Econophysics of Income & Wealth Distributions

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Acknowedging collaborations with:
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CAMBRIDGE UNIVERSITY PRESS

Physics of Society: Econophysics and Sociophysics

Discipline:

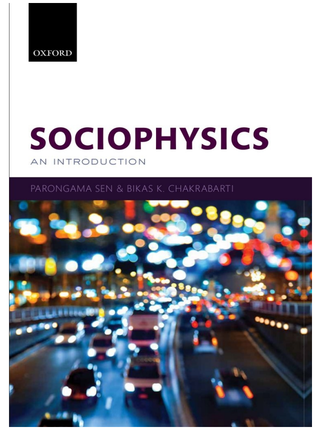
Physics, Economics, Finance, Sociology, Management & Computer Science

This book series is aimed at introducing readers to the recent developments in physics inspired modelling of economic and social systems. Econophysics and sociophysics as interdisciplinary subjects view the dynamics of markets and society in general as those of physical systems. However, they employ the dynamical and statistical methods and techniques used in physics. Social systems are increasingly being identified as 'interacting many-body dynamical systems' very much similar to the physical systems, studied over several centuries now.

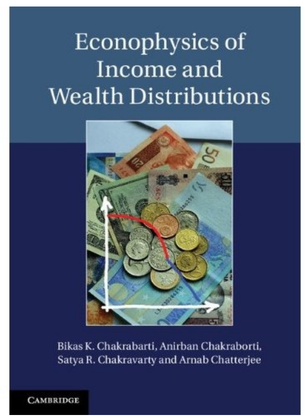
Statistical physics of many-body systems, developed over the last century, in the last three decades in particular, are now quite well established with impressive successes. Inspired by these successes in physics, major attempts have recently been made in extracting the statistical network structure and collective dynamics of various social systems. These include developing models of financial or market systems, correlations of various stock or commodity prices, their dynamics as well as their mapped structures, kinetic exchange models of market dynamics and of spontaneous opinion formations in societies and analysis of election results in these contexts, spin-glass or neural network-like modeling of social (collective and iterative) learning in sharing scarce resources in Minority Games, etc. These are some examples of extensively studied socio-dynamical model systems, often with encouraging results. The research papers, specialized reviews and books which are available till date are mostly addressed to the researchers. This new series brings out introductory books addressed to students across disciplines, both from physical and social science disciplines, including economics. They are all written by pioneers and leading researchers in these interdisciplinary fields. It also brings out research monographs and edited review volumes for already initiated researchers.

Recent books

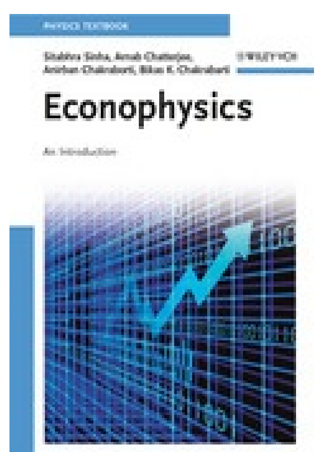
Sociophysics: An Introduction, P. Sen & B. K. Chakrabarti, Oxford University Press, Oxford (2013).



Econophysics of Income & Wealth Distributions, B. K. Chakrabarti, A. Chakraborti, S. R. Chakravarty & A. Chatterjee, Cambridge University Press, Cambridge (2013).



Econophysics: An Introduction, S. Sinha, A. Chatterjee, A. Chakraborti and B. K. Chakrabarti, Wiley-VCH, Berlin (2010).



Econophysics

Universiteit Leiden

Course description Econophysics

Year:	**2012-2013**
Catalog number:	
Teacher(s):	Dhr Dr. D. Garlaschelli
Language:	English
Blackboard:	Yes
EC:	6
Level:	300
Period:	Semester 1 (#part-of)

Description

---Introduction to Econophysics (historical background, interaction between Physics and Economics, past and present aims of the field).
---Stochastic processes and time series.
---Stylised facts of single financial time series.
---Crosscorrelations among multiple time series.
---Complex networks and interactions among economic agents.
---Network models of wealth distribution and market behaviour.
---International economic interactions: the World Trade Web

Literature

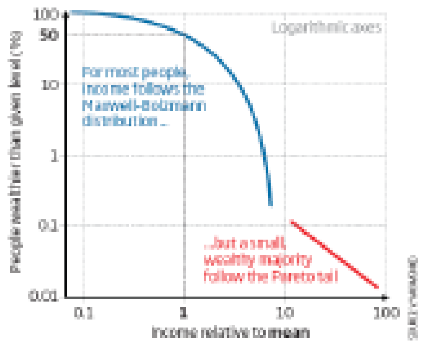
Obligatory: "**Econophysics: An Introduction**" by S. Sinha, A. Chatterjee, A. Chakraborti, B.K. Chakrabarti (Publisher: WileyVCH, 2010; ISBN: 9783527408153)

Year 3; 5th semester

... So, in 2000, Bikas Chakrabarti's team at the Saha Institute of Nuclear Physics in Kolkata, India ...

NewScientist
28 July 2012
P41

In all societies, the division of income follows a pattern similar to the distribution of energy of molecules in a gas – at least for the vast majority of people. But among the very richest, a pattern called the Pareto tail applies



THE PHYSICS OF OUR FINANCES

"The 1 per cent" may be a catchy phrase, but when it comes to understanding how wealth is distributed within society, we should focus on the top 5 or 10 per cent. Those who study income distribution have discovered that there is one rule for the rich and one rule for everybody else. For the masses, income follows a broad curve; for the wealthiest 5 to 10 per cent, the pattern is different, forming the so-called Pareto tail (see graph, above).

The statistical pattern seems to be ubiquitous and unchanging – "from ancient Egypt up to today", says Juan Ferrero, a physicist at the University of Córdoba in Argentina. That implies that there may be a universal mechanism at work.

More than 100 years ago, physicists pointed out that the broad income curve for the majority resembles the distribution of energy among molecules in a gas, a pattern called the Maxwell-Boltzmann distribution. This prompted the idea that the distribution arises because people exchange wealth when they meet, much as gas molecules exchange energy when they collide.

That idea has since been tested using mathematical models that liken human beings to molecules bouncing around in a gas. In the simplest model, people risk surrendering all their wealth at each encounter. That produces a wealth curve that has far more ultra-poor people than we find in the real world. So in 2000, Bikas Chakrabarti's team at the Saha Institute of Nuclear Physics in Kolkata, India, allowed people to retain some of their wealth in each exchange. The result was a wealth curve similar to the broad hump of the Maxwell-Boltzmann distribution.

The next refinement was to allow different people to hold back different percentages of their wealth – effectively setting money aside as savings. With this tweak, the model correctly reproduced the whole wealth distribution curve, including the Pareto tail, which was made up largely of people who saved the most. This finding has been backed up by other similar models, including one developed by Ferrero, in which the richest 10 per cent are once again those most inclined to save.

If these simple models do capture something of the essence of real-world economics, then they offer some good news. It turns out that the main part of the wealth distribution gets narrower, more equal, the more people choose to save. In other words, inequality can't be abolished, but it can be reduced if we all put more money aside for a rainy day. Stephen Battersby

Stephen Battersby

"There is one rule for the rich, and another for everyone else"

last, then I am doing better in life". This also may explain why people are always buying larger houses and larger televisions.

We just don't know any better

Lastly, despite studies contradicting this notion, most people still believe that more money equals more happiness. Consider research I conducted with Elizabeth Dunn and Lara Aknin at the University of British Columbia in Vancouver, Canada. We asked people to predict how happy they would be if their annual income was anything from $5000 up to $1 million. We also asked how much money they actually earned, and how happy they were with their lives. We found that people generally overestimated the impact of money on happiness; for example, those who reported earning $25,0... predicted that their hap... double if they made ... money, much mor...

