Jamming and Gardner, a granular media experiment

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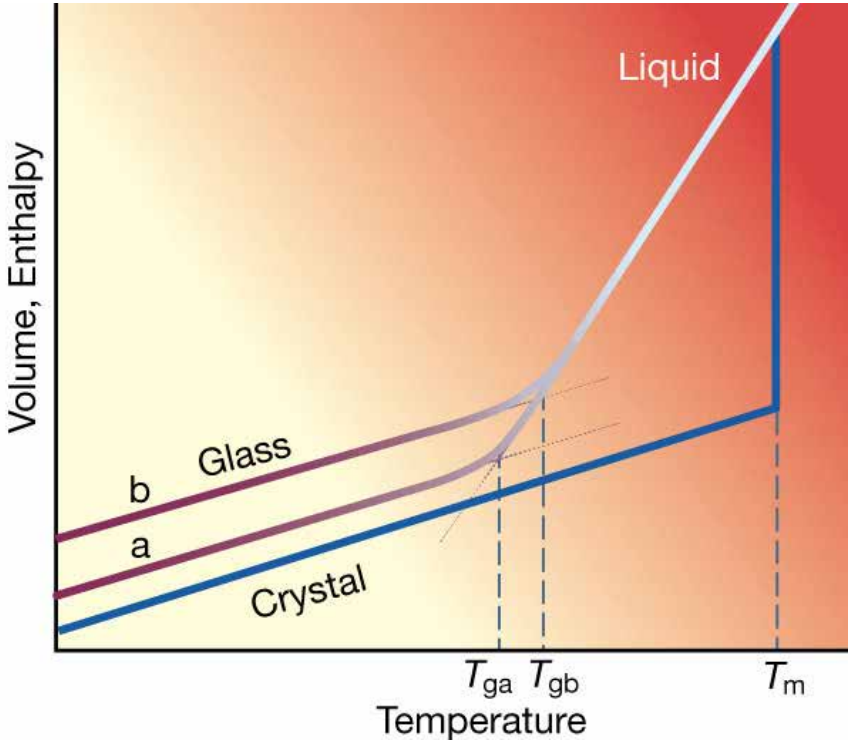
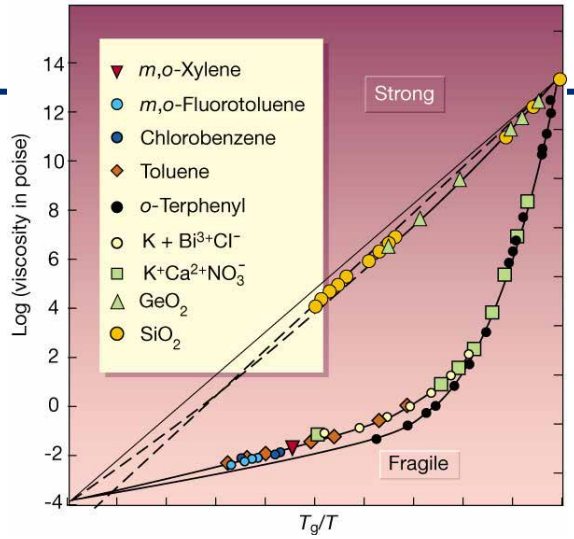
Overview

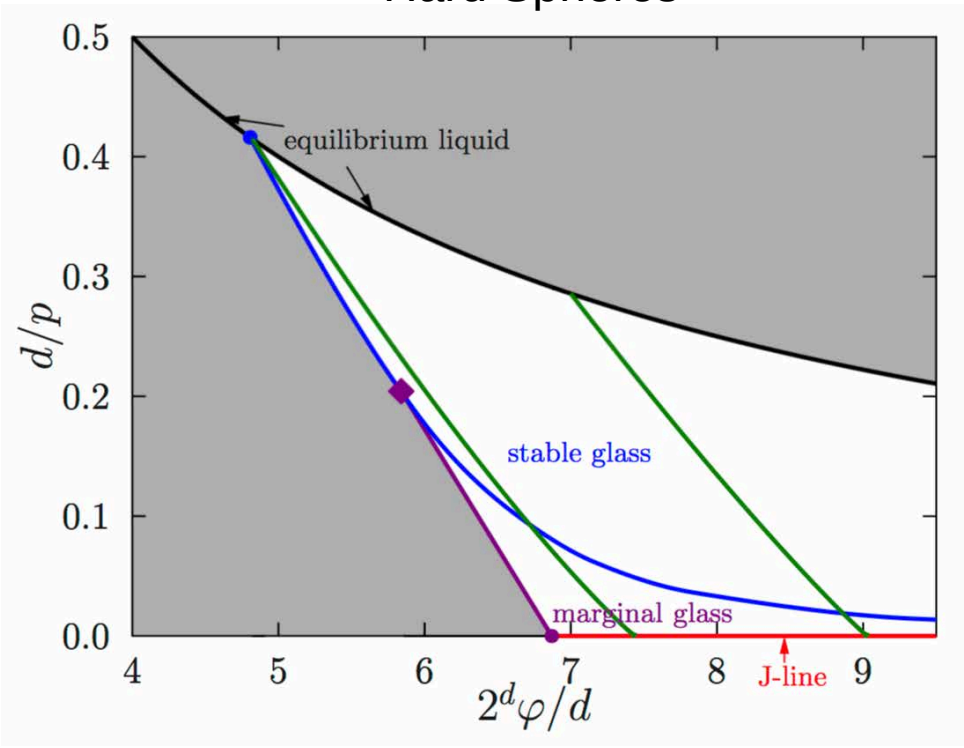
- Introduction
  - What is the Gardner transition ?
- The Gardner transition in a granular system
  - Protocol
  - Results
- Dynamical heterogeneities at minute scales
  - Role of the contact dynamics
- Non linear Mechanical response
  - Shear and Dilatancy weakening
  - Avalanches

Glasses and Gardner



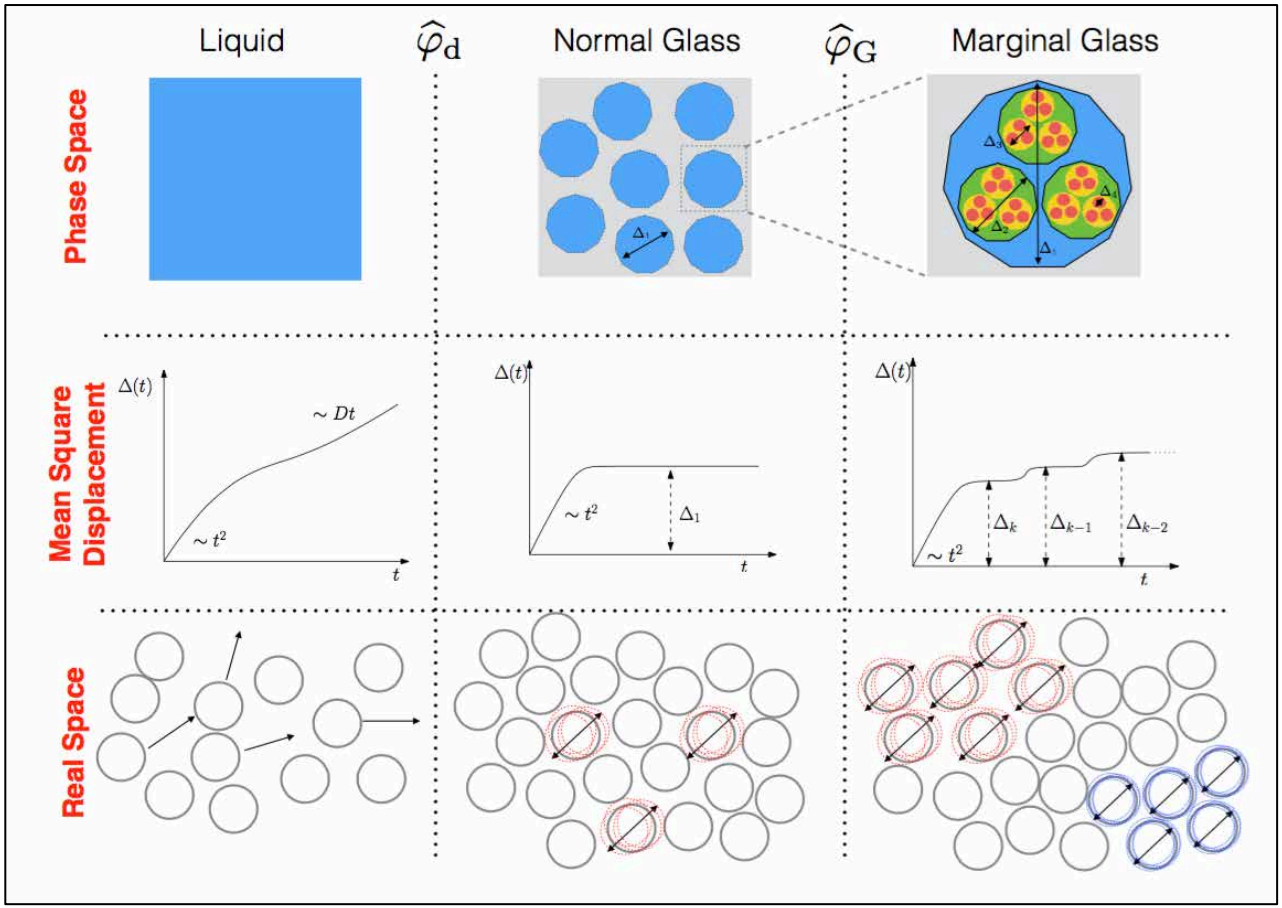


Hard Spheres



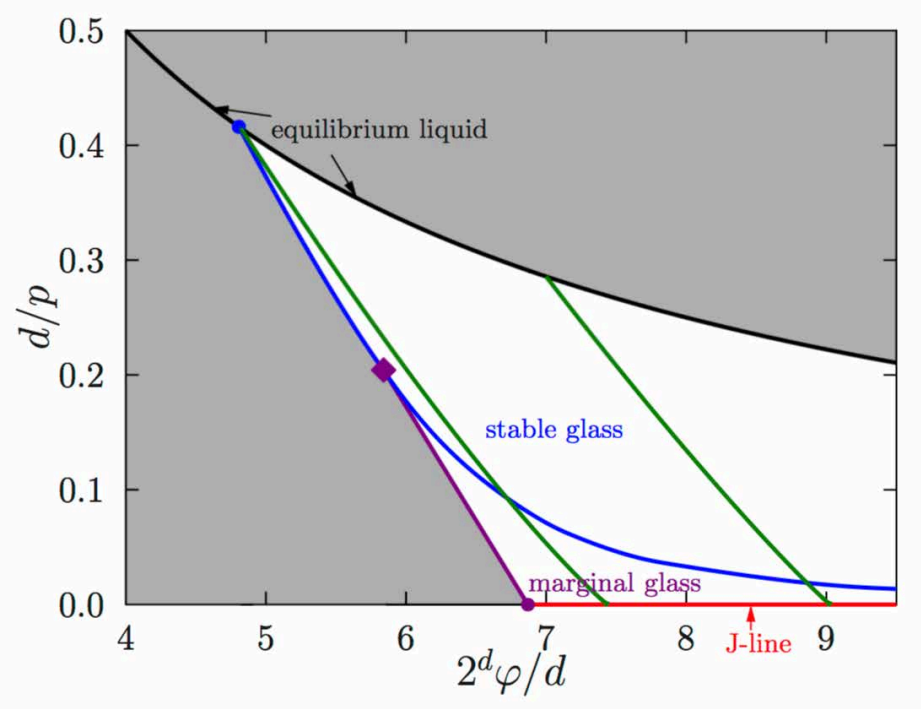
What we are after : the Gardner transition

