Study Of Nuclei at High Angular Momentum – Day 2

Outline

1) Introduction
2) Deformation and Rotation in Nuclei
   Mean Field
   Cranked Shell Model
3) Cranked Shell Model Applied to Exp.
4) Superdeformation

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Nuclear Physics School, Goa, India
Nov. 9-17, 2011

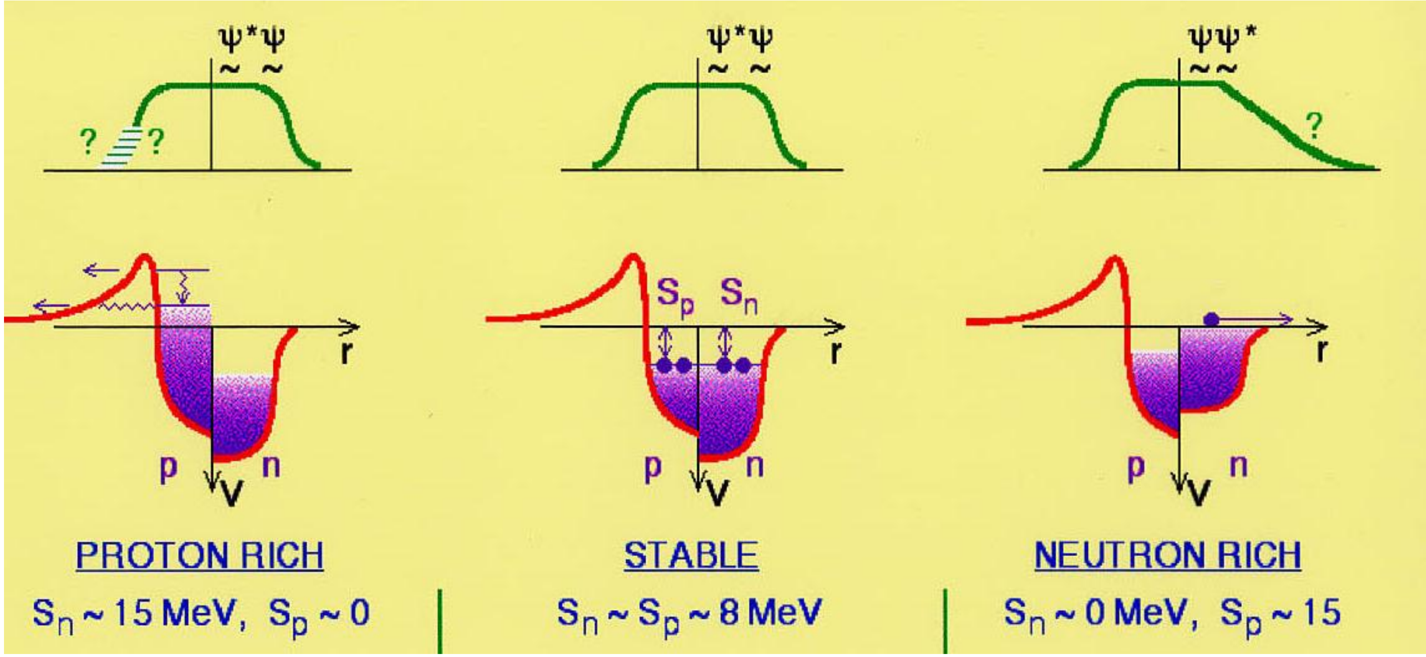
Some of the Physics Questions

How does the asymmetry in the proton and neutron Fermi surfaces impact the nucleus; *i.e.*

What is the impact on the mean field as reflected in:

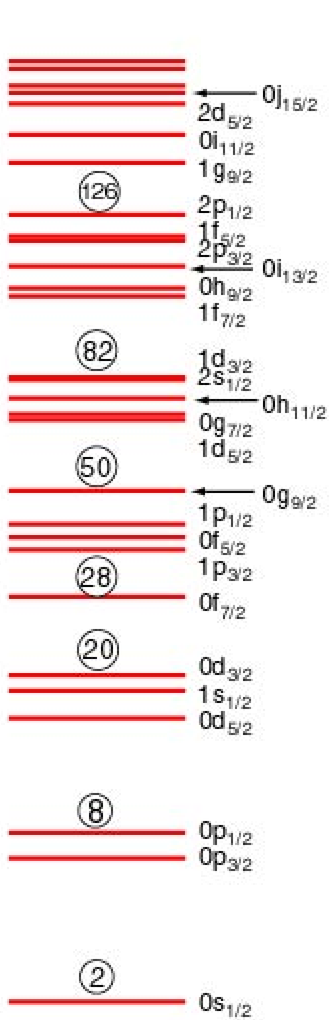
the single particle energies

the shapes and spatial extensions

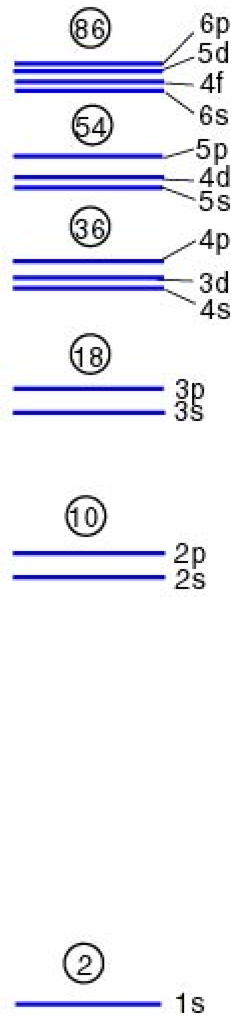


Nuclear Shell Model (text book physics)

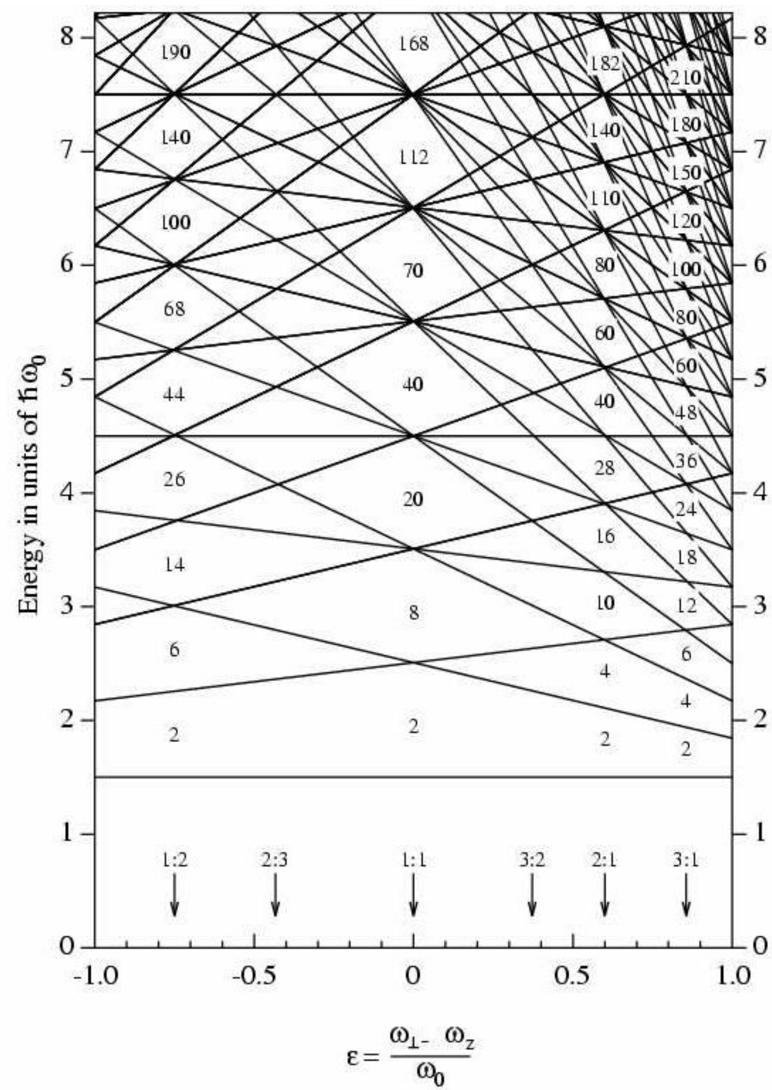
M. Goeppert-Mayer and D. Jensens



Shell Model of Nuclei



Shell Model of Atoms



Nuclear Shapes

Describing the nucleus as a liquid drop leads to the idea of *collective coordinates* – ignore motions of individual nucleons and treat nucleus as a continuous medium

Parameterized Shape of Liquid Drop

$$R(\vartheta,\varphi,t) = R_0\left(1+\sum_{\lambda=0}^{\infty}\sum_{\mu=-\lambda}^{\lambda}\alpha_{\lambda\mu}(t)Y_{\lambda\mu}(\theta,\varphi)\right)$$

where $\alpha_{\lambda\mu}(t)$ are the collective coordinates which are time dependent – this allows for vibrations of the surface

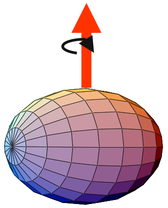
Quadrupole Deformation (Q_0):

Rotation Symmetry:

Rotational Bands,

Enhanced E2 transitions;

$\alpha_{2\mu}(t)$



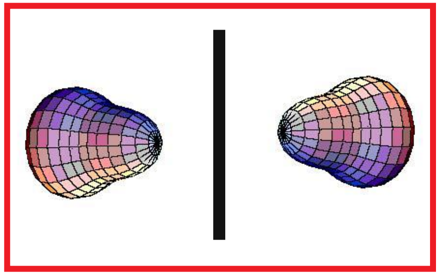
Octupole Deformation (D_0):

Reflection Symmetry:

Parity Doublets,

Enhanced E1 transitions

$\alpha_{3\mu}(t)$



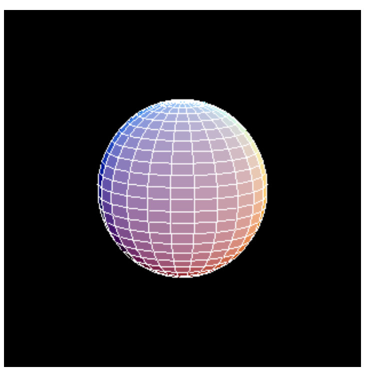
Vibrations of a QM Liquid Drop

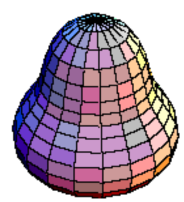
Hamiltonian can be cast into form

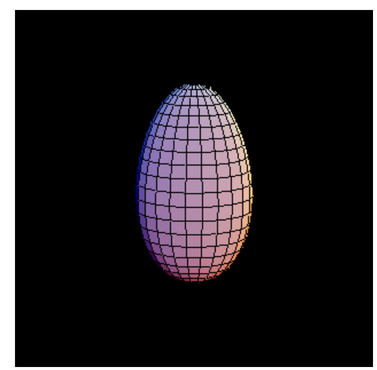
$$\hat{H} = \sum_{\lambda\mu} \left\{ b^{+}_{\lambda\mu} b_{\lambda\mu} + \frac{1}{2} \right\}$$

where $b^{+}_{\lambda\mu}(b_{\lambda\mu})$ is a phonon creation (annihilation) operator

The terms in order of importance (for small amplitude motion and low excitation energies) are λ=2 (quadrupole vibrations), λ=3 (octupole vibrations), etc.





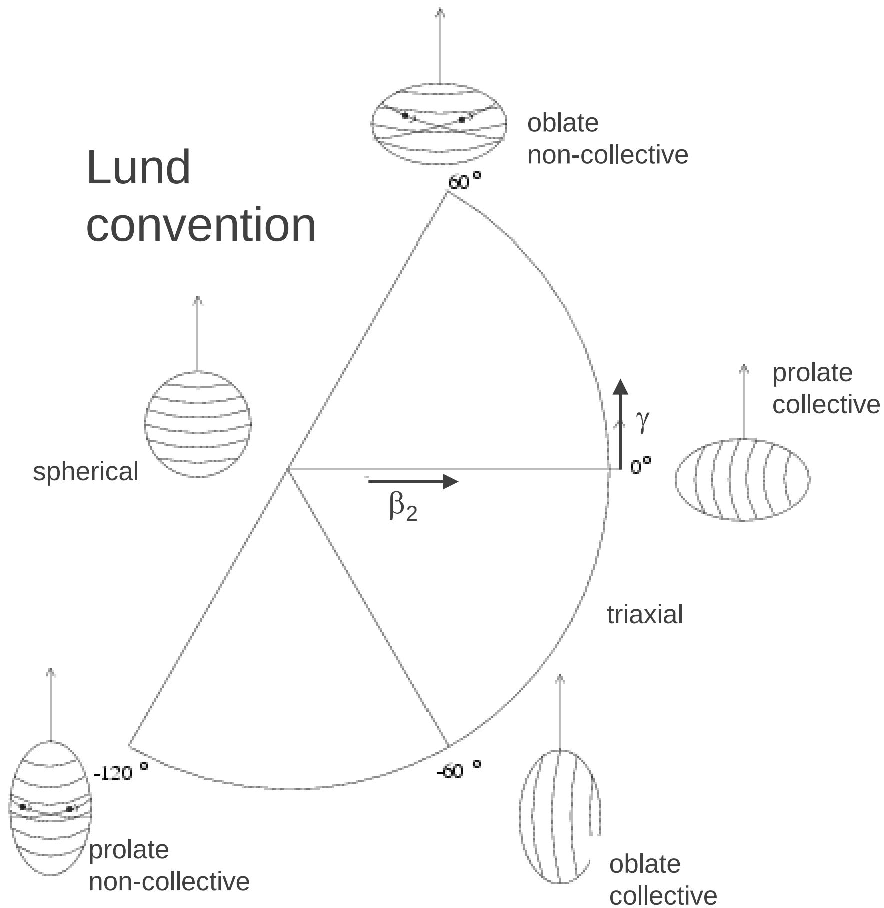


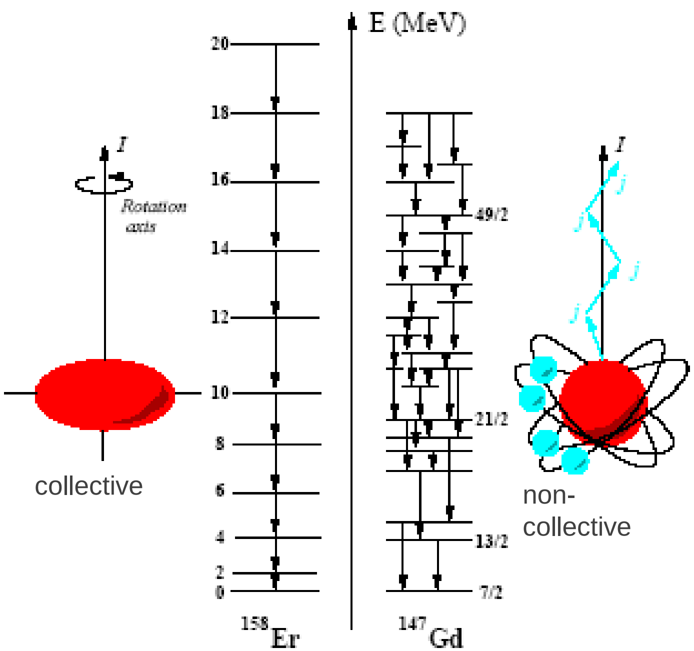
Nuclear Deformation

$$R(\theta,\varphi,t) = R_0\left[1+\sum_{\lambda=0}\sum_{\mu=-\lambda}^{+\lambda} a_{\lambda\mu}(t)\, Y_{\lambda\mu}(\theta,\phi)\right]$$

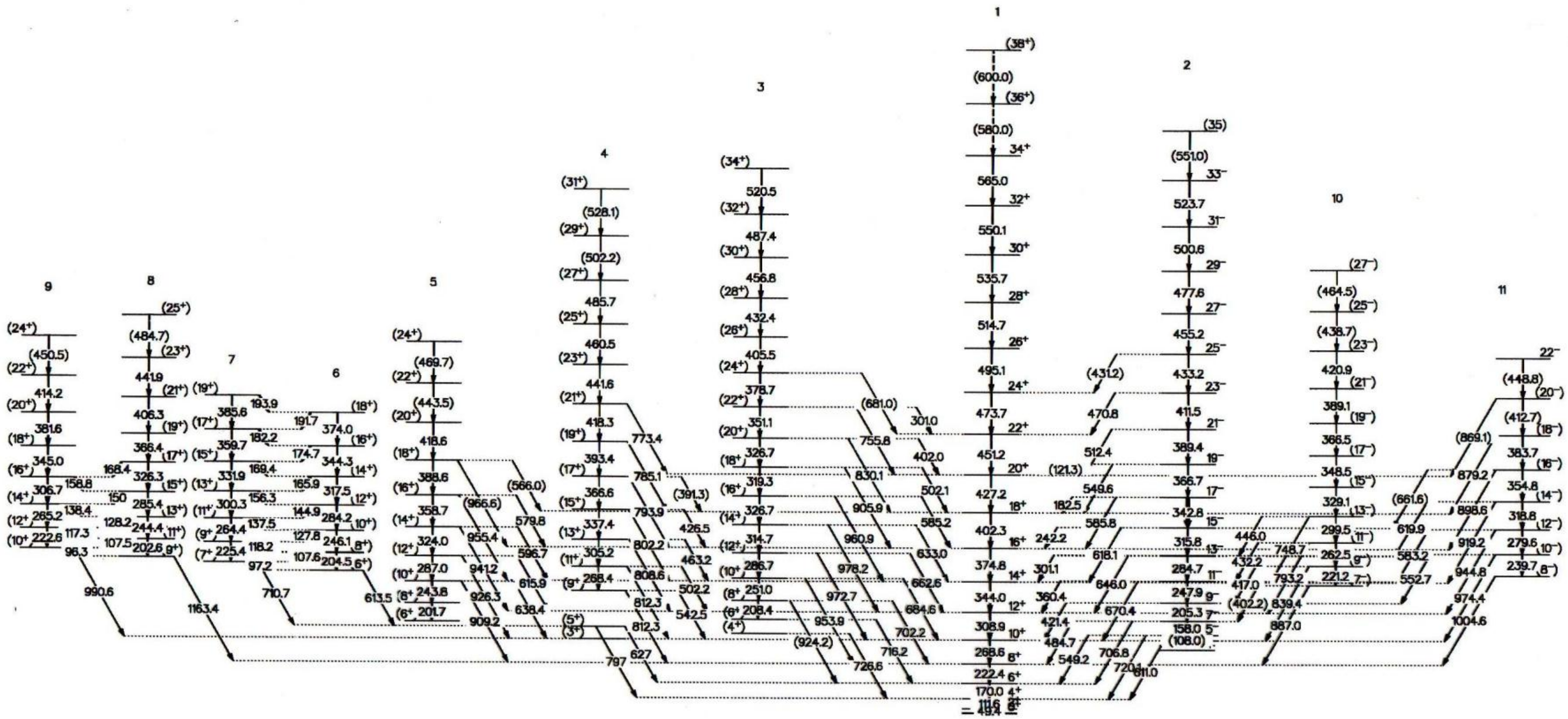
$$a_{20} = \beta\cos\gamma \qquad a_{22} = a_{2-2} = \frac{1}{\sqrt{2}}\beta\sin\gamma$$

Lund convention





Rotation Coupled to Vibration

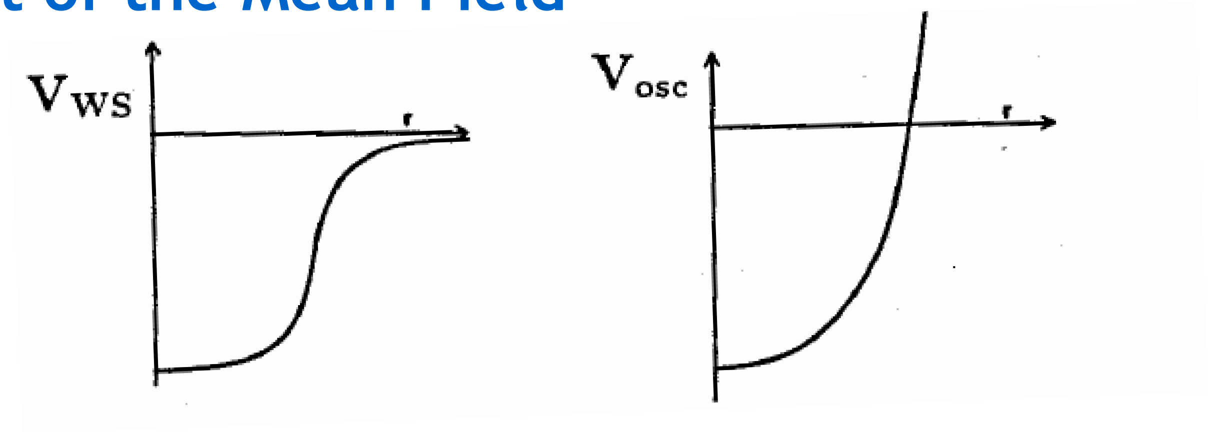


Double Gamma **Beta** **Gamma** **K=0^{-} Octupole** **K=1^{-} Octupole**

^{232}Th

K. Abu Saleem, Ph D. Thesis, Argonne Nat. Lab.

Concept of the Mean Field



Wood Saxon Harmonic Oscillator

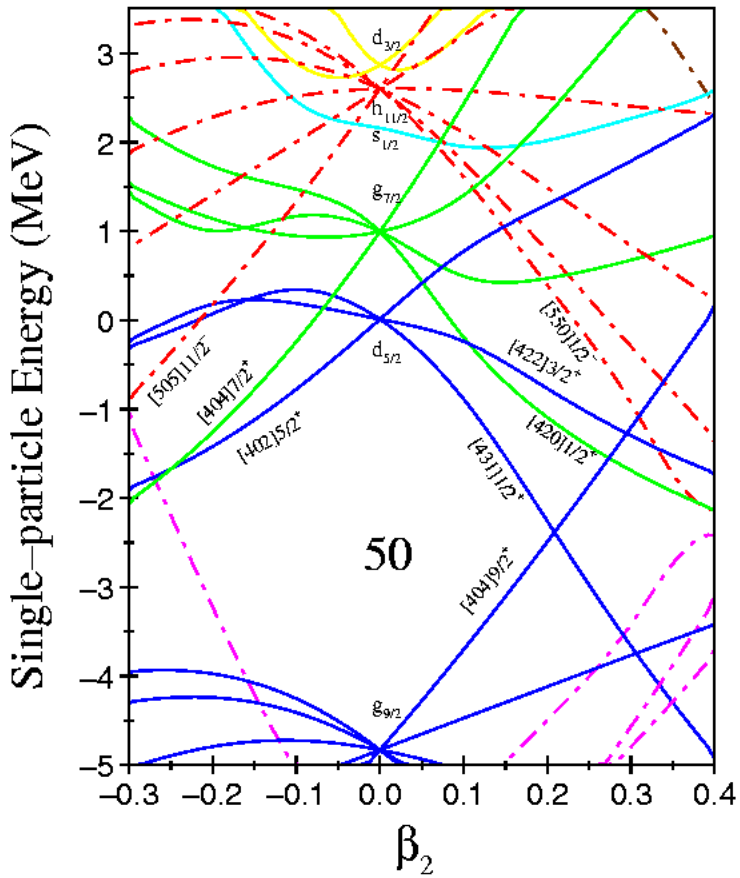
$$V_{ws} = V_c(\mathbf{r};\hat{\beta}) + V_{SO}(\mathbf{r};\hat{\beta}) + \frac{1}{2}(1+\tau_3)V_{coul}(\mathbf{r};\hat{\beta})$$

where $$V_c = \frac{V_0}{1+\exp\left[\frac{r-R}{a}\right]}$$

$$V_{mo} = V_{osc} - 2\kappa\hbar\omega_0\left[l_t \bullet s_t - \mu\left(l_t^2 - \langle l_t^2 \rangle\right)\right]$$

