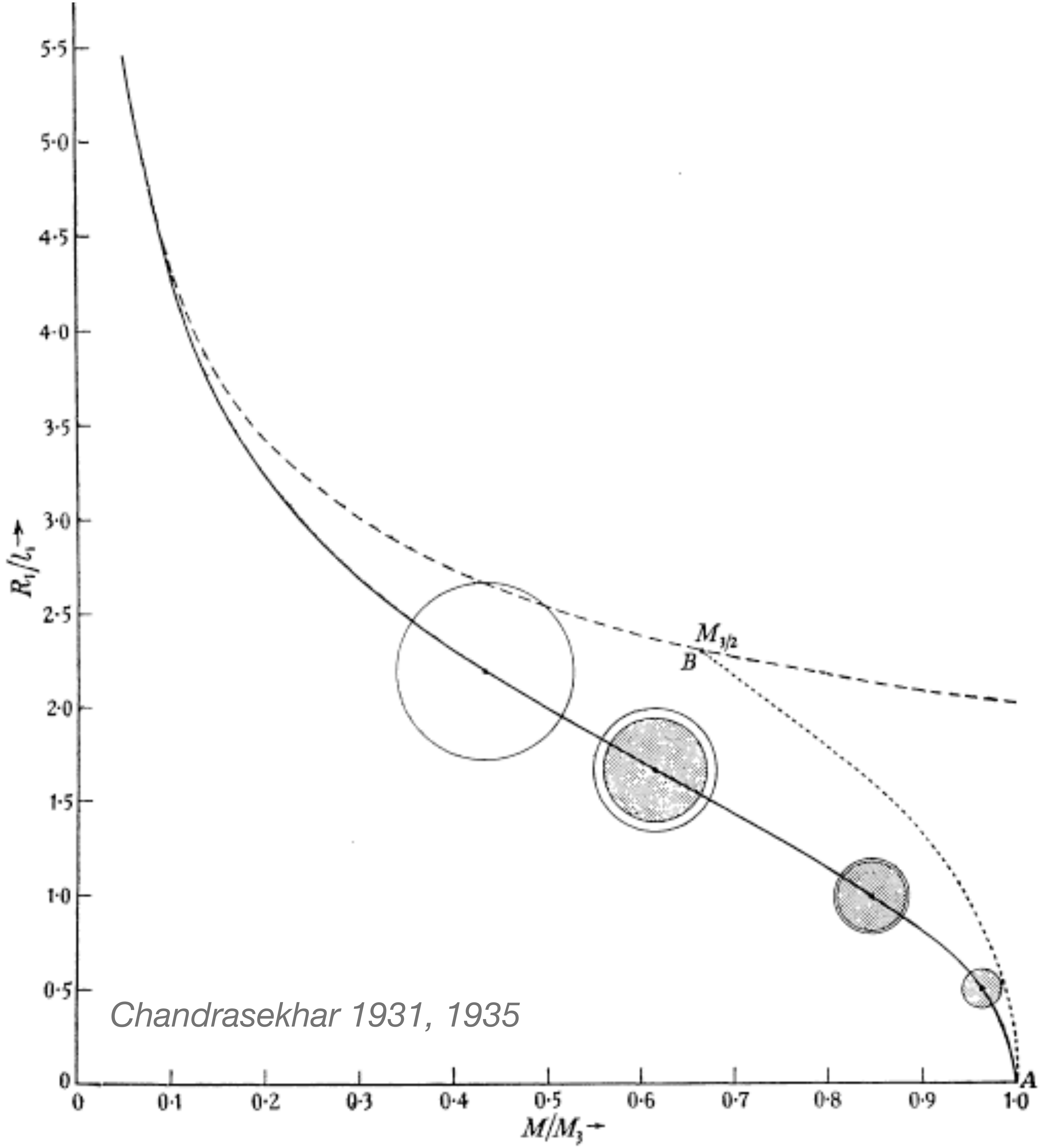
$\Rightarrow$ Non-relativistic: $R \propto m_{\rm e}^{-1} \mu_{\rm e}^{-5/3} M^{-1/3}$ ( $R$ ~ $10^4$ km for $M$ ~ 1 $M_{\rm sun}$ )

Relativistic: $M \sim \left(\frac{K_2}{G}\right)^{3/2} (\mu_{\rm e} m_{\rm p})^{-2}$ : Limiting Mass (Chandrasekhar Mass)

$$M_{\rm Ch} = 5.76 \mu_{\rm e}^{-2} \ \ M_{\odot}$$

# Limiting Mass of White Dwarf





5·5
5·0
4·5
4·0
3·5
3·0
2·5
2·0
1·5
1·0
0·5
0
$R_1/l_1 \rightarrow$
0
0·1
0·2
0·3
0·4
0·5
0·6
0·7
0·8
0·9
1·0
$M/M_3 \rightarrow$
$M_{3/2}$
B
A
Chandrasekhar 1931, 1935

# Neutron Stars

Supported by Neutron degeneracy pressure and repulsive strong interaction

TOV equation + nuclear EOS required for description

Beta equilibrium : $>$ 90% neutrons, $<$ 10% protons and electrons

Uncertainty in the knowledge of nuclear EOS leads to uncertainty in the prediction of Mass-radius relation and limiting mass of neutron stars
*(upon exceeding the max. NS mass a Black Hole would result)*

Inter-nucleon distance ~ 1 fm $\Rightarrow n \sim 10^{39}$ , $\rho \sim 10^{15}$ g cm$^{-3}$

$R \sim$ 10 km for $M \sim 1\, M_{\mathrm{sun}}$

Neutron stars spin fast: $P \sim$ ms - mins
and have strong magnetic field: $B_{\mathrm{surface}} \sim 10^{8} - 10^{15}$ G

Exotic phenomena: *Pulsar, Magnetar activity*

# Equations of stellar evolution

$$\frac{\partial r}{\partial m} = \frac{1}{4\pi r^2 \rho} \ ,$$

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} - \frac{1}{4\pi r^2}\frac{\partial^2 r}{\partial t^2} \ ,$$

$$\frac{\partial T}{\partial m} = -\frac{GmT}{4\pi r^4 P}\nabla \ , \qquad \nabla \equiv d\ln T/d\ln P$$

$$\frac{\partial \mathcal{L}}{\partial m} = \epsilon_{\mathrm{n}} - \epsilon_{\nu} - C_P\frac{\partial T}{\partial t} + \frac{\delta}{\rho}\frac{\partial P}{\partial t} \ , \qquad \delta \equiv -(\partial \ln \rho/\partial \ln T)_P$$

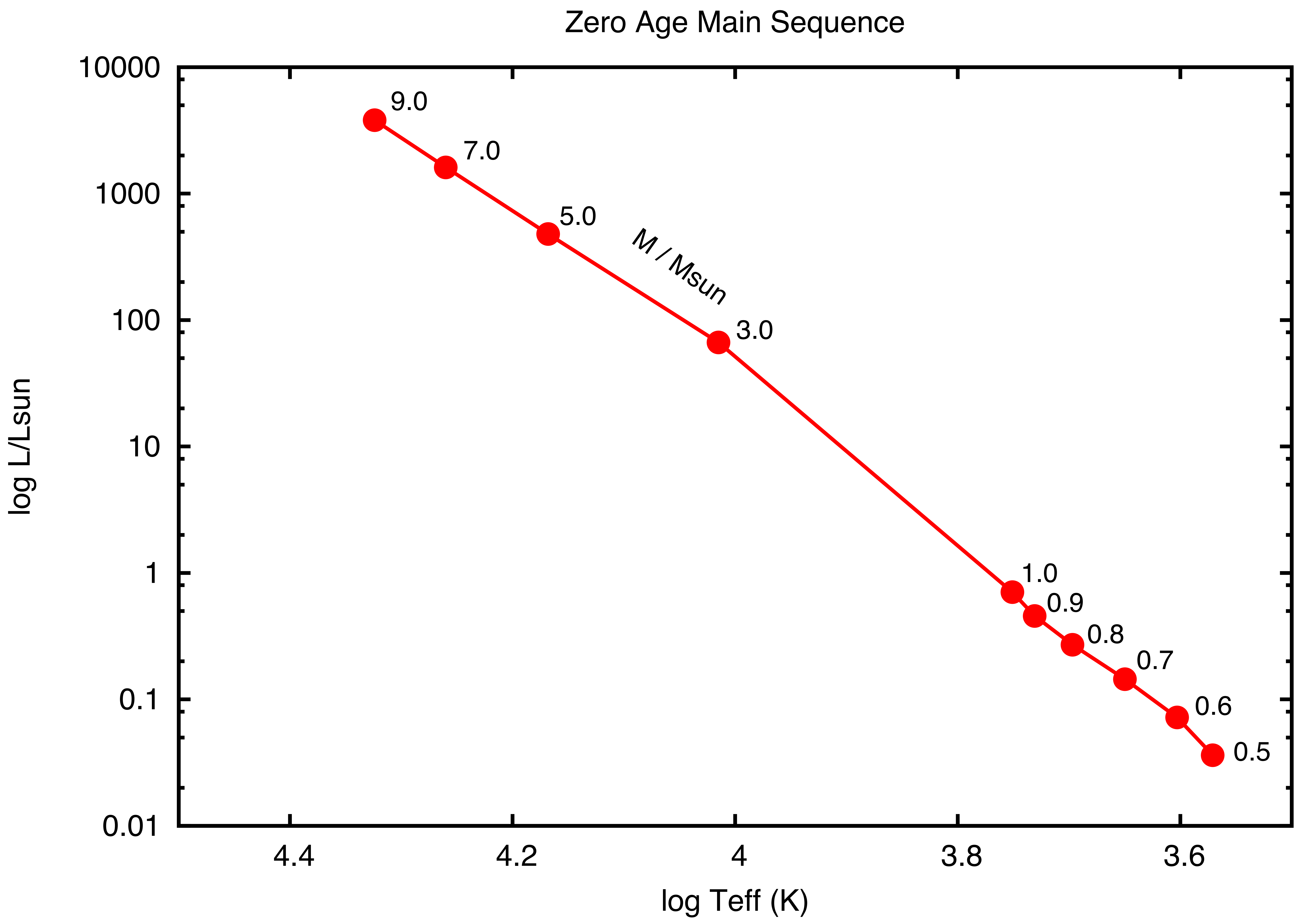
$$\frac{\partial X_i}{\partial t} = \frac{m_i}{\rho}\left(\sum_j r_{ji} - \sum_k r_{ik}\right) \ , \quad i = 1, \ldots, I.$$

# Evolutionary Time Scales

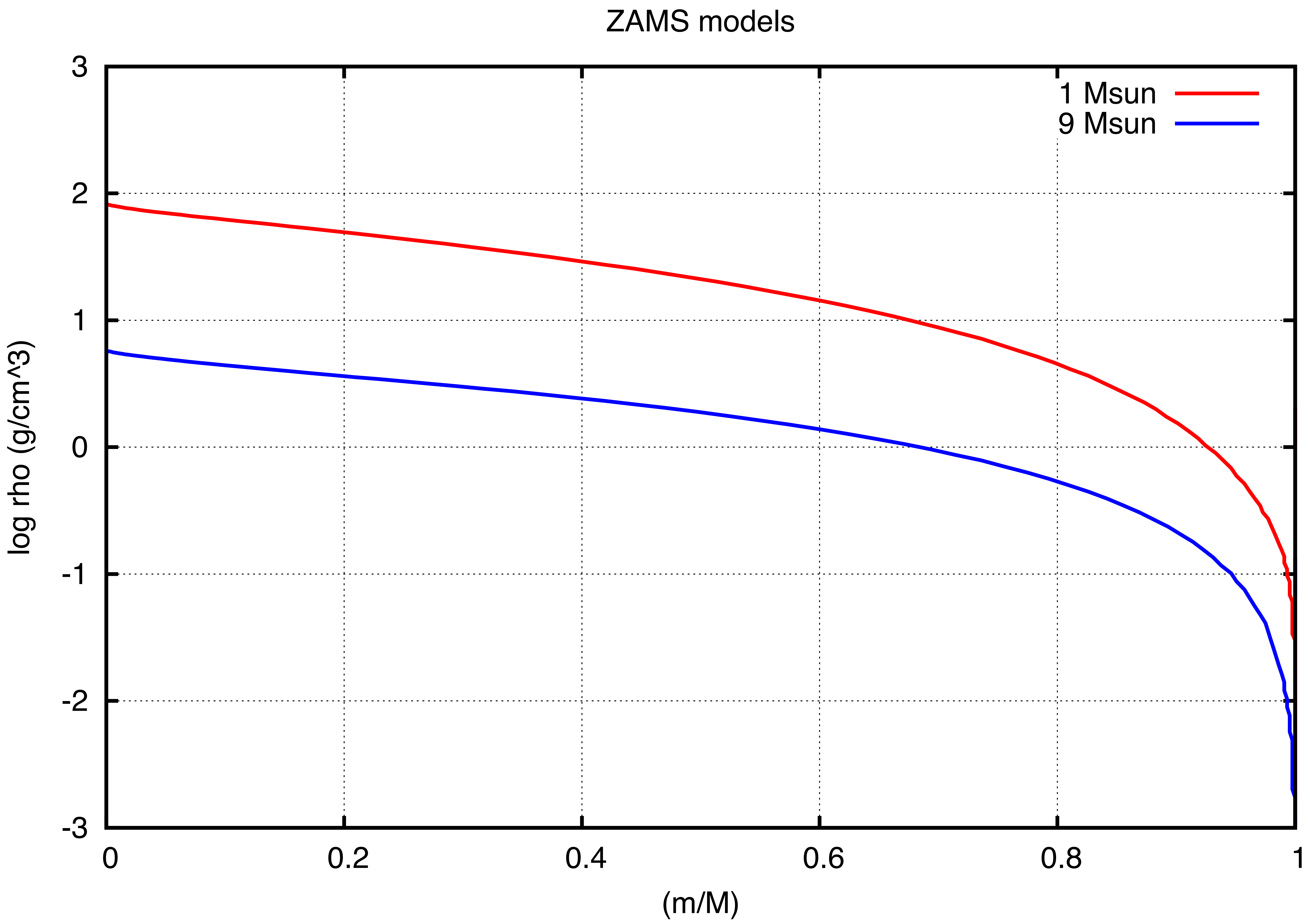
Dynamical: $t_{\mathrm{dyn}} \approx 1/\sqrt{G\bar{\rho}} \sim$ 1 hour for the Sun

Thermal (*Kelvin-Helmholtz*): $t_{\mathrm{th}} \approx E_{\mathrm{th}}/L \sim GM^2/2RL \sim$ 10 Myr for the Sun