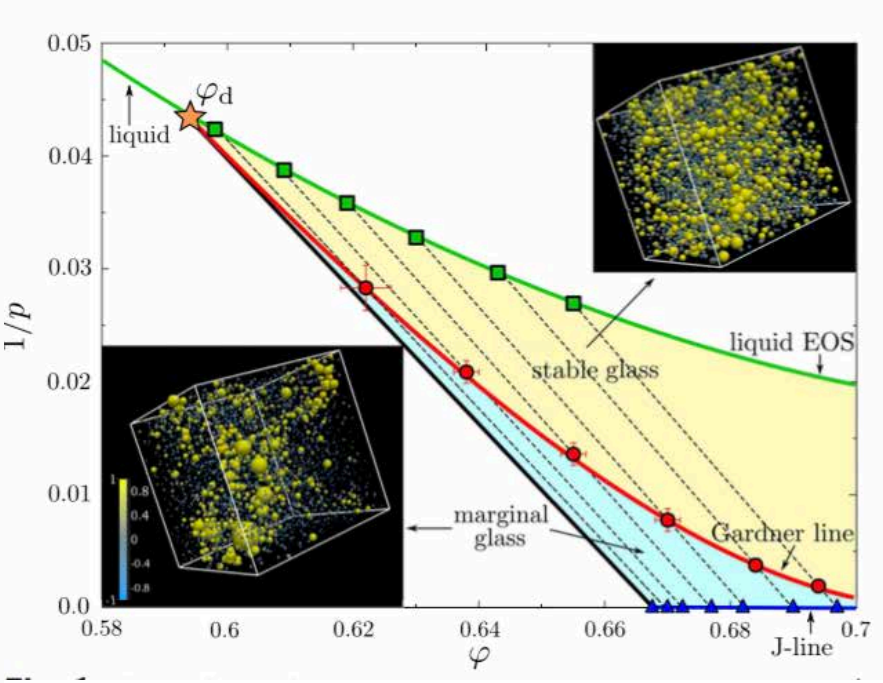
A transition in the vibrational properties of a glass



« Glass and Jamming Transitions: From Exact Results to Finite-Dimensional Descriptions »
Patrick Charbonneau, Jorge Kurchan, Giorgio Parisi, Pierfrancesco Urbani, and Francesco Zamponi.
Annu. Rev. Cond. Matt. Phys. 2016. 8:1–29

Theoretical and numerical results

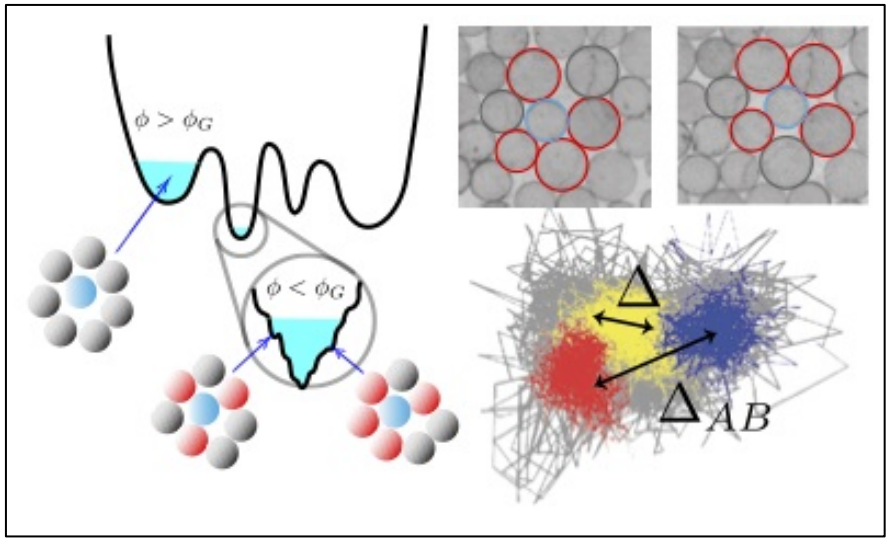


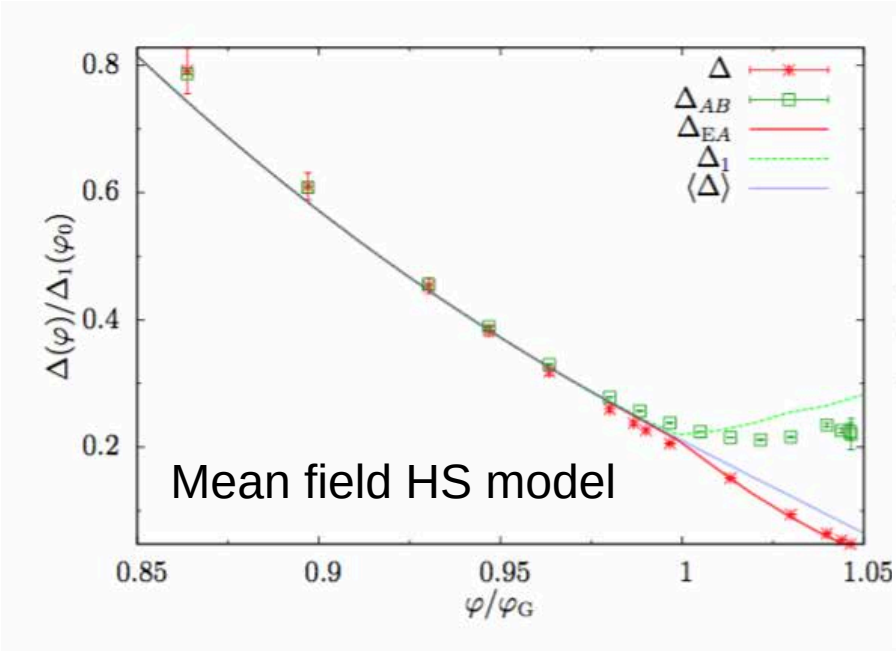


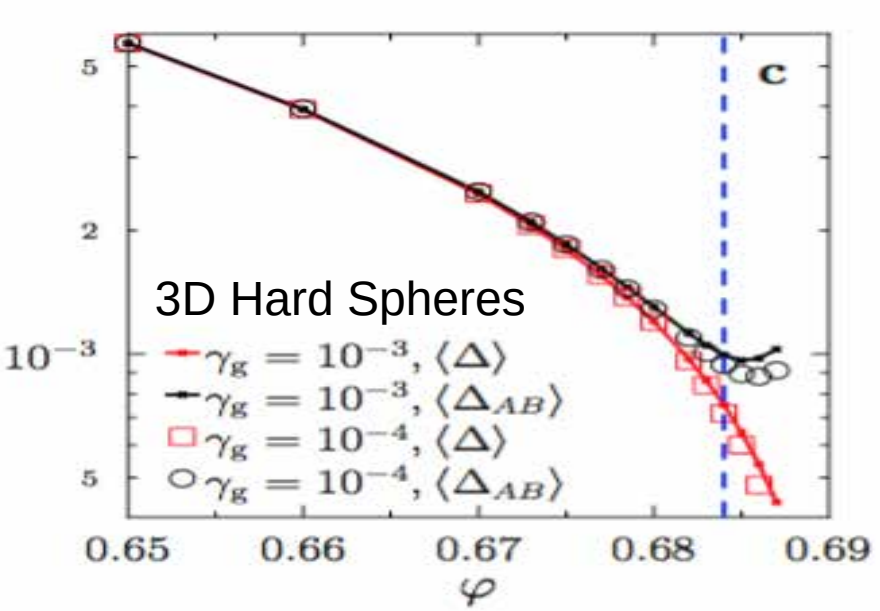
- Theoretically predicted in infinite dimension — Nature Comm 5, 3725 (2014)
- Numerically observed in a mean field glass former — PRE 92, 012316 (2015)
- Numerically observed in 3d and 2d hard spheres — PNAS 113, 8397-8401 (2016)

Here : Evidence of the Gardner transition in a granular glass experiment

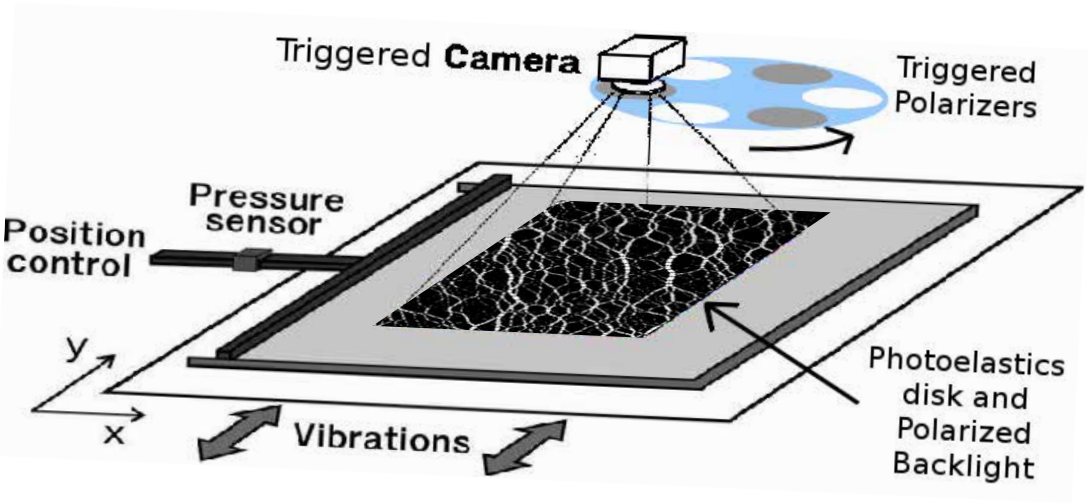
What shall we observe?

