Lensing of CMB : A New Window into Stochastic Gravitational Wave Background

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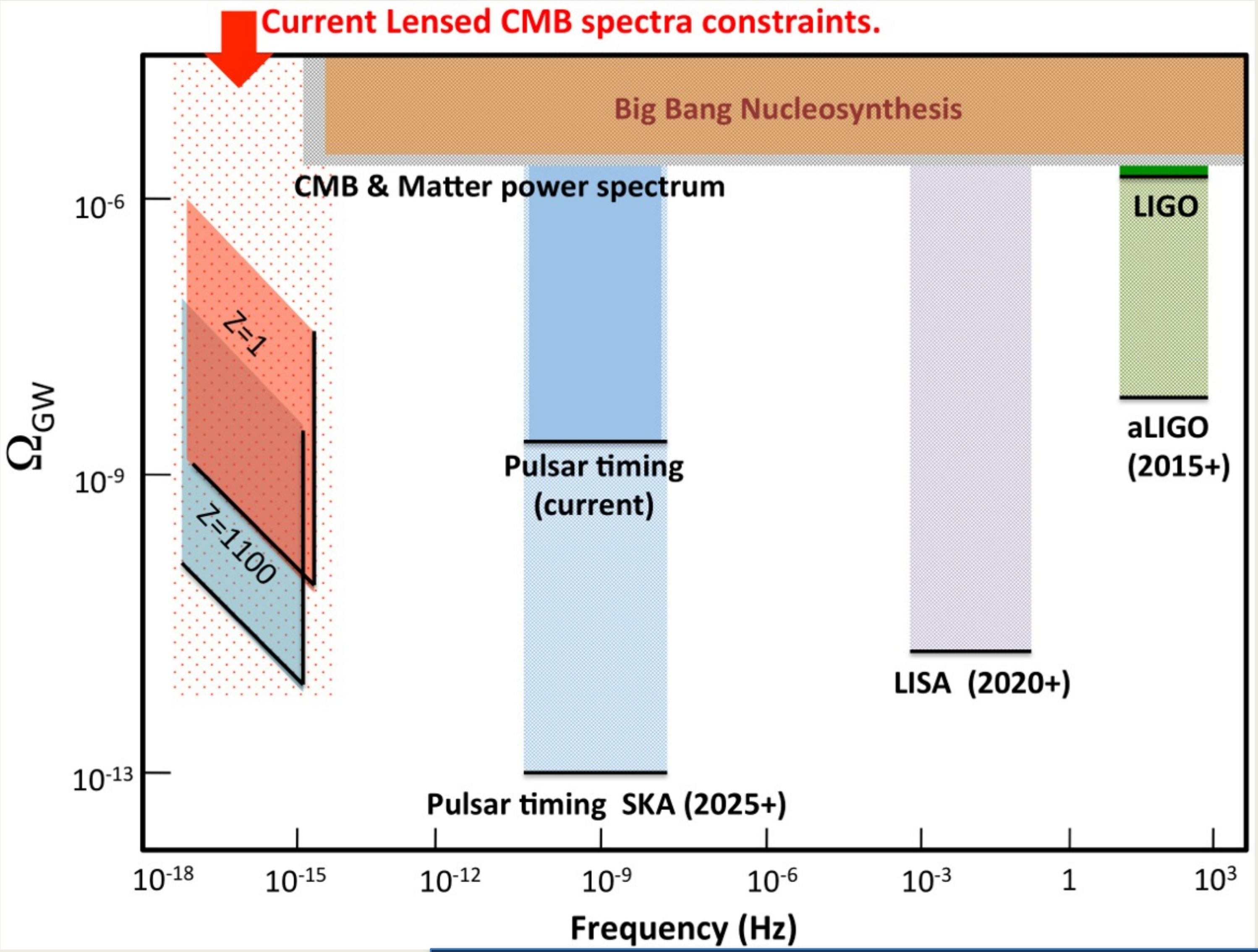
Inter University Centre for Astronomy & Astrophysics (IUCAA), Pune, India.

Cosmology Day
@
Internation Centre for Theoretical Sciences
8th April 2014

The Current Landscape



A. Rotti & T. Souradeep Phys. Rev. Lett. 109, 221301 (2012)

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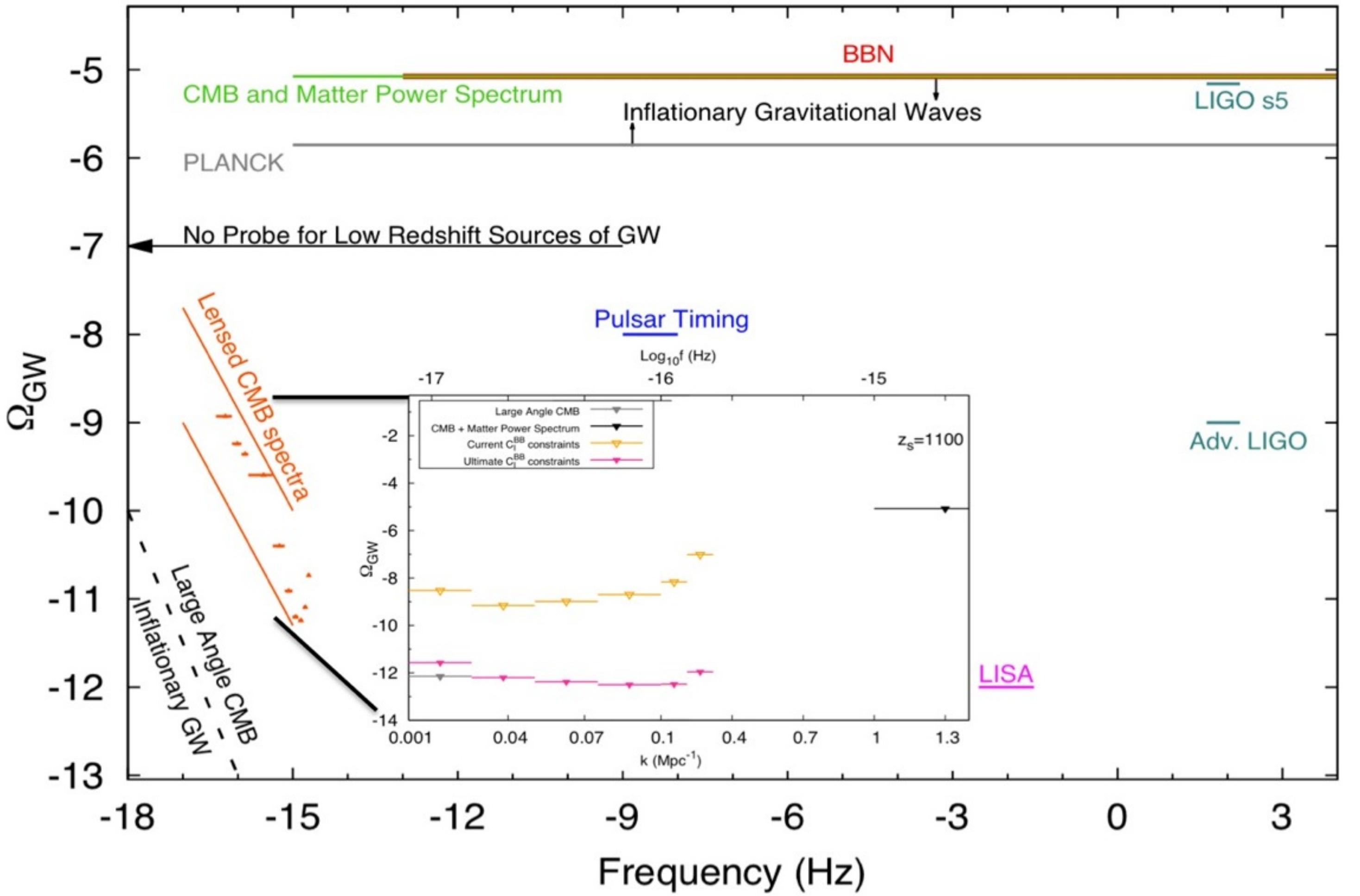
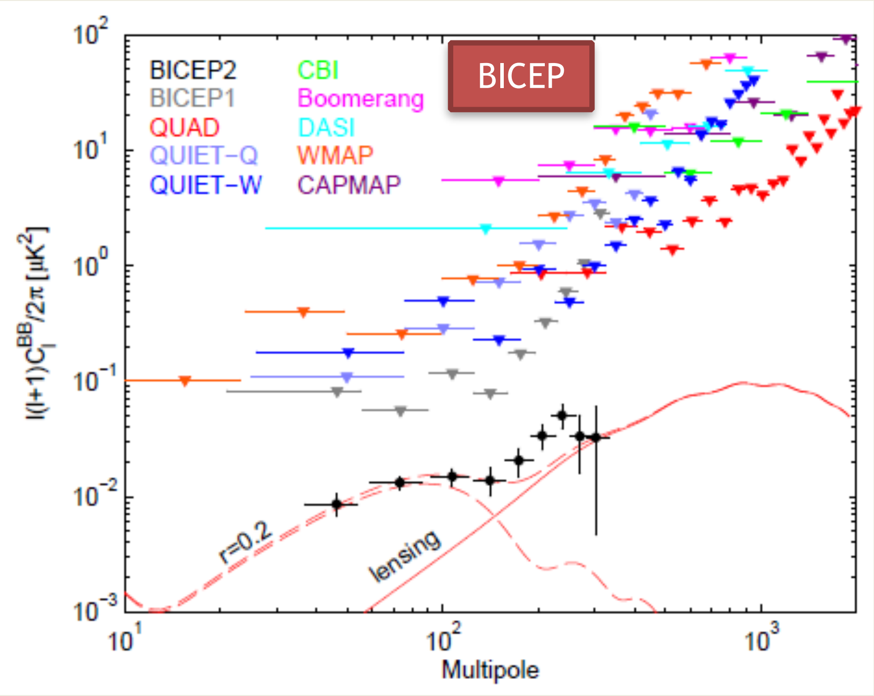
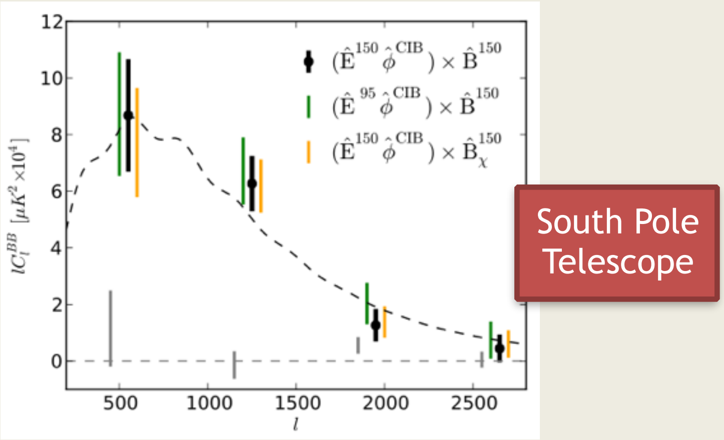