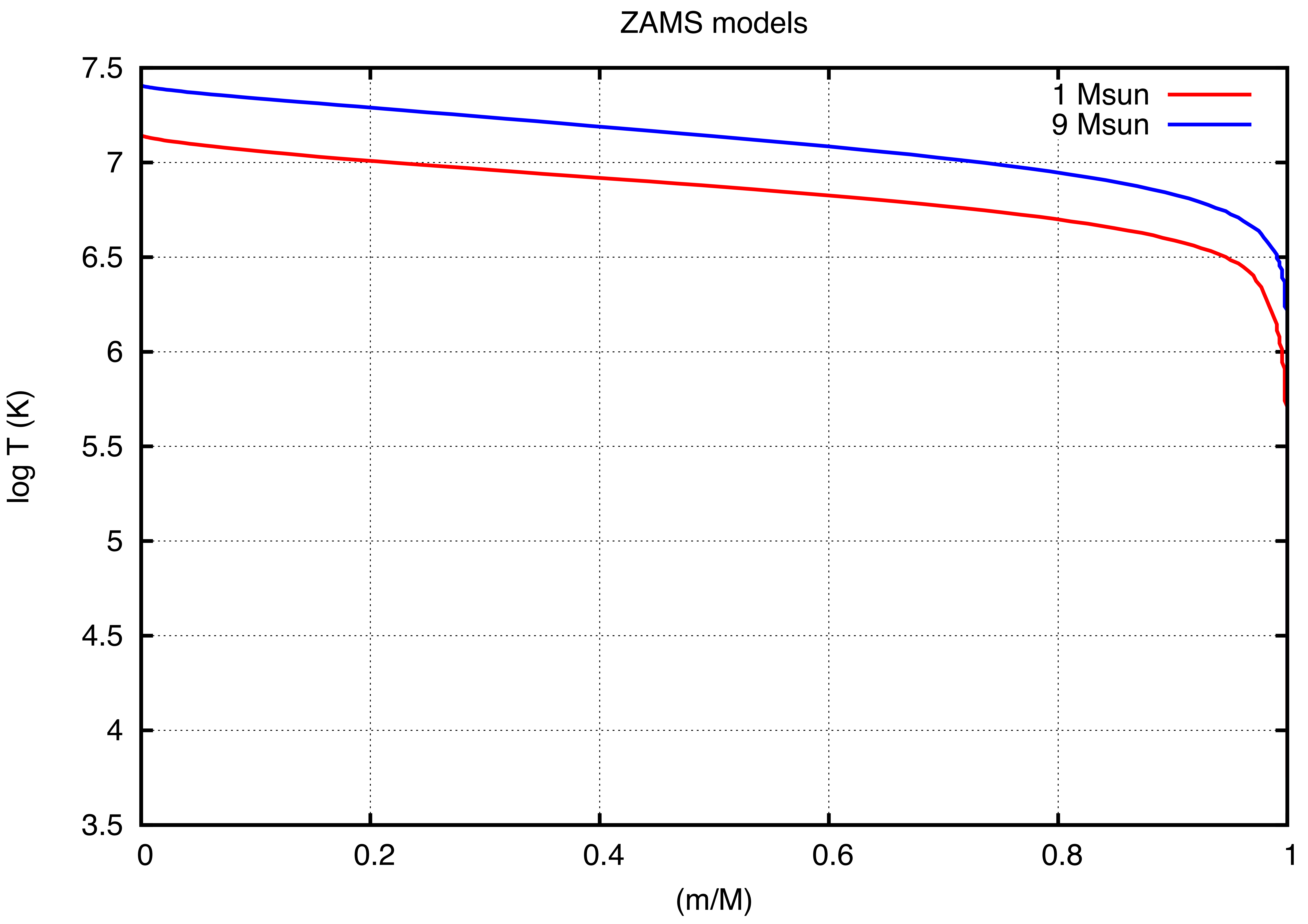
Nuclear: $t_{\mathrm{nuc}} \approx \eta Mc^2/L \sim$ 10 Gyr for the Sun

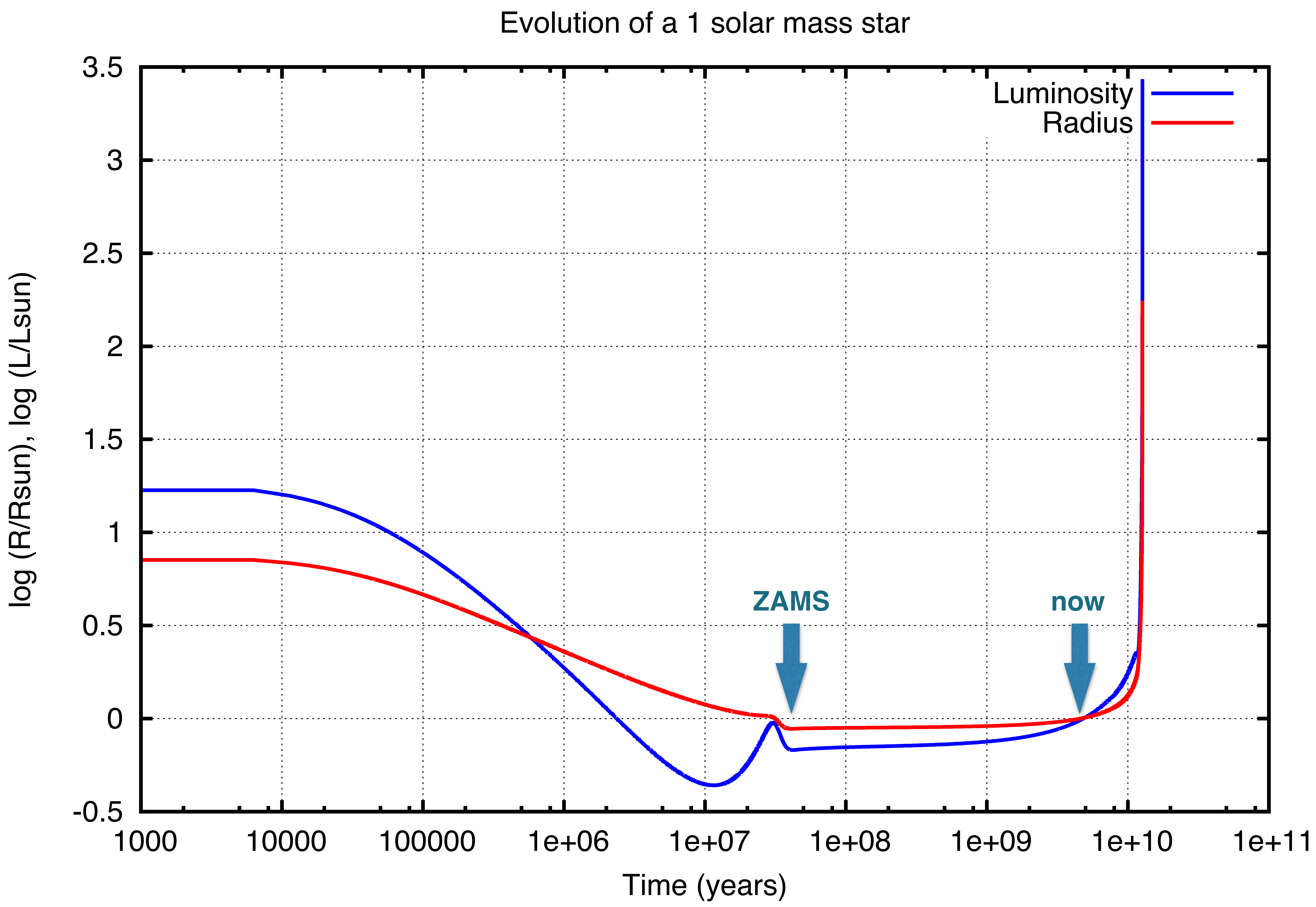


Zero Age Main Sequence
10000
1000
100
10
1
0.1
0.01
log L/Lsun
4.4
4.2
4
3.8
3.6
log Teff (K)
9.0
7.0
5.0
M / Msun
3.0
1.0
0.9
0.8
0.7
0.6
0.5

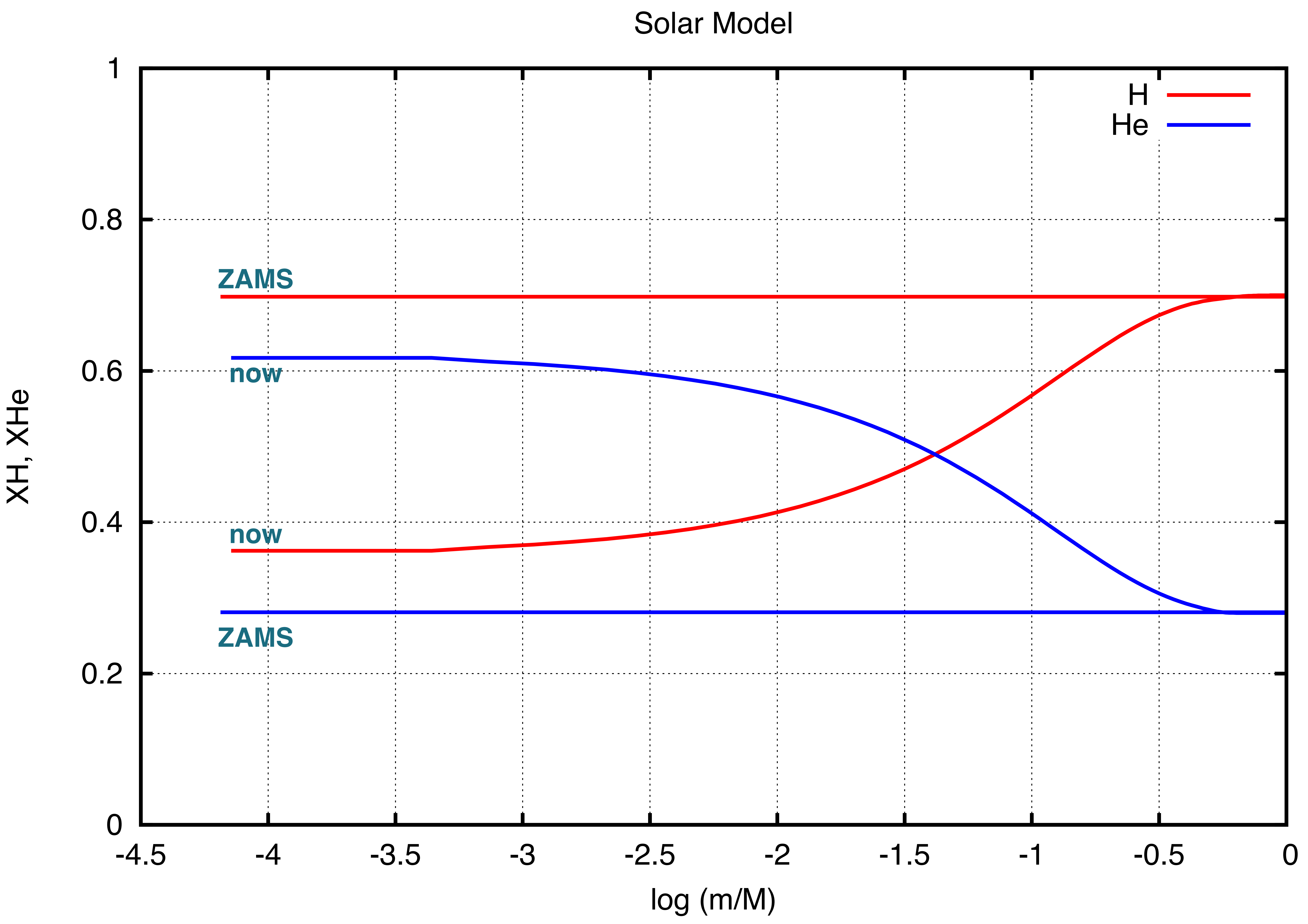


ZAMS models
1 Msun
9 Msun
log rho (g/cm^3)
(m/M)
3
2
1
0
-1
-2
-3
0
0.2
0.4
0.6
0.8
1

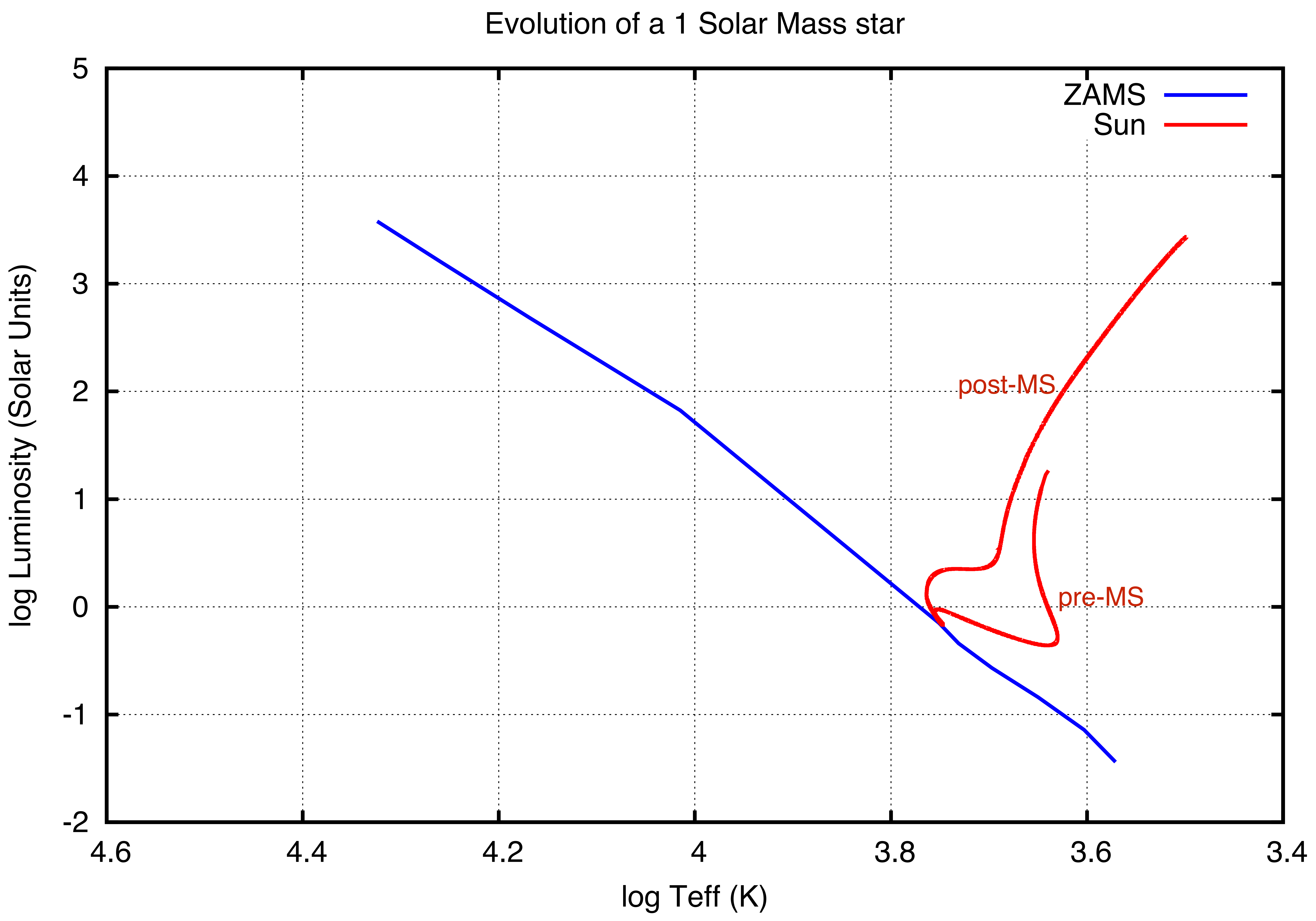

ZAMS models
1 Msun
9 Msun
log T (K)
(m/M)
7.5
7
6.5
6
5.5
5
4.5
4
3.5
0
0.2
0.4
0.6
0.8
1



