

Cuprate High-Temperature Superconductors – How to Get Back to the Future with Phonons

Dennis M. News

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With:

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with thanks to

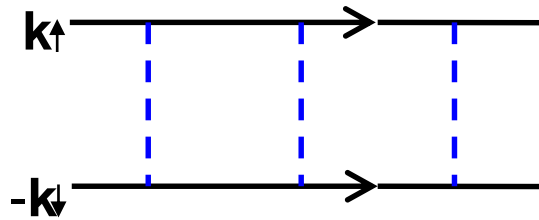
Chang C. Tsuei.

Nature of Superconductivity

- In a metal, electrons near the Fermi Energy are weakly (repulsively) interacting 'quasiparticles' (Landau)



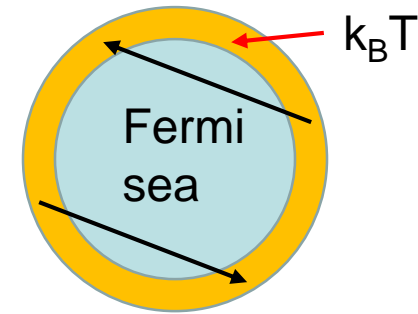
- A source of attraction, typically interaction between the electrons and the lattice, overcomes the repulsion and leads to the electrons pairing up into **Cooper Pairs**



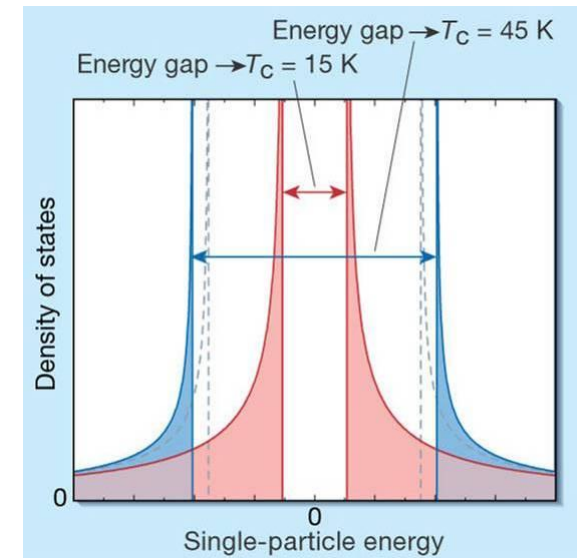
Bardeen, Cooper, Schrieffer (BCS)



- The Cooper pairs are Bosons, and condense into a quantum coherent state – a superfluid – *simultaneously with their formation*, below T_c
- Current in a superconductor is carried by a macroscopic flow of the charged superfluid condensate, involving collective behavior of the electrons. Hence it does not encounter electrical resistance
- The formation of the condensate leads to an **energy gap** in the spectrum of electronic states at the Fermi level – which also carries a macroscopic *phase*.

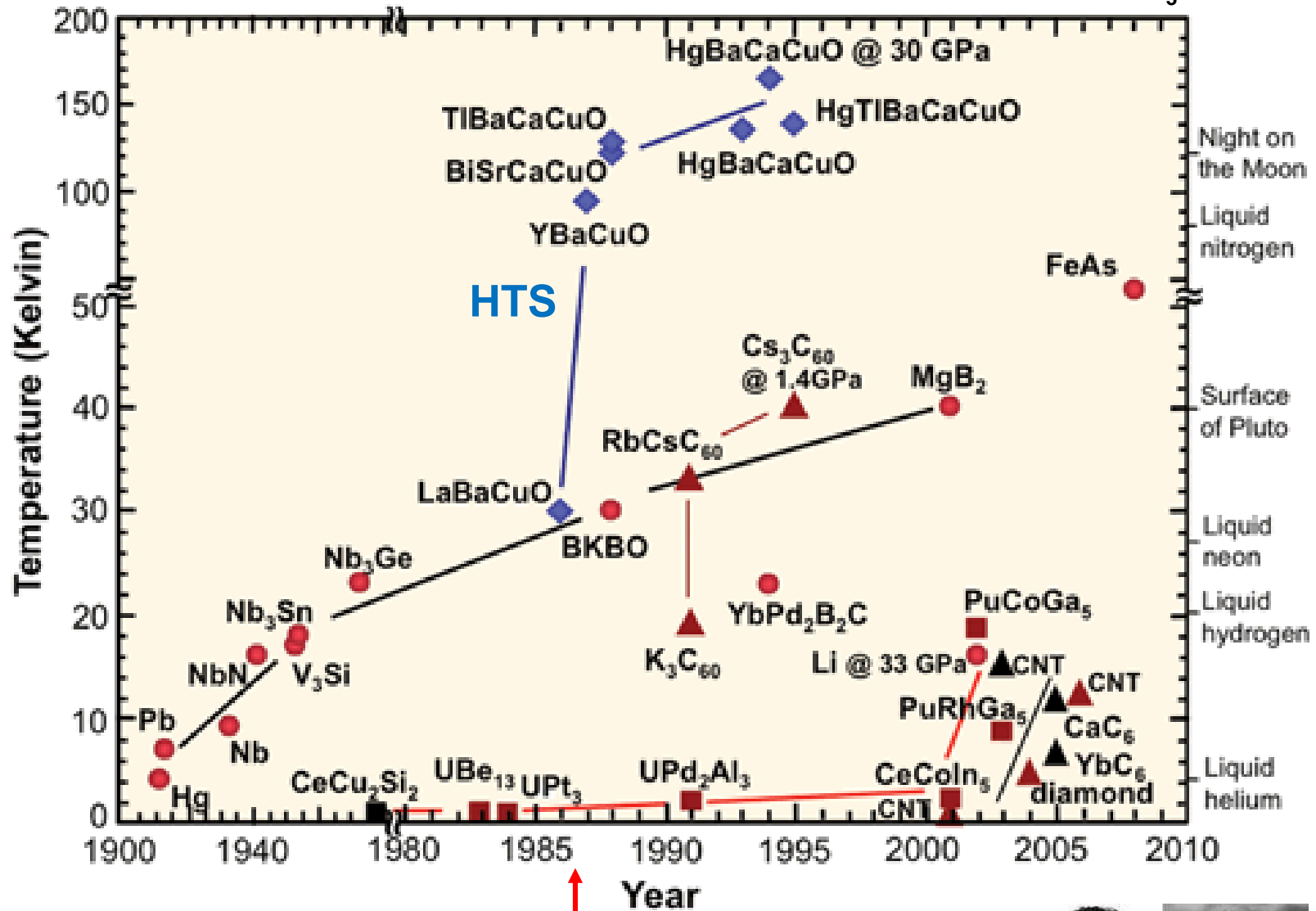


Very few transitions conserving momentum and energy within $k_B T$



Increase in Superconducting T_c with Year

H₃S @ 190 GPa

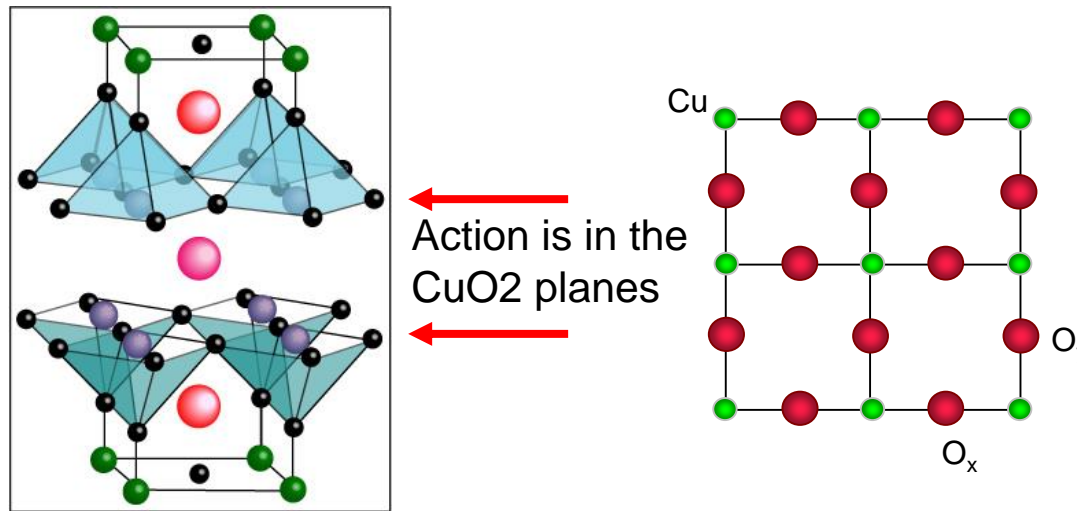


Discovery of LBCO HTS by Muller and Bednorz opened HTS field



The HTS materials: Quasi-2D metallic SC

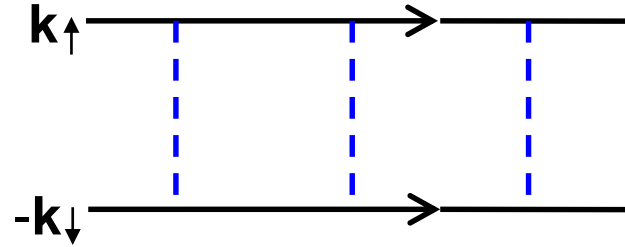
- HTS are *layered perovskite materials* – they contain the O-Cu-O bond in a square symmetry environment



