Random Field Ising Model with Conserved Kinetics

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Outline

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I. Random Field Ising Model

The Random Field Ising Model (RFIM) is the simplest (prototypical) example of a system with disorder.

- Energy function

$$E = -J \sum_{\langle ij \rangle} s_i s_j - \sum_{i=1}^{N} h_i s_i, \quad s_i = \pm 1.$$

  - The interaction $J > 0$ prefers a magnetized structure.
  - The disordering random fields $\{h_i\}$ are generally drawn from:

$$P(h_i) = \frac{1}{\sqrt{2\pi}\Delta} e^{(-h_i{}^2/2\Delta^2)}.$$

  - For $d = 3$, small region of (T, Δ)-values where equilibrium phase is ferromagnetic. $T_c(\Delta = 0) \simeq 4.51$, $\Delta_c(T = 0) \simeq 2.28$.

RFIM with Conserved Dynamics (C-RFIM)

Some Experimental Realizations of the RFIM:

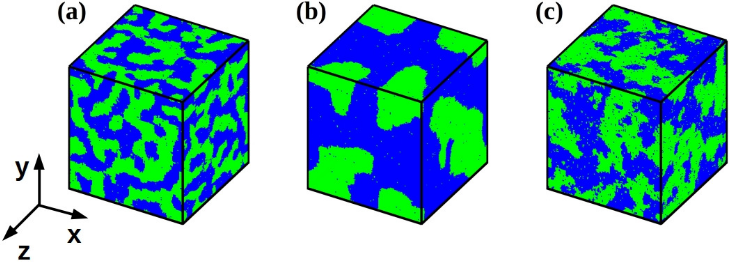
- Diluted antiferromagnets (DAFs) in a uniform field, e.g. $\mathrm{Co_xMn_{1-x}F_2}$. **Fishman & Aharony, J. Phys. C, 1978; Miga et al., PRB, 2009**
- Dipolar quantum magnet $LiHo_xY_{1-x}F_4$ in a transverse field. **Schechter & Stamp, PRL, 2005; Schechter, PRB, 2008**
- Binary mixtures (AB) in porous medium (oil-water, colloid-polymer). **P.G. De Gennes, J. Phys. Chem. Lett., 1984; Vink et al.,PRL, 2006**

Ising spins do not have an intrinsic dynamics. Contact with heat bath generates stochastic spin-flips.

- Glauber model with non-conserved kinetics (DAFs, $\mathrm{LiHo_xY_{1-x}F_4}$).
- Kawasaki model with conserved kinetics (binary mixture with A $\leftrightarrow$ B interchanges).
- Although the two models describe different time-dependent behavior, the equilibrium state is unique.

Domain Growth after a Temperature Quench (Coarsening)

- The lattice size is 128^3 and the temperature $T = 2 < T_c(\Delta)$.
- The green and blue regions correspond to $s_i = 1$ and $s_i = -1$, respectively.



- Domain growth in $d = 3$ C-RFIM: (a) $\Delta = 1.0, \ t = 10^5$ MCS; (b) $\Delta = 1.0, \ t = 10^7$ MCS; (c) $\Delta = 2.0, \ t = 10^7$ MCS.

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Tools for Characterizing Morphologies

- Standard probe is the correlation function:

$$C(r) = \langle \psi(\vec{r}_i)\psi(\vec{r}_j)\rangle - \langle \psi(\vec{r}_i)\rangle\langle \psi(\vec{r}_j)\rangle,$$

where $\psi(\vec{r}_i)$ is an appropriate variable $[\sigma_i]$ and $r = |\vec{r_i} - \vec{r_j}|$. The angular brackets denote an ensemble average.

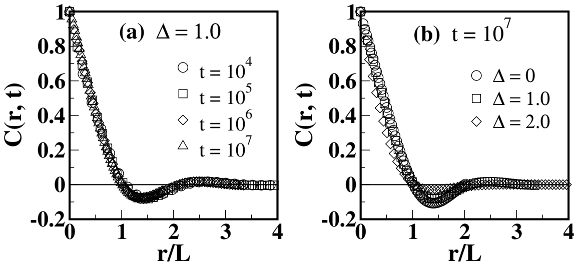
Characteristic length L: Distance over which $C(r)$ decays to (say) $0.2 \times$ maximum value.