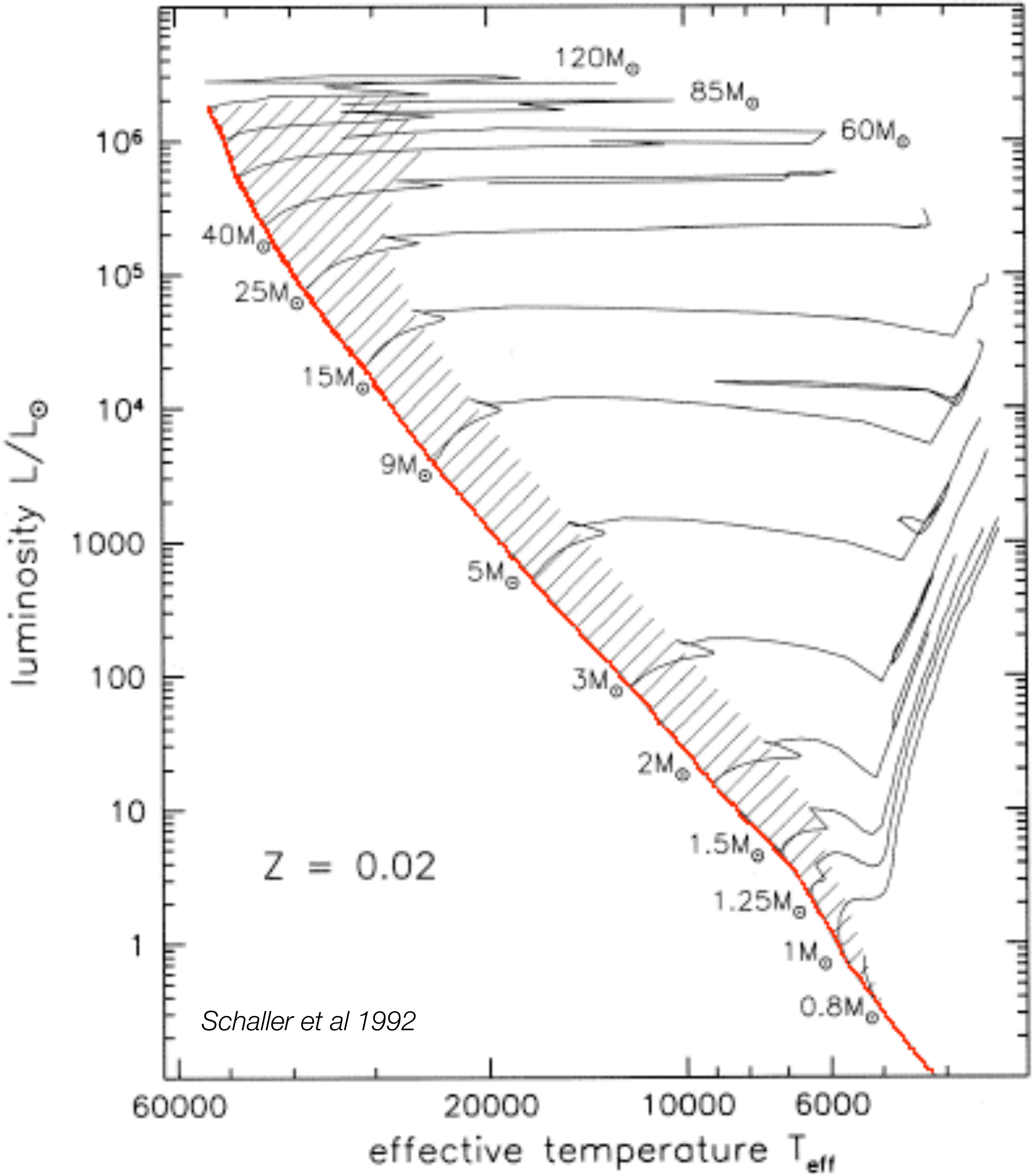
Evolution of a 1 solar mass star
Luminosity
Radius
ZAMS
now
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
log (R/Rsun), log (L/Lsun)
1000
10000
100000
1e+06
1e+07
1e+08
1e+09
1e+10
1e+11
Time (years)



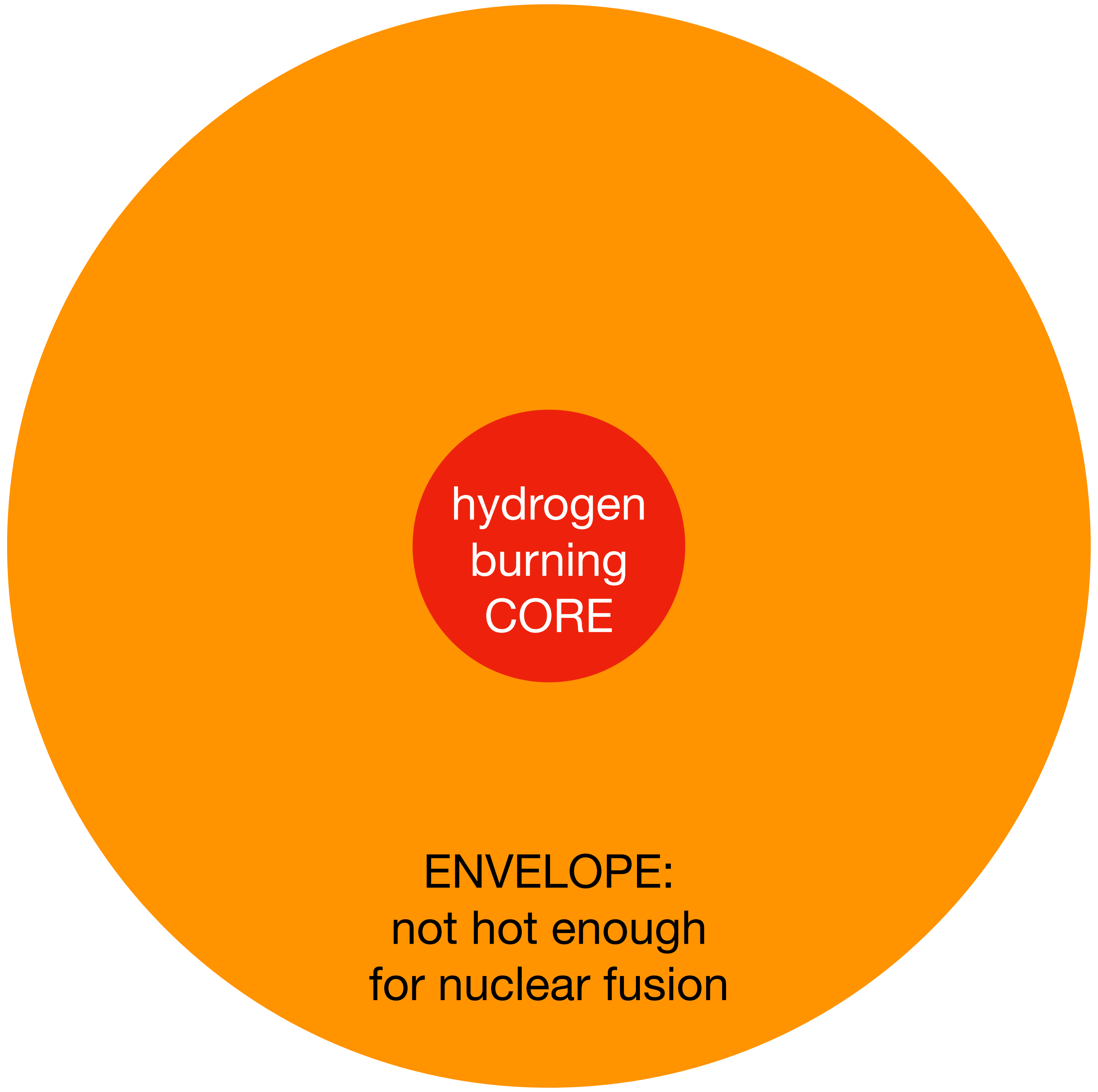
Solar Model
H
He
ZAMS
now
now
ZAMS
XH, XHe
log (m/M)
1
0.8
0.6
0.4
0.2
0
-4.5
-4
-3.5
-3
-2.5
-2
-1.5
-1
-0.5
0


Evolution of a 1 Solar Mass star
ZAMS
Sun
post-MS
pre-MS
log Luminosity (Solar Units)
log Teff (K)
5
4
3
2
1
0
-1
-2
4.6
4.4
4.2
4
3.8
3.6
3.4

# Stellar Evolutionary Tracks



# Evolution of Stars:




Stars begin their lives as homogeneous gas mixtures of
- nearly three quarters Hydrogen,
- about a quarter Helium and
- a small amount of heavier elements

Nuclear fusion occurs in the hottest region: ***The Core***
Stars spend most of their life burning Hydrogen: $4H \rightarrow He$
Composition of the core is thus altered.

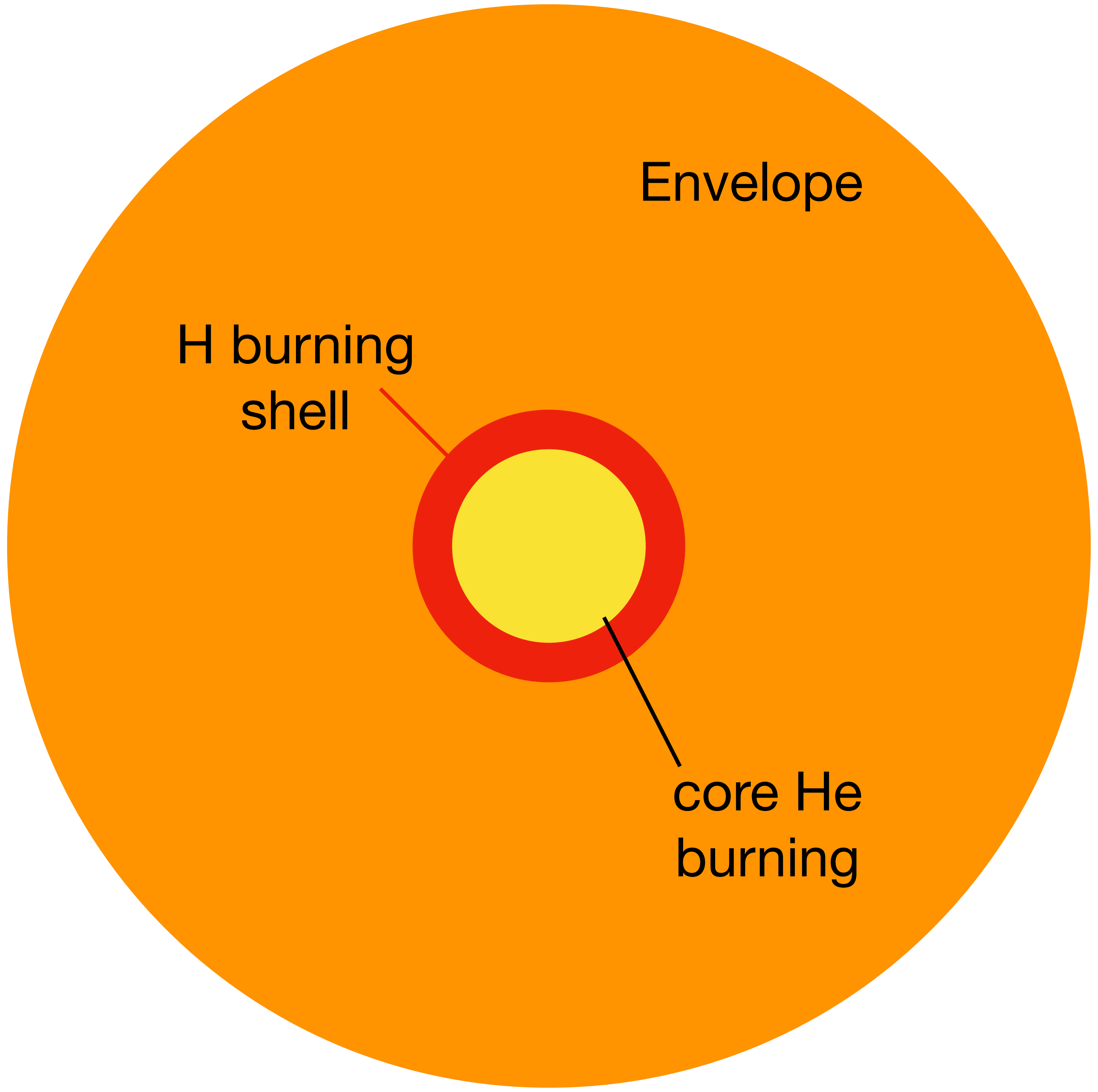
Eventually Hydrogen in the core is exhausted and burning stops.
*Core begins to contract under the influence of gravity*

# Evolution of Stars:

$$E_{\mathrm{Th}} = -\frac{1}{2} E_{\mathrm{grav}}$$ (Virial Theorem)

As gravitational contraction proceeds, the gas gets hotter

$|E_{\mathrm{grav}}|$ increases with contraction



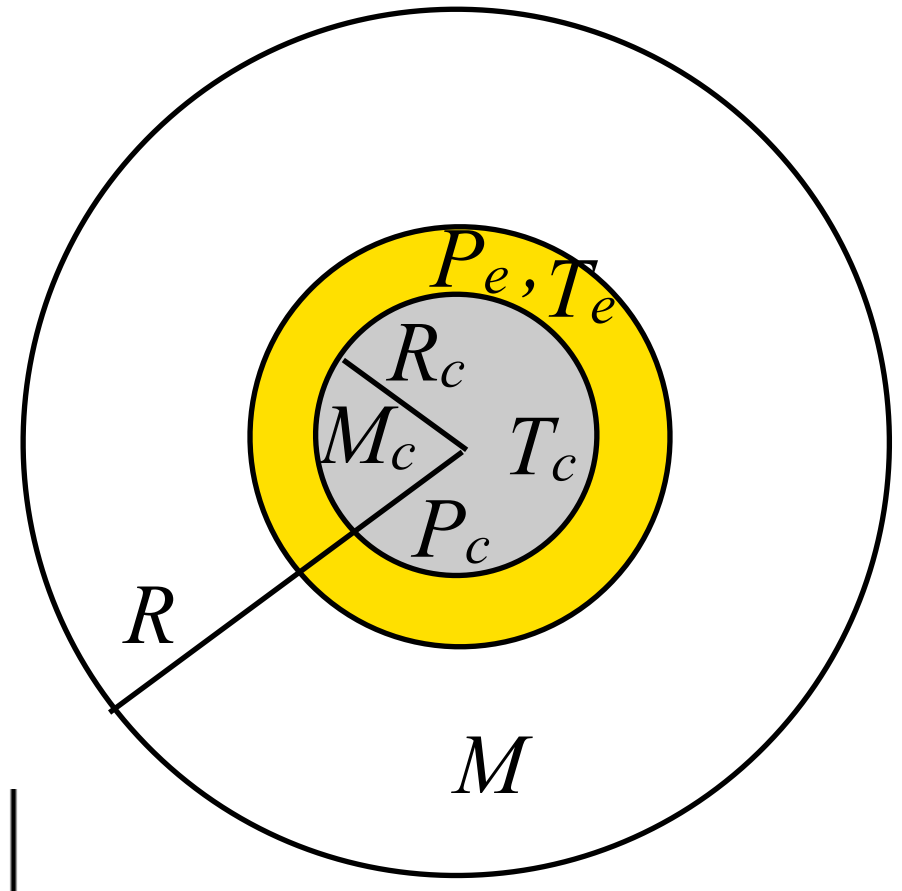

Gas surrounding the contracting core now becomes hot enough to burn Hydrogen in a shell

If contraction proceeds long enough then core temperature rises sufficiently to ignite Helium: $3He \rightarrow C$

Stable equilibrium as long as *He* burns. Core Helium exhaustion would trigger the next stage of core collapse

# Schönberg-Chandrasekhar limit

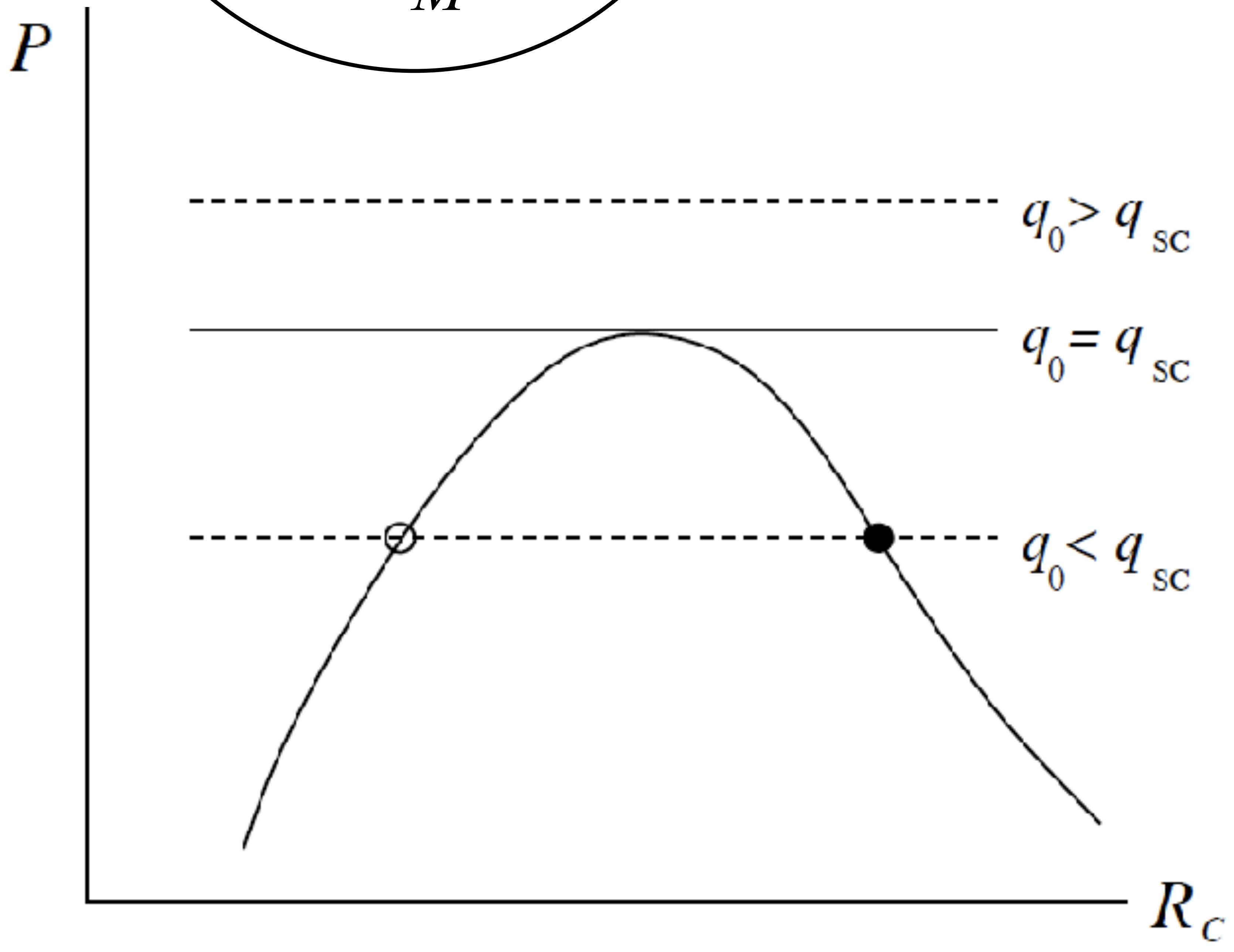
Core-envelope configuration: Inert core surrounded by burning shell