- Action is in the CuO₂ planes
- **Low-*T* state** metallic with de Haas-van Alphen Fermi Surface

Properties of a BCS Superconductor (violated by HTS)

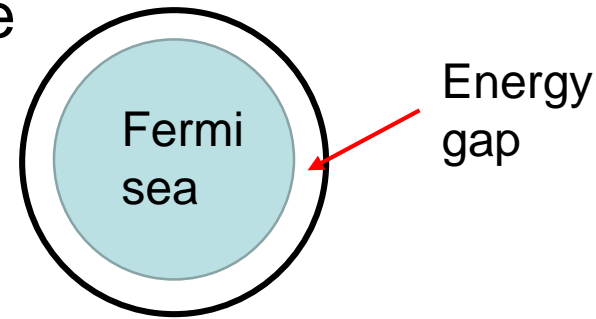
1. Origin of Pairing is electron-phonon interaction



$$T_c = \omega_{ph} e^{-1/(gN(\epsilon_F))}$$

Coupling constant \nearrow DOS at Fermi level \nwarrow

2. Energy gap is isotropic – s-wave



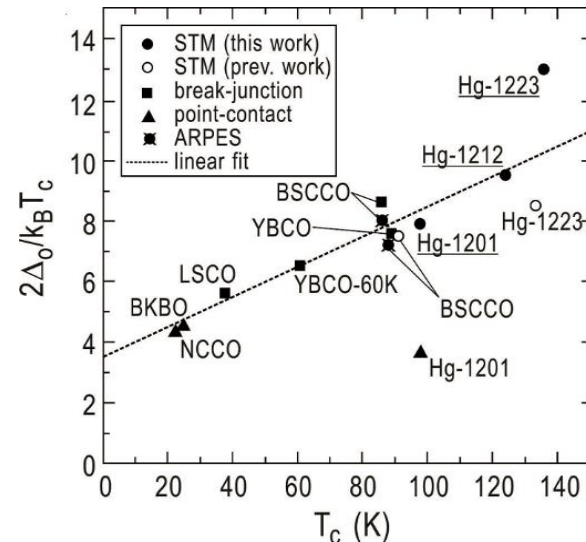
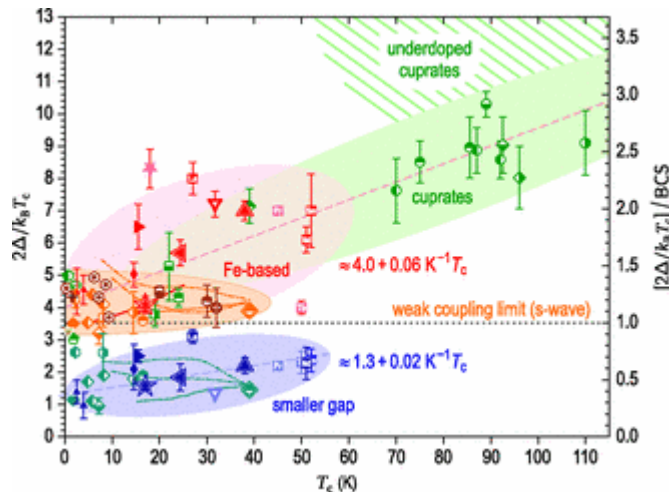
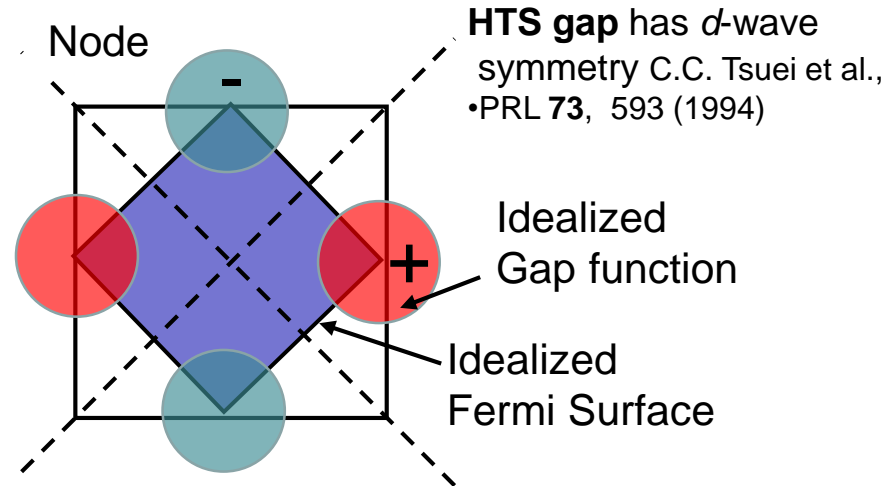
3. Energy gap $2\Delta / k_B T_c = 3.5$: universal value