- Small-angle scattering experiments yield the structure factor:

$$S(\vec{k}) = \int d\vec{r} e^{i\vec{k}\cdot\vec{r}} C(r),$$

where $\vec{k}$ is the wave-vector of the scattered beam.

II. Dynamical Scaling; Super-Universality Violations



(a) Scaled correlation functions, $C(r,t)$ vs. r/L, for disorder $\Delta = 1.0$ and time $t = 10^4, 10^5, 10^6, 10^7$ MCS. The length scale $L(t)$ is the first zero-crossing of $C(r,t)$. Collapse a signature of dynamical scaling; Coarsening morphologies characterized by a unique length scale $L(t,\Delta)$.

(b) Scaled correlation functions for $t = 10^7$ MCS and $\Delta = 0, 1.0, 2.0$. Scaling function not robust with respect to disorder.
(Unlike in the non-conserved case. **VB et al. EPL 2011; PRE 2014, 2016**)

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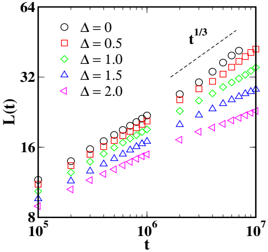
Growth Laws: Algebraic vs. Logarithmic

- Plot of characteristic length scale $L(t)$ vs. t on log-log scale:



For pure systems ($\Delta = 0$), $L(t) \sim t^{1/3}$.
Lifshitz-Slyozov (LS) law

- Notice the slowing down of domain growth at late times for higher values of Δ.

(i) Algebraic growth at early times: $L(t, \Delta) \sim t^{1/\bar{z}}$ with disorder-dependent exponent $\bar{z}(\Delta)$. (For $\Delta = 0$, $\bar{z} = 3$.)

(ii) Cross-over to logarithmic domain growth at late times: $L(t, \Delta) \sim (\ln t)^{1/\psi}$, ψ is a disorder-independent *(barrier)* exponent.

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Scaling Ansatz to determine $\bar{z}$ and ψ

Corberi et al. PRE 2012, 2013

- $$L(t,\Delta) \sim t^{1/z_{\rm eff}} = t^{1/z} F(\Delta/t^{\phi}), \tag{1}$$

  $$F(x) \sim \begin{cases} \text{const.}, & \text{for } x \to 0, \\ x^{1/\phi z}\, \ell\left(x^{-1/\phi}\right), & \text{for } x \to \infty. \end{cases} \tag{2}$$

  $z_{\rm eff}$ is the *effective* growth exponent, ϕ is the crossover exponent.

- The evaluation of $z_{\rm eff}$ is easier using the inverted form:

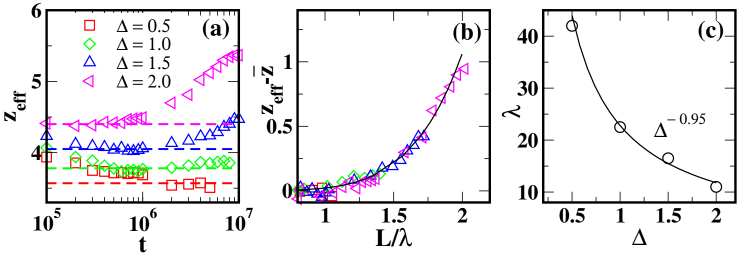
  $$t = L^{z} G(L/\lambda). \tag{3}$$

  Here, $\lambda = \Delta^{1/\phi z}$ is the crossover length scale and $G(y) = [F(x)]^{-z}$.

- The effective exponent as a function of y $(= L/\lambda)$ is then

  $$z_{\rm eff}(y) = \frac{\partial \ln t}{\partial \ln L} = z + \frac{\partial \ln G(y)}{\partial \ln y}. \tag{4}$$

Exponents $\bar{z}$, ϕ and ψ



(a) $z_{\text{eff}} = [d(\ln L)/d(\ln t)]^{-1}$ vs. t (semi-log).
Dashed lines: disorder-dependent exponents $\bar{z}(\Delta)$ of the power law.
This is followed by a late regime where z_{eff} is time-dependent.

(b) Scaling collapse of $z_{\text{eff}} - \bar{z}$ vs. L/λ, where $\lambda = \Delta^{1/\phi\bar{z}}$. The solid line is the best power-law fit: $z_{\text{eff}} - \bar{z} = b(L/\lambda)^{\psi}$ with $b \simeq 0.022$, $\psi \simeq 5.6$.