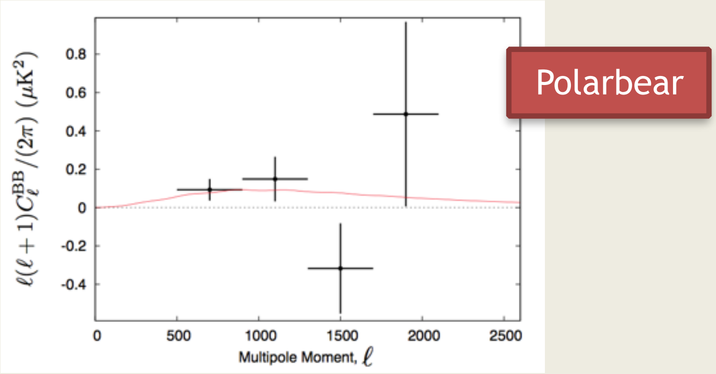
Upper limits on CMB B-mode polarization translate to interesting limits on the spectral energy density of gravitational waves sourced just after recombination. [Rotti & Souradeep 2012]

A. Rotti & T. Souradeep Phys. Rev. Lett. 109, 221301 (2012)

The B-mode connection







Measurement of the spectrum of the B-mode of CMB polarisation will provide the best constraints on the SGWB

Low frequency GW sources ?

- Inflation

Low frequency GW sources ?

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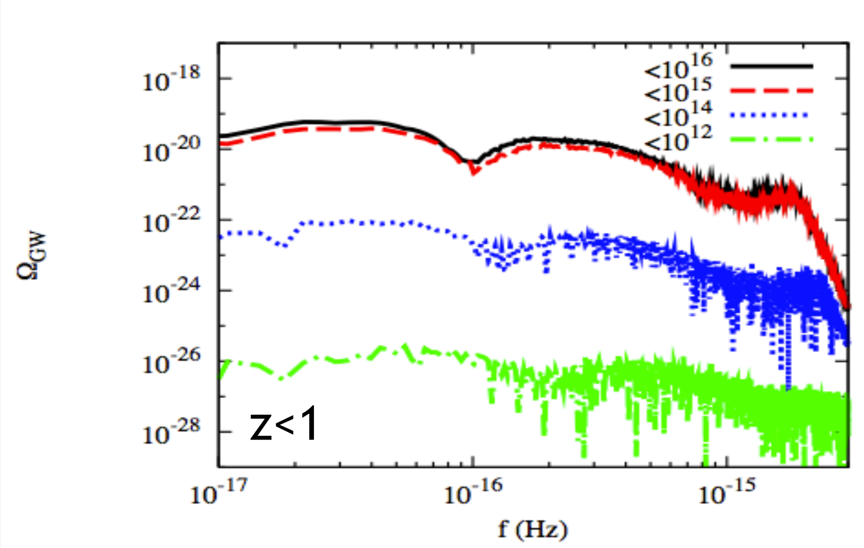
- Stochastic Gravitational Wave Background originating from Halo Mergers.

Takahiro Inagaki, Keitaro Takahashi, Naoshi Sugiyama arXiv:1204.1439v1



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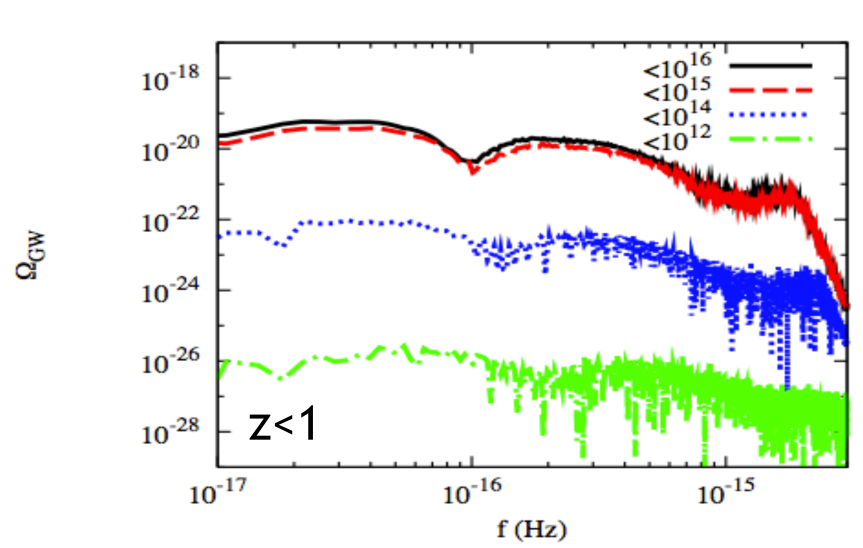
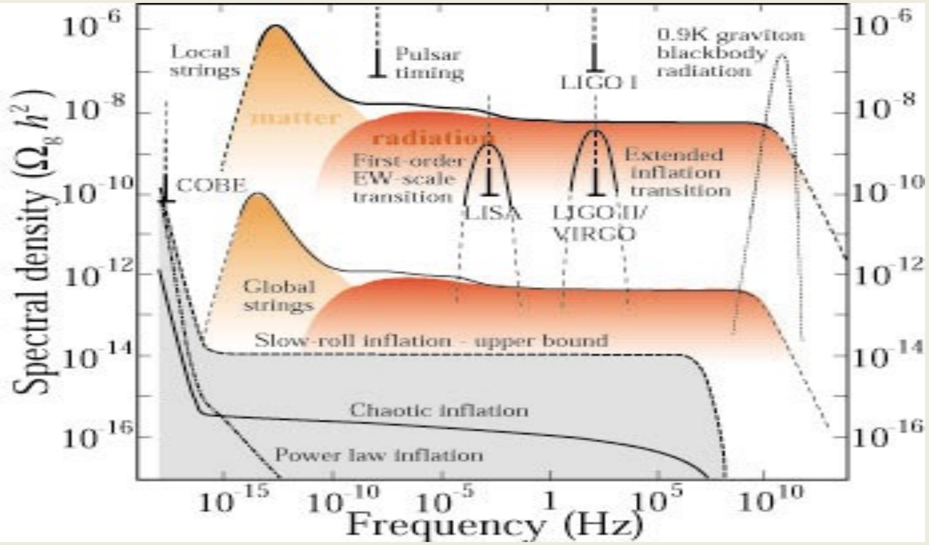


- Strings !!

http://ned.ipac.caltech.edu/level5/March02/Gangui

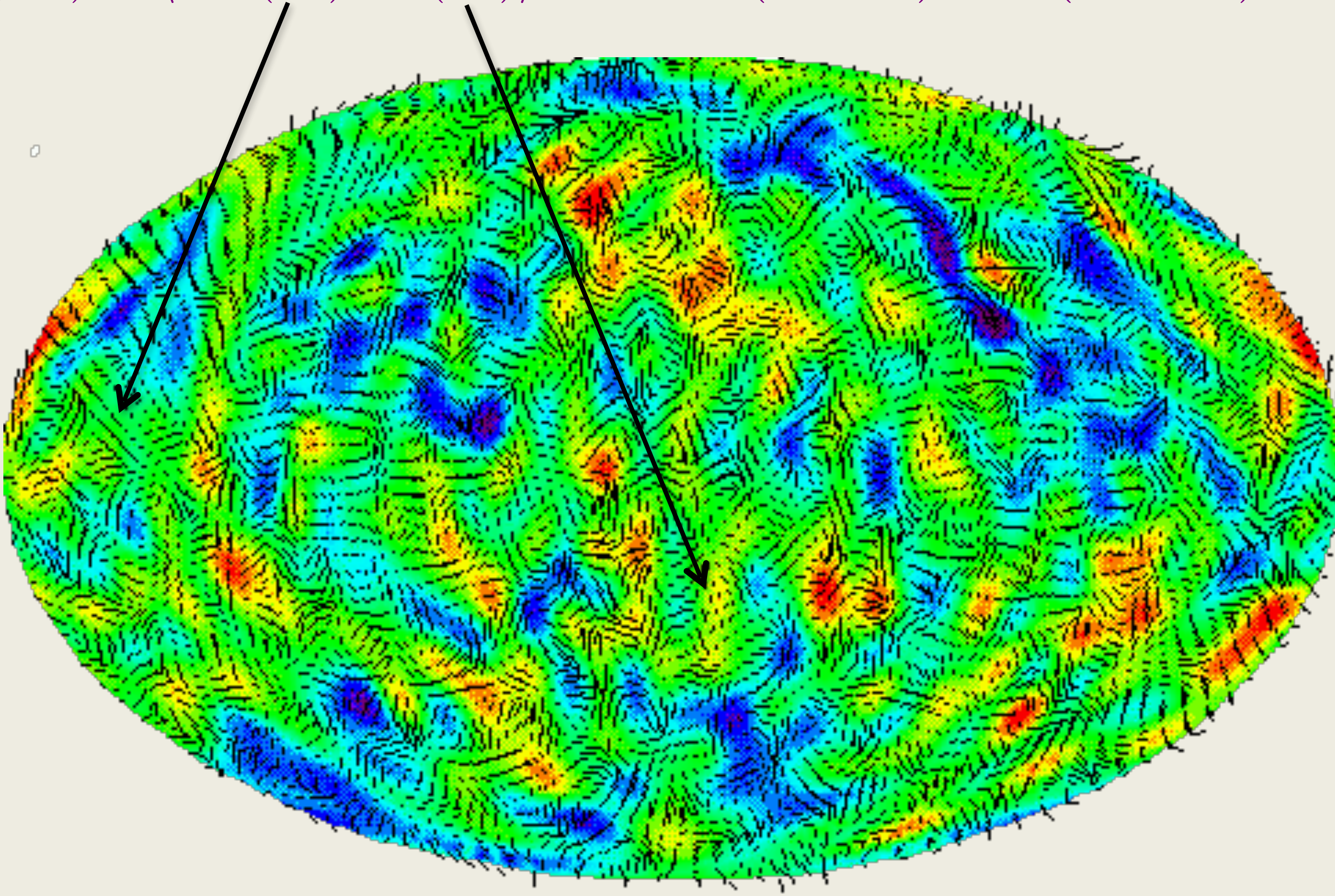


- Other unknown sources ??