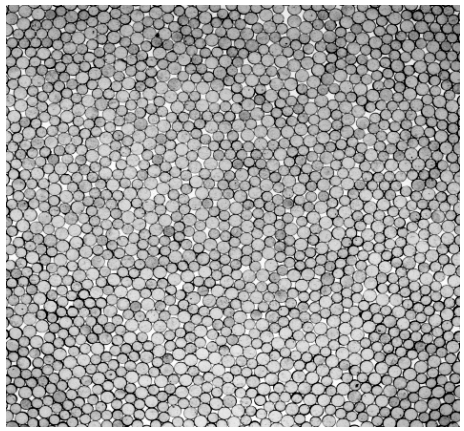


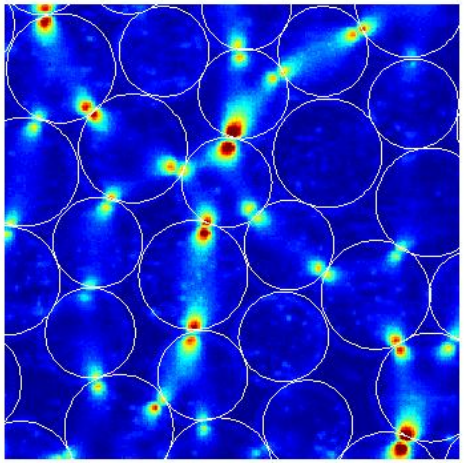




Horizontally shaken photo-elastic discs



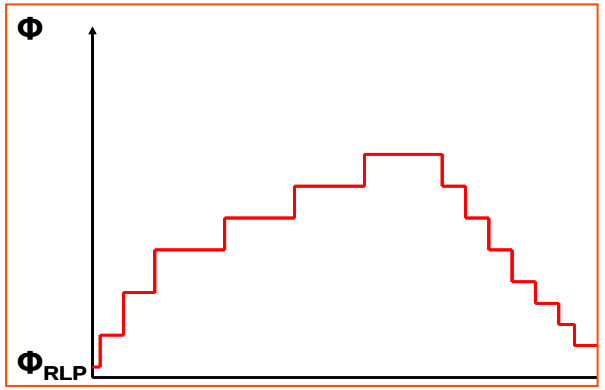


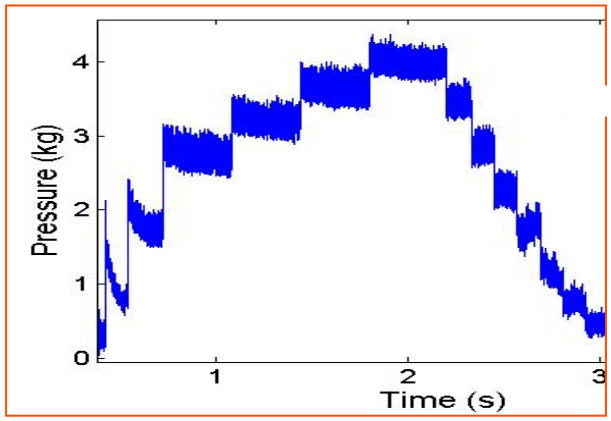


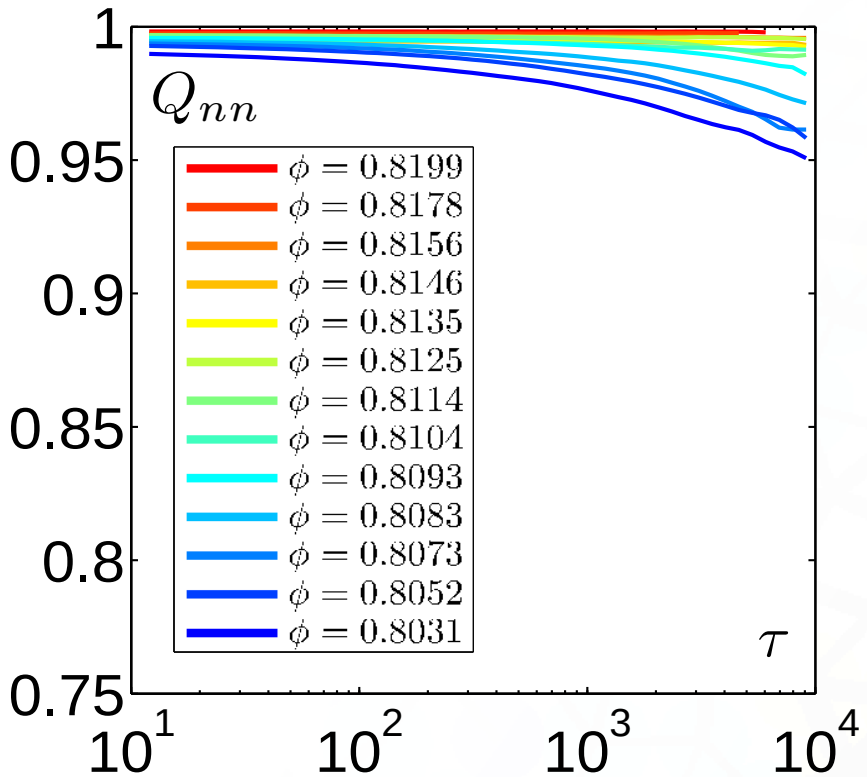
- Horizontal vibration (ω=10 Hz, a=1cm)
- Bi-disperse : d_s = 4mm d_l = 5mm
- 8000 discs in the system (1500 tracked)
- Snapshots in phase with the vibration

Experimental protocol => A granular glass

- Increase packing fraction stepwise:
  - Allow for the slow relaxation of pressure
- Then decrease packing fraction and record dynamics





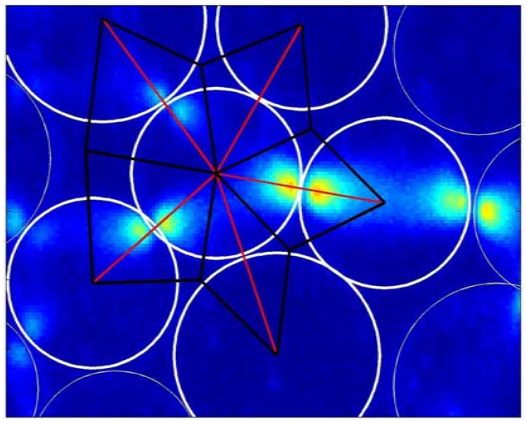


A completely frozen structure

=> A granular glass

Signature of jamming within contacts

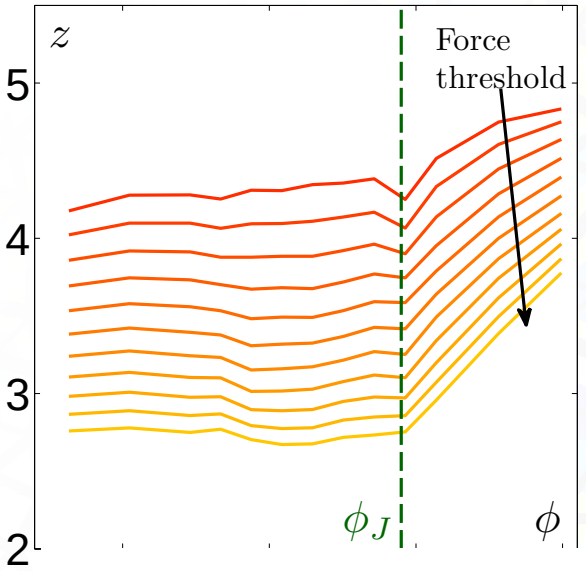
Interparticle force measurement

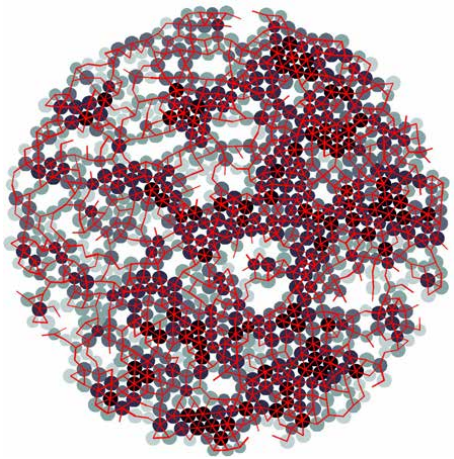


thresholding
gap $< \epsilon$

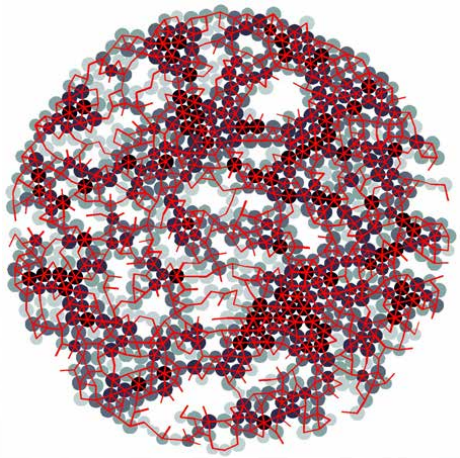
$\rightarrow$

thresholding
force $> f_0$

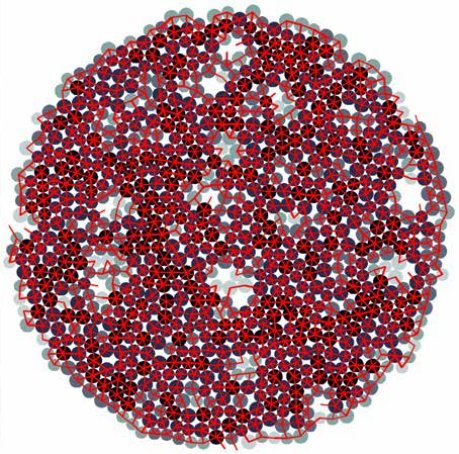




$\phi < \phi_J$



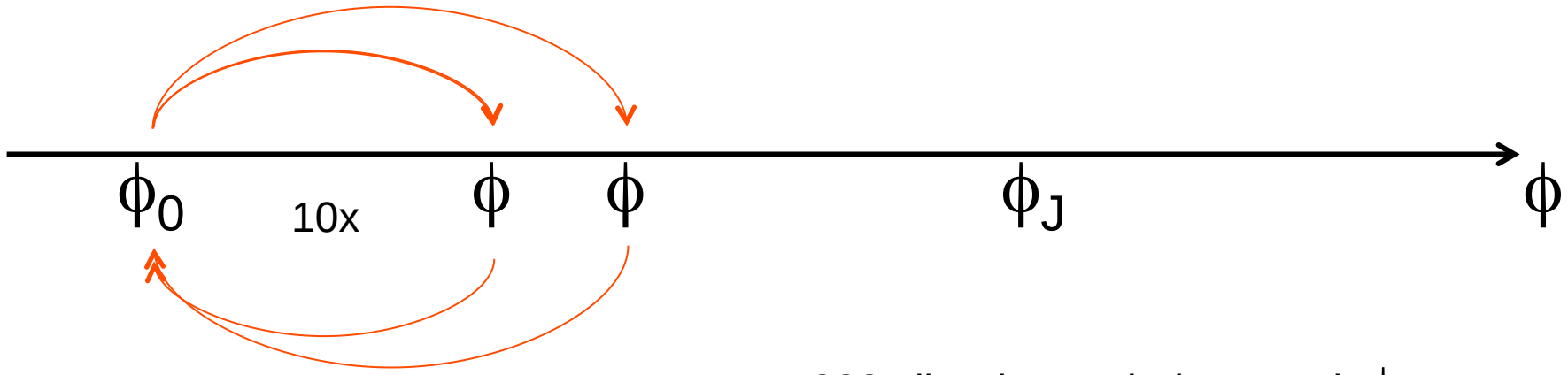
$\phi \sim \phi_J$



$\phi > \phi_J$

Exploring the Gardner transition : Protocol

- Compress up to $\phi > \phi_J$ (i.e. equilibrating a.m.a.p)
- Decompress down to ϕ_J, (lost of contact and P->0)
- Decompress further => ϕ_0 (remaining in the same glass)
- Cycles of compressions towards ϕ (10 times each)

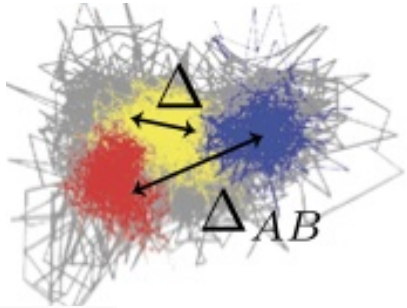


1000 vibration periods at each ϕ

Exploring the Gardner transition : Observables

- Direct access to $r_i(t)$
- However below ϕ_J , convection easily sets in

=> Compute in the centers of mass of the neighbors



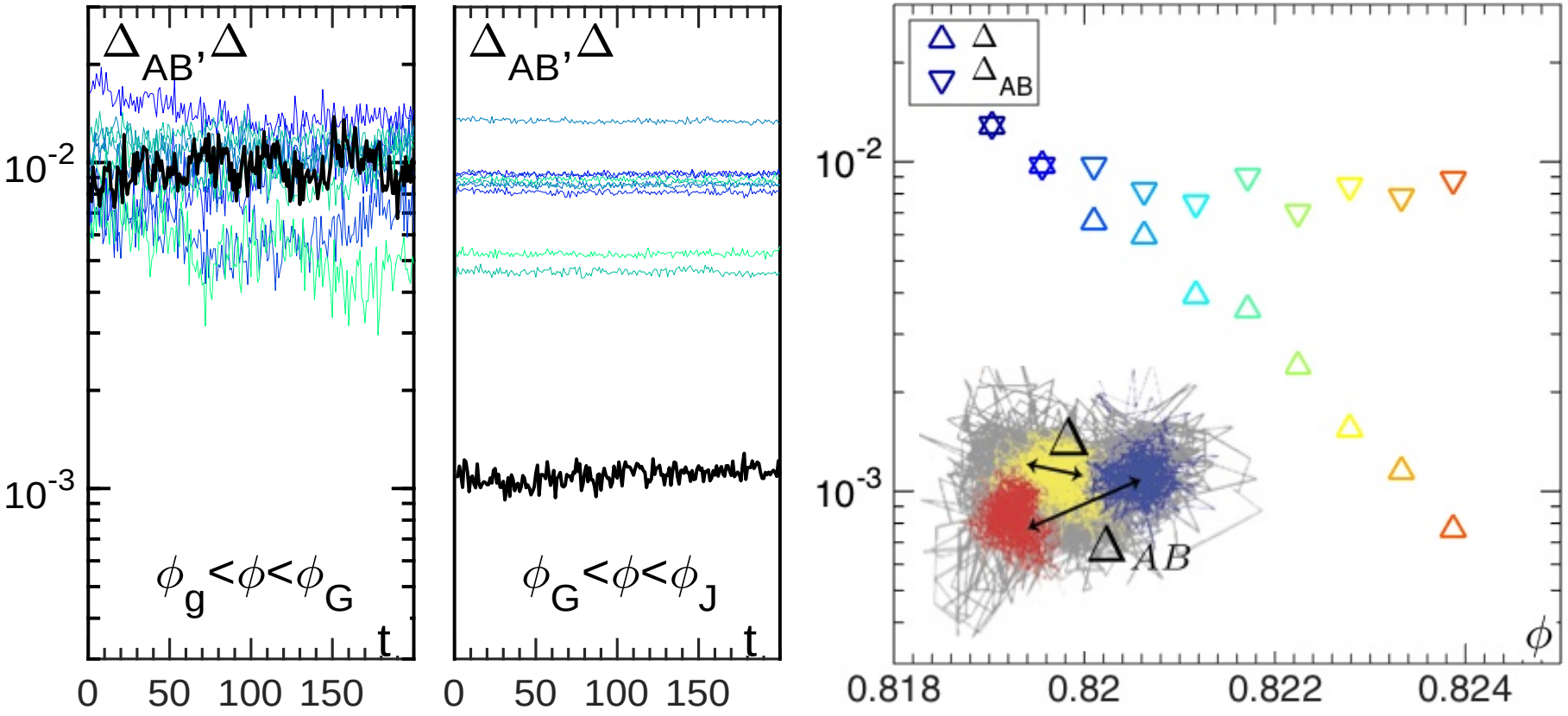
$$\Delta^k(t,t_w) = \frac{1}{N}\sum_{i=1}^{N}\left|\mathbf{r}_i^k(t_w+t)-\mathbf{r}_i^k(t_w)\right|^2$$

$$\Delta^{k,k'}(t) = \frac{1}{N}\sum_{i=1}^{N}\left|\mathbf{r}_i^k(t)-\mathbf{r}_i^{k'}(t)\right|^2 .$$