But when we m... of people at these ... having them rate t... life on a scale from 1 ... the wealthier group w... happier. This shouldn't be ... many of us can think back to tim... when we earned less but were happier. (Remember the good times you had while being broke in college and you all had to chip in just to buy a pizza?)

All of that said, the richest among us are better equipped to turn money into happiness, but perhaps not in the way you would expect. Our research shows that people can gain happiness with money if they do something a bit unusual: give it away. It turns out that spending money on yourself does not make you any happier, but spending on others – from donating to charity to buying coffee for a friend – is an efficient way of turning cash into happiness. The wise hedge-fund manager would do well to take a break from giving investment advice to others, and instead take time to invest in others. ■

Michael I. Norton is an associate professor of business administration at Harvard Business School. He is co-author with Elizabeth Dunn of the forthcoming book, *Happy Money: The science of spending* (Simon & Schuster)

THE NEW PALGRAVE
DICTIONARY OF ECONOMICS
SECOND EDITION 2008

Edited by Steven N. Durlauf and Lawrence E. Blume

Volume 2 command economy – epistemic game theory

econophysics

According to Bikas Chakrabarti (2005, p. 225), the term 'econophysics' was neologized in 1995 at the second Statphys-Kolkata conference in Kolkata (formerly Calcutta), India, by the physicist H. Eugene Stanley, who was also the first to use it in print (Stanley, 1996). Mantegna and Stanley (2000, pp. viii–ix) define 'the multidisciplinary field of econophysics' as 'a neologism that denotes the activities of physicists who are working on economics problems to test a variety of new conceptual approaches deriving from the physical sciences'.

The list of such problems has included distributions of returns in financial markets (Mantegna, 1991; Levy and Solomon, 1997; Bouchaud and Cont, 1998; Gopakrishnan et al., 1999; Sornette and Johansen, 2001; Farmer and Joshi, 2002), the distribution of income and wealth (Drăgulescu and Yakovenko, 2001; Bouchaud and Mézard, 2000; Chatterjee, Yarlagadda and Charkrabarti, 2005), the distribution of economic shocks and growth rate variations (Bak et al., 1993; Canning et al., 1998), the distribution of firm sizes and growth rates (Stanley et al., 1996; Takayasu and Okuyama, 1998; Botazzi and Secchi, 2003), the distribution of city sizes (Rosser, 1994; Gabaix, 1999), and the distribution of scientific discoveries (Plerou et al., 1999; Sornette and Zajdenweber, 1999), among other problems, all of which are seen at times not to follow normal or Gaussian patterns that can be described fully by mean and variance. The main sources of conceptual approaches from physics used by the econophysicists have been from models of

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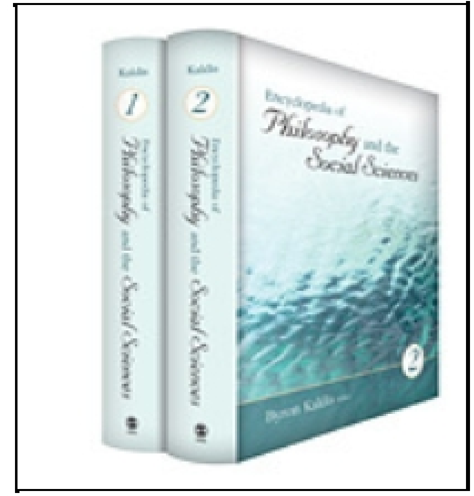


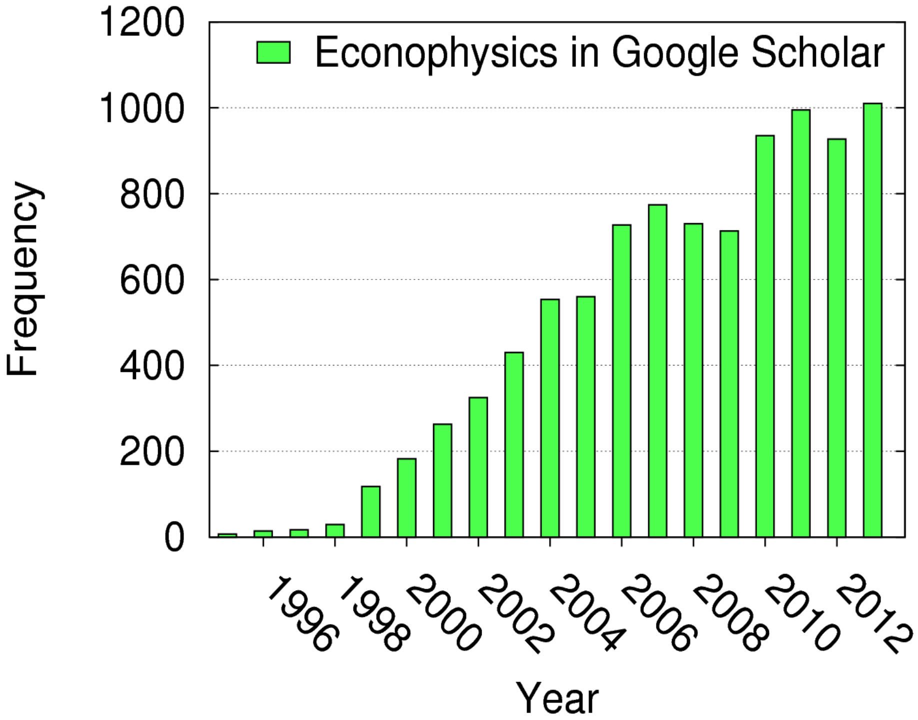
Encyclopedia of Philosophy and the Social Sciences

Byron Kaldis Hellenic Open University, Greece

April 2013 1168 pages *SAGE Publications, Inc*

Contents

Growth in econophysics



Histogram plot of numbers of entries containing the term ‘econophysics’ versus the corresponding year. The data are taken from google scholar site.

“The physicists, however, did not present a parallel perspective of this social science, at least not until recently when eminent physicists like Eugene H. Stanley, Bikas K. Chakrabarti, J. Doyne Farmer, Jean-Philippe Bouchaud and many others having joined the fray to create this new field which has now started to gain academic respect. ... As mentioned, Kolkata, India, occupies a crucial role in the history of this new science which has amongst its pioneers an Indian face, too ...”

In the Editorial (pp. vii-viii)

“... So he (Bikas) started to have meetings on econophysics and I think the first one was probably in 1995 (he decided to start it in 1993–1994). Probably the first meeting in my life on this field that I went to was this meeting. In that sense Kolkata is — you can say — the nest from which the chicken was born and Bikas gets, deservingly so, a lot of credit for that because it takes a lot of work to have a meeting on a field that does not really exist, so to say! ... So he should get a lot of credit for this. ..."

H. Eugene Stanley's Interview (pp. 73-78)

From IIM Kozhikode Society & Management Review, Vol. 2 (July 2013), SAGE Publications

Saha, Srivastava and the income distribution analogy in kinetic theory of gases.

Suppose in a country the assessing department is required to find out the average income per head of the population. They will proceed somewhat in the following way. They will find out the number of persons whose income lies within different small ranges. For example, they will find out the number of persons whose income lies between 10s. and 11s., between 11s. and 12s. and so on. Instead of a shilling, they may choose a smaller interval, say 6d. Then it can be easily seen that the number of persons whose income lies between 10s. and 10s.6d. will be approximately half the number found previously for the range 10s. to 11s. We can generalize by saying that the number whose income lies between x and $x+dx$ is $n_x dx$. It should be noted that the number is proportional to the interval chosen (dx). To get the average income they should choose the interval to be as small as possible, say a penny. When this is not possible they will choose a bigger interval but their results will be proportionately inaccurate.

