IDENTIFYING THE QUARK-HADRON PHASE TRANSITION WITH G-MODE OSCILLATIONS

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Looking forward to more exciting connections between

The Compact Star/GW community in India

&

ICTS

&

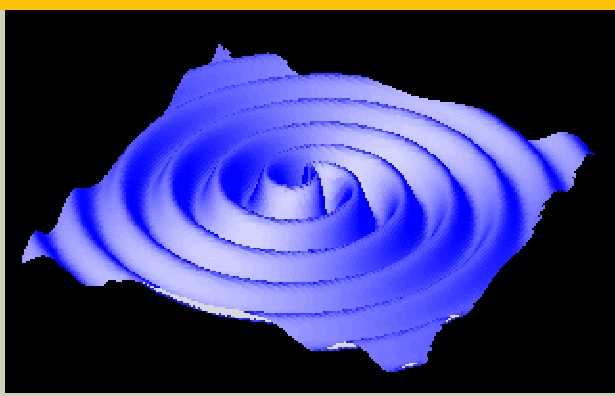
CSQCD

OUTLINE

- Motivation: Gravitational Waves as Discovery Tool
- Nuclear Physics : From Nuclear to Quark Degrees of Freedom
- g-mode Oscillations: The two sound speeds
- Effects of Quark Matter on the g-mode Oscillation Spectrum
- Observational Outlook for g-modes : Damping and SNR

GRAVITATIONAL WAVES (LIGO/VIRGO)

- First detection of BH-BH (GW150914)/NS-NS (GW170817)/?BH-NS(GW190814)?
- Confirmation of short-GRB mechanism
- Bound on graviton mass
- Strong field tests of GR
- Neutron star radii and EOS constraints
- Existence of Black Holes
- Cosmology & Particle Physics

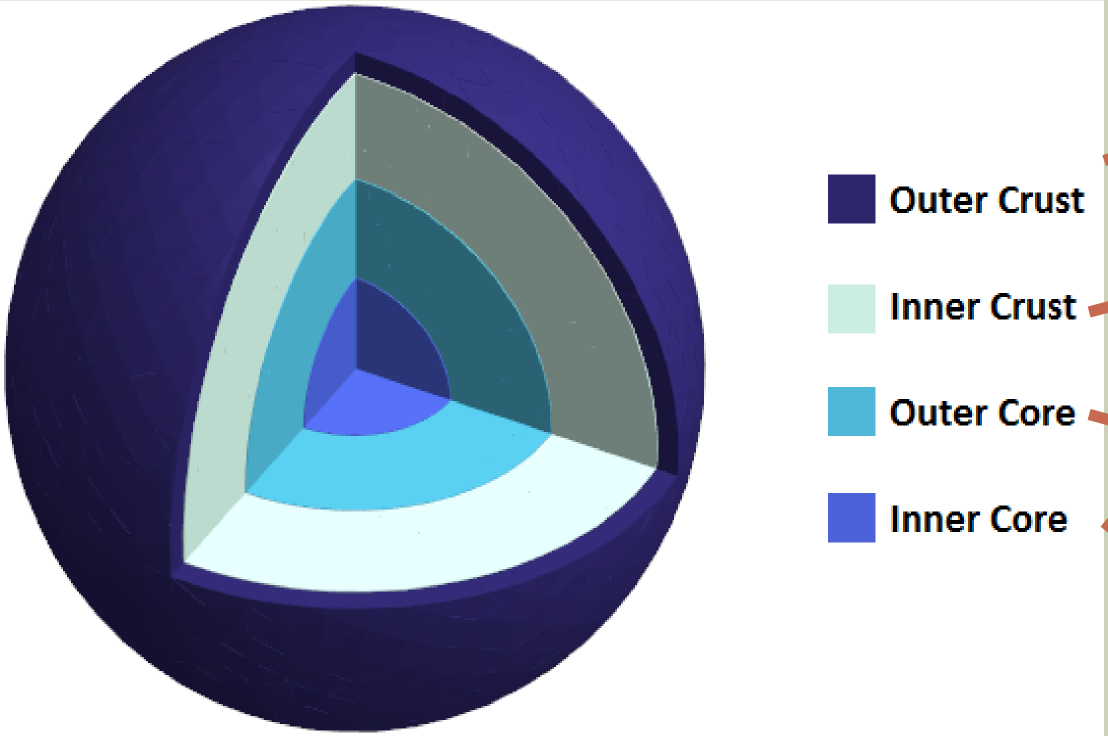


$$G = \frac{8\pi G}{c^4} T$$

$$\left(-\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \nabla^2\right)\bar{h}_{\mu\nu} = 0$$

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} g_{\mu\nu}\, h$$

NEUTRON STAR INTERIOR



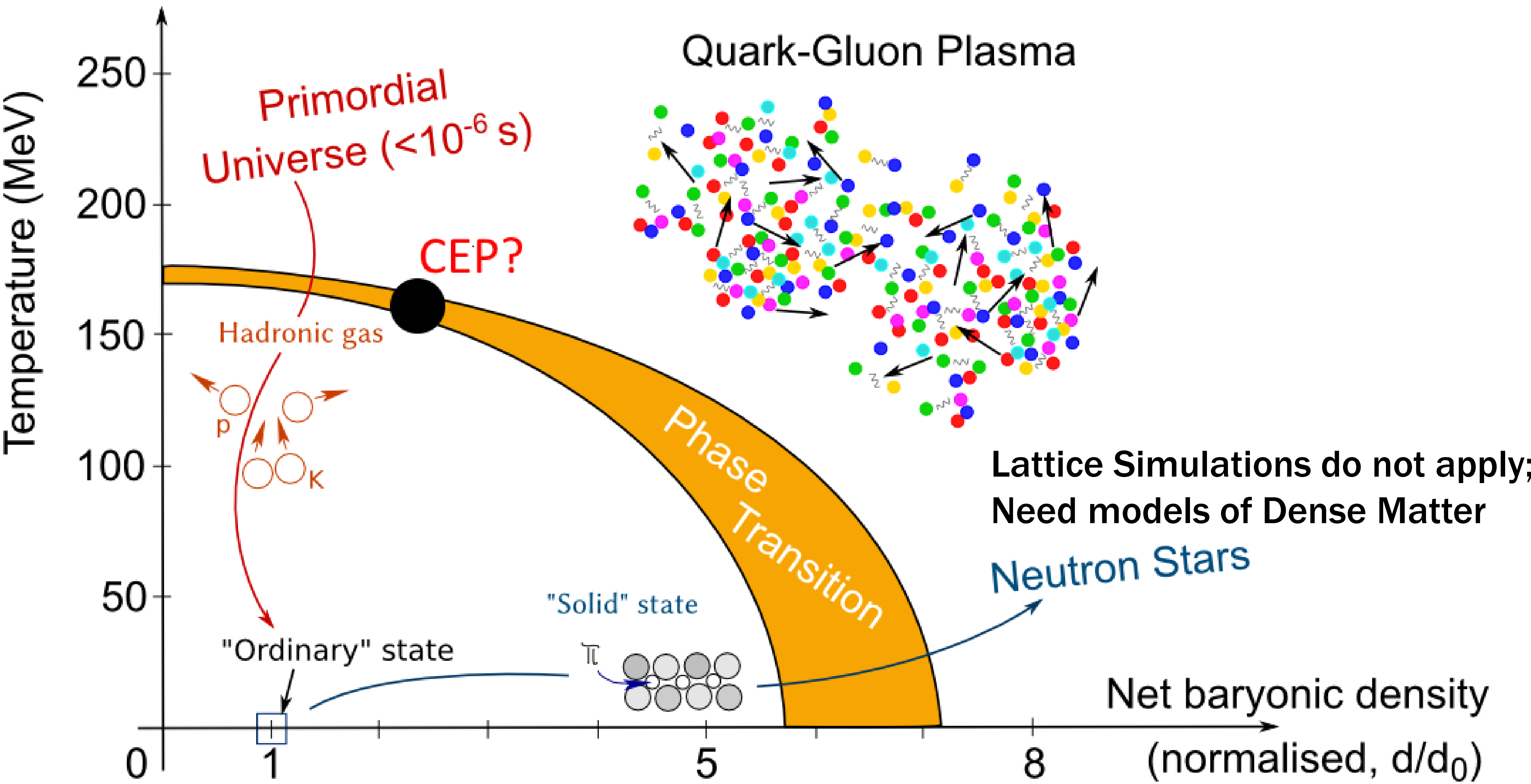
Few 100m,
< neutron drip,
Nuclear Lattice

< saturation
N-rich Nuclei,
dripped neutrons

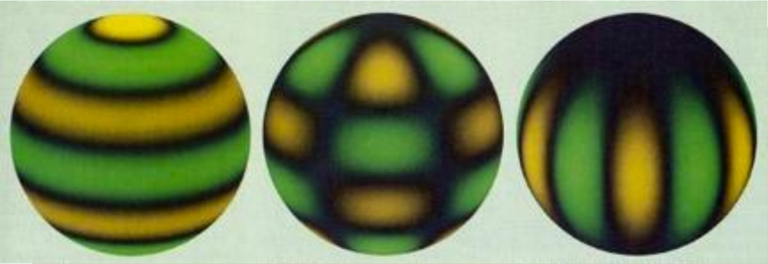
< 2 saturation,
Pasta phases,
N,p,e, μ
Fermi Liquid/Gas

(2-6) saturation
Dense Nuclear
Matter, Quarks?
Quarkyonic?

QCD PHASE DIAGRAM



WHY STUDY STELLAR OSCILLATIONS ?



