Lensing of CMB: A New Window into Stochastic Gravitational Wave Background

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The Current Landscape

Current Lensed CMB spectra constraints.

- Big Bang Nucleosynthesis
- CMB & Matter power spectrum
- Pulsar timing (current)
- LIGO (2015+)
- aLIGO (2015+)
- LISA (2020+)
- Pulsar timing SKA (2025+)

The Current Landscape

A New Window into the Stochastic Gravitational Wave Background

Upper limits on CMB B-mode polarization translate to interesting limits on the spectral energy density of gravitational waves sourced just after recombination. [Rotti & Souradeep 2012]

The B-mode connection

Measurement of the spectrum of the B-mode of CMB polarisation will provide the best constraints on the SGWB
Low frequency GW sources?

- Inflation
Low frequency GW sources?

- Inflation
- Stochastic Gravitational Wave Background originating from Halo Mergers.

Takahiro Inagaki, Keitaro Takahashi, Naoshi Sugiyama  arXiv:1204.1439v1
Low frequency GW sources?

- Inflation

- Stochastic Gravitational Wave Background originating from Halo Mergers.

Takahiro Inagaki, Keitaro Takahashi, Naoshi Sugiyama  arXiv:1204.1439v1

- Strings !!

http://ned.ipac.caltech.edu/level5/March02/Gangui

- Other unknown sources ??
What are the CMB observables?

\[ C(\hat{n}_1, \hat{n}_2) = \langle \Delta T(\hat{n}_1) \Delta T(\hat{n}_2) \rangle \]

\[ C(\hat{n}_1, \hat{n}_2) = C(\hat{n}_1 \cdot \hat{n}_2) \rightarrow C_i \]
What are the CMB observables?

$$C(\hat{n}_1, \hat{n}_2) = \langle \Delta T(\hat{n}_1)\Delta T(\hat{n}_2) \rangle$$

$$C(\hat{n}_1, \hat{n}_2) = C(\hat{n}_1 \cdot \hat{n}_2) \rightarrow C_l$$
CMB measurements
Weak Lensing
Lensing remaps the CMB anisotropies,

\[ \tilde{T}(\hat{n}) = T(\hat{n} + \tilde{\Delta}) \]

The deflection field can be decomposed into a gradient part and a curl part, analogous to the electromagnetic field,

\[ \tilde{\Delta} = \nabla \psi + \nabla \times \Omega \]
Lensing modifications to the CMB anisotropies can be used to reconstruct this field.

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The spectrum for this field is not predicted as the GW background is not well known.

Lensing modifications to the CMB two point correlation function can be used to reconstruct this field.

Before we try to map this field we need to constrain the amount of power in this field. The current and future CMB observations will allow us to do exactly that!!
Curl component consistent with zero!!
Direction of photon arrival changed

\[(\theta_0, \phi_0) \rightarrow (\theta_0 + \delta\theta, \phi_0 + \delta\phi)\]

Let \(\Delta\) denote the displacement on the sphere.

\[
\Delta_a = - \sum_{lm} (h_{lm}^{\oplus} Y_{lm}^{a} + h_{lm}^{\otimes} Y_{lm}^{b} \epsilon^b_a)
\]

Angular power spectrum of photon displacements

\[
C_l^{h^{\oplus}} = \frac{1}{2l + 1} \sum_{m=-l}^{m=l} h_{lm}^{\oplus} h_{lm}^{\oplus*}
\]

\[
C_l^{h^{\otimes}} = \frac{1}{2l + 1} \sum_{m=-l}^{m=l} h_{lm}^{\otimes} h_{lm}^{\otimes*}
\]

Lensing Modifications to CMB power spectra

Lensing induces power transfer between the two polarization spectra

\[
\begin{pmatrix}
\tilde{C}_l^{TT} \\
\tilde{C}_l^{EE} \\
\tilde{C}_l^{BB} \\
\tilde{C}_l^{TE}
\end{pmatrix} =
\begin{pmatrix}
A_{11} & 0 & 0 & 0 \\
0 & A_{22} & A_{23} & 0 \\
0 & A_{32} & A_{33} & 0 \\
0 & 0 & 0 & A_{44}
\end{pmatrix}
\begin{pmatrix}
C_l^{TT} \\
C_l^{EE} \\
C_l^{BB} \\
C_l^{TE}
\end{pmatrix}
\]

\[
\mathcal{F}(C_l^{\phi\phi}, F^{\oplus}, 2F^{\oplus}, C_l^{\omega\omega}, F^{\otimes}, 2F^{\otimes})
\]

Note:

\[
C_l^{\psi\psi} \equiv C_l^{\phi\phi} \equiv C_l^{\oplus} \quad C_l^{\omega\omega} \equiv C_l^{\otimes}
\]
CONCLUSIONS:

For a primordial background of gravitational waves from inflation with an amplitude corresponding to a tensor-to-scalar ratio below the current upper limit of $r \sim 0.3$, the resulting modifications to the angular power spectra of CMB temperature anisotropy and polarization are below the cosmic variance limit.
The CORRECTED lensing kernels

\[
\begin{align*}
C_l^E & = C_l^E - (l^2 + l - 4) R C_l^E + \frac{1}{2(2l + 1)} \\
& \times \sum_{l_1 l_2} \left[ C_{l_2}^{h\oplus} \left( 2 F_{ll_1 l_2}^{\oplus} \right)^2 + C_{l_2}^{h\otimes} \left( 2 F_{ll_1 l_2}^{\otimes} \right)^2 \right] \left[ (C_{l_1}^E + C_{l_1}^B) + / - (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]
\end{align*}
\]

\[
\begin{align*}
C_l^B & = C_l^B - (l^2 + l - 4) R C_l^B + \frac{1}{2(2l + 1)} \\
& \times \sum_{l_1 l_2} \left[ C_{l_2}^{h\oplus} \left( 2 F_{ll_1 l_2}^{\oplus} \right)^2 + C_{l_2}^{h\otimes} \left( 2 F_{ll_1 l_2}^{\otimes} \right)^2 \right] \left[ (C_{l_1}^E + C_{l_1}^B) - / + (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]
\end{align*}
\]

\[
\begin{align*}
\tilde{C}_l^{TE} & = C_l^{TE} - (l^2 + l - 2) R C_l^{TE} + \frac{1}{2l + 1} \\
& \times \sum_{l_1 l_2} \left[ C_{l_2}^{h\oplus} \left( F_{ll_1 l_2}^{\oplus} \right) (+2 F_{ll_1 l_2}^{\oplus}) - C_{l_2}^{h\otimes} \left( F_{ll_1 l_2}^{\otimes} \right) (+2 F_{ll_1 l_2}^{\otimes}) \right] C_{l_1}^{TE}
\end{align*}
\]
The CORRECTED lensing kernels

\[ C_l^{\tilde{E}} = C_l^{E} - (l^2 + l - 4)RC_l^{E} + \frac{1}{2(2l + 1)} \]
\[ \times \sum_{l_1 l_2} \left[ C_{l_2}^{h \oplus} \left(2 F_{l l_1 l_2}^{\oplus}\right)^2 + C_{l_2}^{h \otimes} \left(2 F_{l l_1 l_2}^{\otimes}\right)^2 \right] \left[ (C_{l_1}^{E} + C_{l_1}^{B}) \mp (-1)^L (C_{l_1}^{E} - C_{l_1}^{B}) \right] \]

\[ C_l^{\tilde{B}} = C_l^{B} - (l^2 + l - 4)RC_l^{B} + \frac{1}{2(2l + 1)} \]
\[ \times \sum_{l_1 l_2} \left[ C_{l_2}^{h \oplus} \left(2 F_{l l_1 l_2}^{\oplus}\right)^2 + C_{l_2}^{h \otimes} \left(2 F_{l l_1 l_2}^{\otimes}\right)^2 \right] \left[ (C_{l_1}^{E} + C_{l_1}^{B}) \pm (-1)^L (C_{l_1}^{E} - C_{l_1}^{B}) \right] \]

\[ \tilde{C}_l^{TE} = C_l^{TE} - (l^2 + l - 2)RC_l^{TE} + \frac{1}{2l + 1} \]
\[ \times \sum_{l_1 l_2} \left[ C_{l_2}^{h \oplus} \left(F_{l l_1 l_2}^{\oplus}\right)(+2 F_{l l_1 l_2}^{\oplus}) - C_{l_2}^{h \otimes} \left(F_{l l_1 l_2}^{\otimes}\right)(+2 F_{l l_1 l_2}^{\otimes}) \right] C_{l_1}^{TE} \]
Lensing generates B-mode power
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\[ C_{\Omega \Omega} = C_{\psi \psi} \]
Lensing generates B-mode power
Why are those the correct kernels?

Independent real space calculations!!

\[ \tilde{\xi}_{XY}(\theta) = \langle X(\hat{n}_1)Y(\hat{n}_2) \rangle \quad X, Y \rightarrow \{T, E, B\} \]

\[ \tilde{\xi}_{XY}(\theta) = \mathcal{F}(C_{i}^{XY}, \langle \alpha_i \alpha_j \rangle) \]

\[ \tilde{\alpha} = \nabla \psi + \nabla \times \Omega \]

\[ \tilde{C}_{i}^{XY} \]

\[ \mathcal{G}(A_0, A_2) \]

Only difference!!

\[ A_0(r) = \int_0^\infty \frac{dl}{2\pi} \frac{l^3}{2} \left[ C_{i}^{\psi\psi} + C_{i}^{\Omega\Omega} \right] J_0(rl) \]

\[ A_2(r) = -\int_0^\infty \frac{dl}{2\pi} l^3 C_{i}^{\Omega\Omega} J_2(rl) \]

\[ A_2(r) = \int_0^\infty \frac{dl}{2\pi} l^3 C_{i}^{\psi\psi} J_2(rl) \]

CAMB evaluates lensed spectra using this method.

