Phase Transitions and Applications of a Few Model Low-dimensional Optical Lattices

Swapan K Pati

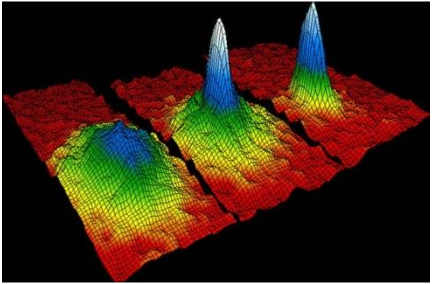
Theoretical Sciences Unit

JNCASR, Bangalore

http://www.jncasr.ac.in/pati/

Bose-Einstein Condensation of Bosons

Experimentally first Bose-Einstein Condensation of ^{87}Rb atoms was shown by laser cooling down to $\approx$ 20 nanoK.



Momentum distribution of ^{87}Rb atoms at $T > T_c$, $T = T_c$ and $T < T_c$.
(Here peak represents a group of atoms with the same velocity)

M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Science, 269, 198 (1995).

Bose-Einstein Condensation of Fermions

- Fermions can also form BEC by making composite Bosons (pairing of even number of Fermions).
- In an optical lattice one can form composite Bosons from Fermionic atoms through Magnetic Field Feshbach Resonance
- By applying external magnetic field B, one can tune the scattering length of atoms.

Bound and unbound state of Fermions have different total electron spin state.

Experimentally BEC of Fermions has been shown for potassium atoms (^{40}K).

M. Greiner, C. A. Regal, and D. S. Jin, Nature, 426, 537 (2003)

Science Vol 322 10 October 2008

A High Phase-Space-Density Gas of Polar Molecules

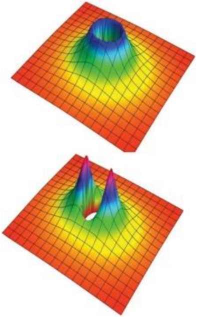
K.-K. Ni, S. Ospelkaus, M. H. G. de Miranda, ..., P. S. Julienne, D. S. Jin, J. Ye

Eenable explorations of a large class of many-body physics phenomena but also could be used for quantum information processing. We report on the creation of an ultracold dense gas of potassium-rubidium ($^{40}K^{87}Rb$) polar molecules.

Nature, Vol 464 | 29 April 2010

Dipolar collisions of polar molecules in the quantum regime

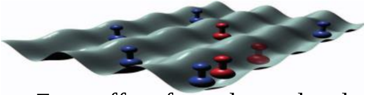
K.-K. Ni, S. Ospelkaus, D. Wang, ..., J. Ye & D. S. Jin



A p-wave centrifugal barrier for dipolar collisions between Fermionic polar molecules (zero & moderate E-field).

Nature VO L 5 0 1 26 September 2013

Observation of dipolar spin-exchange interactions with lattice-confined polar molecules



Quantum Zeno effect for polar molecules in a 3D lattice

The lattice depths along X and Z are kept at 40Er, whereas the lattice depth along Y is reduced to allow tunnelling along the Y direction

Vol 89, Num13 Physical Review Letters 23 2002

Stefano Giovanazzi, Axel Gorlitz, and Tilman Pfau

Tunability of interaction between permanent dipoles in BECs. Even very weak magnetic dipole coupling in alkali gases can be used to excite collective modes.

Physical Review Letters 107, 190401 (2011)

Strongly Dipolar Bose-Einstein Condensate of Dysprosium

Mingwu Lu, Nathaniel Q. Burdick, Seo Ho Youn, and Benjamin L. Lev

We report the Bose-Einstein condensation (BEC) of the most magnetic element, dysprosium.

Nature Physics | VOL 13 January 2017

New frontiers for quantum gases of polar molecules

Steven A. Moses, Jacob P., ..., Deborah S. Jin and Jun Ye

Compared to atoms, molecules possess additional degrees of freedom that can be exploited in fundamental tests, ultracold chemistry, and engineering new quantum phases in many-body systems. Here, we review the recent progress in creating and manipulating ultracold bialkali molecules to study quantum gases of polar molecules.

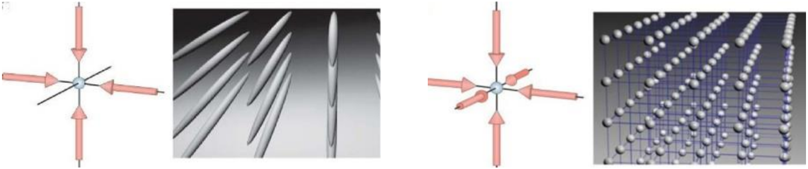
Outline of this Talk

- Introduction to the Optical Lattice Systems
- Quantum Phases of Hardcore Bosons in two Coupled Chains
- Triangular dipolar Fermion Lattice: Triplet SF
- AB field Breaking of electron-pairs in a SC ring

Outline of this Talk

- Introduction to the Optical Lattice Systems
- Quantum Phases of Hardcore Bosons in two Coupled Chains
- Triangular dipolar Fermion Lattice: Triplet SF
- AB field Breaking of electron-pairs in a SC ring

Optical Lattices

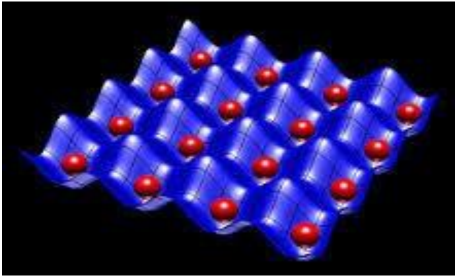


- Optical lattice is an artificial crystal of light created by the interference patterns of counter propagating laser beams.

