# Can There be Bounce in Weakly Broken Galileon Theories?

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# Why we need Bounce?

[M. Novello and S. E. P. Bergliaffa, Phys. Rep. 463 (2008)]

- **Problems with SCM**:initial singularity, horizon and flatness problem, entropy problem....
- Inflation answers some$\rightarrow$**initial singularity remains unsolved** $\rightarrow$ spacetime description breaks down there.
- **Potential solution to singularity problem/alternative to inflation$\rightarrow$ non-singular bouncing cosmology$\rightarrow$scale factor never reaching zero**.

# Approaches to produce Bounce

• **Necessary conditions**: $\dot{a} = 0$ and $\ddot{a} > 0$ at bounce (or $H = 0, \ \dot{H} > 0$).

$\odot$ Possible in modified gravity $\rightarrow$ Pre-Big-Bang, Ekpyrotic, higher order corrections, braneworld, loop quantum cosmology, etc..

• **Cyclic Universe-periodic sequence of contractions and expansions**$\rightarrow$ bounce and turnaround during each cycle$\rightarrow$scale factor goes through minimum and maximum.

$\odot$ Explains scale invariant power spectrum and moderate non-Gaussianities.

# Cosmic History in a Bouncing Universe

- Time axis→ $-\infty < t < \infty \rightarrow$ bounce occurs at $t = 0$.
- **Three main phases:** the initial **contracting phase** → a **bounce phase** (transition takes place) → usual **expanding phase** of Friedmann Universe.
- Solves **horizon, flatness, entropy** problems .....
- **Perturbations generated by matter sector in the contracting phase** → go outside the horizon → re-enter again after bounce.
- Power spectrum is scale invariant → apply matching conditions across bounce → dominating **modes after bounce remains scale invariant**.

# Weakly Broken Galileon Theories(WBG)

▶ **Properties**:

• Scalar field coupled to gravity→ subclass of **Horndeski scalar tensor theory** → presence of higher derivatives.

• **Second order eq. of motion**.

• Satisfy **weakly broken Galilean symmetry** $\phi \to \phi + b_\mu x^\mu$.

• Symmetry-breaking interaction terms suppressed.

• **Satisfy solar system constraints** due to **Vainshtein mechanism**.[D. Pirtskhalava, *et al.*, JCAP. 09 (2015)].

▶ Action:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2 R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \sum_{I=2}^{5}\mathcal{L}_I^{\rm WBG} + \ldots\right] + S_m \quad (1)$$

■ **Few definitions**:

$$\mathcal{L}_2^{\rm WBG} = \Lambda_2^4 G_2(X),\ \mathcal{L}_3^{\rm WBG} = \frac{\Lambda_2^4}{\Lambda_3^3} G_3(X)[\Phi],\ \mathcal{L}_4^{\rm WBG} = \frac{\Lambda_2^8}{\Lambda_3^6} G_4(X) R + 2\frac{\Lambda_2^4}{\Lambda_3^6} G_{4X}(X)\left([\Phi]^2 - [\Phi^2]\right)\ ,$$

$$\mathcal{L}_5^{\rm WBG} = \frac{\Lambda_2^8}{\Lambda_3^9}\, G_5(X) G_{\mu\nu}\Phi^{\mu\nu} - \frac{\Lambda_2^4}{3\Lambda_3^9}\, G_{5X}(X)\left([\Phi]^3 - 3[\Phi][\Phi^2] + 2[\Phi^3]\right) \quad (2)$$

□ $X$ is a dimensionless variable given by $X \equiv -\frac{1}{\Lambda_2^4} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ and $[\Phi] = g^{\mu\nu}\nabla_\mu\nabla_\nu\phi$.

■ $\Lambda_3 \rightarrow$ scale **suppressing invariant galileon interactions** $\rightarrow \Lambda_2 \rightarrow$ suppresses single-derivative operators.

□ $G_I$'s are dimensionless functions$\rightarrow$ $G_2, G_4$ and $G_5$ start at **least quadratic** in $X$, while $G_3$ have **linear piece**.

