Probing the Netrino Mass Hierarchy & the CPV Phase δ in the Foreseeable Future Experiments

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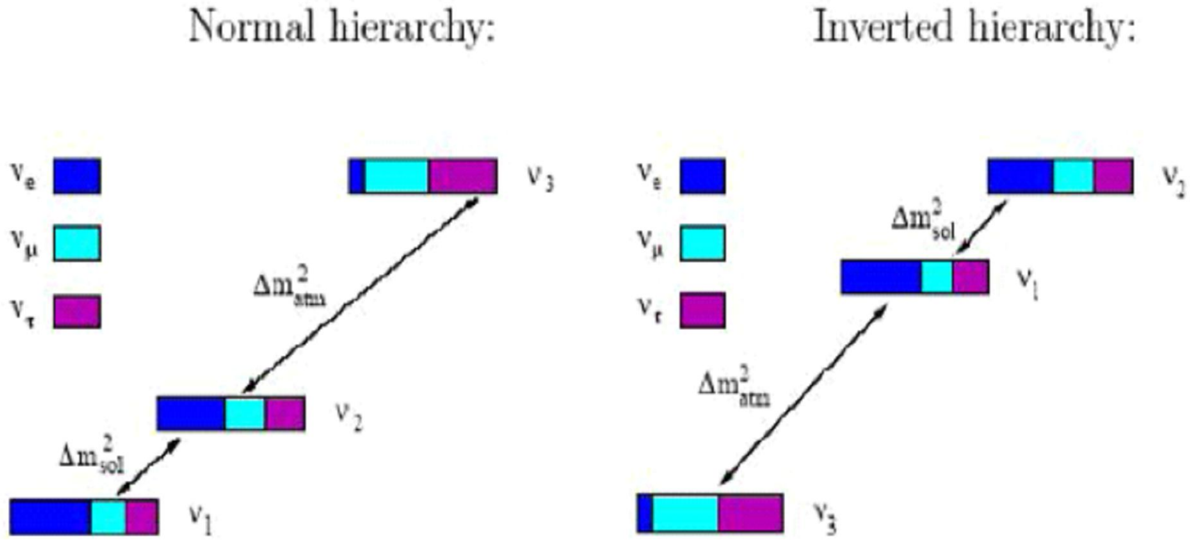
Outline

- Introduction
- Three Neutrino Mixing & Oscillation Formalism
- Determination of θ_{13} from SBL Reactor (Anti)neutrino Expts.
- Implications for Determining Mass Hierarchy & CPV δ in LBL Accelerator Neutrino Expts.
- Implications for Atmospheric Neutrino Expts.

Introduction:



Our Knowledge of Neutrino Mass And Mixing Parameters till 2010

Atmos. & LBL Accl. v Expt: $\Delta m_{atm}{}^2 = \Delta m_{32}{}^2 \approx \Delta m_{31}{}^2 \approx \pm 2.4\text{x}10^{-3}\ \text{eV}^2,\ \sin^2 2\theta_{23} \approx 1.0;$

Sol. & LBL Reactor v Expt: $\Delta m_{sol}{}^2 = \Delta m_{21}{}^2 \approx 7.6\text{x}10^{-5}\ \text{eV}^2,\ \sin^2 \theta_{12} \approx 0.3.$

SBL Reactor v Expt: $\sin^2 2\theta_{13} < 0.15$ at 90% CL,

3 Unknown v Osc Parameters: $\sin^2 2\theta_{13}$, Sign of $\Delta m_{31}{}^2$ & CPV Ph. δ

2010 - 2012: Det of $\sin^2 2\theta_{13} \approx 0.1$ => Det of the Sign of $\Delta m_{31}{}^2$ & δ

Three Nutrino Mixing and Oscillation:

$$\nu_\alpha = \sum U^*_{\alpha i} \nu_i, \alpha = e, \mu, \tau$$

$$U = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$s_{ij} = \sin\theta_{ij}$ & $c_{ij} = \cos\theta_{ij}$

$$\frac{|U_{e2}|^2}{|U_{e1}|^2} = \tan^2\theta_{12}, \frac{|U_{\mu 3}|^2}{|U_{\tau 3}|^2} = \tan^2\theta_{23}, |U_{e3}|^2 = \sin^2\theta_{13}.$$

$$P(\nu_\alpha \to \nu_\beta) = \left| \sum_j U_{\beta j} e^{\frac{-im_j^2 L}{2E_\nu}} U^*_{\alpha j} \right|^2$$

$$P(\nu_\alpha \to \nu_\beta) = \delta_{\alpha\beta} - 4\sum_{i>j} \mathrm{Re}\left[U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j}\right] \sin^2 \Delta_{ij}$$
$$- 2\sum_{i>j} \mathrm{Im}\left[U^*_{\alpha i} U_{\alpha j} U_{\beta i} U^*_{\beta j}\right] \sin 2\Delta_{ij} ,$$

where

$$\Delta_{ij} = \Delta m^2_{ij} L / 4E_\nu .$$

Last term contains the CPV cont. $\propto$ sin δ : vanishes for $\alpha=\beta$ (Disappear. Expt.).