- Atoms can be trapped by optical dipole traps: $V_{dip}(r) = -d \cdot E$
  d = induced dipole moment by oscillating laser field (E).

I. I. Bloch, Nature Physics, 1, 23 (2005).
2. M. Greiner, O. Mandel, T. Esslinger, T. W. Hansch and I. Bloch, Nature, 415, 39 (2002).

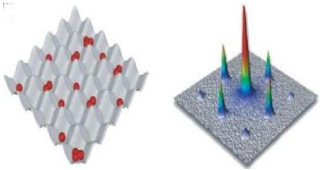
Optical Lattices



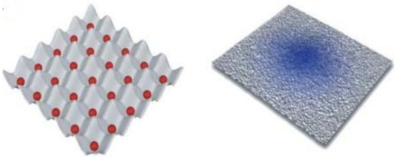
- Optical lattices are an ideal quantum systems to study various quantum phases of mater.

- In an optical lattice, one can easily manipulate and control parameters, which gives rise to novel quantum phases…..quantum magnets, supersolids, superfluids, insulators, topological phases, etc…

Superfluid and Mott Insulator



Superfluid



Mott insulator

In superfluid phase, Bosons form a macroscopic wavefunction with long-range phase coherence; Well defined, but number of atoms fluctuate in each lattice site.

In Mott insulator phase, each lattice site is filled with a fixed number of atoms, but the phase remains uncertain.

Low Dimensional Lattices

- Low-dimensional quantum systems are quite unique, as quantum fluctuations destroy the true long range order (LRO).

- They either show short range order (SRO) or quasi-LRO.

  The Correlation functions decay exponentially -> SRO
  Algebraically (nonzero up to the system size)→ QLRO

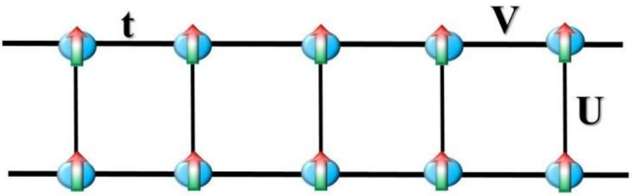
- For low-dimensional systems, mean-field and Fermi liquid theories do not yield accurate results.

We have developed finite size DMRG codes for Bosonic, Fermionic and Spin systems, which give accurate results for Ground state and its properties, Excited states. We also have developed tDMRG, which are used to study various t-dependent phenomena.

Outline of this Talk

- Introduction to the Optical Lattice Systems
- Quantum Phases of Hardcore Bosons in two Coupled Chains
- Triangular dipolar Fermion Lattice: Triplet SF
- AB field Breaking of electron-pairs in a SC ring

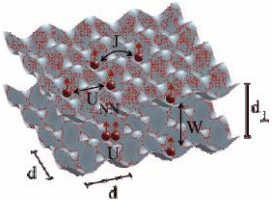
Quantum phases of hardcore Bosons in two coupled chains: A DMRG study



Introduction

- Trefzger et al. have looked at polarized dipolar particles in two decoupled 2D layers, in the presence of repulsive interactions in the planes and attractive interactions between the two layers.

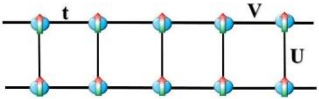
- They have shown the existence of PSS and PSF phases by solving the effective extended Bose-Hubbard Hamiltonian in the low-energy subspace of pairs, by means of a mean-field Gutzwiller approach and exact diagonalization methods.



C. Trefzger, C. Menotti,et al, Phys. Rev. Lett. 103, 035304 (2009)

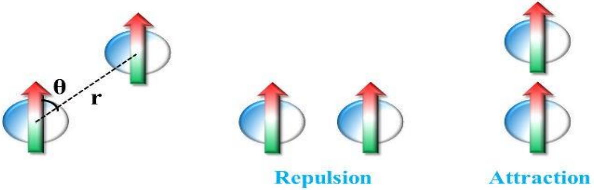
Dipolar Hardcore Bosons

- We have considered hardcore Bosons with dipolar interactions on two coupled one-dimensional chains.