Core surface pressure $P_c = \dfrac{2E_{\rm th} + E_g}{4\pi R_c^3} = c_1 \dfrac{M_c T_c}{R_c^3} - c_2 \dfrac{M_c^2}{R_c^4}$

Envelope base pressure $P_e = c_3 \dfrac{T_e^4}{M^2}$

For mechanical and thermal balance $T_e = T_c$ and $P_e = P_c$




But $P_c$ has a maximum as a function of $R_c$

$$P_{c,\rm max} = c_4 \frac{T_c^4}{M_c^2}$$

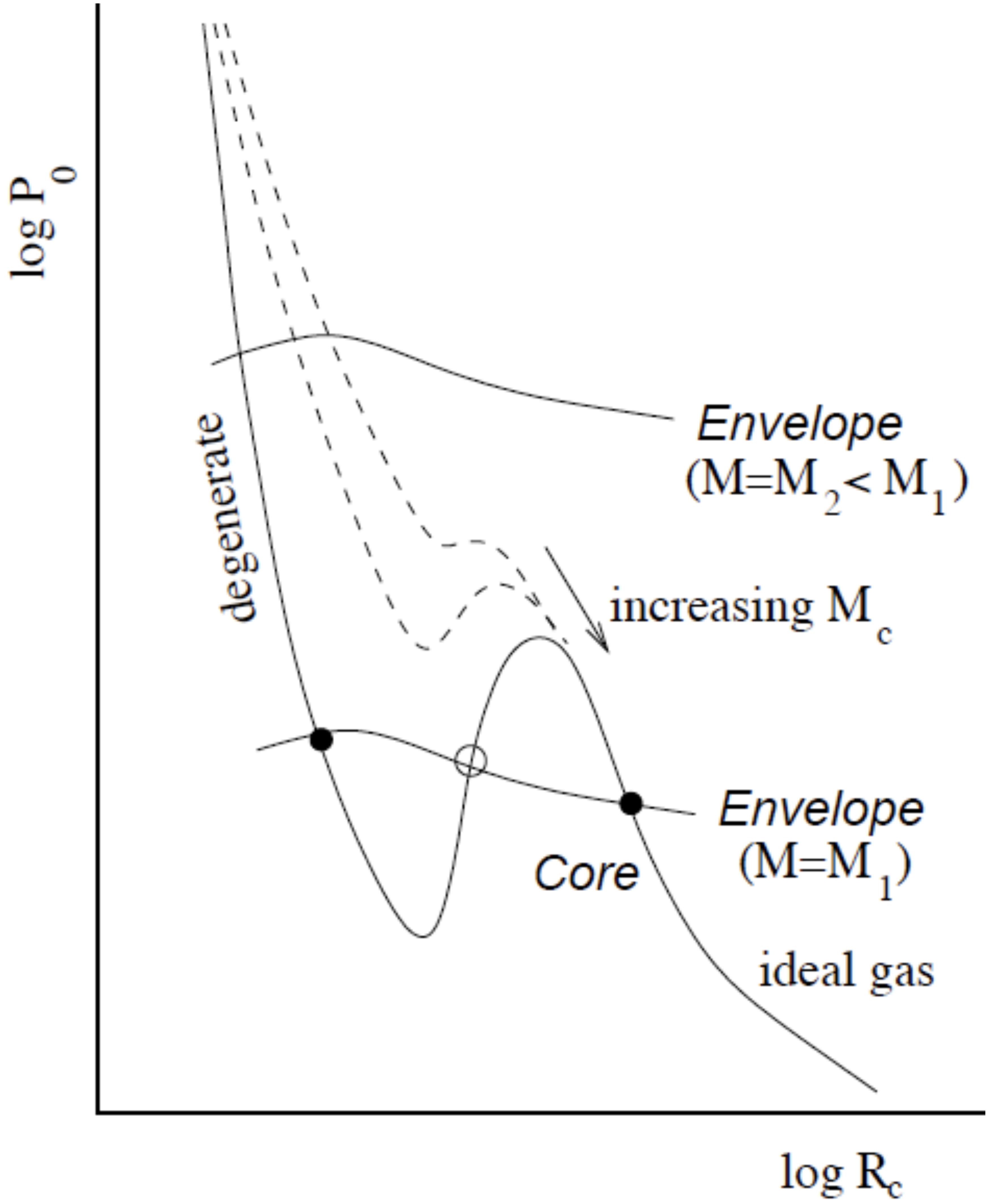
So balance is possible only if $P_e \leq P_{c,\rm max}$

i.e. $q_0 \equiv \dfrac{M_c}{M} \leq \sqrt{\dfrac{c_4}{c_3}} \equiv q_{\rm sc} \approx 0.37 \left(\dfrac{\mu_{\rm env}}{\mu_{\rm core}}\right)^2$

if core mass grows beyond this, then core collapse would occur.

$\Rightarrow$ *contraction until degeneracy support*

# Post Main Sequence Evolution




Low mass star (< 1.4 $M_\odot$):

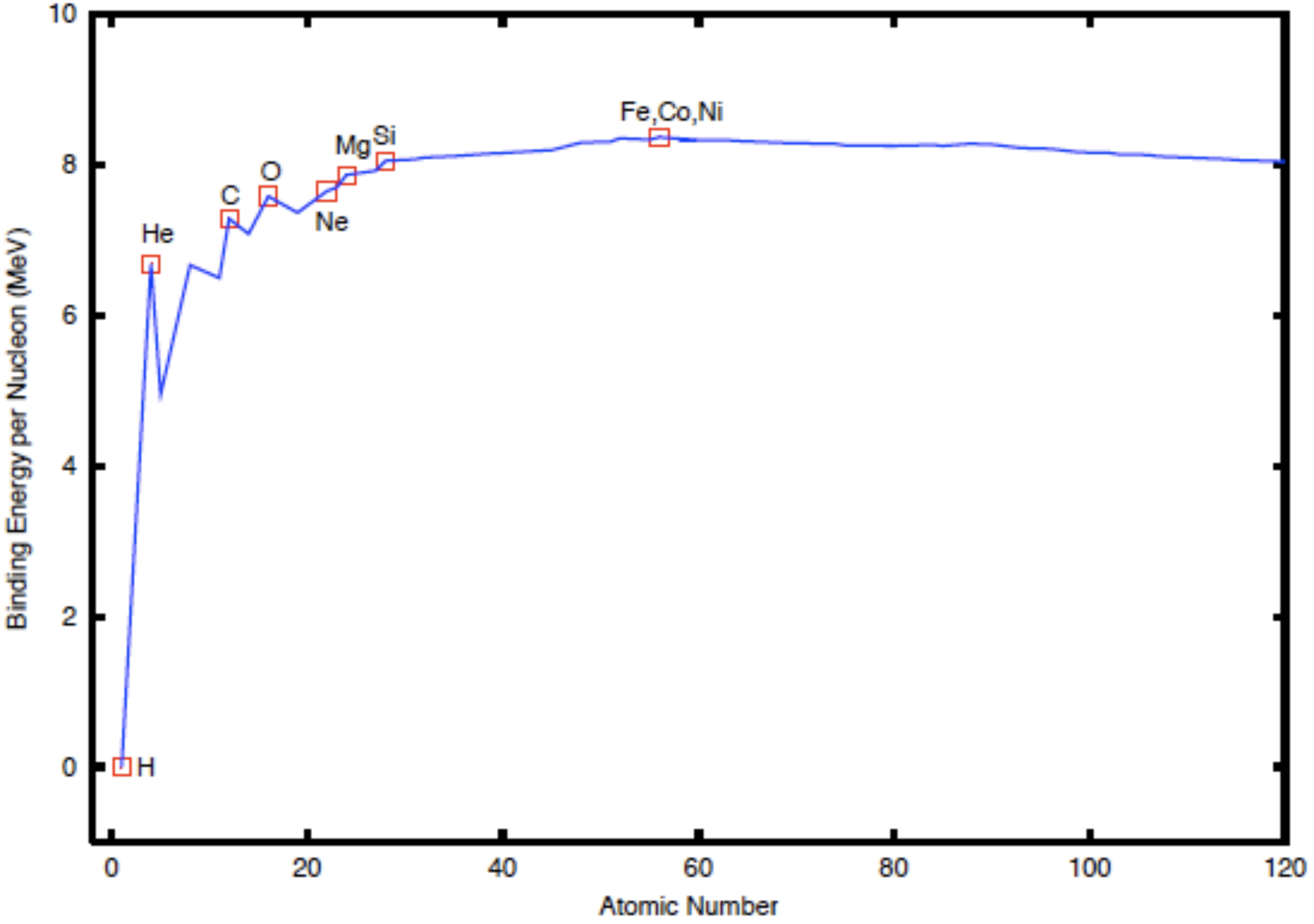
gradual shrinkage of the core to degenerate configuration
He ign at $M_c$ = 0.45 $M_\odot$ : L=100 $L_\odot$
varied mass loss; *horizontal branch*
later AGB ⇒ WD+*planetary nebula*

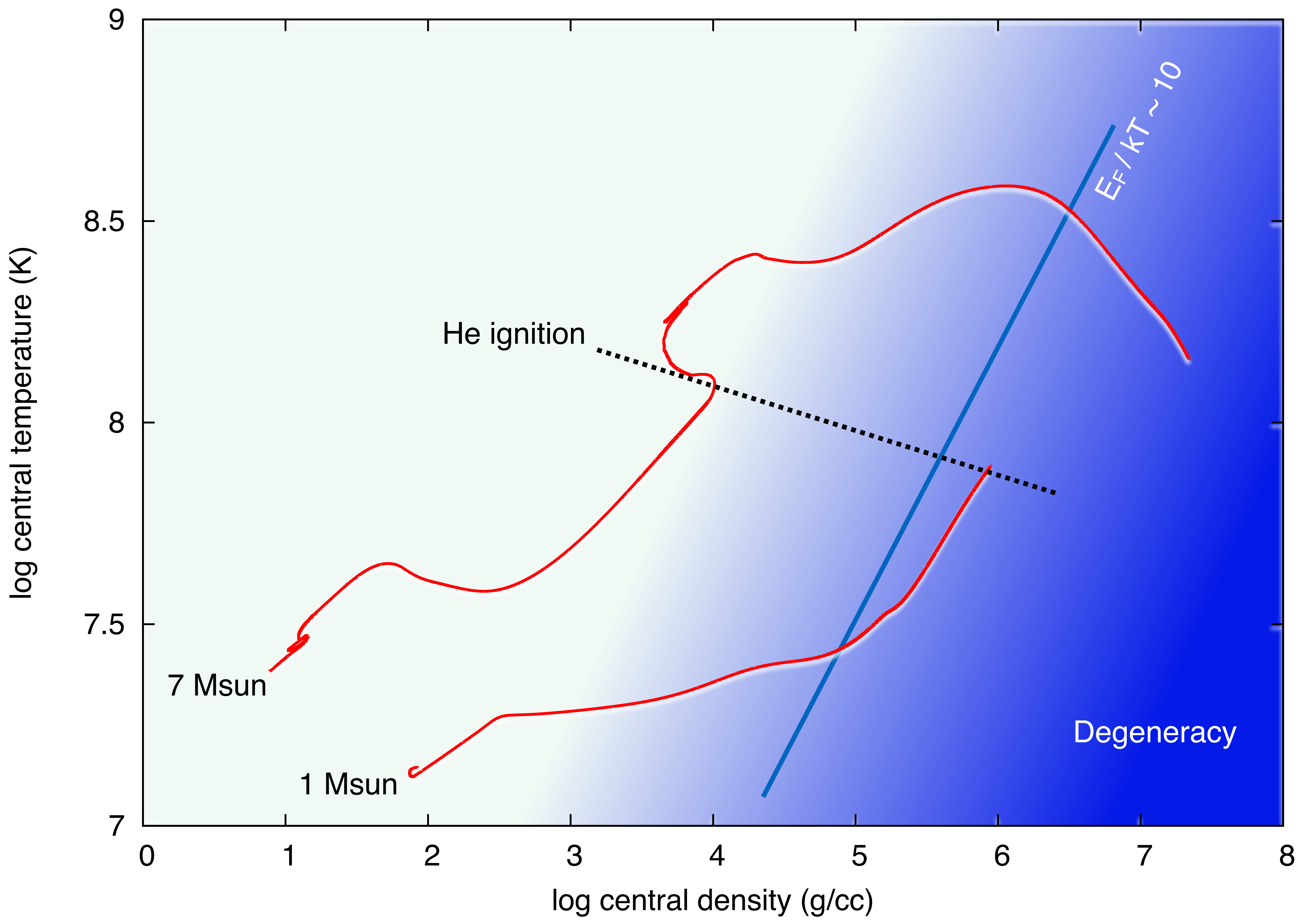
High mass star:

sudden collapse of the core from thermal to degenerate branch
⇒ quick progress to giant

Multiple burning stages
If final degen. support at $M_c < M_{ch}$ ,
WD+PN will result
Else burning all the way to Fe core.
Once $M_{ch}$ exceeded : collapse, neutronization, supernova