What are the CMB observables ?

$$C(\hat{n}_1, \hat{n}_2) = \langle \Delta T(\hat{n}_1) \Delta T(\hat{n}_2) \rangle$$

$$C(\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2) = C(\hat{\mathbf{n}}_1 \cdot \hat{\mathbf{n}}_2) \longrightarrow C_l$$



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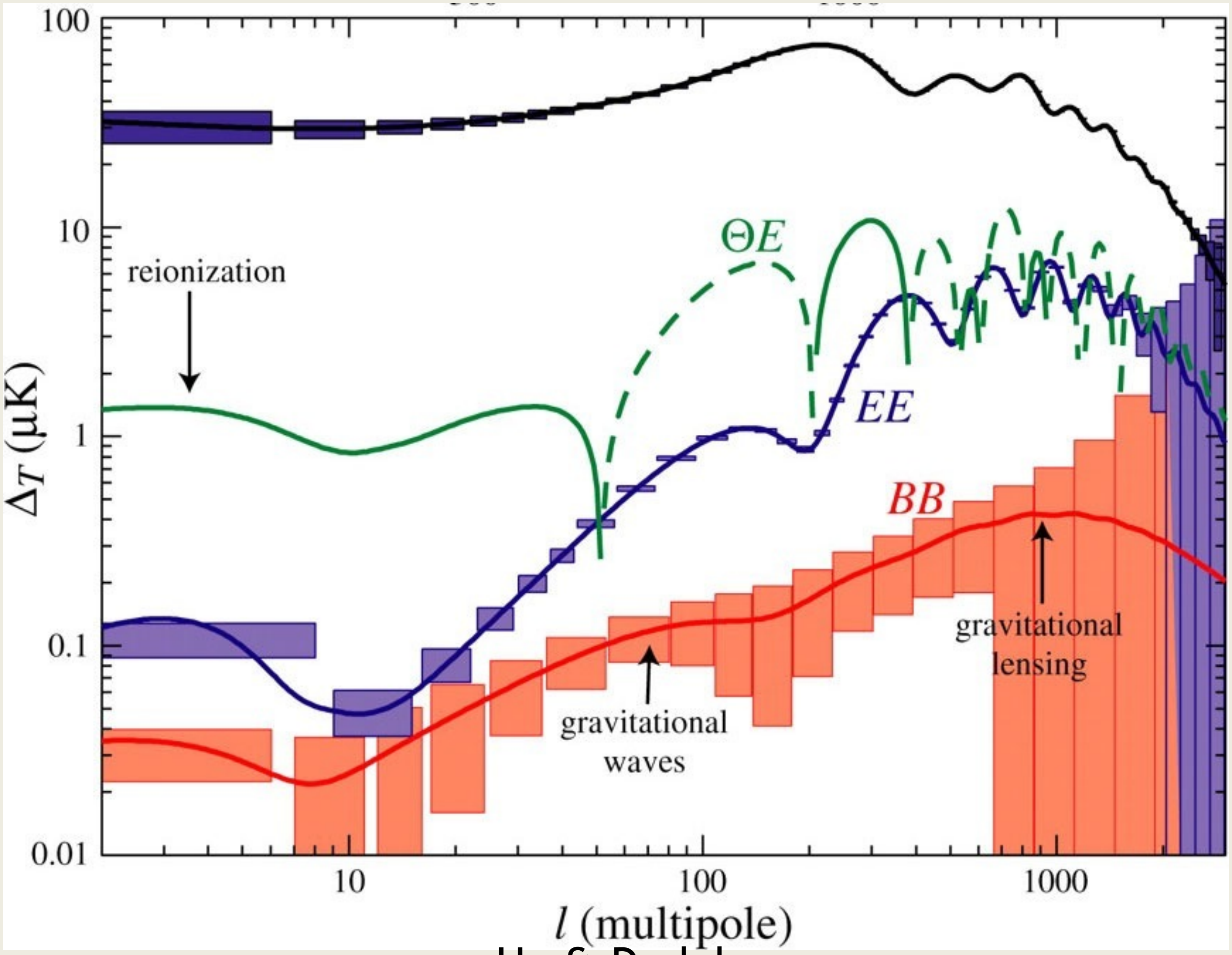
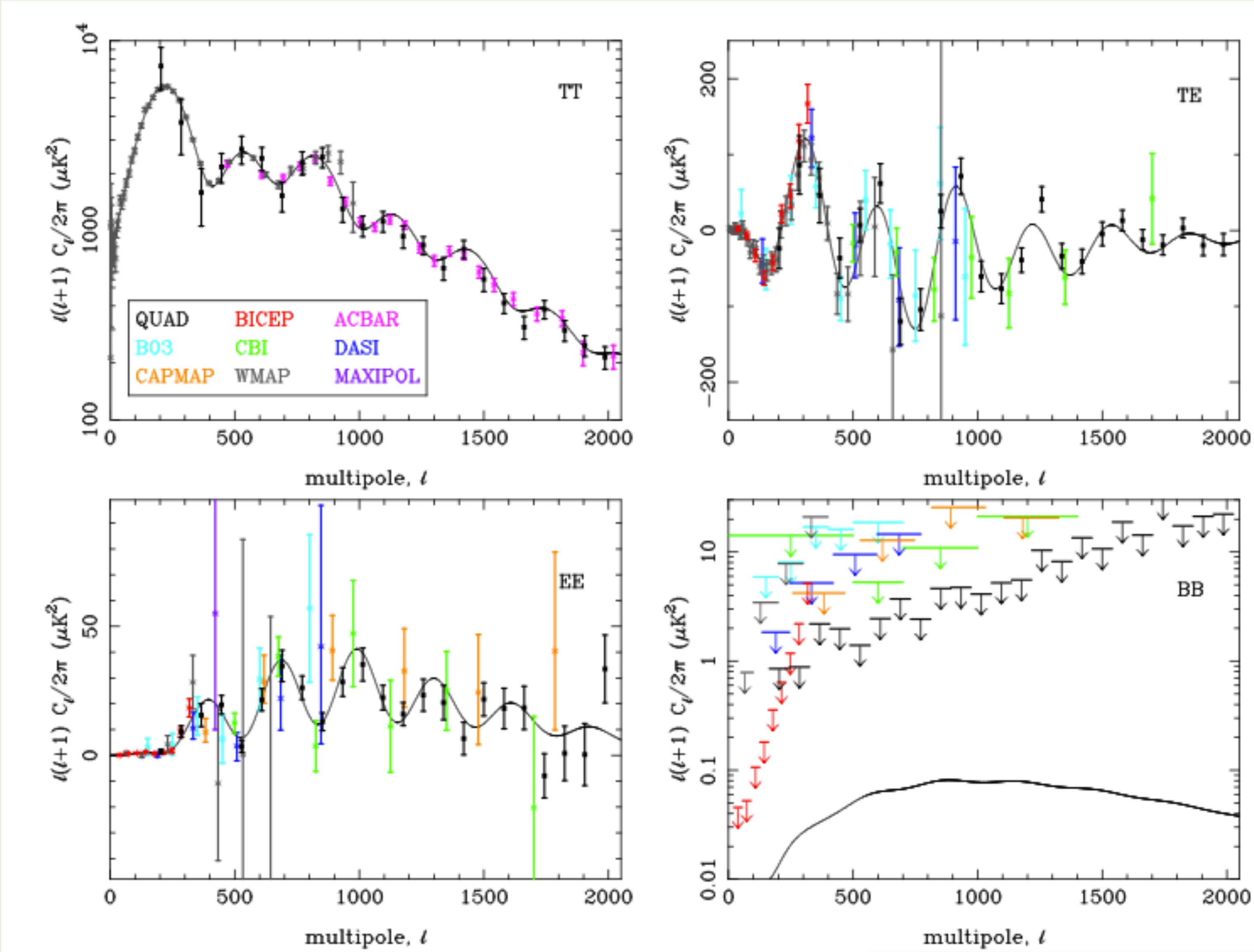


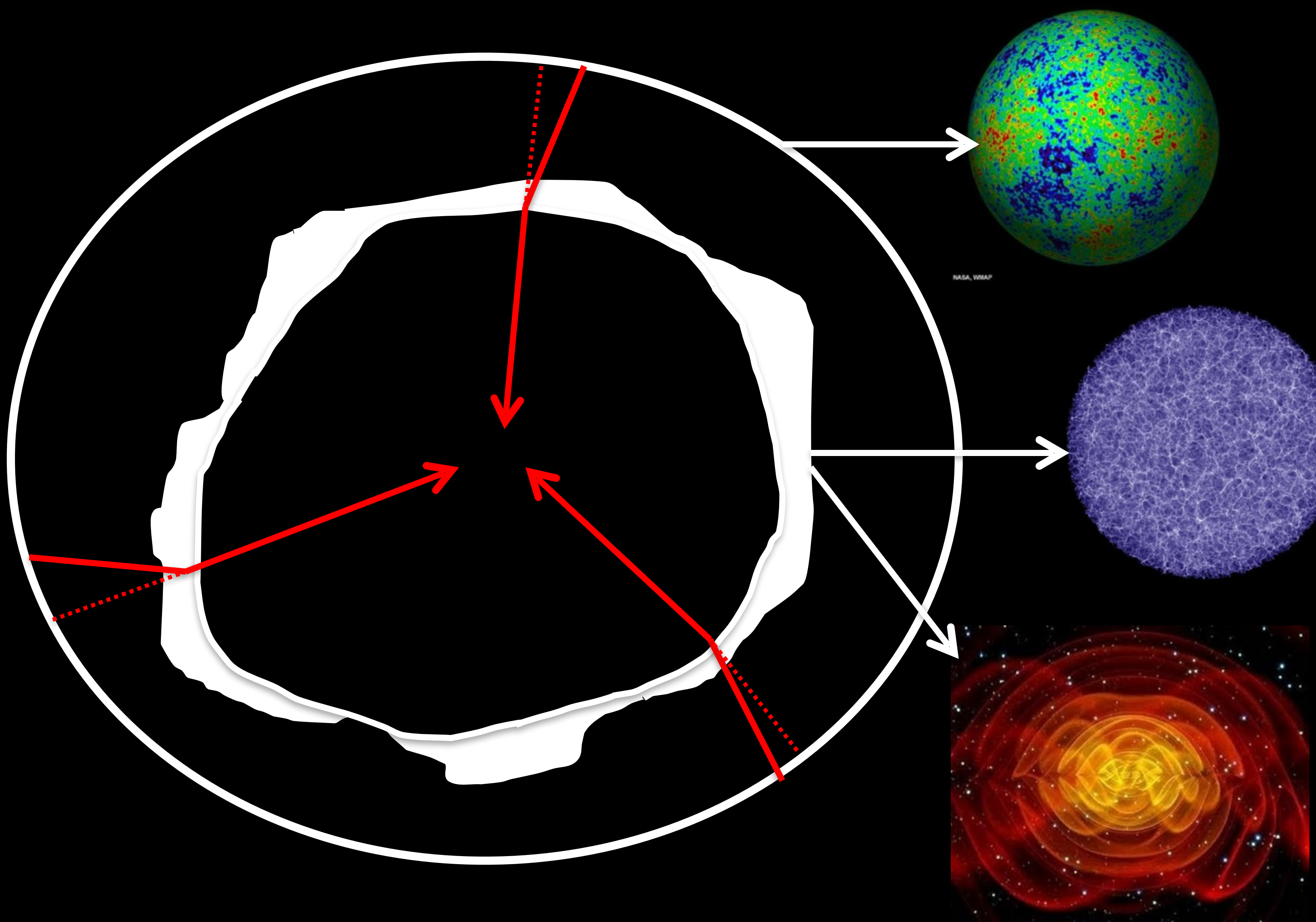
Hu & Dodelson

CMB measurements



QUAD (2009) arXiv:0906.1003v3

Weak Lensing



Weak Lensing

Lensing remaps the CMB anisotropies,

$$\tilde{T}(\hat{n}) = T(\hat{n} + \vec{\Delta})$$

The deflection field can be decomposed into a gradient part and a curl part, analogous to the electromagnetic field,