- The dipolar interaction can be defined as

$$V_d(r) = \frac{(1 - 3cos^2(\theta))}{r^3}$$

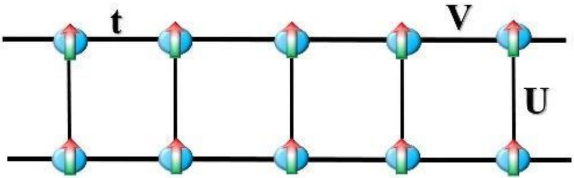


The Model Hamiltonian: Hardcore Boson

- The effective Hamiltonian of the system, without taking into account the interchain hopping, can be written as,

$$H = -t \sum_{\alpha,<i,j>} \left(b^{\dagger}_{\alpha,i} b_{\alpha,j} + h.c \right) + V \sum_{\alpha,<i,j>} \hat{n}_{\alpha,i} \hat{n}_{\alpha,j} - U \sum_{i} \hat{n}_{1,i} \hat{n}_{2,i},$$

where t (hopping) and V (nearest-neighbour repulsion) are intrachain, whereas U is interchain attractive term along the rungs.



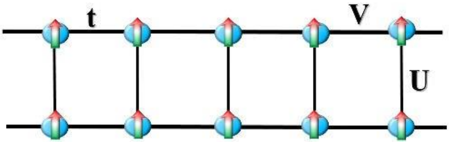
Model Hamiltonian: Spin -1/2

- Hardcore Bosonic operators can be mapped onto Spin-1/2 operators (factors missing; exact only for the GS properties).

$$s_i^+ \to b_i^\dagger, \quad s_i^- \to b_i, \quad s_i^z \to n_i - 1/2$$

- The resulting final spin Hamiltonian is equivalent to coupled chain of spin-1/2 *XXZ* model with ferromagnetic interchain coupling.

$$H = -t \sum_{\alpha,<i,j>} \left(s_{\alpha,i}^+ s_{\alpha,j}^- + h.c \right) + V \sum_{\alpha,<i,j>} s_{\alpha,i}^z s_{\alpha,j}^z - U \sum_i s_{1,i}^z s_{2,i}^z$$



Method

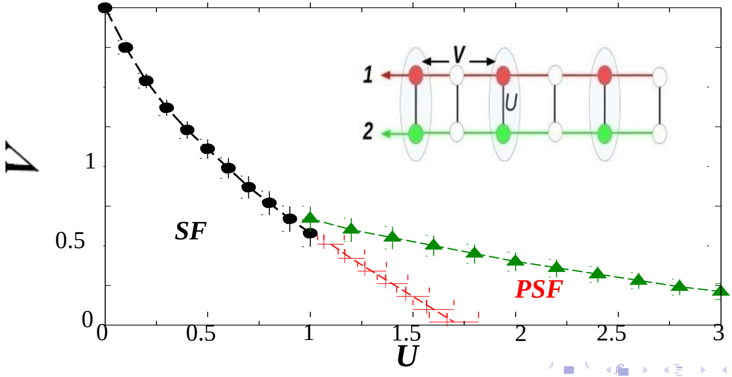
We considered spin-1/2 at every site. Varied DMRG cut-off from 250 to 400.

At times, we have carried out calculations with Bosonic Hamiltonian and have compared the two.

We compare our DMRG results with exact diagonalization up to 28 lattice sites.

Phase Diagram: Hardcore Bosons

- Small values of U and V $\rightarrow$ Superfluid Phase (SF).
- Small V and large U $\rightarrow$ Pair-Superfluid phase (PSF).
- Large values of U and V $\rightarrow$ Density Wave Phase (DW)



Order Parameters

- Superfluid Phase (*SF*): Power law decay of two-point correlation function $C_\alpha(r)$ and divergence of the correlation length, ξ^α.

$$C_\alpha(r) = \langle S^+_{\alpha,0} S^-_{\alpha,r} \rangle, \quad \xi^\alpha_L = \sqrt{\frac{\sum_r r^2 C_\alpha(r)}{\sum_r C_\alpha(r)}}$$

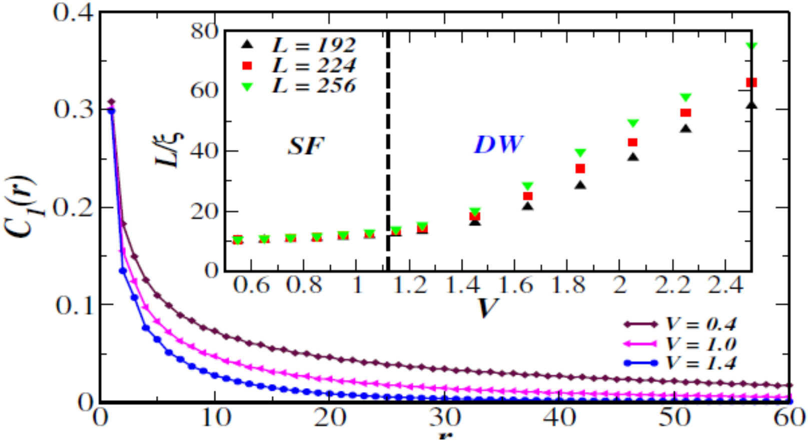
- Density Wave Phase (*DW*): Non-zero values of order parameter $O_G(L)$.