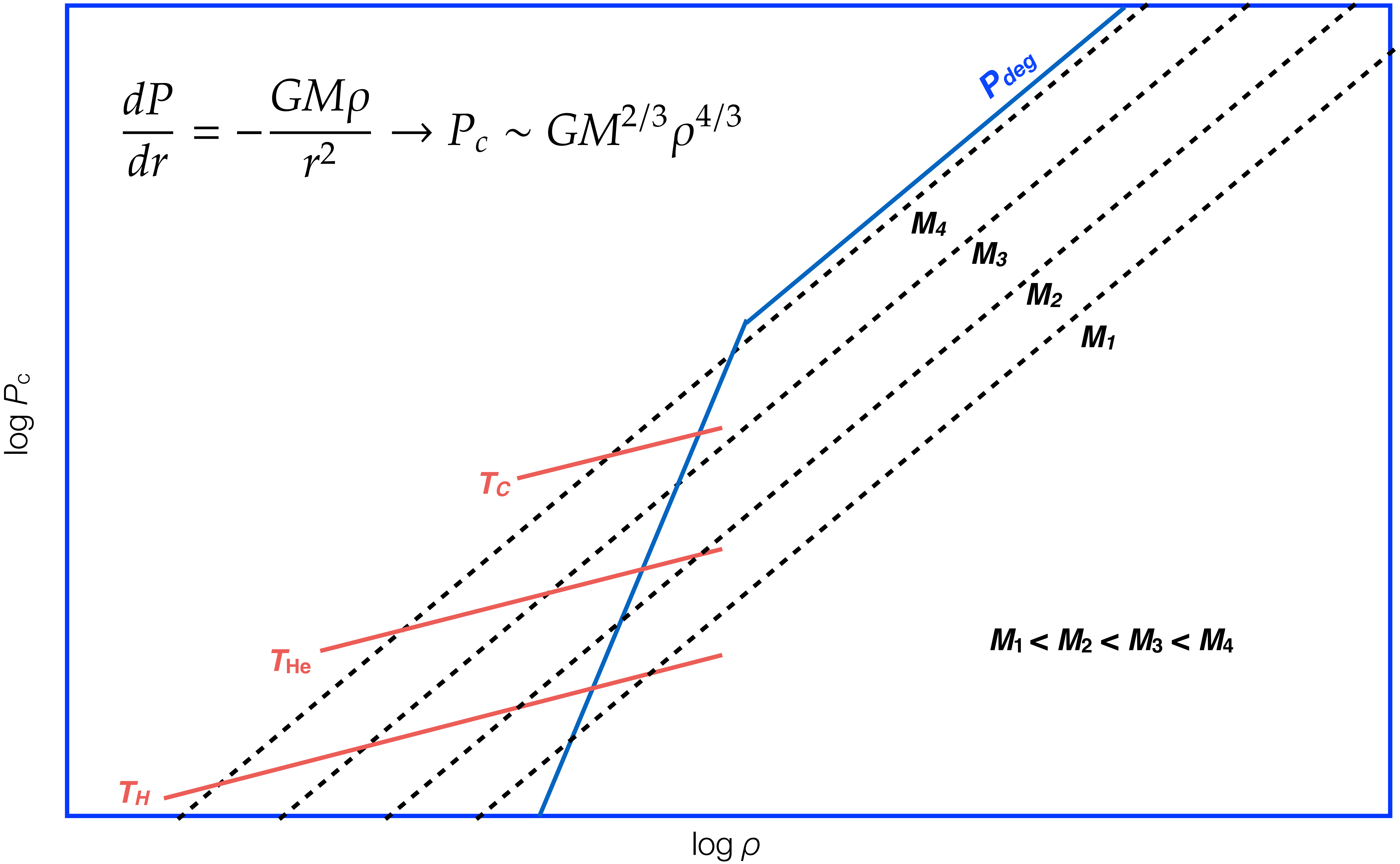
# Nuclear burning stages

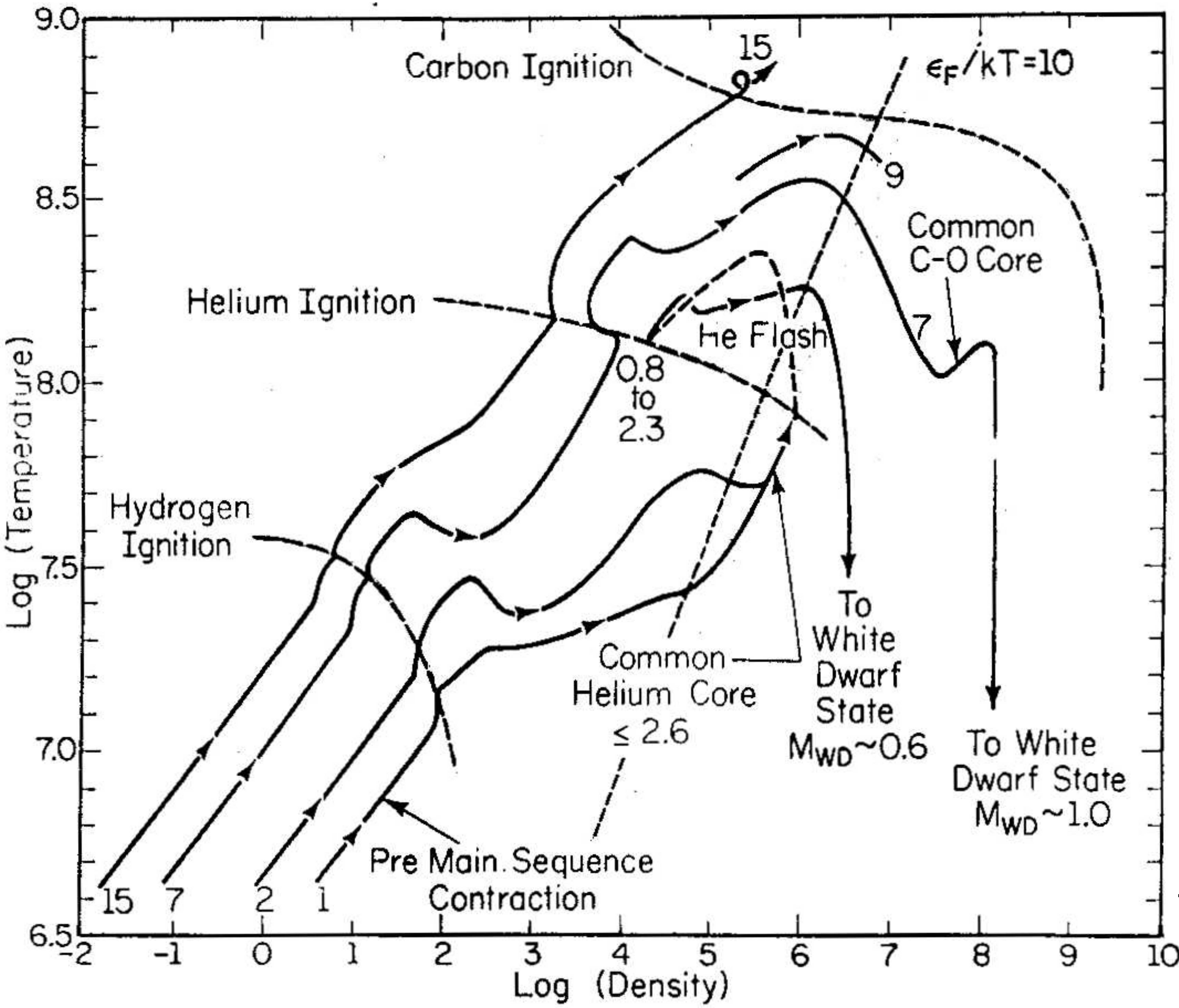




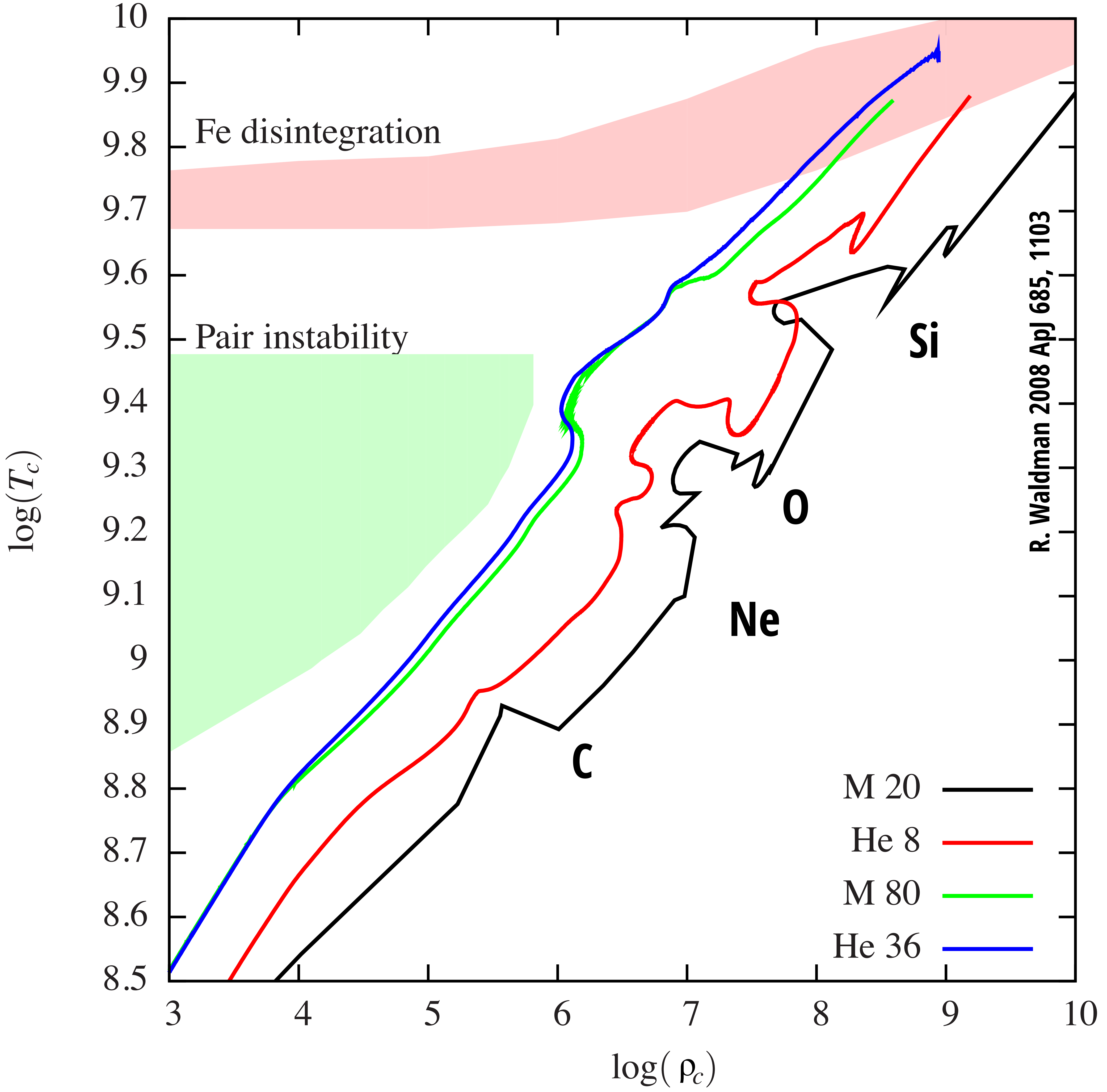
9
8.5
8
7.5
7
0
1
2
3
4
5
6
7
8
log central temperature (K)
log central density (g/cc)
He ignition
7 Msun
1 Msun
EF / kT ~ 10
Degeneracy

# Degeneracy-limited nuclear burning






I. Iben 1985 QJRAS 26, 1

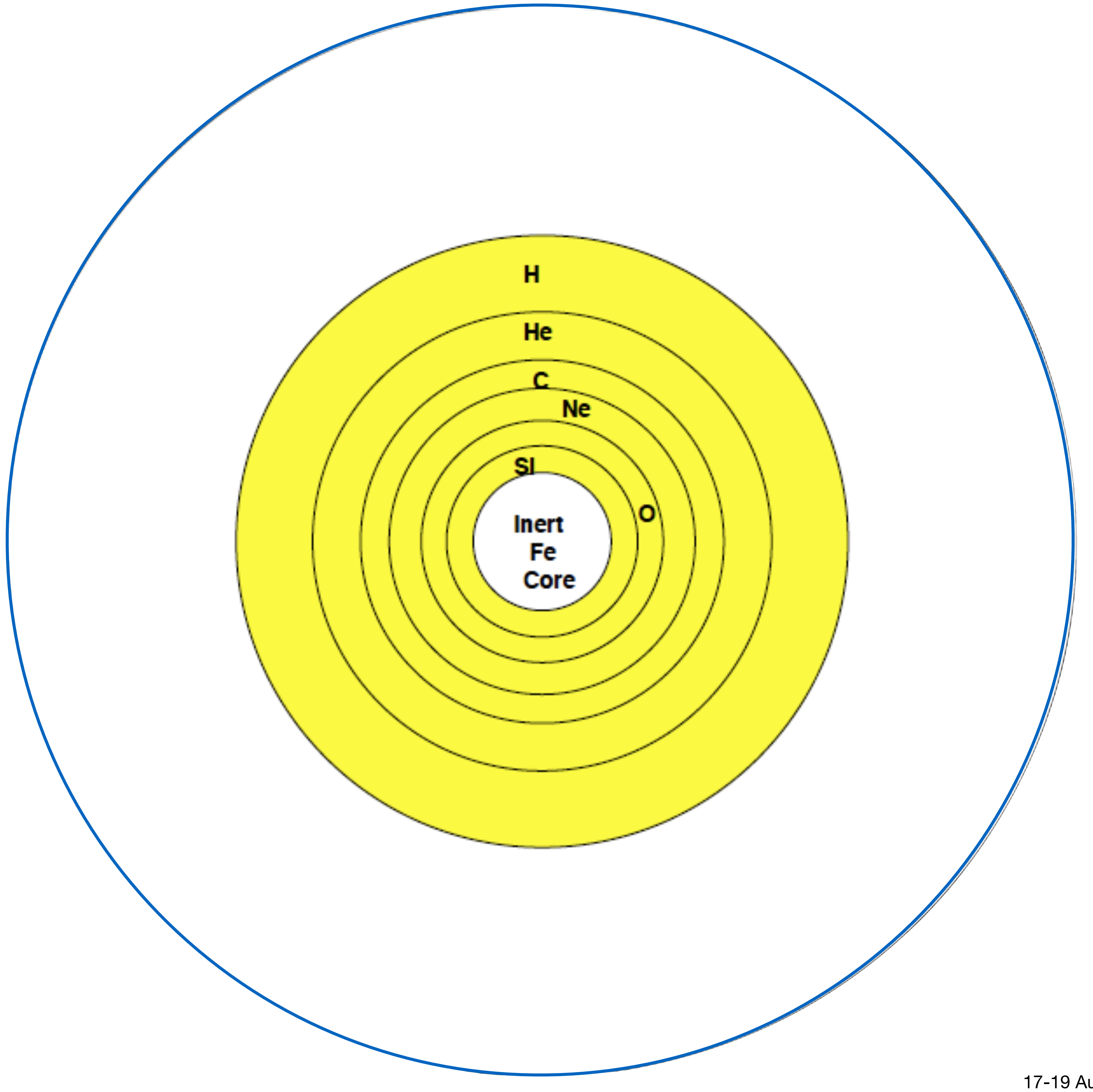



$^{12}C+^{12}C \rightarrow ^{23}Na+p$ ; $^{20}Ne+\alpha$

$^{20}Ne+^{20}Ne+\gamma \rightarrow ^{16}O+^{24}Mg+\gamma$

$^{16}O+^{16}O \rightarrow ^{28}Si+\alpha$

$^{28}Si+^{28}Si \rightarrow ^{56}Fe+2e^+$



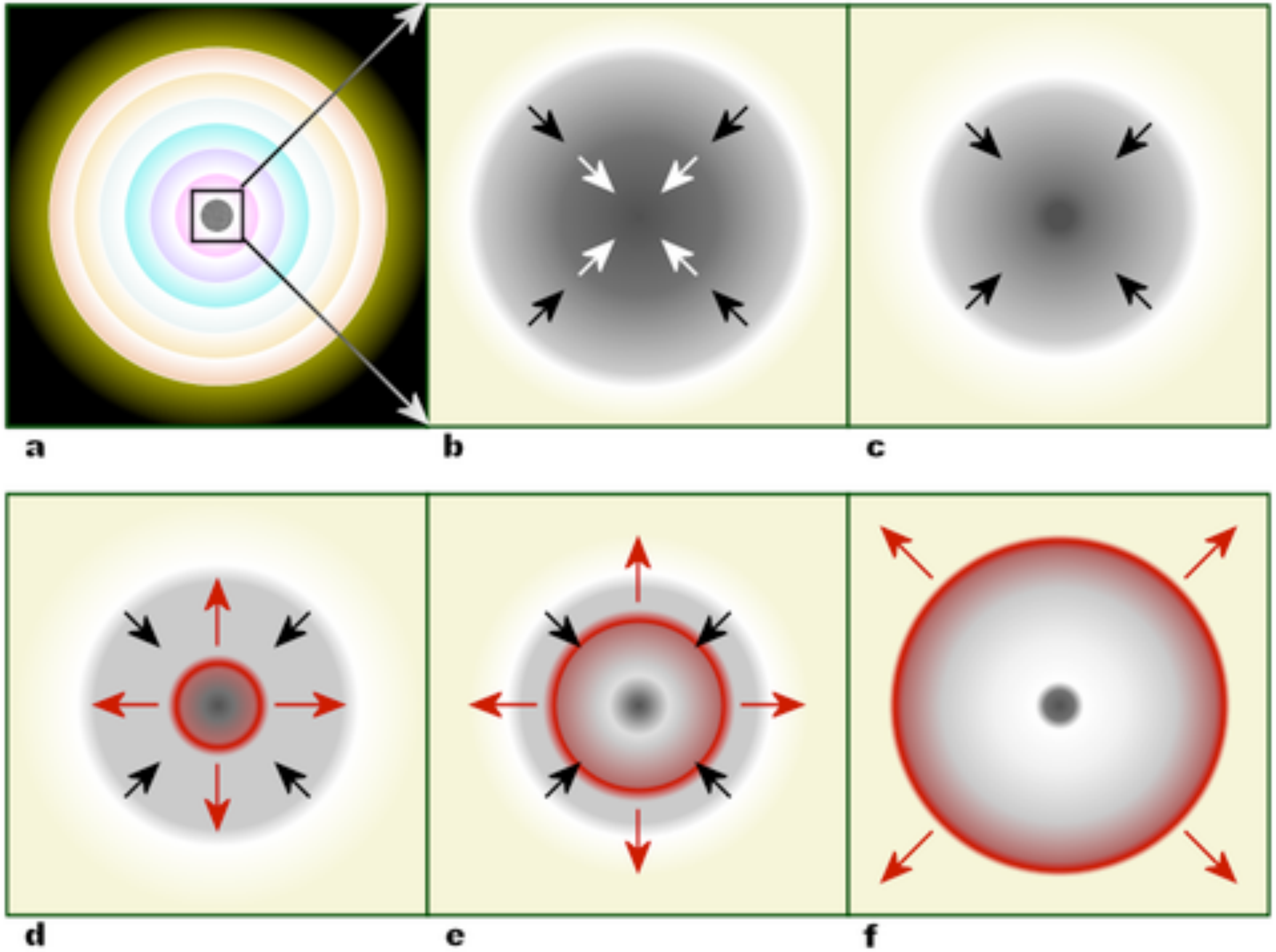

**Core-collapse Supernovae**:

$E_{tot} \sim 10^{53}$ erg; $E_{kin} \sim 10^{51}$ erg; $E_{rad} \sim 10^{49}$ erg