It changes sign in going from $P(\nu_\alpha \to \nu_\beta)$ to $P(\nu_\beta \to \nu_\alpha)$ or to $P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ since $P(\nu_\beta \to \nu_\alpha) = P(\bar{\nu}_\alpha \to \bar{\nu}_\beta)$ by CPT invariance.

$\Delta m^2_{32} = m^2_3 - m^2_2 \Rightarrow \Delta_{32} = \Delta_{31} - \Delta_{21} \rightarrow$ to rewrite $P(\nu_\alpha \to \nu_\beta)$ in terms of Δ_{31} & Δ_{21}

$\alpha = |\Delta m^2_{21}| / |\Delta m^2_{31}| = |\Delta_{21}| / |\Delta_{31}| \cong 0.03 \rightarrow$ to approximate $P(\nu_\alpha \to \nu_\beta)$ in terms of a single Δ

$\Delta_{ij} = 1.27 \Delta m_{ij}^2 L / E_\nu$, with Δm_{ij}^2 in eV^2, L in km (m) & E_ν in GeV (MeV)

Atmos. & LBL Accl. *ν* Expts: $E_\nu \approx GeV, L \approx 10^3\, km \Rightarrow \Delta_{31} \approx 1, \Delta_{21} \approx \alpha$ ≈ 1/30

$$P(\nu_\mu \to \nu_\mu) \cong 1 - (c_{13}^4 \sin^2 2\theta_{23} + s_{23}^2 \sin^2 2\theta_{13}) \sin^2 \Delta_{31}$$

$\cong 1 - \sin^2 2\theta_{23} \sin^2 \Delta_{31}$, neglecting terms of ~ $\cos 2\theta_{23}$ & $\sin^4 \theta_{13}$ in last step

⇒$\sin^2 2\theta_{23}$ & Δm_{31}^2 determined using this formula hold to a very good approx.
⇒These Expts are not good for determining the small angle θ_{13}.

SBL Reactor *ν* Expt: *$E_\nu \approx MeV$, $L \approx 10^3\, m$ =>* $P(\nu_e \to \nu_e) \cong 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{31}$
(2012)

LBL Reactor *ν* Expt (KamLAND): $E_\nu \approx MeV, L \approx 10^5\, m \Rightarrow \Delta_{31} \approx 1/\alpha, \Delta_{21} \approx 1$ =>$\sin^2 \Delta_{31} \approx 1/2$

$$P(\nu_e \to \nu_e) \cong 1 - \tfrac{1}{2} \sin^2 2\theta_{13} - c_{13}^4 \sin^2 2\theta_{12} \sin^2 \Delta_{21}$$

$\cong c_{13}^4 (1 - \sin^2 2\theta_{12} \sin^2 \Delta_{21})$, neglecting $\sin^4 \theta_{13}$ term in the last step

MSW formula for solar matter effect => $P_{solar}(\nu_e \to \nu_e) \cong c_{13}^4 \sin^2 \theta_{12}$ (SK, SNO)

Nonzero θ_{13} => $c_{13} < 1$ => θ_{12} (solar) < θ_{12} (KamLAND) assuming $c_{13} = 1$.

SNO (2010) : $s_{13}^2 = \sin^2 \theta_{13} = 2.0^{+2.1}_{-1.6} \times 10^{-2}$ Fogli et al. (2010) : $\sin^2 \theta_{13} = 2 \pm 1 \times 10^{-2}$

$$P(\nu_\mu \to \nu_e) = \sin^2 2\theta_{13} s_{23}^2 \sin^2 \Delta_{31}$$ ~ 0.1 ~ $(1/3)^2$

$$+\alpha \sin 2\theta_{13} \sin 2\theta_{12} \cos(\Delta_{31}+\delta)\Delta_{31} \sin\Delta_{31} + \alpha^2 \sin^2 2\theta_{12} c_{23}^2 \Delta_{31}^2 .$$

~ (1/30)x(1/3) ~ $(1/30)^2$

Nonzero $P(\nu_\mu \to \nu_e)$ => Nonzero $\sin 2\theta_{13}$; but its value depends on the CPV ph. δ.
With $\sin 2\theta_{13}$ known from SBL Reactor ν expt. => CPV δ from $P(\nu_\mu \to \nu_e)$ at LBL Accl ν expt.
But the CPV term ~ 20% of the leading term => Require $P(\nu_\mu \to \nu_e)$ to ~ 5% to measure δ(~25%)

$P(\bar{\nu}_\mu \to \bar{\nu}_e) \to P(\nu_\mu \to \nu_e)$ => δ → -δ => Their difference $\propto$ sin δ.