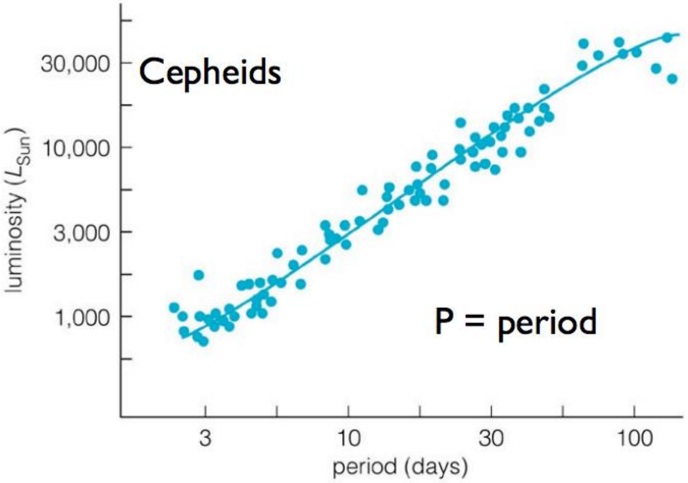
(ACOUSTIC)

(OPTICAL)

1. Dynamo origin of solar magnetic field
   (Provided proof for differential rotation in the Sun)

2. Verified age of Sun ~ 4.6 billion yrs
   (sound speed depends on He/H ratio)

3. Pointed to neutrino oscillations
   (Ruled out solar physics solution to the neutrino problem)



$$P \approx \frac{4R}{\sqrt{\gamma GM/R}} \approx \frac{4R}{\sqrt{\gamma G}}\sqrt{\frac{R^3}{M}} \approx \frac{2\sqrt{3}}{\sqrt{\gamma G\pi}} < \rho >^{-1/2}$$

Non-adiabatic radial oscillations

Change of Ionization state drives pulsations

HeI ↔ HeII
(less opaque) (more opaque)

Standard Candle for Extra-galactic scales on the Cosmic distance ladder

GRAVITATIONAL WAVES

CAN WE USE GRAVITATIONAL WAVES

TO TELL THE COMPOSITION OF A NEUTRON STAR?

Modes (Non-Rotating, Zero-B and Temperature)

- Fluid Displacement (Spheroidal)

$$\xi(r,\theta,\phi,t) = \mathcal{R}\left\{\sum_{lm}\left[\xi_r(r) Y_{lm}(\theta,\phi)\hat{r} + \xi_h(r)\left(\frac{\partial Y_{lm}}{\partial\theta}\hat{\theta} + \frac{1}{\sin\theta}\frac{\partial Y_{lm}}{\partial\phi}\hat{\phi}\right)\right] \mathrm{e}^{i\omega t}\right\}$$

- Newtonian for simplicity: (primes = Eulerian perturbations)

$$\text{Continuity}: \quad \rho' = -\nabla.(\rho_0 \xi)\,,$$

$$\text{Euler}: \quad \rho_0\,\xi_{tt} = -\nabla\, p' - \rho_0 \nabla\phi' - \rho'\nabla\phi_0\,,$$

~~$$\text{Poisson}: \quad \nabla^2\phi' = 4\pi G\rho'$$~~ **(Cowling Approximation)**

$$\text{Energy}: \quad p' + \xi.\nabla p_0 = \frac{\Gamma_1 p_0}{\rho_0}(\rho' + \xi.\nabla\rho_0)$$

Boundaries

- (Fluid) Center ($r = 0$) : Regularity
  $\implies p', \phi' \sim \mathcal{O}(r^l)\,,\, \xi_r \sim \mathcal{O}(r^{l-1})$

$$\left(\frac{\rho_{\rm av}}{\rho_c}\right)\frac{\Omega^2 \xi_r}{l} + \frac{p'}{\rho_c g} = 0 \tag{1}$$

- (Fluid) Surface ($r = R$) : Free surface $\implies \delta p|_{r=R} = 0$

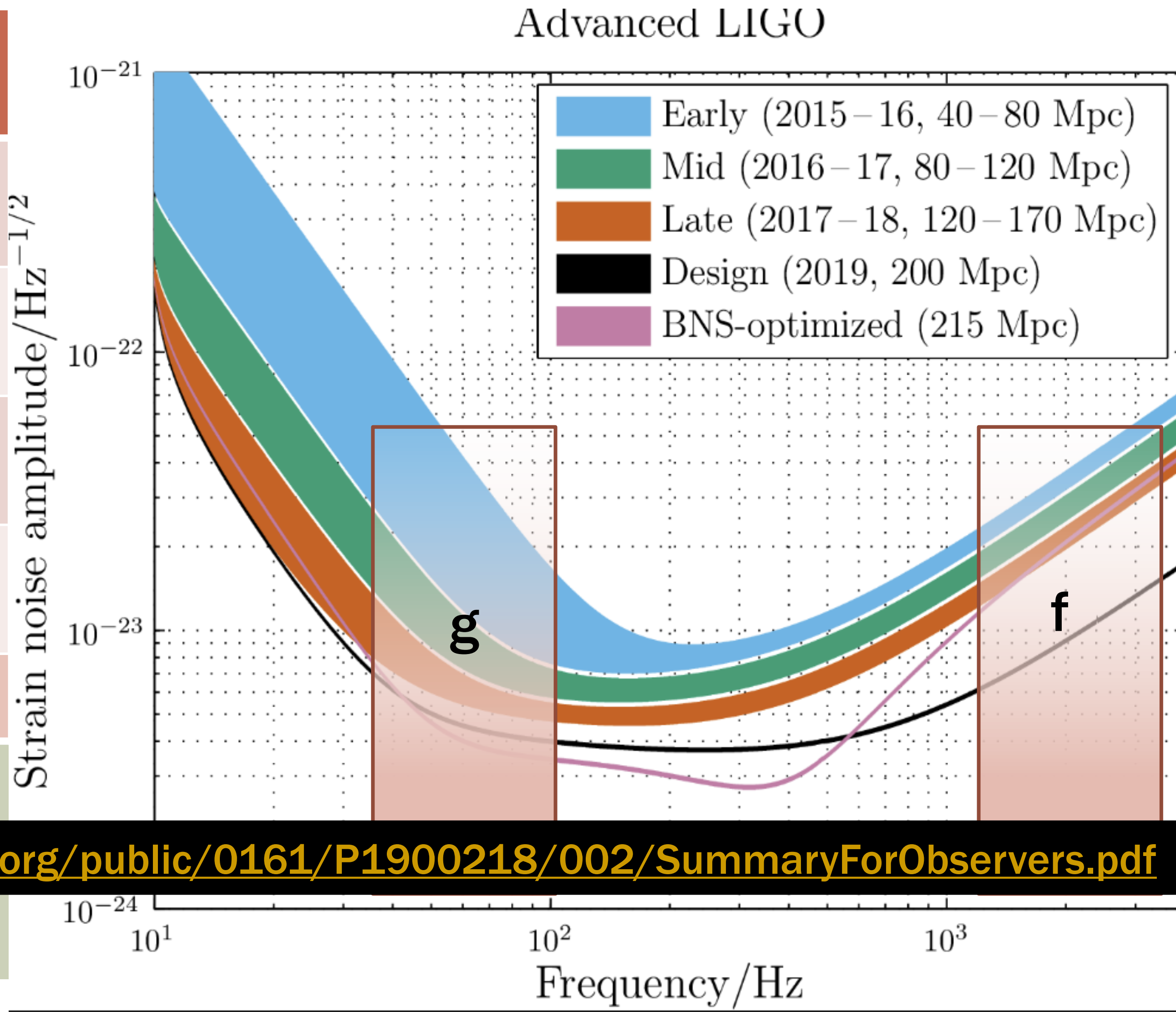
$$p' + \left(\frac{dp_0}{dr}\right)\xi_r = 0 \tag{2}$$

- (Solid) Interface ($r{=}r_i$) : Traction ($\mathrm{T}{=}\tau.\hat{n}$) $\implies T_h|_{r=r_c} = 0$

$$\delta\tau = \mu\left(\nabla\xi + (\nabla\xi)^T\right) + \left(\kappa - \frac{2\mu}{3}\right)\mathbb{1}\nabla.\xi \tag{3}$$

TYPES OF MODES

f
p
g
s , i
r



https://dcc.ligo.org/public/0161/P1900218/002/SummaryForObservers.pdf

GENERAL RELATIVITY

Spherically Symmetric Background: Schwarzschild Metric

$$ds^2 = e^{2\nu}(dt)^2 - e^{2\mu_2}(dr)^2 - e^{2\mu_3}(d\theta)^2 - e^{2\psi}(d\phi)^2$$

Metric perturbations

$$g_{\mu\nu} = g^0_{\mu\nu} + h_{\mu\nu} \quad \rightarrow \quad \delta G_{\mu\nu} = 8\pi\delta T_{\mu\nu}$$