4. Isotope shift $\alpha = -\frac{\Delta T_c / T_c}{\Delta M_o / M_o} = 0.5$: universal value

5. No ancillary phase transition expected

High Temperature Superconductivity Contrasts with BCS

1. Origin of Pairing in dispute
2. Energy gap is *d*-wave
3. Energy gap/ T_c ratio increases with T_c up to 10

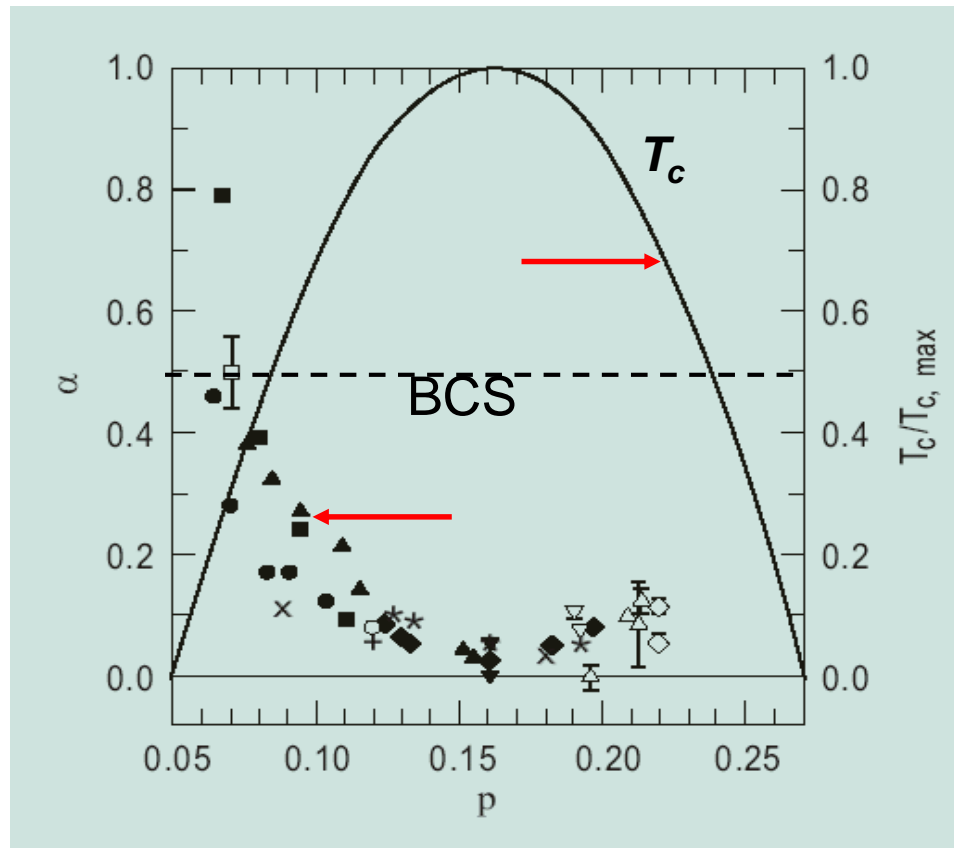


D. S. Inosov, J. T. Park, A. Charnukha, Yuan Li, A. V. Boris, B. Keimer, and V. Hinkov, Phys. Rev. B 83, 214520 (2011).

J. Wei, C. Tsuei, P. van Bentum, Q. Xiong, C. Chu, and M. Wu, Phys. Rev. B 57, 3650 (1998).

High Temperature Superconductivity

4. Isotope shift *large*, varies strongly with doping

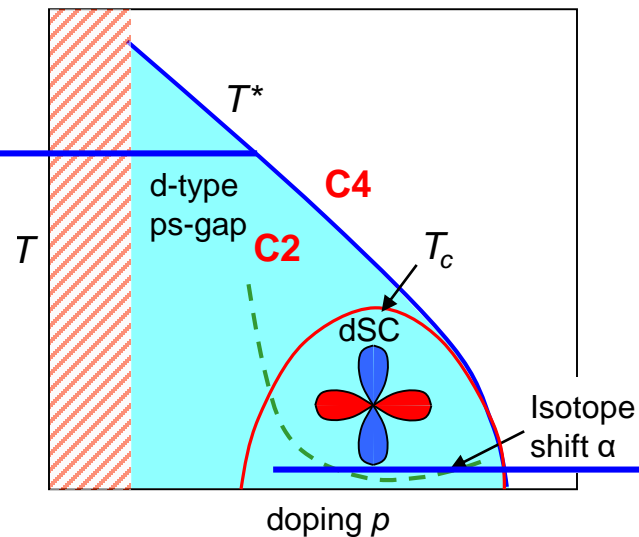
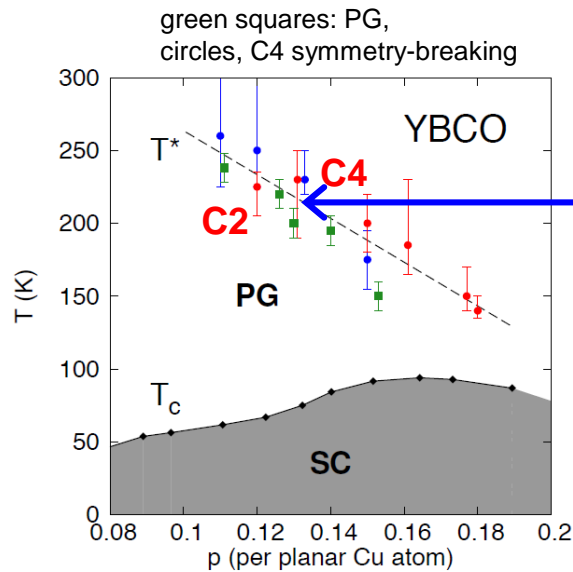


5. Ancillary phase transition, the *pseudogap*, involving $C4 \rightarrow C2$ symmetry breaking = *d*-wave CDW

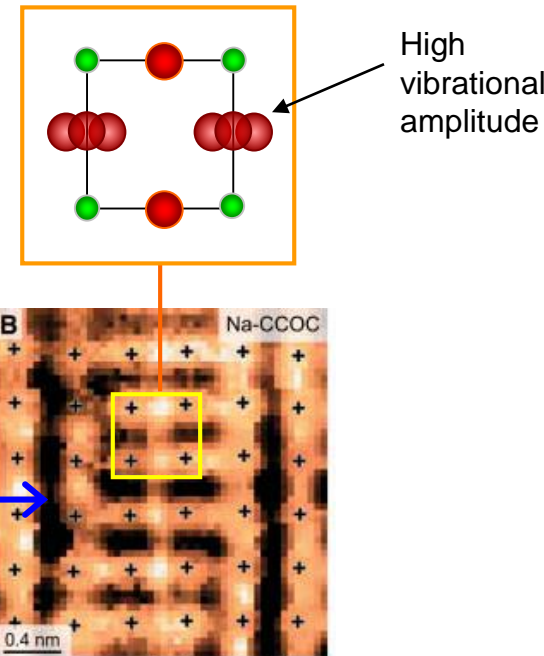
Identity of T^* with $C4$ symmetry-breaking based on Nernst effect (R. Daou et al., Nature **463**, 519 (2010)).

Phase Diagram

Low-T STM :Y. Kohsaka et. al. *Science* **315**, 1380 (2007), Y. Kohsaka et. al., *Nature* **454**, 1072 (2008).

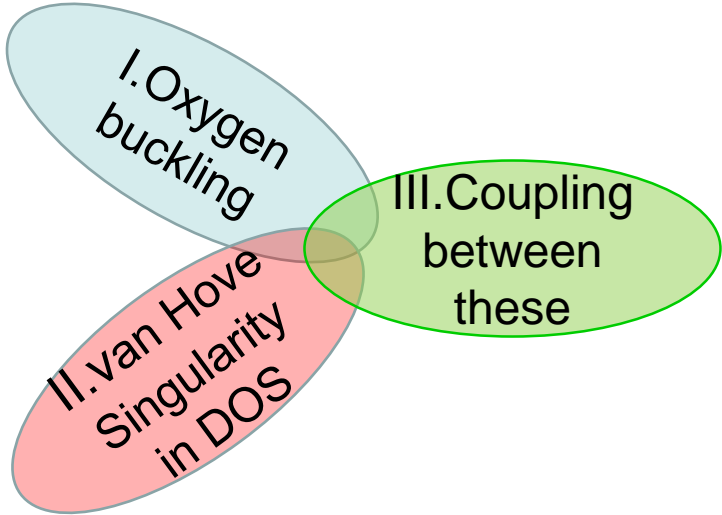


+ other data ...

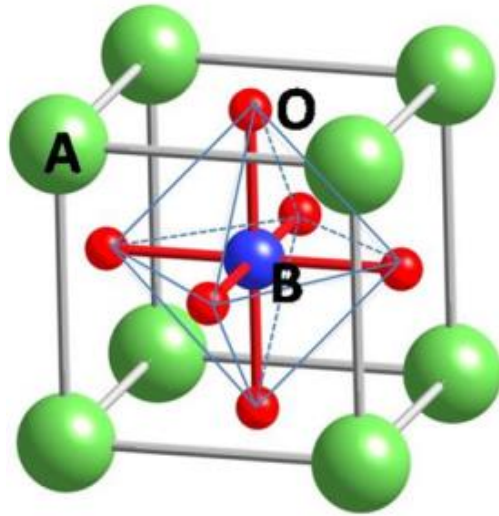


- The pseudogap transition is associated with a $C4 \rightarrow C2$ symmetry breaking (in most materials with *short coherence length*)
- The symmetry breaking includes the vibrational amplitude of the planar oxygens

Where does this lead us?

- What basic physics underlies the anomalous phenomenology of HTS?
- There is a strongly felt view that it is spin-based, can be described by Hubbard model – but evidence shows this can't be the only component
- If we take into account *three basic phenomena*:
 - I. Oxygen buckling
 - II. van Hove Singularity in DOS
 - III. Coupling between these
- The phenomenology could be understood in a *modified conventional electron-vibration model*.

I. Oxygen Buckling in Cubic ABO_3 Perovskites leading to *Highly Anharmonic Potential*



tolerance factor t

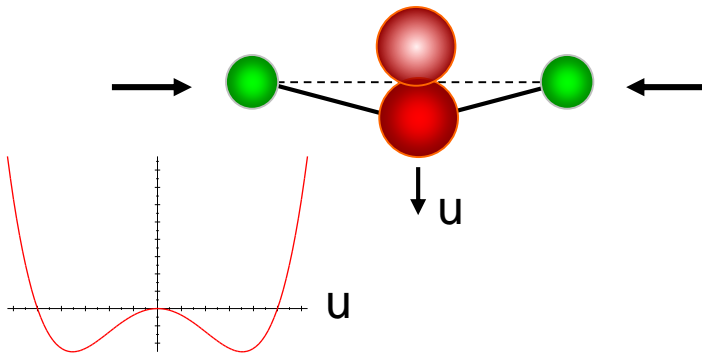
$$t = \frac{R_{\text{A-O}}}{\sqrt{2} (R_{\text{B-O}})}$$

e.g. from sum of radii

Benedek & Fennie, J. Phys. Chem C, **117**, 13339 (2013)

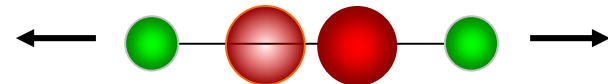
$$t < 1$$

Cu-O-Cu bond under compression
oxygen **buckle** - affects magnetism



$$t > 1$$

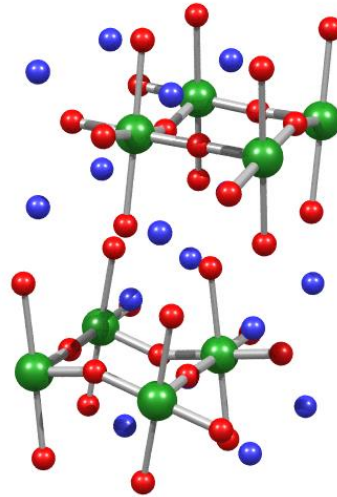
Cu-O-Cu bond under tension
ferroelectricity



The HTS Cu-O-Cu bond **also buckles**
(i.e. under effective compression)

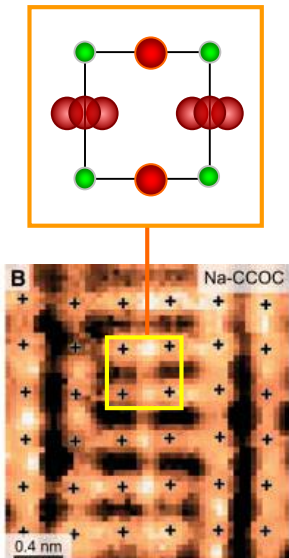
Oxygen Buckling in HTS Perovskites: Experiment and Theory