Additional complications due to earth matter effect => Opportunity to determine $Sg(\Delta m_{31}^2)$

CC int. of ν_e with electron => $V = \sqrt{2} G_F N_e \cong 7.6 \times 10^{-14} (\frac{\rho}{g/cm^3}) Y_e \,\text{eV}, \quad \rho \cong 3 g/cm^3, Y_e \cong 0.5.$

$$i \frac{d}{dt} |\nu(t)\rangle = H |\nu(t)\rangle, \qquad H \approx \frac{1}{2E_\nu} U diag(0, \Delta m_{21}^2, \Delta m_{31}^2) U^\dagger + diag(V,0,0).$$

$$(\because E = \sqrt{p^2 + m_i^2} \approx p + m_i^2 / 2E)$$

For antineutrinos: $U \to U^*, V \to -V.$

Perturbative diagonalisation of the effective Hamiltonian => $H = U' diag(E_1, E_2, E_3) U'^\dagger$
Akhmedov Johansson, Lindner, Ohlsson, Schwetz (2004),

$$P(\nu_\alpha \to \nu_\beta) = \left|\sum_j U'_{\beta j}\, e^{-iE_j L}\, U'^{*}_{\alpha j}\right|^2$$

$$P(\nu_\mu \to \nu_e) = 4 s_{13}^2 s_{23}^2 \frac{\sin^2 (A-1)\Delta_{31}}{(A-1)^2} + \alpha^2 \sin^2 2\theta_{12}\, c_{23}^2 \frac{\sin^2 A\Delta_{31}}{A^2}$$

$$+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta_{31}+\delta) \frac{\sin A\Delta_{31}}{A} \frac{\sin(A-1)\Delta_{31}}{A-1},$$

where

$$A = \frac{VL}{2\Delta_{31}} = \frac{2E_\nu V}{\Delta m_{31}^2} \cong \pm \frac{E_\nu (GeV)}{10}.$$

Sign of A changes with sign of $\Delta m_{31}{}^2$
and with neutrino → antineutrino

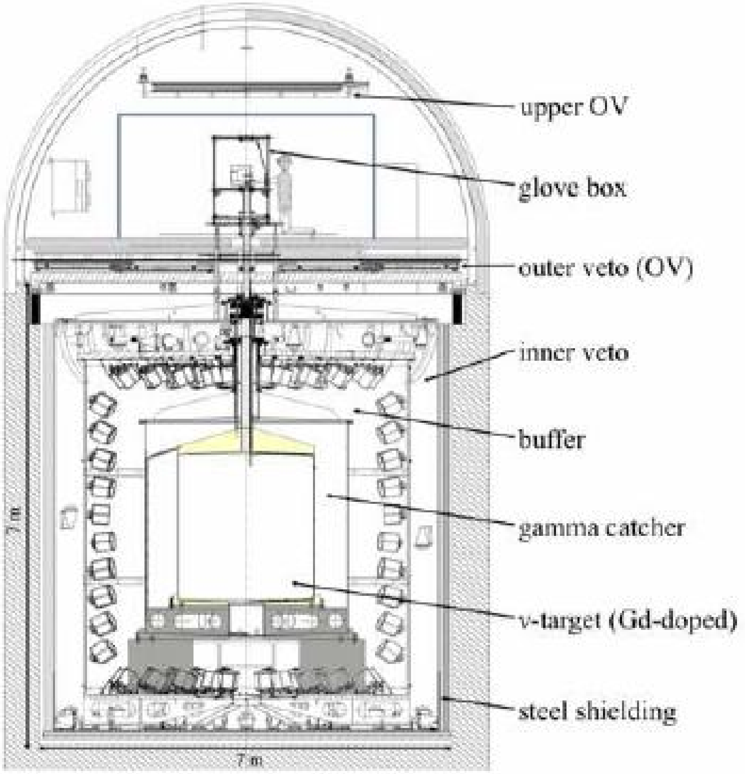
Off-axis Expts. T2K & NOvA have $E_\nu \sim 1$ GeV & $\Delta_{31} \approx \pi/2$ =>Rel. size of matter term ~ 2A

$$P(\nu_\mu \to \nu_e) = 4 s_{13}^2 s_{23}^2 [\sin^2 \Delta_{31} + A(2\sin^2 \Delta_{31} - \Delta_{31} \sin 2\Delta_{31})] + \alpha^2 \sin^2 2\theta_{12}\, c_{23}^2 \Delta_{31}^2$$

$$+ 2\alpha s_{13} \sin 2\theta_{12} \sin 2\theta_{23} \cos(\Delta_{31}+\delta)\Delta_{31}[\sin \Delta_{31} + A(\sin \Delta_{31} - \Delta_{31} \cos \Delta_{31})].$$

Determination of θ_{13} by SBL Reactor (Anti)neutrino Expts:

Double Chooz: Target containing 10 m^3 of Gd doped Liquid scintillator placed at L = 1050 m from 2x4.25 GW Chooz Reactor complex in France



$\bar{\nu}_e + p \to e^+ + n$, $n + \mathrm{Gd} \to \gamma$ (~8 MeV)

$$E_{\mathrm{prompt}} = E_\nu + m_p - m_n + m_e \approx E_\nu - 0.8\ \mathrm{MeV}$$

PRL2012: 4121 events/ 4344 ± 165 (pred.)

$$R = 0.944 \pm 0.016\ (\mathrm{stat}) \pm 0.040\ (\mathrm{syst}),$$

A similar detector to be installed near the reactor to measure antineutrino flux and reduce syst. err.

+ Distortion of E_{prompt} spectrum => $\sin^2 2\theta_{13} = 0.086 \pm 0.041\ (\mathrm{stat}) \pm 0.034\ (\mathrm{syst}).$

ICHEP2012: ~ 8000 events
=> (~ 3σ signal)

$$\sin^2 2\theta_{13} = 0.109 \pm 0.030\ (\mathrm{stat}) \pm 0.025\ (\mathrm{syst})$$

RENO: Two identical near and far detectors placed at L = 294 m & 1383 m from the centre of an array of 6x2.8 GW Reactors in S. Korea.