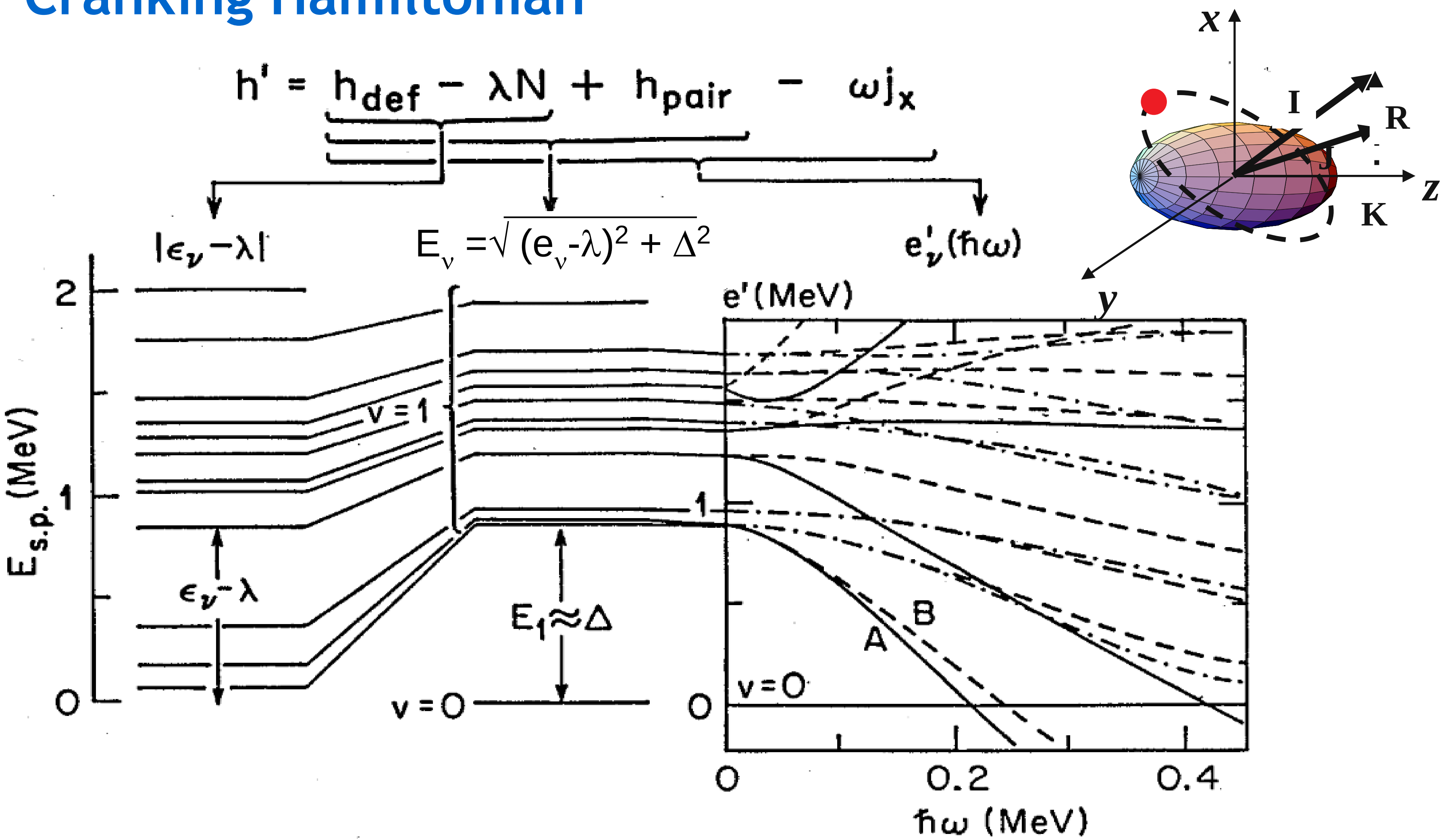
For axial symmetry $$V_{osc} = \frac{1}{2}\hbar\omega_0\rho^2\left[1-\frac{2}{3}\varepsilon_2 P_2(\cos\vartheta_t)\right]$$

Nilsson Diagrams



- Asymptotic labels ($[Nn_z\Lambda]\Omega$)valid at extreme deformations
- Spherical labels quickly break down with deformation, but are fairly good for the unique-parity states
- The slope of a Nilsson level is related to the intrinsic s.p. moment
- For odd-A deformed systems, the angular momentum of the band-head of a rotational band usually coincides with Ω

Cranking Hamiltonian



- Cranking Hamiltonian supplies energies in the rotating frame for each quasiparticle
- Note how rotation (ωj_x) breaks time reversal symmetry of quasiparticle pairs
- Alignment of quasiparticle along rotational axis is give by $i = \partial e' / \partial \omega$.

Rotation of Deformed Nuclei

rotational axis ⊥ symmetry axis

$$E(I)=\frac{\hbar^2}{2\mathrm{J}}\left[I(I+1)-K^2\right]$$

kinematic moment of inertia

$$\mathrm{J}^{(1)}=I\left(\frac{\partial E}{\partial I}\right)^{-1}=\frac{I}{\hbar\omega}\approx\frac{\Delta I\,\langle I\rangle}{E_\gamma}$$

dynamic moment of inertia

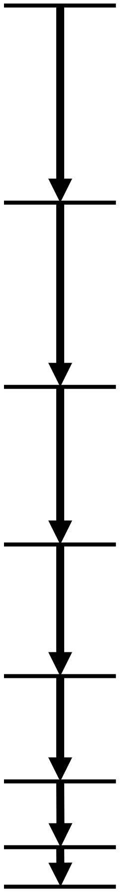
$$\mathrm{J}^{(2)}=\left(\frac{\partial^2 E}{\partial I^2}\right)^{-1}=\frac{\partial I}{\hbar\partial\omega}\approx\frac{(\Delta I)^2}{\Delta E_\gamma}$$

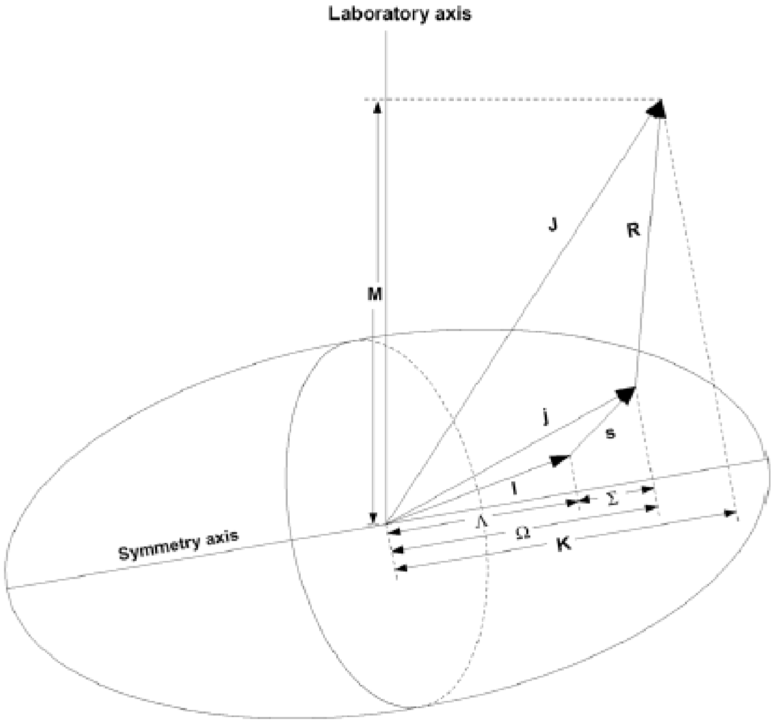
$$\mathrm{J}^{(2)}=\mathrm{J}^{(1)}+\omega\frac{\partial \mathrm{J}^{(1)}}{\partial\omega}$$

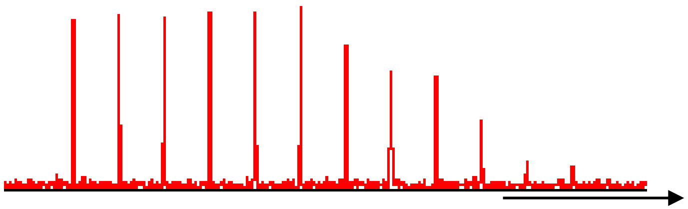
$\mathrm{J}^{(2)}$ measures the variation of J
rigid rotor: $\mathrm{J}^{(2)}=\mathrm{J}^{(1)}$

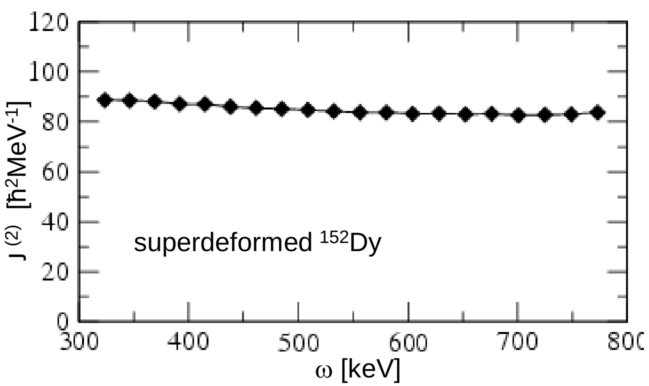
rotational frequency

$$\hbar\omega=\frac{\partial E}{\partial I}\approx\frac{E_\gamma}{\Delta I}$$

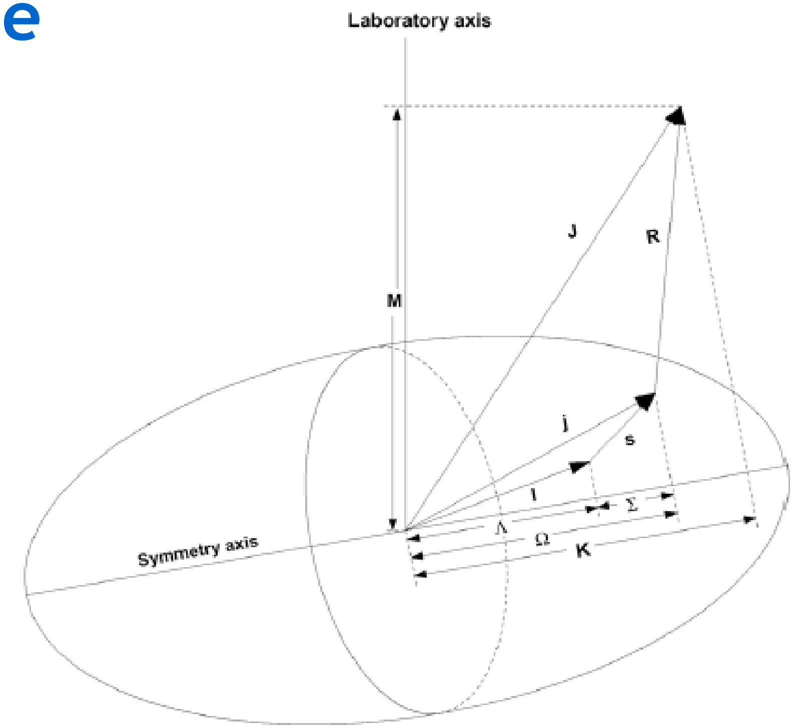








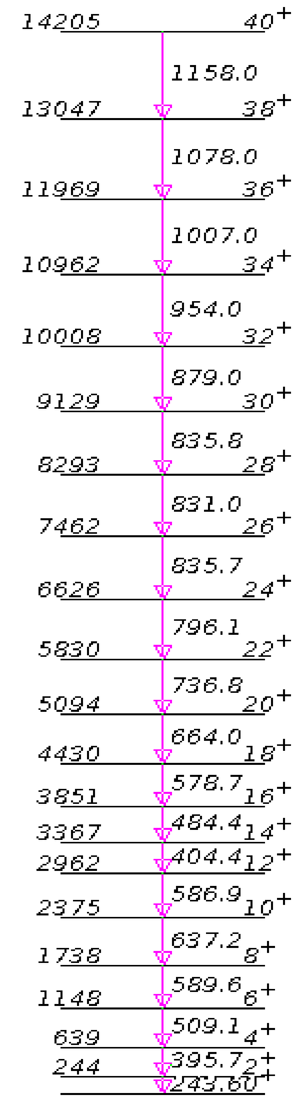
Nuclear Level Schemes Transformed to Rotating Frame



$$\omega(I) = \frac{E(I+1) - E(I-1)}{I_x(I+1) - I_x(I-1)}$$

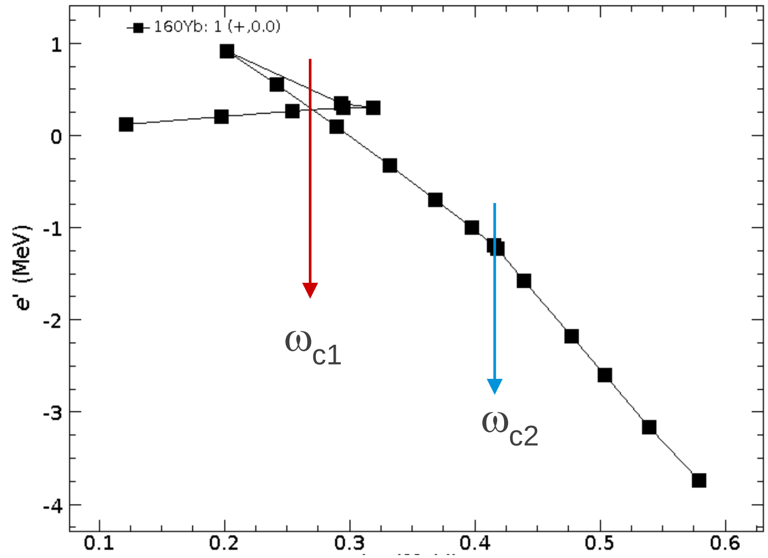
$$I_x(I) = \sqrt{(I+1/2)^2 - K^2}$$

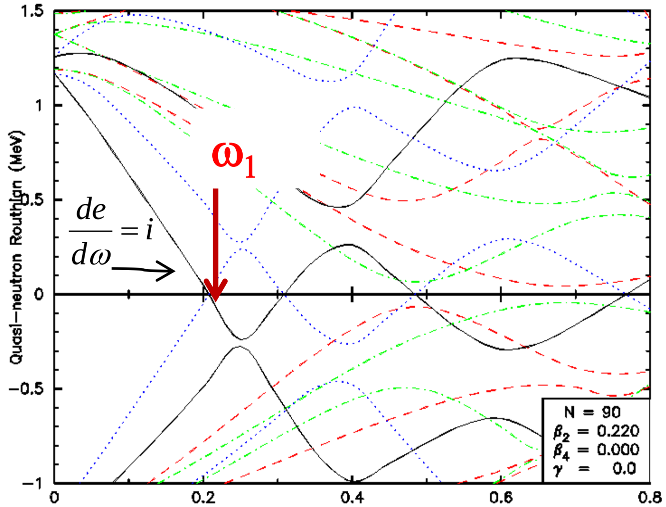
$$E'(\omega) = (E(I+1) + E(I-1))/2 - \omega I_x(I)$$

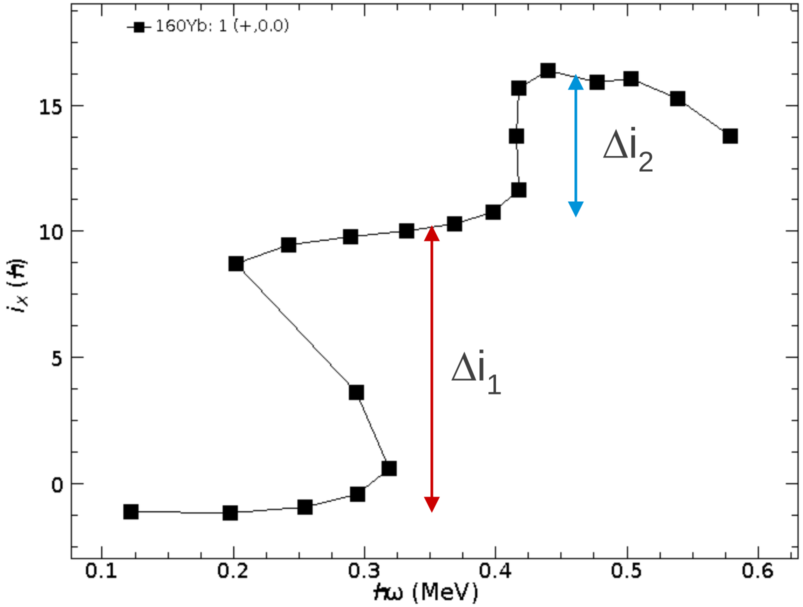


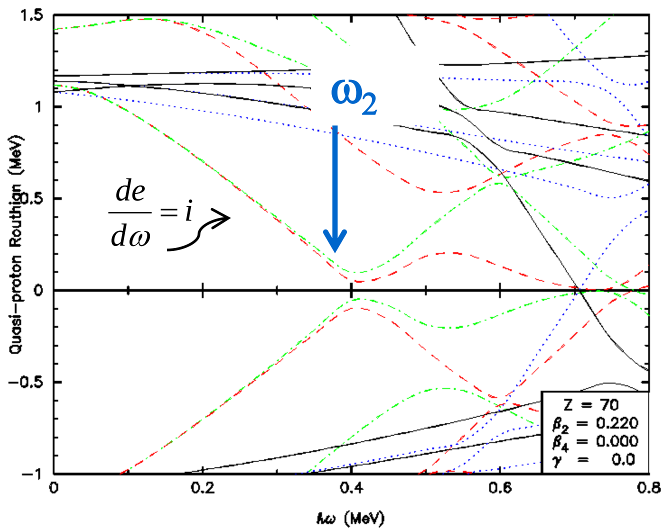
^{160}Yb

Alignments and Crossing Frequencies

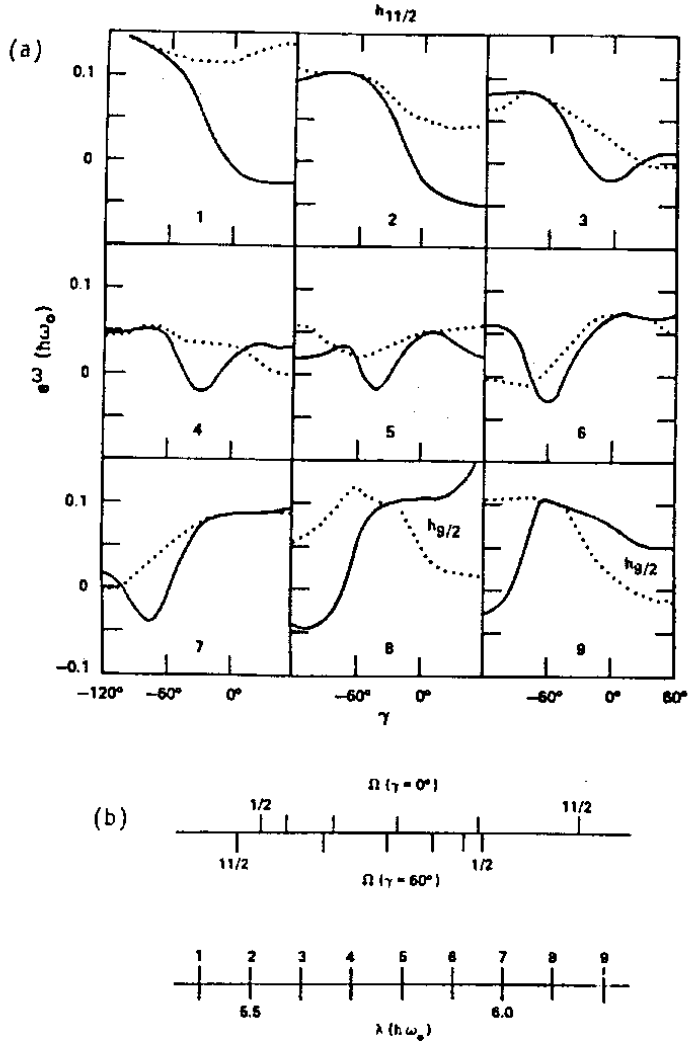








Deformation Driving Effects of Quasiparticles



G.A. Leander. S. Frauendorf, F.R. May, in *High Angular Momentum Properties,* ed. by N.R. Johnson (1982) p.281

Shell Correction Method

Total Energy of nuclei can be calculated using two-body interaction:

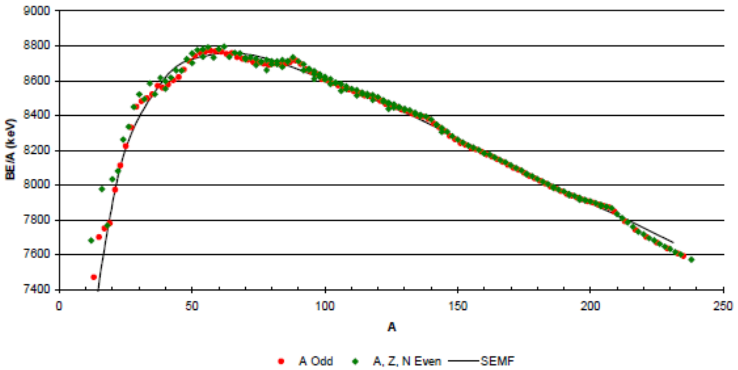
$$E_0 = \sum_m e_m^{(HF)} - \frac{1}{2}\int \rho(\mathbf{r})\rho(\mathbf{r}')V(\mathbf{r},\mathbf{r}')d^3\mathbf{r}d^3\mathbf{r}'$$

Strutinsky showed that total energy could be calculated from mean field

$$E_0 \approx E_{SC} = E_{LD} + \delta E_{sh}$$

where

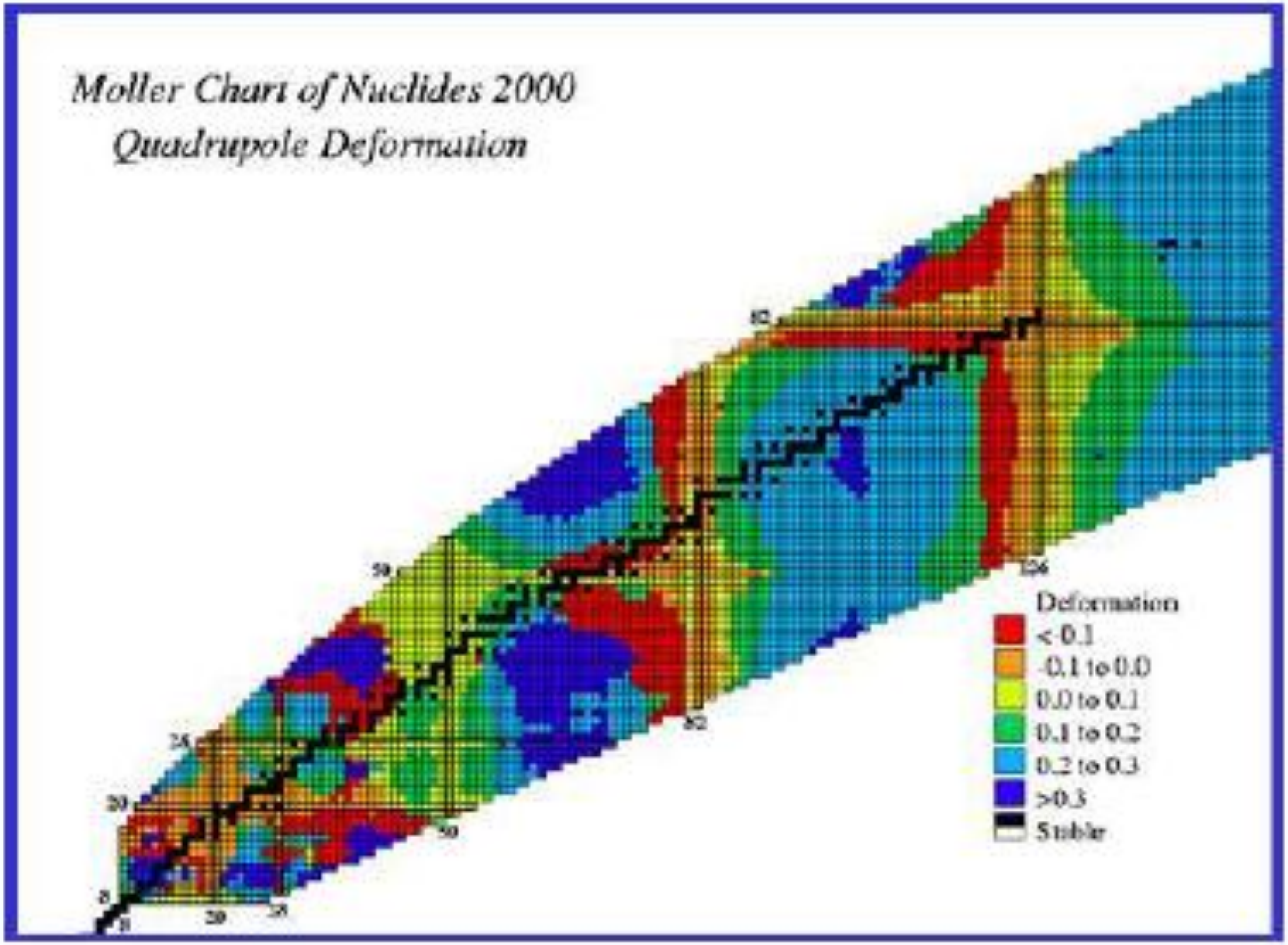
$$\delta E_{sh} = E_{sh} - \left\langle \tilde{E} \right\rangle$$