R. A. Nistor, et al.,
Phys. Rev. B **83**, 144503 (2011)
Predicted this structure
theoretically

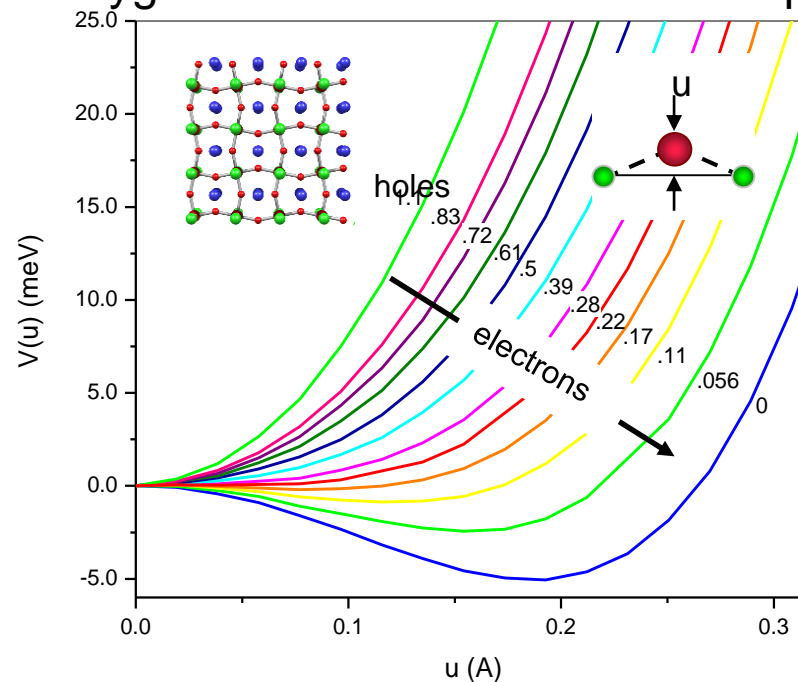


It is also seen
Experimentally as the
LTT structure.

Oxygen buckling

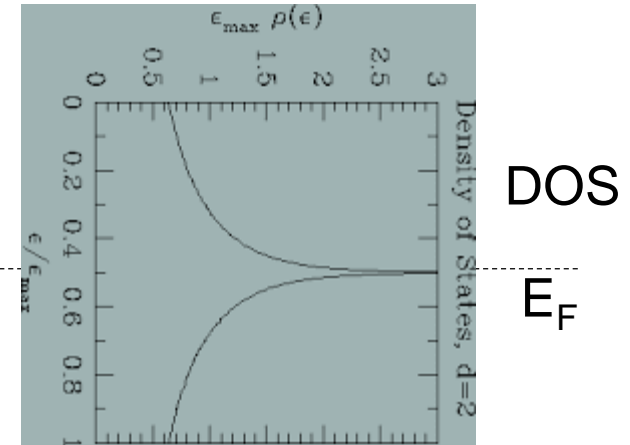
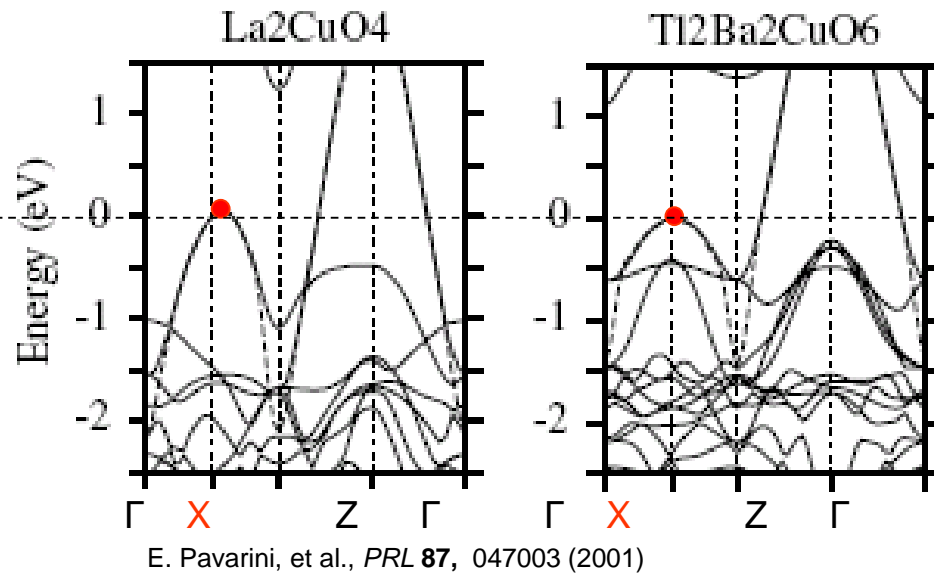


We did a detailed calculation of the
Oxygen PE surface for different dopings



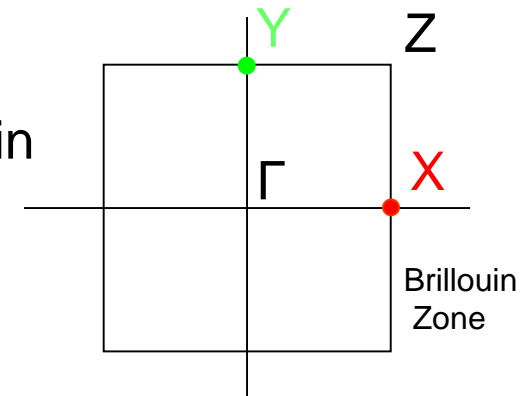
It clearly showed
buckling towards the
undoped side.

II. 2D Band structure has Log van Hove Singularity



Theorem by van Hove –
Powerful effect when at E_F

Two vHs is located at points X and Y in
Brillouin Zone (simplest type)



The log singularity in the DOS enhances superconductivity
- but can also lead to a Peierls distortion, the ancillary phase transition.

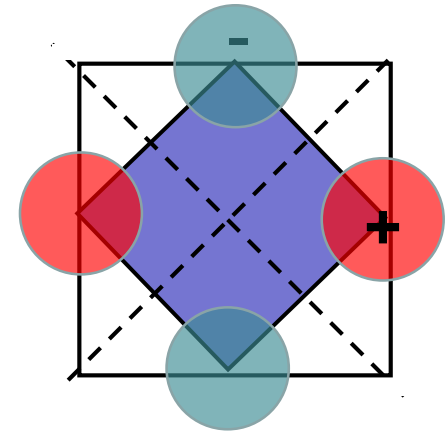
III. Coupling to Low Frequency Vibrational Modes

- From Eliashberg Equation:

$$\nu_n Z(\mathbf{k}, n) = \nu_n - T \sum_{\mathbf{q}, m} W(\mathbf{k}, \mathbf{k} + \mathbf{q}, m) \\ \times G_2(\mathbf{k} + \mathbf{q}, n + m) \nu_{n+m} Z(\mathbf{k} + \mathbf{q}, n + m),$$

$$\Phi(\mathbf{k}, n) = -T \sum_{\mathbf{q}, m} W(\mathbf{k}, \mathbf{k} + \mathbf{q}, m) \\ \times G_2(\mathbf{k} + \mathbf{q}, n + m) \Phi(\mathbf{k} + \mathbf{q}, n + m),$$

$$G_2 = \left(\nu^2 Z^2 + \Phi^2 + \varepsilon_k^2 \right)^{-1}; \quad \Phi = \Delta Z$$

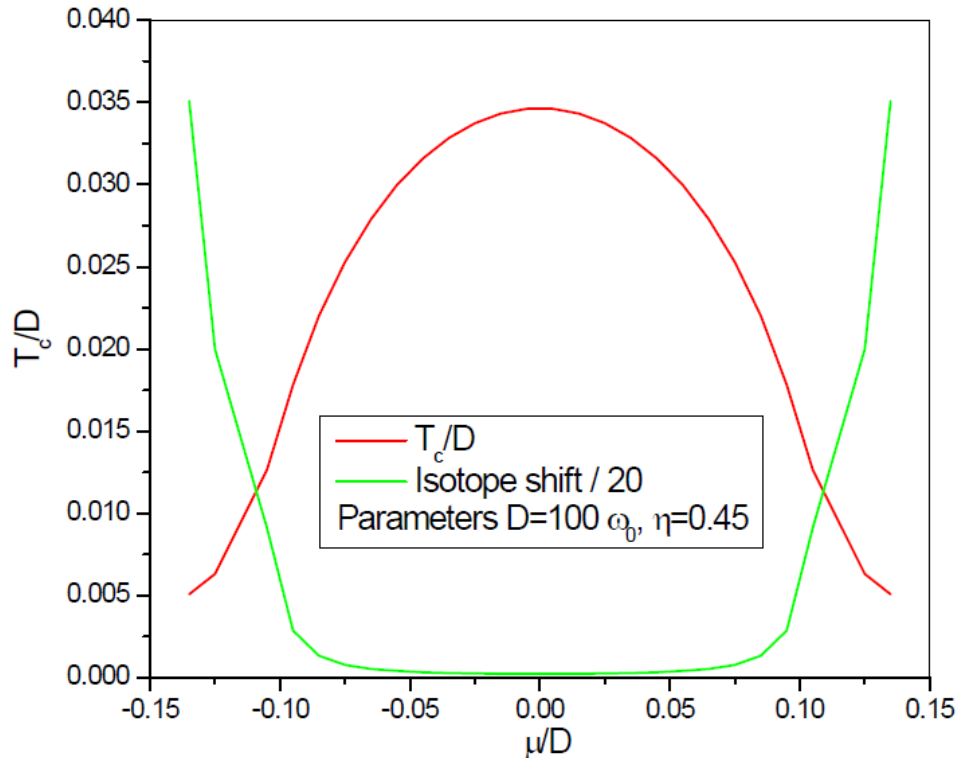


$$W(\mathbf{k}, \mathbf{k} + \mathbf{q}) =$$

$D(q)$ and $V(q)$ being a phononic propagator and a repulsive electronic interaction. d -wave HTS is generated with a **bond-centric** $D(q)$. $V(q)$ is counter- d -wave.