5 non-linear coupled PDE inside, 2 outside – a computationally intensive problem

AXIAL MODES OF BLACK HOLES

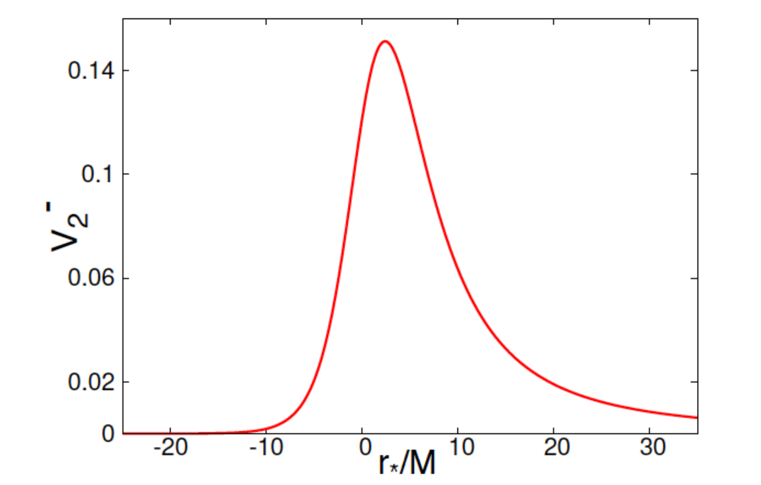
Chandrasekhar and Detweiler (Proc. Roy. Soc. A. 344 1639 1975):

- radial part of the perturbation equation is a Schrodinger equation

Zerilli Equation:

$$\frac{d^2 Z}{dr_*^2} + [\omega^2 - V_2^-(r)]Z = 0$$

$$r_* = r + 2M \ln(r/2M - 1)$$



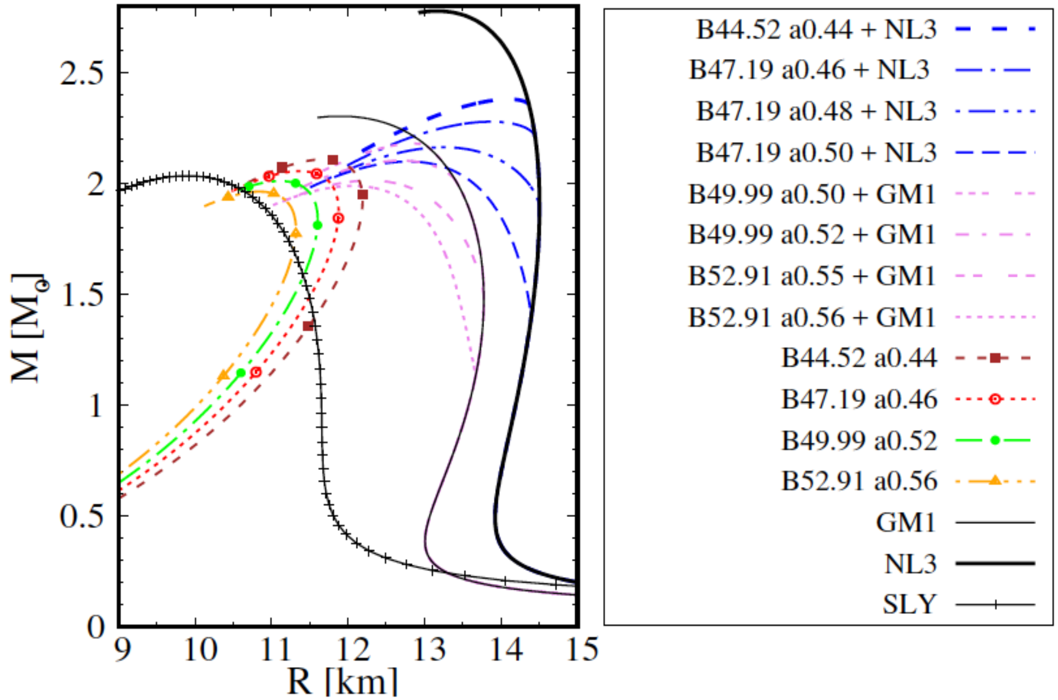
Finding Quasi-Normal Modes =>

Solving 1D Schrodinger equation for scattering from a central potential

Various Methods : Resonances, WKB, Continued Fraction

Neutron Stars / Strange Stars - Core EOS

Vasquez, Hall & Jaikumar, Phys. Rev. C 96, 065803 (2017)



Quark Matter EOS (Bag + a_4)

$$P_{q,\mathrm{core}} = \frac{1}{3}(\epsilon - 4B) - \frac{m_s^2}{3\pi}\sqrt{\frac{\epsilon - B}{a_4}} + \frac{m_s^4}{12\pi^2}\left[2 - \frac{1}{a_4} + 3\ln\left(\frac{8\pi}{3m_s^2}\sqrt{\frac{\epsilon - B}{a_4}}\right)\right]$$

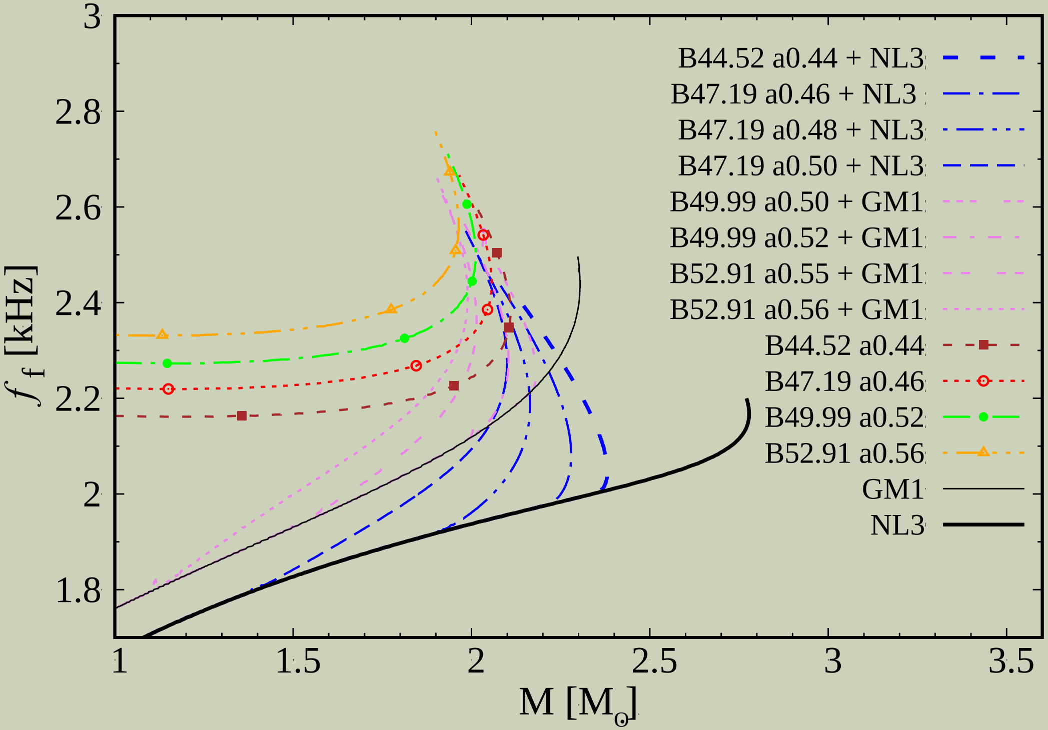
Hadronic EOS (SLy)

$10^5 \leq \rho(g/cc) \leq 10^8$; BPS

$10^8 \leq \rho(g/cc) \leq 5.10^{10}$; HP

$5.10^{10} \leq \rho(g/cc) \leq \rho_c$; SLy

F-MODE :
NEUTRON MATTER VS QUARK MATTER

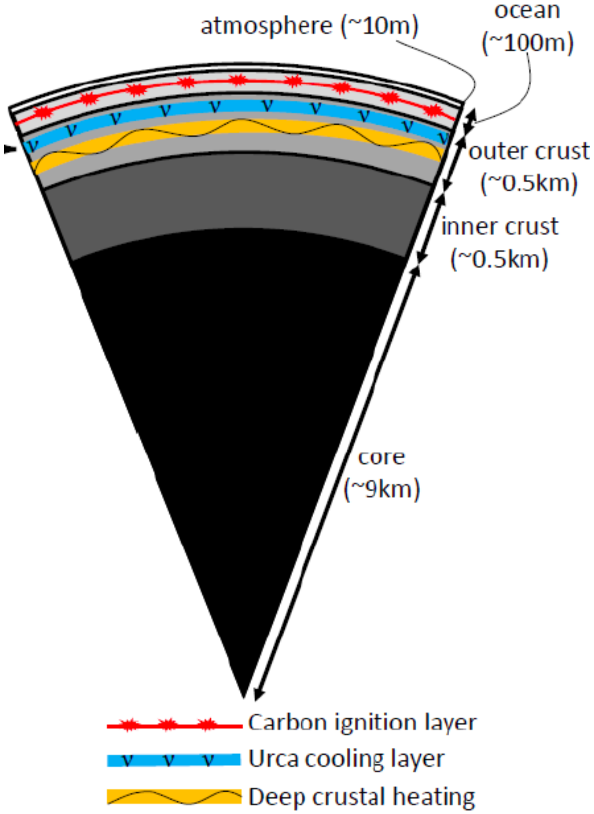


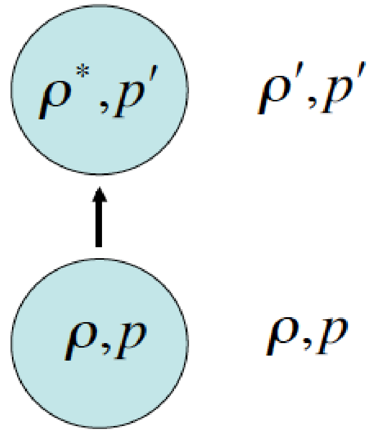
- f-mode frequencies approximately constant for pure quark matter up to 1.8 Msun
- Detection in AdLIGO would support Neutron/Hybrid stars
- Detection in Schenberg/Mini-Grail (NEMO?) would support some fraction of quark matter in neutron stars

Vasquez, Hall & Jaikumar, Phys. Rev. C 96, 065803 (2017)