$$G_\alpha(r) = \langle s^z_{\alpha,i} s^z_{\alpha,i+r} \rangle, \quad O_G(L) = \frac{1}{L}(-1)^r G_\alpha(r)$$

- Pair-superfluid phase (*PSF*): Power law decay of four-point correlation function $P(r)$.

$$P(r) = \left\langle S^+_{1,0} S^+_{2,0} S^-_{1,r} S^-_{2,r} \right\rangle - \left\langle S^+_{1,0} S^-_{1,r} \right\rangle \left\langle S^+_{2,0} S^-_{2,r} \right\rangle$$

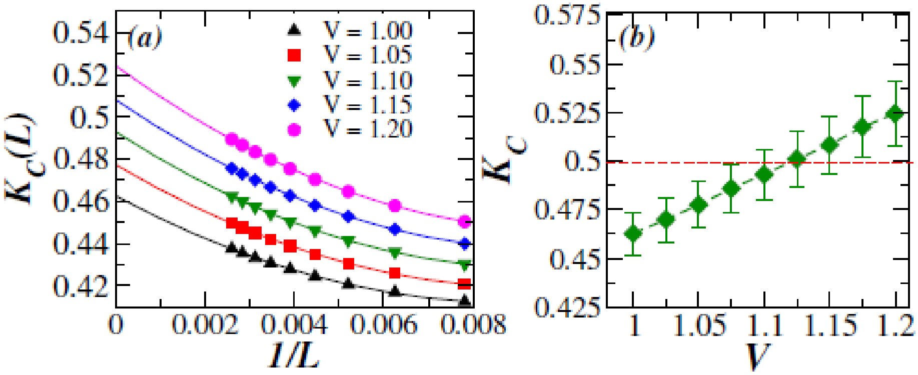
SF to DW Transition



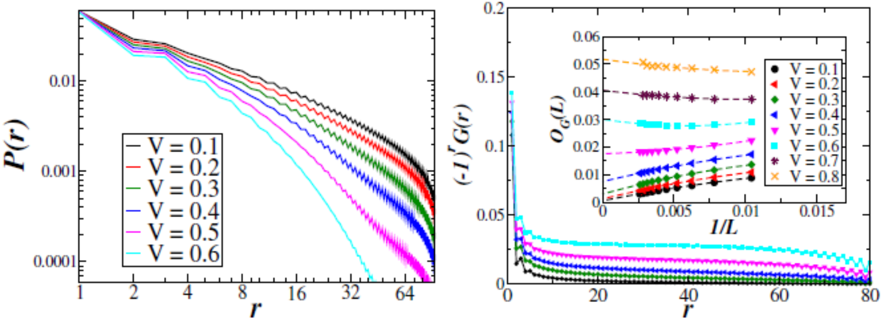
- $C_\alpha(r)$ shows power law behavior for $V < 1.1$.
- In SF phase, L/ξ^α is constant for all lengths.
- *SF to DW* transition at $V = 1.1 \pm 0.05$ for $U = 0.5$.

SF To DW phase transition (Berezinskii-KT type)

- Transition between SF to DW here is BKT type.
- $K_C(L)$ can be obtained by fitting the $C_1(r)$ with A/r^{2K_C}.
  - The transition point is where $K_c = 1/2$.



PSF to DW Transition



- For $U = 2$, $P(r)$ shows power law behavior upto $V = 0.4 \pm 0.08$.
- $O_G(L) = \frac{1}{L}(-1)^r G_1(r)$, has some finite values at $V = 0.4 \pm 0.08$.

Conclusion

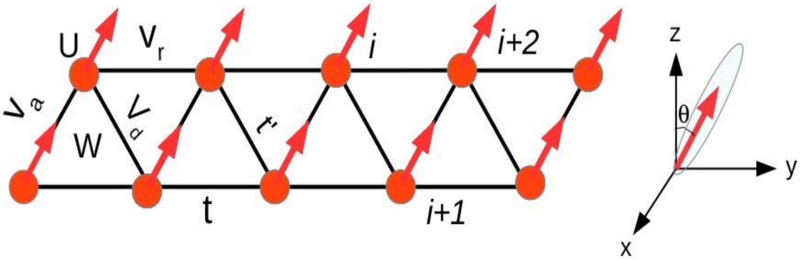
- Found various phases of hardcore Bosons in two coupled chains: interchain attraction and intrachain repulsion.
- Lower values of U and V $\rightarrow$ Superfluid Phase (SF).
- Lower values of V and large values of U $\rightarrow$ Pair-superfluid phase (PSF).
- Larger values of U and V $\rightarrow$ DW Phase of Dimers.