- When pairing interaction $D(q)$ is **on a bond**, leads to **d-wave pairing**
- Significant d -wave pairing can occur with coupling to **low frequency vibrations** (e.g. limit $\pi T > \omega_{\text{phonon}}$ vibrators classical, d -wave not s -wave).

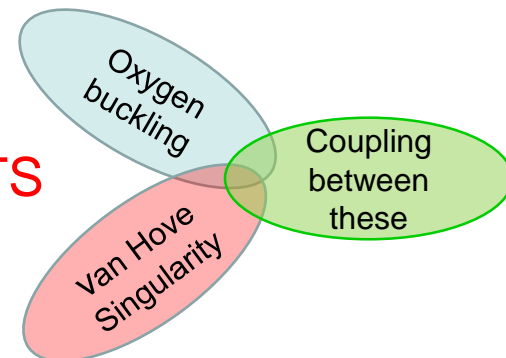
Coupling to Low Frequency Vibrational Modes



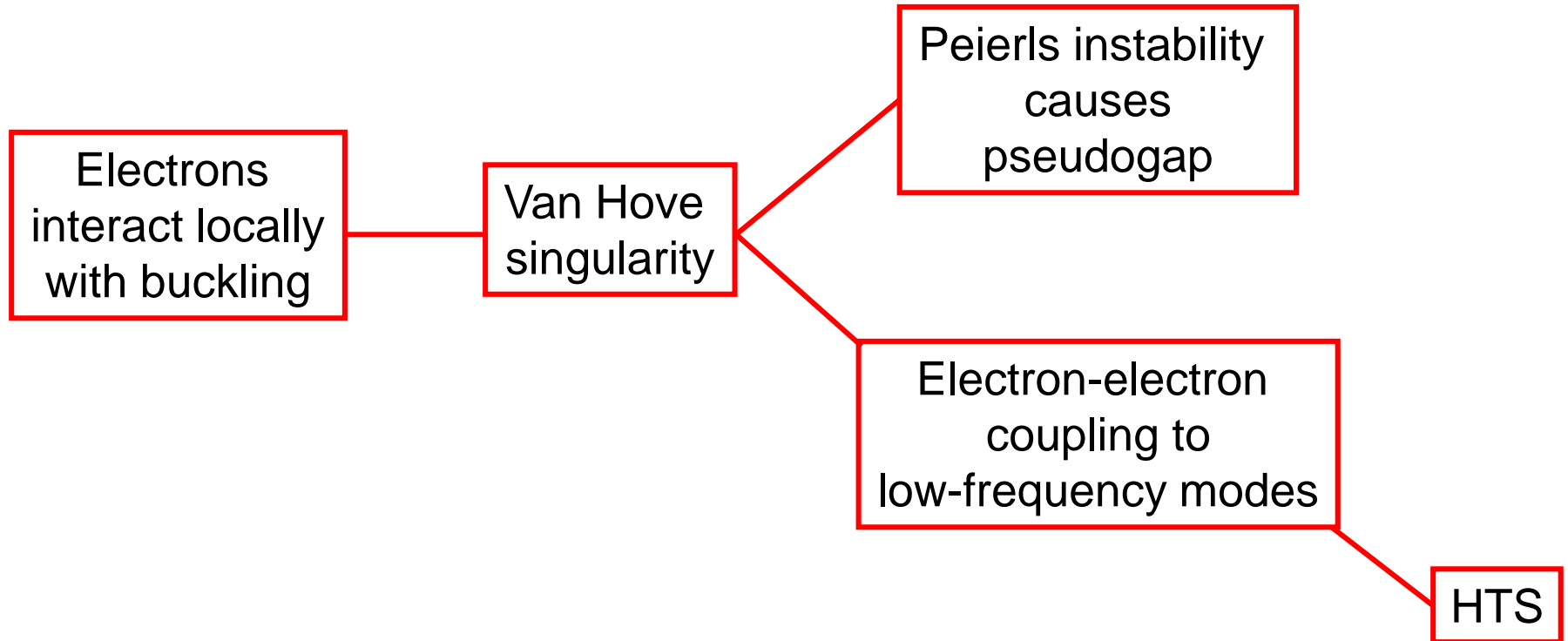
D = electronic bandwidth
 η = dimensionless coupling

- If we look at the Eliashberg equation in this regime at a van Hove singularity, we see remarkably similar T_c and isotope shift behavior to that in the HTS experimental data.

These 3 concepts can generate the phenomenology of HTS

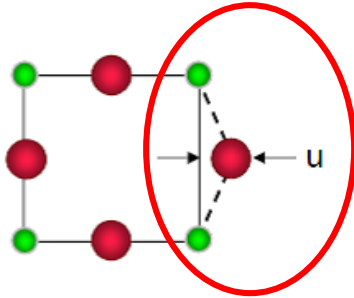


Fluctuating Bond Concept

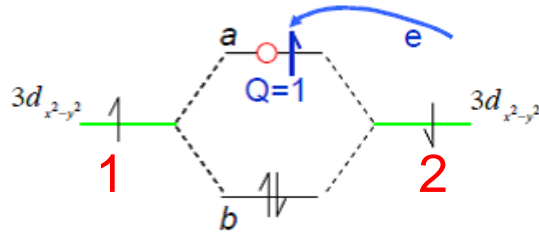


Fluctuating Bond Model: Coupling Buckling to Electrons

One bond example

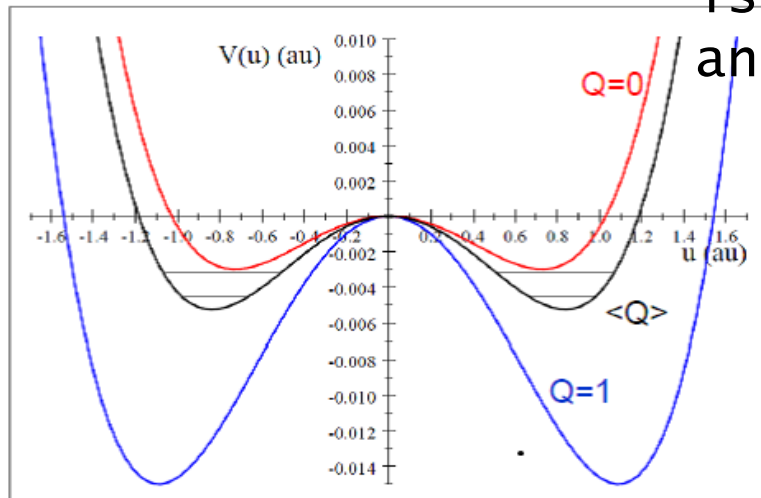


Oxygen vibrator coordinate u transverse to Cu-O-Cu bond.



Electronic energy levels of Cu-O-Cu bond (1-band picture), b = bonding, a = antibonding level.

$$Q = \frac{1}{2} \sum_{\sigma} (c_1^{\dagger} - c_2^{\dagger})(c_1 - c_2) = n^a$$



is electronic occupation of antibonding level
buckling strongly enhanced by electron occupation Q of antibonding level

$$V(u) = \frac{1}{2}u^2 + \frac{1}{8}wu^4 - \frac{1}{\sqrt{2}}vu^2Q$$

Fluctuating Bond Model

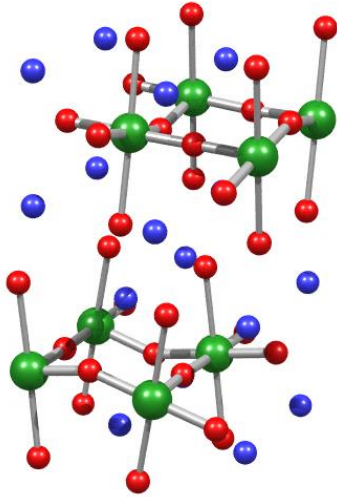
$$H = \sum_{\mathbf{k}, \sigma} \varepsilon_{\mathbf{k}} n_{\mathbf{k}, \sigma} + \sum_b \left[\frac{p_b^2}{2M_o} + \frac{\chi_0}{2} u_b^2 + \frac{w}{8} u_b^4 \right] - \frac{v}{\sqrt{2}} \sum_b u_b^2 n_b^a$$