To represent graphically[1] the income of the population they will plot a curve with n_x as ordinate and x as abscissa. The curve will be similar to that given in Fig. 6. This will begin with a minimum at 0, rise to a maximum at some point, and thereafter approach the axis of x, meeting it at a great distance. The curve will have this shape because the number of absolute beggars is very small, and the number of millionaires is also small, while the majority of population have average income.

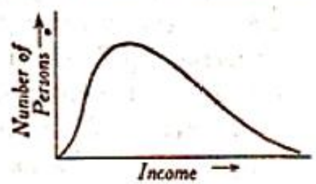


Fig. 6.—Distribution of income among persons.

In their textbook ***A Treatise on Heat*** (1931) Meghnad Saha and B. N. Srivastava used the example of reconstructing a distribution curve for incomes to illustrate the problem of determining the distribution of molecular velocities in kinetic theory. The relevant extract from page 105 of their book prefigures developments in the first decade of this century showing this indeed the bulk of the income distribution follows a Gibbs-like distribution.

Distribution P(m) of Individual Weekly Income in UK / USA / Japan

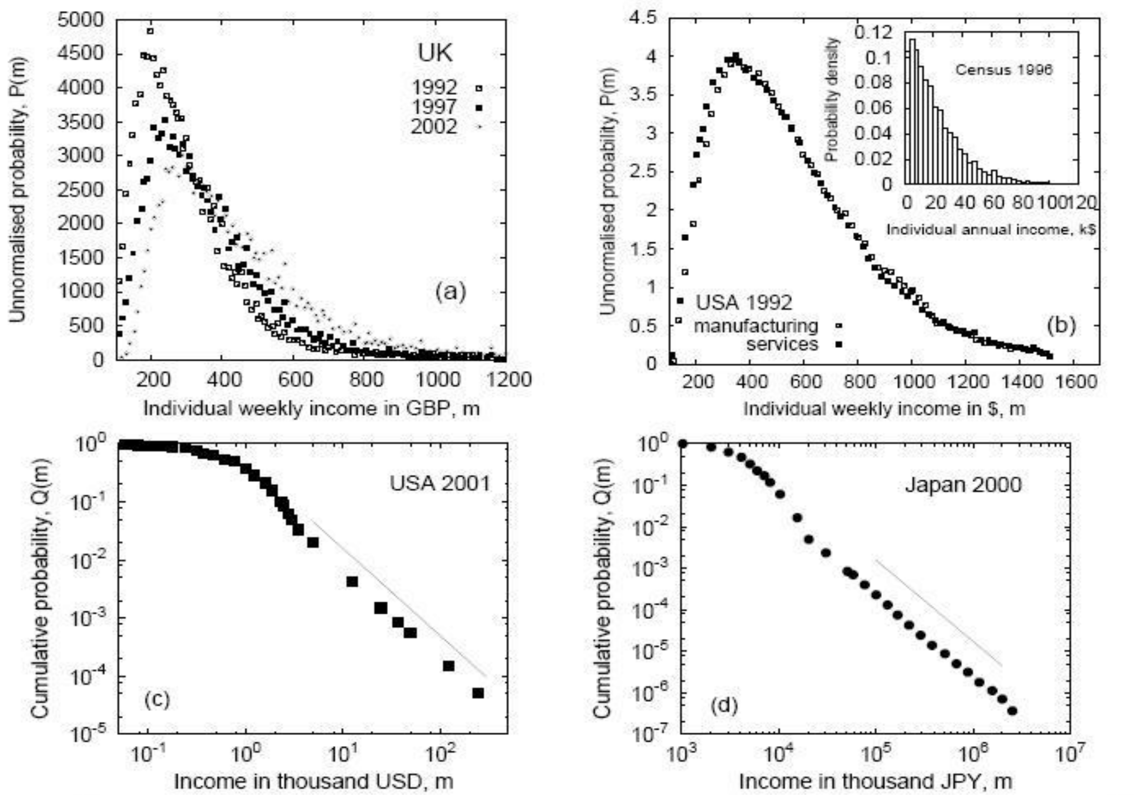
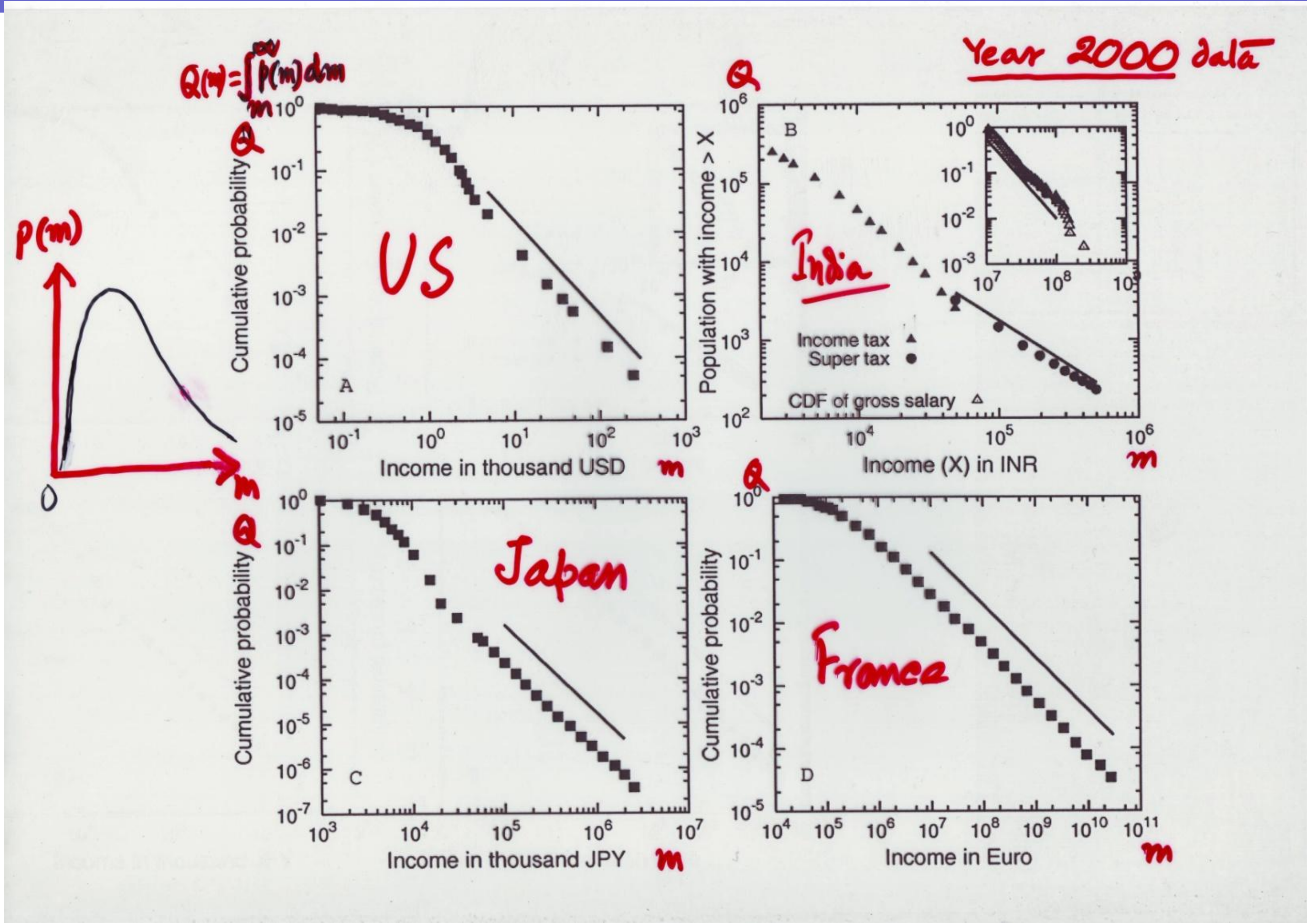