Curl type displacements; induced by GW

Gradient type displacements; induced by GW

A. Challinor and A. Lewis, PRD 71 (2005) 103010

GW Lensing > LSS Lensing!!
GW Lensing  >  LSS Lensing !!

\[ \frac{C_{\ell}^{BB}}{C_{\ell}^{\text{LSS}}} = C_{\ell}^{\psi \psi} \]
GW also induce a rotation of the polarization of photons in addition to deflecting them.

\[ \ell(\ell + 1)C^B_\ell/(2\pi)^2 \left(\mu K^2\right) \]

Expected BB signal from lensing of CMB.

- Current lensing considerations are only due to scalar perturbations (LSS).
- The matter power spectrum is already well constrained.
  - A huge difference between current upper limits on the BB spectra and the expected signal (as of 2011 when this work was done)

QUAD arXiv:0906.1003v3

South Pole Telescope

BICEP2
Constraining procedure

- Choose a form for the GW power spectra.
- Divide the power spectra in bins and evaluate the deflection spectra.
- Use the deflection spectra to evaluate the lensed CMB spectra.
- Keep the cosmological parameters fixed and vary the amount of power in each bin till the CMB spectra are consistent with current data.
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Constraints for GW sourced at different $z$
Constraints for GW sourced at different \( z \)

![Graph showing constraints for GW sourced at different \( z \).](image-url)
Constraints for GW sourced at different $z$
CMB spectra lensed by GW

$z=1$
The lensing kernels for scalar and tensor lensing have been shown to have a very similar form.

The correct lensing kernel has been derived for lensing due to GW.

Tensors are seen to be equally efficient at transferring power between E-mode and B-mode of CMB polarization.

This probe provides a new window into Gravitational Waves which has not been previously explored.

If the current measurements of B-modes are to be believed, then we have already saturated the best constraints!!
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GW Power Spectra \quad \leftrightarrow \quad GW Energy Density

\[ \Omega_{GW} = \frac{\rho_{GW}}{\rho_0} \]

\[ \rho_{GW} = \frac{c^2}{32\pi G} \langle \dot{h}_{ij} \dot{h}^{ij} \rangle \]

\[ \rho_0 = \frac{3c^2 H_0^2}{8\pi G} \]

Critical energy density of the Universe

\[ \Omega_{GW}(k) = \frac{4\pi}{3} \left( \frac{c}{H_0} \right)^2 k^3 P_T(k)[k\mathcal{T}']^2 \]

\[ \Omega_{GW} \bigg|_{bin} = \int_{bin} d\ln k \: \Omega_{GW}(k) \]
Weak Lensing.

Lensing Remaps the CMB anisotropies

\[ \tilde{T}(\hat{n}) = T(\hat{n} + \Delta) \]

Deflection field on the sphere,

\[ \Delta = \nabla \psi + \nabla \times \Omega \]

Projected Lensing potential:

\[ \psi(\hat{n}) = -2 \int_{0}^{\eta_0} \, d\eta \frac{\eta_0 - \eta}{\eta_0 \eta} \Psi(\hat{n}, \eta_0 - \eta) \]
Weak Lensing.

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\[ \psi(\hat{n}) = -2 \int_0^{\eta_0} d\eta \frac{\eta_0 - \eta}{\eta_0 \eta} \Psi(\hat{n}, \eta_0 - \eta) \]
\[
\tilde{\mathcal{C}}^{TT}_l = C^{TT}_l - l(l+1)RC^{TT}_l + \sum_{l_1 l_2} \frac{C^{TT}_{l_1}}{2l+1} \left[ C^{\oplus}_{l_2} (F^{\oplus}_{ll_1 l_2})^2 + C^{\otimes}_{l_2} (F^{\otimes}_{ll_1 l_2})^2 \right]
\]
\[
\tilde{C}_{l}^{TT} = C_{l}^{TT} - l(l + 1) R C_{l}^{TT} + \sum_{l_{1}l_{2}} \frac{C_{l_{1}}^{TT}}{2l + 1} \left[ C_{l_{2}}^{\oplus} (F_{l_{1}l_{2}}^{\oplus})^2 + C_{l_{2}}^{\otimes} (F_{l_{1}l_{2}}^{\otimes})^2 \right]
\]