(c) Δ-dependence of λ. Power-law fit: $\lambda \sim \Delta^{-0.95}$.

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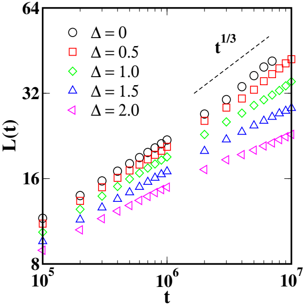
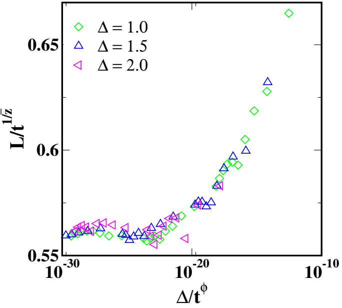
Exponents and Data Collapse

Δ	0	0.5	1.0	1.5	2.0
$\bar{z}$	3.0	3.57	3.78	4.05	4.40
$\lambda\ (= \Delta^{1/\phi\bar{z}})$	∞	42.1	22.5	16.5	11.0





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Why the Logarithmic Domain Growth?

Generalizing Eqs. (1)-(4) by replacing $z \to \bar{z}$,

$$\frac{\partial \ln G(y)}{\partial \ln y} = z_{\text{eff}} - \bar{z} = by^{\psi} \quad \Rightarrow \quad G(y) \sim \exp\left(\frac{b}{\varphi} y^{\psi}\right). \tag{5}$$

Substituting in Eq. (3) results in the asymptotic logarithmic growth form:

$$\frac{L}{\lambda} \simeq \left[\frac{\psi}{b} \ln(t/\lambda^{\bar{z}})\right]^{1/\psi}. \tag{6}$$

The disorder-independent exponent ψ has great physical significance:

- Domain growth in disordered systems (e.g. RFIM) proceeds via activation over barriers of energy $E_B \sim \epsilon_B L^{\psi}$,
  ϵ_B: barrier energy per unit length; ψ: barrier exponent.
- The asymptotic growth law is then logarithmic:
  $L(t) \sim (T/\epsilon_B)^{1/\psi} (\ln t)^{1/\psi}$.

Huse & Henley, PRL 1985; Lai et al., PRB 1988

Rough Interfaces, Cusp Singularities and Non-Porod Tails

- Interfaces separating correlated regions of up and down spins are rough in disordered systems.
- The signature is a *cusp singularity* in the small-r behavior of the correlation function:

$$C\left(r, t; \Delta\right) = 1 - A(x)^{\alpha} + O(x^{2+\alpha}),$$

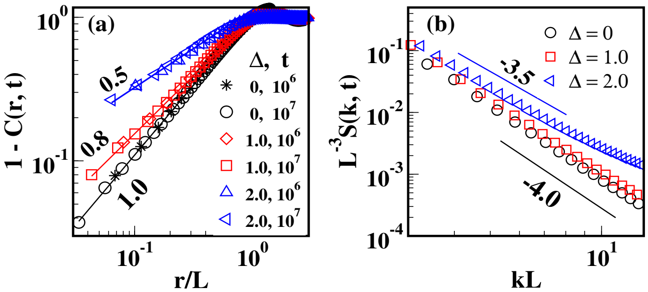
  where $x = r/L$, A is a constant, and α is the *cusp* exponent.
- For smooth interfaces, $\alpha = 1$. For rough (fractal) interfaces, $0 < \alpha < 1$ and the fractal dimension $d_f = d - \alpha$.
- The corresponding structure factor exhibits a *non-Porod tail* indicative of scattering off rough interfaces:

$$S\left(k, \Delta\right) \simeq \tilde{A}\left(kL\right)^{-(d+\alpha)}$$

- For $\alpha = 1$, $S\left(k, \Delta\right) \sim k^{-(d+1)}$ yielding the *Porod law* due to scattering from smooth interfaces.

VB et al., EPL 2013, PRE 2014, PRE 2016

Interfacial Characteristics during Domain Growth



(a) Data collapses for fixed Δ and different values of t, but not for different values of Δ, as the system exhibits dynamical scaling but not super universality. Solid lines: Disorder-dependent roughness exponent $\alpha(\Delta) \simeq 1.0, 0.8, 0.5$ for $\Delta = 0, 1.0, 2.0$, respectively.