- Timescales
  - Finite t_w = > ageing is expected
  - => t = 500 < t_w
  - Average over a small time-window Δt = 200 vibration periods
  - Restrict the analysis to k' = k+1

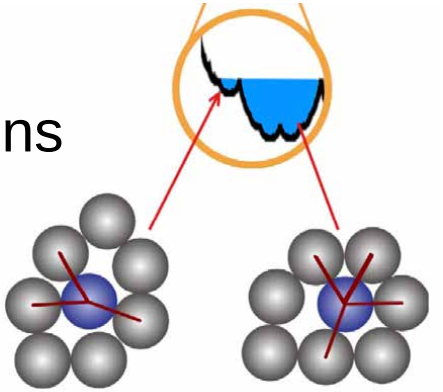
Direct evidence of Gardner transition

Averaging over time and cycles

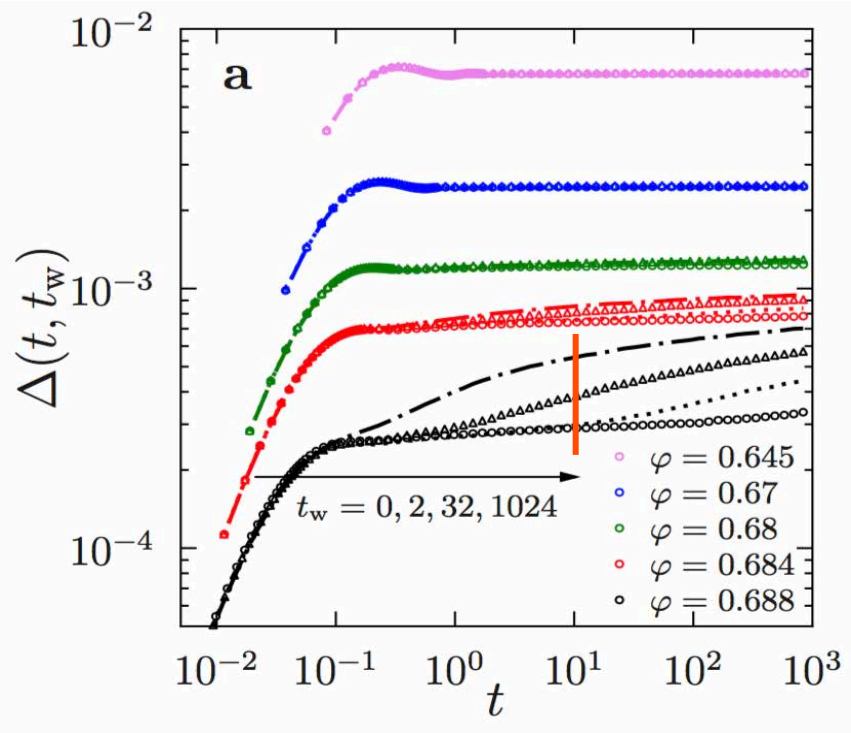


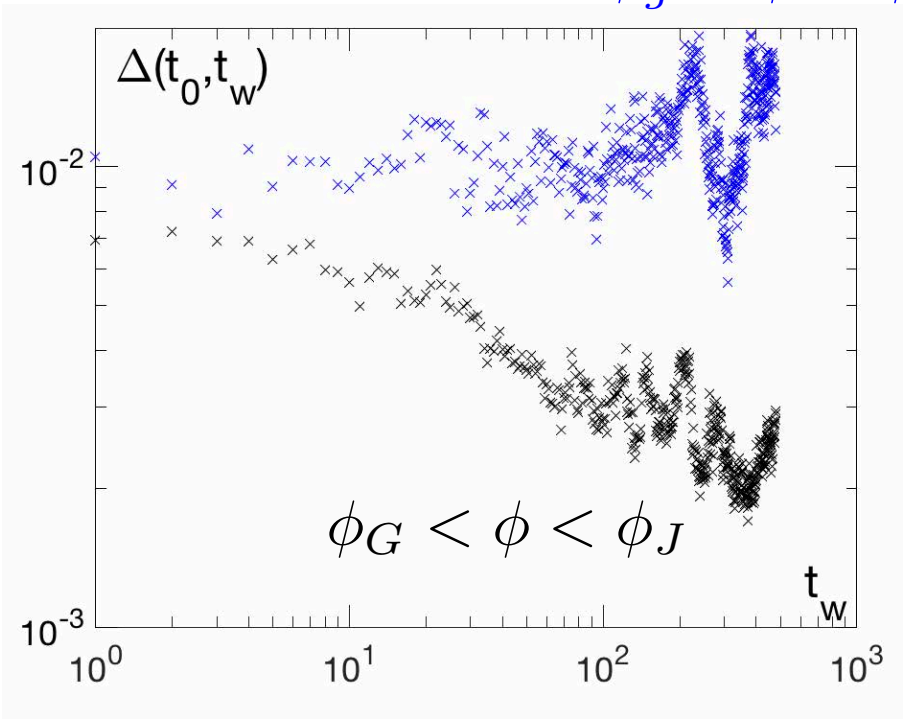
Aging

- **Aging in the Gardner phase**, hierarchical structure of the bassins => system always ages



$\phi_g < \phi < \phi_G$

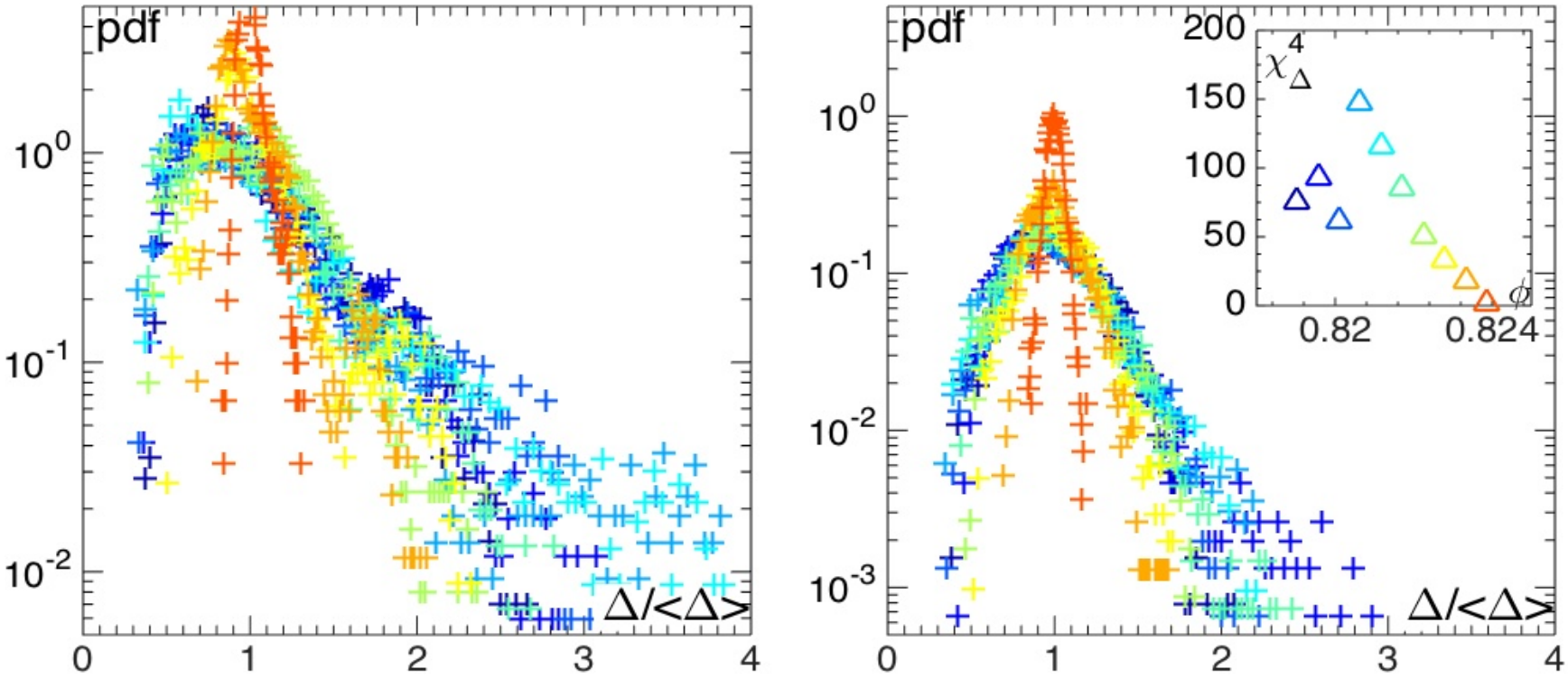




Discussion I : Vibrational heterogeneities

- **Spatial heterogeneities of the vibrations**

Non Gaussian cage size (also intra-state)

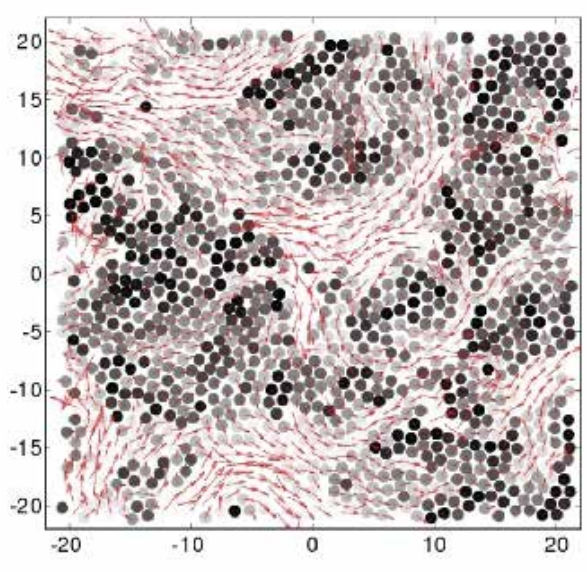


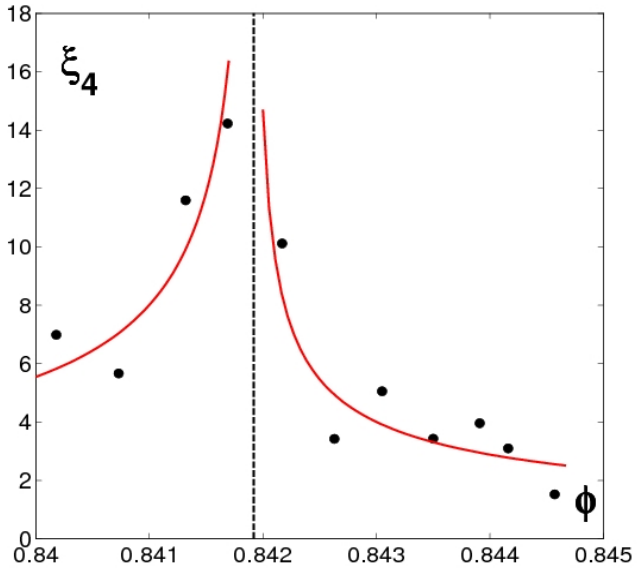
=> Vibrational heterogeneities

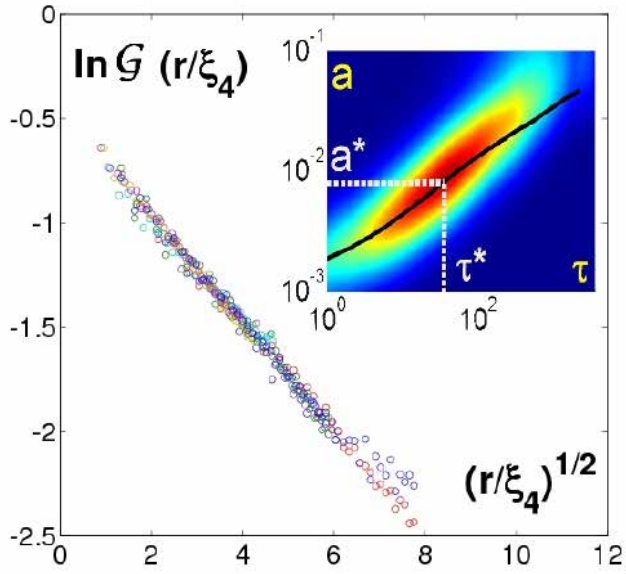
Discussion I : Vibrational heterogeneities

$$Q_a(t,t_w) = \frac{1}{N}\sum_i Qi = \frac{1}{N}\sum_i e^{-\frac{\Delta_i(t,t_w)^2}{2a^2}}$$

$$\chi_4(a,t) = NVar(Q_a(t)) \qquad G_4^*(r,\phi) = \langle QiQj\rangle_{d_{ij}=r} = \frac{\lambda_\phi}{r^\alpha} G\left(\frac{r}{\xi_4}\right)$$