Each detector contains 16 tons (18.6 m^3) of Gd-doped liquid scintillator target.

=> Flux x target size = 2x2 times larger than Double Chooz => 4 times larger signal

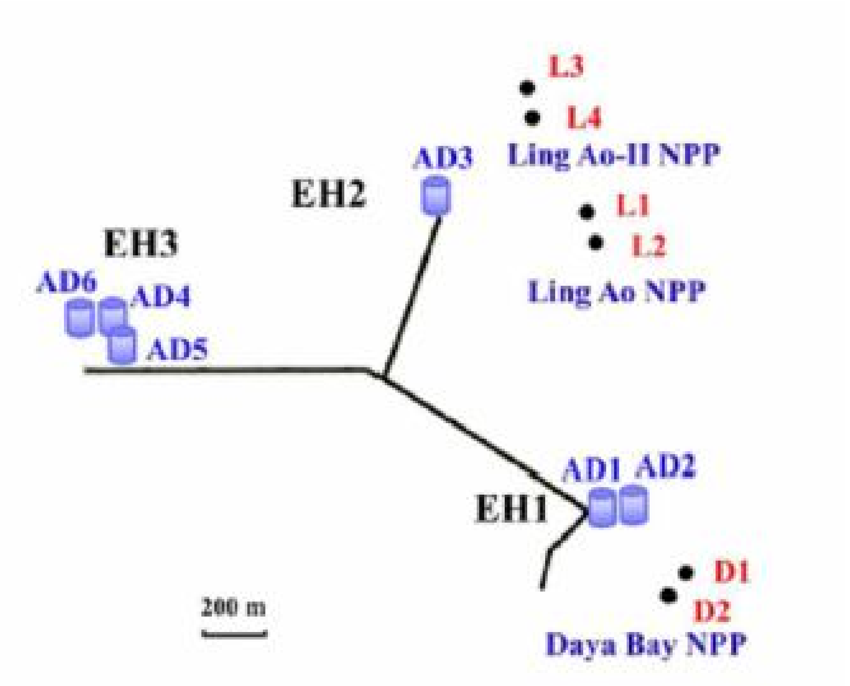
PRL2012: Ratio of observed to predicted # of events in the far detector (~ 5σ signal)

$R = 0.920 \pm 0.009\ (\text{stat}) \pm 0.014\ (\text{syst})$ => $\sin^2 2\theta_{13} = 0.113 \pm 0.013\ (\text{stat}) \pm 0.019\ (\text{syst})$

Daya Bay: 3 near and 3 far detectors detecting the antineutrinos from an array of 6x2.9 GW Reactors in China. 2 more to be added to the near and far Experimental Halls EH1 and EH3.

Each detector contains 20 tons of Gd-doped Liquid scintillator target.

⇒Target and the resulting signal size 4 (16) Times Larger than RENO (DC) !!!!



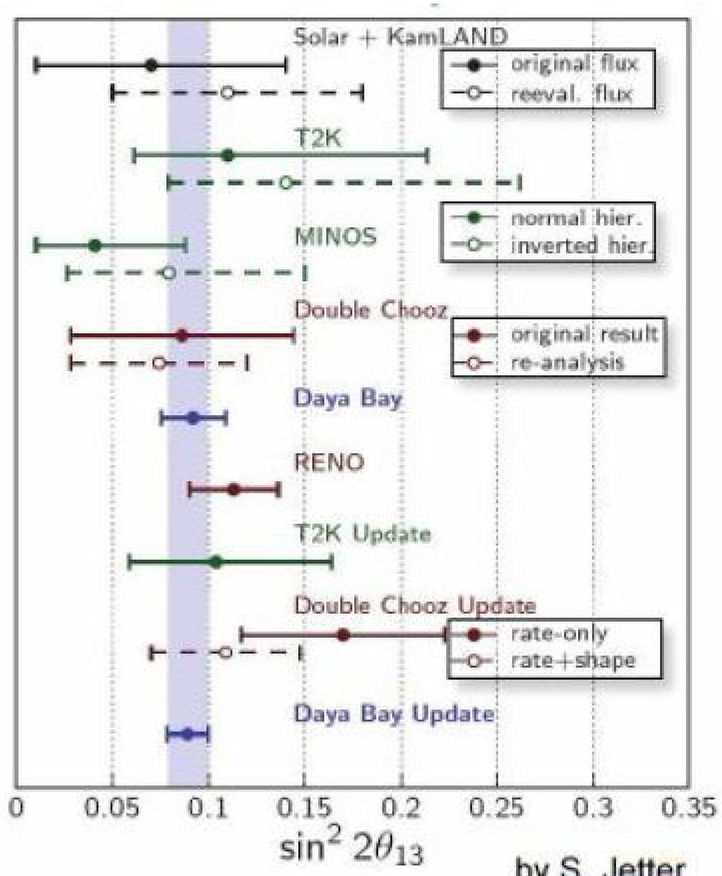
PRL2012:The Ratio of observed to predicted # of events from only 55 days data (5.2σ sig)

$R = 0.94 \pm 0.011\ (\mathrm{stat}) \pm 0.004\ (\mathrm{syst})$ => $\sin^2 2\theta_{13} = 0.092 \pm 0.016\ (\mathrm{stat}) \pm 0.005\ (\mathrm{syst})$

ICHEP2012: 140 days Daya Bay data => ~ 8σ sig.