Hinderer et al., Nature Communications 11 2553 (2019) – f_2 > 1.4 kHz from GW170817

LOCALIZED MODES – OCEAN (g-MODE)





$$\rho \frac{d^2}{dt^2}(\delta z) = -(\rho^* - \rho')g$$

$$\frac{d^2}{dt^2}(\delta z) + N^2(\delta z) = 0$$

The Brunt-Väisälä Frequency

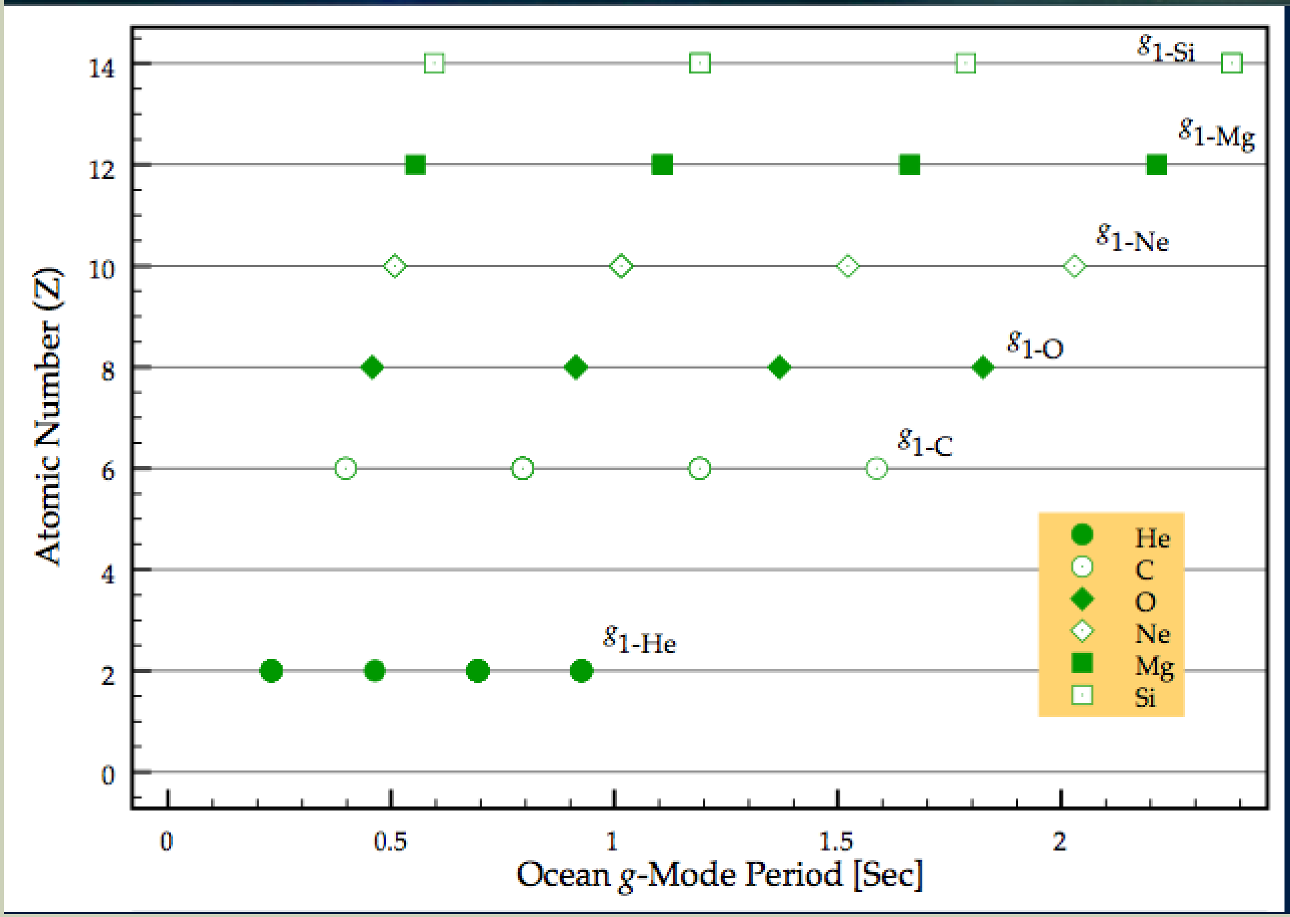
$$N^2 = -\frac{g}{r}\left(\frac{1}{\Gamma_1}\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} - \frac{\mathrm{d}\ln \varrho}{\mathrm{d}\ln r}\right)$$

Buoyancy → gravity waves

Restoring force from Buoyancy (density dependent)

Evidence for g-modes :
10-100 Hz frequencies may explain modulation of X-ray flux during accretion events

G-MODES AND COMPOSITION



two-component NS star, l = 2.
M = 1.4 M_{sun}
R = 10 km

$$N^2 = -Ag$$

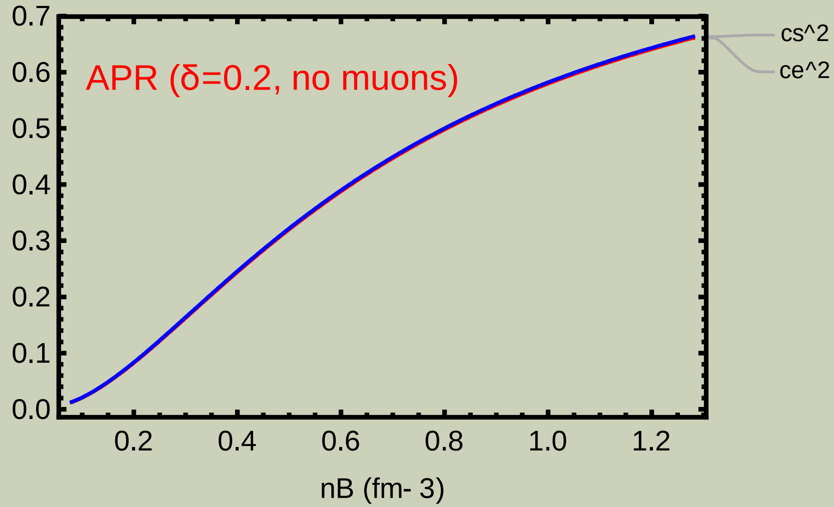
Modes depend on atomic mass number, A as $1/A^{1/2}$

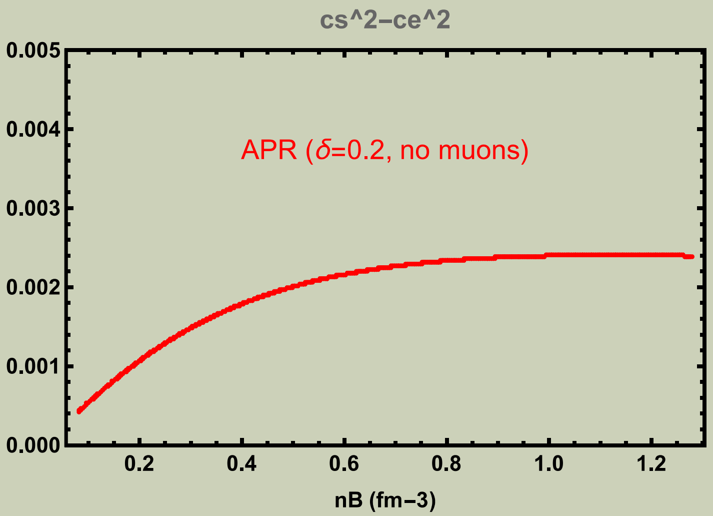
CORE G-MODES

$$N^2 = g^2 \left(\frac{1}{c_e^2} - \frac{1}{c_s^2} \right) e^{\nu - \lambda}$$

Cs : The adiabatic sound speed : beta-equilibrium timescale > oscillation timescale

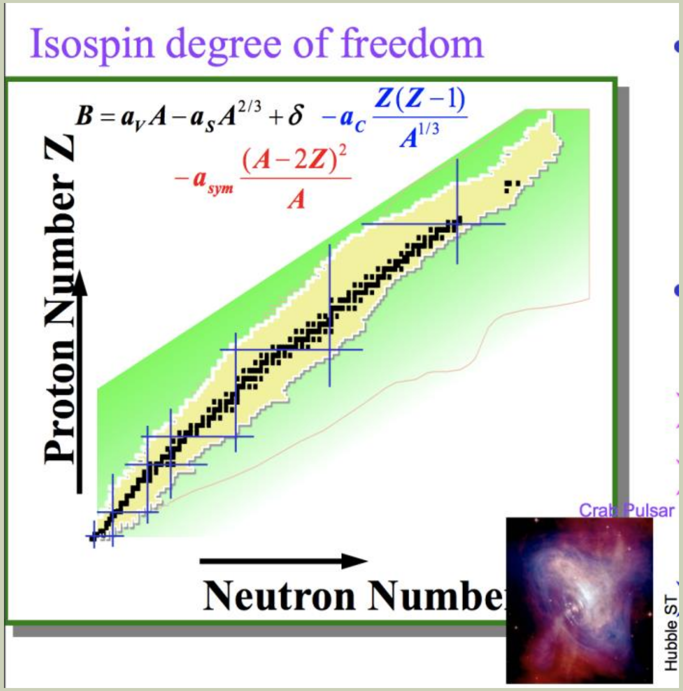
$$c_s^2 = \left(\frac{\partial p}{\partial \varepsilon} \right)_{x_x}$$