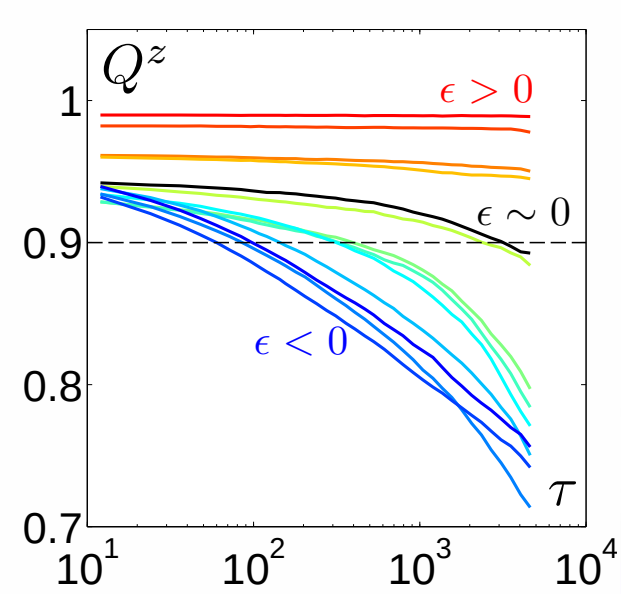


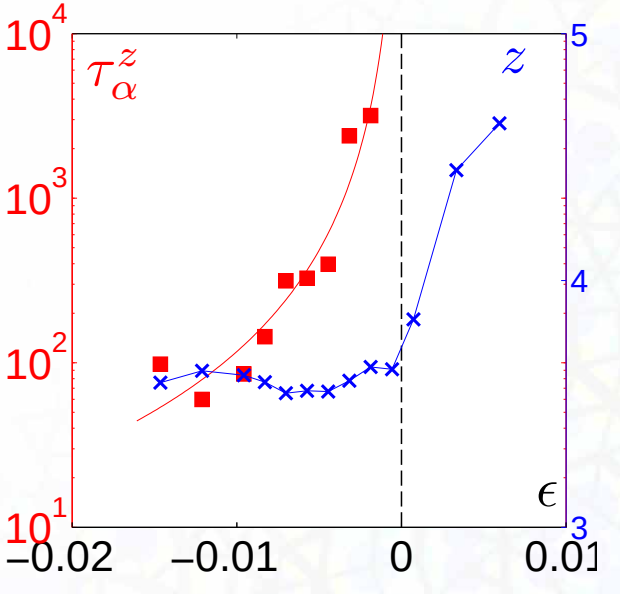
a* = 5.10^{-3} d
no cage jumps
no change of neighbours

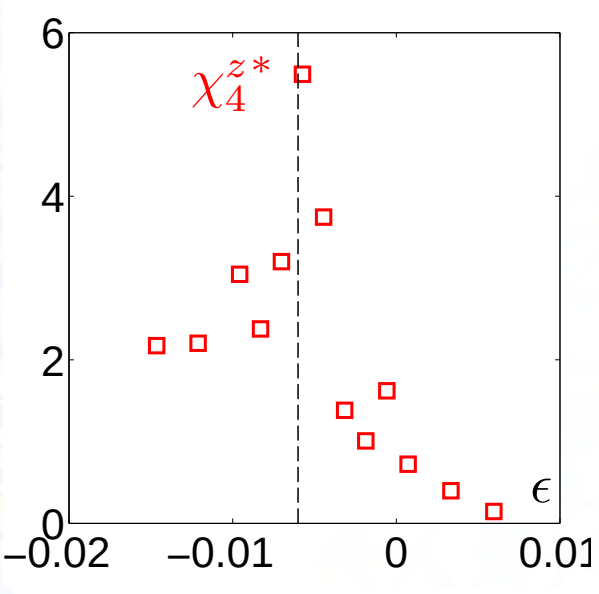
Also in the contact dynamics

$$Q^z(t,\tau) = \frac{1}{N}\sum_i Q_i^z(t,\tau) \quad \text{where } Q_i^z(t,\tau) = \begin{cases} 1 \text{ if } |z_i(t+\tau) - z_i(t)| \leq 1 \\ 0 \text{ if } |z_i(t+\tau) - z_i(t)| > 1 \end{cases}$$

$$Q_z(\tau) = \langle Q^z(t,\tau)\rangle_t \qquad \chi_4^{z,r}(\tau) \equiv N\mathrm{Var}\left(\left\langle Q_i^{z,r}\right\rangle_i\right)_t$$





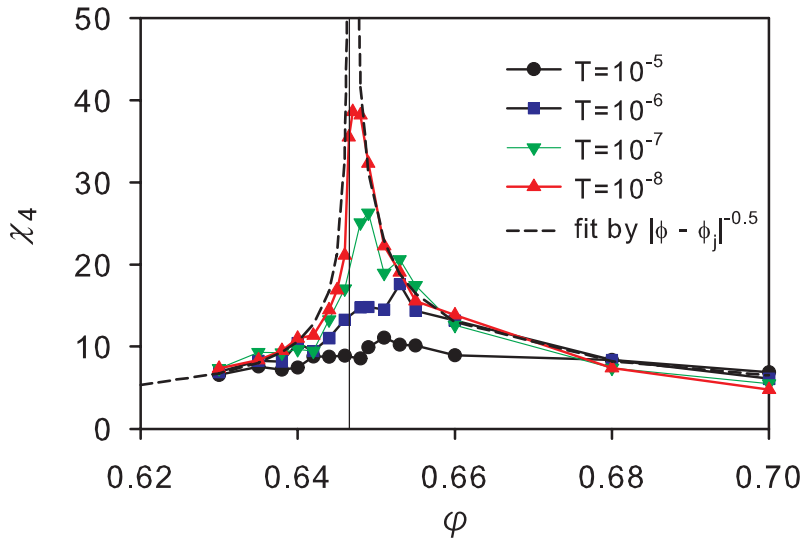


Maximal vibrational heterogeneities at a packing fraction

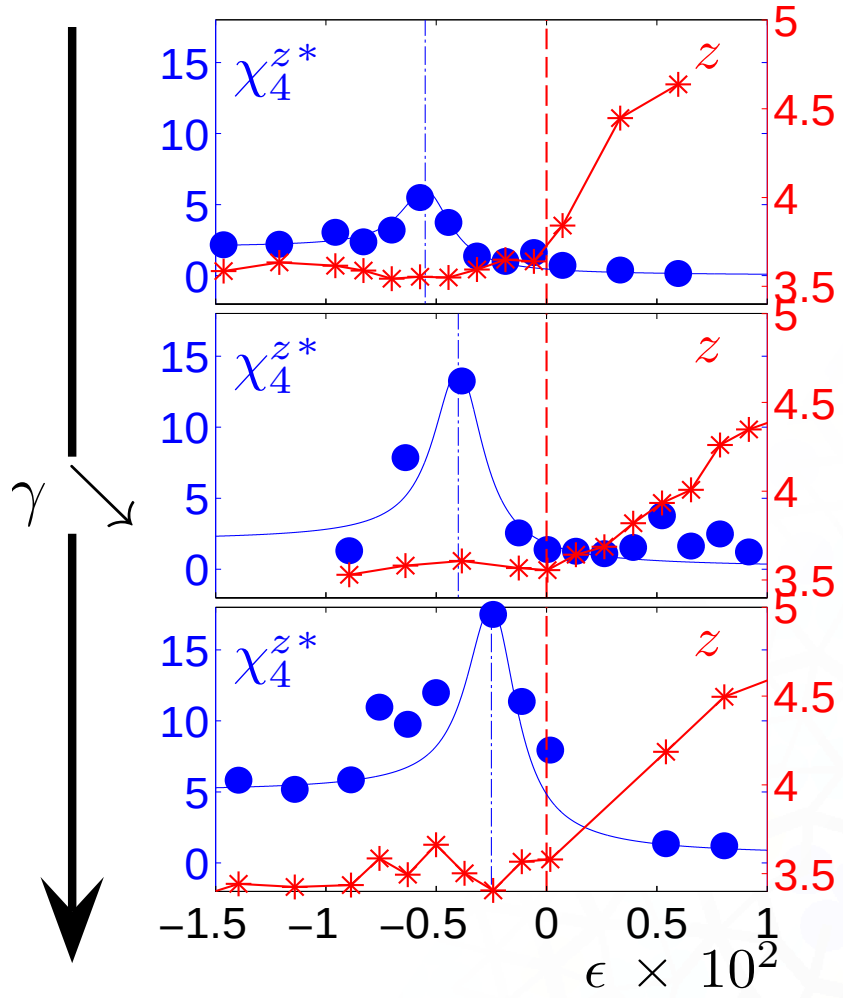
$\neq$ onset of permanent contacts

Discussion I : Vibrational heterogeneities

- For soft spheres, as well as in the present experiment,



How are these maxima related to the Gardner transition ?