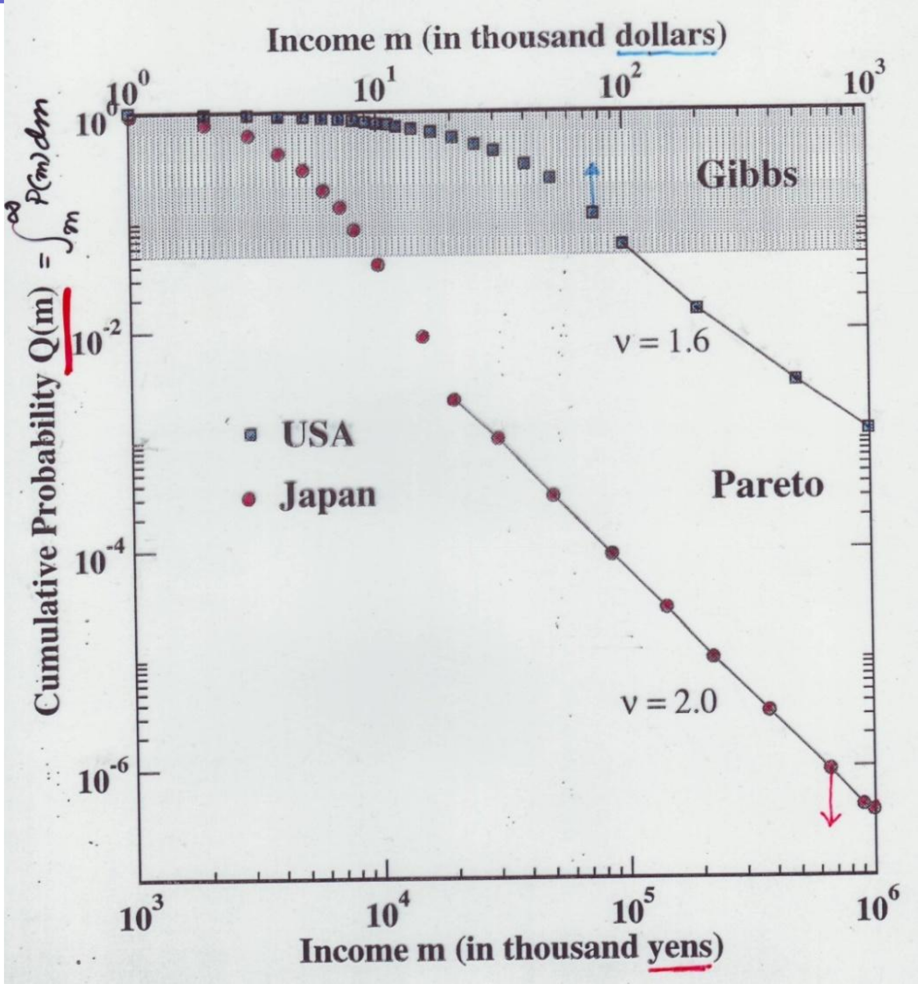
Fig. 1. (a) Distribution $P(m)$ of individual weekly income in UK for 1992, 1997 and 2002; data adapted from Ref. [7]. (b) Distribution $P(m)$ of individual weekly income for manufacturing and service sectors in USA for 1992; data for US Statistical survey, taken from Ref. [7]. The inset shows the probability distribution of individual annual income, from US census data of 1996. The data is adapted from Ref. [8]. (c) Cumulative probability $Q(m) = \int_m^\infty P(m)dm$ of rescaled adjusted gross personal annual income in US for IRS data from 2001 (adapted from Ref [6]), with Pareto exponent $\nu \approx 1.5$ (given by the slope of the solid line). (d) Cumulative probability distribution of Japanese personal income in the year 2000 (data adapted from Ref. [9]). The power law (Pareto) region approximately fits to $\nu = 1.96$.

$$Q(m) = \int_m^\infty P(m)\,dm$$



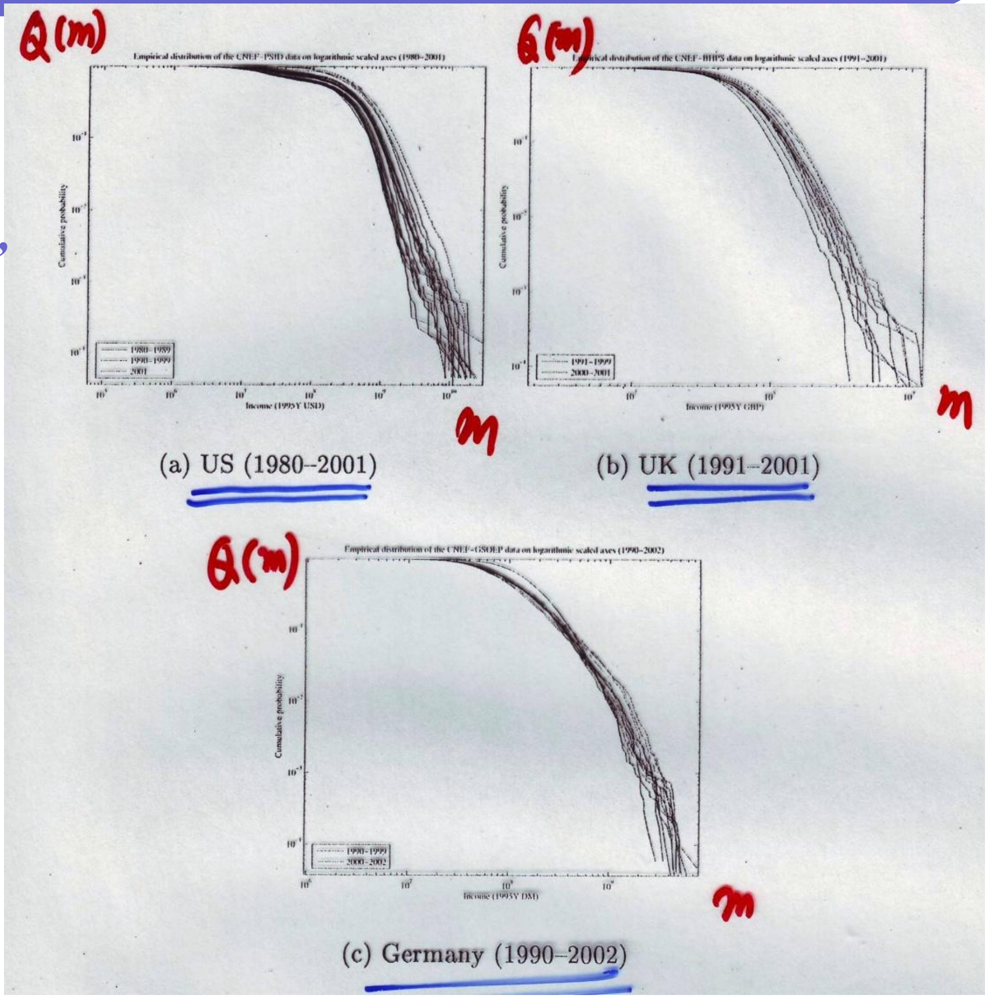
$$P(m) \sim m^{-(1+\nu)}\Big|_{m \to \infty}$$

2002 Data



Pareto's Law of Income Distribution–Graph

Source :
M. Galegatti et. al.
"Econophysics of Wealth Distribution"
Springer (2005)
Eds. A. Chatterjee et. al.