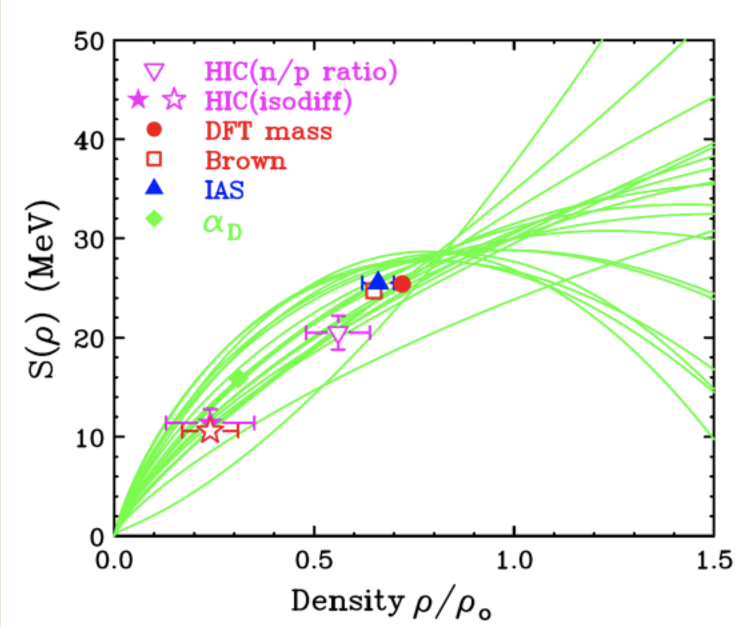


SYMMETRY ENERGY

$$\omega_{BV} = g\left(\frac{1}{c_e^2} - \frac{1}{c_s^2}\right)^{1/2} \approx 2\left(\frac{g}{c_e}\right)\left(\frac{x}{3}\right)^{1/2} \frac{(3nE_s' - E_s)}{\sqrt{E_s(\frac{10}{9}E_0 + 2nE_s' + n^2E_S'')}}$$

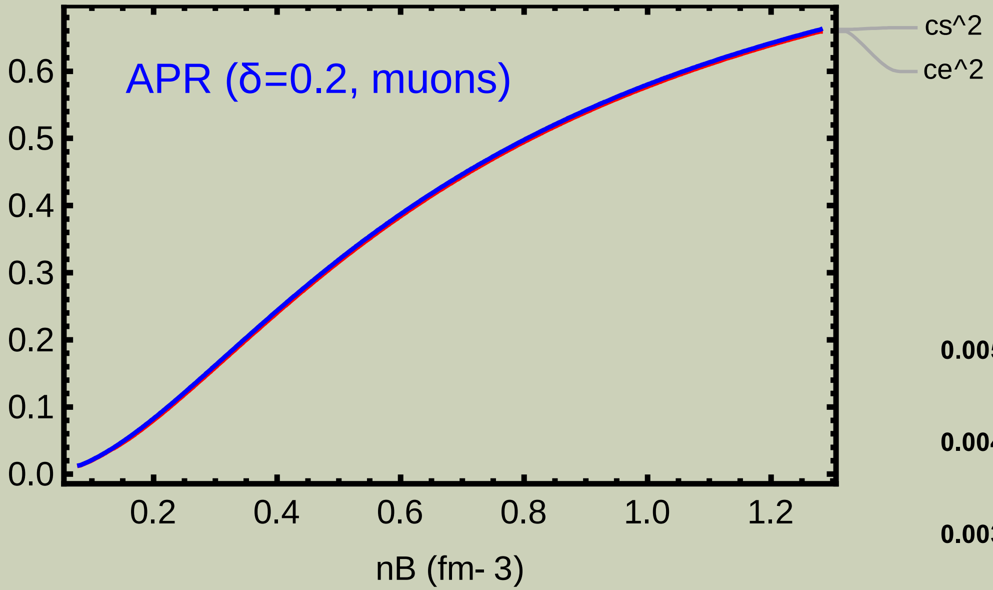


Models constrained at low density only



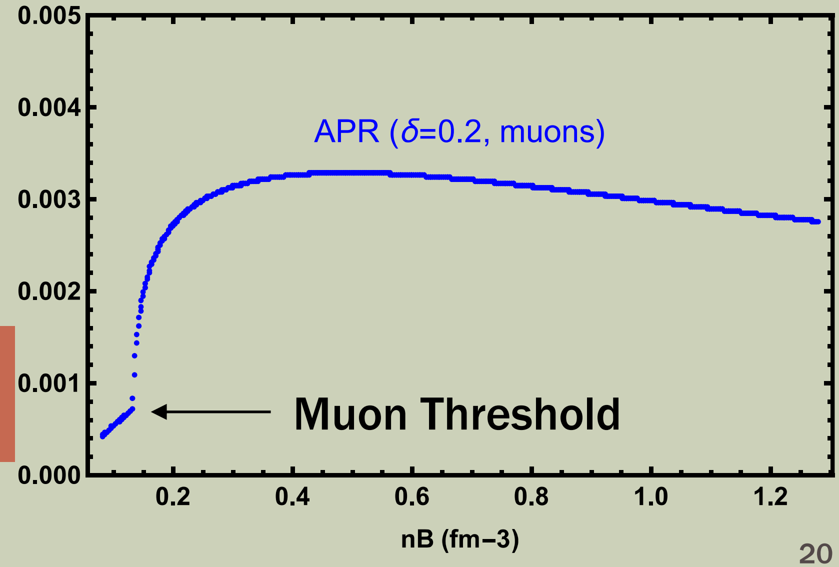
Lynch and Tsang,
e-Print: 1805.10757 [nucl-ex]

SOUND SPEED AND COMPOSITION

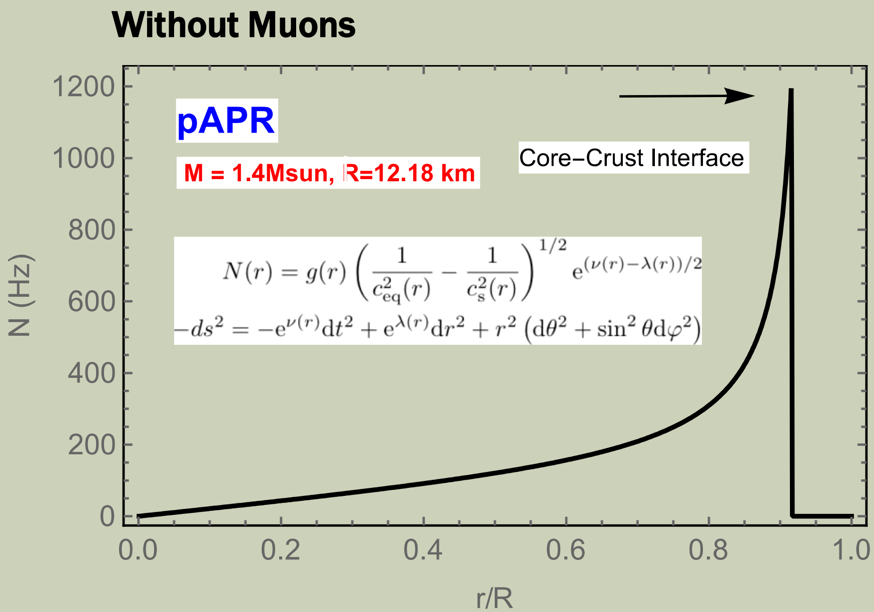


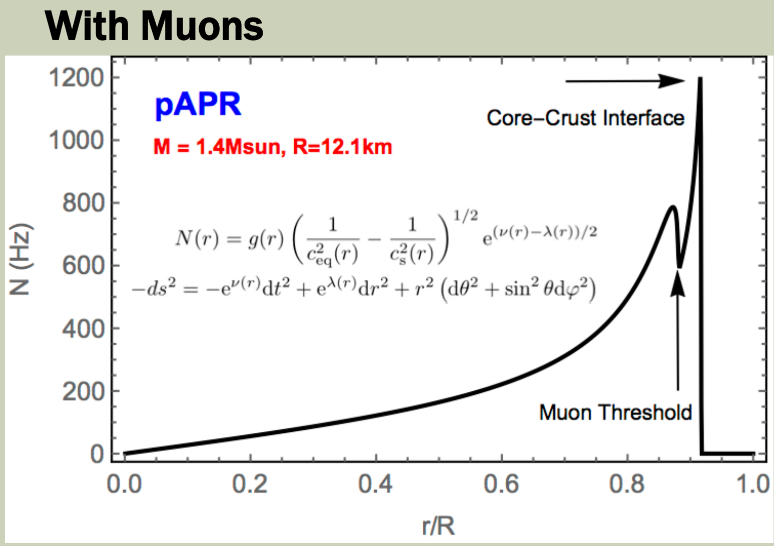
Despite the two sound speeds being very close ...