r-process nucleosynthesis

Massive, fast spinning stars ⇒ jets ⇒ GRB

# Core collapse and bounce




Wikipedia

# Key results

**Main Sequence:**

The large majority of stars we see are in their hydrogen burning phase, lifetime $\propto 1/M^3$

Heavier stars have lower density, and higher temperature

**Virial theorem:**

Thermally supported stars get hotter (and more compact) as they lose heat

**Degeneracy Pressure:**

Quantum origin, present even at zero temperature, EoS becomes softer as matter turns relativistic

Decides which nuclear burning stages will occur in a stellar core

Provides long-term support to evolutionary end products - brown dwarfs, white dwarfs, [*neutron stars*]

The heavier the configuration, the smaller it is

**The two limits of Chandrasekhar:**

Degeneracy pressure support not possible above a certain mass - ~1.4 $M_\odot$ for white dwarfs

Thermal support of stellar core not possible if core mass fraction exceeds ~ 15%

**End products:**

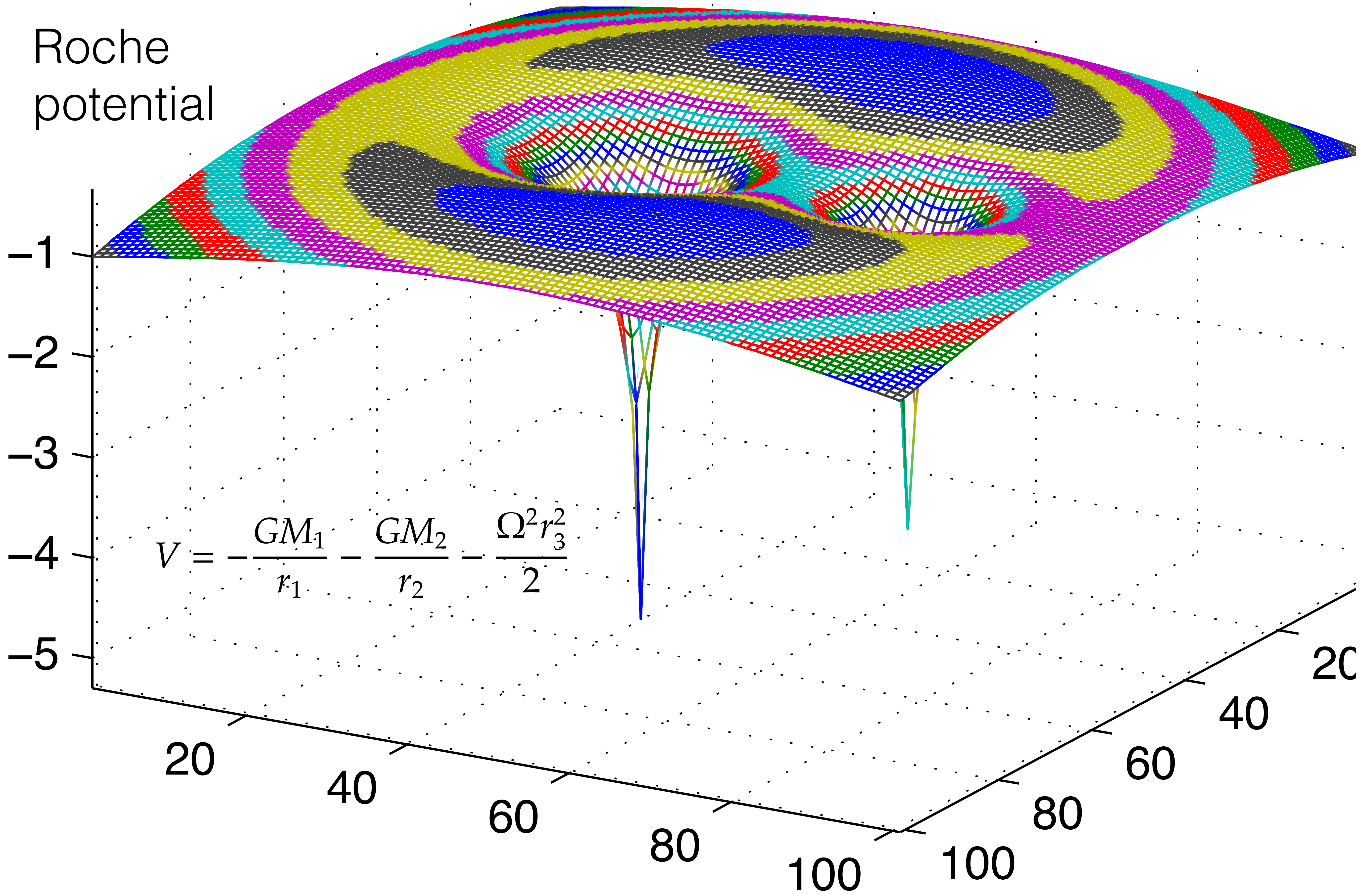
~0.08-0.5 $M_\odot$ : He WD, ~0.5-8 $M_\odot$ : C-O WD, ~8-10 $M_\odot$ : ONeMg WD, ~10-40 $M_\odot$ : NS, >40 $M_\odot$ : BH

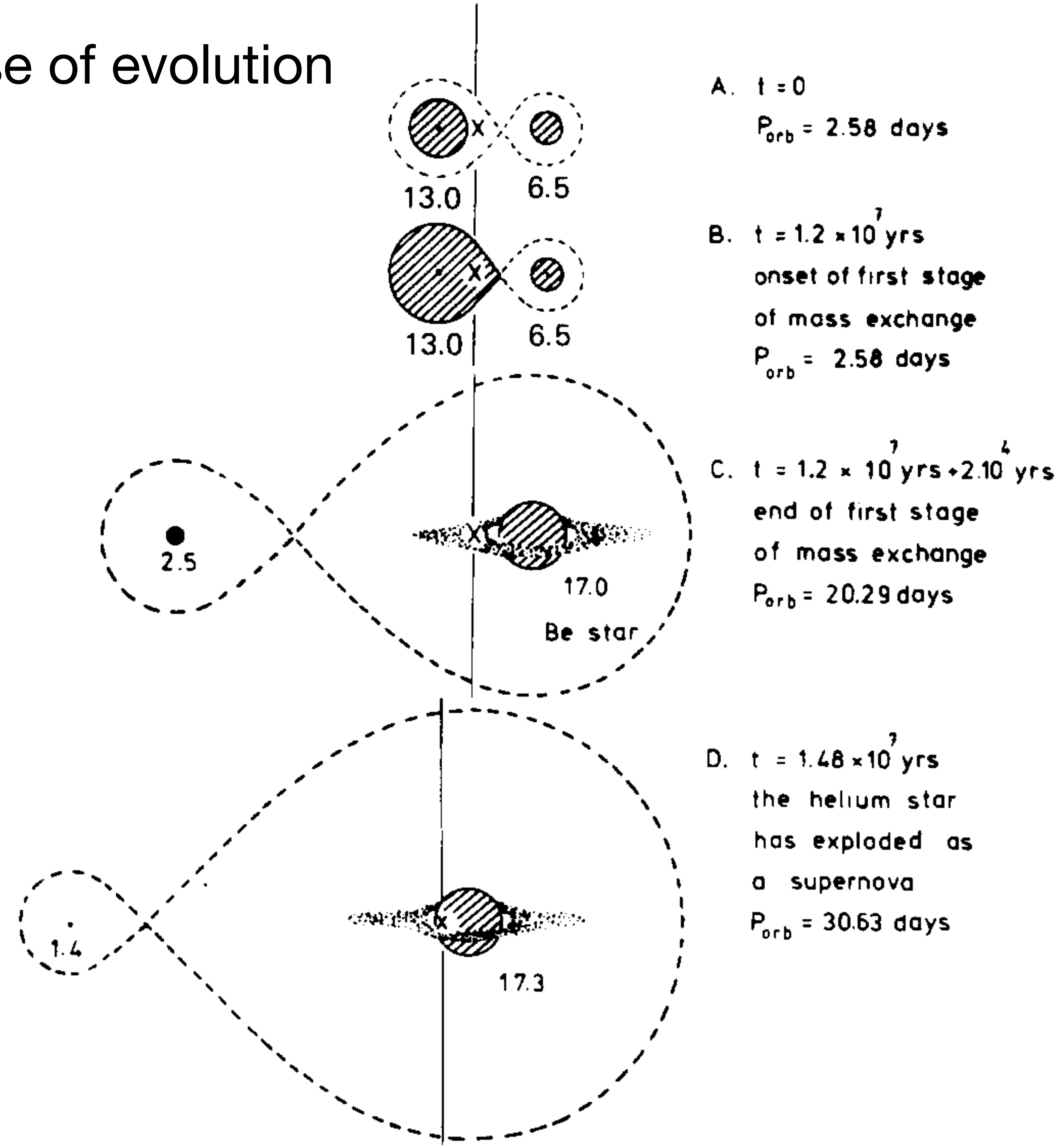
# Stellar Evolution in Close Binary Systems

Mass exchange between components alters the course of evolution

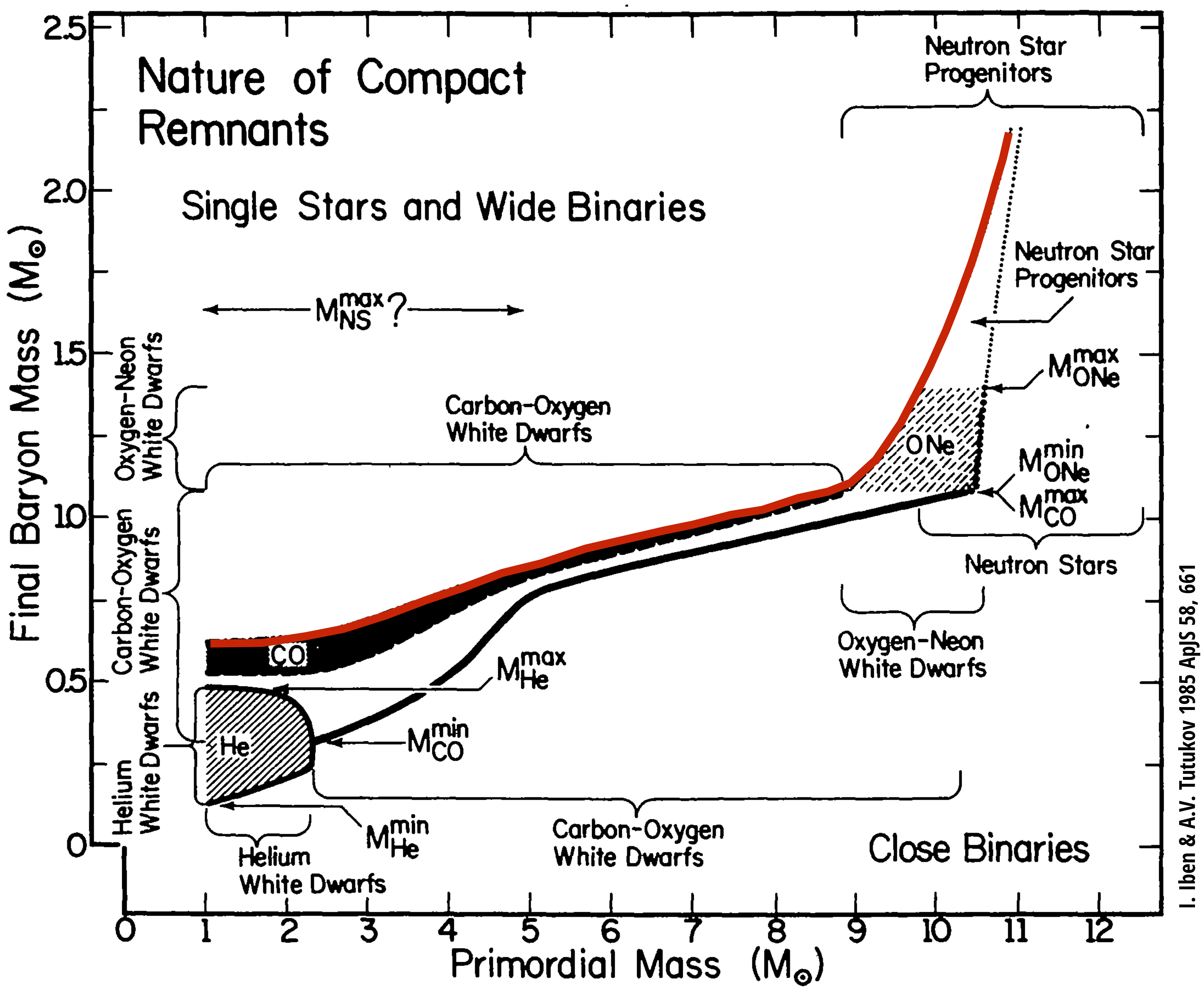
Stellar masses and orbital separation both change