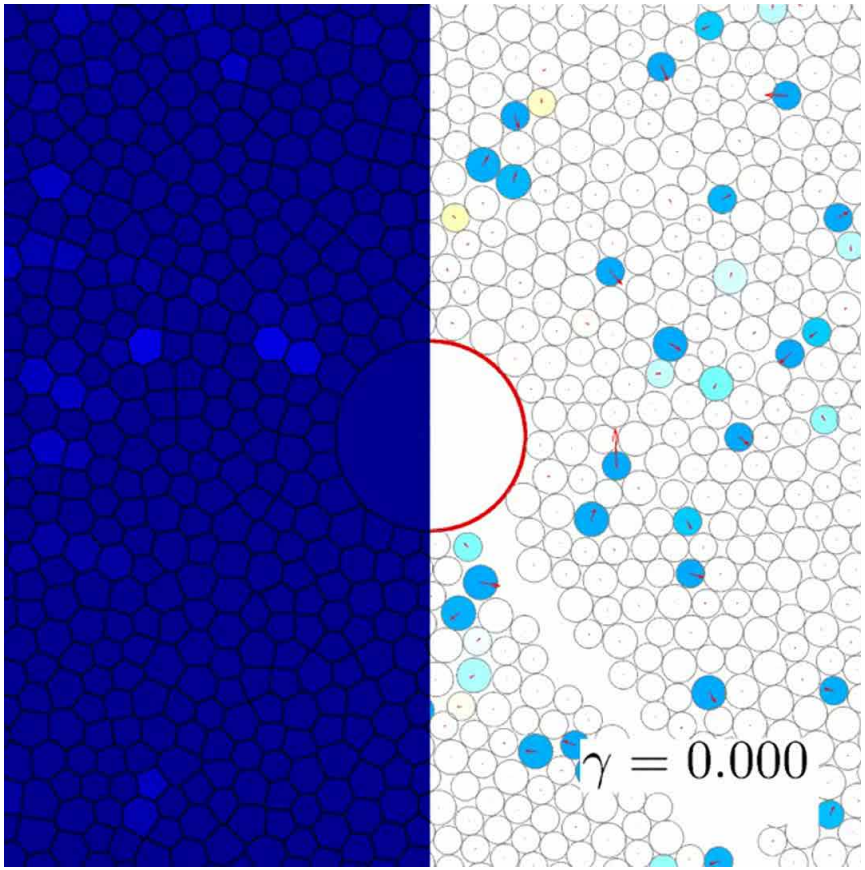


Discussion II : Probing elasticity

- Prepare the system at large packing fraction under vibration
- Stop vibration
- Inflate an intruder in the center
- Measure stress and strain
- Decrease the packing fraction while vibrating
- iterate

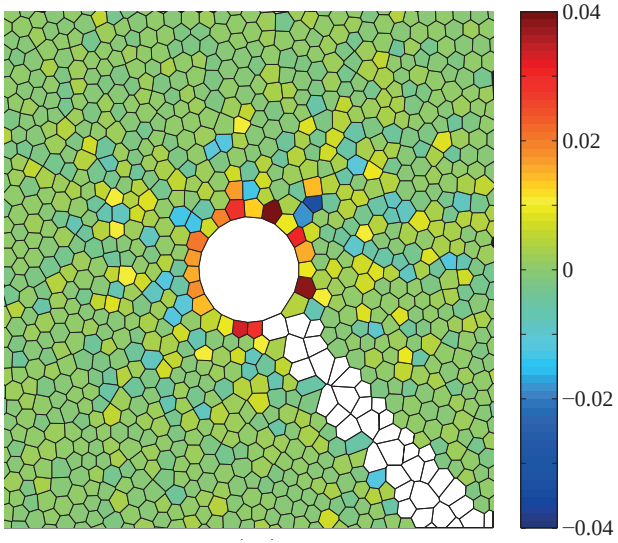
R0 - > R0 + a

γ = a/R0

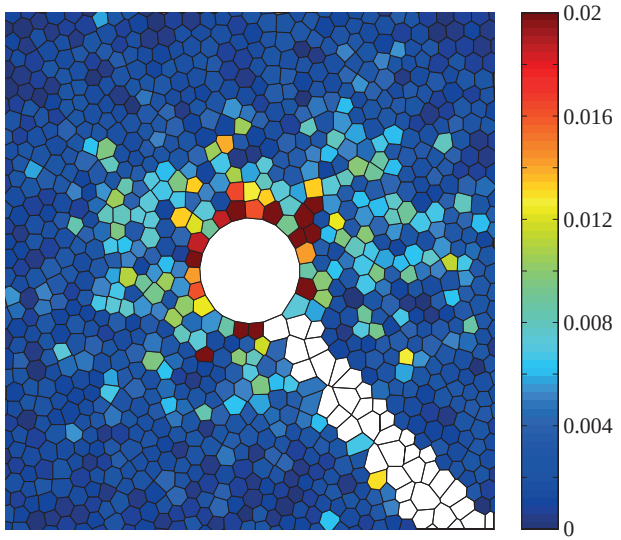


For each packing fraction and each a/R_0

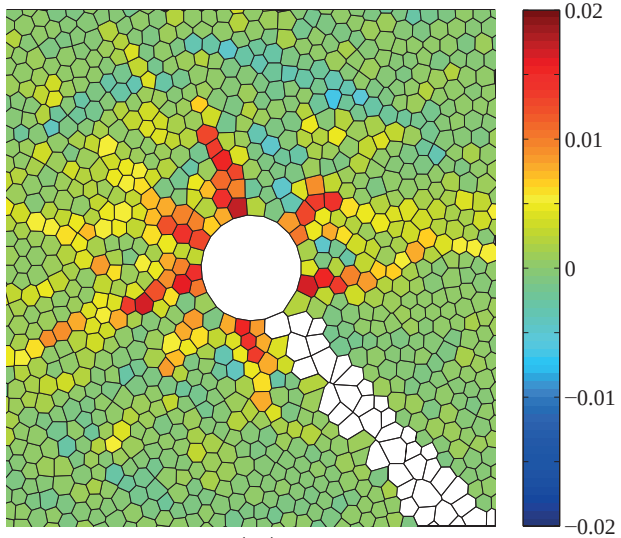
$\delta=Tr(\varepsilon)$



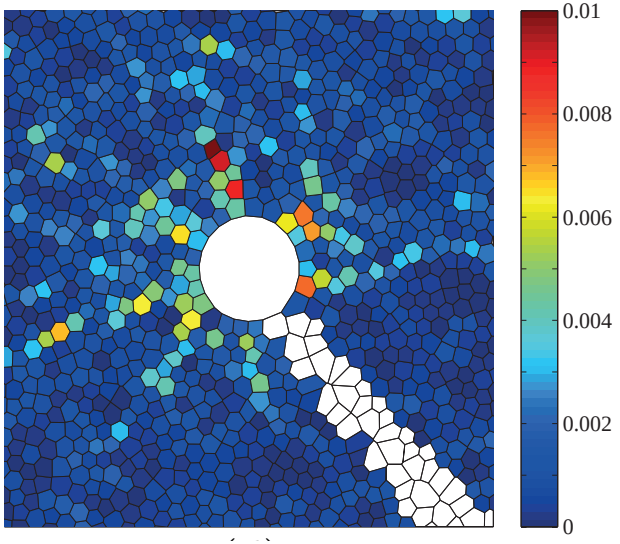
$\gamma=J_2(\varepsilon)$



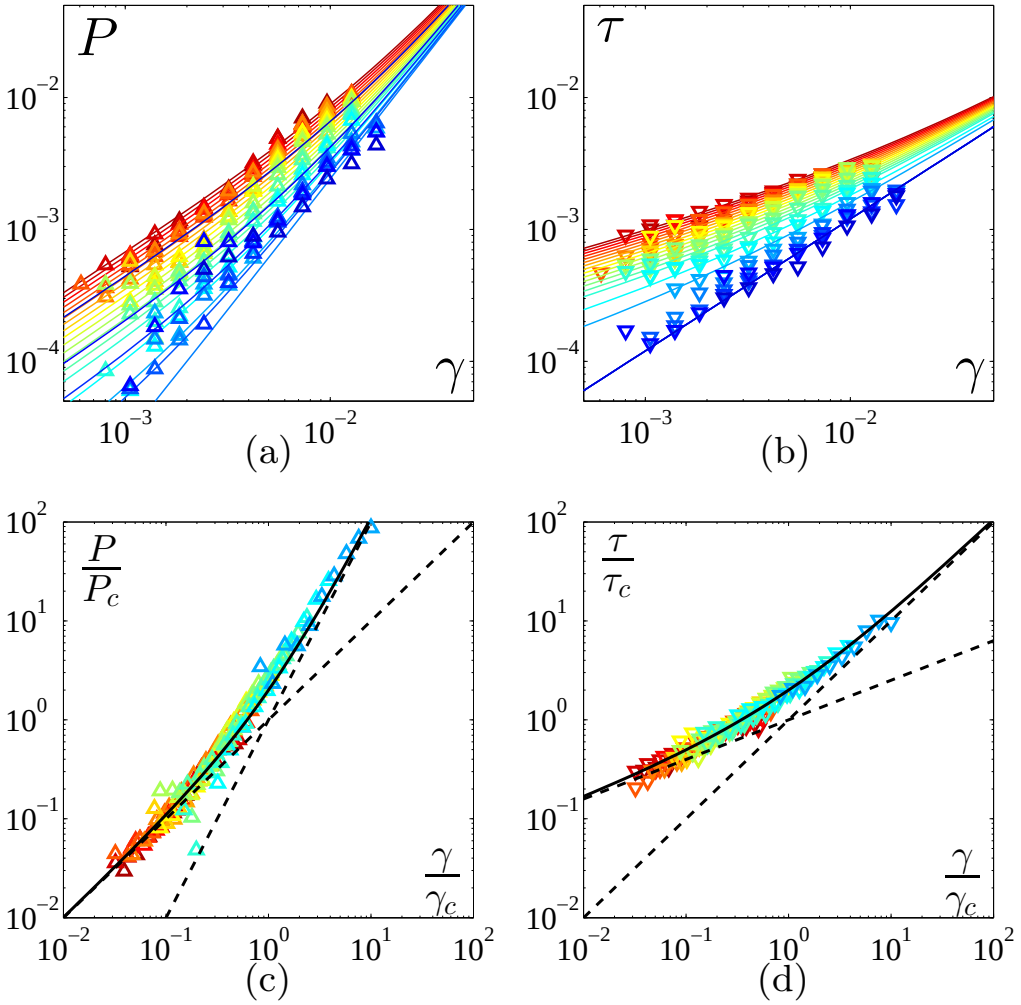
$P=Tr(\sigma)$



$\tau=J_2(\sigma)$



Parametric plot of stress vs. strain



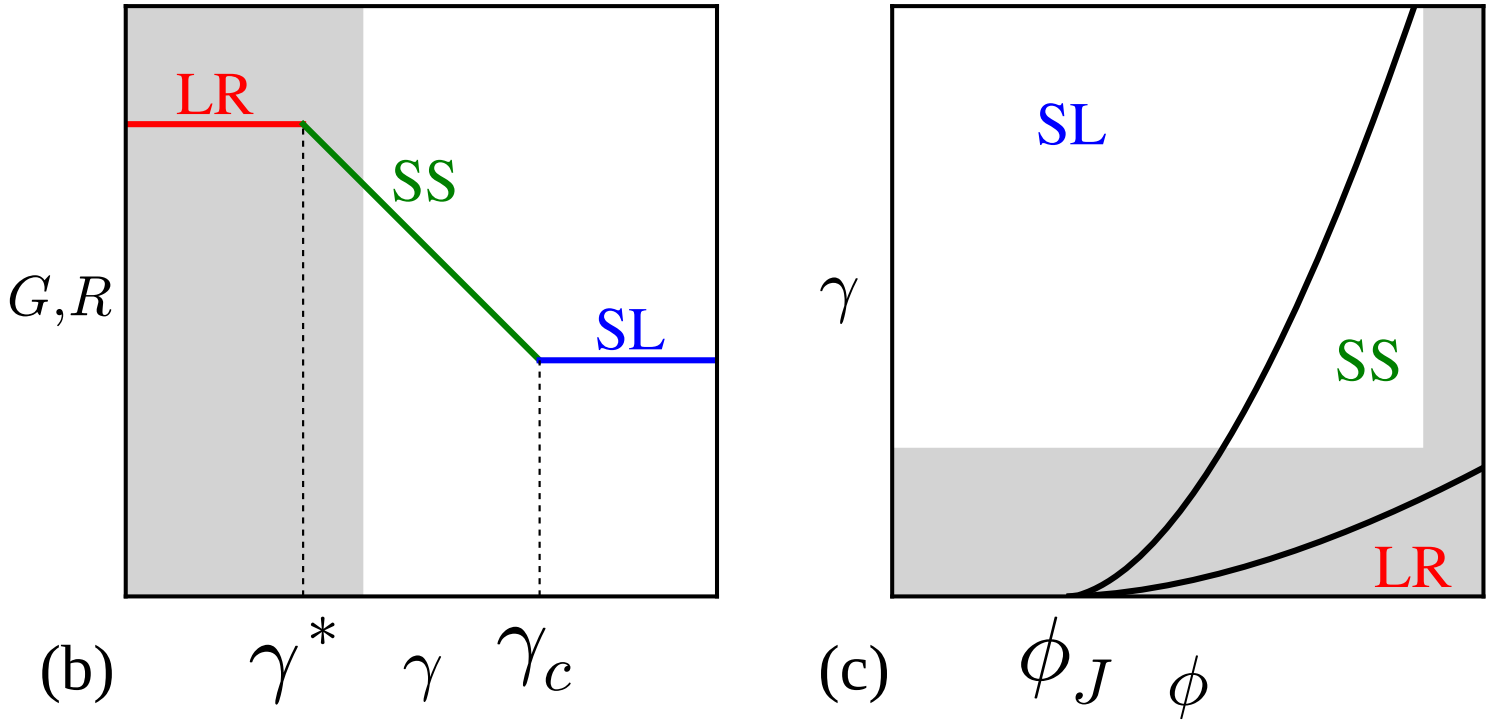
$$P = [R_0 + R_{nl}(\Delta\phi, \gamma)]\ \gamma^2$$
$$\tau = 2\,[G_0 + G_{nl}(\Delta\phi, \gamma)]\ \gamma$$

$$R_{nl}(\Delta\phi, \gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ a\Delta\phi^{\mu}\gamma^{\alpha-2} & \text{for } \phi > \phi_J \end{cases}$$

$$G_{nl}(\Delta\phi, \gamma) = \begin{cases} 0 & \text{for } \phi < \phi_J \\ b\Delta\phi^{\nu}\gamma^{\beta-1} & \text{for } \phi > \phi_J \end{cases}$$

- $\mu = 1.7 \quad \alpha = 1.0$
- $\nu = 1.0 \quad \beta = 0.4$
- => $\gamma_c \sim \Delta\phi^{\zeta}$, ζ=1.7

Discussion II : Probing elasticity



- Is the transition from the shear softening regime to the saturated linear regime controled by the Gardner transition?