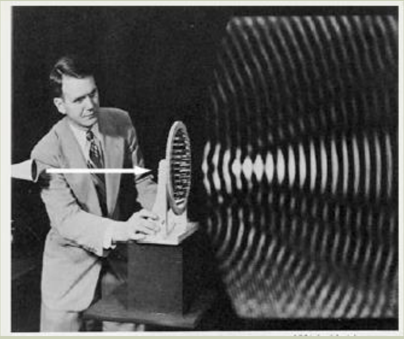
..their difference shows a sharp change when a new species enters



BRUNT VAISALA FREQUENCY

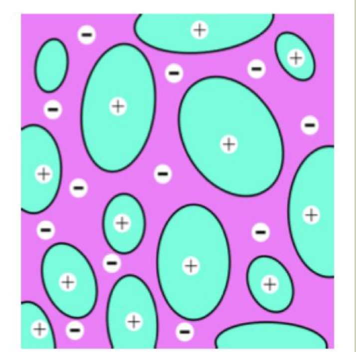




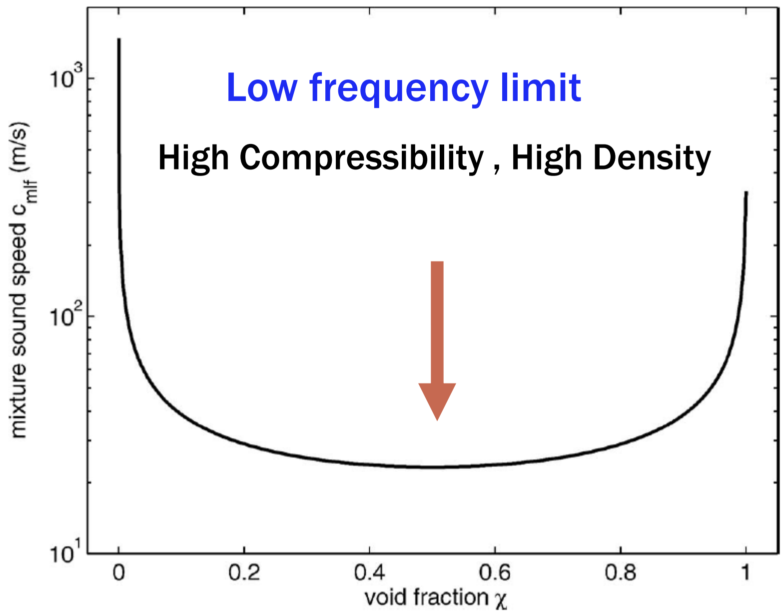


Homogeneous phases : g-mode frequency ~ 100 Hz

How does sound propagate in a mixed phase?



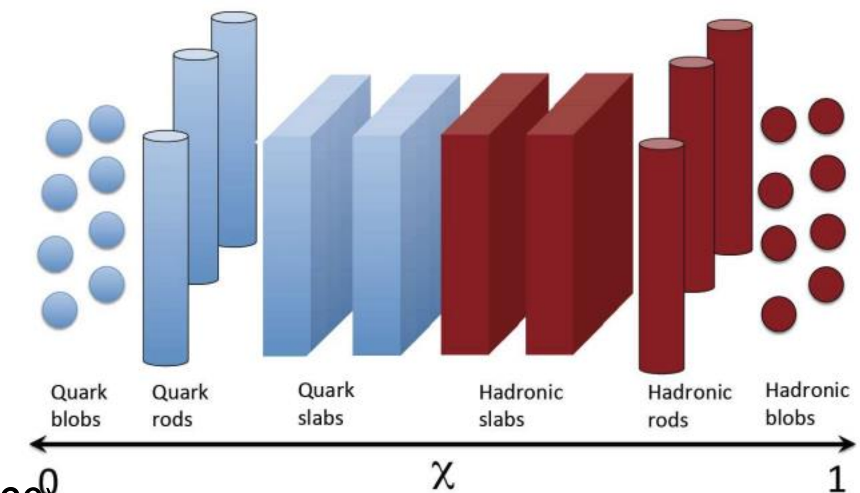
SOUND IN BUBBLY FLUID



Wilson & Roy, AmJ. Phys. 76, 975 (08)

$$\frac{1}{c_{\mathrm{mlf}}^2} = \frac{(1-\chi)^2}{c_\ell^2} + \frac{\chi^2}{c_g^2} + \chi(1-\chi)\frac{\rho_g^2 c_g^2 + \rho_\ell^2 c_\ell^2}{\rho_\ell \rho_g c_\ell^2 c_g^2}$$

In a neutron star – quark hadron mixed phase



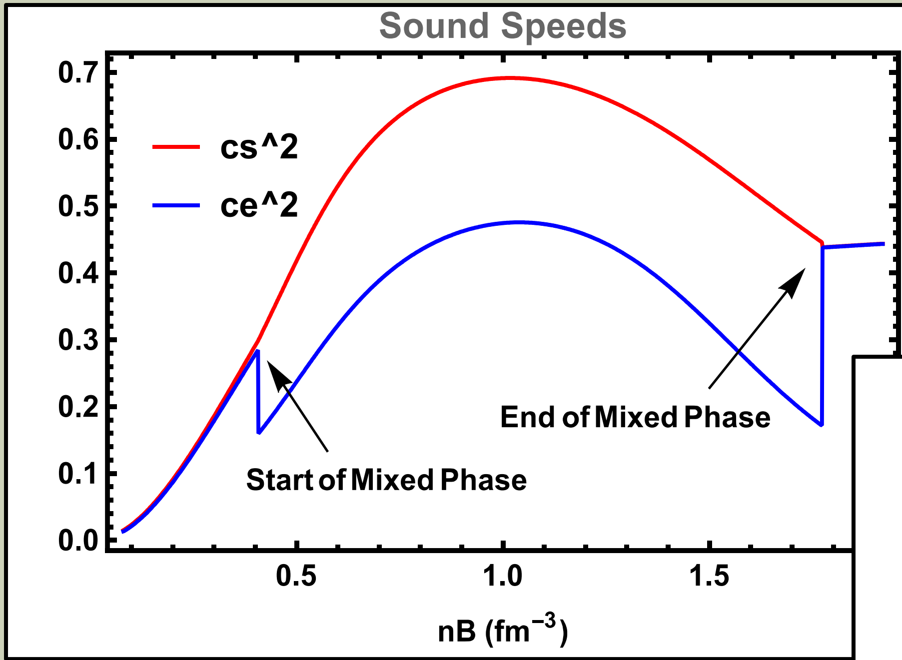
Two conserved charges :

Electric charge and baryon number

$$c_{\mathrm{mix}}^2 = \frac{dP_{\mathrm{mix}}(\mu_B, \mu_Q)}{d\rho} = \frac{\partial P_{\mathrm{mix}}}{\partial \mu_B}\left(\frac{d\mu_B}{d\rho}\right) + \frac{\partial P_{\mathrm{mix}}}{\partial \mu_Q}\left(\frac{d\mu_Q}{d\rho}\right)$$

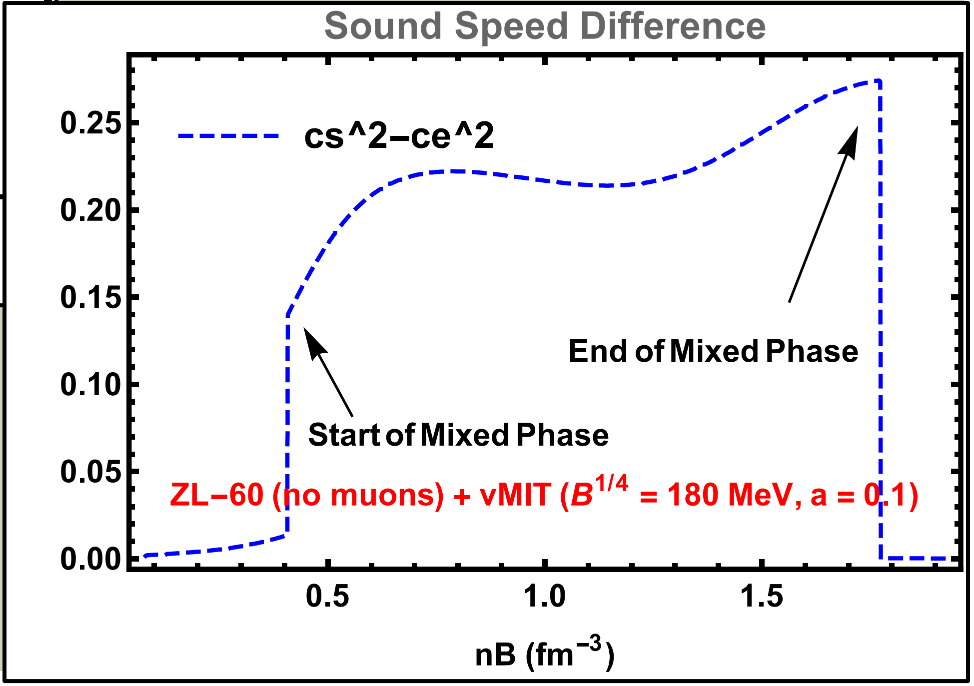
Barry, Salinas, Wei, Klaehn & Jaikumar, submitted to ApJ (2020)

MIXED PHASE IDENTIFICATION



Nuclear : ZL (Zhao & Lattimer, PRD102 023021 2020)

Quark : vMIT (Schramm et al., ApJ 877 139 2019)



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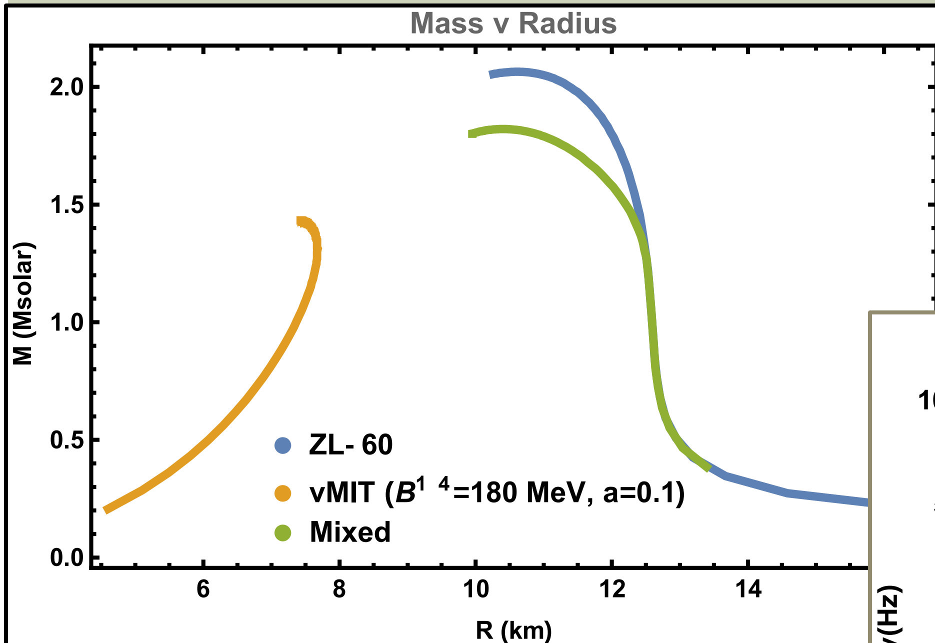