(b) Plot of scaled structure factor, $L(t)^{-d} S(k, t; \Delta)$ vs. $kL(t)$, for $t = 10^7$ MCS and $\Delta = 0, 1.0, 2.0$. The solid lines denote relevant Porod and non-Porod tails.

Generalized Tomita's Rule

- Using conditions of continuity and differentiability for $S(k,\Delta)$, some algebra yields:

$$\int_0^\infty \mathrm{d}p\, p^{1-\alpha}\left[p^{d+\alpha}f(p)-\mathcal{C}\right]=0, \tag{7}$$

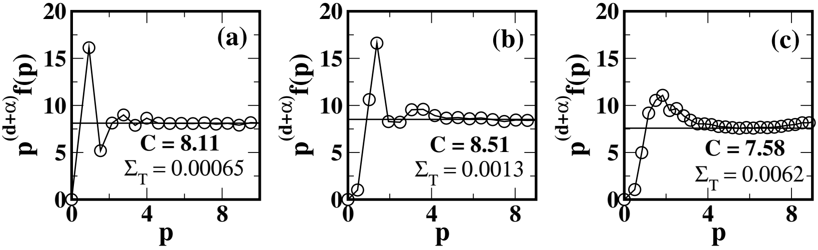
where $p=kL$, $f(p)$ is the scaled structure factor & $\mathcal{C}$ is a constant.

  - The result with $\alpha=1$ (case with sharp interfaces) is referred to as *Tomita's sum rule*. **Tomita, Prog. Theor. Phys. 1984, 1986**
  - Eq. (7) constitutes a generalization to the case with rough or fractal interfaces. **Kumar, VB & Puri, EPL 2017**

- To date, there is no theory available for the complete scaling function in the case with conserved kinetics even for the pure case.

- The Tomita sum rule sets a useful constraint on reasonable functional forms for the correlation function or structure factor.

C-RFIM Obeys the Generalized Tomita's Rule



- Plot of $p^{(d+\alpha)} f(p)$ vs. p to demonstrate the *generalized Tomita sum rule* for (a) $\Delta = 0$, (b) $\Delta = 1.0$, and (c) $\Delta = 2.0$.
- The solid line in each plot indicates the value of the constant $\mathcal{C}$ in Eq. (7).
- The values of $\mathcal{C}$ and the Tomita sum Σ_T, obtained using numerical integration, are also specified in each frame.

Kumar, VB & Puri, EPL 2017

Summary

- Comprehensive MC study of domain growth in the RFIM with conserved dynamics (C-RFIM) in $d = 3$. (128^3, $t = 10^7$ MCS).
- Observe *clean* cross-overs from a disorder-dependent power-law growth to a disorder-independent logarithmic growth.
- There is dynamical scaling, signifying the presence of a unique length-scale. However, super-universality (SU) is violated indicating that system is not robust to disorder.
- The small-r behavior of the correlation function exhibits a cusp singularity: $1 - C(r) \simeq A(r/L)^{\alpha(\Delta)}$. The cusp exponent α yields the interfacial fractal dimension: $d_f = d - \alpha$.
- The corresponding structure factor exhibits a non-Porod decay: $S(k, t, \Delta) \sim k^{-(d+\alpha)}$, signifying scattering off fractal interfaces. Further, the scaling function for the structure factor obeys a *generalized Tomita sum rule*.

Implications of Fractal Interfaces

- Growth of domain of size R is by overcoming the barrier energy E_B separating local minima of the complex energy landscape.
- For curvature-driven growth in non-conserved systems,
  - $dR/dt = a(R,t)/R$ **(Lai, Mazenko & Valls, PRB, 1988)**
- In systems with quenched disorder,
  - $a(R,t) = a_0 \exp(-\epsilon_B R^{\psi}/T)$,
  - $R(t) \sim (T\epsilon_B^{-1})^{1/\psi}(\ln t)^{1/\psi}$,

  where ϵ_B is the energy barrier per unit length.
- Villain argued that the barrier energy $\epsilon_B(R) \sim R^{\psi}$, where $\psi = 2 - \alpha$. **(Villain, PRL, 1984)**

Fractal interfaces lead to slow logarithmic domain growth.