band structure

O-vibrator: KE + PE + quartic term

quadratic electron-lattice coupling

Coupling v quantified by *ab initio* calculations



R. A. Nistor, et al.,
Phys. Rev. B **83**, 144503
(2011).

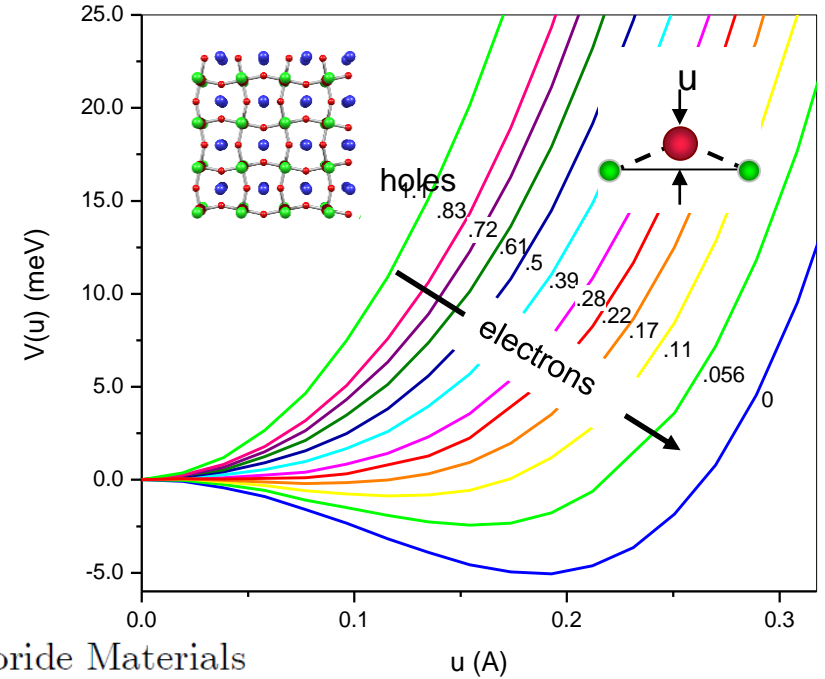
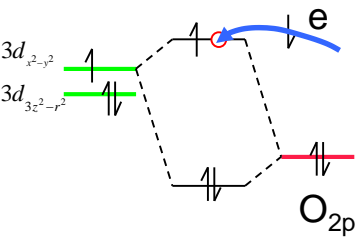


Table I: FBM Parameters for 214 and Oxychloride Materials

Polarization	v_{214} (au)	v_{oxy} (au)	w_{214} (au)	w_{oxy} (au)	K_{214} (eV)	K_{oxy} (eV)
$xy \perp$ to bond	0.0163	0.0182	0.053	0.090	0.14	0.10
$z \perp$ to bond	0.0174	0.0202	0.122	0.106	0.068	0.104

Mean Field Solution to the Fluctuating Bond Model



Leads to anharmonic oscillator where antibonding level occupation softens harmonic force constant

$$H_{vib} = \sum_b \left[\frac{p_b^2}{2M_O} + \left(\frac{\chi_0}{2} - \frac{v}{\sqrt{2}} \langle n_b^a \rangle \right) u_b^2 + \frac{w}{8} u_b^4 \right]$$

anharmonic oscillator with harmonic force constant controlled by $\langle n_b^a \rangle$

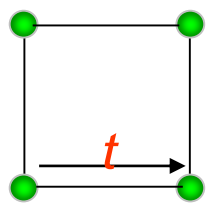
Decouple interaction term in 2 possible ways

$$H = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}, \sigma} + \sum_b \left[\frac{p_b^2}{2M_O} + \frac{\chi_0}{2} u_b^2 + \frac{w}{8} u_b^4 \right] - \frac{v}{\sqrt{2}} \sum_b u_b^2 \langle n_b^a \rangle - \frac{v}{\sqrt{2}} \sum_b \langle u_b^2 \rangle n_b^a$$

band structure

O-vibrator KE + PE + quartic term

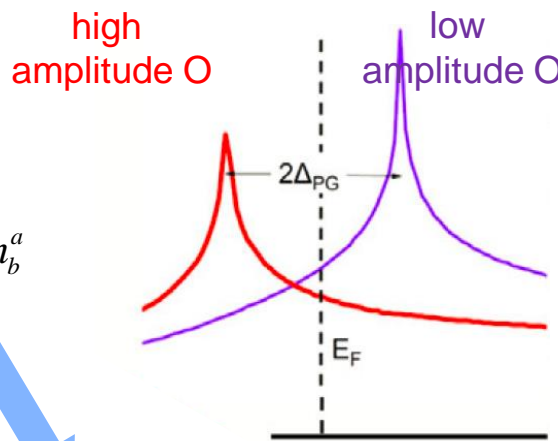
quadratic electron-lattice coupling



Leads to electronic Hamiltonian where vibrator amplitude modifies band structure (lowers t and hence reduces band width)

$$H_{el} = \sum_{\mathbf{k}, \sigma} \epsilon_{\mathbf{k}} n_{\mathbf{k}, \sigma} - \frac{v}{\sqrt{2}} \sum_b \langle u_b^2 \rangle n_b^a$$

band structure modified by term in $\langle u_b^2 \rangle$



- Assume spatially-uniform solution, then each piece, H_{vib} and H_{el} can be exactly solved.
- The electronic and vibrational problems are coupled via the expectation values $\langle n_b^a \rangle$ and $\langle u_b^2 \rangle$
- These are solved for self-consistently

A Generalized RPA Defines the Interaction $W(\mathbf{k}, \mathbf{k}+\mathbf{q}, m)$

RPA $1/n_e$ expansion in (modified C-int.) x (e-h response function R)

$$V_{\text{int}} = \frac{\text{Coulomb interaction}}{1 + \text{polarization bubble}} = \text{Coulomb interaction} - \text{Coulomb interaction} \times \text{polarization bubble} + \text{Coulomb interaction} \times \text{R} \times \text{polarization bubble} \times \text{polarization bubble} \dots$$

modified C-int = V_q - (FBM coupling² mediated by complete vibrator response function)

complete vibrator response function = $\frac{\text{vibrator bubble D}}{1 + \text{vibrator interaction w} \times \text{vibrator bubble D}}$ = $\text{vibrator bubble D} + \text{vibrator bubble D} \times \text{vibrator interaction w} \times \text{vibrator bubble D} + \dots$

1/n expansion in (interaction w) x (2 vibrator bubble D)

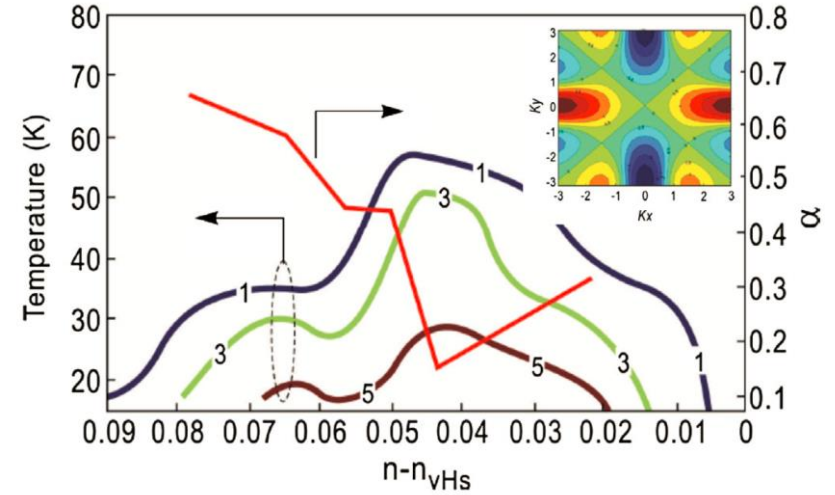
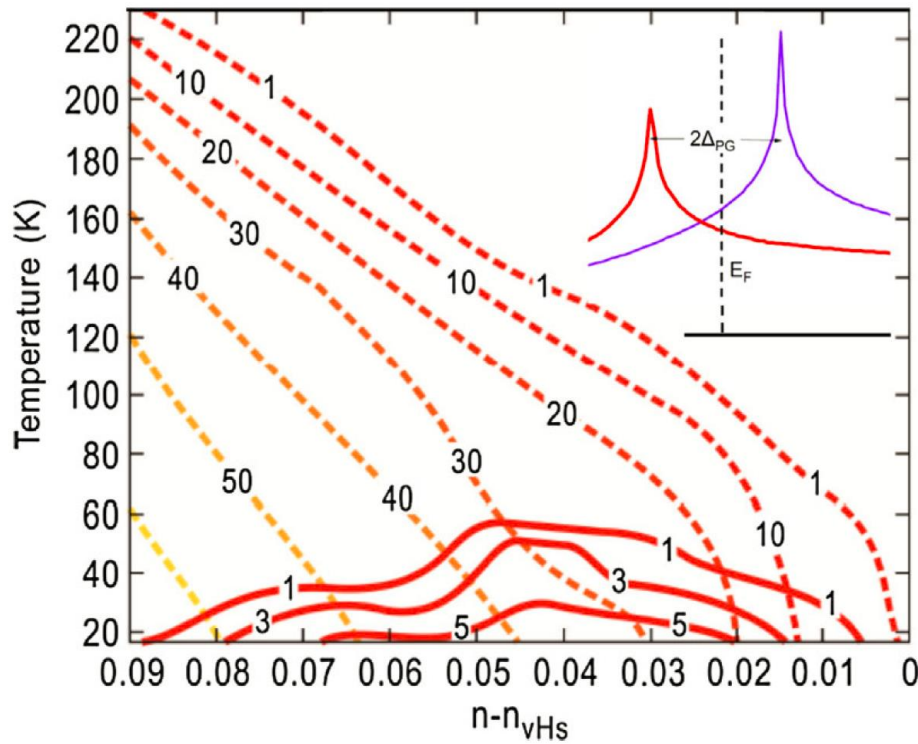
———— = electron propagator g - - - - - = vibrator propagator d
 □ = vibrator interaction w ● = FBM interaction v ~~~~~ = Coulomb interaction V_q