B. Pandey, S. Sinha, and SKP, Phys. Rev. B **91**, 214432 (2015).

Outline of this Talk

- Introduction to the Optical Lattice Systems
- Quantum Phases of Hardcore Bosons in two Coupled Chains
- Triangular dipolar Fermion Lattice: Triplet SF
- AB field Breaking of electron-pairs in a SC ring

Triplet Superfluidity on a Triangular Lattice with Dipolar Fermions



Introduction

- At low-T, liquid ^3He forms p-wave SFs, where ^3He atoms (or quasi particles) form pairs in spin triplet state.
- Chromium based quasi-1D superconductors and strontium based oxide, Sr_2RuO_4, are considered to be good candidates for triplet pairing.
- Unconventional SF, particularly with triplet pairing is of huge interest, due to its intrinsic connection to topological phases.

Introduction

- Recent progress in dipolar Fermi gas gives opportunity to realize Fermionic superfluids with triplet pairing.

- In presence of attractive head to tail arrangement of dipolar interactions, the one and two dimensional dipolar fermions become unstable and they undergo either collapse or phase separation.

- To overcome these difficulties, bilayer system has been proposed, where dipoles are aligned perpendicular to the layers, giving more stable paired phases.

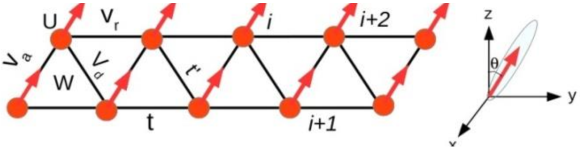
A. C. Potter, et al., Phys. Rev. Lett. 105, 220406 (2010)

Model Hamiltonian

- We consider dipolar Fermions on a two-leg triangular ladder.
- The effective Hamiltonian of the system can be written as:

$$H = \sum_{\sigma,i} \left(t c^{\dagger}_{\sigma,i} c_{\sigma,i+2} + t_{\perp} c^{\dagger}_{\sigma,i} c_{\sigma,i+1} + H.C. \right)$$

$$+ U \sum_{i} \hat{n}_{i,\uparrow} \hat{n}_{i,\downarrow} + \sum_{<ij>} V(i,j) \tilde{n}_i \tilde{n}_j$$



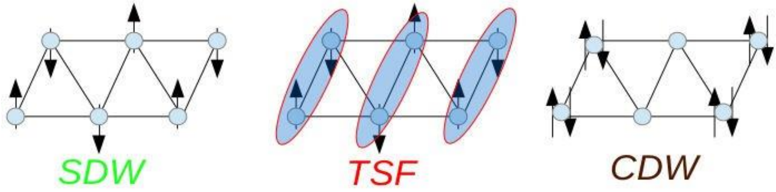
$$V(i,j) = \begin{cases} V_r & \text{repulsive term on each chain.} \\ V_d & \text{repulsive term for even rungs.} \\ -V_a & \text{attractive term for odd rungs.} \end{cases}$$

Fermionic DMRG with max=300 to 600.

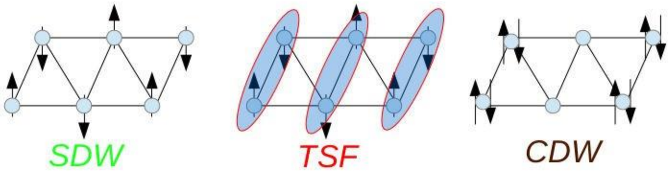
SDW → TSF → CDW Transition

We consider a fixed value of $U = 2$ and vary V_a.

- For lower values of $V_a \rightarrow$ SDW phase.
- For moderate values of $V_a \rightarrow$ TSF-phase ($S = 1$ and $s_z = 0$).
- At large values of $V_a \rightarrow$ CDW-phase.