$$\vec{\Delta} = \nabla\psi + \nabla \times \Omega$$

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Lensing modifications to the CMB two point correlation function can be used to reconstruct this field.

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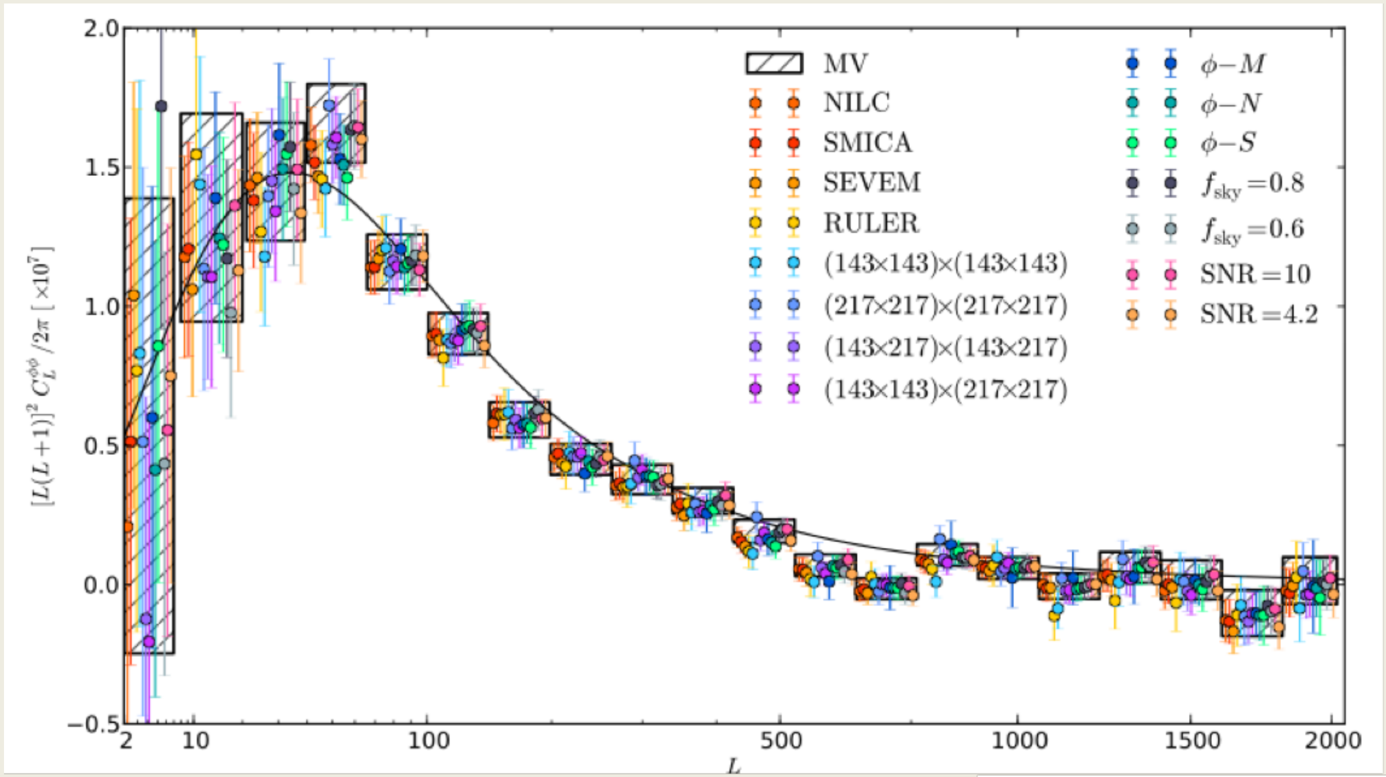
The spectrum for this field is not predicted as the GW background is not well known.

Lensing modifications to the CMB two point correlation function can be used to reconstruct this field.

Before we try to map this field we need to constrain the amount of power in this field. The current and future CMB observations will allow us to do exactly that !!

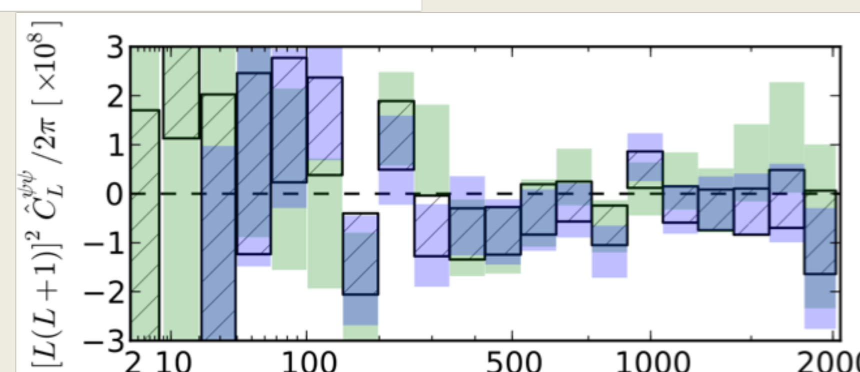
Curl component consistent with zero !!

PLANCK XVII



$$\nabla \phi$$

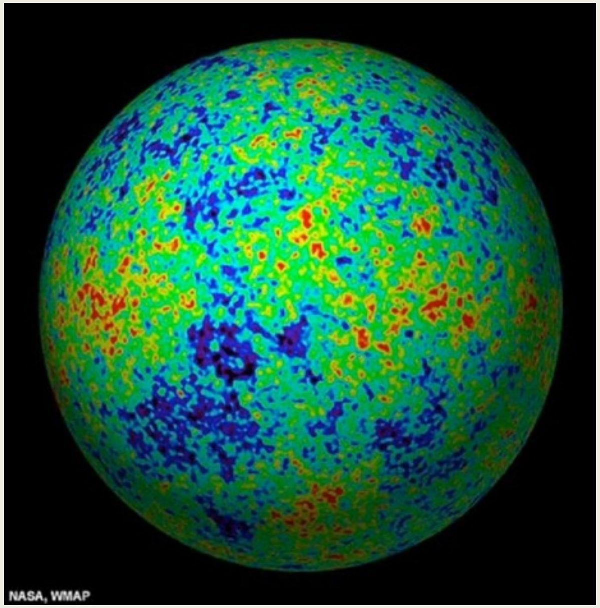
$$\nabla \times \Omega$$



Lensing (Harmonic space)

Direction of photon arrival changed

$$(\theta_{\mathbf{o}}, \phi_{\mathbf{o}}) \rightarrow (\theta_{\mathbf{o}} + \delta\theta, \phi_{\mathbf{o}} + \delta\phi)$$



Let $\triangle$ denote the displacement on the sphere.

$$\mathbf{\Delta_a} = -\sum_{\mathbf{lm}} (\mathbf{h^{\oplus}_{lm} Y_{lm:a}} + \mathbf{h^{\otimes}_{lm} Y_{lm:b} \epsilon^{b}{}_{a}})$$

Angular power spectrum of photon displacements

$$C_l^{h^{\oplus}} = \frac{1}{2l+1} \sum_{m=-l}^{m=l} h^{\oplus}_{lm} h^{\oplus *}_{lm}$$

Gradient Spectrum

$$C_l^{h^{\otimes}} = \frac{1}{2l+1} \sum_{m=-l}^{m=l} h^{\otimes}_{lm} h^{\otimes *}_{lm}$$

Curl Spectrum

A. Stebbins, arXiv:astro-ph/p609149

Lensing Modifications to CMB power spectra

Lensing induces power transfer between the two polarization spectra

$$\begin{pmatrix} \tilde{C}_l^{TT} \\ \tilde{C}_l^{EE} \\ \tilde{C}_l^{BB} \\ \tilde{C}_l^{TE} \end{pmatrix} = \underbrace{\begin{pmatrix} A_{11} & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & 0 \\ 0 & A_{32} & A_{33} & 0 \\ 0 & 0 & 0 & A_{44} \end{pmatrix}}_{\mathcal{F}(C_l^{\phi\phi}, F^{\oplus}, {}_2F^{\oplus}, C_l^{\Omega\Omega}, F^{\otimes}, {}_2F^{\otimes})} \begin{pmatrix} C_l^{TT} \\ C_l^{EE} \\ C_l^{BB} \\ C_l^{TE} \end{pmatrix}$$

Note :

$$C_l^{\psi\psi} \equiv C_l^{\phi\phi} \equiv C_l^{\oplus} \qquad C_l^{\Omega\Omega} \equiv C_l^{\otimes}$$