Roche potential

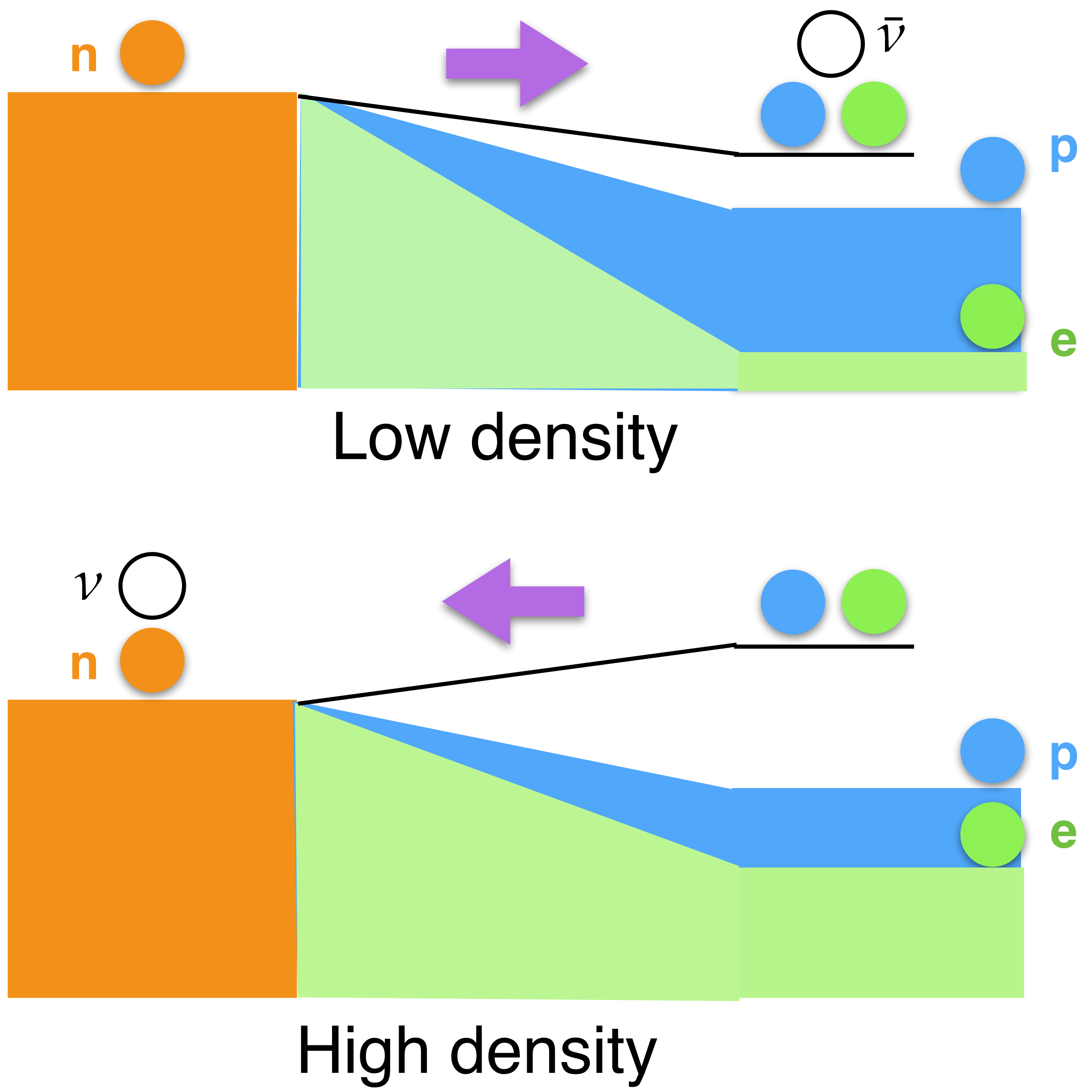



Bhattacharya and van den Heuvel 1991


Nature of Compact Remnants
Single Stars and Wide Binaries
Close Binaries
Final Baryon Mass (M⊙)
Primordial Mass (M⊙)
$M_{NS}^{max}$?
Neutron Star Progenitors
Neutron Star Progenitors
$M_{ONe}^{max}$
$M_{ONe}^{min}$
$M_{CO}^{max}$
Neutron Stars
Oxygen-Neon White Dwarfs
Carbon-Oxygen White Dwarfs
Oxygen-Neon White Dwarfs
Carbon-Oxygen White Dwarfs
Helium White Dwarfs
ONe
CO
He
$M_{He}^{max}$
$M_{CO}^{min}$
$M_{He}^{min}$
Helium White Dwarfs
Carbon-Oxygen White Dwarfs
0
0.5
1.0
1.5
2.0
2.5
0
1
2
3
4
5
6
7
8
9
10
11
12
I. Iben & A.V. Tutukov 1985 ApJS 58, 661

# Inverse β decay: neutronization



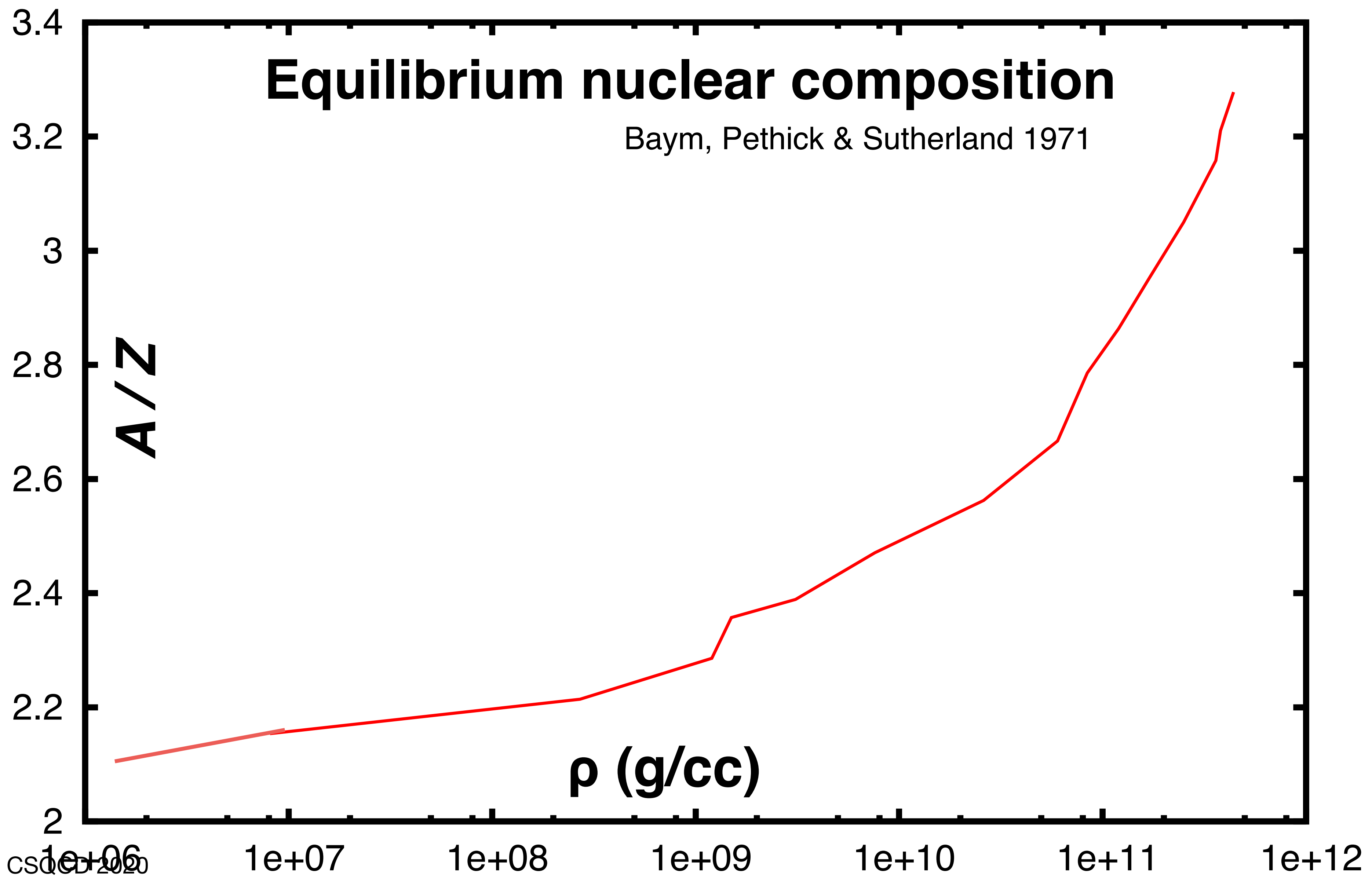

Pycnonuclear Reactions : $X_Z^A + X_Z^A = Y_{2Z}^{2A}$
and electron capture cause transmutations to produce equilibrium composition of Cold Catalysed Nuclear Matter ⇒

At high density inverse β decay depletes electrons and softens electron degenerate Equation of State

Beta equilibrium:

$$\mu_n = \mu_p + \mu_e \quad ; \quad \mu = \sqrt{p_F^2 c^2 + m^2 c^4}$$

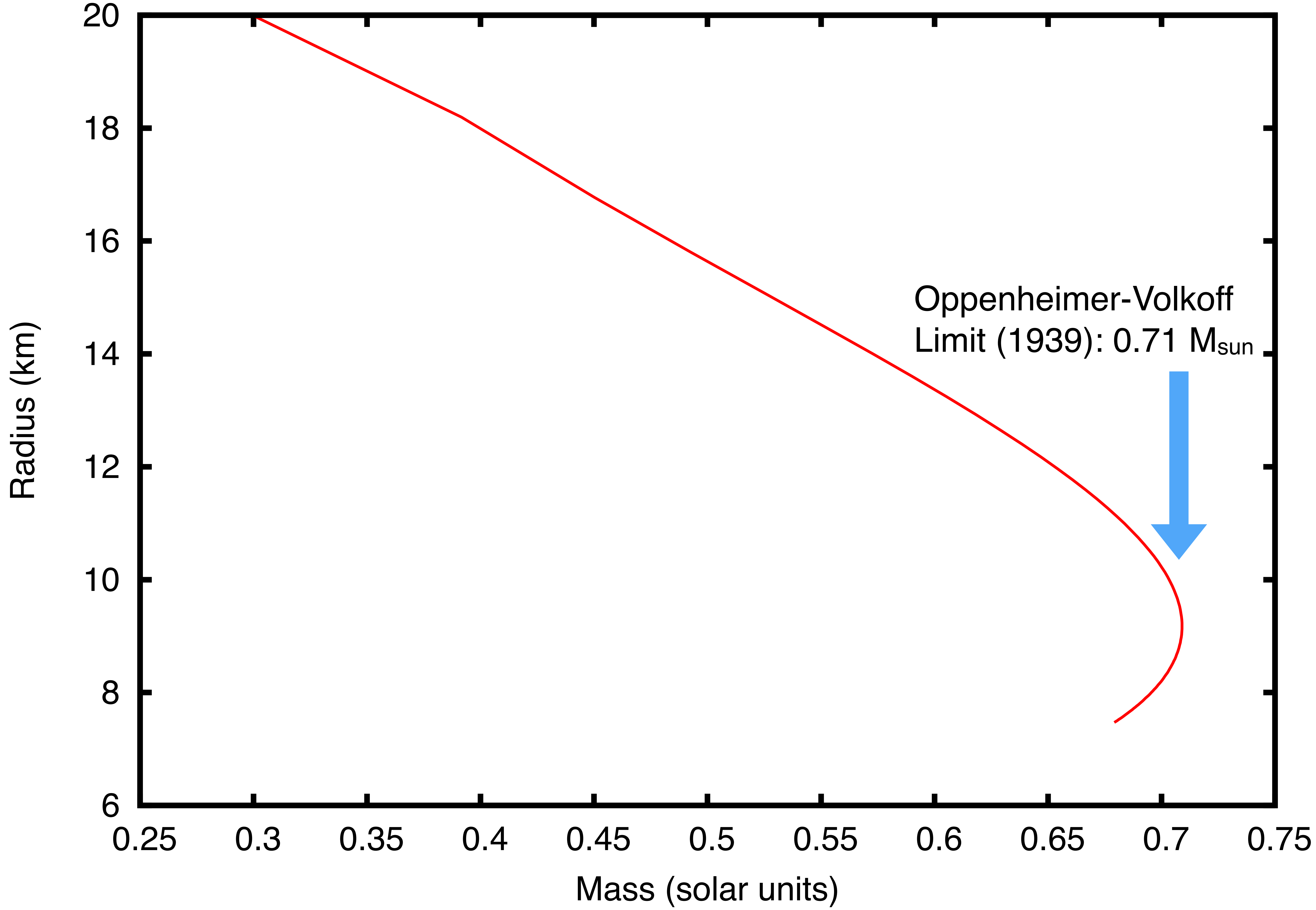
Equilibrium nuclear composition may be derived by combining with nuclear mass formula including shell effects



# Structure supported by pure neutron degeneracy

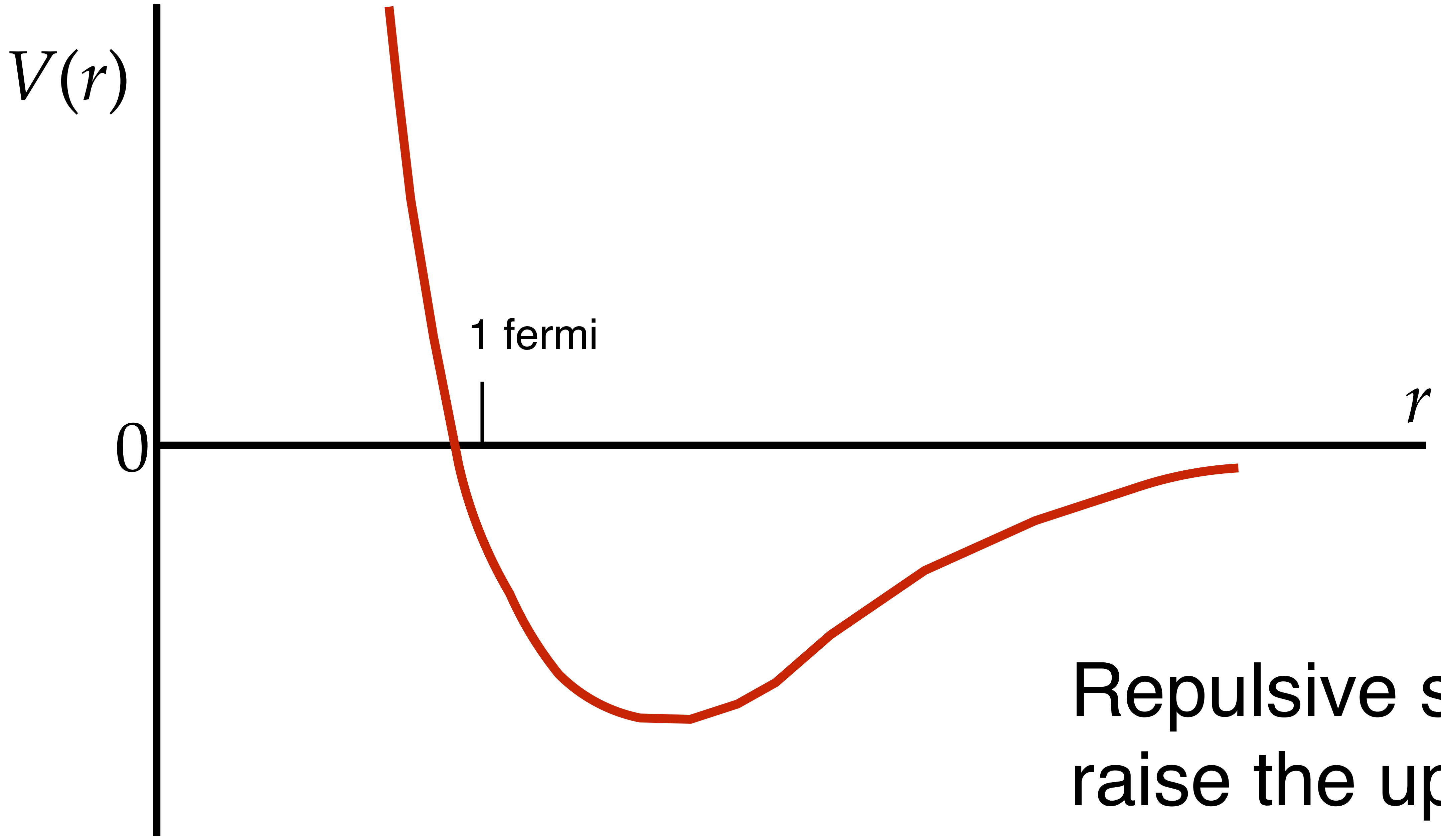
$$-\frac{dP}{dr} = \frac{G(M + 4\pi r^3 P/c^2)(\rho + P/c^2)}{r^2(1 - 2GM/rc^2)}$$ (TOV Equation)