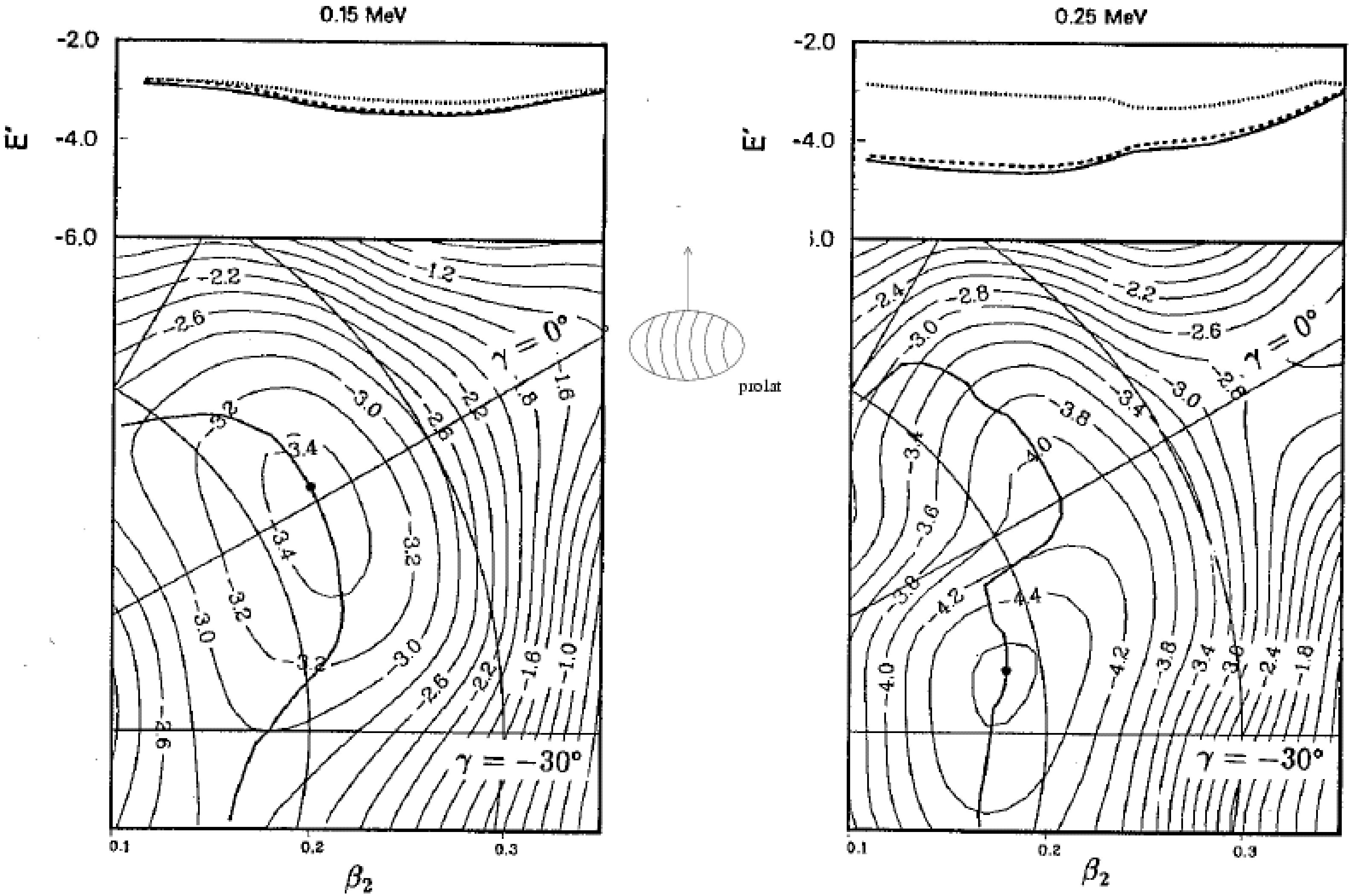
It can then be shown

$$R^{\omega} = E^{LD} + \delta E_{sh}^{\omega=0} + \sum\left(e_i^{\omega} - e_i^{\omega=0}\right)$$

Global Calculation of Ground State Deformations



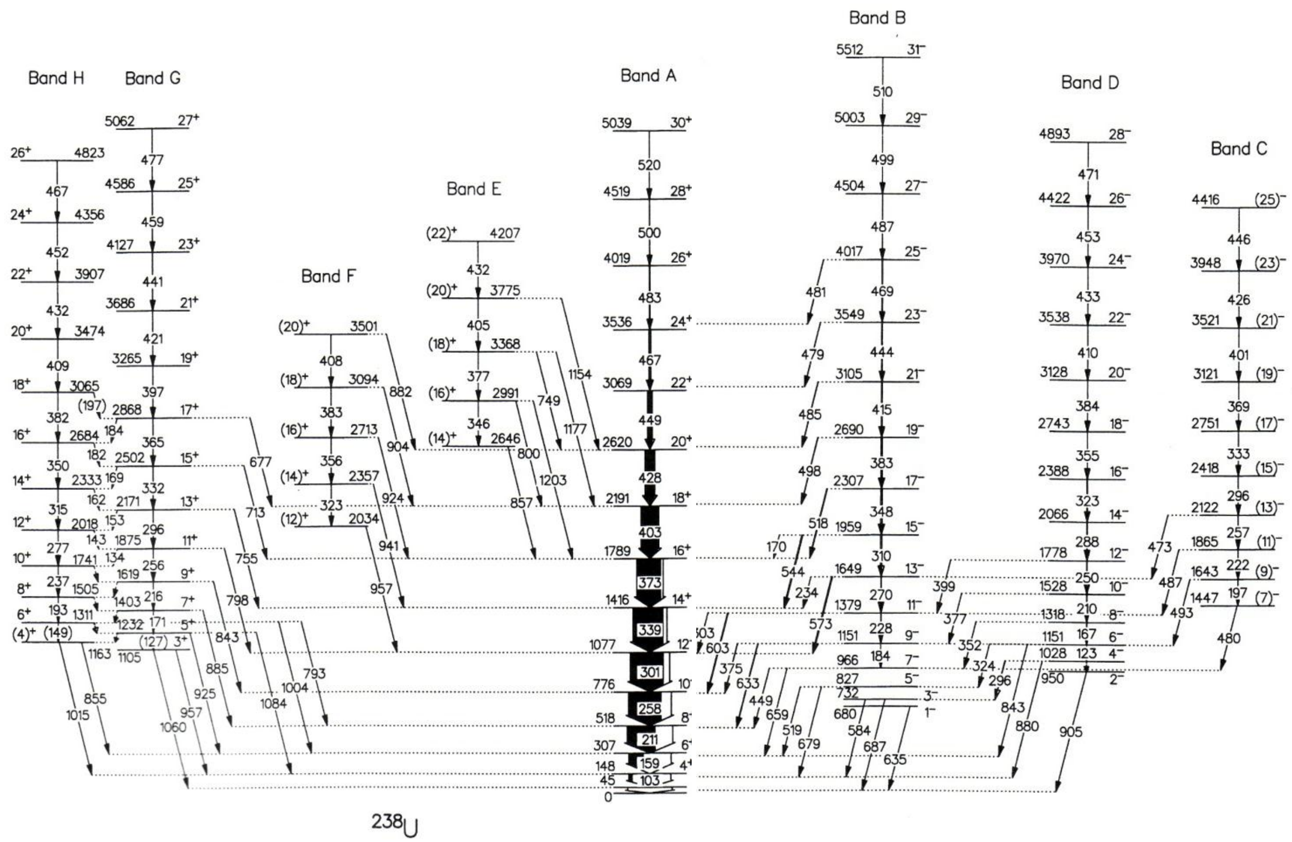
Total Routhian Surfaces



^{184}Pt Yrast

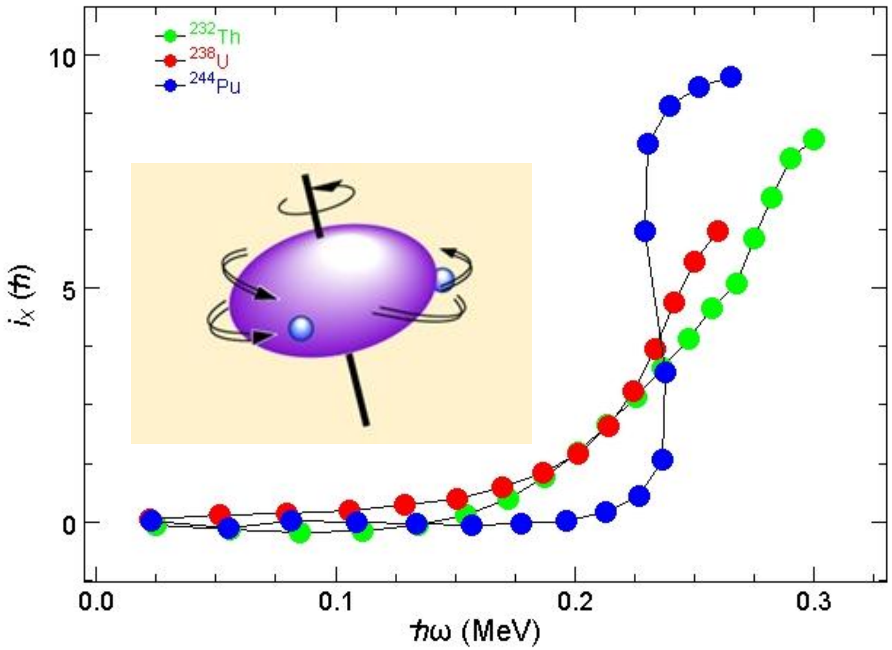
A Practical Example

Actinide Region Show Deformed Systems

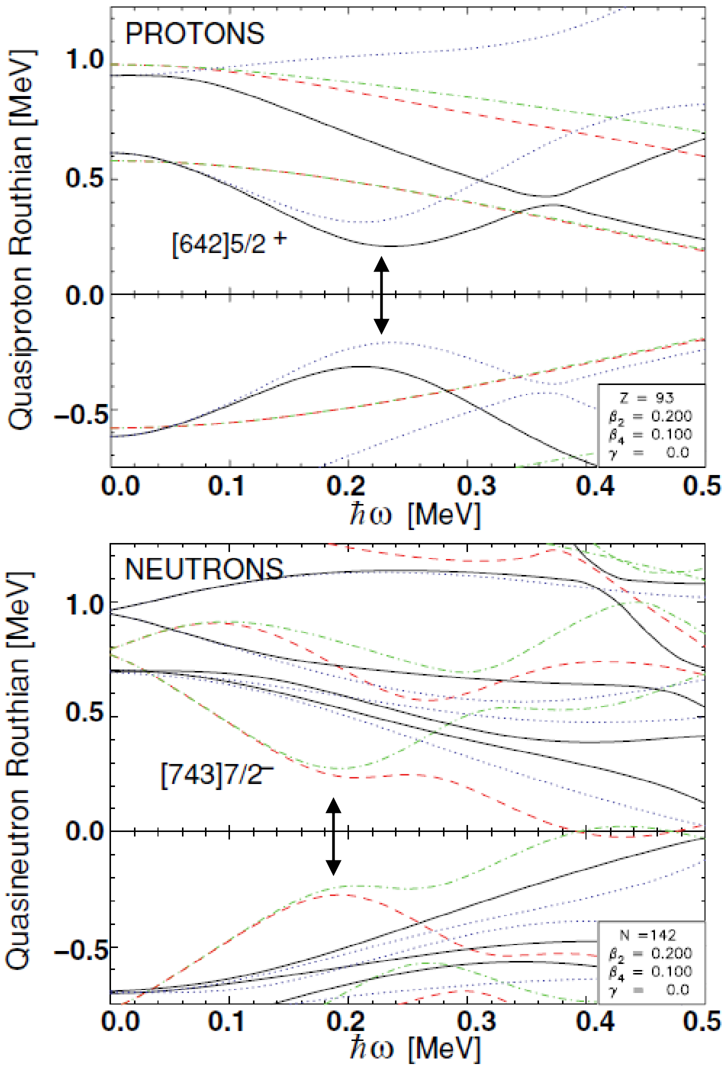


D. Ward *et al.,* Nucl. Phys. A 600 (1996) 88.

Selected Even-Even Yrast Bands



$$i(\omega) = I_x(\omega) - I_x^g(\omega)$$



CSM calculations indicate that the single band crossing observed results from simultaneous alignment of neutrons ($j_{15/2}$) and protons ($i_{13/2}$)

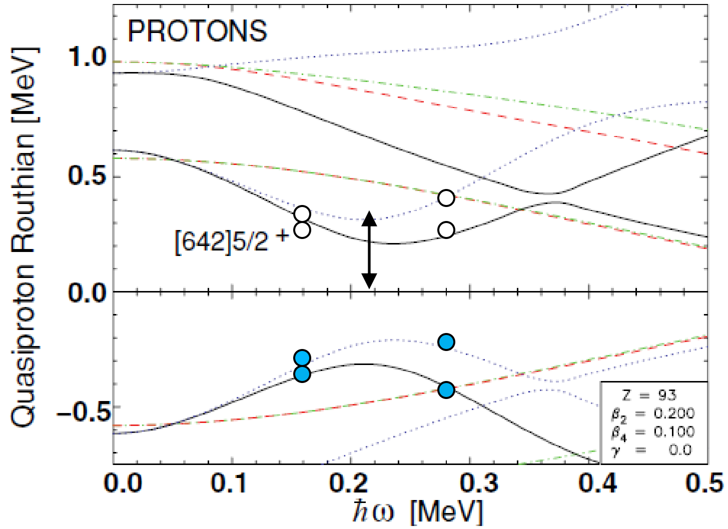
How Do We Confirm This Experimentally?

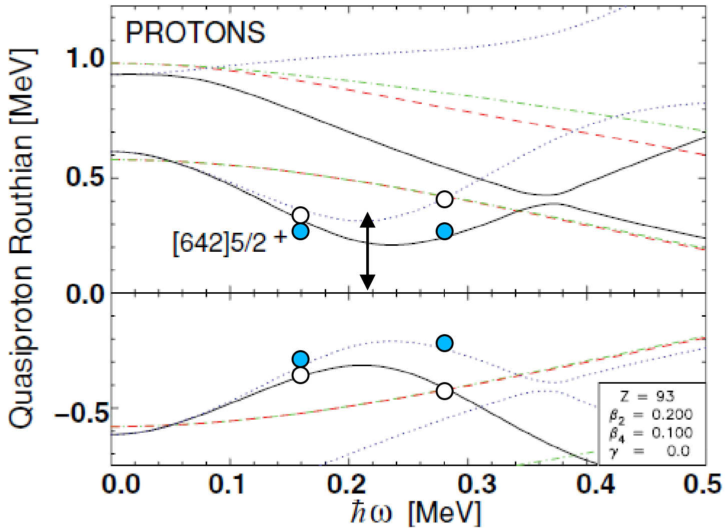
- In the even-even system, the slope of the occupied Routhians change after the crossing corresponding to the alignment of the quasiparticle pair.

- In an odd-A system excitations in the CSM are made by promoting particles into the appropriate quasiparticle orbital.

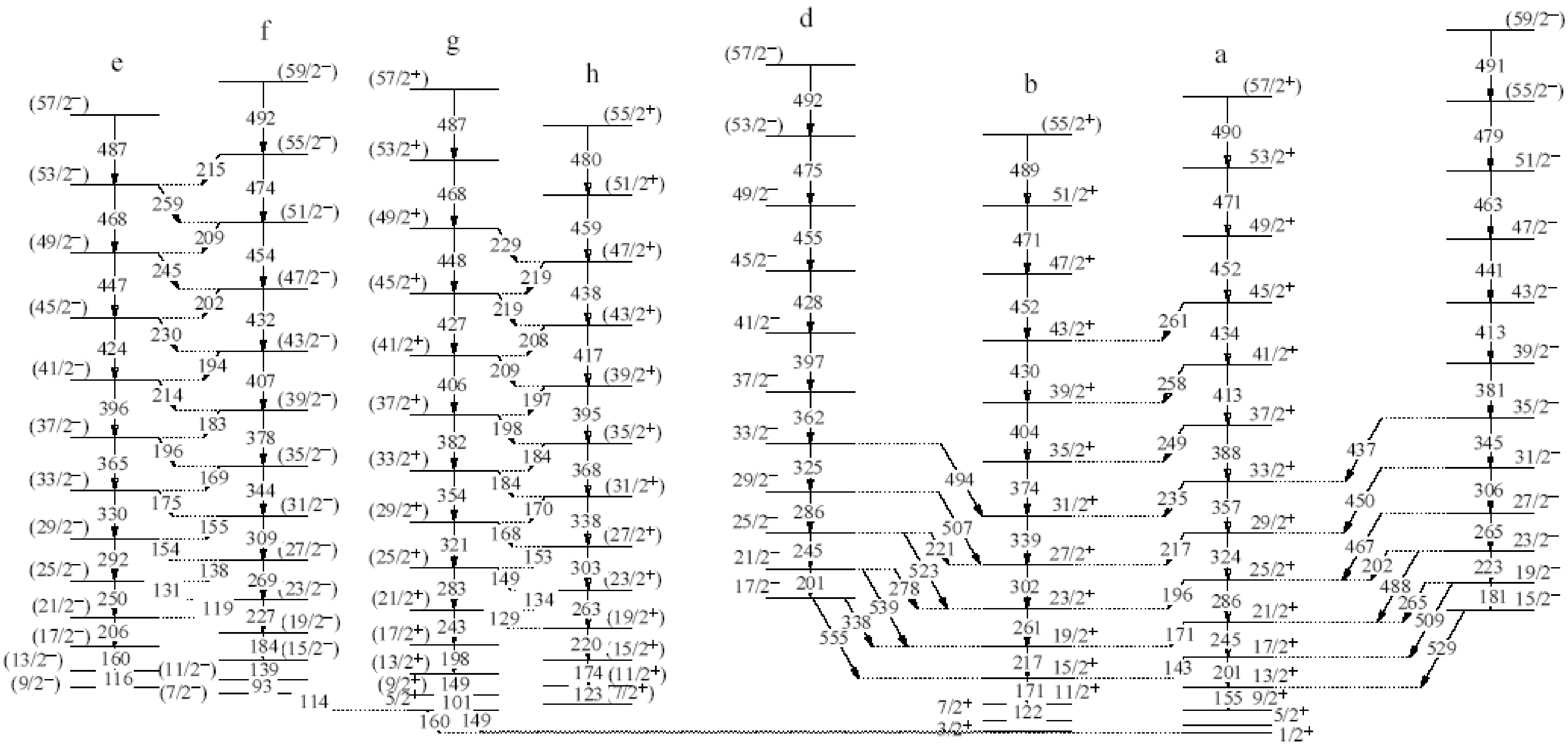
- After the band crossing the occupied configurations are identical; we say that the alignment is blocked (Pauli Blocking).

- In addition, the occupied orbital results in reduction of pairing correlations.





Identify Single-particle States in Odd Neutron Nuclei



$7/2^-[743]$

$j_{15/2}$

$5/2^+[622]$

$i_{11/2}$

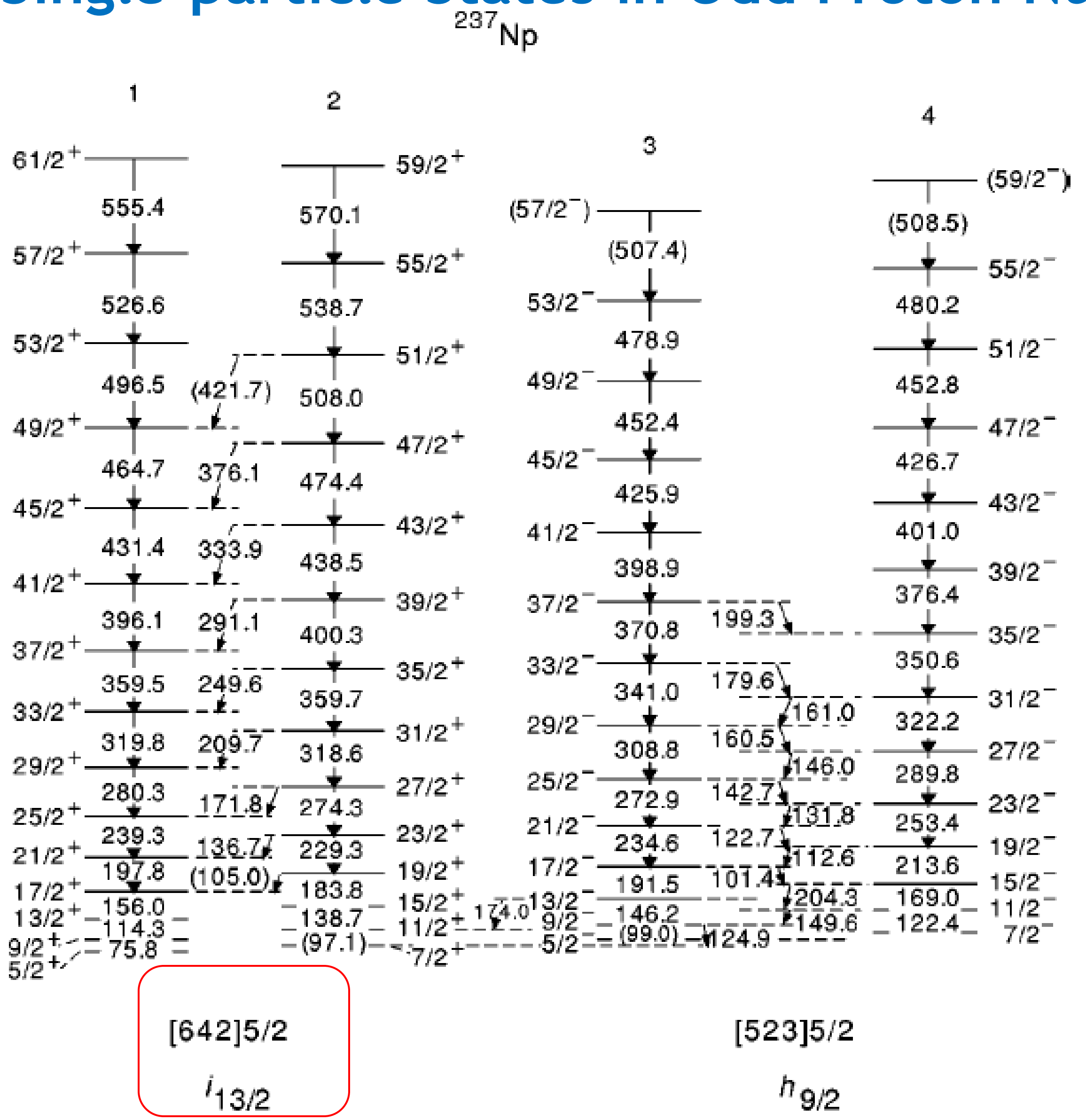
^{237}U

$1/2^+[631]$

$d_{5/2}$

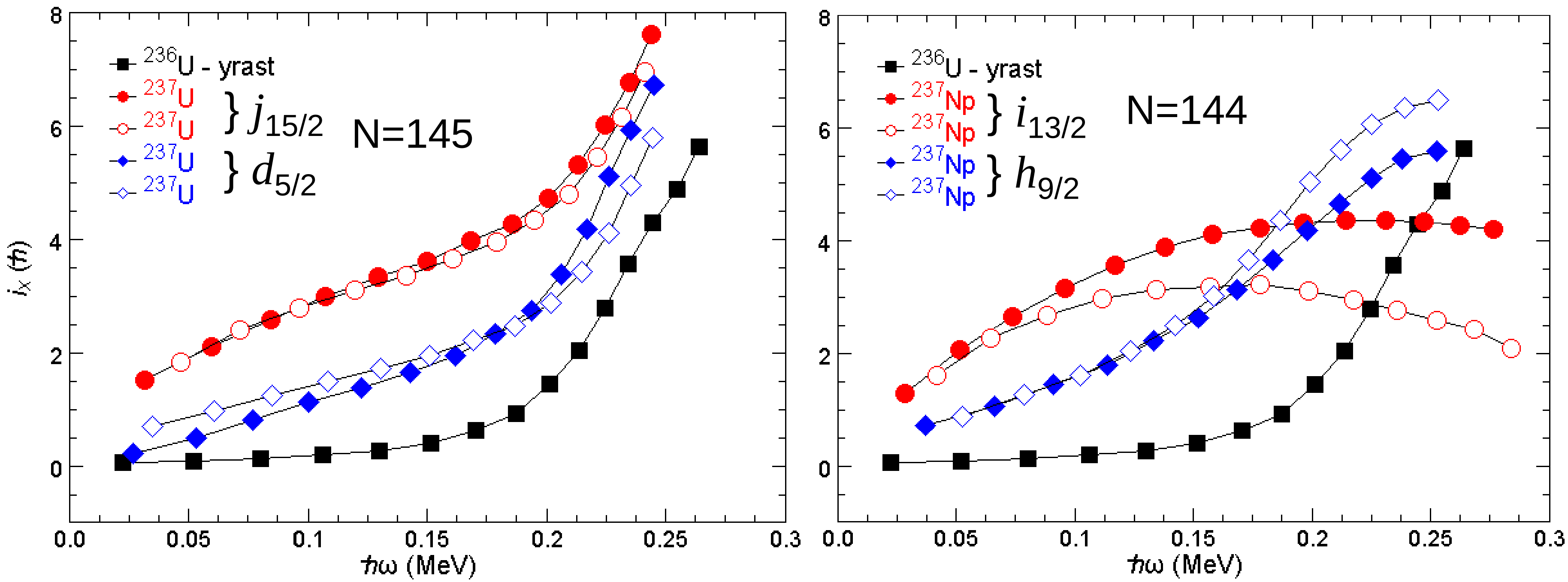
S. Zhu *et al*., Phys. Lett. B618, 51 (2005)

Identify Single-particle States in Odd Proton Nuclei



K. Abu Saleem *et al*, Phys. Rev. C 70, 024310

Blocking Arguments using Alignment Plots



- ^{237}U – odd-*N*, $j_{15/2}$ band similar to $d_{5/2}$ band ($\pi i_{13/2}$ alignment) N=145.
- ^{237}Np – odd-*Z*, $i_{13/2}$ band no upbend (no $\nu j_{15/2}$ alignment) N=144.