$R = 0.944 \pm 0.007\ (\mathrm{stat}) \pm 0.003\ (\mathrm{syst})$ => $\sin^2 2\theta_{13} = 0.089 \pm 0.010\ (\mathrm{stat}) \pm 0.005\ (\mathrm{syst})$

Daya Bay => 5% precision in 3 yrs.



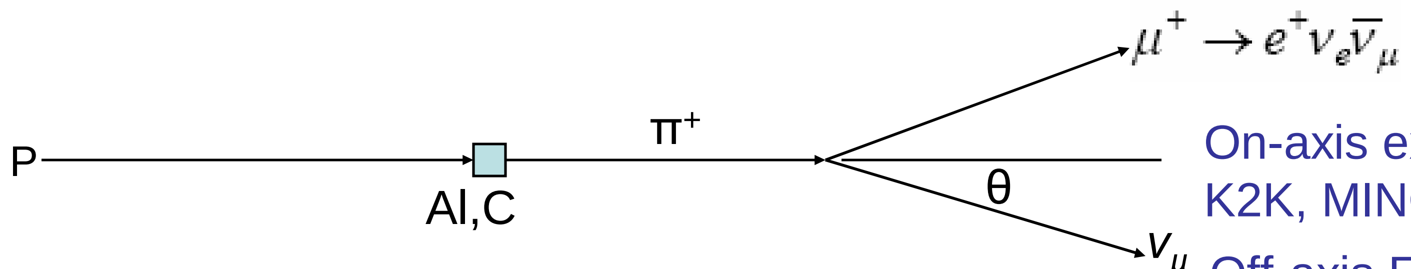
by S. Jetter

Weighted average of the final Reno, Double Chooz & Daya Bay Results give

$\mathrm{Sin}^2\, 2\theta_{13} = 0.10 \pm 0.01$

Fig 3. Global summary of the consistent evolution of a nonzero θ_{13} signal, culminating in the latest Daya Bay result [21].

Determination of Mass Hierarchy and CPV Ph δ in LBL Accl. *ν* Expts.



On-axis expts.
K2K, MINOS

Off-axis Expts
T2K, NOvA
↓

$\nu_\mu \rightarrow \nu_e$ Appearance expts. are off-axis

$$E_\nu \cong \left(1-\frac{m_\mu^2}{m_\pi^2}\right)\frac{E_\pi}{1+\gamma^2\theta^2} \cong \frac{1}{2}\frac{E_\pi}{1+\gamma^2\theta^2}, \gamma = E_\pi / m_\pi.$$

On-axis (θ = 0) beam => E_ν (≈ E_π /2) large & large tail

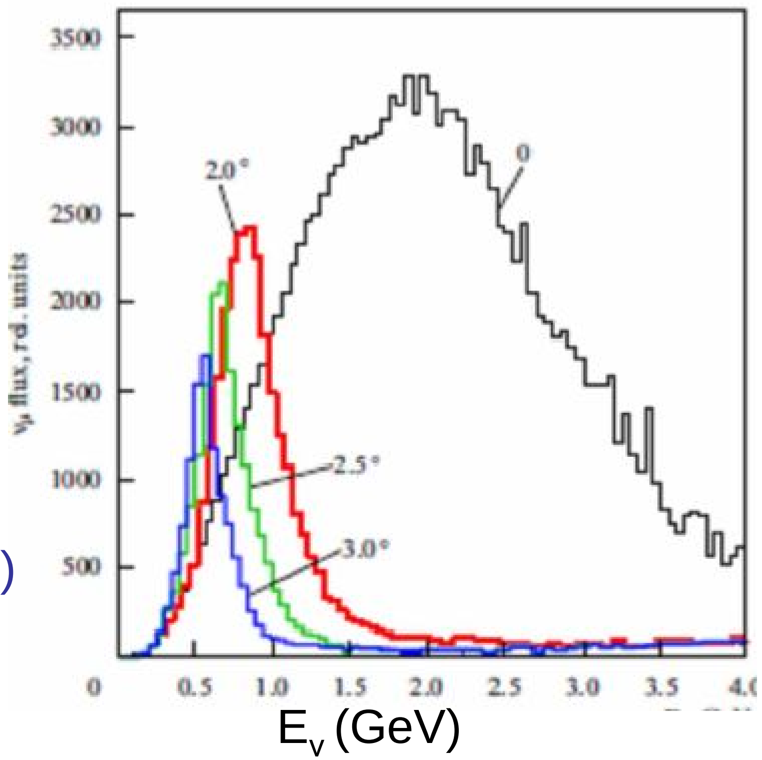
$$\nu_\mu p \rightarrow \nu_\mu p \pi^0,\ \pi^0 \rightarrow \gamma\gamma,\ \gamma \rightarrow e^+ e^-$$

$$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

2 serious Bg from large E_ν tail.

$$\frac{dE_\nu}{dE_\pi} \cong \left(1-\frac{m_\mu^2}{m_\pi^2}\right)\frac{1-\gamma^2\theta^2}{(1+\gamma^2\theta^2)^2}$$