Discussion III : Yielding close to jamming …

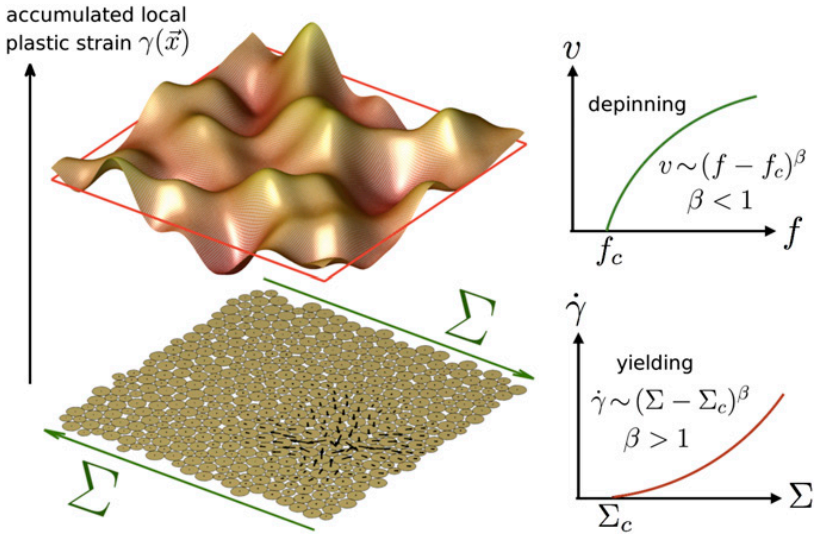
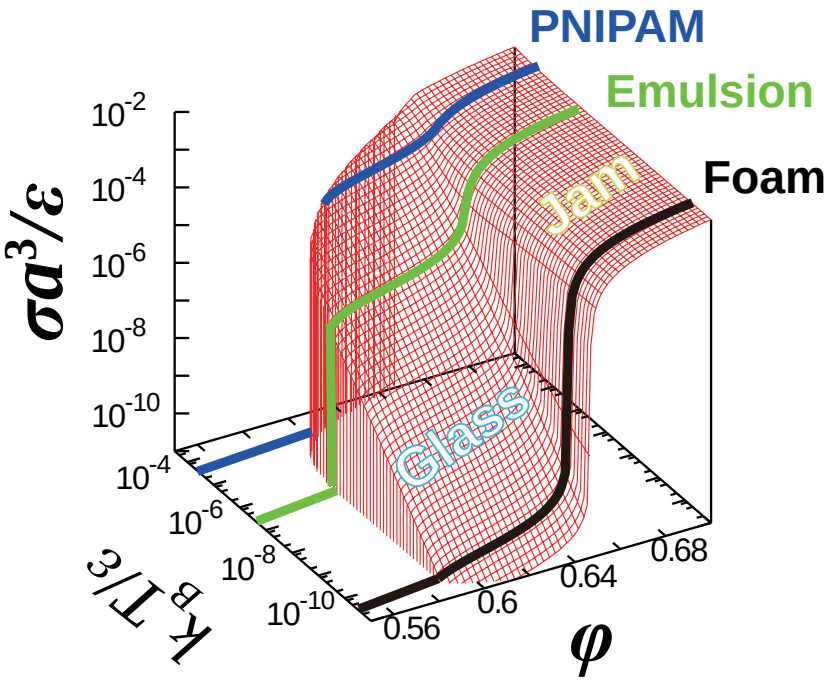
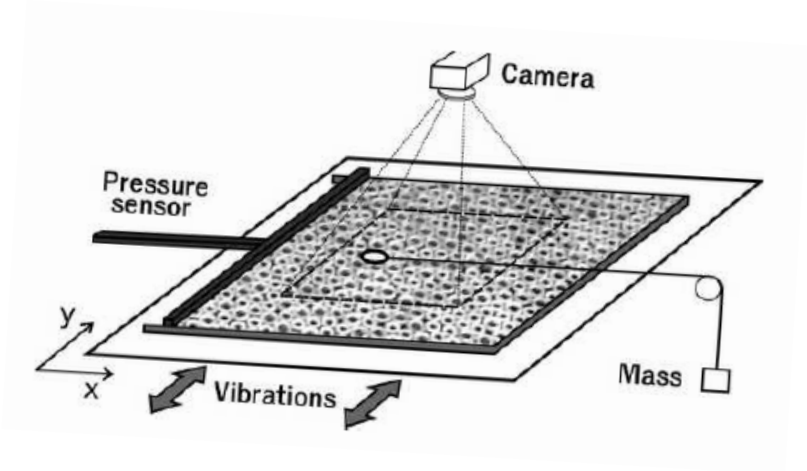


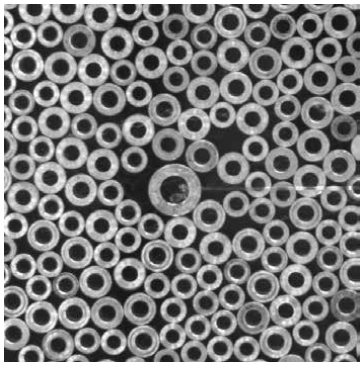
Fig. 1. (*Left*) Analogy between the yielding transition of a *d*-dimensional amorphous solid and the depinning transition of an elastic interface of *d* dimensions in a space of *d* + 1 dimensions, illustrated here for *d* = 2. The

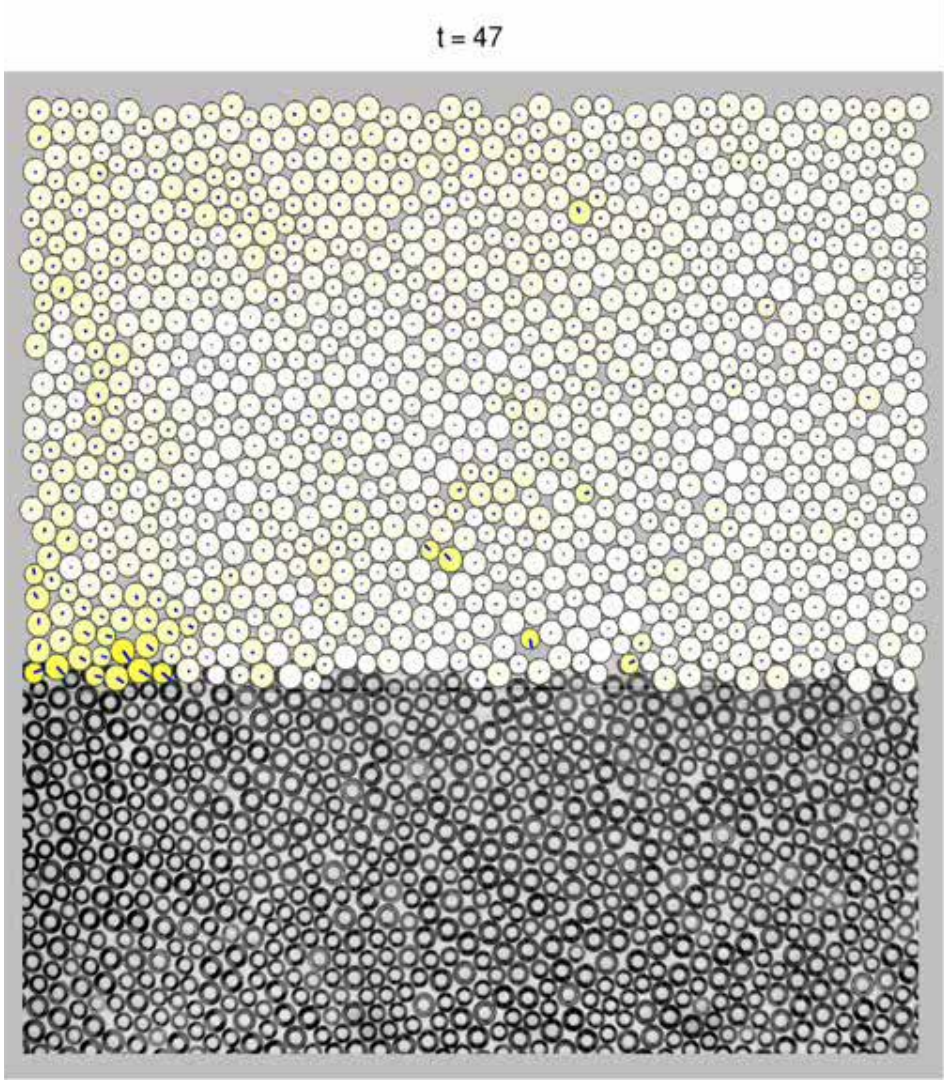


Scaling relations between
- yielding exponent
- avalanche statistics
- density of STZ

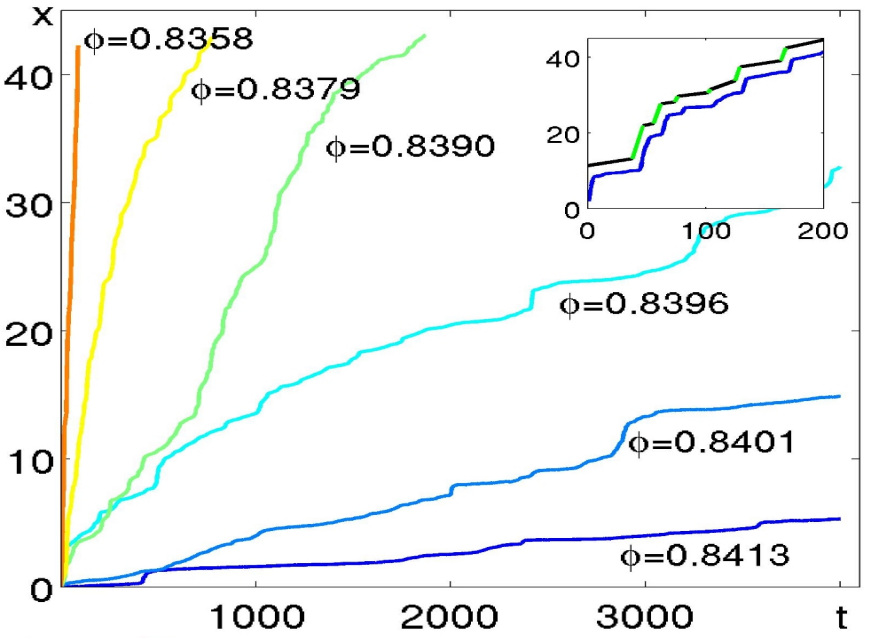
Discussion III : Yielding close to jamming …