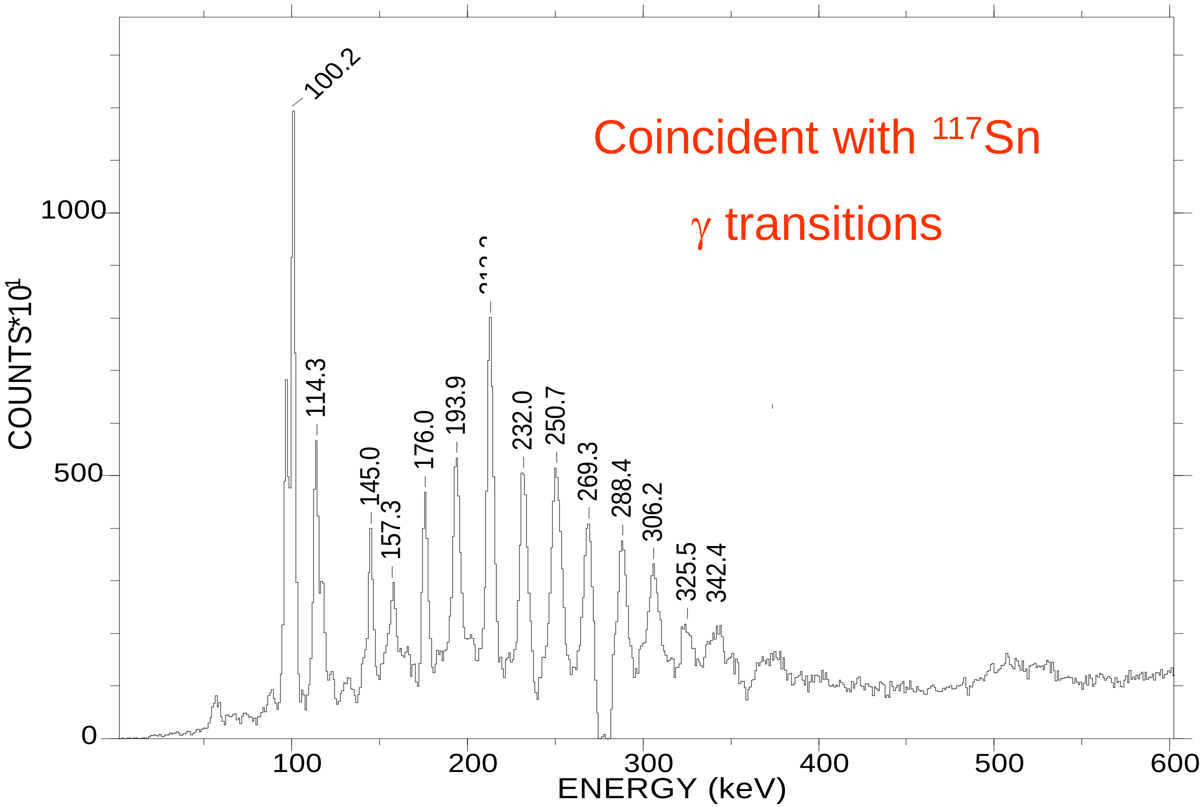
Where Did the $j_{15/2}$ Alignment Go?

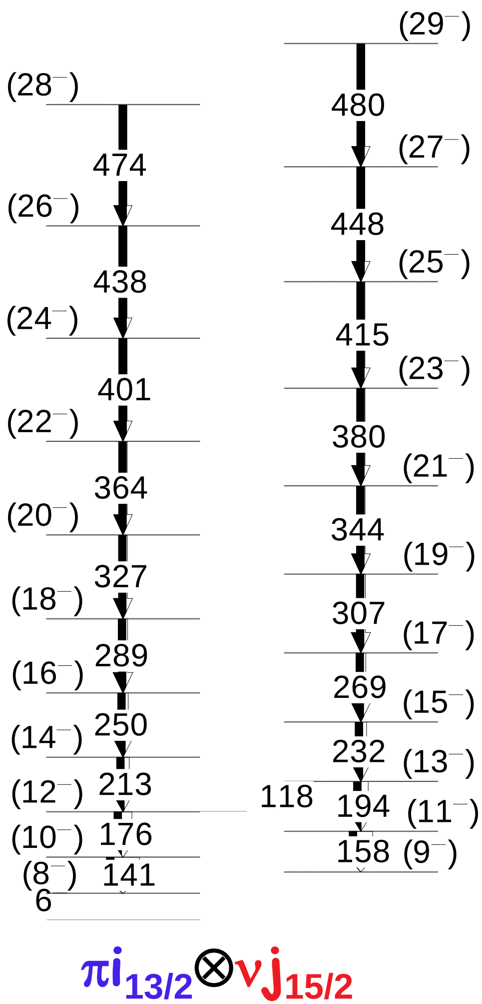
1. In CSM calculations the neutron pairing is found to be quite weak which could result in slow decoupling of $j_{15/2}$ pairs as function of spin – superdeformed 190 region.

2. CSM calculations are wrong and actual alignment is at higher rotational frequency.

- Current data cannot distinguish which is correct interpretation
- Odd-Odd nuclei might aid in distinguishing 1 from 2.

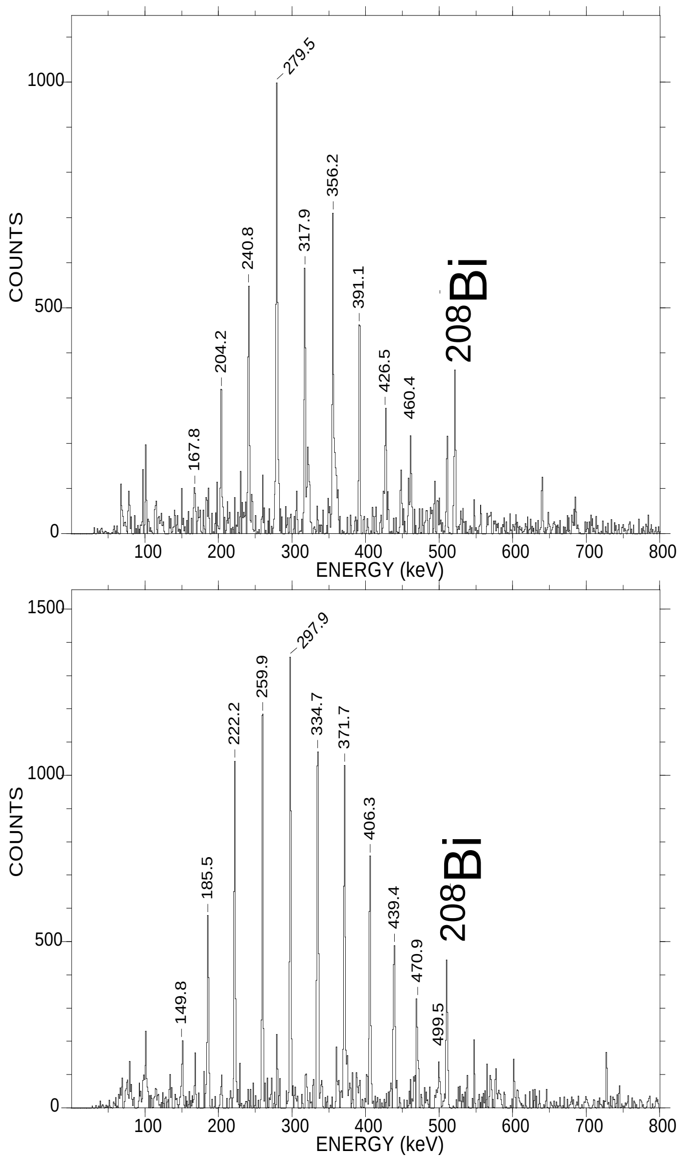
New Results for ^{236}Np

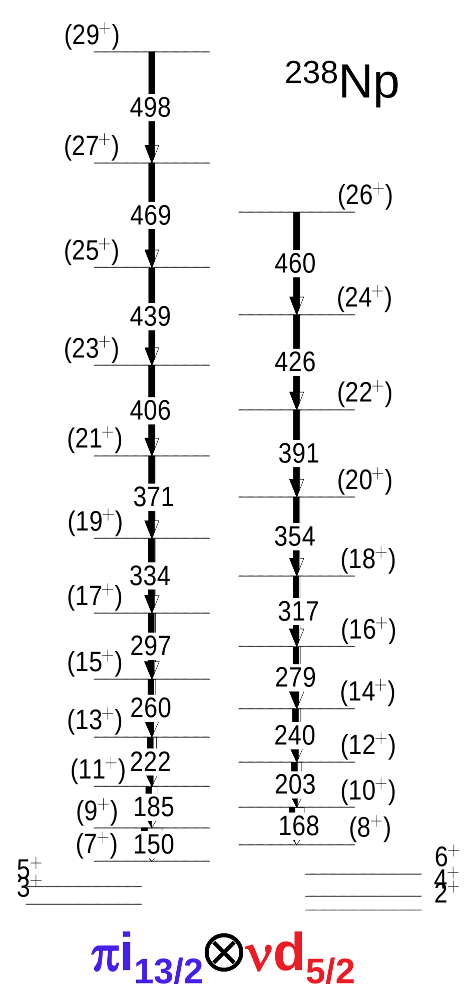




- Reaction: ^{116}Sn + ^{237}Np $\rightarrow$ ^{117}Sn + ^{236}Np
- Thin target using Gammasphere + Chico
- Doppler correct for target-like and projectile-like fragments.

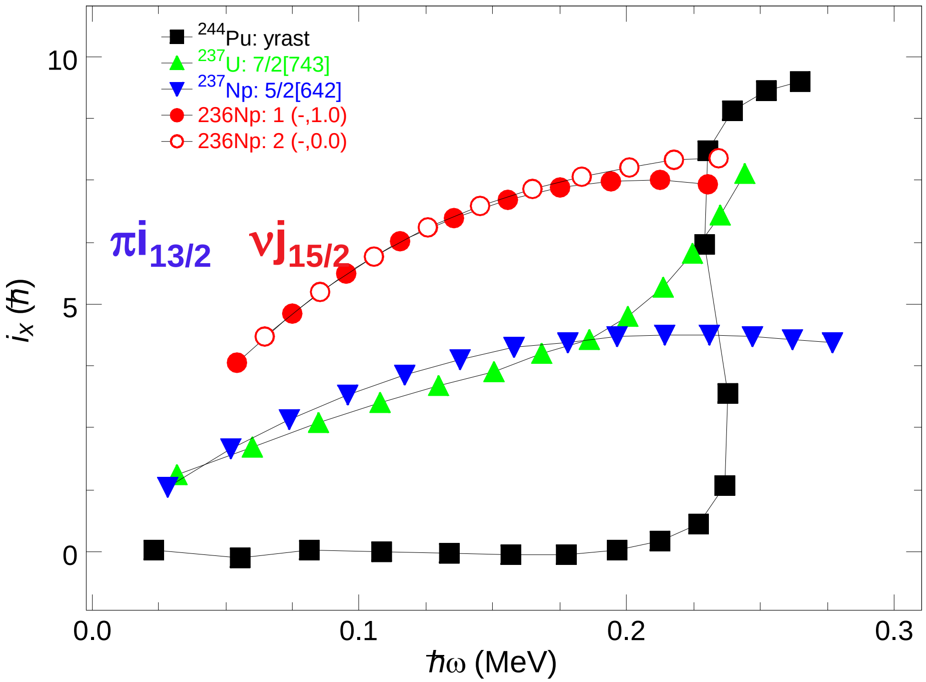
New results on ^{238}Np

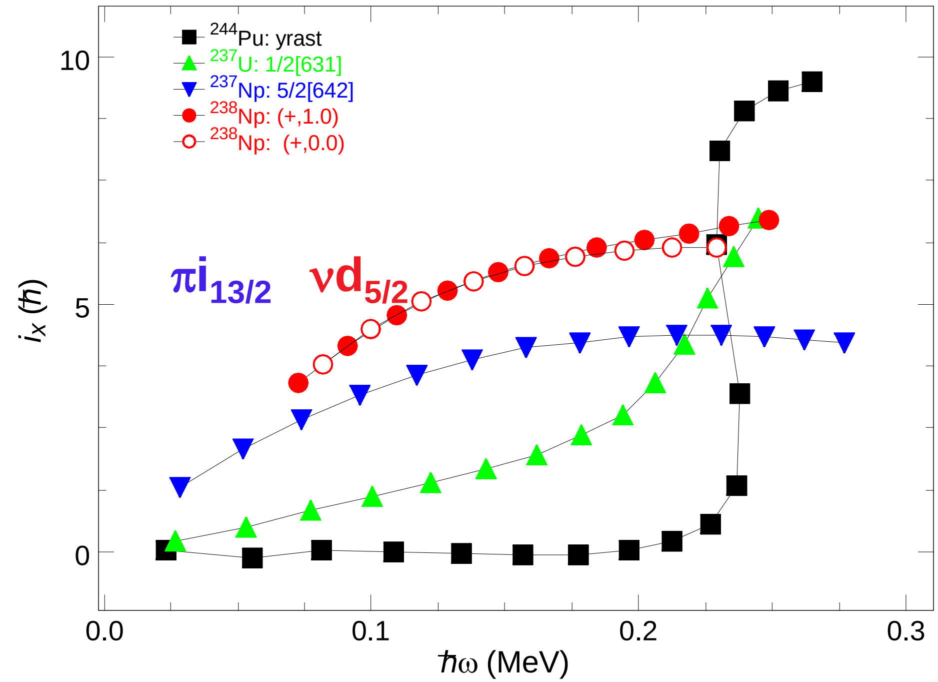




- Thick target ^{209}Bi + ^{237}Np @ 1450 MeV
- Identification to ^{238}Np by cross coincidences with ^{208}Bi.

What do the Odd-Odd Nuclei Tell Us?





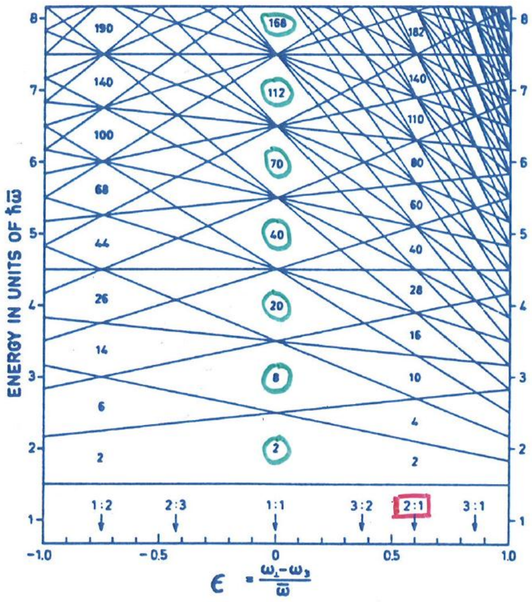
- The alignment characteristics of the ^{236,238}Np bands can only be understood if the proton configuration is 5/2[642] *i.e*. the $i_{13/2}$ proton alignment is blocked.
- The difference in total alignment between bands of ^{236}Np and ^{238}Np results from different neutron configurations ($j_{15/2}$ vs $d_{3/2}$).
- The near identical trajectories of the ^{236,238}Np bands contradict the CSM calculations which predict the alignment of a $\nu j_{15/2}$ pair for $\hbar\omega < 0.25$ MeV.

Lessons Learned

- In the study of nuclei at high-spin, it is essential to explore the rotational properties of not only the even-even, but odd-A and odd-odd nuclei as well.
- Placing particles in different orbitals and observing their response to rotation allows for a deep understanding of the underlying nuclear structure.
- The reason for the missing $j_{15/2}$ alignment is unknown and still and open questioned to be pursed.
- You should not consider the breakdown of the CSM as a failure but rather and opportunity to solve an outstanding problem.
- By digging deeper. solutions to this problem may result in deeper understanding.
- I will show another example tomorrow to illustrate this.

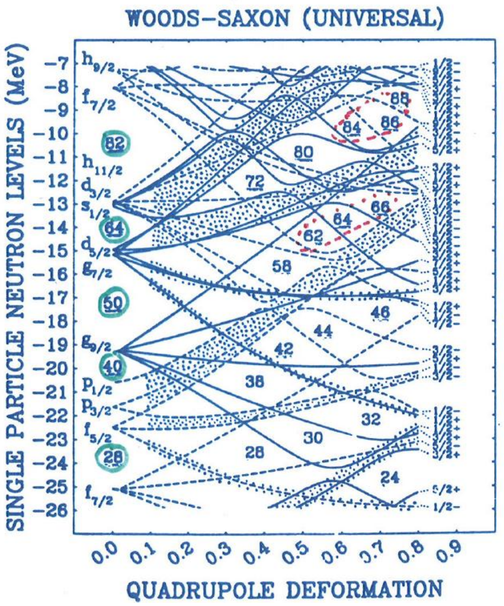
Superdeformation : The Ultimate in High Spin

Superdeformation: Shell Effects at Large Deformation



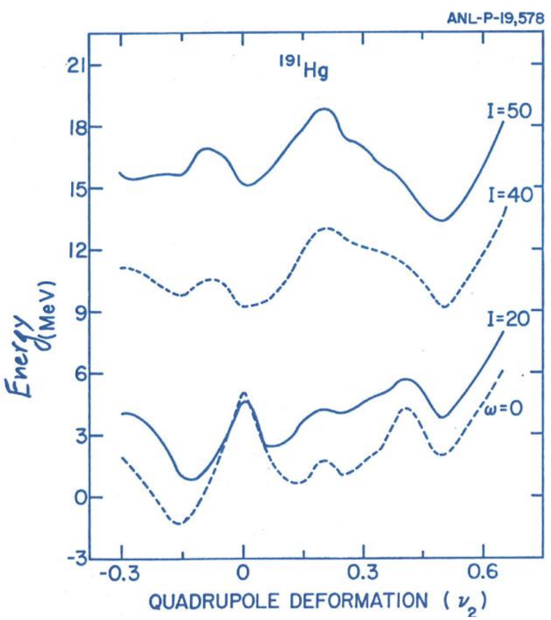
Single-particle levels for an Harmonic Oscillator potential as a function of elongation → Shell gaps at large deformation (2:1, 3:1)

Single-particle levels for a Woods-Saxon potential (high level density regions are shaded) → Shell gaps remain, but not necessarily at 2:1 or 3:1 exactly



Role of Rotation: deepening of the SD minimum :

- Due to large moments of inertia SD becomes yrast at high spin
- Must use fusion-evaporation with heavy ions to feed in to SD well at high spin.



SUPERDEFORMATION

A UNIQUE OPPORTUNITY

(1) TO TEST OUR UNDERSTANDING OF THE IMPORTANCE OF SHELL STRUCTURE AT LARGE DEFORMATIONS

(2) TO STUDY THE DECAY FROM ONE POTENTIAL WELL INTO ANOTHER

A FEW OF THE MANY INTERESTING QUESTIONS:

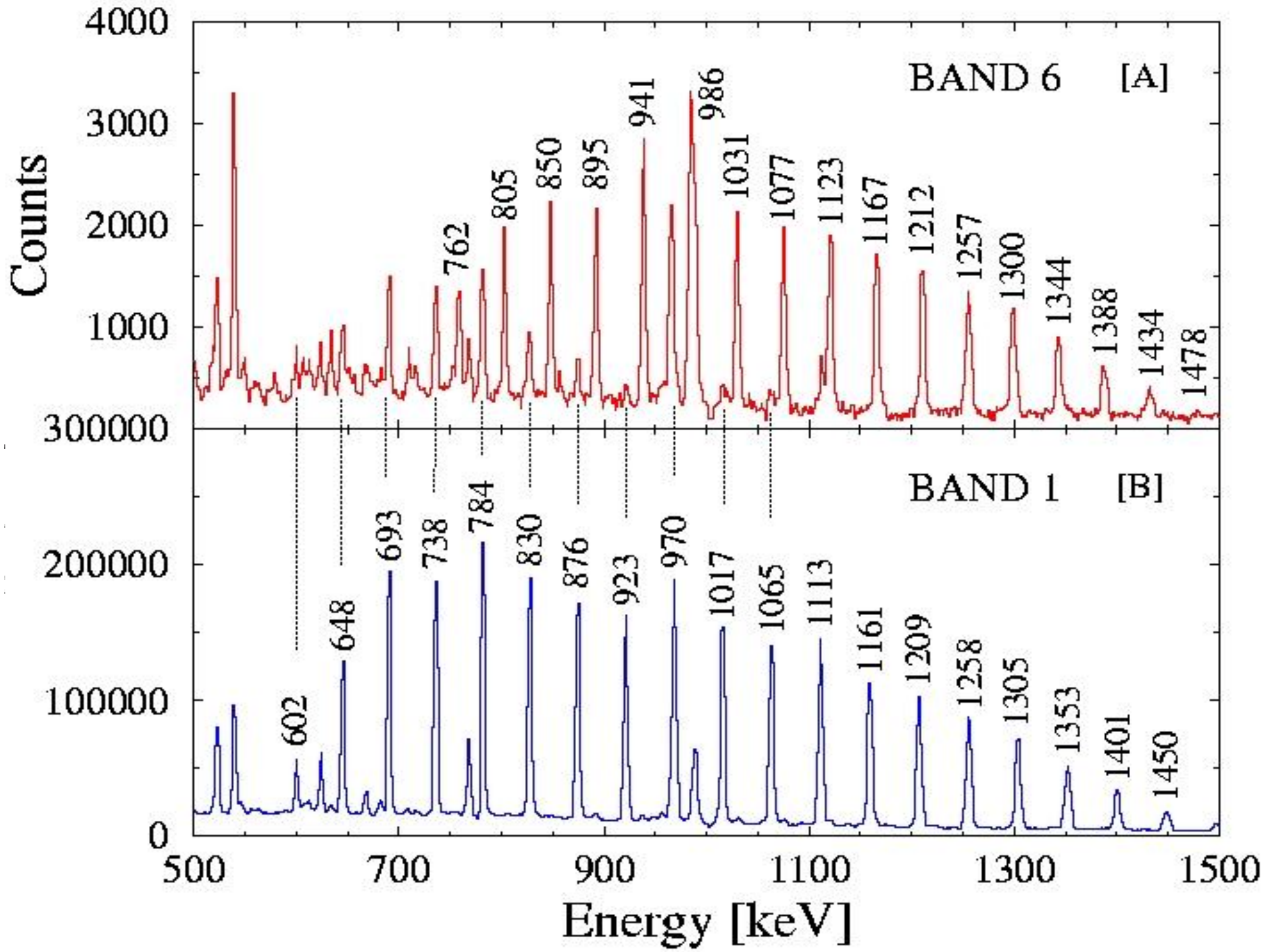
- IDENTIFICATION OF ORBITALS NEAR THE FERMI SURFACE IN THE SD WELL
- ROLE OF SPECIFIC SHAPE DRIVING ORBITALS
- STIFFNESS OF THE SD WELL AND POSSIBLE COLLECTIVE EXCITATIONS
- PAIRING AT LARGE β_2
- CORIOLIS INTERACTION AND PARTICLE ALIGNMENTS AT LARGE β_2 ?
- NEW SYMMETRIES ?
- MORE SURPRISES (IDENTICAL BANDS, $\Delta I = 4$ BIFURCATION ...) ?
- WHICH PARAMETERS GOVERN FEEDING INTO & DECAY OUT OF SD WELL ?
- • • •