Lensing by Inflationary GW

$$C_l^{\tilde{E}} = C_l^E - (l^2 + l - 4) R C_l^E + \frac{1}{2(2l+1)} \times \sum_{l_1 l_2} \left[C_{l_2}^{h^\oplus} ({}_2F_{ll_1l_2}^{\oplus})^2 + C_{l_2}^{h^\otimes} ({}_2F_{ll_1l_2}^{\otimes})^2 \right] \left[(C_{l_1}^E + C_{l_1}^B) + (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]$$

$$C_l^{\tilde{B}} = C_l^B - (l^2 + l - 4) R C_l^B + \frac{1}{2(2l+1)} \times \sum_{l_1 l_2} \left[C_{l_1}^{h^\oplus} ({}_2F_{ll_1l_2}^{\oplus})^2 + C_{l_1}^{h^\otimes} ({}_2F_{ll_1l_2}^{\otimes})^2 \right] \left[(C_{l_2}^E + C_{l_2}^B) - (-1)^L (C_{l_2}^E - C_{l_2}^B) \right]$$

$$C_l^{\tilde{T}\tilde{E}} = C_l^{TE} - (l^2 + l - 2) R C_l^{TE} + \frac{1}{2l+1} \times \sum_{l_1 l_2} \left[C_{l_1}^{h^\oplus} (F_{ll_1l_2}^{\oplus})({}_{+2}F_{ll_1l_2}^{\oplus}) + C_{l_1}^{h^\otimes} (F_{ll_1l_2}^{\otimes})({}_{+2}F_{ll_1l_2}^{\otimes}) \right] C_{l_2}^{\theta E}$$

CONCLUSIONS :

For a primordial background of **gravitational waves from inflation** with an amplitude corresponding to **a tensor-to-scalar ratio below the current upper limit of r~0.3**, the resulting modifications to the angular power spectra of CMB temperature anisotropy and polarization are **below the cosmic variance limit.**

C. Li and A. Cooray, PRD 74, 023521(2006)

The CORRECTED lensing kernels

$$C_l^{\tilde{E}} = C_l^E - (l^2 + l - 4) R C_l^E + \frac{1}{2(2l+1)} \times \sum_{l_1 l_2} \left[C_{l_2}^{h^\oplus} ({}_2F^{\oplus}_{l l_1 l_2})^2 + C_{l_2}^{h^\otimes} ({}_2F^{\otimes}_{l l_1 l_2})^2 \right] \left[(C_{l_1}^E + C_{l_1}^B) +/- (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]$$

$$C_l^{\tilde{B}} = C_l^B - (l^2 + l - 4) R C_l^B + \frac{1}{2(2l+1)} \times \sum_{l_1 l_2} \left[C_{l_2}^{h^\oplus} ({}_2F^{\oplus}_{l l_1 l_2})^2 + C_{l_2}^{h^\otimes} ({}_2F^{\otimes}_{l l_1 l_2})^2 \right] \left[(C_{l_1}^E + C_{l_1}^B) -/+ (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]$$

$$\tilde{C}_l^{TE} = C_l^{TE} - (l^2 + l - 2) R C_l^{TE} + \frac{1}{2l+1} \times \sum_{l_1 l_2} \left[C_{l_2}^{h^\oplus} (F^{\oplus}_{l l_1 l_2})({}_{+2}F^{\oplus}_{l l_1 l_2}) - C_{l_2}^{h^\otimes} (F^{\otimes}_{l l_1 l_2})({}_{+2}F^{\otimes}_{l l_1 l_2}) \right] C_{l_1}^{TE}$$

H. Padmanabhan, A. Rotti and T. Souradeep, Phys. Rev. D 88, 063507 (2013)

The CORRECTED lensing kernels

$$C_l^{\tilde{E}} = C_l^E - (l^2 + l - 4)RC_l^E + \frac{1}{2(2l+1)} \times \sum_{l_1 l_2} \left[C_{l_2}^{h^\oplus} ({}_2F^{\oplus}_{l l_1 l_2})^2 + C_{l_2}^{h^\otimes} ({}_2F^{\otimes}_{l l_1 l_2})^2 \right] \left[(C_{l_1}^E + C_{l_1}^B) +/- (-1)^L (C_{l_1}^E - C_{l_1}^B) \right]$$

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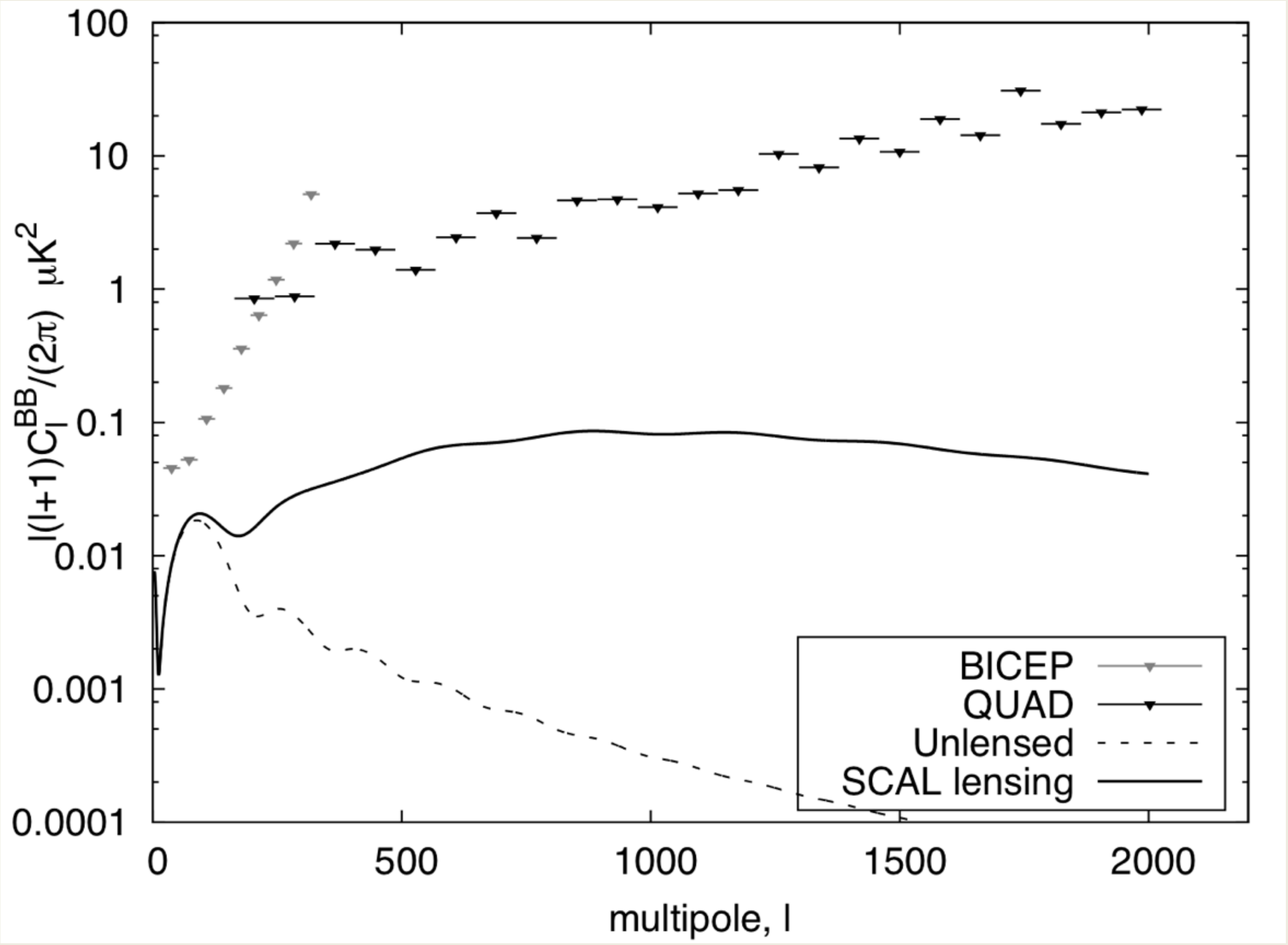
Lensing generates B-mode power



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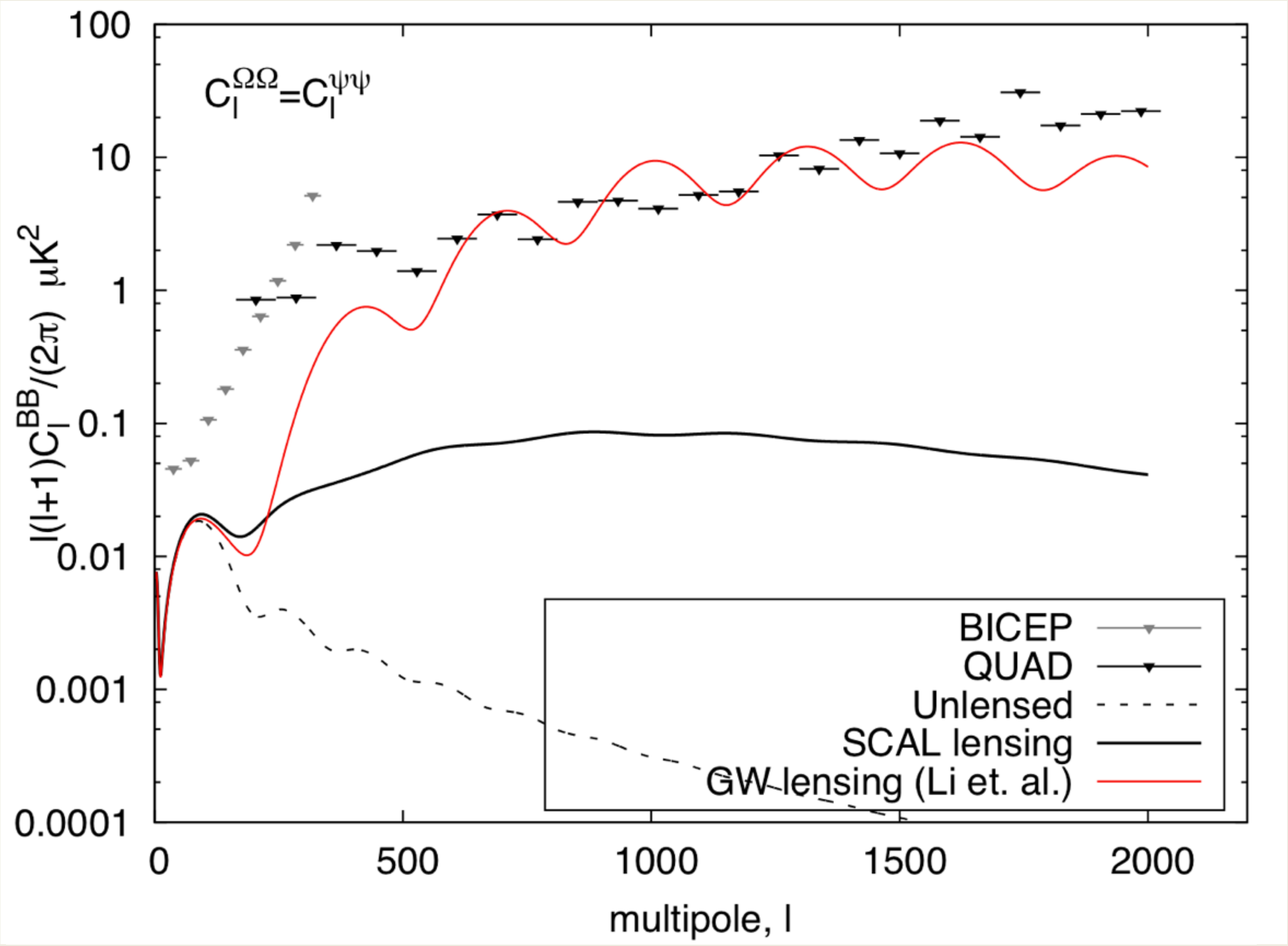