- **Metric field eqs**:

$$3M_{pl}^2H^2 = \rho_m + V + \Lambda_2^4 X\left[\frac{1}{2} - \frac{G_2}{X} + 2G_{2X} - 6ZG_{3X} - 6Z^2\left(\frac{G_4}{X^2} - 4\frac{G_{4X}}{X} - 4G_{4XX}\right)\right.$$
$$\left. + 2Z^3\left(5\frac{G_{5X}}{X} + 2G_{5XX}\right)\right], \quad (3)$$

$$M_{pl}^2\dot{H} = -\frac{\Lambda_2^4 XF + M_{pl}\ddot{\phi}(XG_{3X} - 4ZG_{4X} - 8ZXG_{4XX} - 3Z^2G_{5X} - 2Z^2XG_{5XX})}{1 + 2G_4 - 4XG_{4X} - 2ZXG_{5X}}$$
$$-\frac{\rho_m}{2} - \frac{p_m}{2}, \quad (4)$$

where

$$F(X,Z) = \frac{1}{2} + G_{2X} - 3ZG_{3X} + 6Z^2\left(\frac{G_{4X}}{X} + 2G_{4XX}\right) + Z^3\left(3\frac{G_{5X}}{X} + 2G_{5XX}\right), \quad (5)$$

with $Z \equiv \frac{H\dot{\phi}}{\Lambda_3^3}$.

- Matter conservation equation satisfied.

# Analytical conditions for bounce

- **Eqn satisfied by $H$:**

  [S. Banerjee, E. N. Saridakis, Phys. Rev. D **95** (2017)]

$$aH^3 + bH^2 + cH + d = 0, \tag{6}$$

  where

$$a = \frac{2\dot{\phi}}{\Lambda_3^9}\left(\frac{5G_{5X}}{X} + 2G_{5XX}\right); \quad b = -\frac{6\dot{\phi}^2}{\Lambda_3^6}\left(\frac{G_4}{X^2} - \frac{4G_{4X}}{X} - 4G_{4XX}\right) - 3M_{pl}^2;$$
$$c = -\frac{6\dot{\phi}}{\Lambda_3^3}G_{3X}\Lambda_2^4 X; \quad d = V + \frac{\Lambda_2^4 X}{2} - G_2\Lambda_2^4. \tag{7}$$

- In and around bounce ($\rightarrow H = 0$):

$$b^2 = 3ac; d = 0 \quad \text{or} \quad b^2 = 3ac; d = \frac{b^3}{18a^2} \tag{8}$$
$$b < 0 \text{ (for expansion, } H > 0); \tag{9}$$
$$b > 0 \text{ (for contraction, } H < 0). \tag{10}$$

- Second requirement $\dot{H} > 0$ gives

$$\frac{\left(\Lambda_2^4 X + 2\Lambda_2^4 X G_{2X} + 2M_{pl}\ddot{\phi} X G_{3X}\right)}{(4XG_{4X} - 2G_4 - 1)} > 0. \tag{11}$$

- **Simplest possible choice of the galileon functions for generating bounce:** $\boxed{G_2 \neq 0, G_3 \neq 0, G_4 \neq 0 \text{ and } G_5 = 0}$
- Conditions simplify as

$$\dot{\phi}^2|_b = \frac{M_{pl}^2 \Lambda_3^6}{2\left(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2}\right)}; \quad V(\phi)|_b = G_2 \Lambda_2^4 - \frac{\Lambda_2^4 X}{2}$$

$$\dot{\phi}^2 < \frac{M_{pl}^2 \Lambda_3^6}{2(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2})} \quad \text{(for expansion)}$$

$$\dot{\phi}^2 > \frac{M_{pl}^2 \Lambda_3^6}{2(\frac{4G_{4X}}{X} + 4G_{4XX} - \frac{G_4}{X^2})} \quad \text{(for contraction).} \tag{12}$$

# Numerical Analysis [S. Banerjee, E. N. Saridakis, Phys. Rev. D **95** (2017)]

▶ **Background Solution**:

• **Correction terms generate bounce, cyclicity** for suitable choices of $G_I's$, $V(\phi)$.

⊙ **Reconstructed scalar potential for a not so tuned** choice of $G_I's \to$ using simplest model $G_2 = G_4 = X^2, G_3 = X, G_5 = 0 \to$ for bouncing and cyclic scale factor.

• **Reconstructed** $G_I's$ for a zero or non-zero given potential.

⊙ Showed we have **enhanced freedom to satisfy the relevant requirements**.

## Reconstructing $V(\phi)$

□ For **bouncing scale factor**- $\boxed{a(t) = a_b(1 + Bt^2)^{1/3}}$ $\rightarrow$ choosing simplest class of WBG model-$G_2 = G_4 = X^2; \quad G_3 = X; \quad G_5 = 0.$

■ **Reconstructed potential** which generates bounce- $\boxed{V(\phi) = V_0 + (\phi - \phi_0)^2}$ $\rightarrow$ verified using reverse reconstruction.