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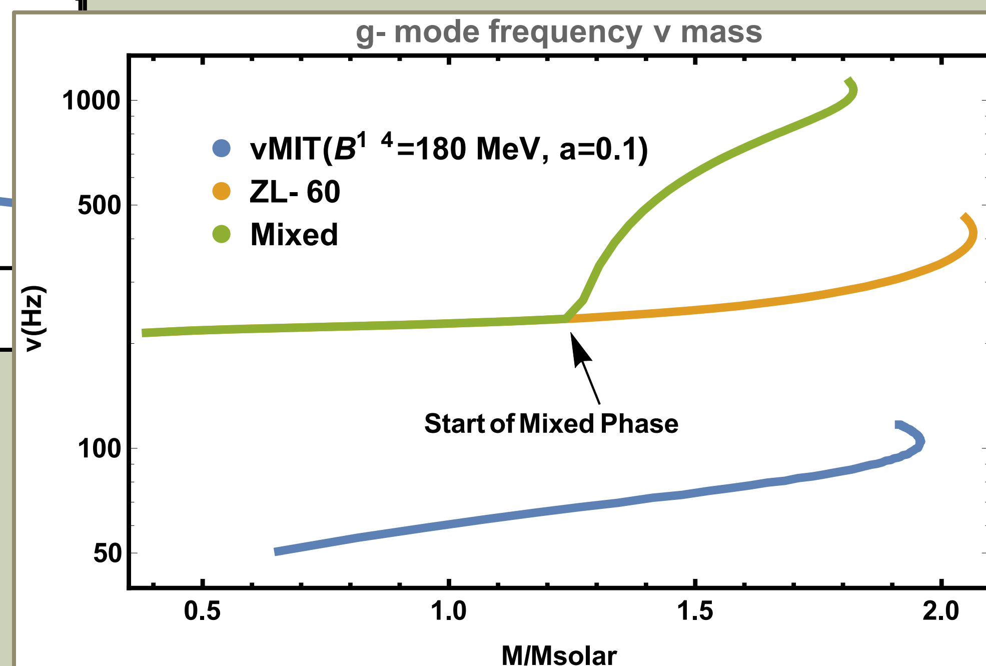
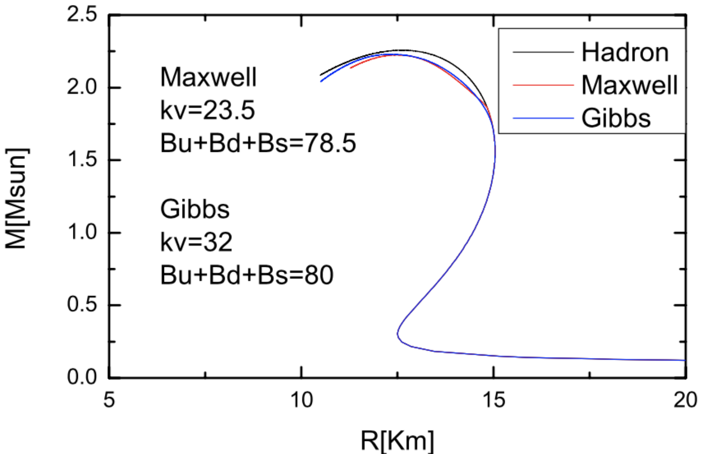
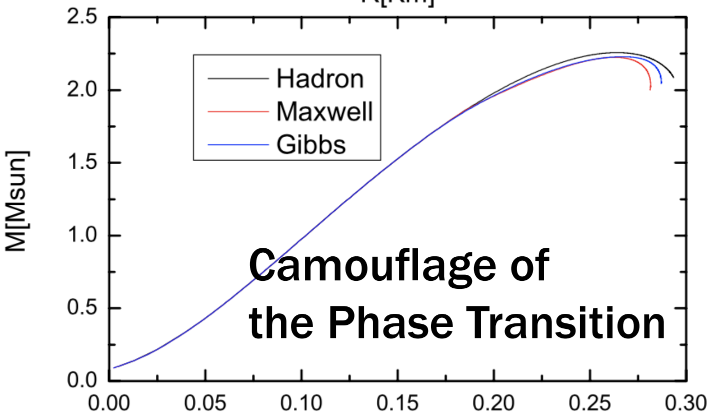


QUARK-HADRON MIXED PHASE





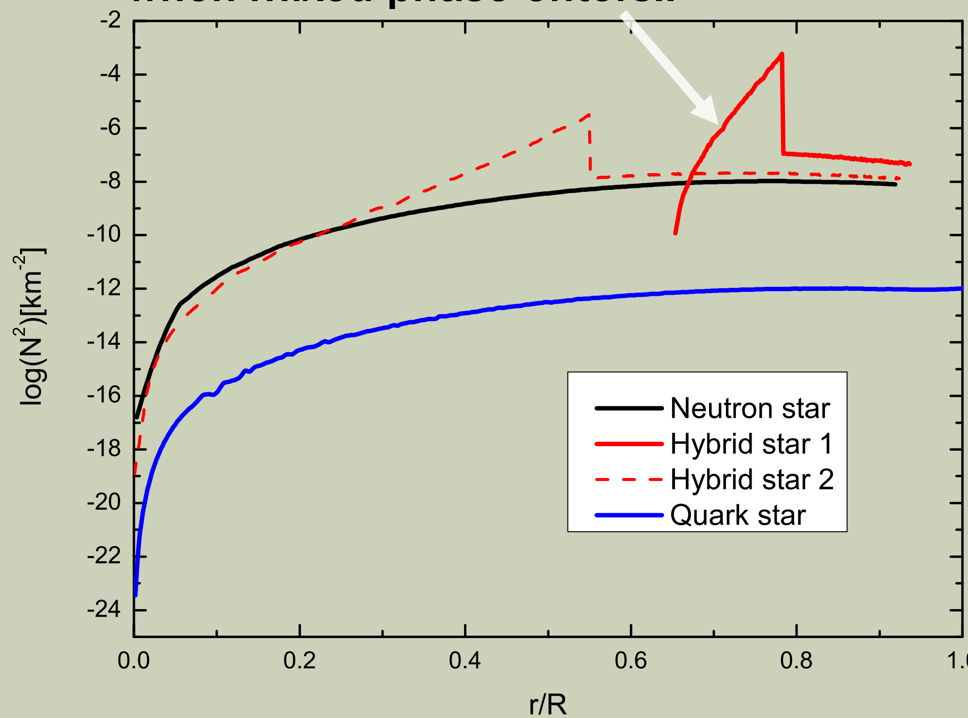
- Mixed phase is preferred if surface tension between Quark/hadron matter is small

EOS = DBHF (nuclear) + vBag (quark)

Wei, Salinas, Irving, Klaehn and Jaikumar, ApJ 887 (2), 151 (2019)

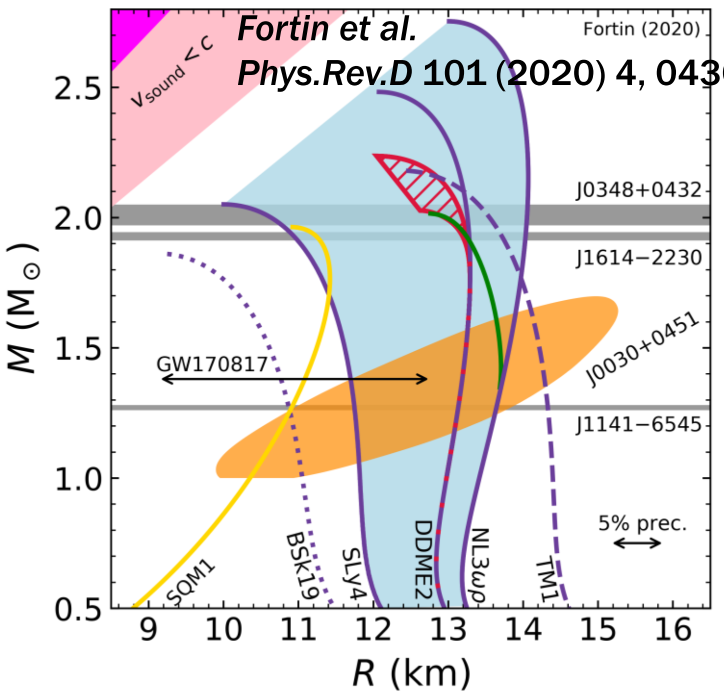
Distinctive signature of a continuous phase transition

Local oscillation frequency shows a peak when mixed phase enters..