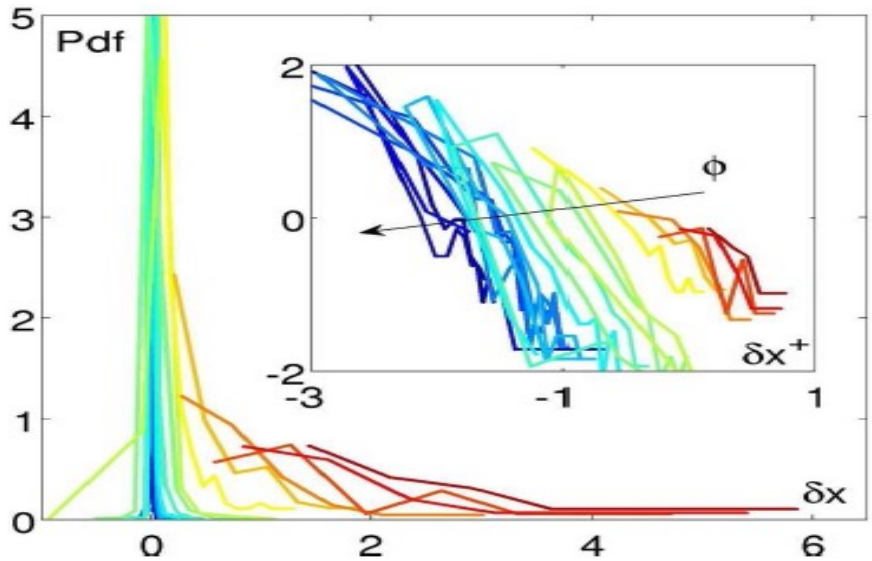




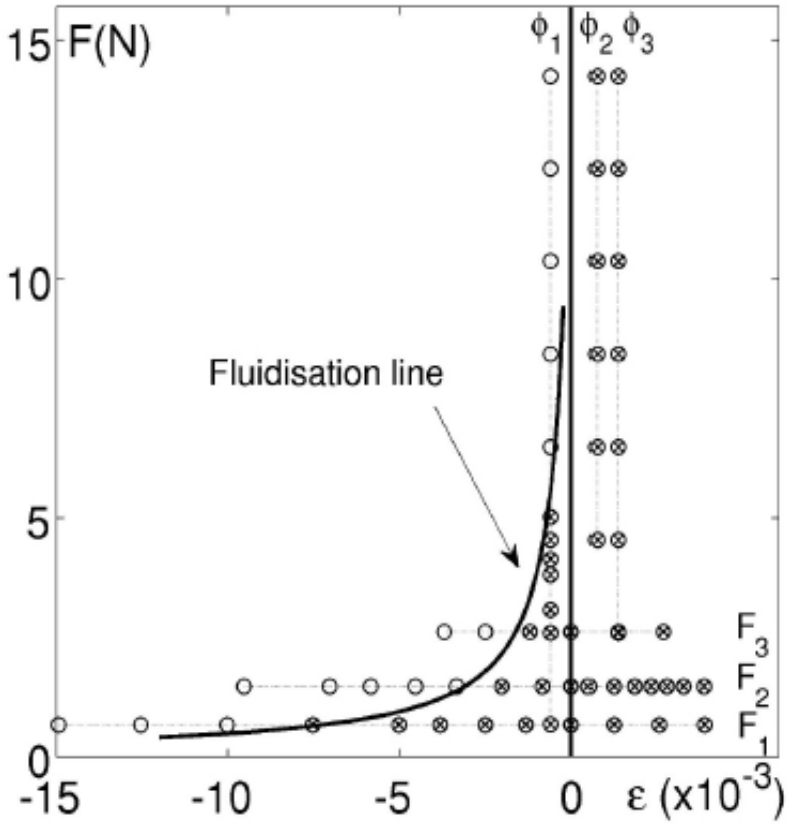


Evidence of a fluidization transition

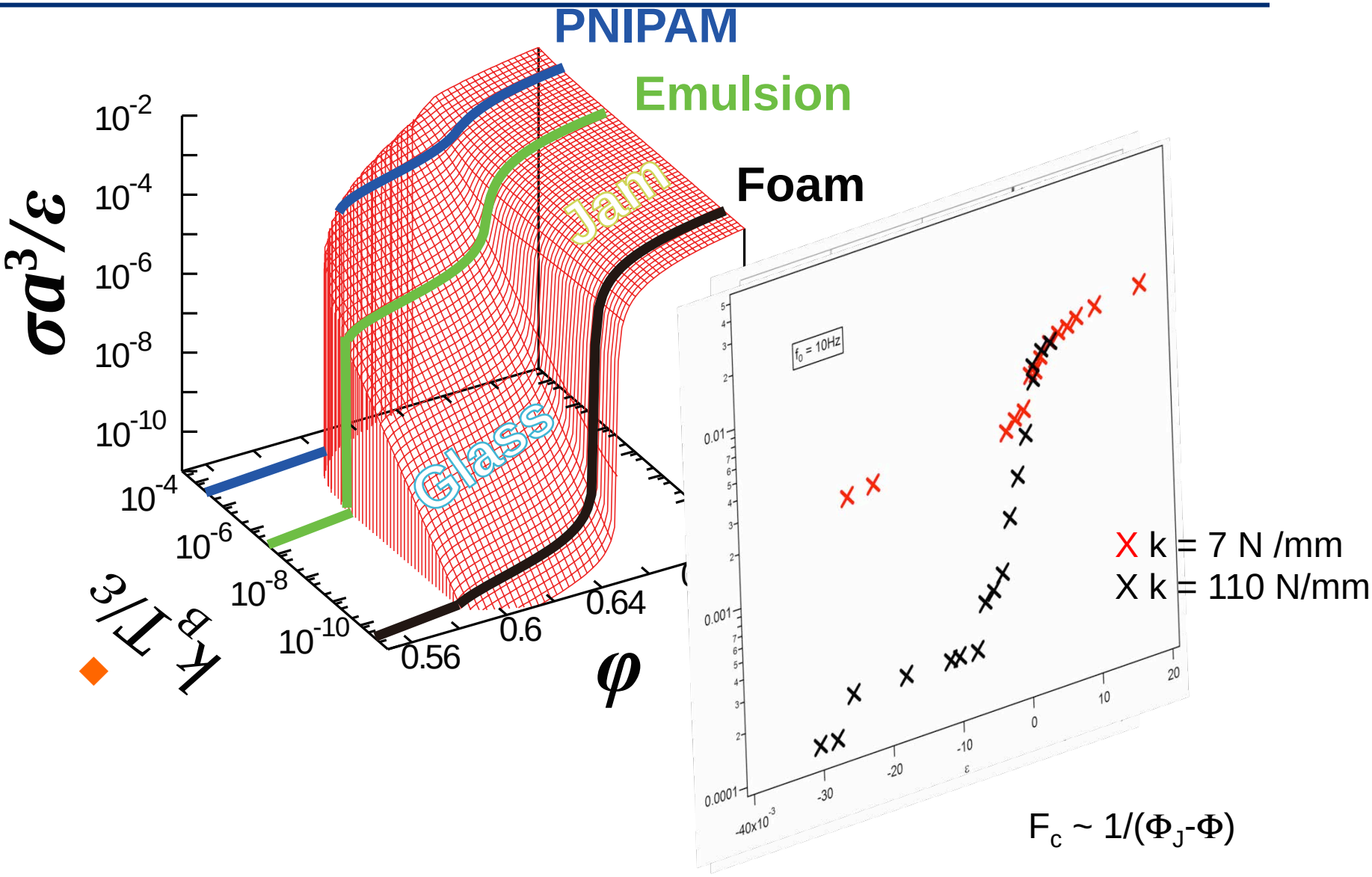




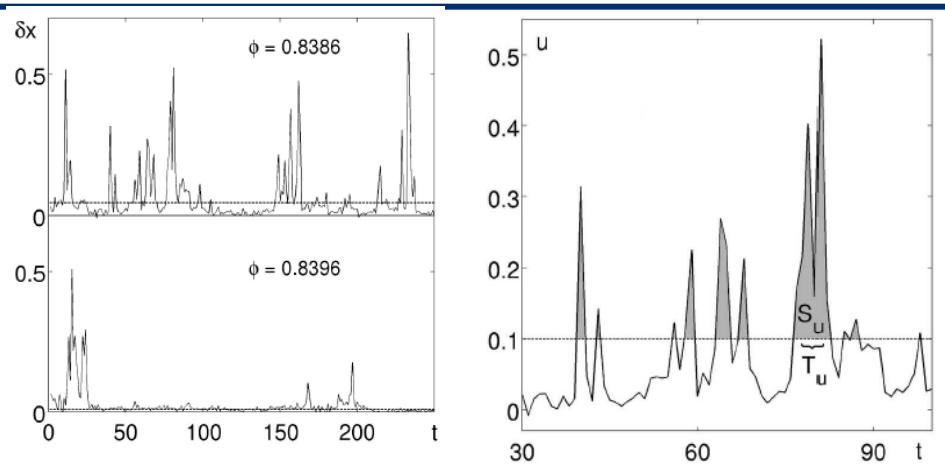
Transition : % $\delta x < 0$ --> 0



Critical force : “thermal” yield stress



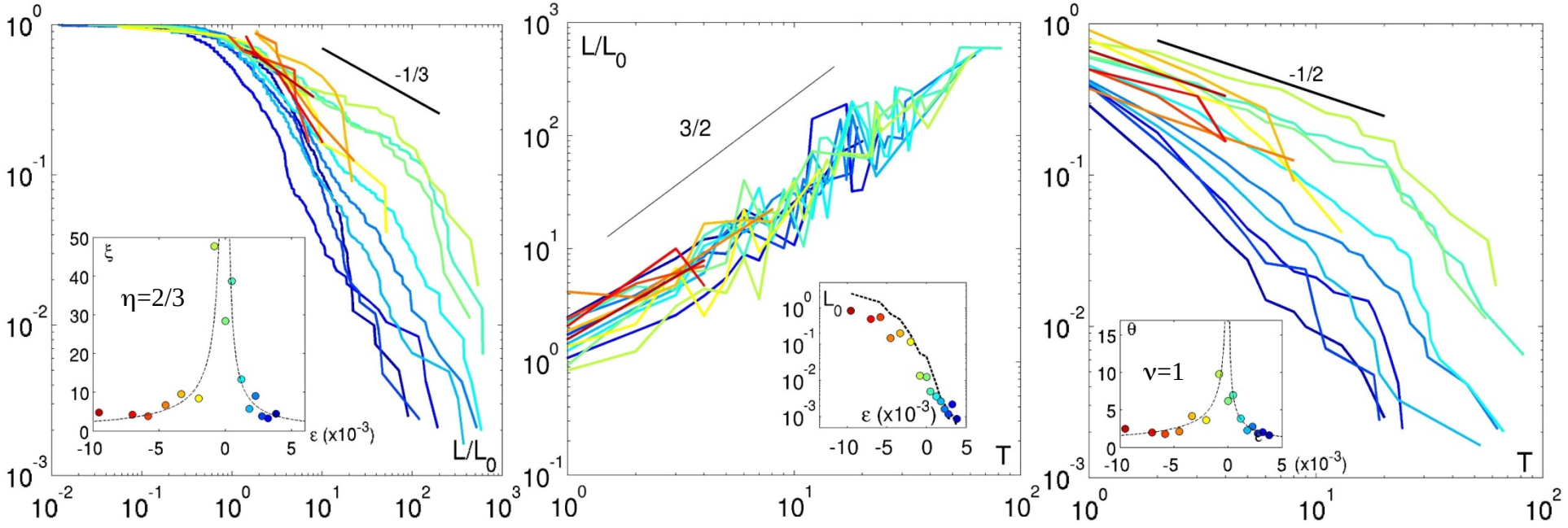
In the intermittent regime : avalanche dynamics



$$pdf(T) \propto T^{-(1+\alpha)} f\left(\frac{T}{\theta(\phi)}\right);\ \theta(\phi) \propto \left|\phi - \phi_J\right|^{-\eta}$$

$$L \propto T^{1/z}$$

$$pdf(L) \propto L^{-(1+\beta)} f\left(\frac{L}{\xi(\phi)}\right);\ \xi(\phi) \propto \left|\phi - \phi_J\right|^{-\nu}$$



Conclusion

- A clear signature of the Gardner transition
- Further comparison with numerical results are needed to clarify
  - Finite time and aging dynamics
  - Effect of softness (crossover …)
- Need for better characterizing the Gardner phase :
  - Avalanche dynamics in local rheology ?
  - Non-linear elasticity ?

- Further readings :
  - **Gardner : Phys Rev Lett 117 (22), 228001 (2016).**
  - **Shear Softening : Phys Rev Lett 113 (19), 198001 (2014).**
  - **Yielding and Avalanches : Phys Rev Lett 103 12800 (2009).**
  - **Early vibrational heterogeneities : EPL, 83, 46003, (2008).**