□ Above procedure applied in the presence of $\rho_m = \rho_{m0}/a^3$ (pressureless)$\rightarrow$ gives rise to **matter bounce**.

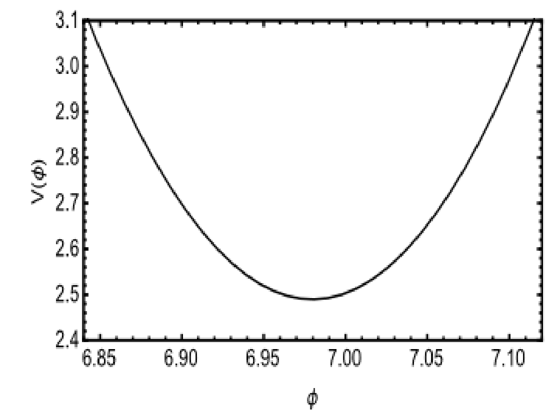


Figure: *The reconstructed scalar potential* $V(\phi)$ *that generates the bouncing scale factor, in the case where* $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. *The bouncing parameters have been chosen as* $a_b = 0.2$, $B = 10^{-5}$, *while* $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, *in* $M_{pl}$ *units.*

## → Similarly reconstructed $G_3$ for the simplest model-

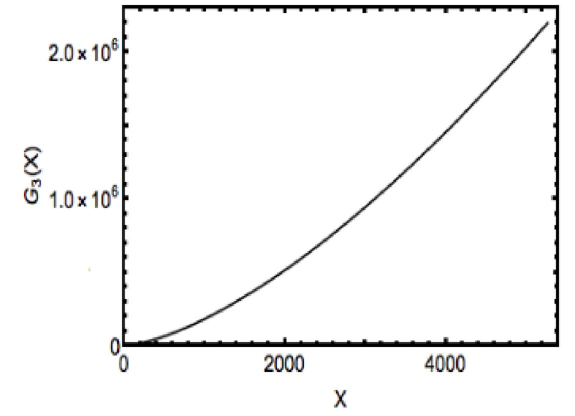


Figure: *The reconstructed galileon function $G_3(X)$ that generates the bouncing scale factor, in the case where $V(\phi) = 0$, and with $G_2 = G_4 = X^2$, $G_5 = 0$. The bouncing parameters have been chosen as $a_b = 0.2$, $B = 10^{-5}$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in $M_{pl}$ units.*

# Cyclic Solution

• Choose cyclic scale factor- $\boxed{a(t) = A\sin(wt) + a_c}$.

• **Applying reconstruction procedure**, with $\rho_m = \rho_{mb}(a_c - A)^3/a^3$ (dust)$\rightarrow$ for the simplest possible WBG model $\rightarrow$ one gets **the potential that generates cyclic bounce**.

• Reverse reconstruction show simple oscillatory potential $\boxed{V(t) = V_1\sin(w_V t) + V_2}$ generate cyclic universe.

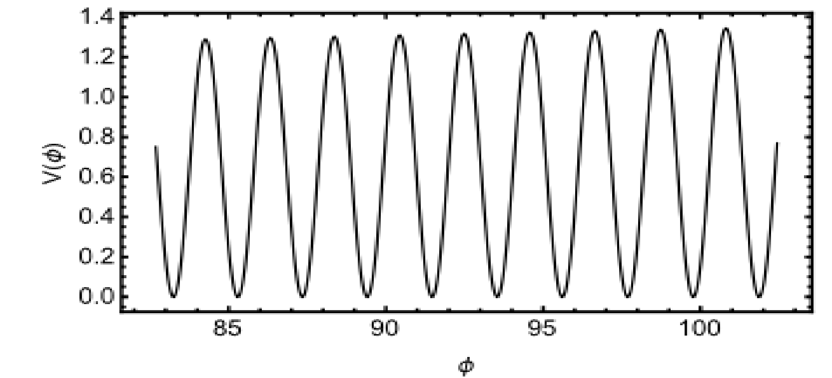


Figure: *The reconstructed scalar potential $V(\phi)$ that generates the cyclic scale factor, in the case where $G_2 = G_4 = X^2$, $G_3 = X$, $G_5 = 0$. The model parameters have been chosen as $a_c = 0.01$, $A = 10^{-4}$, $w = 15$, $\rho_{mb} = 0.01$, while $\Lambda_2 = 0.9$, $\Lambda_3 = 0.01$, in $M_{pl}$ units.*