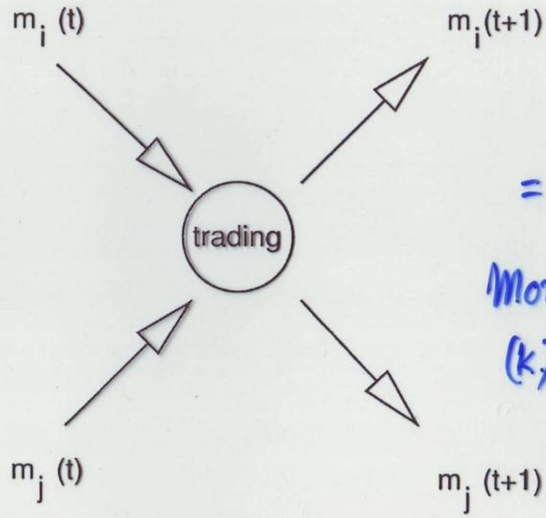


(a) US (1980–2001)

(b) UK (1991–2001)

(c) Germany (1990–2002)

Market Exchange ≡ Scattering Process ("CC-CCM" Models)



$$m_i(t) + m_j(t) = m_i(t+1) + m_j(t)$$

Money Conservation ≡ (k) energy conservation

$$\sum_{i=1}^{N} m_i(t) = M \text{ (const.)}$$

FIG. 2: Schematic diagram of the trading process. Agents i and j redistribute their money in the market: $m_i(t)$ and $m_j(t)$, their respective money before trading, changes over to $m_i(t+1)$ and $m_j(t+1)$ after trading.

$$m_i(t+1) = \lambda_i m_i(t) + \epsilon \left[(1-\lambda_i) m_i(t) + (1-\lambda_j) m_j(t)\right]$$

$$m_j(t+1) = \lambda_j m_j(t) + (1-\epsilon)\left[(1-\lambda_i) m_i(t) + (1-\lambda_j) m_j(t)\right]$$

ϵ: random fraction (annealed)

λ_i: Saving propensity of i-th agent (quenched)

Steady State Money Distribution P(m)

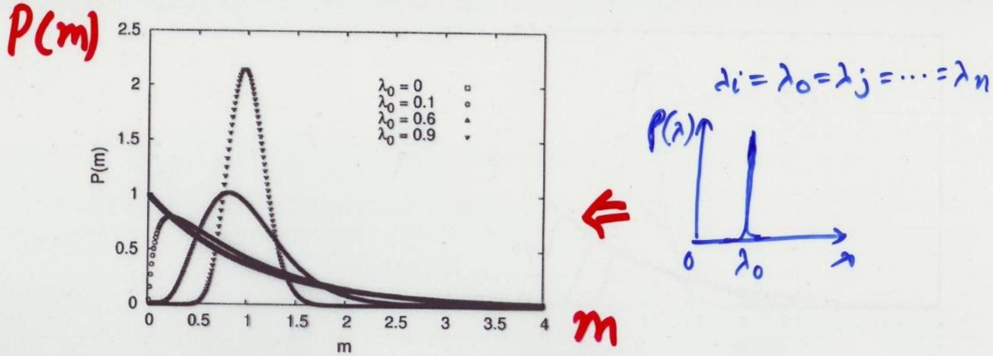


FIG. 3: Steady state money distribution $P(m)$ for the model with uniform savings. The data shown are for different values of λ: $0, 0.1, 0.6, 0.9$ for a system size $N = 100$. All data sets shown are for average money per agent $M/N = 1$.

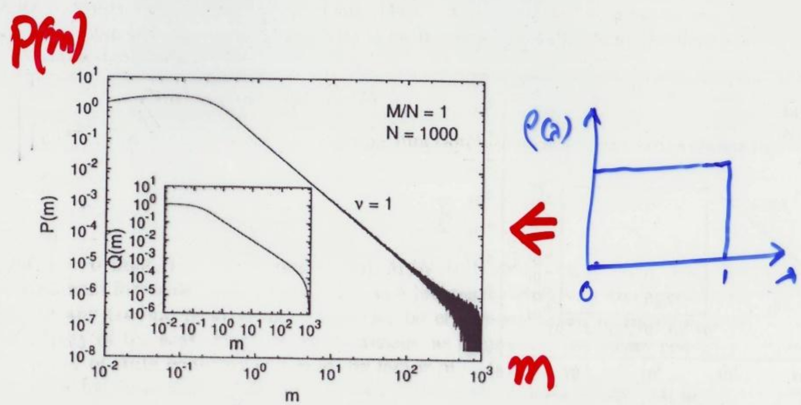
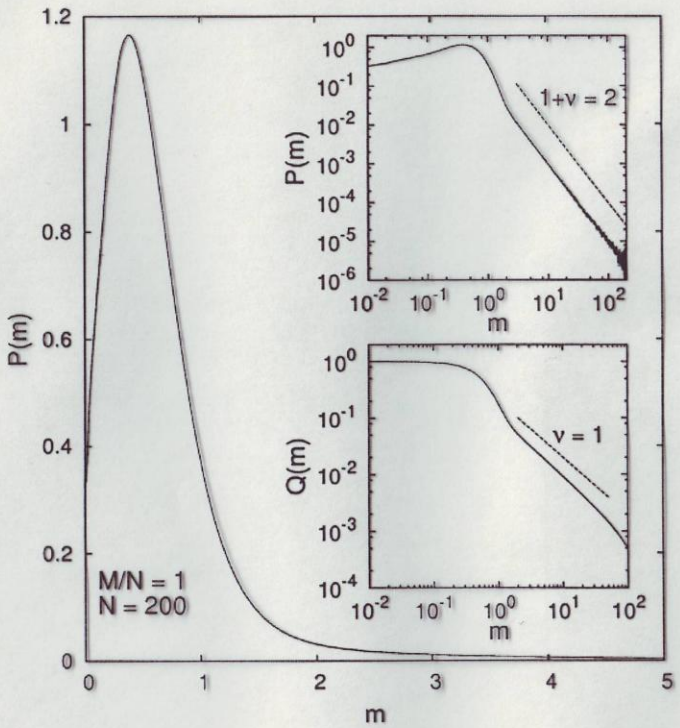


FIG. 4: Steady state money distribution $P(m)$ for the distributed λ model with $0 \leq \lambda < 1$ for a system of $N = 1000$ agents. The x^{-2} is a guide to the observed power-law, with $1 + \nu = 2$. Here, the average money per agent $M/N = 1$.

Steady State Money Distribution P(m)



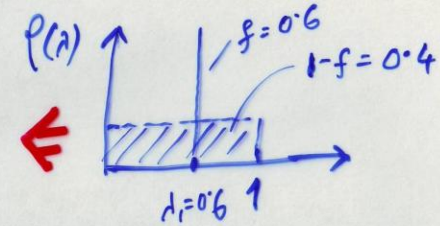


FIG. 5: Steady state money distribution $P(m)$ for a model with $f = 0.6$ fraction of agents with a uniform saving propensity $\lambda_1 = 0.6$ and the rest $1 - f$ fraction having random uniformly distributed (quenched) savings, in $0 \leq \lambda < 1$ for a system of $N = 200$ agents. Here, the average money per agent $M/N = 1$. The top inset shows $P(m)$ in log-log scale for the full range, while the bottom inset shows the cumulative distribution $Q(m)$. In addition to the power law tail in $P(m)$ and $Q(m)$ (as in the basic, distributed savings model), $Q(m)$ resembles a behavior similar to observed in empirical data (see Fig. 1).