The First SD Rotational Band

^{152}Dy

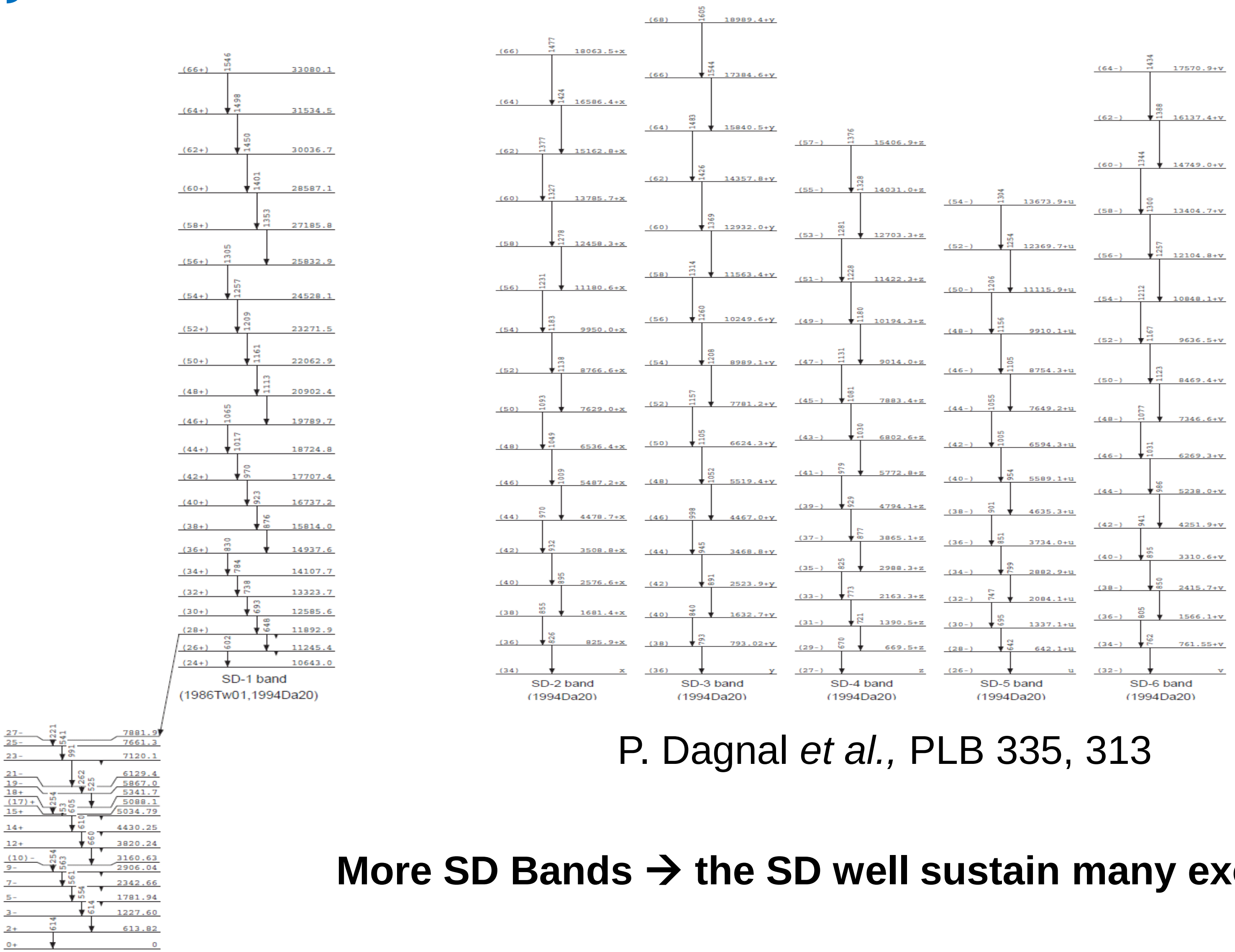
P. Twin *et al.*,

PRL 57 (1986)



- T. Lauritsen *et al.*, Phys. Rev. Lett. **88**, (2002). 042501
- T. Lauritsen *et al.*, Phys. Rev. Lett. **89** (2002) 282501

Many Bands Have Been Identified

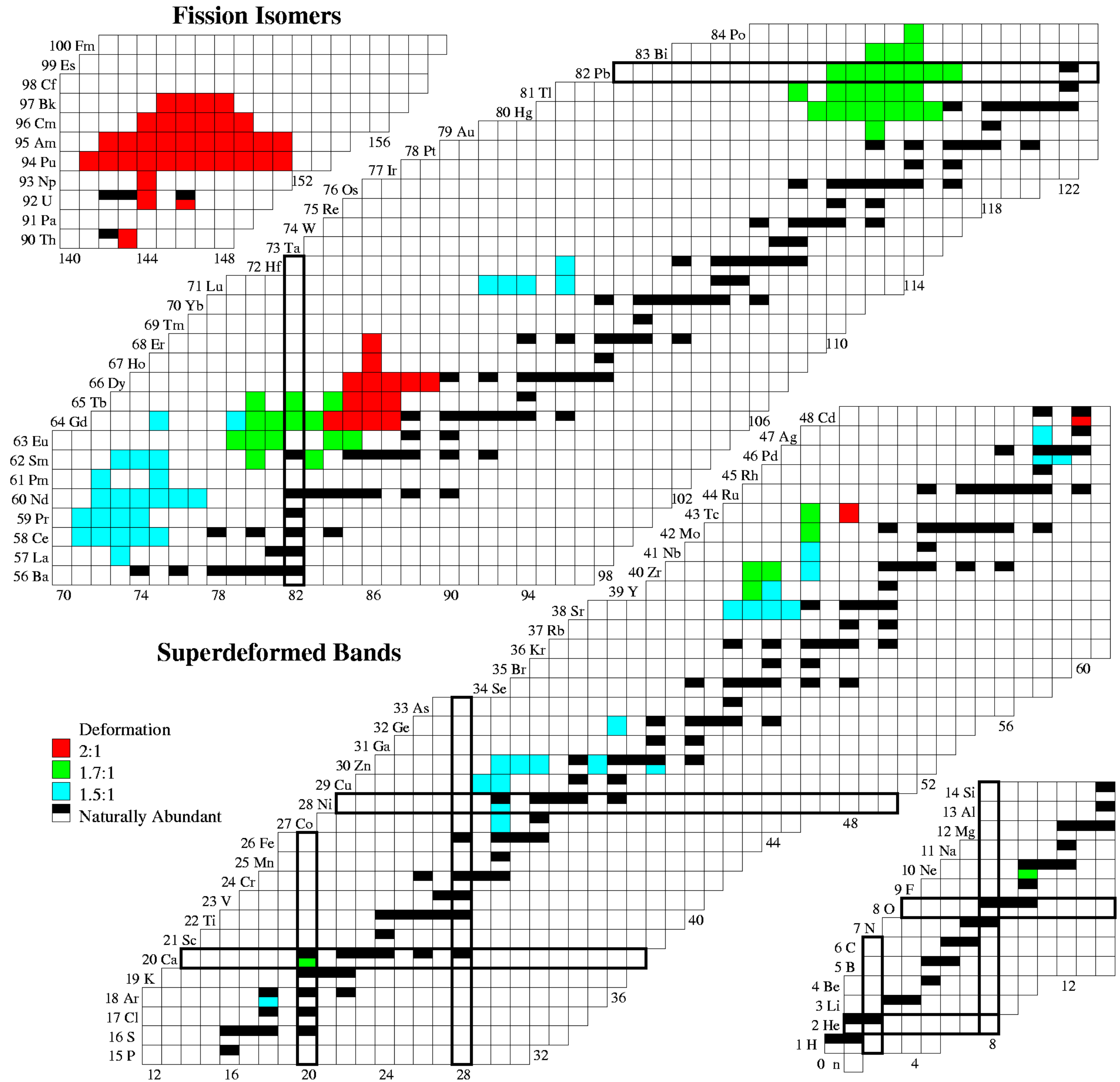


P. Dagnal *et al.*, PLB 335, 313

More SD Bands → the SD well sustain many excitations

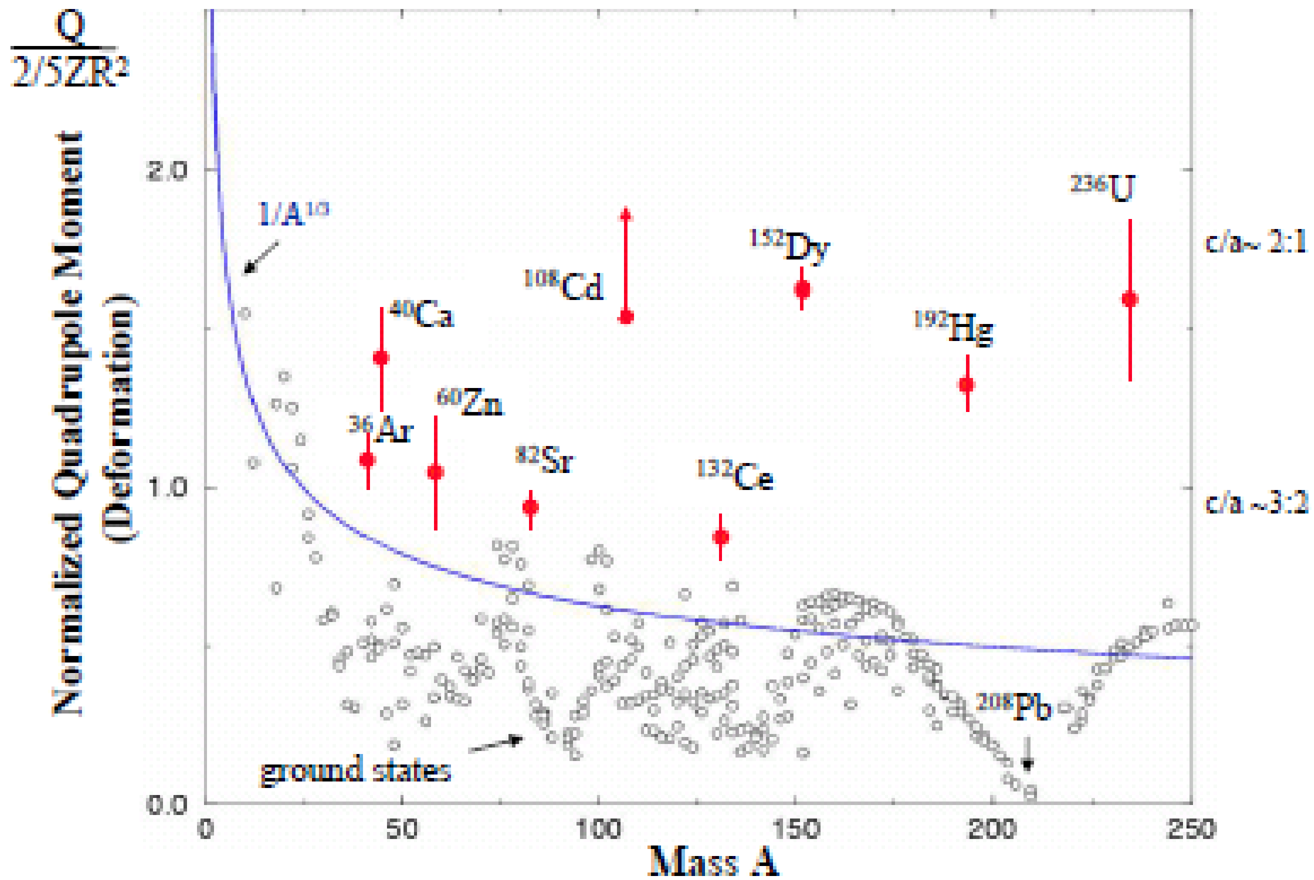
^{152}Dy

SD Regions



Fission Isomers
Superdeformed Bands
Deformation
2:1
1.7:1
1.5:1
Naturally Abundant
100 Fm
99 Es
98 Cf
97 Bk
96 Cm
95 Am
94 Pu
93 Np
92 U
91 Pa
90 Th
140
144
148
152
156
84 Po
83 Bi
82 Pb
81 Tl
80 Hg
79 Au
78 Pt
77 Ir
76 Os
75 Re
74 W
73 Ta
72 Hf
71 Lu
70 Yb
69 Tm
68 Er
67 Ho
66 Dy
65 Tb
64 Gd
63 Eu
62 Sm
61 Pm
60 Nd
59 Pr
58 Ce
57 La
56 Ba
70
74
78
82
86
90
94
98
102
106
110
114
118
122
48 Cd
47 Ag
46 Pd
45 Rh
44 Ru
43 Tc
42 Mo
41 Nb
40 Zr
39 Y
38 Sr
37 Rb
36 Kr
35 Br
34 Se
33 As
32 Ge
31 Ga
30 Zn
29 Cu
28 Ni
27 Co
26 Fe
25 Mn
24 Cr
23 V
22 Ti
21 Sc
20 Ca
19 K
18 Ar
17 Cl
16 S
15 P
12
16
20
24
28
32
36
40
44
48
52
56
60
14 Si
13 Al
12 Mg
11 Na
10 Ne
9 F
8 O
7 N
6 C
5 B
4 Be
3 Li
2 He
1 H
0 n
4
8
12

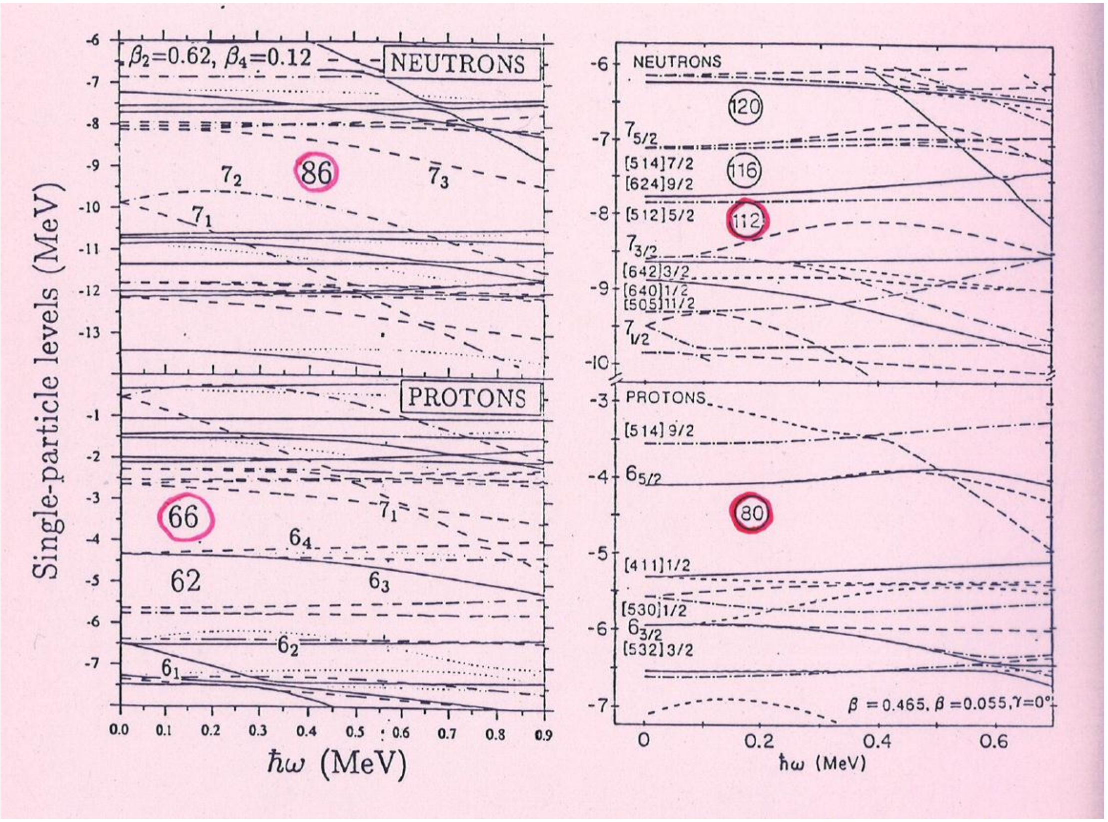
Measured Quadrupole Moments Establish Deformation



Physics of Shell Gaps

Doubly Magic Superdeformed Nuclei

A ~ 150 **A ~ 190**

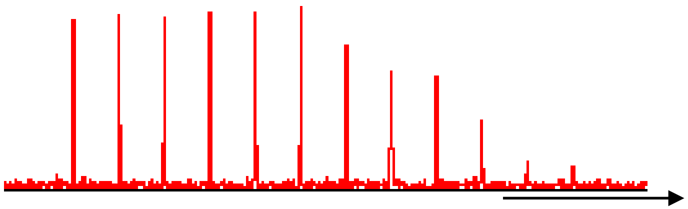


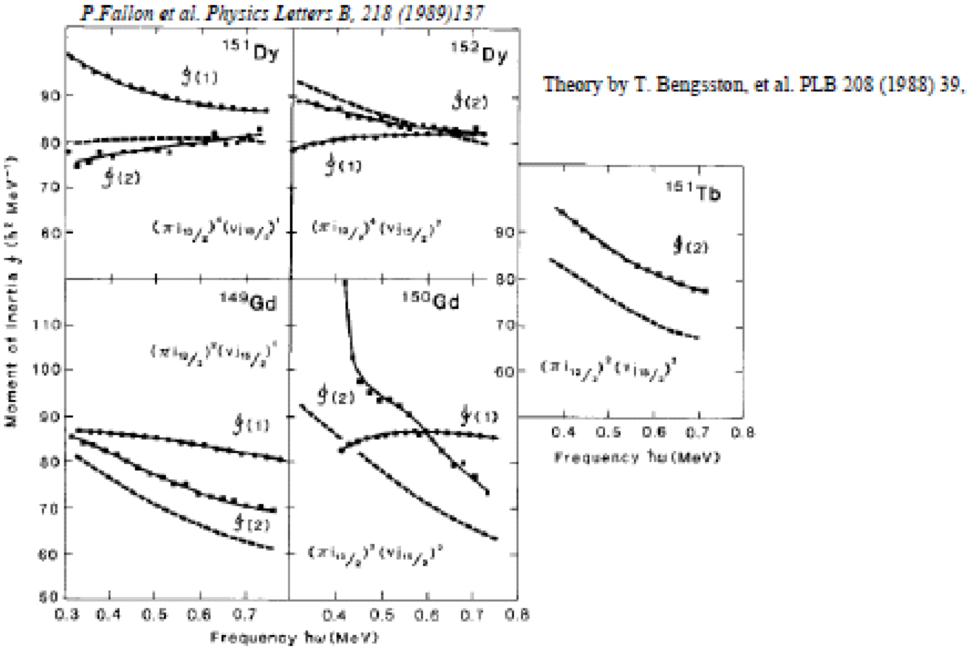
^{152}Dy
$(\pi i_{13/2})^4(\nu j_{15/2})^2$

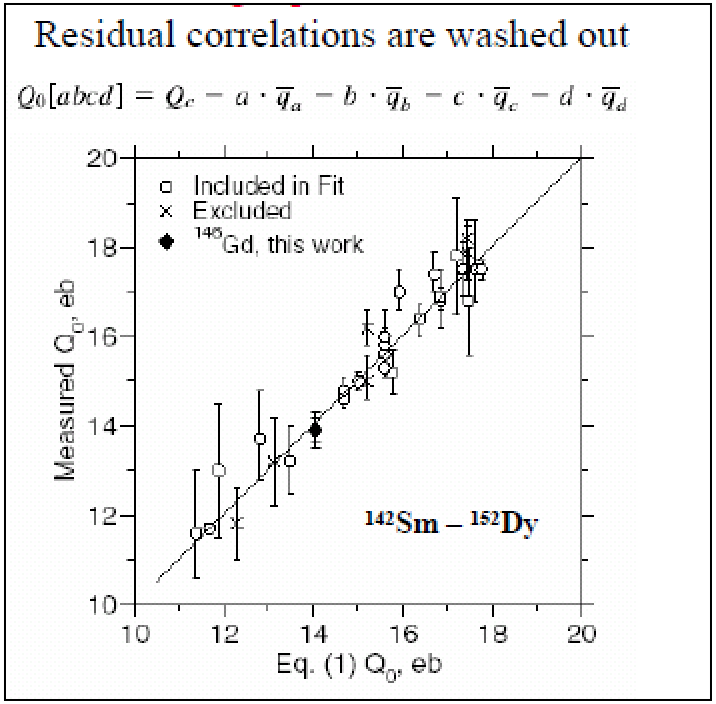
^{192}Hg
$(\pi i_{13/2})^4(\nu j_{15/2})^4$

Examples of Extreme Single Particle Motion

Even though the bands appear extremely regular, differences in moments of inertia are present

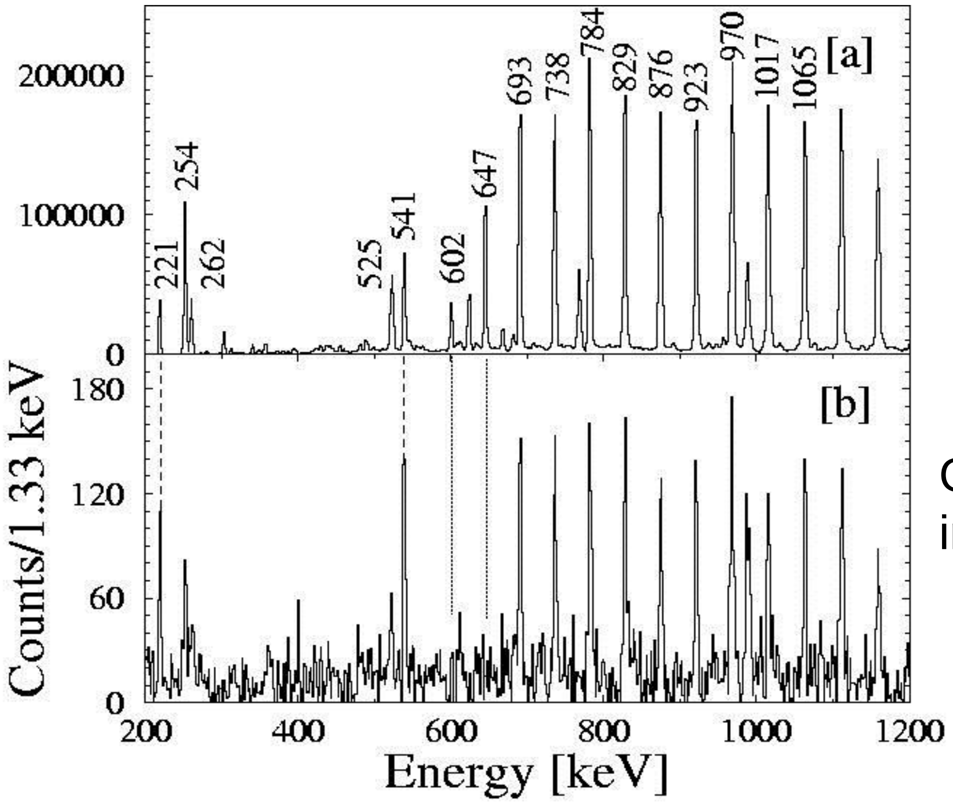






Both the difference in moments of inertia and quadrupole moments can be explained by differences in occupied intruder orbitals.

Linking of SD Band with Yrast States



Sum of two-fold clean SD gates (Band-1)

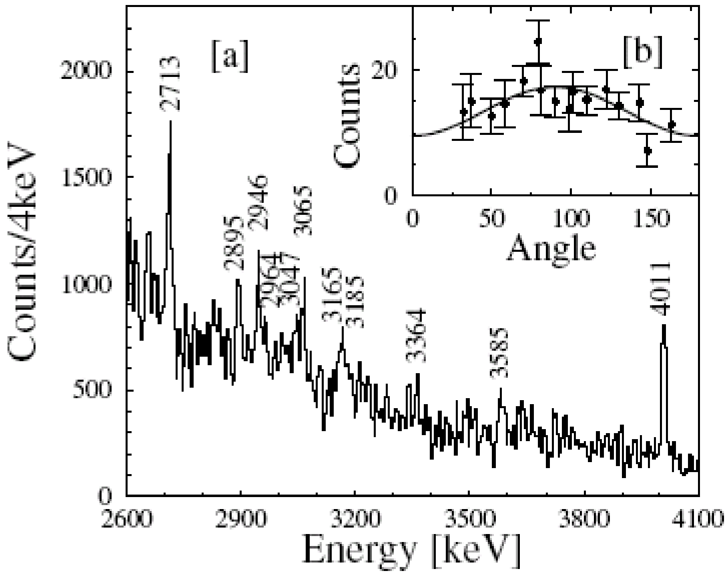
Gate requiring 4011 keV linking transition in coincidence with members of SD band.