Suppressed with Off-axis beam (QMC)



$$\theta = 1/\gamma = m_\pi/E_\pi \ \Rightarrow E_\nu \cong \left(1-\frac{m_\mu^2}{m_\pi^2}\right)\frac{m_\pi}{2\theta} \cong \frac{30 MeV}{\theta}$$

Peak at $E_\nu \approx$ 2 GeV => $E_\pi \approx$ 4 GeV => $\theta = 0.035 = 2^\circ$, $E_\nu \approx 0.85$ GeV, $\theta = 2.5^\circ$ => $E_\nu \approx 0.68$ GeV

QMC

(Osc. Max)

T2K: J-PARC ν_μ *(0.7 MW)* $\xrightarrow[\text{L = 295 km, } E_\nu \approx 0.68 \text{ GeV}]{}$ SK (50 kt WCD)

$L/E_\nu \approx 450$ km/GeV => $|\Delta_{31}| \approx 80^\circ$

Osc. Max

MINOS(10.7×10^{20} POT): ICHEP2012
=> 88 ν_e events (BG 69 ± 9) =>2σ sig

$$\sin^2 2\theta_{13} = 0.06\ (0.10) \text{ for +ve (-ve) } \Delta_{31}{}^2$$

Detection via QE proc. ν_e (ν_μ) p→ e (μ) n
ICHEP2012
(3×10^{20} POT) =>11 ν_e events (BG 3.2 ± 0.4)
=> 3.2σ signal for nonzero θ_{13}

$$\sin^2 2\theta_{13} = 0.094^{+0.053}_{-0.040}\left(0.116^{+0.063}_{-0.049}\right) \text{ for +ve (-ve) } \Delta m_{31}{}^2$$

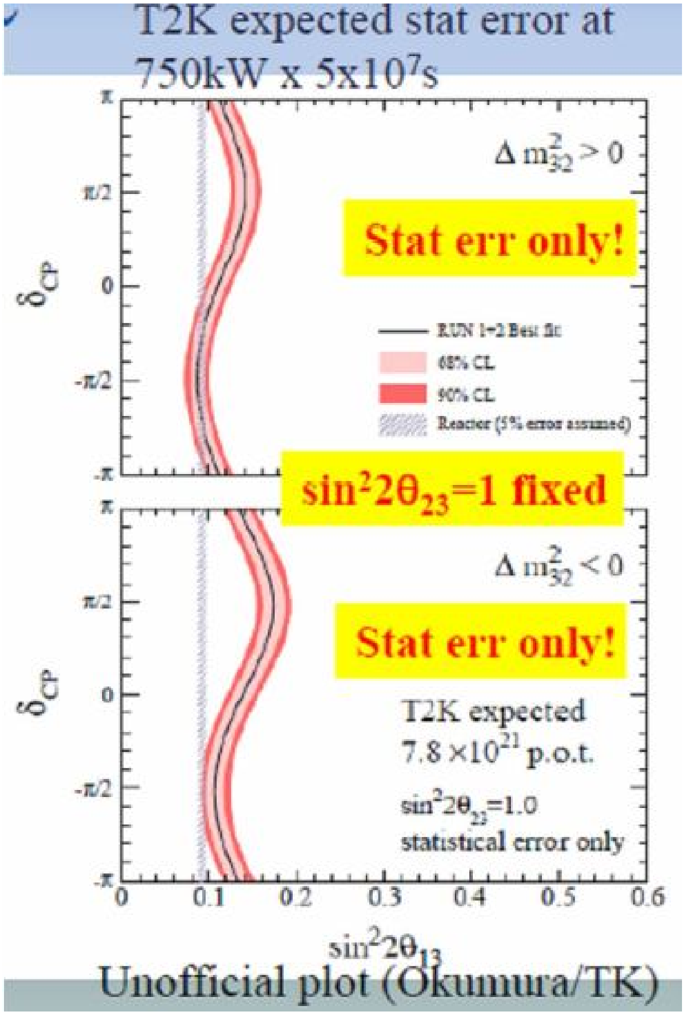
assuming δ = 0 (± 20% variation over the δ)

A ≈ ± 6.8% => ± 10% matter effect

78×10^{20} POT data expected in 5 yrs => Comparison with reactor result can find nonzero δ sig at 90%CL over about half the δ cycle.

1. Second far detector at L = 658 km & $E_\nu \approx 2$ GeV to determine sign of $\Delta m_{31}{}^2$ via matter effect.

2. Install a ~ 1Mt (HK) detector to determine sign of $\Delta m_{31}{}^2$ from atmospheric ν data and δ from T2K ν data.



T2K expected stat error at 750kW x 5×10^7s

Unofficial plot (Okumura/TK)

NOvA: 2013→

L = 810 km, $E_\nu \approx 2$ GeV

Fermilab ν_μ ⟶ NOvA (14 kt liq. Scintillator)
0.7 MW

$L/E_\nu \approx 405$ km/GeV => $|\Delta_{31}| \approx 70^\circ$,

$E_\nu \approx 2$ GeV => ± 30% matter effect
(& the ± 20% variation with δ)