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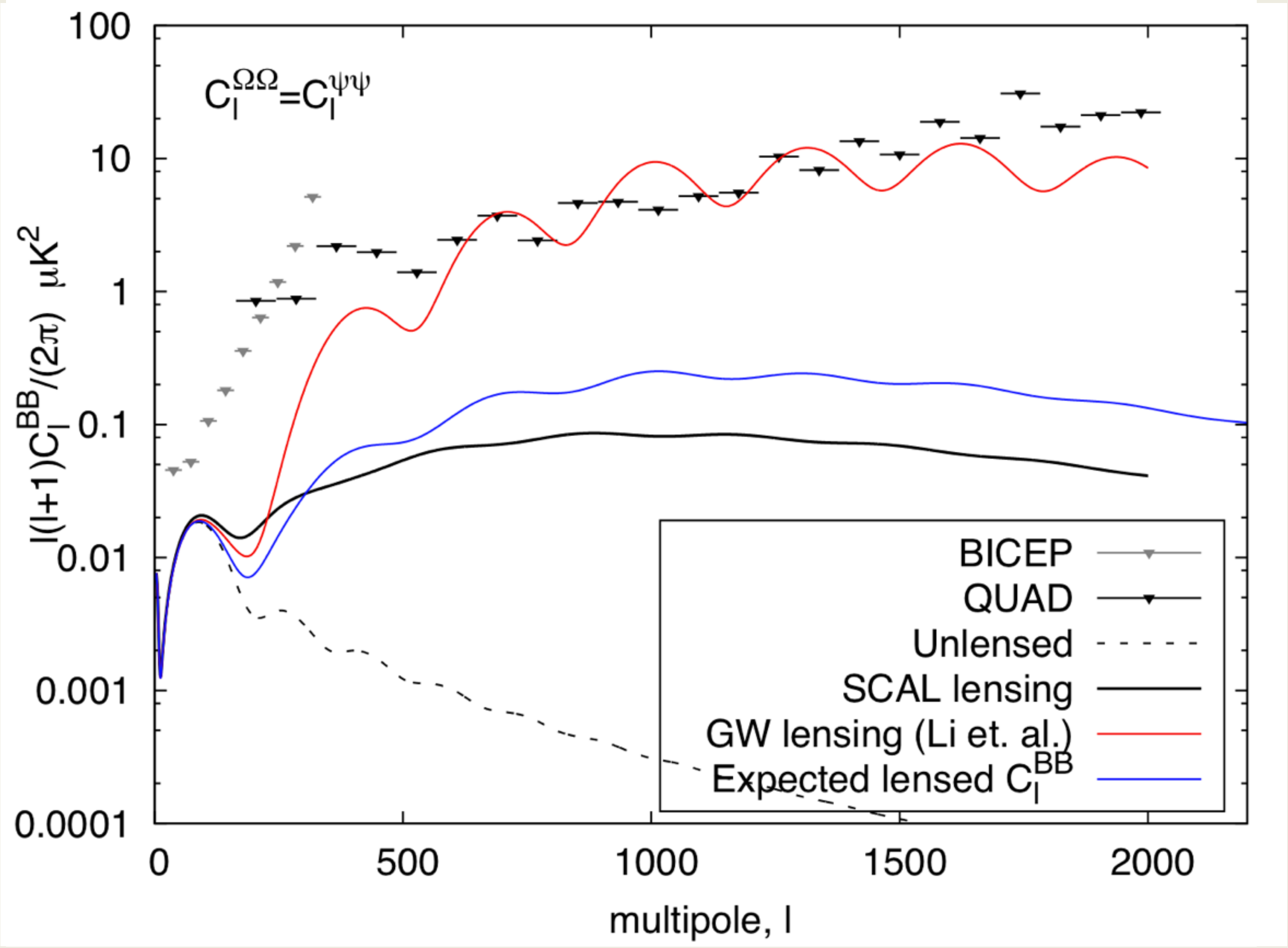


Why are those the correct kernels ?

Independent real space calculations !!

A. Challinor and A. Lewis, PRD 71 (2005) 103010

$$\tilde{\xi}_{XY}(\theta) = \langle X(\hat{n}_1) Y(\hat{n}_2) \rangle \qquad X, Y \to \{T, E, B\}$$

$$\tilde{\xi}_{XY}(\theta) = \mathcal{F}(C_l^{XY}, \langle \alpha_i \alpha_j \rangle) \qquad \vec{\alpha} = \nabla\psi + \nabla \times \Omega$$

$$\tilde{\xi}_{XY}(\theta) \to \tilde{C}_l^{XY}$$

$$\mathcal{G}(A_0, A_2) \to \langle \alpha_i \alpha_j \rangle$$

$$A_0(r) = \int_0^\infty \frac{dl\; l^3}{2\pi} \left[C_l^{\psi\psi} + C_l^{\Omega\Omega} \right] J_0(rl)$$

Only difference !!

$$A_2(r) = -\int_0^\infty \frac{dl}{2\pi} l^3 C_l^{\Omega\Omega} J_2(rl)$$

Curl type displacements ; induced by GW

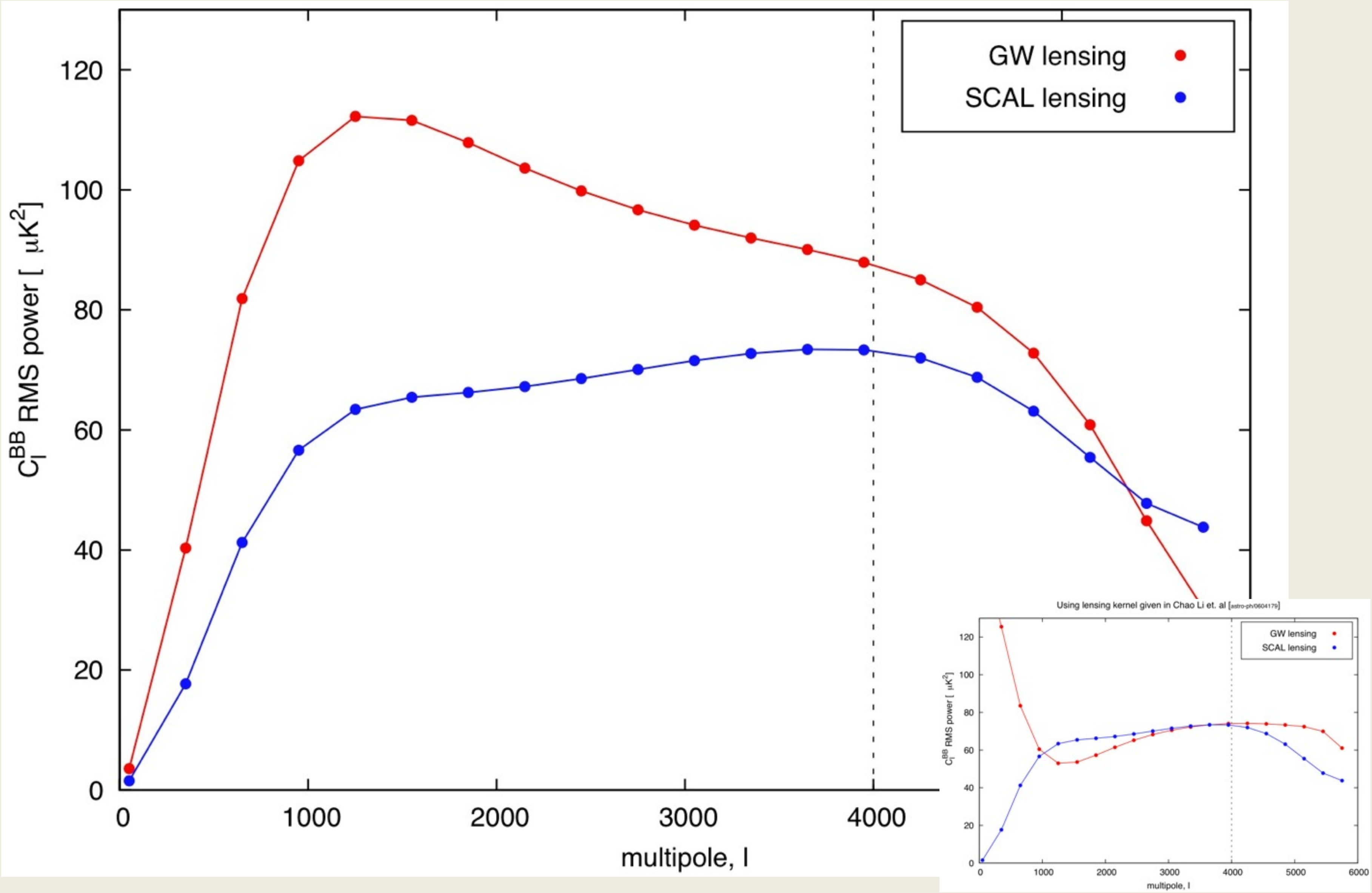
$$A_2(r) = \int_0^\infty \frac{dl\; l^3}{2\pi} C_l^{\psi\psi} J_2(rl)$$

Gradient type displacements ; induced by GW

CAMB evaluates lensed spectra using this method.

H. Padmanabhan, A. Rotti and T. Souradeep, Phys. Rev. D 88, 063507 (2013)

GW Lensing > LSS Lensing !!