\[
|F_{l_{1}l_{2}}^{\oplus}| = \frac{1}{2} [l_{1}(l_{1} + 1) + l_{2}(l_{2} + 1) - l(l + 1)] \sqrt{\frac{(2l + 1)(2l_{1} + 1)(2l_{2} + 1)}{4\pi}} \begin{pmatrix} l & l_{1} & l_{2} \\ 0 & 0 & 0 \end{pmatrix}
\]

\[
|F_{l_{1}l_{2}}^{\otimes}| = \frac{1}{2} \sqrt{l_{1}(l_{1} + 1)l_{2}(l_{2} + 1)} \sqrt{\frac{(2l + 1)(2l_{1} + 1)(2l_{2} + 1)}{4\pi}} \begin{pmatrix} l & l_{1} & l_{2} \\ 0 & -1 & 1 \end{pmatrix} [1 - (-1)^{l+l_{1}+l_{2}}]
\]

\[
F_{l_{1}l_{2}}^{\oplus/\otimes} = \frac{1}{2} \sqrt{l_{1}(l_{1} + 1)l_{2}(l_{2} + 1)} \sqrt{\frac{(2l + 1)(2l_{1} + 1)(2l_{2} + 1)}{4\pi}} \begin{pmatrix} l & l_{1} & l_{2} \\ 0 & -1 & 1 \end{pmatrix} [1+/-(-1)^{l+l_{1}+l_{2}}]
\]
Likewise for lensing of polarization power spectra

\[
|_{2}F_{l_{1}l_{2}}^{\oplus} = \frac{1}{2}[l_{1}(l_{1} + 1) + l_{2}(l_{2} + 1) - l(l + 1)]\sqrt{\frac{(2l + 1)(2l_{1} + 1)(2l_{2} + 1)}{4\pi}} \begin{pmatrix} l & l_{1} & l_{2} \\ 2 & -2 & 0 \end{pmatrix}
\]

\[
|_{2}F_{l_{1}l_{2}}^{\otimes} = \sqrt{\frac{l_{2}(l_{2} + 1)(2l + 1)(2l_{1} + 1)(2l_{2} + 1)}{8\pi}} \\
\times \left( \sqrt{\frac{(l_{1} + 1)(l_{1} - 1)}{2}} \begin{pmatrix} l & l_{1} & l_{2} \\ 2 & -1 & -1 \end{pmatrix} - \sqrt{\frac{(l_{1} - 2)(l_{1} + 3)}{2}} \begin{pmatrix} l & l_{1} & l_{2} \\ 2 & -3 & 1 \end{pmatrix} \right)
\]
Similarity of lensing kernels!

Likewise for lensing of polarization power spectra

\[ |2F_{\ell_1 \ell_2}^\oplus| = \frac{1}{2} [l_1(l_1 + 1) + l_2(l_2 + 1) - l(l + 1)] \sqrt{\frac{(2l + 1)(2l_1 + 1)(2l_2 + 1)}{4\pi}} \left( \begin{array}{ccc} l & l_1 & l_2 \\ 2 & -2 & 0 \end{array} \right) \]

\[ |2F_{\ell_1 \ell_2}^\otimes| = \frac{\sqrt{l_2(l_2 + 1)(2l + 1)(2l_1 + 1)(2l_2 + 1)}}{8\pi} \times \left( \sqrt{\frac{(l_1 + 2)(l_1 - 1)}{2}} \left( \begin{array}{ccc} l & l_1 & l_2 \\ 2 & -1 & -1 \end{array} \right) - \sqrt{\frac{(l_1 - 2)(l_1 + 3)}{2}} \left( \begin{array}{ccc} l & l_1 & l_2 \\ 2 & -3 & 1 \end{array} \right) \right) \]

\[ |2F_{\ell_1 \ell_2}^{\oplus/\otimes}| = \frac{\sqrt{l_2(l_2 + 1)(2l + 1)(2l_1 + 1)(2l_2 + 1)}}{8\pi} \times \left( \sqrt{\frac{(l_1 + 2)(l_1 - 1)}{2}} \left( \begin{array}{ccc} l & l_1 & l_2 \\ 2 & -1 & -1 \end{array} \right) +/\!- \sqrt{\frac{(l_1 - 2)(l_1 + 3)}{2}} \left( \begin{array}{ccc} l & l_1 & l_2 \\ 2 & -3 & 1 \end{array} \right) \right) \]

Gravitational waves also induce gradient type displacements, but we ignore them in this study.

Choosing the form of the tensor power spectrum
Choosing the form of the tensor power spectrum

\[ P(k) \sim k^n \rightarrow C_l^{\otimes} = Al^{n-3} \]
Choosing the form of the tensor power spectrum