$^{108}Pd(^{48}Ca,4n)^{152}Dy$

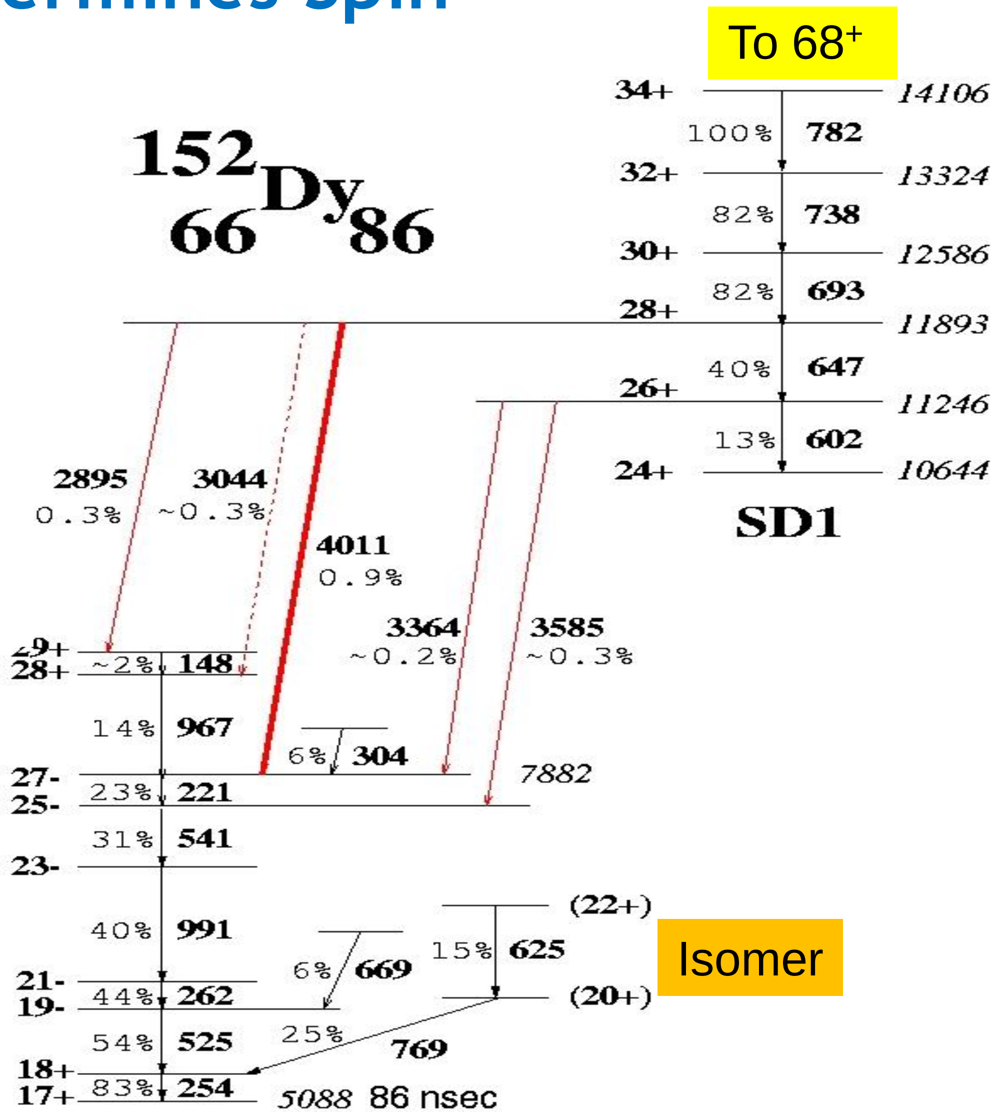
38 shifts (12 days)

Isomer tagging (87 nsec isomer)

Angular Distribution Determines Spin



- Angular distribution of 4011 keV transitions establishes I of 26 or 28 for 11893 SD level
- Feeding of 29$^+$ state from 11893 level establishes spin to be 28.
- Parity comes from fact that E1's should dominate over M1's at these energies.
- Assigned parity consistent with single particle interpretation of band.



T. Lauritsen *et al*. PRL 88, 42501

Comparison to Theory

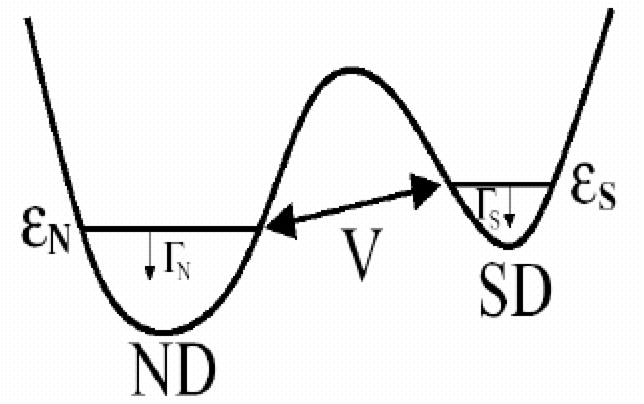
Experiment: **Lowest State in SD band:**

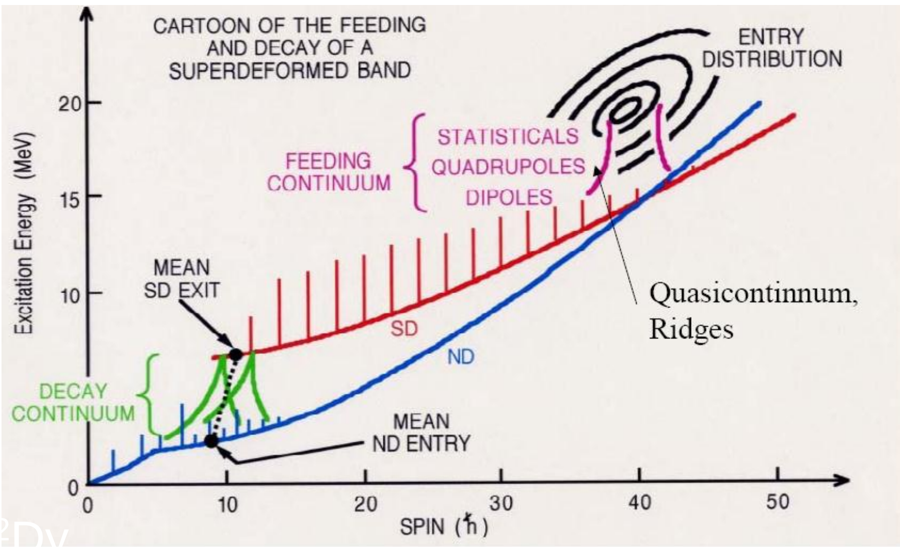
$I^{\pi} = 24^{+}$ $E(0^{+}) = 7.5$ MeV

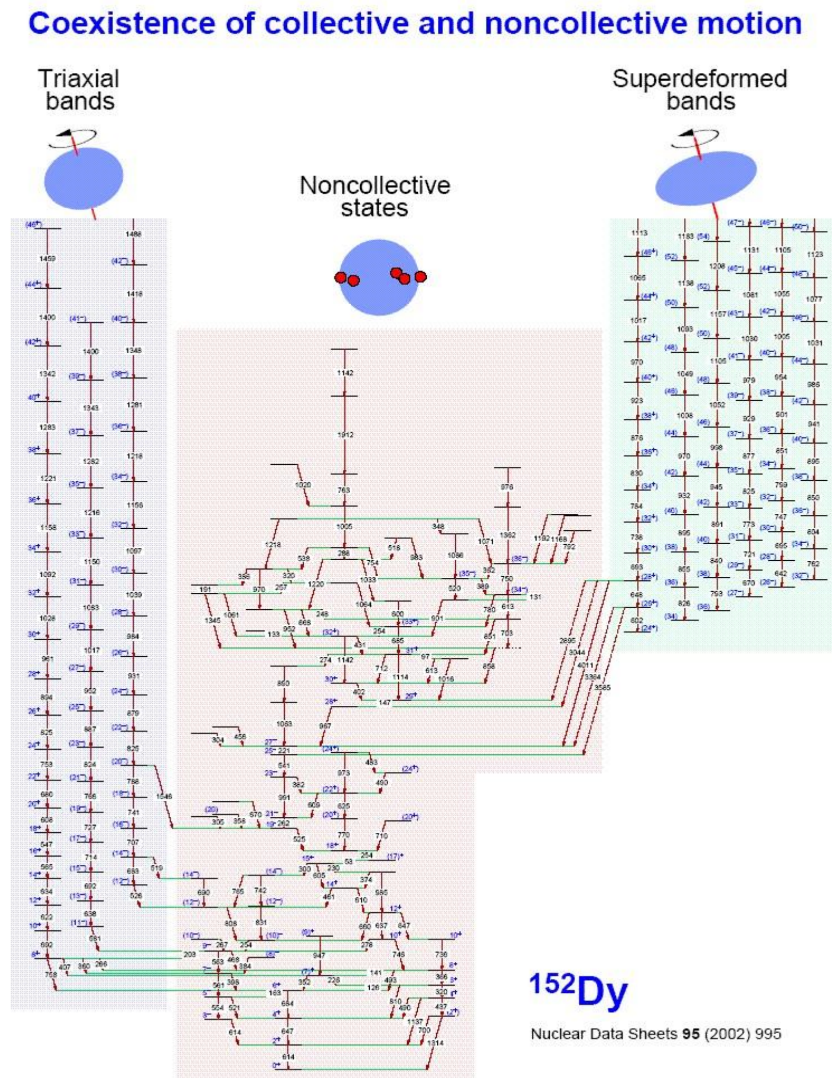
Calculations:

Nilsson – Strutinsky	**26^{+}**	**8.8 MeV**	**I. Ragnarsson NP A557, 167**
Woods – Saxon	**22^{+}**	**8.4 MeV**	**J. Dudek et al., PR C38, 940**
Relativistic Mean Field	**24^{+}**	**8.3 MeV**	**A.V. Afanasjev et al., NP A634, 395**
Hartree Fock Bogoliubov	**24^{+}**	**7.1 MeV**	**J.L. Egido et al., PRL 85, 26**

SD Minima is Distinct from ND Minima



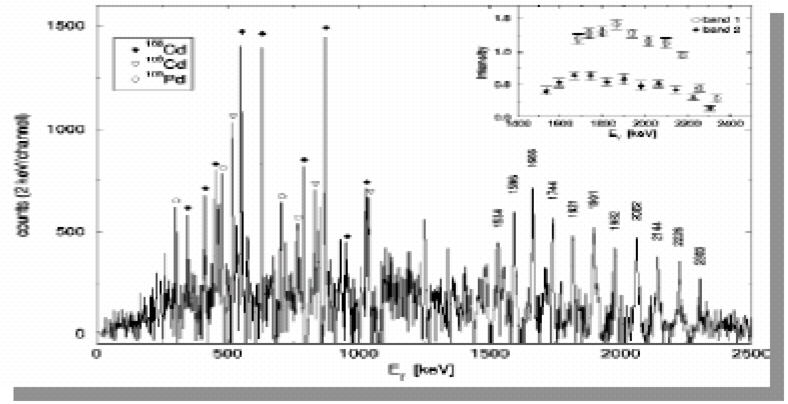


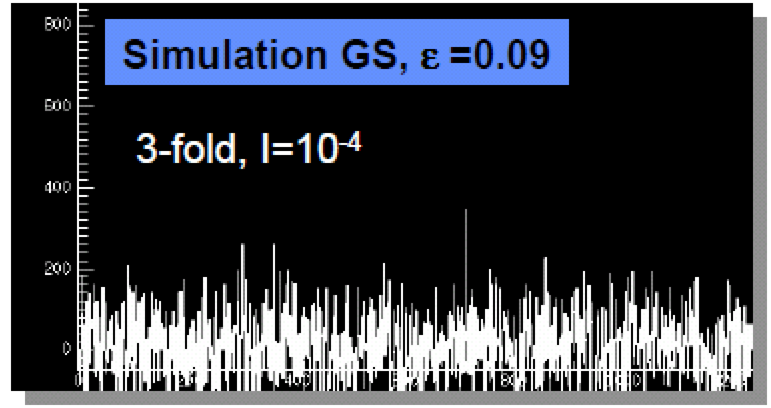


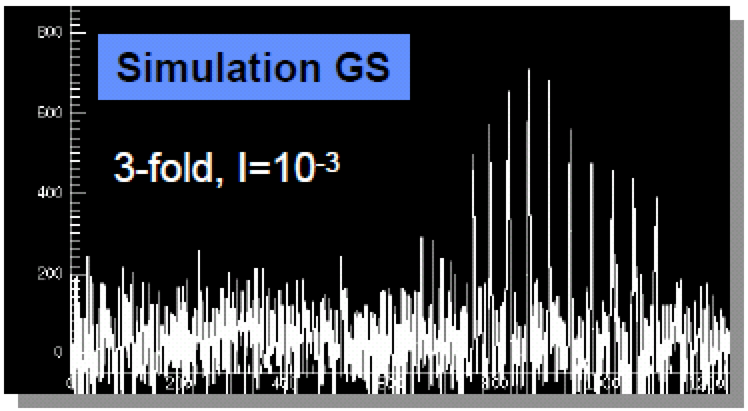
Chaos to order to Chaos to Order

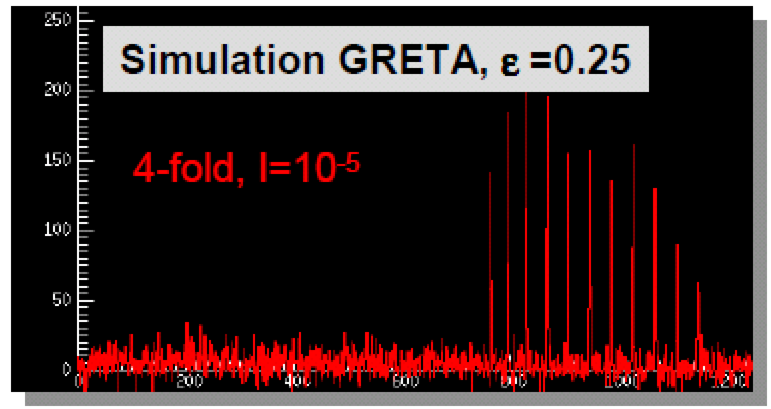
What Might the Future Hold?

^{64}Ni (^{48}Ca, 4n) ^{108}Cd, Gammasphere v/c=0.04





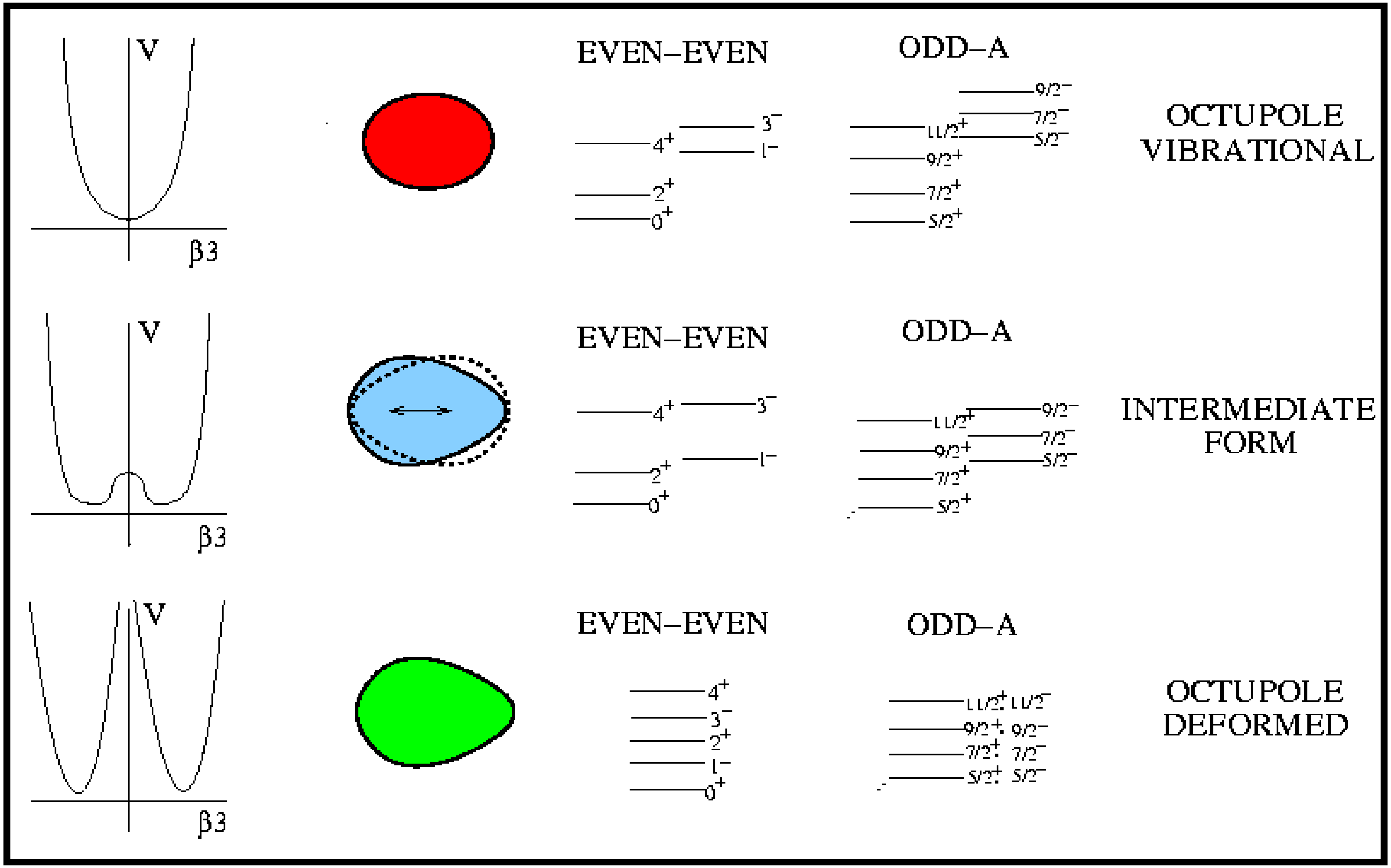




- Greta and Agata will allow us to look for cascades which are populated two orders of magnitude weaker than currently achievable
- Possibility of observing even more exotic shapes *e.g.* Hyperdeformation (3:1).

Octupole Collectivity:
Do We have the Right Picture?

Octupole Collectivity

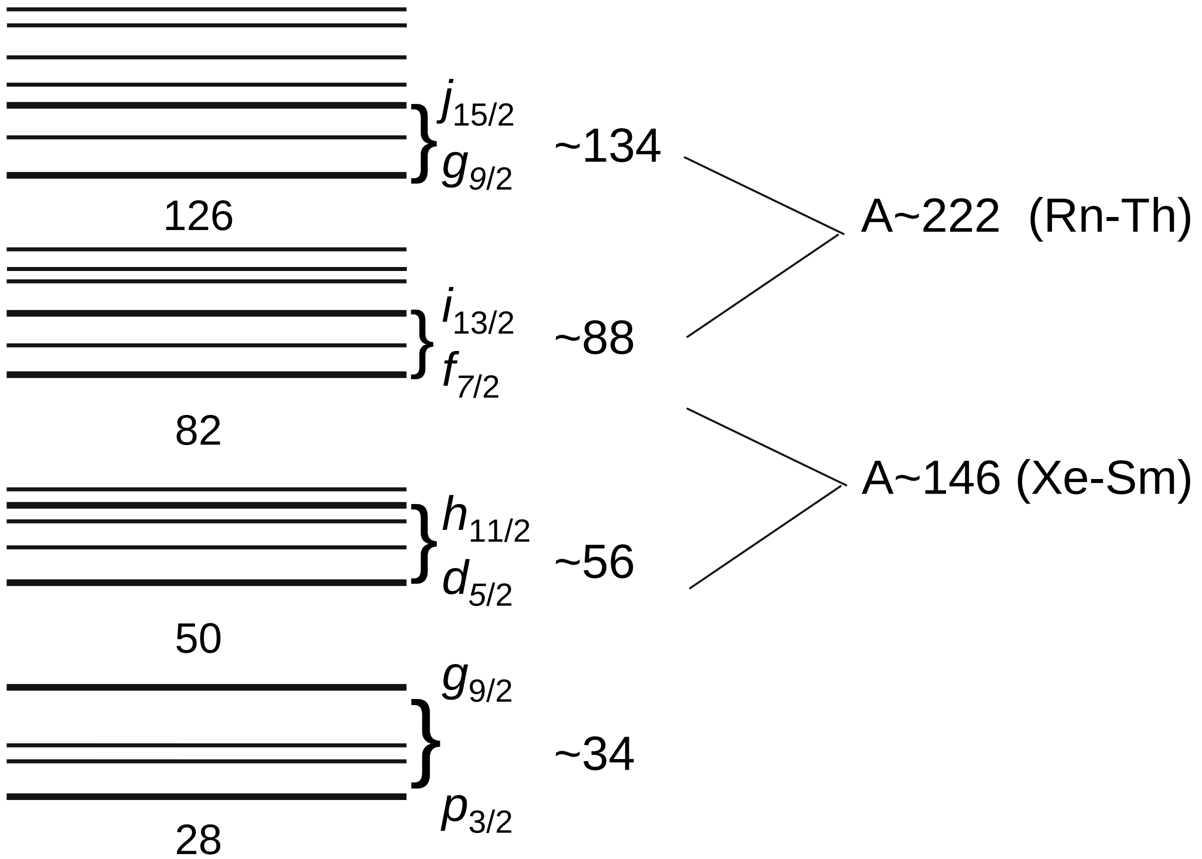


I. Ahmad & P. A. Butler, Annu. Rev. Nucl. Part. Sci. **43**, 71 (1993).

P. A. Butler and W. Nazarewicz, Rev. Mod. Phy. **68**, 349 (1996).

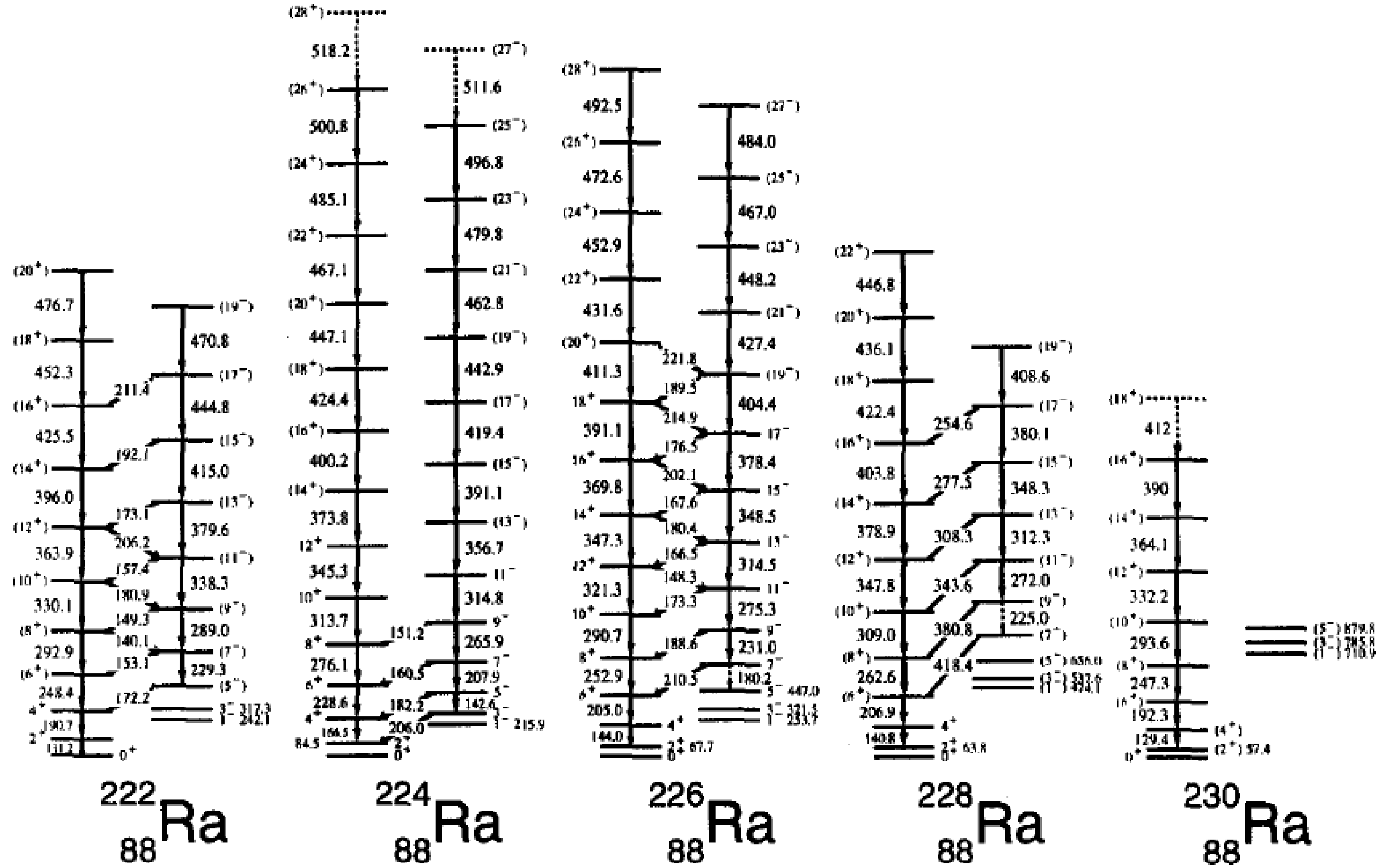
Where to find enhanced octupole collectivity