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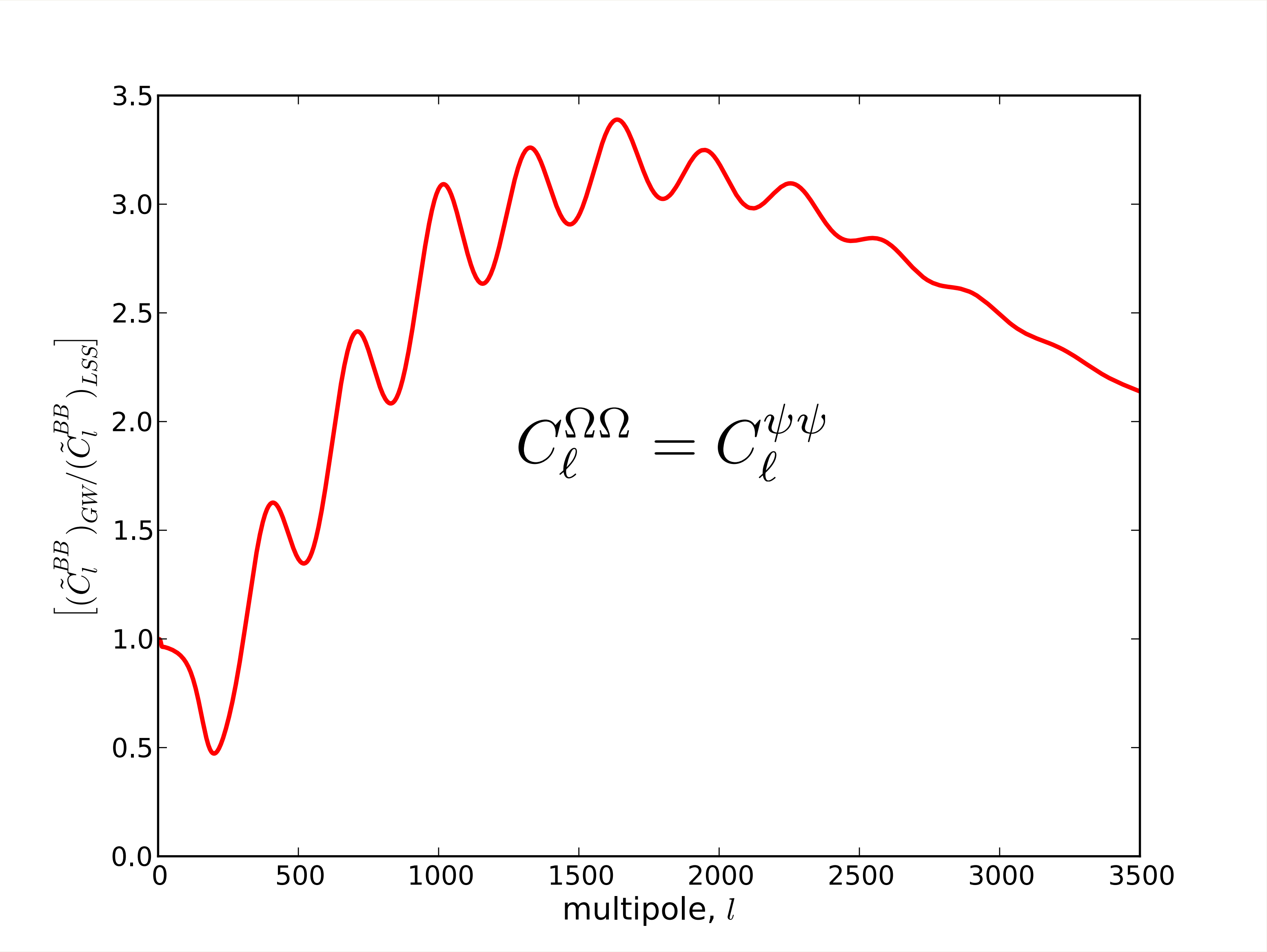
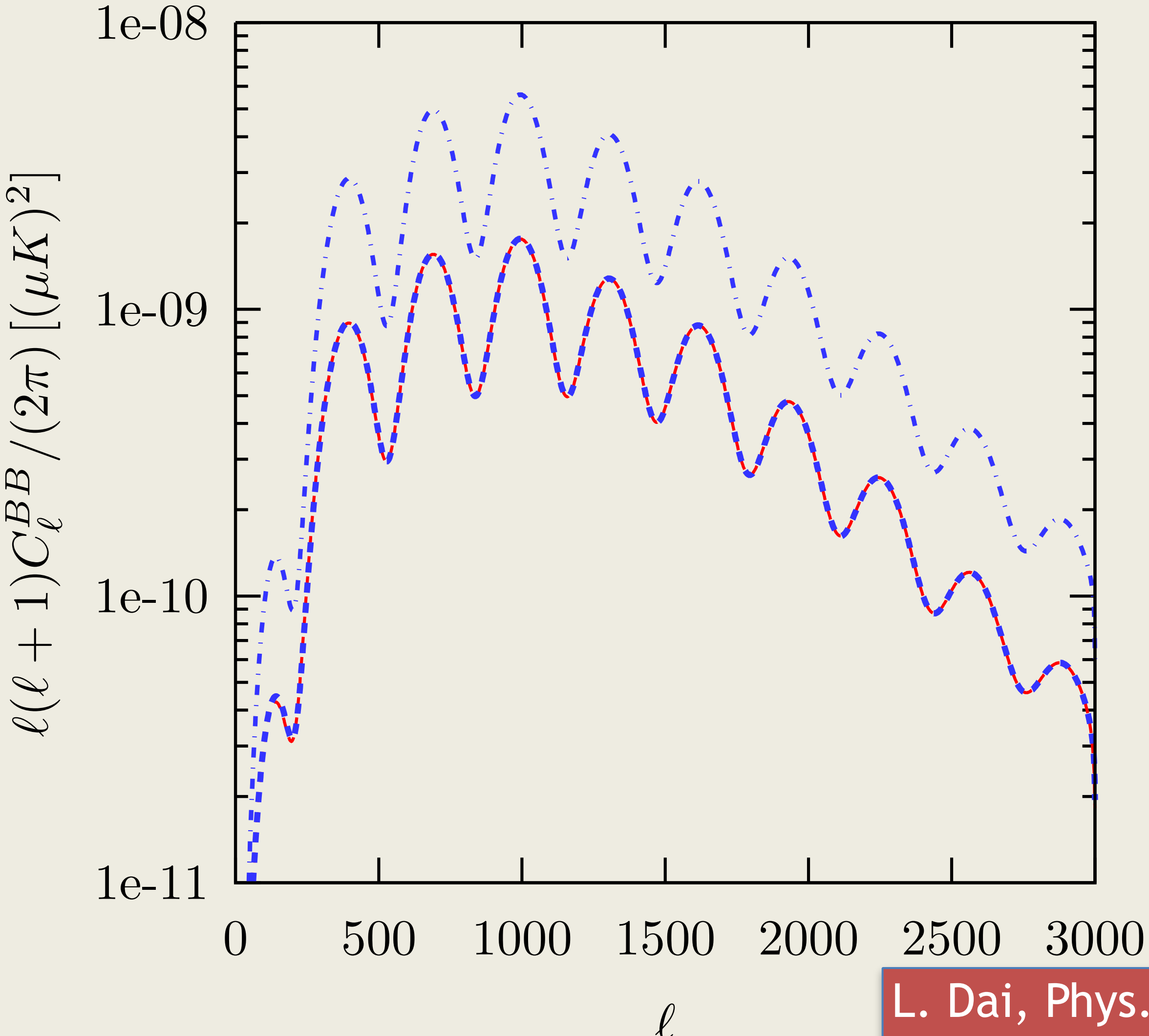


LSS Lensing = GW Lensing

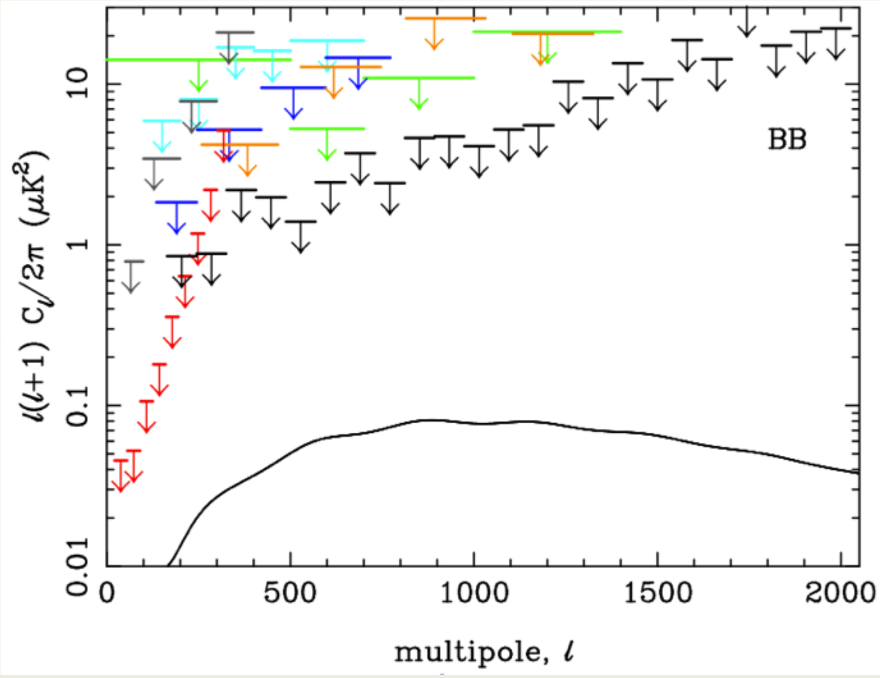
GW also induce a rotation of the polarization of photons in addition to deflecting them.



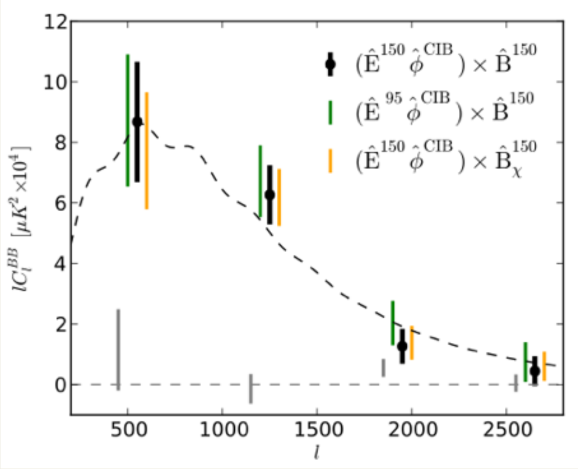
L. Dai, Phys. Rev. Lett. 112, 041303 (2014)

Expected BB signal from lensing of CMB.