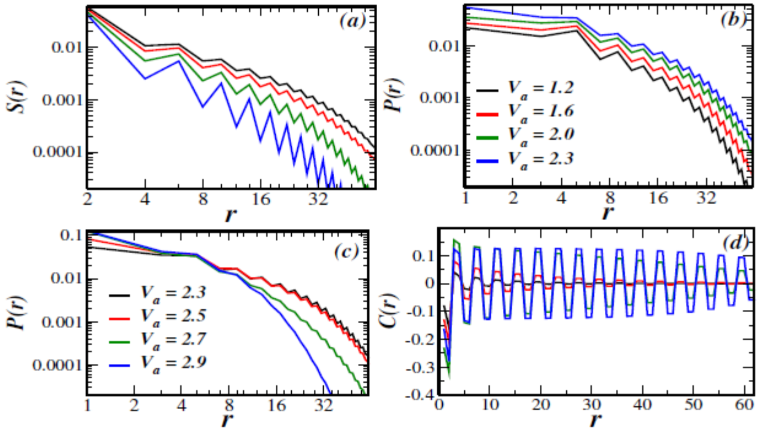


Correlation Functions



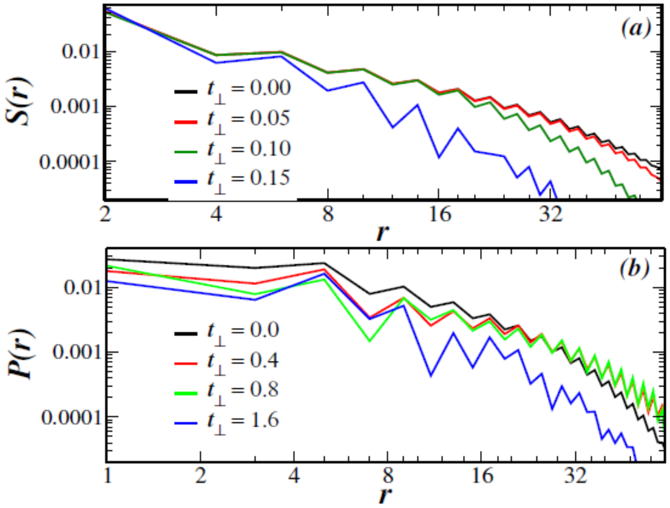
- Spin-spin correlation function :
  $S(r) = 1/N(r)\sum_r \left(\langle s_i^z s_{i+r}^z\rangle\right)$
- Triplet pair-correlation function: $P(r) = 1/N(r)\sum_r \langle \Delta_i^\dagger \Delta_{i+r}\rangle$,
  where $\Delta^\dagger(i) = \left(c_{i,\uparrow}^\dagger c_{i+1,\downarrow}^\dagger + c_{i,\downarrow}^\dagger c_{i+1,\uparrow}^\dagger\right)$.
- Density-density correlation function:
  $C(r) = \langle (n(i) - \bar{n})(n(j) - \bar{n})\rangle$

Effect of attractive interaction: V_a



- For $V_a < 1.7 \rightarrow$ S(r) dominates.
- For $1.7 < V_a < 2.5 \rightarrow$ P(r) dominates.
- For $V_a > 2.5 \rightarrow$ C(r) dominates.

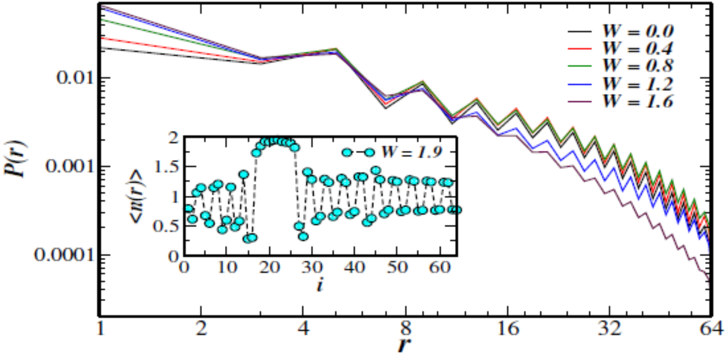
Effect of interchain hopping $t_\perp$: On SDW Phase



- The SDW-phase is unstable in presence of $t_\perp$.
- Transition to TSF phase at $t_\perp \sim 0.1$ ($U = 2; V_a = 1.6$).

Effect of Three-body Interaction: *W*

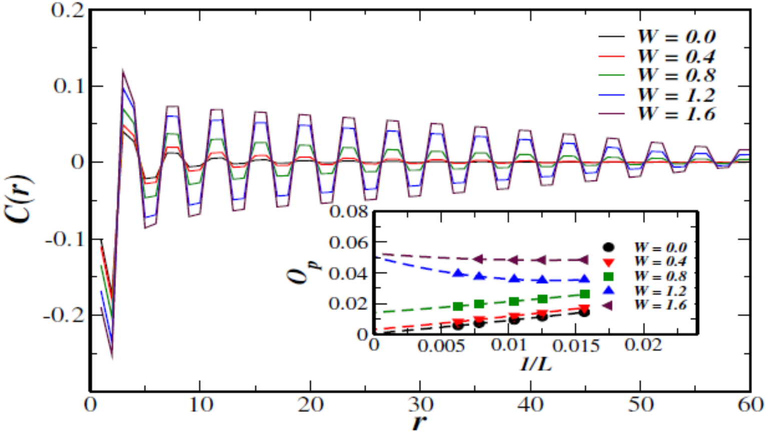
- Due to triangular geometry and dipolar interactions, an additional three-body interaction term appears in each of the triangular plaquette. $H = H_{old} + W_i n_i n_{i+1} n_{i+2}$



$$U = 2, \; V_a = 1.8, \; V_r = 0.3, \; V_d = 0.1$$

With increase in W, correlation function, P(r) increases
Further increase leads to a phase-separated state

Effect of Three-body Interaction: *W*



- With increase in W (>0.6), periodic modulation in $C(r)$ appears
- Coexistence of periodic modulation in $C(r)$ and algebraic decay of $P(r)$ shows Fermionic supersolid (TSF+DW) phase for $0.6 < W < 1.6 \pm 0.1$.

Conclusion

- Quantum phases of dipolar Fermions, at half filling, on a triangular lattice has been studied in details.