▶ **Perturbations**:

• Actions for the scalar and tensor modes

$$S_\zeta = \int d^4xa^3A(t)M_{pl}^2\left[\dot{\zeta}^2 - c_s^2\frac{(\nabla\zeta)^2}{a^2}\right]\ ;\ S_\gamma^{(2)} = \int d^4xa^3\frac{M_{pl}^2}{8}\left[\dot{\gamma}_{ij}^2 - \frac{1}{a^2}(\partial_k\gamma_{ij})^2\right] \quad (13$$

• Power spectra

$$\boxed{P_\zeta \equiv \frac{k^3}{2\pi^2}\left|\frac{\sigma_k}{z}\right|^2 \approx \frac{H_{bp}^2}{48\pi^2M_{Pl}^2};\quad P_T \equiv \frac{k^3}{2\pi^2}\left|\frac{\sigma_k}{z}\right|^2 \approx \frac{H_{bp}^2}{48\pi^2M_{Pl}^2}} \qquad (14)$$

∗ **Observations**:

· Both the spectra are **of the same order of magnitude** → Provides a **large tensor-to-scalar ratio**, a problem faced by all bouncing models.

· Possible solution-**kinetic amplification** [Y. F. Cai, R. Brandenberger and X. Zhang, JCAP **1103** (2011)].

· For example, presence of a massless scalar $\chi$ and considering it to couple to the galileon field $\phi$ as $g^2\phi^2\chi^2$ → tensor-to-scalar ratio can be reduced to values $r \simeq 10^{-3}$.

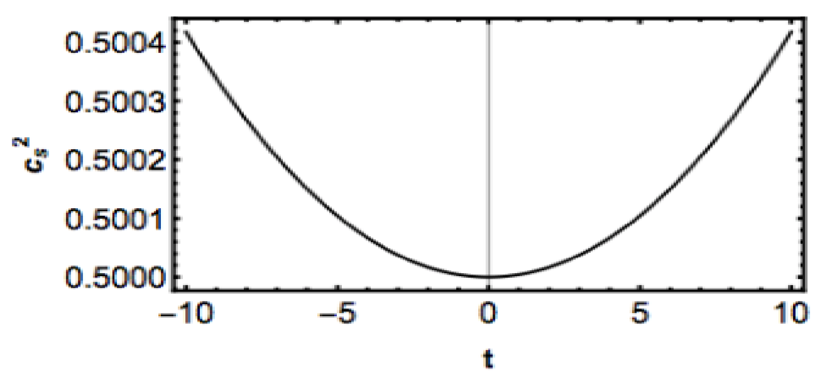


Figure: *The evolution of the sound speed square, for the bouncing scale factor, in the case where* $G_2 = G_4 = X^2,\ G_3 = X,\ G_5 = 0$*, and with* $a_b = 0.2,\ B = 10^{-5},\ \Lambda_2 = 0.9,\ \Lambda_3 = 0.01$*, in mpl units..*

# Summary/Discussions

- We have investigated the **bounce and cyclicity** realization in the framework of **weakly broken galileon theories**.
- We have shown that bounce and cyclicity **can be easily realized** in the framework of weakly broken galileon theories for a **suitably constructed potential or Galileon function**.
- We proceeded to a detailed investigation of the perturbations where we saw that the scenario at hand **shares the disadvantage of all bouncing models**, namely that it provides a large tensor-to-scalar ratio. Hence, we discussed about possible solution.

## Summary/Discussions *contd...*

- We also studied the stability of the perturbations in this model in the context of **the No-Go theorem** [T. Kobayashi, Phys. Rev. D **94** (2016)] . We found that this model **evades No-Go theorem and hence is free from ghosts and instabilities** [S. Banerjee, Y. F. Cai and E. N. Saridakis (submitted to Phys. Rev. D, 2018)] .

- Extended the above analysis to **new gravitational scalar tensor theories** [A. Naruko, D. Yoshida and S. Mukohyama, Class. Quant. Grav. **33** (2016)] $\rightarrow$ theories with Lagrangian containing Ricci scalar and its first and second derivatives, in a specific combination that makes them free of ghosts $\rightarrow$ proved to be a subclass of **bi-scalar extensions of GR** [E. N. Saridakis, S. Banerjee, R. Myrzakulov, Phys. Rev. D **98** (2018)].