Long-range interactions between single particle states with $\Delta j = \Delta l = 3$;



Difficult regions to study experimentally

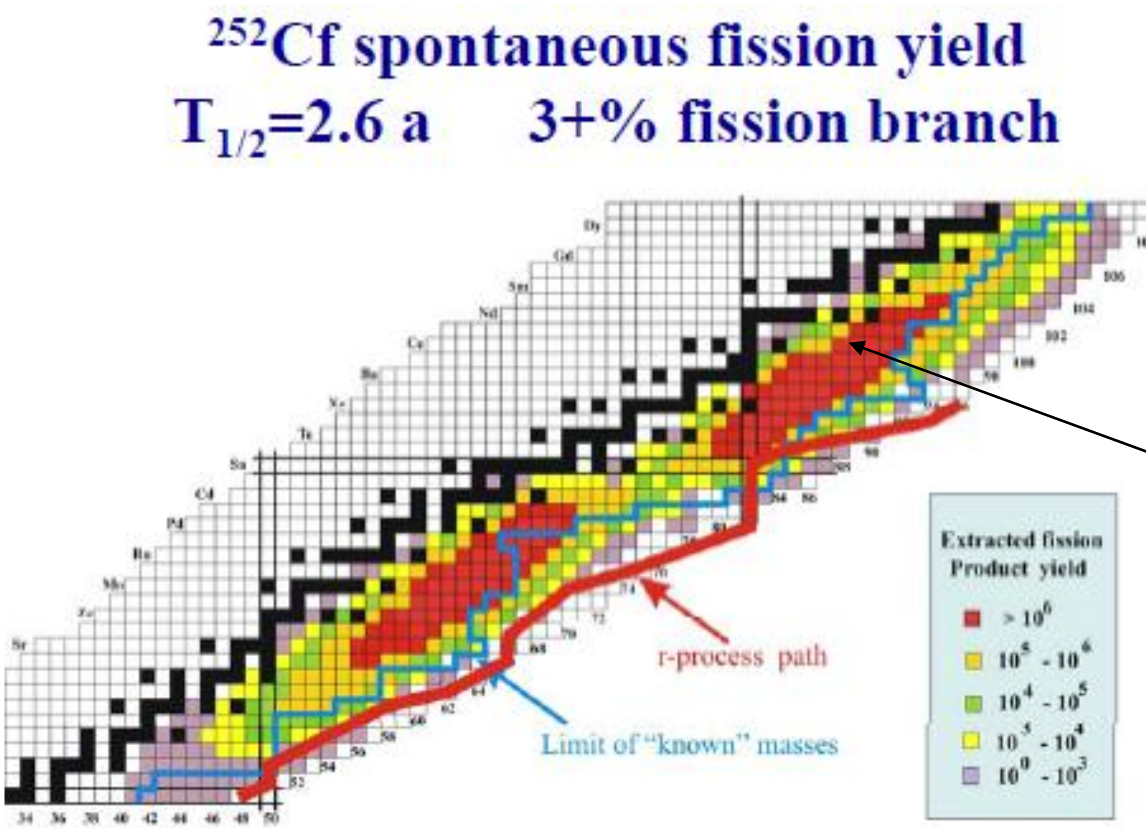
Multi-Nucleon Transfer: ^{136}Xe + ^{232}Th (Gammasphere)

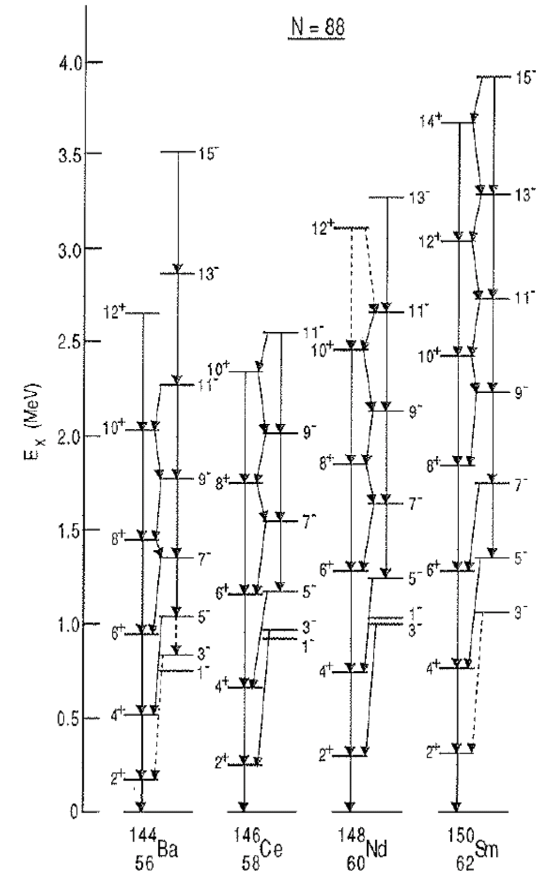


In addition, ^{226,228,230,232,234}Th

J.F.C. Cocks et at., Nuc. Phys. A 645 (1999) 61

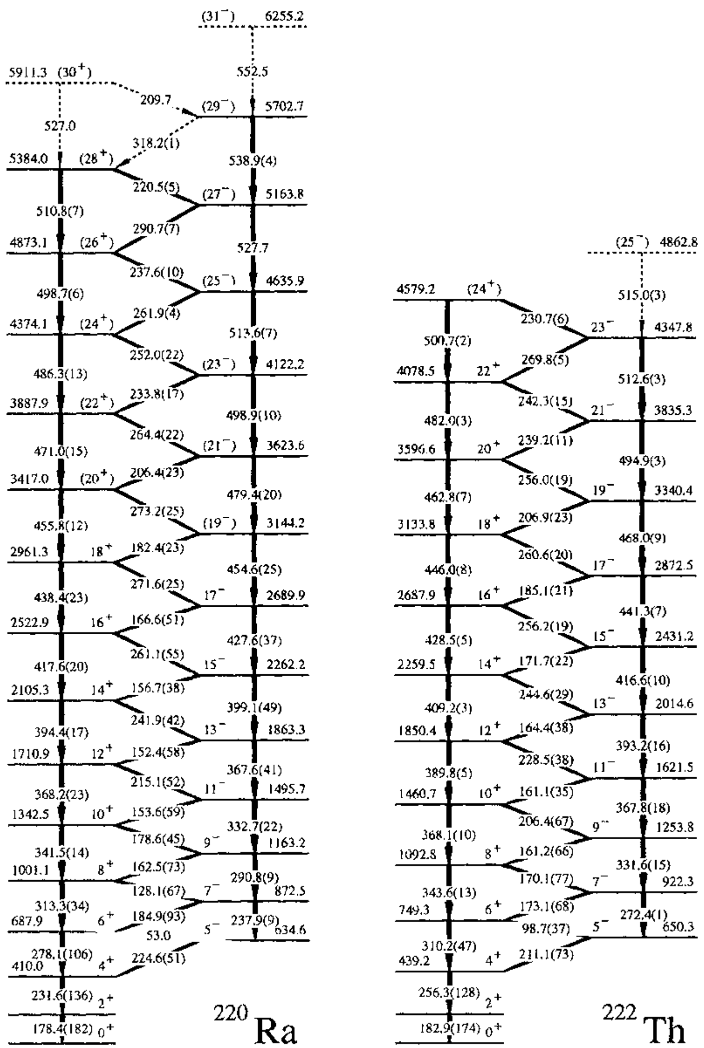
A~146 Region (Ba, Ce, Nd, Sm)



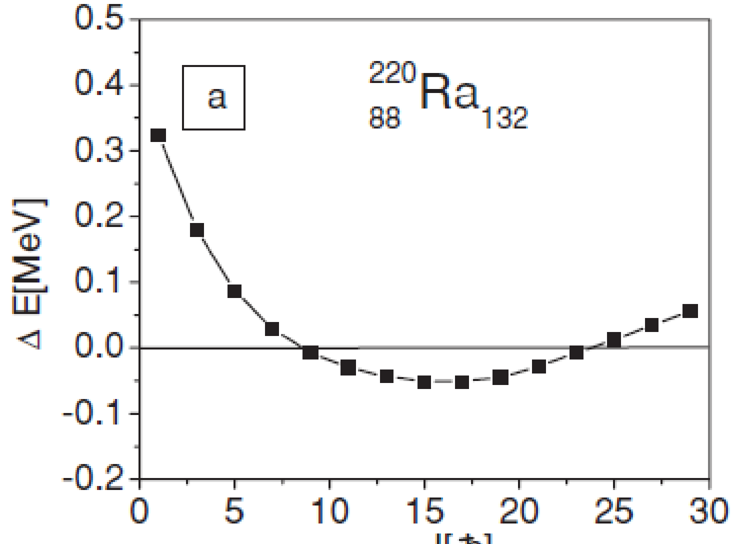


- Neutron rich region, excited states populated by spontaneous fission.
- CARIBU at ANL will provide reaccelerated beams of neutron-rich Ba isotopes for Coulomb excitation studies.

Rotation appears to stabilize static octupole shape



J. Smith *et al.,* Phys. Rev. Lett. 75 (1995) 1050.



$$S(I) \equiv E_I - \frac{(I+1)\cdot E_{I-1} + I\cdot E_{I+1}}{2I+1}$$

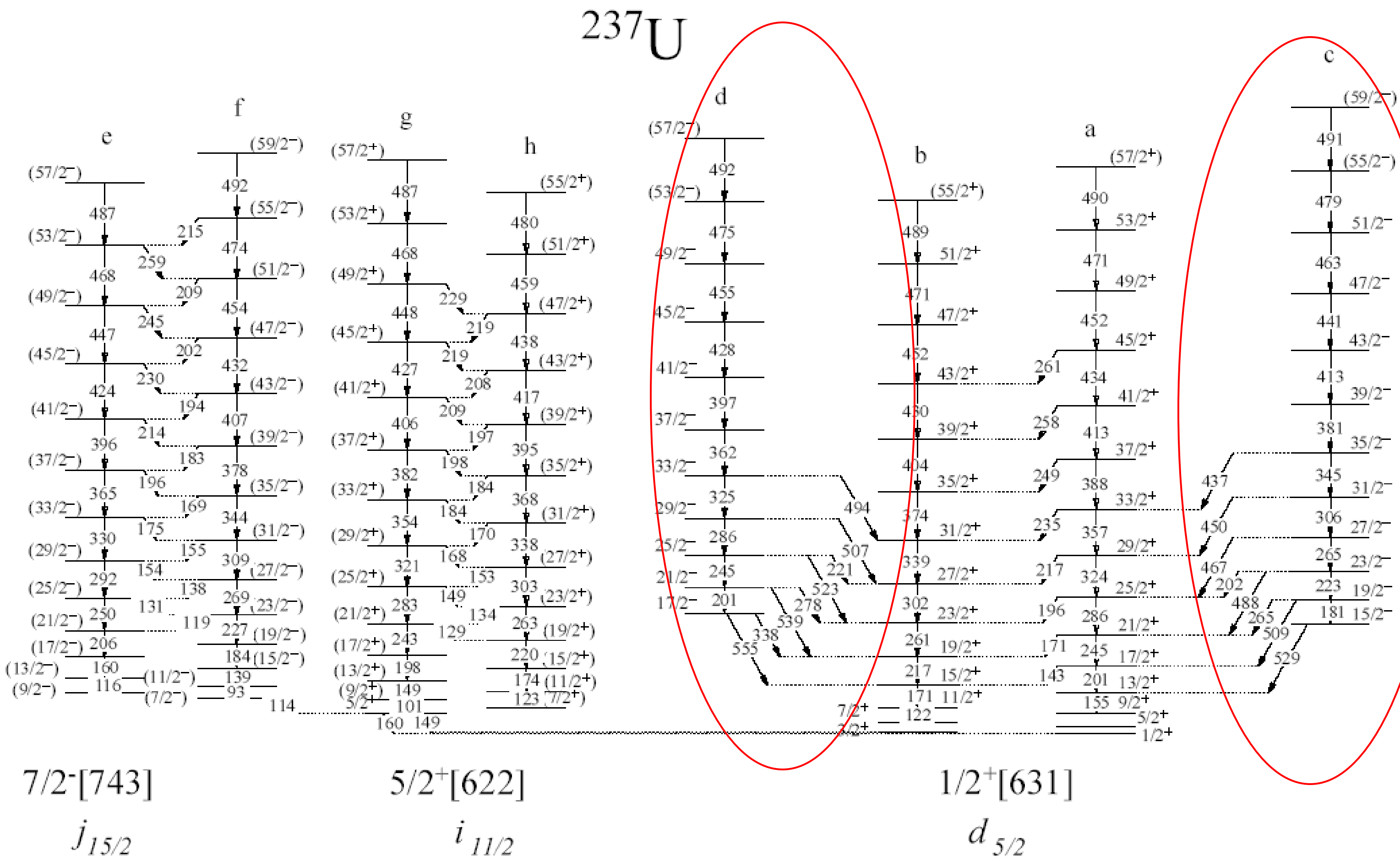
Large Dipole Moments

$D_0 \sim [B(E1)/\langle I_i 010 | I_f 0\rangle]^{1/2}$

where

$D_0 > 0.2$ eb-fm

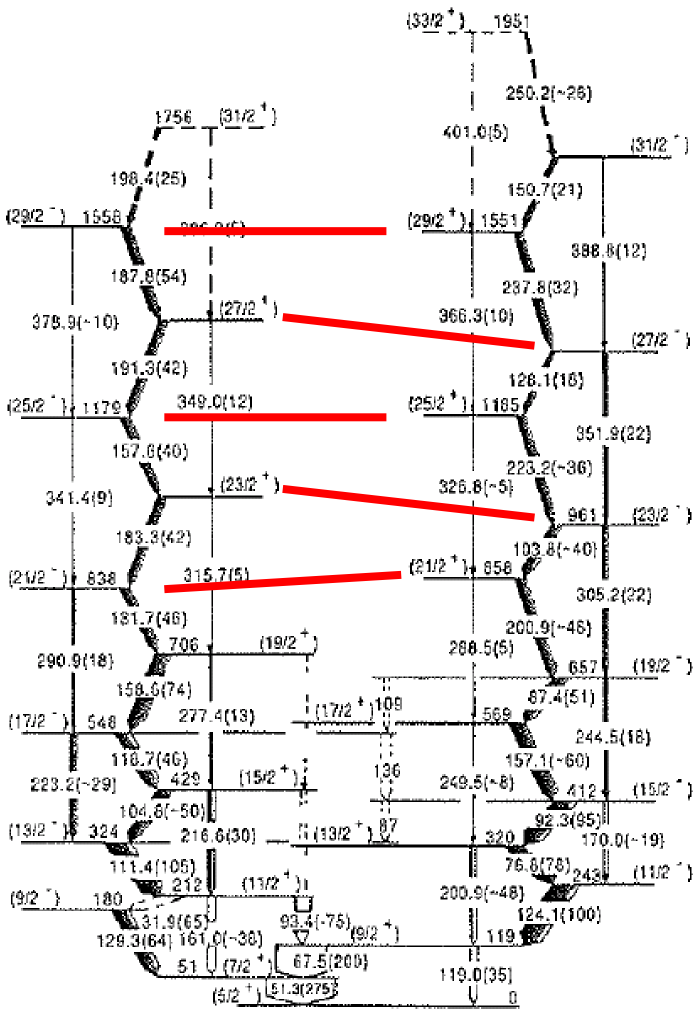
Odd-A Nuclei Exhibit Evidence of Octupole Collectivity



Octupole phonon coupled to quasiparticle

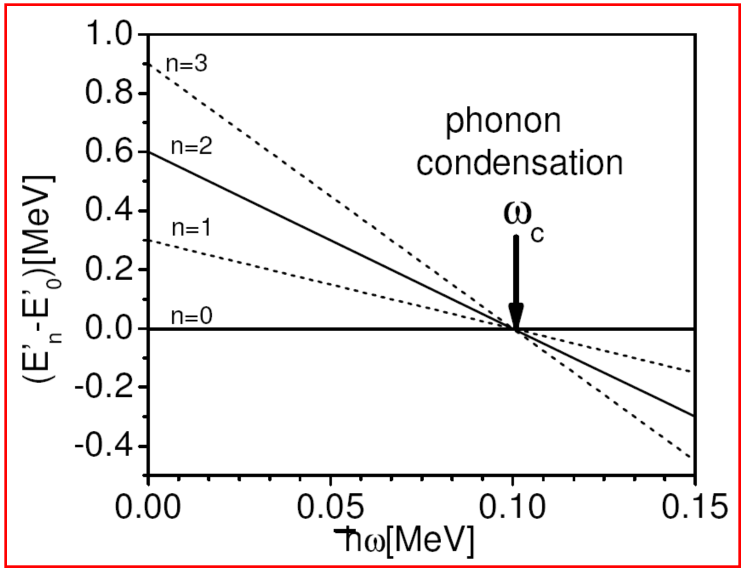
S. Zhu *et al.*, Phys. Lett. B618, 51 (2005

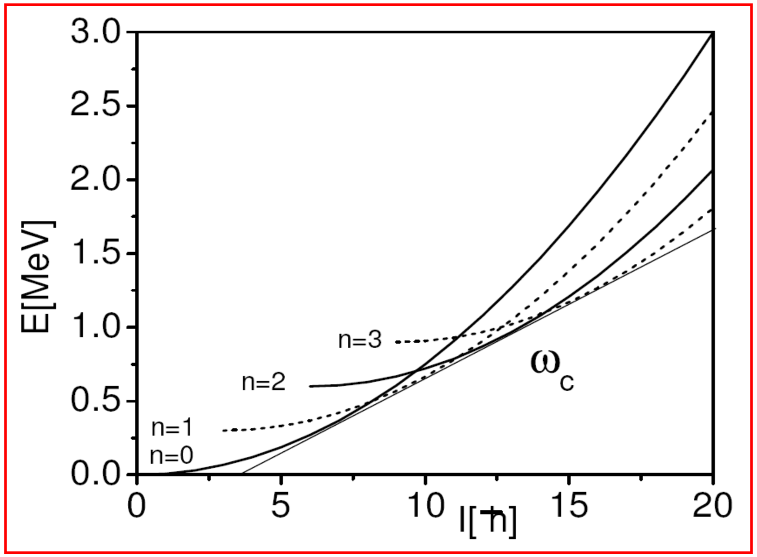
Signature of Octupole Collectivity in Odd-A Systems



Parity Doublets (Octupole deformed)

Alternative Explanation of static Octupole Deformation





Energy = Vibrational + Rotational

$$E_n = \hbar\Omega_3(n+1/2) + \Im\omega^2/2$$

Lowest excitation when phonon's align with rotational axis.

$$E_n^{'}(\omega) = \hbar\Omega_3(n+1/2) - ni\omega - \Im\omega^2/2$$

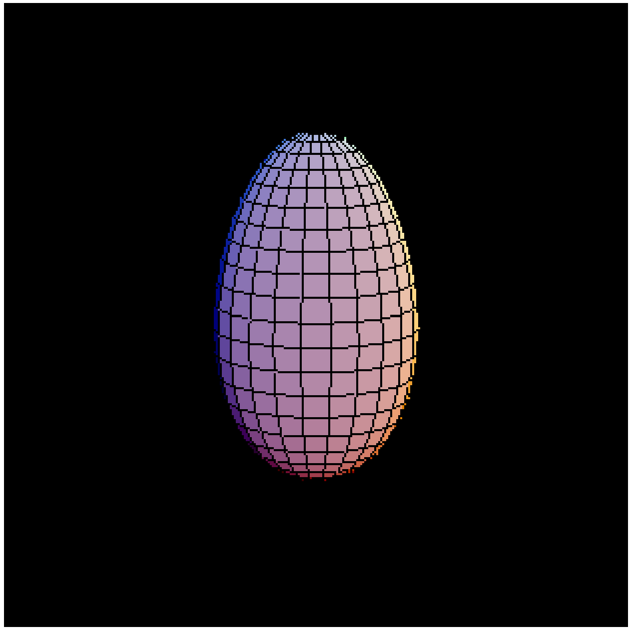
At critical frequency, ω_c, all phonon Routhians converge resulting in phonon condensation.

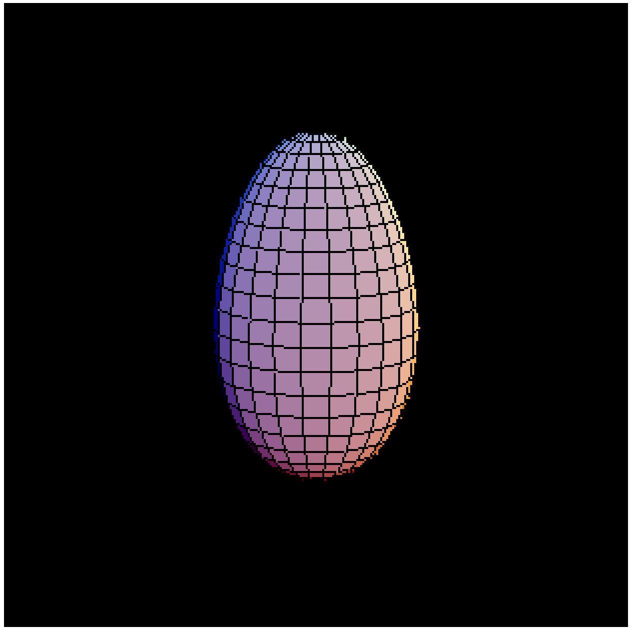
$$\omega_c = \Omega_3 / i$$

$$\omega_c : E_n(\omega) = E_{n+1}(\omega)$$

S. Frauendorf, Phys Rev. C 77 (2008) 021304(R).

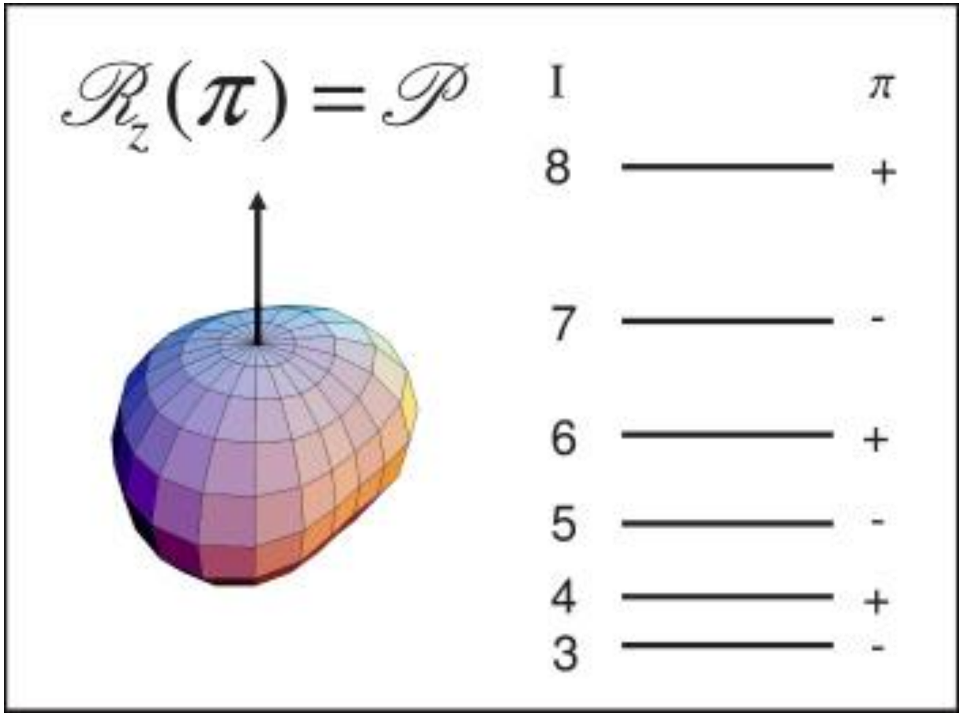
What is the Resultant Shape at Condensations

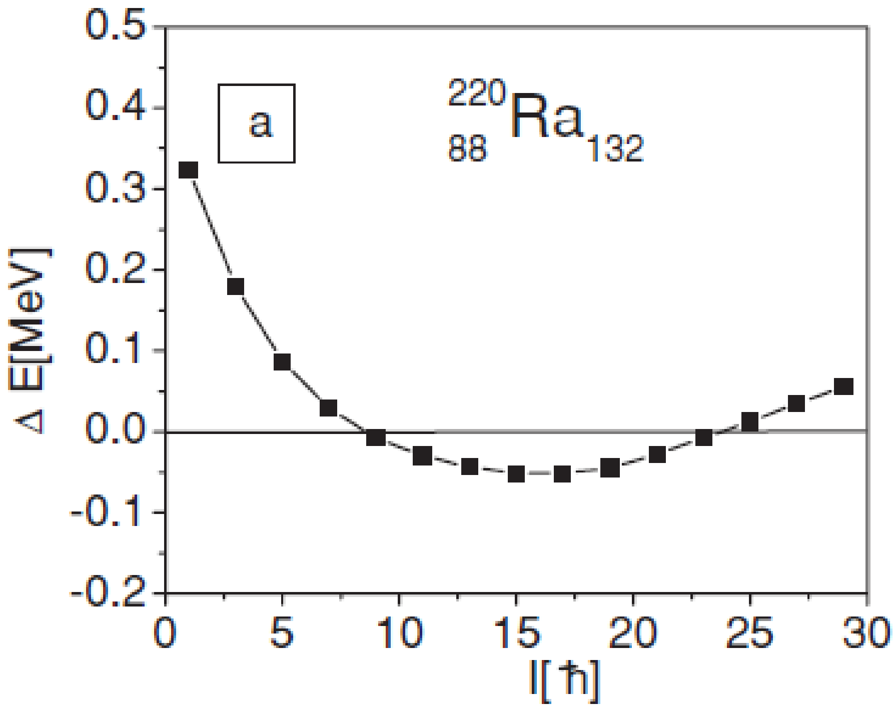




$$\omega_c = \Omega_3 / 3$$

Consequence of Anharmocities.