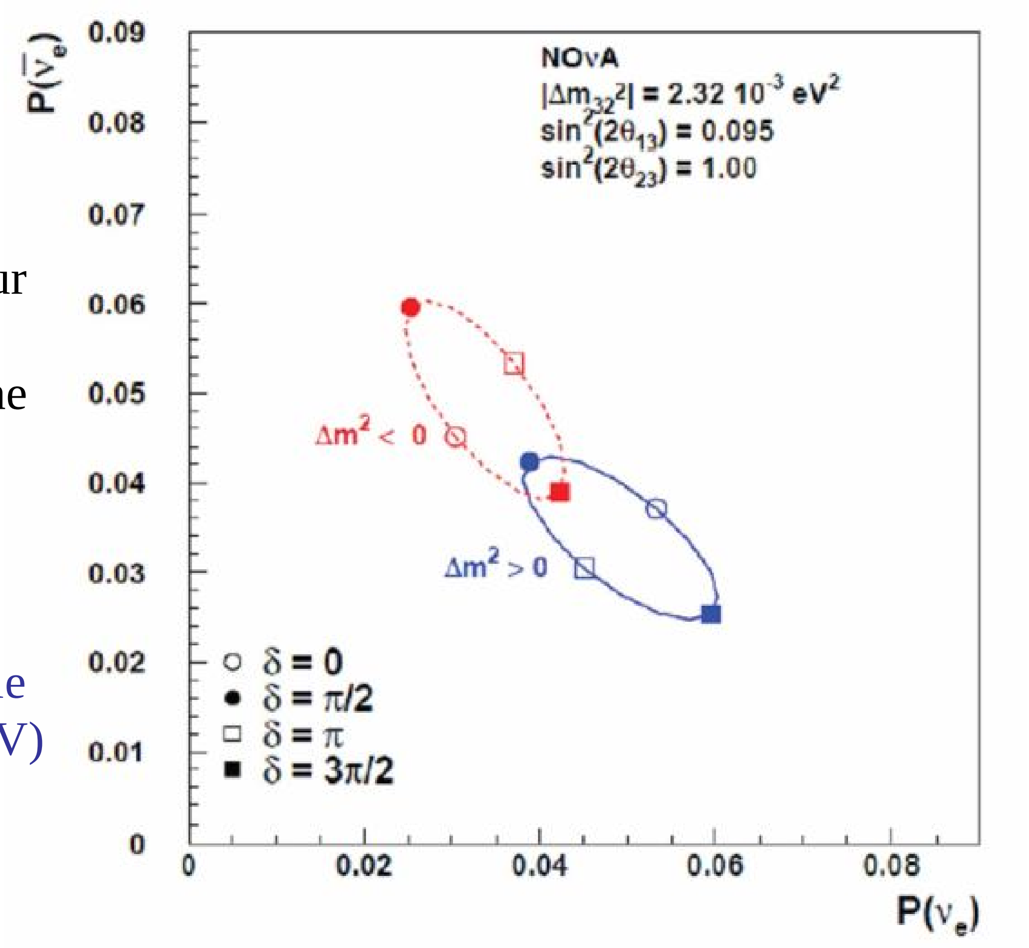
complete 3+3 years of $\nu_\mu \to \nu_e + \bar{\nu}_\mu \to \bar{\nu}_e$ appearance

J. M. Paley (NOvA & LBNE)
ICHEP2012

2σ error bars ≈ ± 0.015 =>
Effective overlap ~ half of each contour

⇒2σ Res. Mass hierarchy over ~ half the δ cycle
=>2σ sig for nonzero δ not possible.

NOvA+T2K:
=>1σ Res. Mass hierarchy → full δ cycle
=>1.5σ (90%CL) sig for nonzero δ (CPV) over most of the δ cycle.



LBNE Prop. Fermilab ν_μ $\xrightarrow[L = 1300 \text{ km}]{\hspace{6cm}}$ 10 kt liquid Ar TPC
0.7→2.2 MW

- 2σ Res. Mass hierarchy over full δ cycle
- 4σ Res. Mass hierarchy with (NOvA+T2K)
- 2σ Sig. for nonzero δ (CPV) over $.2\pi < \delta < .8\pi$
- 3σ Sig. for nonzero δ (CPV) with (NOvA+T2K)

- Thanks to the sizable value of θ_{13}, it seems feasible to resolve the neutrino mass hierarchy and detect signal of nonzero δ (CPV) in the T2K & NOvA experiments along with their proposed extensions in the foreseeable future.

Implications for Hierarchy Res. In Atmospheric Neutrino Expts.

PRO

- The $\nu_\mu \rightarrow \nu_e$ & $\nu_e \rightarrow \nu_\mu$ appearance probabilities of core traversing neutrinos experience larger matter effect than in LBL accelerator expts.

- They are insensitive to δ unlike in LBL expts.

CON

- Huge BG to the atmospheric $\nu_\mu \rightarrow \nu_e$ & $\nu_e \rightarrow \nu_\mu$ appearance from the ν_e & ν_μ survival probabilities, which are unsuppressed by any $\sin^2 2\theta_{13}$ factor.

- Energy and direction of the incoming neutrino has to be inferred from the measured energies and directions of the outgoing particles.

- Likewise the nature of the incoming neutrino has to be inferred from the identification of the outgoing lepton (e/μ) and its charge.