Steady State Money Distribution P(m)

$M = C p_0$

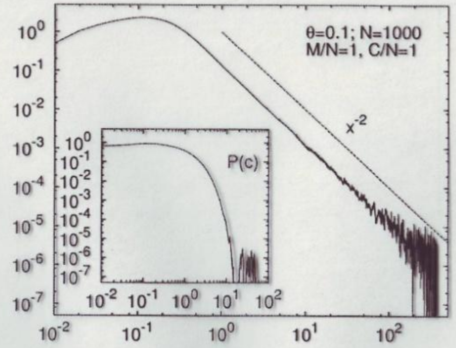
$\sum_{i=1}^{N} m_i(t) = M$ (const.)

$\sum_{i=1}^{N} c_i(t) = C$ (const)

$m_i(t+1) - m_i(t) \geq p(t)\left[c_i(t) - c_i(t+1)\right]$

$p(t) = p_0 \pm \delta(t)$

arbitrary



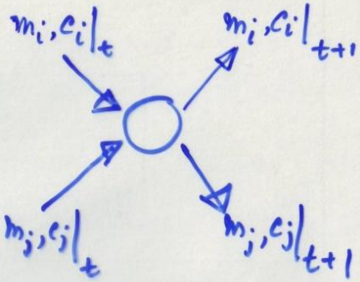


FIG. 15: Steady state distribution $P(m)$ of money m in the commodity market with distributed savings $0 \le \lambda < 1$. $P(m)$ has a power-law tail with Pareto exponent $\nu = 1 \pm 0.02$ (a power law function x^{-2} is given for comparison). The inset shows the distribution $P(c)$ of commodity c in the same commodity market. The graphs show simulation results for a system of $N = 1000$ agents, $M/N = 1$, $C/N = 1$.

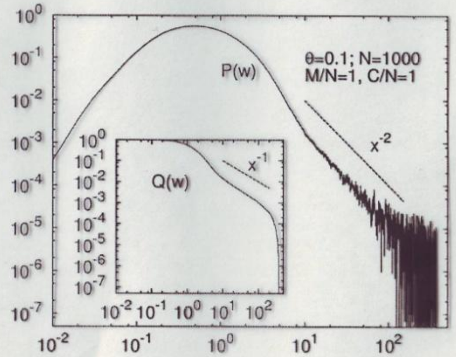


FIG. 16: Steady state distribution $P(w)$ of total wealth $w = m + c$ in the commodity market with distributed savings $0 \le \lambda < 1$. $P(w)$ has a power-law tail with Pareto exponent $\nu = 1 \pm 0.05$ (a power law function x^{-1} is given for comparison). The inset shows the cumulative distribution $Q(w) \equiv \int_w^\infty P(w)\mathrm{d}w$. The graphs show simulation results for a system of $N = 1000$ agents, $M/N = 1$, $C/N = 1$.

Asset transfer.. Contd.

- Utility maximization:

$$L=(x_1)^{\alpha}(x_2)^{\beta}(m_1)^{\lambda}+\omega(M_1+p_1Q_1-p_1x_1-p_2x_2-m_1)$$

utility function — Lagrange multiplier — budget eqn.

- FOC: $\delta L/\delta x=0$ where $x=x_1,x_2,m_1$ and ω.
- Let us assume that $\alpha+\beta+\lambda=1$.
- Demand functions: $x_1=\alpha(M_1+p_1Q_1)/p_1$, $x_2=\beta(M_1+p_1Q_1)/p_2$, $m_1=\lambda(M_1+p_1Q_1)$;

Asset transfer.. Contd.

- Similarly dd. Functions for the 2nd agent:
  $y_1 = \alpha(M_2+p_2Q_2)/p_1$, $y_2 = \beta(M_2+p_2Q_2)/p_2$,
  $m_2 = \lambda(M_2+p_2Q_2)$
- Market clearing ⟹ $x_1+y_1=Q_1$ & $x_2+y_2=Q_2$
- Equilibrium prices:
  $p_1=(\alpha/\lambda)(M_1+M_2)/Q_1$ & $p_2=(\beta/\lambda)(M_1+M_2)/Q_2$

Asset transfer.. Contd.: CC model

- Money transfer equations (plugging p_1 and p_2 in the money dd. functions) :

$$m_1(t+1)= \lambda m_1(t)+\epsilon(1- \lambda)(m_1(t)+m_2(t))$$

$$m_2(t+1)= \lambda m_2(t)+(1- \epsilon)(1- \lambda)(m_1(t)+m_2(t))$$

where $m_i(t+1)= m_i$ and $m_i(t)=M_i$ for i=1,2

and $\epsilon = \alpha /(\alpha + \beta)$;

- Let λ be fixed and α~uni[0,1- λ]~ β.

Hence, $\epsilon = \alpha /(\alpha + \beta)= \alpha /(1- \lambda) \sim$ uni[0,1].

Ref**: Physica A 388** (2009) 4151-4158

A mean field analysis & Pareto distribution

The trading equation can be written as

$$m_i \to \lambda_i m_i + \varepsilon[(1-\lambda_i)m_i + (1-\lambda_j)m_j]$$

$$m_j \to \lambda_j m_j + (1-\varepsilon)[(1-\lambda_i)m_i + (1-\lambda_j)m_j]$$

Suppose, $\bar{m}_i = \frac{\sum m_i}{n_i}$ where n_i is the total number of agents having same λ_i

$$\bar{m}_i = \lambda_i \bar{m}_i + \bar{\varepsilon}(1-\lambda_i)\bar{m}_i + c$$

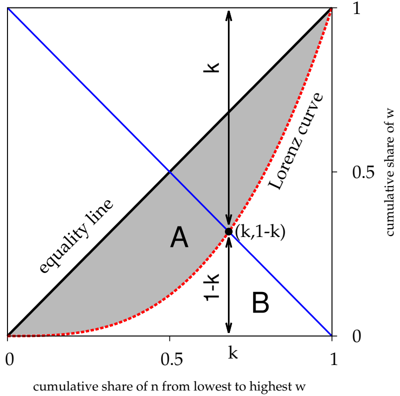
$$\bar{m}_i = \frac{2c}{1-\lambda_i}$$

c is a constant, depending on average money in the system

Considering $p(\bar{m})d\bar{m} = g(\lambda)d\lambda$ we get $p(\bar{m}) = \frac{2cg(1-2c/\bar{m})}{\bar{m}^2}$

Ref: P. K. Mohanty**, PRE 74** (2006) 011117

A new measure of social inequality: k-index



Gini index $\Rightarrow g = \frac{A}{(A+B)}$
k-index $\Rightarrow$ k fraction of wealth are possessed by $1-k$ fraction of people.

$k = 1/2$ (or $g = 0$) $\Rightarrow$ equality; $k = 1$ (or $g = 1$) $\Rightarrow$ monarchy

Country	g	k
Brazil	0.62	0.73
Denmark	0.36	0.63
India	0.45	0.66
Japan	0.31	0.61
Malaysia	0.50	0.68
NewZeland	0.37	0.63
Panama	0.44	0.66
Sweden	0.38	0.64
Tunisia	0.50	0.69
Uruguay	0.49	0.68
Columbia	0.55	0.70
Finland	0.47	0.67
Indonesia	0.44	0.65
Kenya	0.61	0.73
Netherlands	0.44	0.66
Norway	0.36	0.63
SriLanka	0.40	0.65
Tanzania	0.53	0.70
United Kingdom	0.36	0.63
Uruguay	0.49	0.68

g-index and k-index values for income distribution of various countries of the world during the years 1963 to 1983 as obtained analyzing data reported in A. F. Shorrocks, *Ranking income distributions*, Economica **50**, pp. 3-17 (1983).