Generates low frequency coupled vibrational/electronic modes associated with pseudogap formation

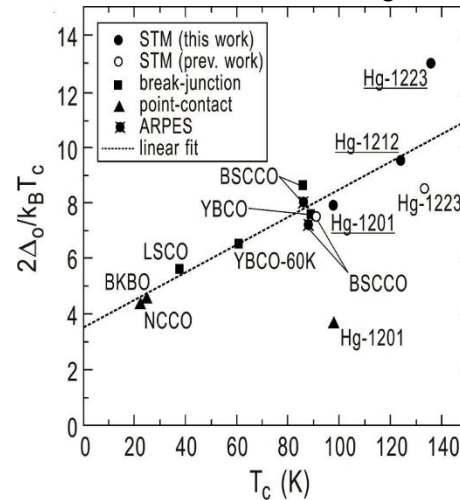
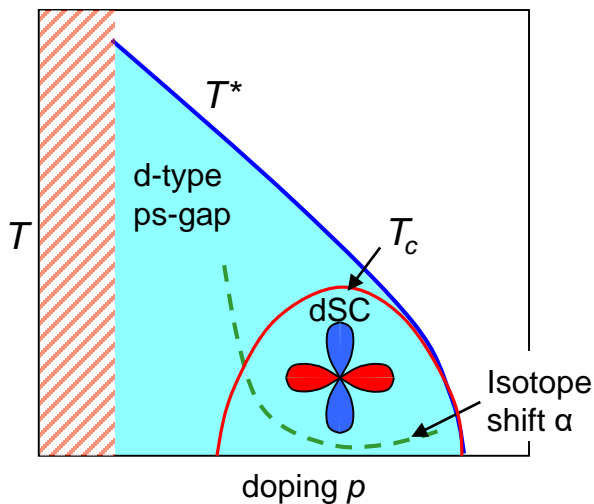
Solve Eliashberg Equation with this interaction

Hsiao, Martyna, Newns, PRL 114, 107001 (2015)

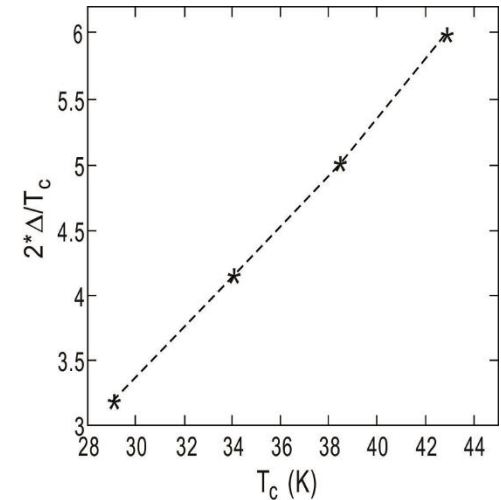
Phase Diagram with all gaps, symmetries reproduced



Anomalous Gap/ T_c ratio reproduced
– T_c varied via increased v



(a)

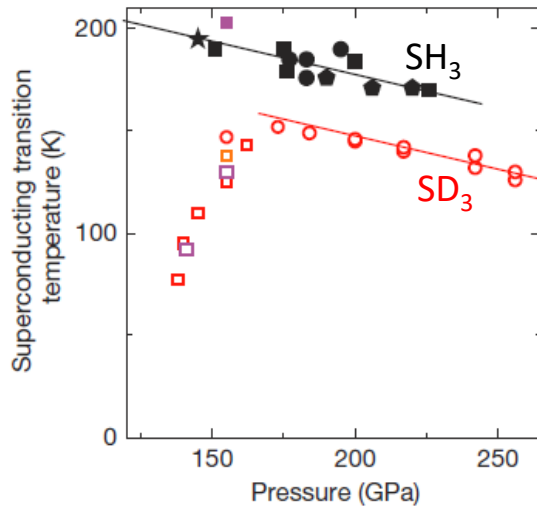


(b)

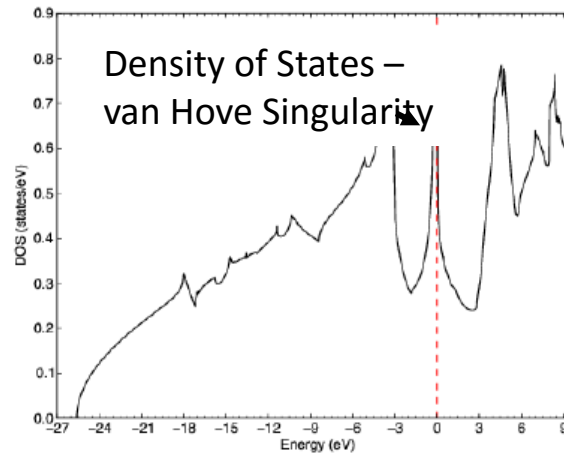
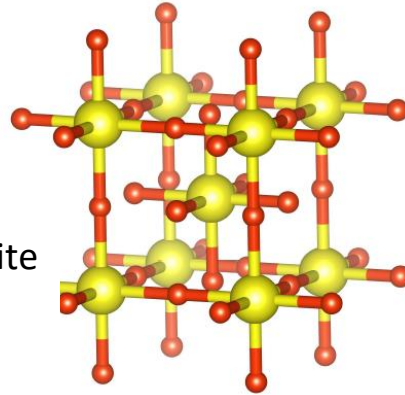
Conclusion

- The FBM, based on anharmonic oxygens coupled nonlinearly to the electron gas, captures the key pseudogap and SC properties of the cuprate HTS.
- The model produces a *d*-wave SC gap distorted by the $C4 \rightarrow C2$ symmetry breaking of the pseudogap, on average it is *d*-wave. There is some evidence for this.
- The model has no spatial variation of the pseudogap – like certain YBCO compositions. As in YBCO, a double-hump T_c is predicted.
- A more accurate minimum isotope shift value should be produced by going beyond the quasi-harmonic mean-field + RPA approximation
- It is important to calculate dynamics for comparison e.g. with the shift of O vibration frequencies with temperature below T_c .
- A key next step is to incorporate the on-site Cu Hubbard U to complete the picture.

Conventional superconductivity at 203 kelvin at high pressures in the sulfur hydride system



Perovskite
Planes



Anharmonicity

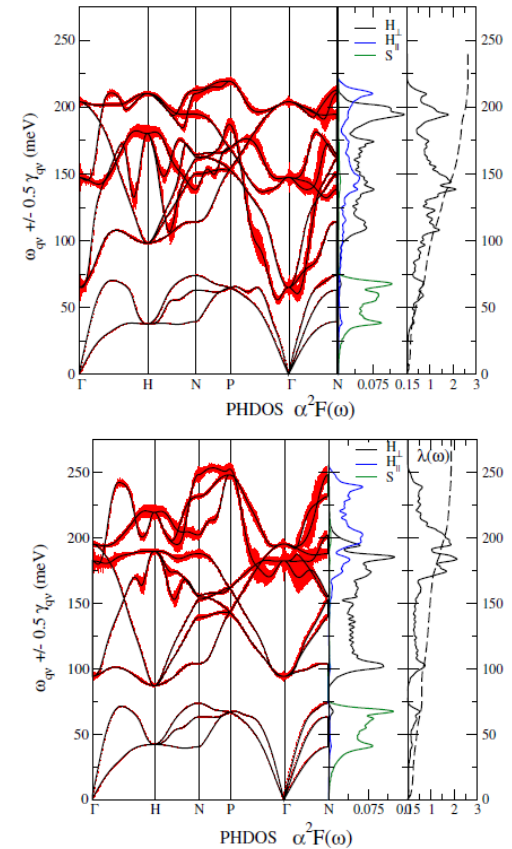
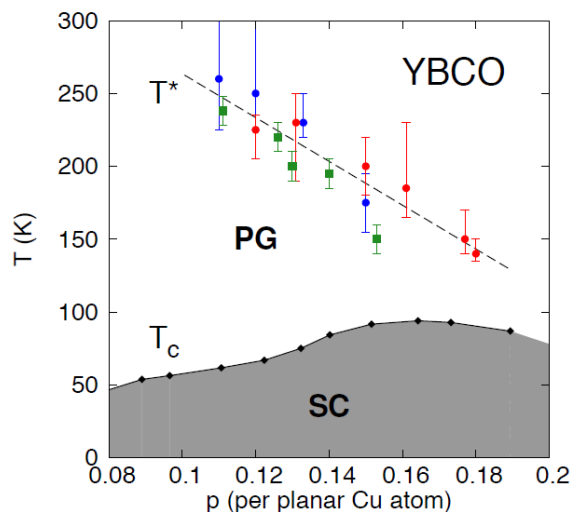