Mass-radius relation for neutron degeneracy alone



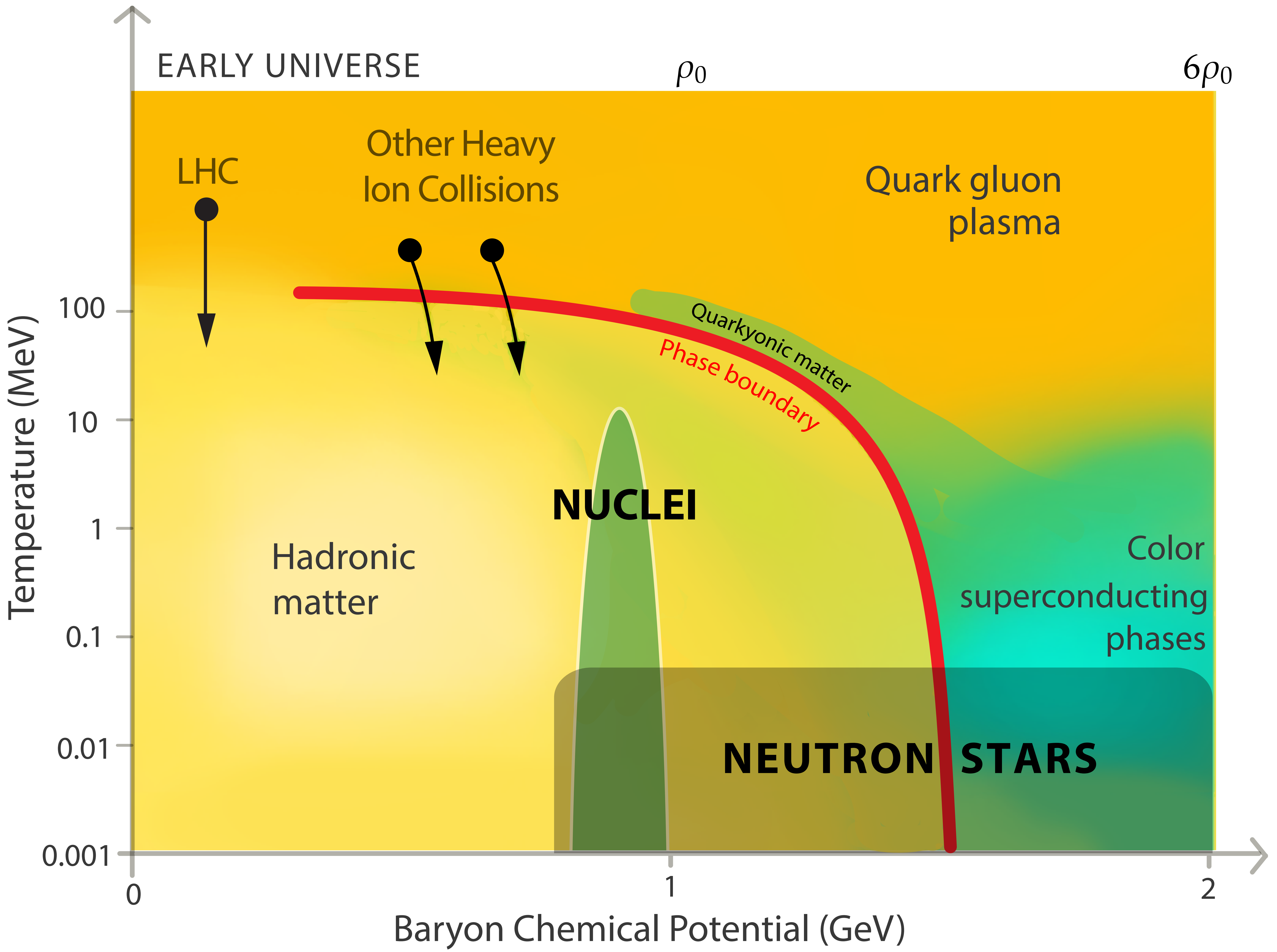
# Importance of nuclear forces

- In realistic neutron stars nucleons are squeezed within inter-particle distance of ~1 fermi
- Strong nuclear forces become important contributor to EoS
- Nature of this force still uncertain, hence uncertainty in EoS




Repulsive strong forces would raise the upper mass limit

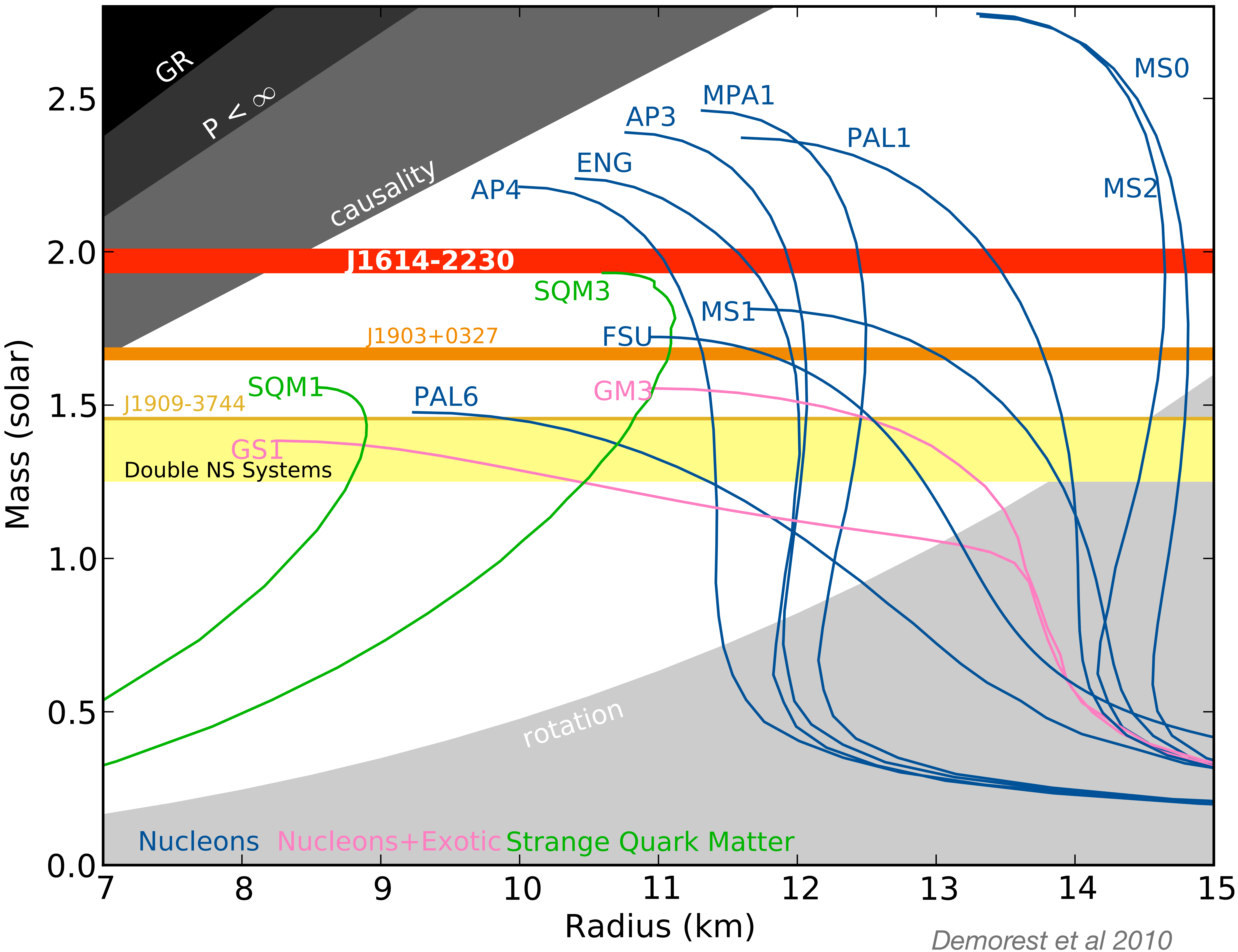
# The regime of Neutron Star matter (cold, dense) cannot be probed in terrestrial experiments



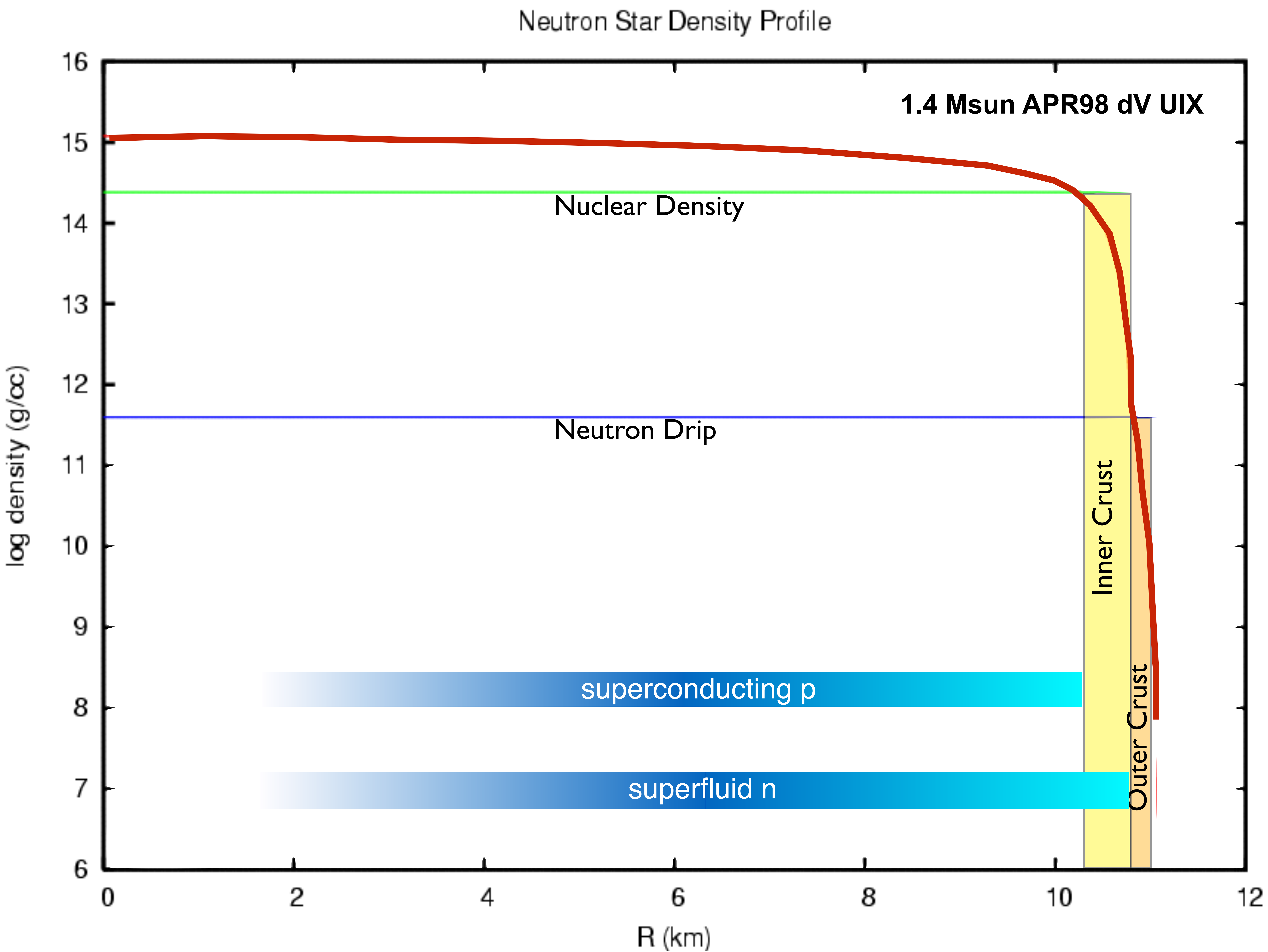

Watts et al 2016

- A large number of different models have been devised to describe matter in the neutron star regime
- It is hoped that astrophysical observations of neutron stars will provide constraints on the nature of strong interaction in the cold, dense matter regime.
- Observables: Mass, Radius, Oscillations, Deformability….

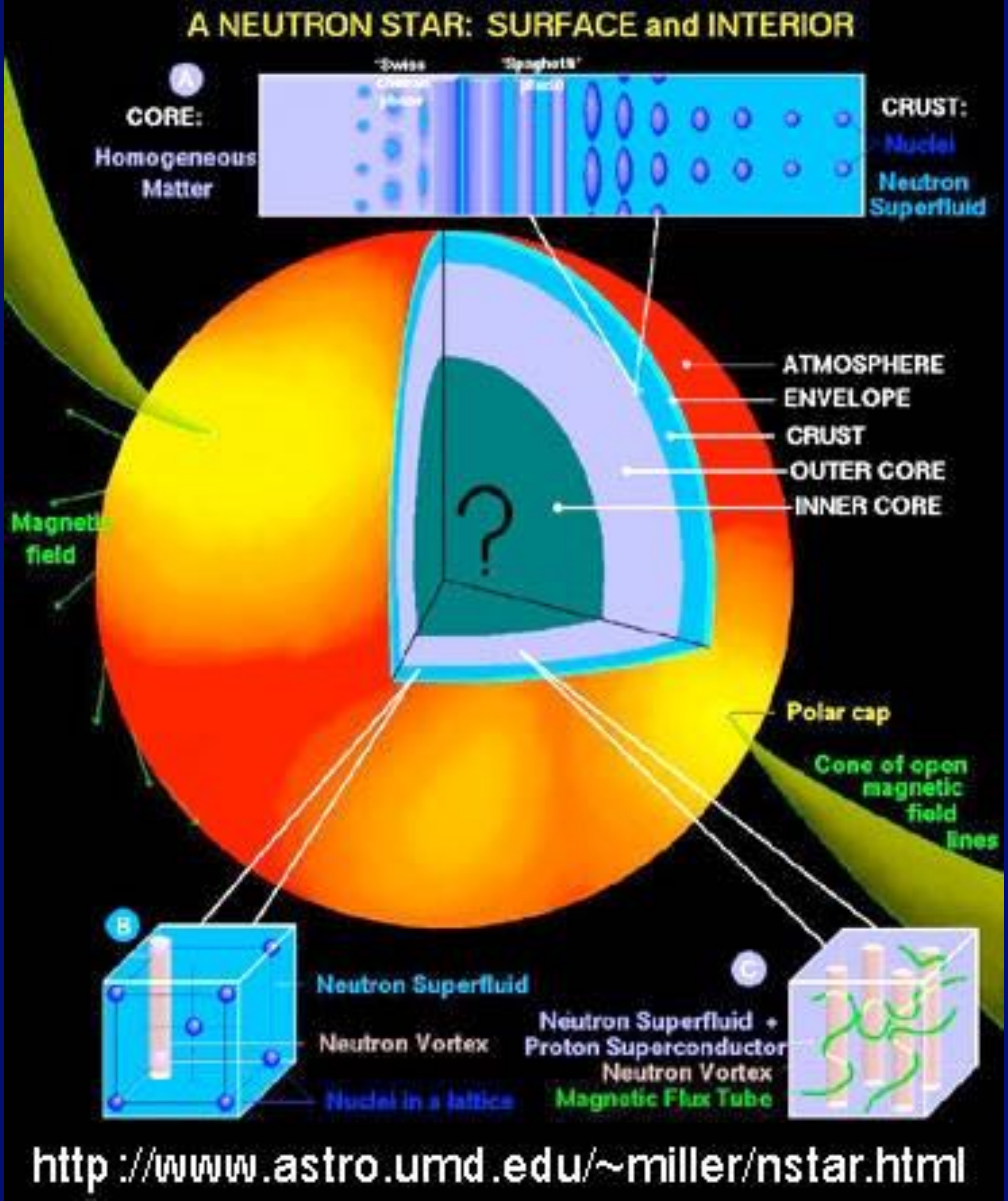
# Neutron Star Mass-Radius Relation




*Demorest et al 2010*



Neutron Star Density Profile
1.4 Msun APR98 dV UIX
Nuclear Density
Neutron Drip
Inner Crust
Outer Crust
superconducting p
superfluid n
log density (g/cc)
R (km)
16
15
14
13
12
11
10
9
8
7
6
0
2
4
6
8
10
12




Fast spin (P ~ ms to s)

Strong Magnetic Field

Highly active magnetosphere

Ultrarelativistic charged particles

Strongly beamed broadband radiation: **Pulsars**

$M \sim 1.5\ M_{\odot}$ , $R \sim 10$km
strong surface gravity
X-rays from accretion: **X-ray Binaries**