- In presence of moderate values of repulsive onsite interaction and attractive intersite interactions, the Fermions form spin triplet superfluid phase.

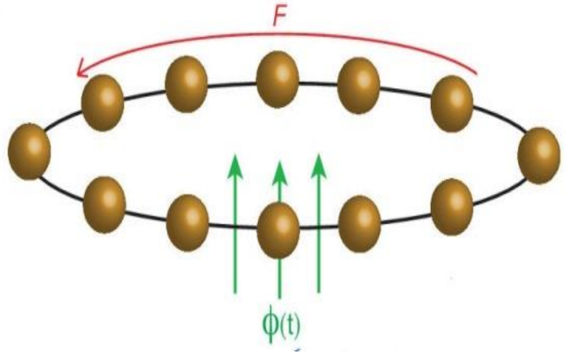
- In presence of three body interaction, a Fermionic supersolid phase appears, where both TSF and CDW coexist.

B. Pandey and SKP: Phys. Rev. B **95**, 085105 (2017)

Outline of this Talk

- Introduction to the Optical Lattice Systems
- Quantum Phases of Hardcore Bosons in two Coupled Chains
- Triangular dipolar Fermion Lattice: Triplet SF
- AB field Breaking of electron-pairs in a SC ring

Breakdown of Electron-pairs in a Superconducting ring: Effect of Electric Field



Introduction

- In a recent experiment, on exposure to photon flux, breaking of electron-pairs (Cooper-pairs) of a superconducting Aluminium thin film is shown [1].
- In a superconductor, electron can form pairs by coupling with lattice vibrations (electron-phonon interaction) [BCS theory] .
- Electron can also form local pairs (real space pairing) via short-range attractive potential. (Polaronic or coupling between electrons and excitons) [2].
- The electron pairing can be recognized by flux quantization[3] and by off-diagonal long-range order [4].
- At low enough temperature, superconductors are condensate of electron pairs which are sensitive towards the external perturbations.

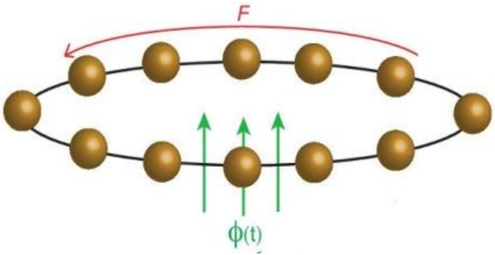
1. P. J. de Visser *et al.*, Nature Communications, 5, 3130 (2014).
2. R. Micnas, *et al*, Rev. Mod. Phys., 62, 113 (1990).
3. L. Onsagar, Phys. Rev. Lett. 7, 50 (1961).
4. C. N. Yang, Rev. Mod. Phys. 34, 694 (1962).

Model Hamiltonian

- We consider a finite ring in presence of time dependent Aharonov Bohm flux, $\varphi(t) = eFLt$ (AB-flux).
- We model the system within attractive Hubbard model U.

$$H(t) = -\gamma \sum_{i,\sigma} \left(\exp^{2\pi i\phi(t)/N} \, c^{\dagger}_{i+1,\sigma} c_{i,\sigma} + h.c. \right) - U \sum_{i} n_{i,\uparrow} n_{i,\downarrow}$$

- The electric field, F is produced by the flux, $\phi(t)$: $F = \frac{1}{L}\frac{d\phi(t)}{dt}$.



Method

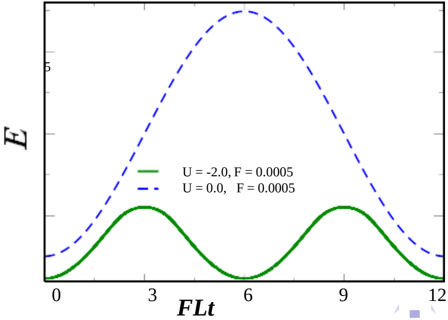
- Exact diagonalization method to get ground state wavefunction, $|\psi(0)>$, at $t = 0$.

- Crank-Nicolson's algorithm to obtain $|\psi(t)>$.

- Using $|\psi(t)>$, we calculate E(t), J(t) and other time dependent quantities.

Flux Quantization

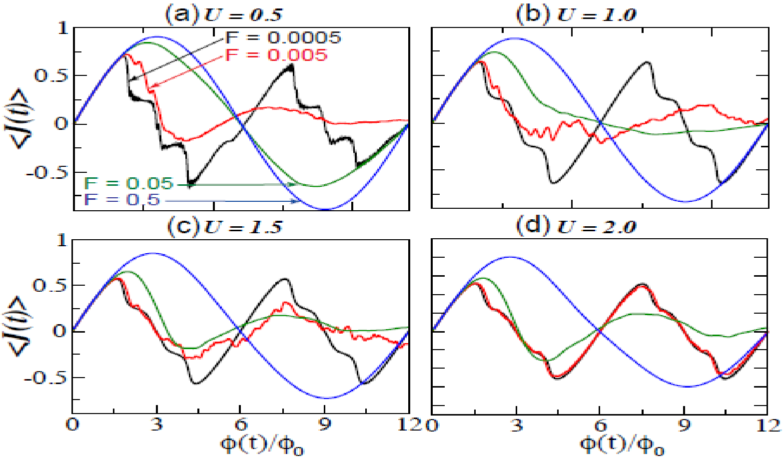
- Depending on the state of the ring, the presence of *AB*-flux gives rise to different periodicity of energy (E).
- For non-interacting case, the periodicity is equal to the extended AB-period, $\phi(t) = L\phi_0$.
- For finite values of $|U|$, due to pairing of electrons, the periodicity becomes halved, $\phi(t) = L\phi_0/2$.