FIG. 2 (color online). Phonon dispersion, phonon density of states projected onto selected atoms and directions, and the Eliashberg function of H_3S in the harmonic approximation (top) and with the inclusion of anharmonic effects (bottom) for H_3S at 200 GPa. H_\perp and H_\parallel label displacements of an H atom in the

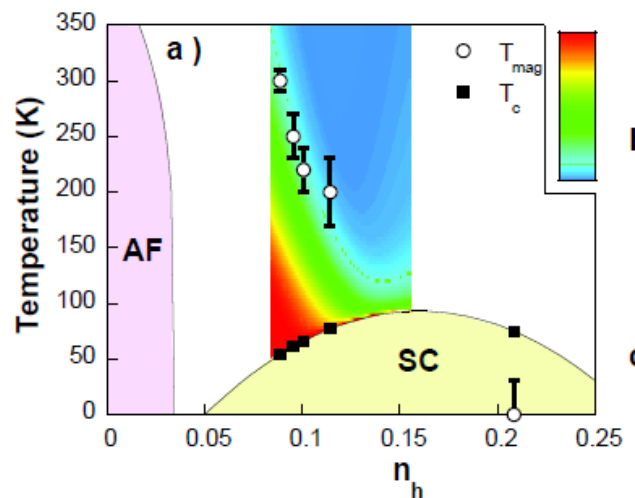
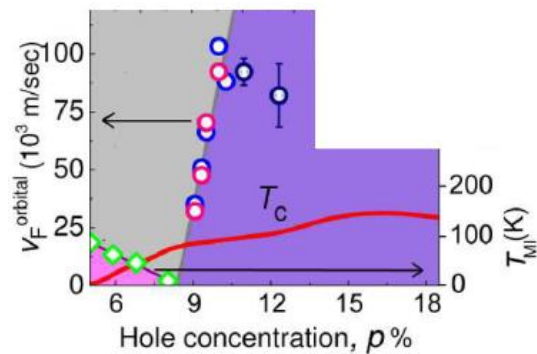
has some features in common with HTS - ? relevance

Backup Foils

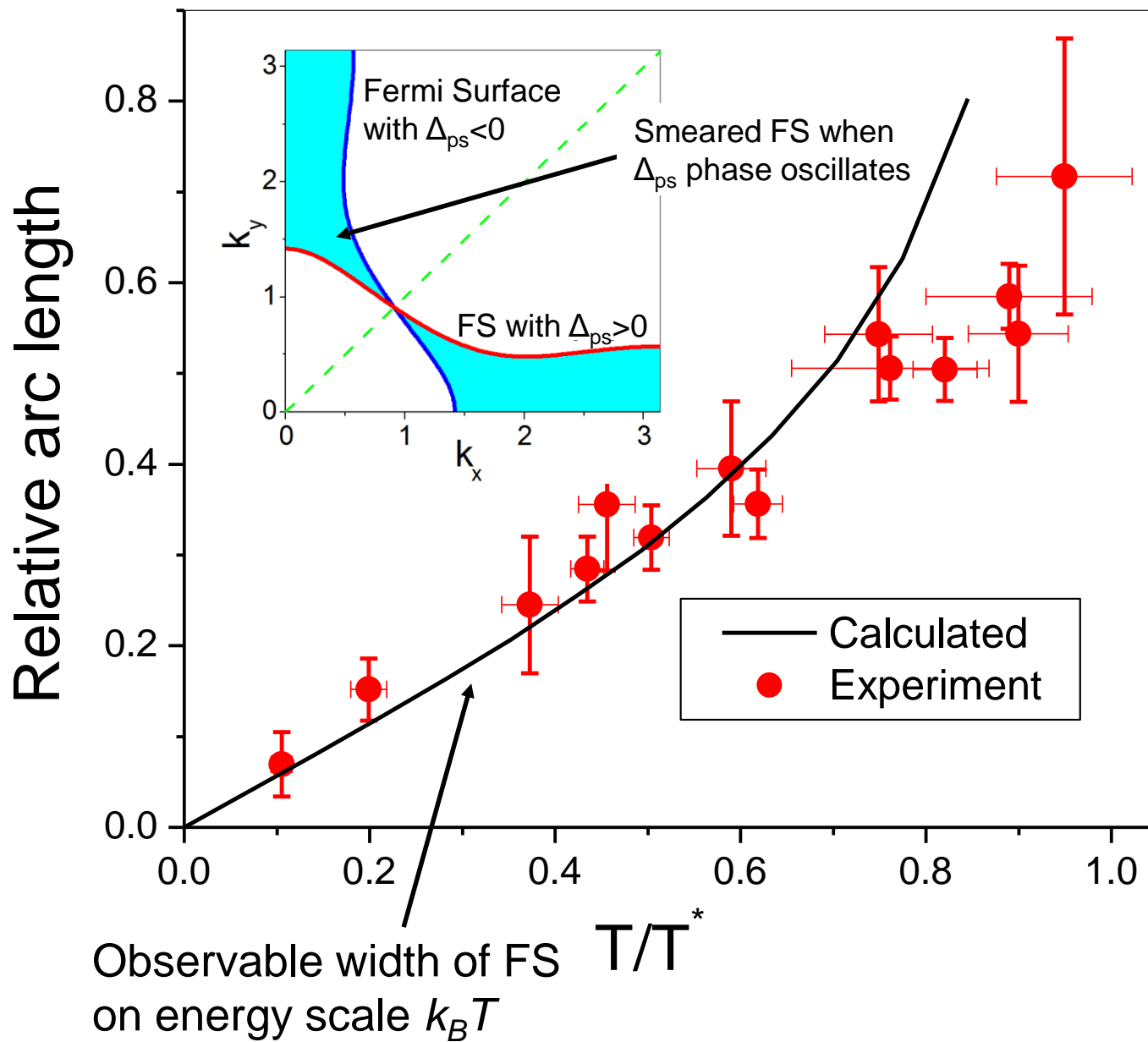
Magnetism in Metallic Phase only near Mott Transition



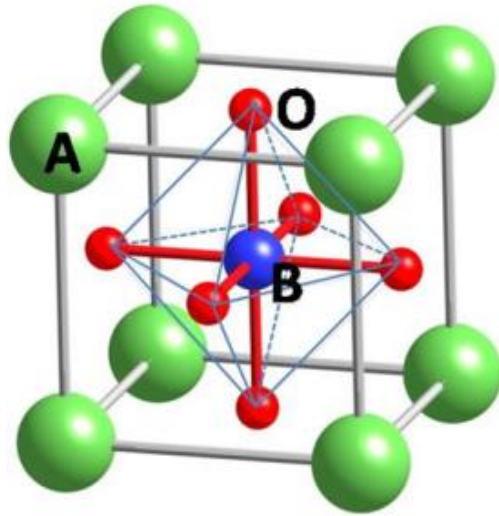
Identity of T^* with C4 symmetry-breaking based on Nernst effect (R. Daou et al., Nature **463**, 519 (2010)).



Fermi Surface Arc



I. Oxygen Buckling in Cubic ABO_3 Perovskites leading to *Highly Anharmonic Potential*

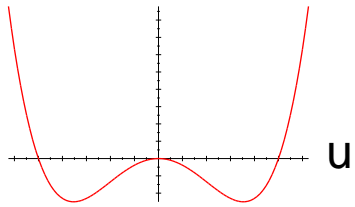


tolerance factor t

$$t = \frac{R_{\text{A-O}}}{\sqrt{2} (R_{\text{B-O}})}$$

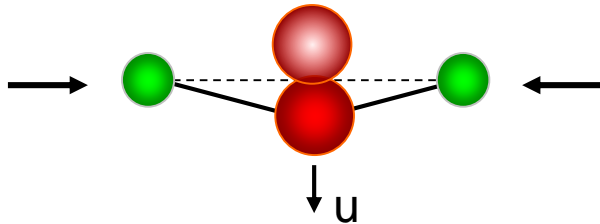
e.g. from sum of radii

Benedek & Fennie, J. Phys. Chem C, **117**, 13339 (2013)



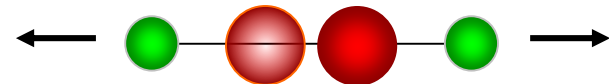
$t < 1$

Cu-O-Cu bond under compression
oxygen **buckle** - affects magnetism



$t > 1$

Cu-O-Cu bond under tension
ferroelectricity



The HTS Cu-O-Cu bond **also buckles** (i.e. under effective compression)