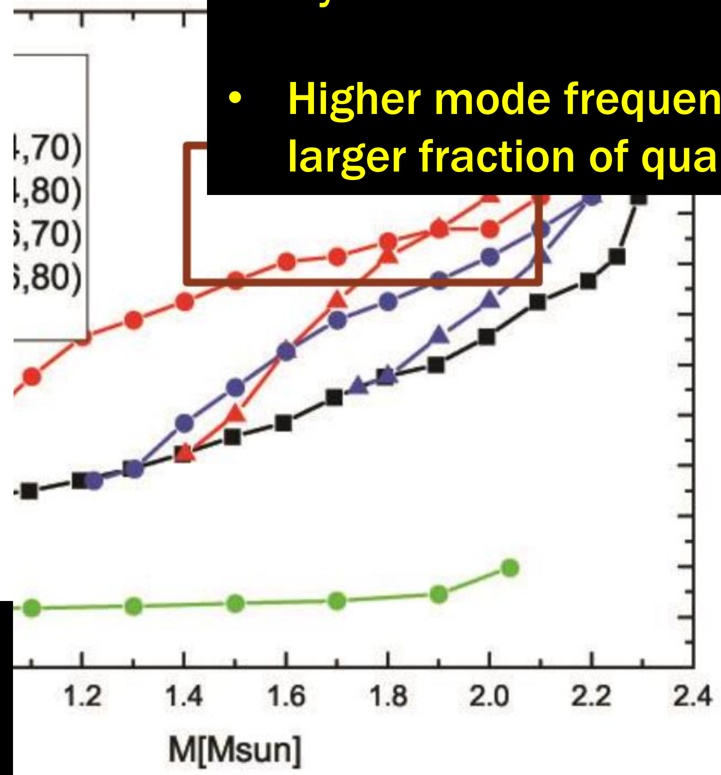


IDENTIFYING A MIXED PHASE

Fortin et al.
Phys.Rev.D 101 (2020) 4, 043021

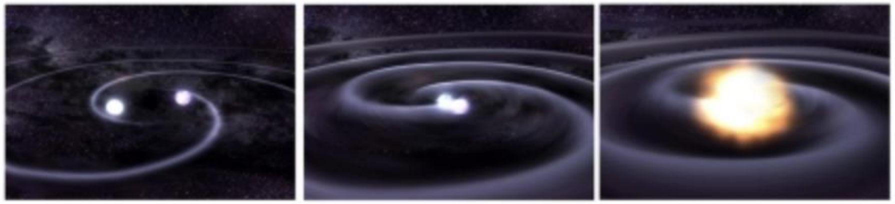


- g-mode > 0.8 kHz only for Hybrid stars
- Higher mode frequency => larger fraction of quark matter

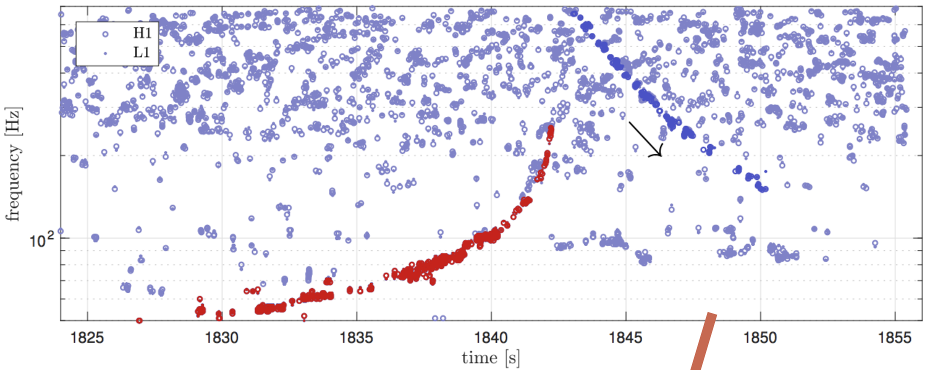


- Improving constraints on radii
- One or both components could be Hybrid stars.

OBSERVATIONAL OUTLOOK



GW170817 : A global astronomical event



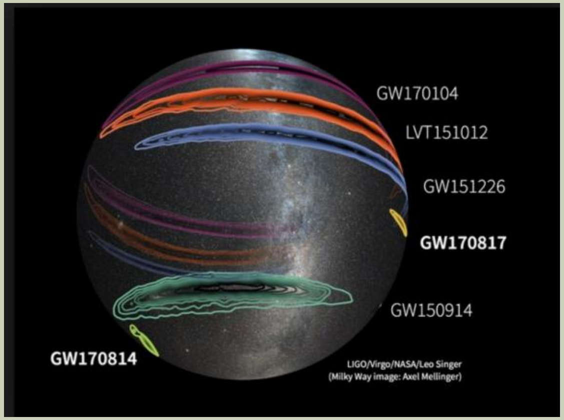
Spectrogram of GW170817

(Abbott et al. PRL 119, 161101 (2017))

$$f_m(t) = A(t_c - t)^{-3/8} \quad (t < t_c)$$

$$M_{\rm chirp} \approx 1.1382 F(e)^{-3/5} M_\odot \approx 1.188 M_\odot$$

$$D = \frac{5}{96\pi^2}\frac{c}{h}\frac{f_{\rm GW}}{\dot{f}_{\rm GW}^3}$$



G-MODE DAMPING

- Neutrino damping ($\delta\mu(n_B, x_e) = \mu_n - \mu_p - \mu_e$)

$$\tau_\beta(\mathrm{yr}) \approx 8.2\, T_9^{-4} \left(\frac{n_{\mathrm{sat}}}{n}\right)^{2/3} \frac{1}{(\delta\mu/\mathrm{MeV})}$$

- Shear damping

$$\tau_{\mathrm{visc}}(\mathrm{yr}) \sim \frac{L^2}{\nu} \approx 1.5 \times 10^3 L_6^2 T_9^{5/3} \left(\frac{n_{\mathrm{sat}}}{n}\right)^{5/9}$$

- GW damping (growth)

$$\tau_{\mathrm{gw}}(\mathrm{yr}) \sim \frac{1+\mathcal{E}}{25} \hat{\omega}_i^{-5} \hat{\omega}_r \frac{R_{10}^4}{M_{1.4}^3} \left(\frac{10^{-4}}{\delta D_{22}}\right)^2$$

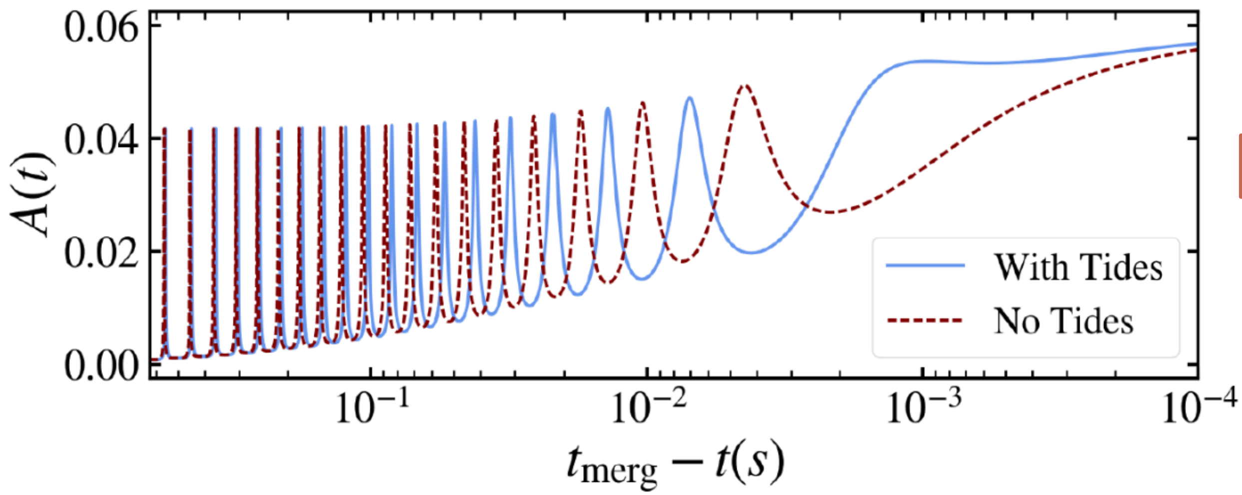
$$\tau = (\tau_\beta^{-1} + \tau_{\mathrm{visc}}^{-1} + \tau_{\mathrm{gw}}^{-1})^{-1}$$

$\tau < 0$ for instability

G-mode can be driven unstable in rotating stars if 0.1 < T9 < 10 and $\omega_{\mathrm{rot}} > 2\omega_g$

DETECTION PROSPECTS

Vick & Lai, PRD 100, 063001 (2019)



Example of phase shift due to f-mode oscillations (Vick 2019)

$$\Delta\phi_{\rm stat} \approx \sqrt{D-1}/({\rm SNR})$$

$$SNR \geq 30\,,$$
$$f = \omega/(2\pi) \approx 0.5{\rm kHz}$$

$$\Delta\Phi(\tau) \approx 2\times 10^{-2}\left[\frac{0.33}{\tau^{3/8}} - 1\right]\left(\frac{\omega_g}{2\omega_{\rm dyn}}\right)^{1/3}\left(\frac{S}{10^{-2}}\right)^2$$

Single Detector : tens of Mpc
Network : hundreds of Mpc

CONCLUSIONS

- ❖ Non-radial modes of compact stars carry imprints of the phase of matter through resonant excitation frequencies
- ❖ g-modes can probe stratification : mixed phase / crust of neutron stars
- ❖ Oscillation modes as or more sensitive to composition than tidal polarizability (but may need continuous wave sources)
- ❖ Detection of oscillation modes is worth pursuing with improved sensitivity and more detectors