Effect of Electric Field for Different U

- The current density $J(t)$ as a function of the applied AB-flux.

$$J(t) = -\gamma \sum_{i,\sigma} i \left(\exp^{2\pi i \phi(t)/N} c^{\dagger}_{i+1,\sigma} c_{i,\sigma} - h.c. \right)$$

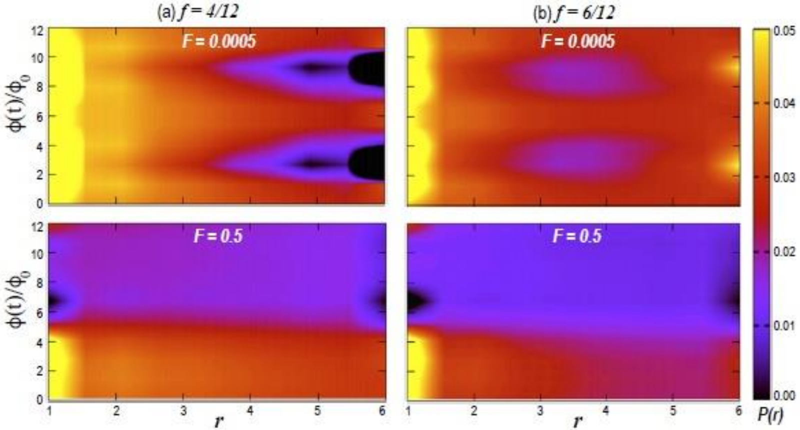


Effect of F on Pair-Correlation Function

- Time evolution of the pair-correlation function is given as:

$$P(r,t) = \langle c^{\dagger}_{1,\uparrow} c^{\dagger}_{1,\downarrow} c_{r,\downarrow} c_{r,\uparrow} \rangle - \langle c^{\dagger}_{1,\uparrow} c_{r,\uparrow} \rangle \langle c^{\dagger}_{1,\downarrow} c_{r,\downarrow} \rangle$$

- For large values of F , $P(r\ ,\ t)$ shows short range behavior.

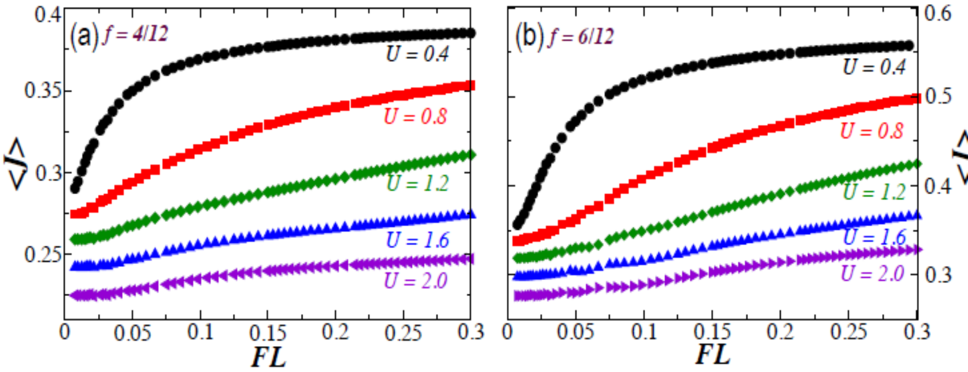


Effect of F on Average Current J

- The time-averaged current

$$\langle J\rangle = \frac{1}{T}\int_0^T \langle J(t)\rangle dt$$

With integration over quarter of the extended AB period ($\phi(t) = L/4$)



Conclusion

- The time-dependent non-equilibrium properties of superconducting ring has been studied.
- The applied field breaks the electron-pairs driving the system from superconducting to metallic phase.
- The required strength of the applied field depends on the strength of the attractive interaction potential.

B. Pandey, S. Dutta and SKP, J. Phys.: Condens. Matter **28**, 195601 (2016).

Acknowledgements:

Thank you for your attention

Geetanjali, Exciton, Onsagar (Ours),
Sampige, Booruga, Taavare, Mallige (TUE-CMS Clusters)

SERB and J C Bose National Fellowship; KIST and EU for Funding

Postdoc@Paris-Sud

Collaborators (in this talk)

Prof. Subhasis Sinha (IISER/Kolkata)

Dr. Sudipta Dutta (IISER/Tirupati)