A. Ghosh *et al.* Physica A, **410** (2014) 30

k-index for citations

Journals	Year	total papers/citations	g	k
Nature	1980	2904/178927	0.80	0.81
	1990	3676/307545	0.86	0.85
	2000	3021/393521	0.81	0.82
	2010	2577/100808	0.79	0.81
Science	1980	1722/111737	0.77	0.80
	1990	2449/228121	0.84	0.84
	2000	2590/301093	0.81	0.82
	2010	2439/85879	0.76	0.79
PNAS (USA)	1980	-	-	-
	1990	2133/282930	0.54	0.70
	2000	2698/315684	0.49	0.68
	2010	4218/116037	0.46	0.66
Cell	1980	394/72676	0.54	0.70
	1990	516/169868	0.50	0.68
	2000	351/110602	0.56	0.70
	2010	573/32485	0.68	0.75
PRL	1980	1196/87773	0.66	0.74
	1990	1904/156722	0.63	0.74
	2000	3124/225591	0.59	0.72
	2010	3350/73917	0.51	0.68
PRA	1980	639/24802	0.61	0.73
	1990	1922/54511	0.61	0.72
	2000	1410/38948	0.60	0.72
	2010	2934/26314	0.53	0.69

University	Year	total papers/citations	g	k
Harvard	1980	4897/225626	0.73	0.78
	1990	6036/387244	0.73	0.78
	2000	9566/571666	0.71	0.77
	2010	15079/263600	0.69	0.76
MIT	1980	2414/101929	0.76	0.79
	1990	2873/156707	0.73	0.78
	2000	3532/206165	0.74	0.78
	2010	5470/109995	0.69	0.76
Cambridge	1980	1678/62981	0.74	0.78
	1990	2616/111818	0.74	0.78
	2000	4899/196250	0.71	0.77
	2010	6443/108864	0.70	0.76
Oxford	1980	1241/39392	0.70	0.77
	1990	2147/83937	0.73	0.78
	2000	4073/191096	0.72	0.77
	2010	6863/114657	0.71	0.76
SINP	1980	32/170	0.72	0.74
	1990	91/666	0.66	0.73
	2000	148/2225	0.77	0.79
	2010	238/1896	0.71	0.76
IISC	1980	450/4728	0.73	0.78
	1990	573/8410	0.70	0.76
	2000	874/19167	0.67	0.75
	2010	1624/11497	0.62	0.73

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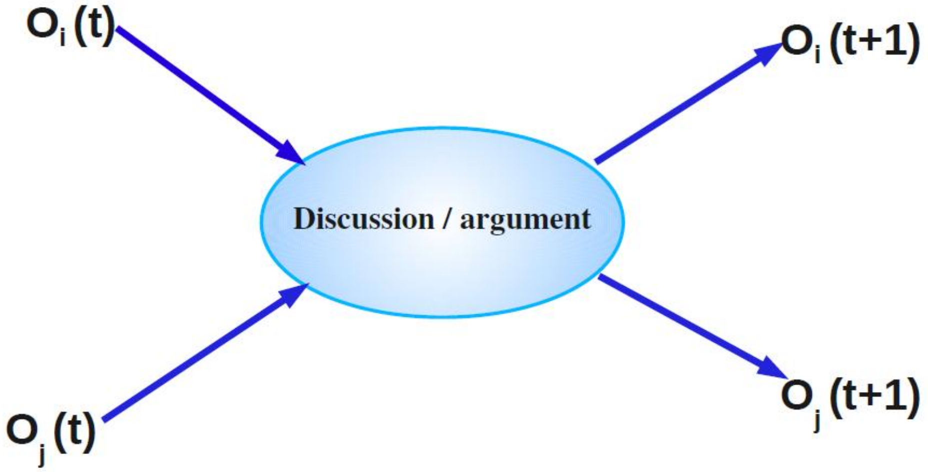
Social opinion formation & kinetic exchange model



$$O_i(t+1) = \lambda_i O_i(t) + \varepsilon \mu_j O_j(t)$$
$$O_j(t+1) = \lambda_j O_j(t) + \varepsilon' \mu_i O_i(t)$$

$$-1 \le O_i(t) \le 1 \qquad (1)$$

$$0 \le \varepsilon(t), \varepsilon'(t) \le 1 \quad \& \quad 0 \le \lambda_i, \mu_i \le 1$$

Uniform conviction-influence case

Let us consider

$$\lambda_i = \lambda = \mu_i$$

$$O_i(t+1) = \lambda \left(O_i(t) + \varepsilon O_j(t) \right)$$
$$O_j(t+1) = \lambda \left(O_j(t) + \varepsilon' O_i(t) \right) \quad (2)$$

$$\bar{O}(t) = (1/N) \left| \sum_i O_i(t) \right|$$

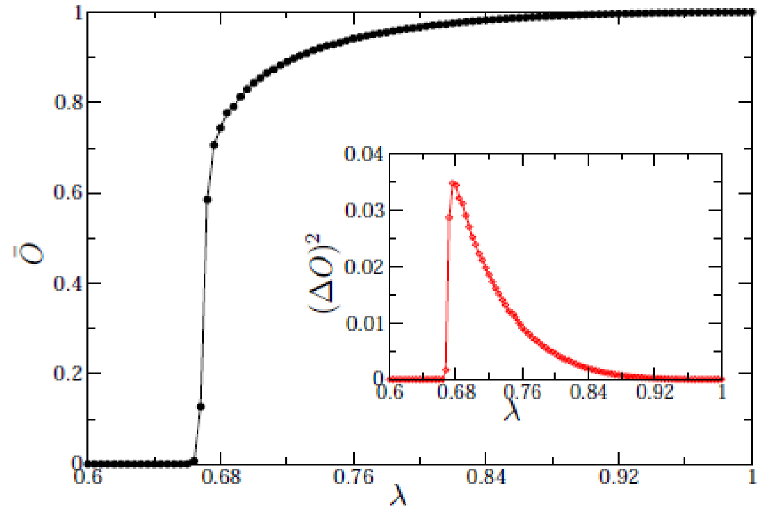


FIG. 1. Numerical results for the variation of the average opinion $\bar{O}(t)$ for large t (steady state value of $\bar{O}$) against λ, following dynamics of Eq. (2). (Inset) Numerical results for the variation of the variance $(\Delta O)^2 \equiv \overline{(O-\bar{O})^2}$ against λ, following dynamics of Eq. (2).

Consider the stochastic multiplier map

$$O(t+1) = \lambda(1+\varepsilon_t)O(t) \qquad (3)$$

$$\ln O\big|_{t+1} = \ln \lambda(1+\varepsilon) + \ln O\big|_t$$

$$\rightarrow \overline{\ln \lambda_c(1+\varepsilon)} = 0$$

$$\rightarrow \ln \lambda_c = -\int_0^1 \ln(1+\varepsilon)\,d\varepsilon$$

$$= -[2\ln 2 - 1] \rightarrow \lambda_c \simeq 0.679$$

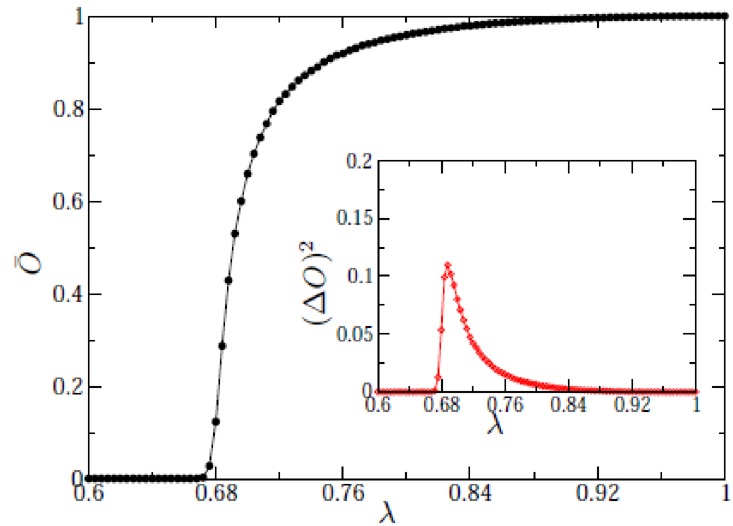


FIG. 2. Numerical results for the variation of the average opinion $\bar{O}(t)$ for large t (steady state value of $\bar{O}$) against λ, following dynamics of Eq. (3). (Inset) Numerical results for the variation of the variance $(\Delta O)^2 \equiv \overline{(O-\bar{O})^2}$ against λ, following dynamics of Eq. (3).