- Due to anharmocities, phonons bands will cross at different frequencies
- Phonon's of same parity will interact.
- Resulting yrast line will have states on average which appear to be interweaved as expected for static octupole shape.

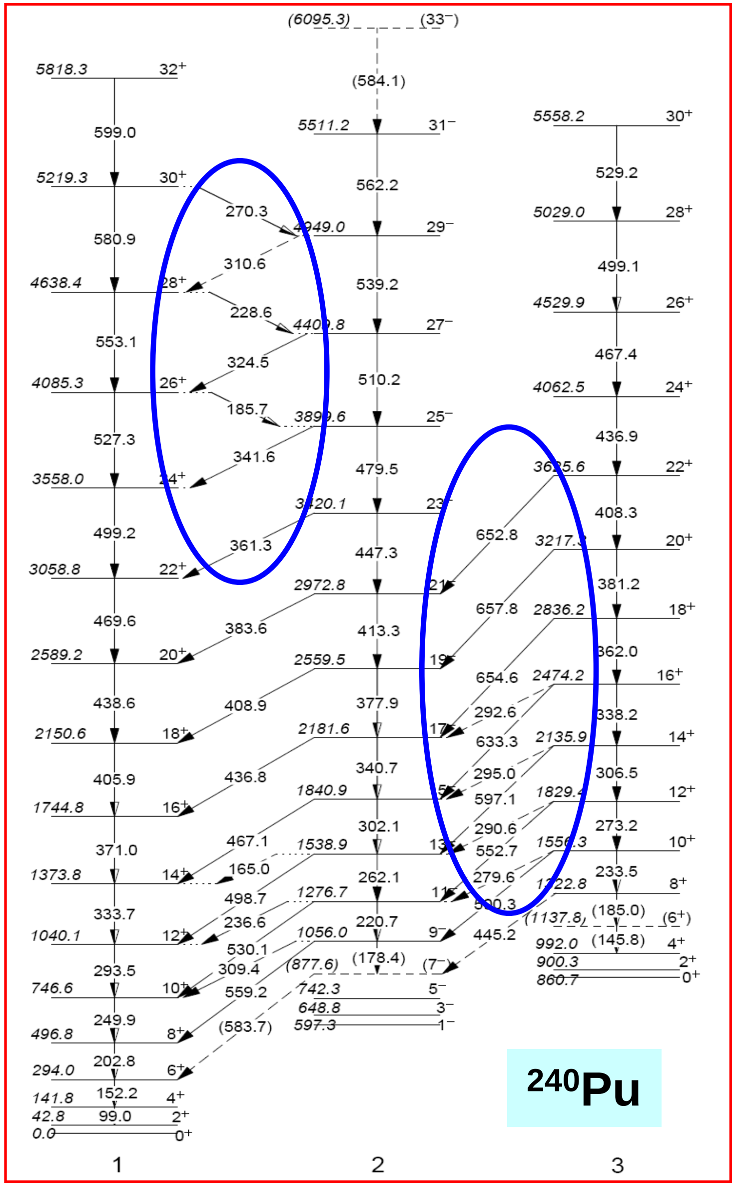
S. Frauendorf, Phys. Rev. C 77 021304(R).

Can One Distinguish Between the Two Pictures?

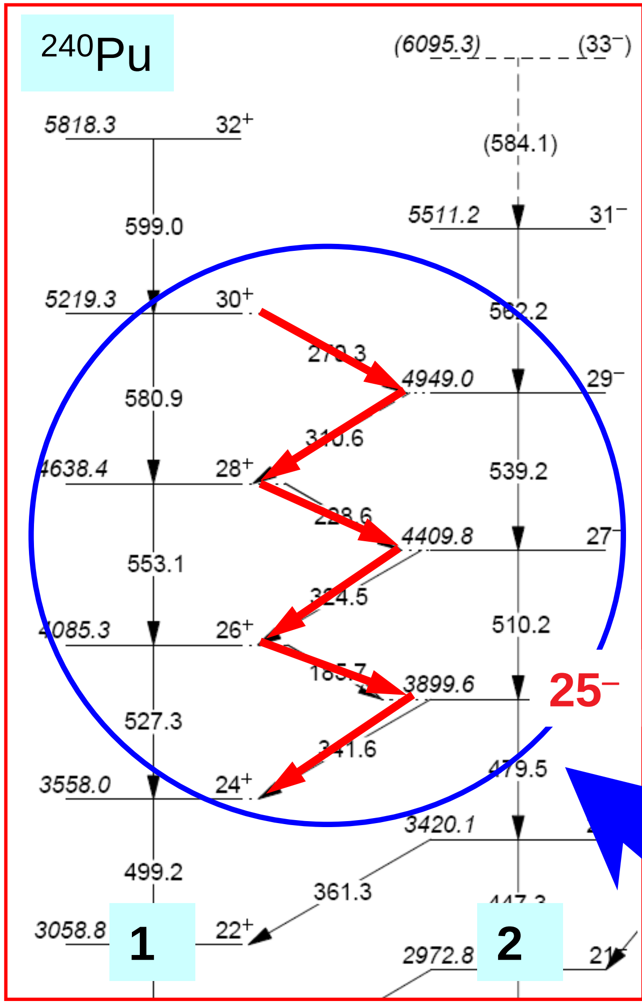
New Results on ^{240}Pu

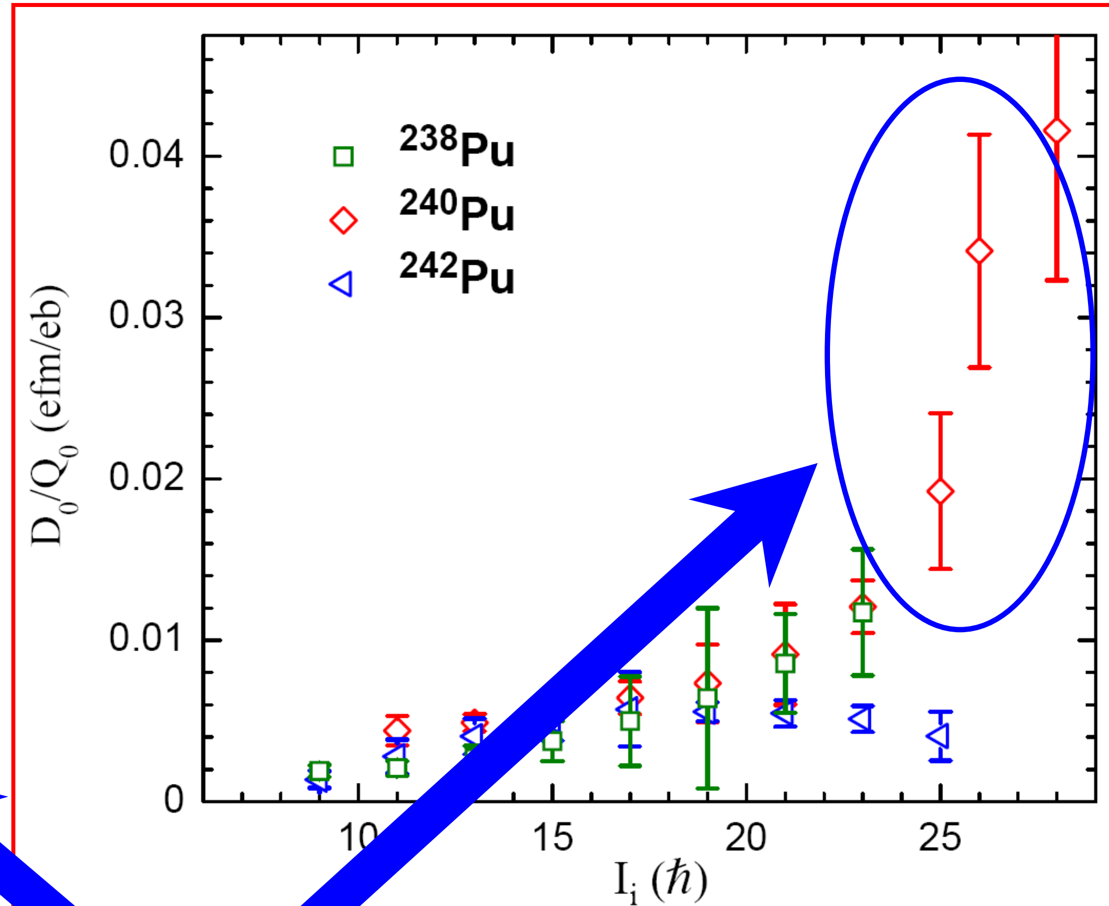
- Interleaving E1 transitions now observed between yrast (1) and octupole (2) bands.
- Positive parity band built on 0^+ state at 861 keV is observed to high-spin.
  - Competes in excitation energy with yrast band.
  - Decays exclusively to octupole band
  - Large B(E1)/B(E2) ratios: ~1×10^{-8} fm^{-2}
  - Band with these characteristics not seen before

X. Wang *et al.*, Phys. Rev. Lett. 102, 122501 (2009)



Dipole Moment at High-Spin is Large

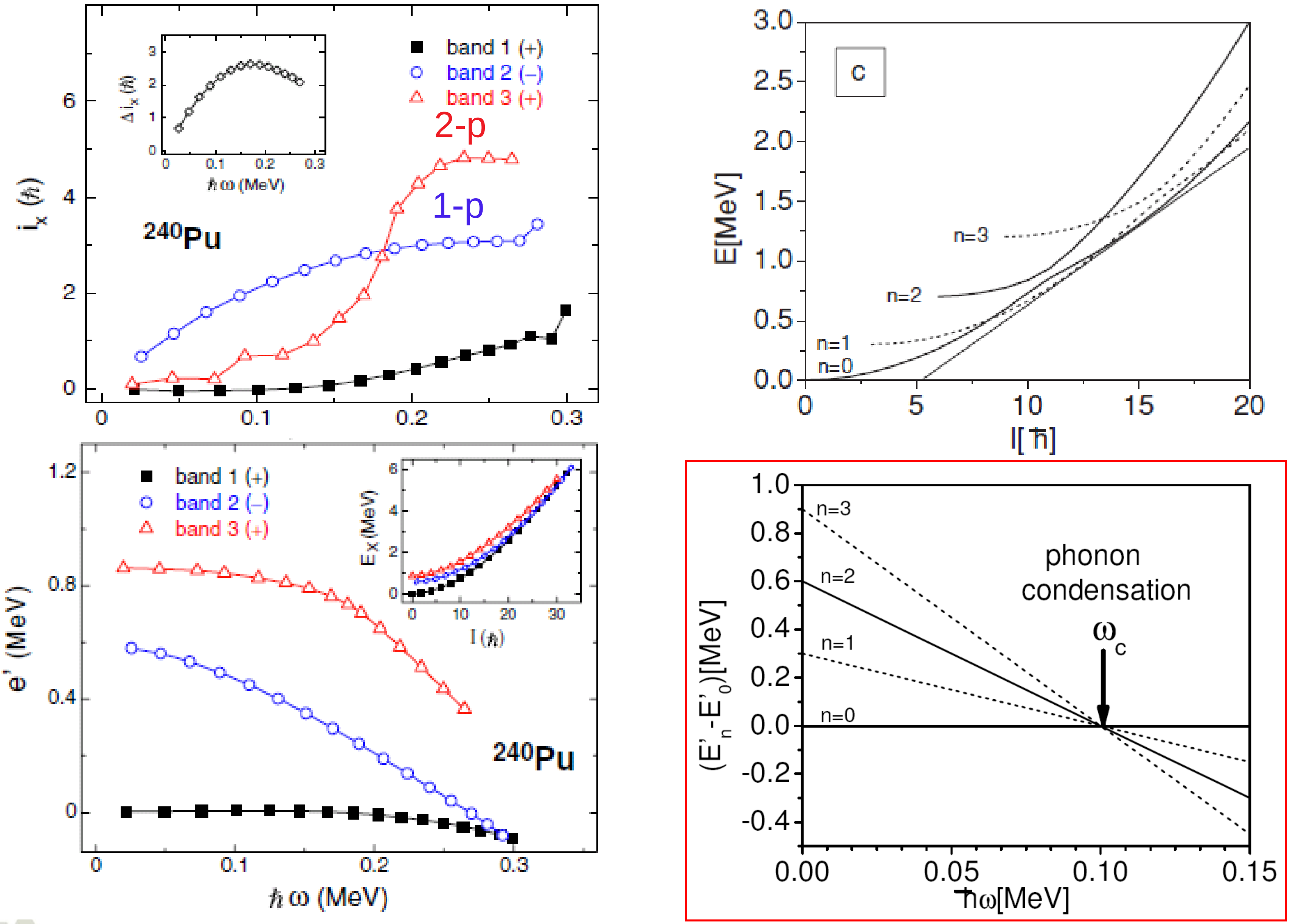




$D_0 > 0.2$ e fm (assuming $Q_0 \sim 11$eb)

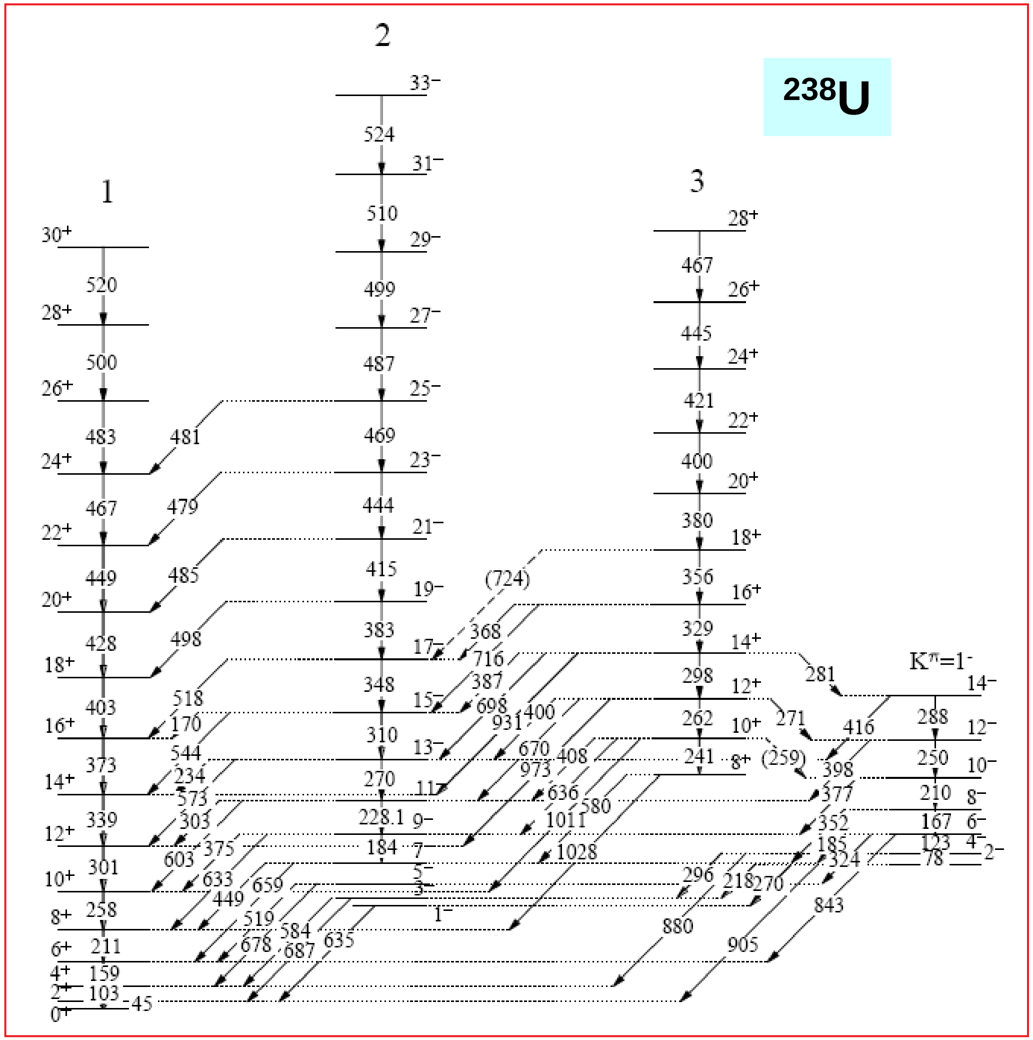
X. Wang *et al.*, Phys. Rev. Lett. 102 122501 (2009)

Strong evidence for phonon condensation.



X. Wang *et al.*, Phys. Rev. Lett. 102, 122501 (2009)

Is There More Evidence for Octupole Condensation?

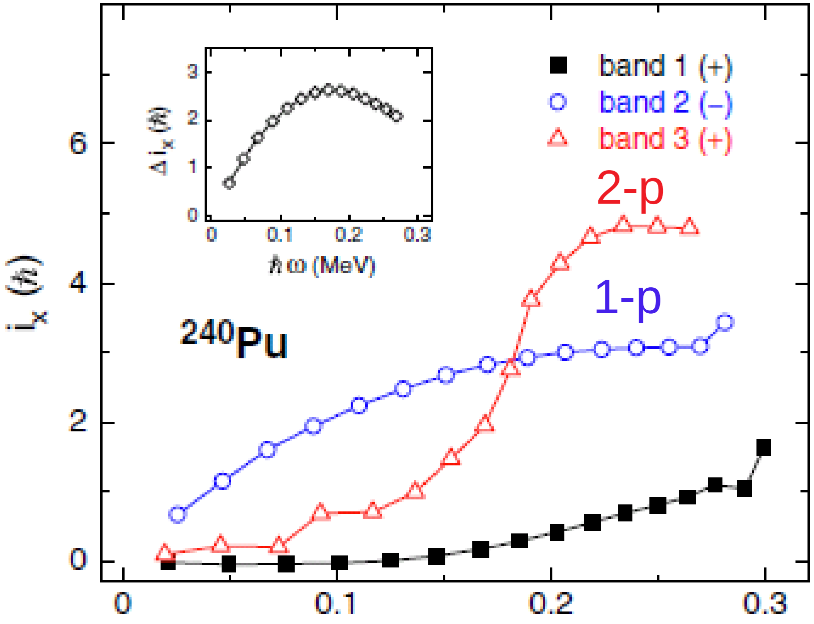


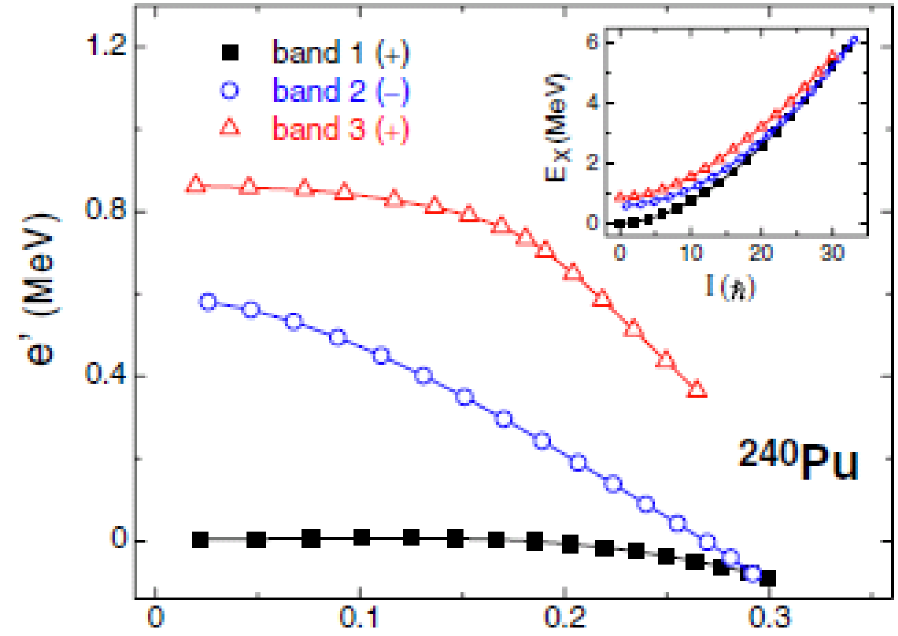
New Results on ^{238}U

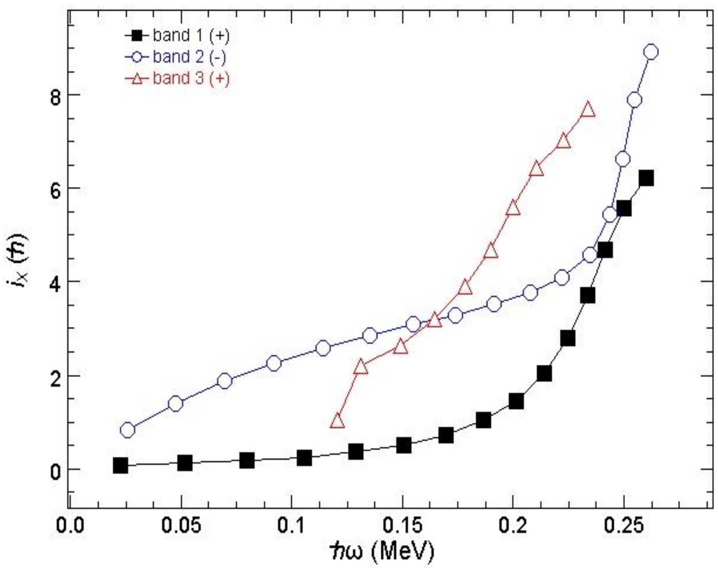
- New positive parity band in ^{238}U
  - Strongly decays to one-phonon band;
  - Sizable $B(E1)_{out}/B(E2)_{in}$ ratios: $\sim 2\times10^{-9}$ fm^{-2}
  - Weakly decays to yrast band
- Negative parity band never interleaves with yrast band.

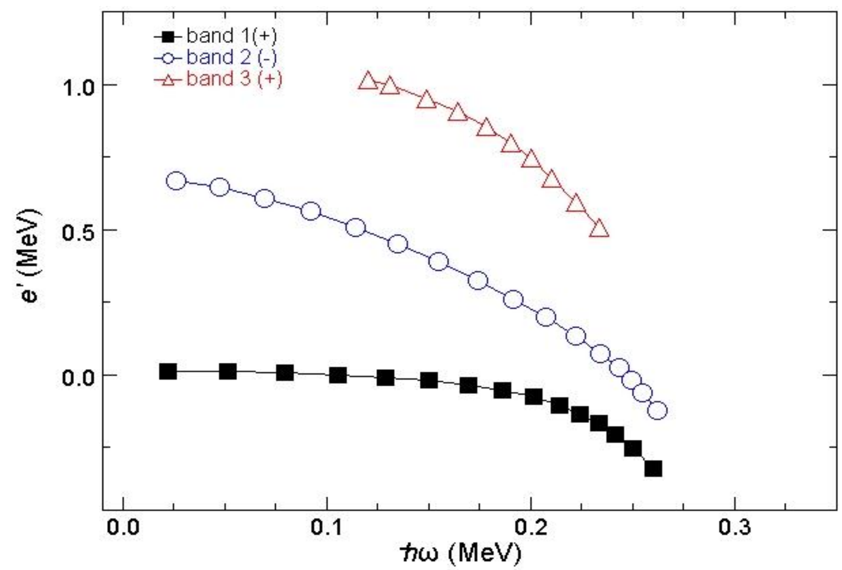
S. Zhu et al., Phys. Rev. C 81 (2010) 041306(R)

Contrast Octupole Phonon bands in ^{238}U and ^{240}Pu









S. Zhu et al., Phys. Rev. C 81 (2010) 041306(R)