- They make very challenging demands on the detector performance of atmospheric neutrino experiments.

SK Expt. (ICHEP2012): 3900 days data (240 kt.yr)

- $\sin^2 2\theta_{13} \approx 0.1$: $\nu_\mu \rightarrow \nu_e$ appearance => ~ 12% (5%) excess of core traversing ν_e events for normal (inverted) mass hierarchy & the other way around for $\bar{\nu}_e$ events.

- SK data has over 2000 multi-GeV $\nu_e/\bar{\nu}_e$ events.

- Yet they are unable to detect any statistically significant excess of events signaling nonzero $\sin^2 2\theta_{13}$, which does not require $\nu_e/\bar{\nu}_e$ separation.

- They do not have good $\nu_e/\bar{\nu}_e$ separation. So they are unable to resolve mass hierarchy even at a fraction of 1σ level, which requires $\nu_e/\bar{\nu}_e$ separation.

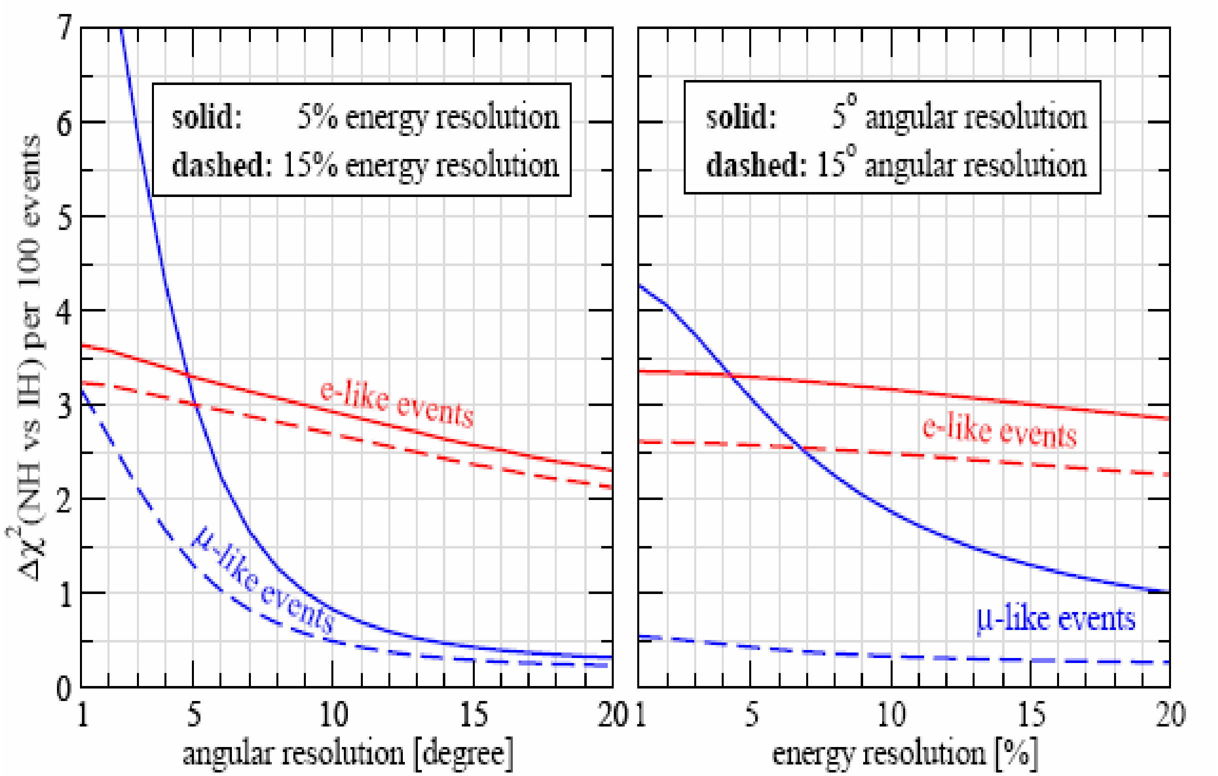
- A 3σ resolution of mass hierarchy possible at the proposed 1 Mt scale HK detector with 10 years of atmospheric $\nu_e/\bar{\nu}_e$ data.

INO (50 kt magnetized iron tracking calorimeter): 2017→

Can collect 200 - 300 $\nu_\mu/\bar{\nu}_\mu$ events in 2-3 years with good $\nu_\mu/\bar{\nu}_\mu$ separation.
Can it resolve mass hierarchy? Petcov and Schwetz, NP 2006

Possible with $\sigma(\theta,E_\nu)$ = 5%
But not with $\sigma(\theta,E_\nu)$ = 15%

Blennow and Schwetz,2012
⇒INO can achieve 2σ mass
Resolution by itself in 10 yrs
and with T2K+NOvA in 5 yrs
with $\sigma(\theta,E_\nu)$ = 10%.
But no significant cont. to MH
Resolution with $\sigma(\theta,E_\nu)$ =15%.



MINOS: $\sigma(E_\nu)$ = 15-20%.
INO Passive (iron) layers are
5 cm thick, against 2.5 cm of
MINOS => $\sigma(E_\nu)$ poorer than
MINOS => Hierarchy res. seems
unlikely at INO unless it can
improve $\sigma(E_\nu)$ significantly.