stochastic multiplier map ... cont'd

$$\bar{O} = (1-p)O_{av} + p.1, \quad (4)$$

where $O_{av} = (O_{min} + 1)/2$. We have assumed that the value $O(t)$ stays in those two regions (from λ to 1 and exactly at 1) with probability $(1-p)$ and p. Hence, the corresponding equations are

$$O(t+1) = \lambda(1+\epsilon)O(t) \qquad \text{with probability } 1-p,$$

and

$$O(t+1) = 1 \qquad \text{with probability } p.$$

Note that the first equation is realized only if $\lambda(1+\epsilon)O(t) < 1$ or $\epsilon < \epsilon_{max} = \frac{1}{\lambda O_{av}} - 1$. This cut-off implies that $p = \int_0^{\epsilon_{max}} d\epsilon = \frac{1}{\lambda O_{av}} - 1$, since $\epsilon \sim$uni[0,1]. By substituting O_{av} and p in Eq. (4), we derive the result that

$$\bar{O} = \frac{5\lambda + 2\lambda^2 - \lambda^3 - 2}{2\lambda(1+\lambda)} \quad (5)$$

which is compared with the numerical simulations for $\lambda \to 1$ in Fig. 6. It is evident that the approximation holds well, only for $\lambda \to 1$.

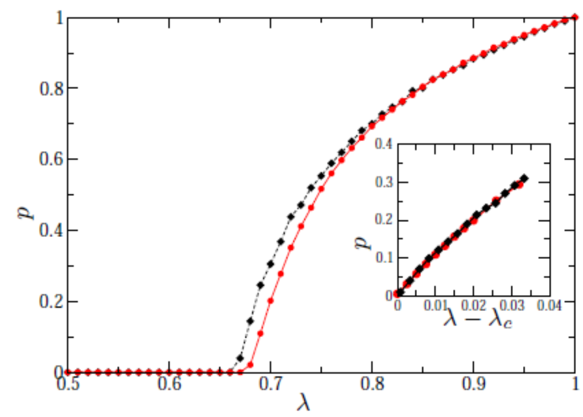
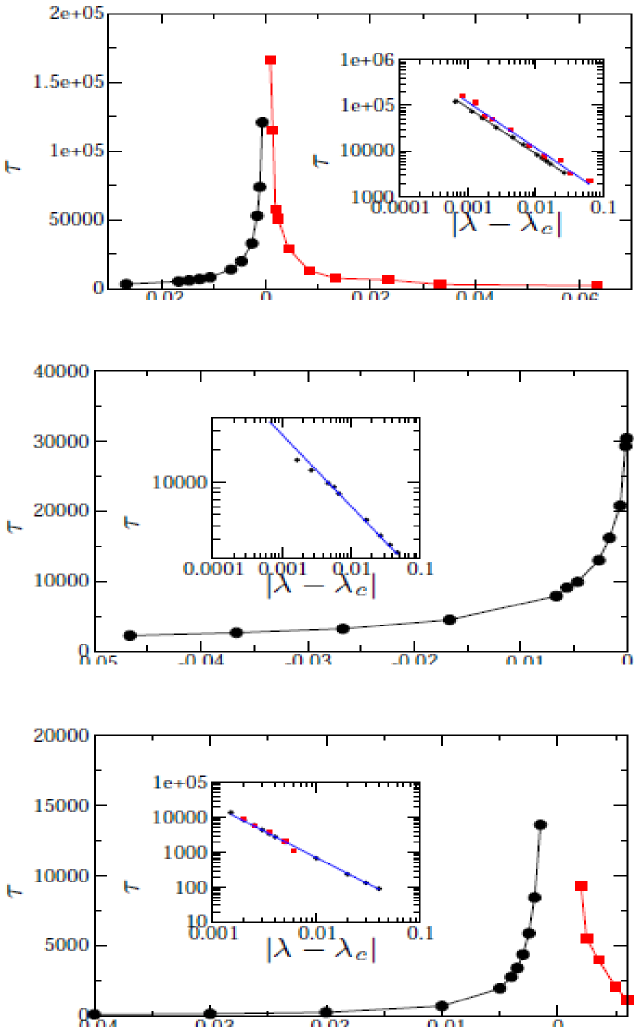


FIG. 3. Numerical results for the variation of the average condensate fraction $p(t)$ for large t (steady state value of p) against λ, following dynamics of Eq. (2) in black diamonds, and dynamics of Eq. (3) in red circles. (Inset) Numerical results for the growth of p, following dynamics of Eq. (2) in black diamonds, and dynamics of Eq. (3) in red circles, close to λ_c.

stochastic multiplier map ... cont'd



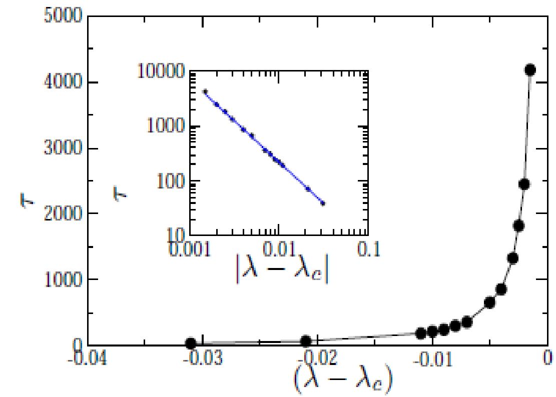


FIG. 4. Numerical results for relaxation time behaviors τ versus $\lambda - \lambda_c$, for (a) Multi-agent model with $\bar{O}$ (b) Multi-agent model with p (c) Map with $\bar{O}$ (d) Map with p . (Insets) Determination of exponent z from numerical fits of $\tau \sim |\lambda - \lambda_c|^{-z}$.

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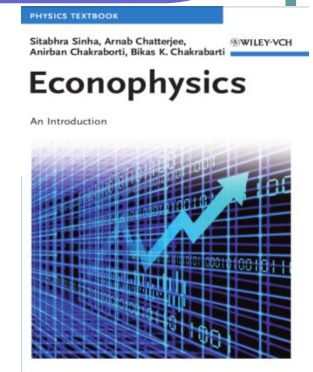

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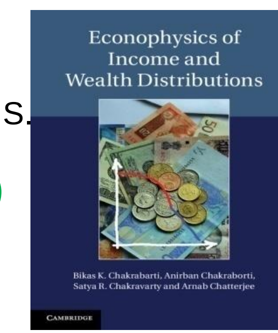

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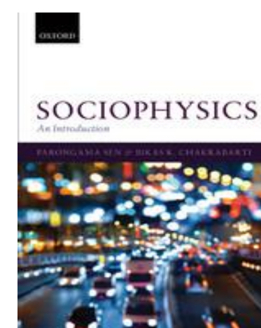

THANK YOU