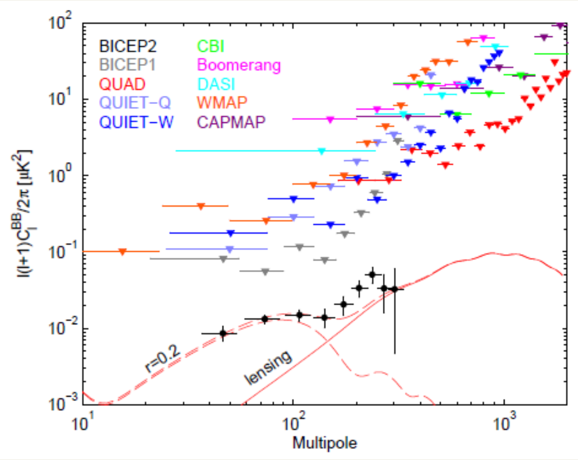
- Current lensing considerations are only due to scalar perturbations (LSS).
- The matter power spectrum is already well constrained.
- A huge difference between current upper limits on the BB spectra and the expected signal (*as of 2011 when this work was done)*



QUAD arXiv:0906.1003v3



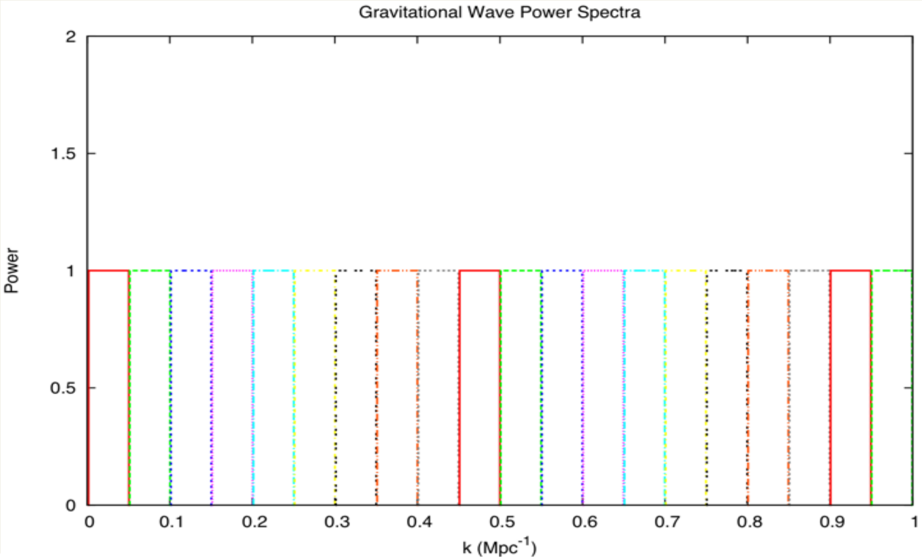
South Pole Telescope

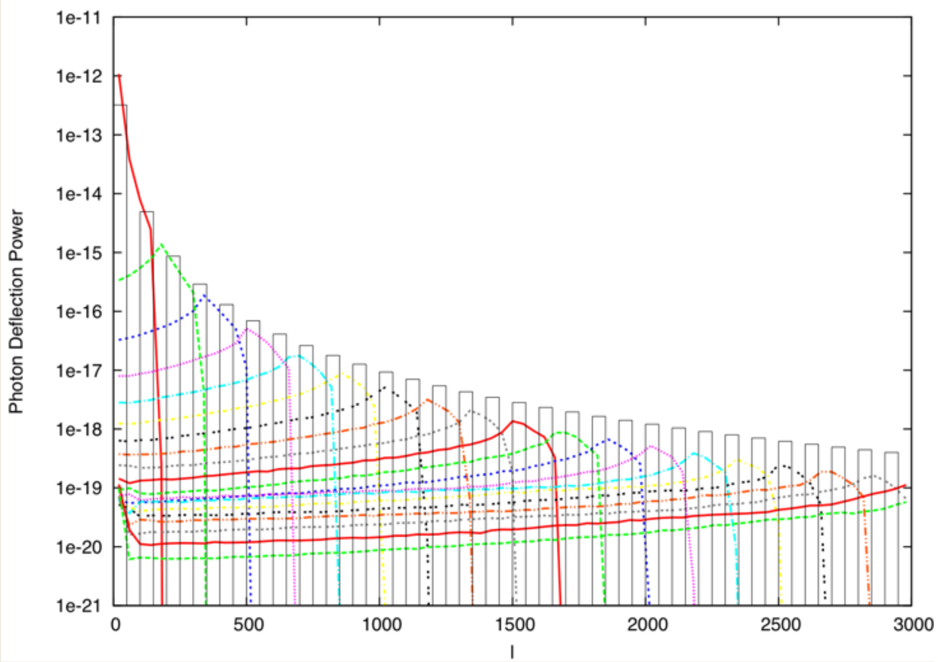


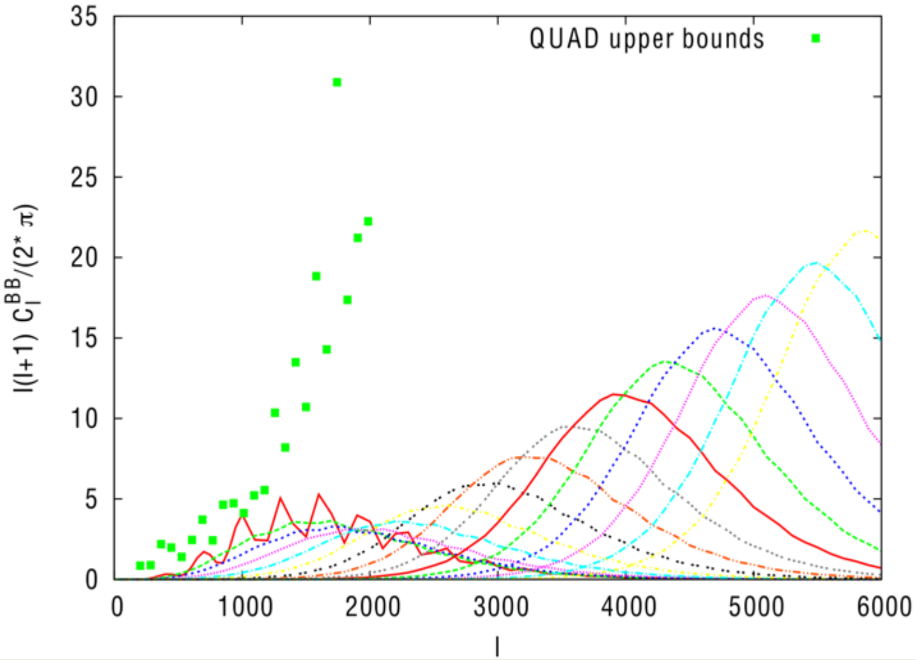
BICEP2

Constraining procedure

- Choose a form for the GW power spectra.
- Divide the power spectra in bins and evaluate the deflection spectra.
- Use the deflection spectra to evaluate the lensed CMB spectra.
- Keep the cosmological parameters fixed and vary the amount of power in each bin till the CMB spectra are consistent with current data.







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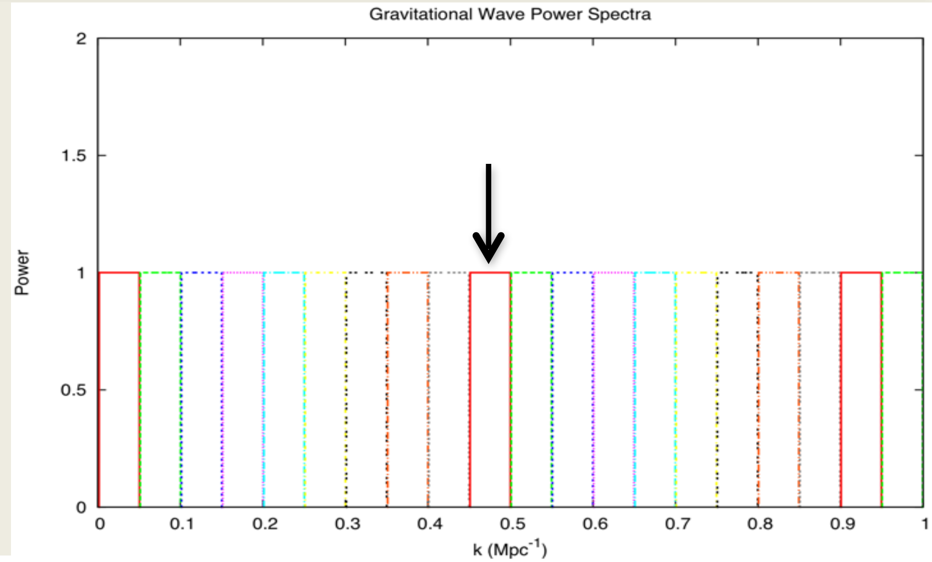
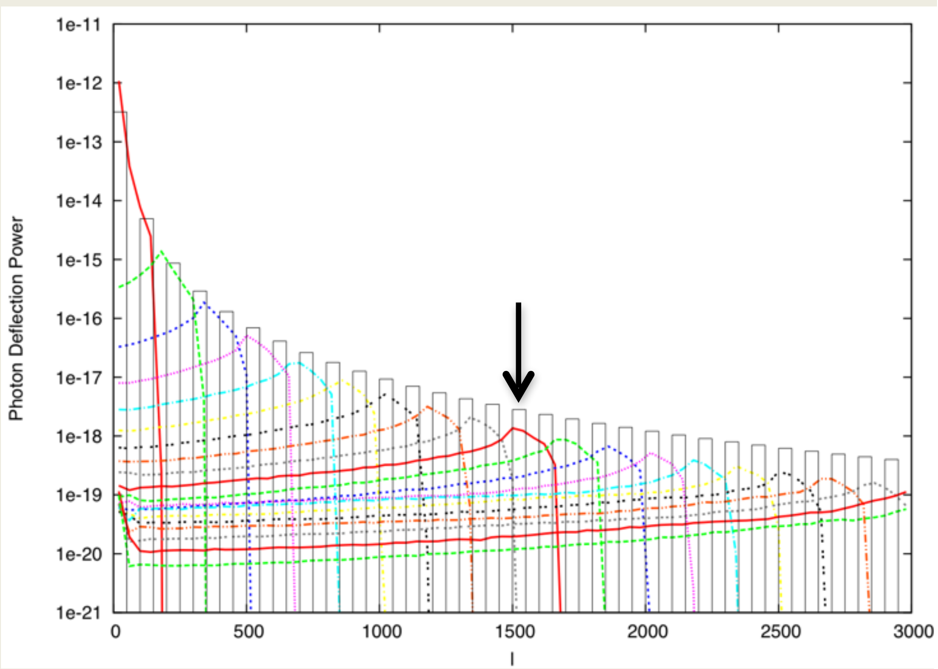
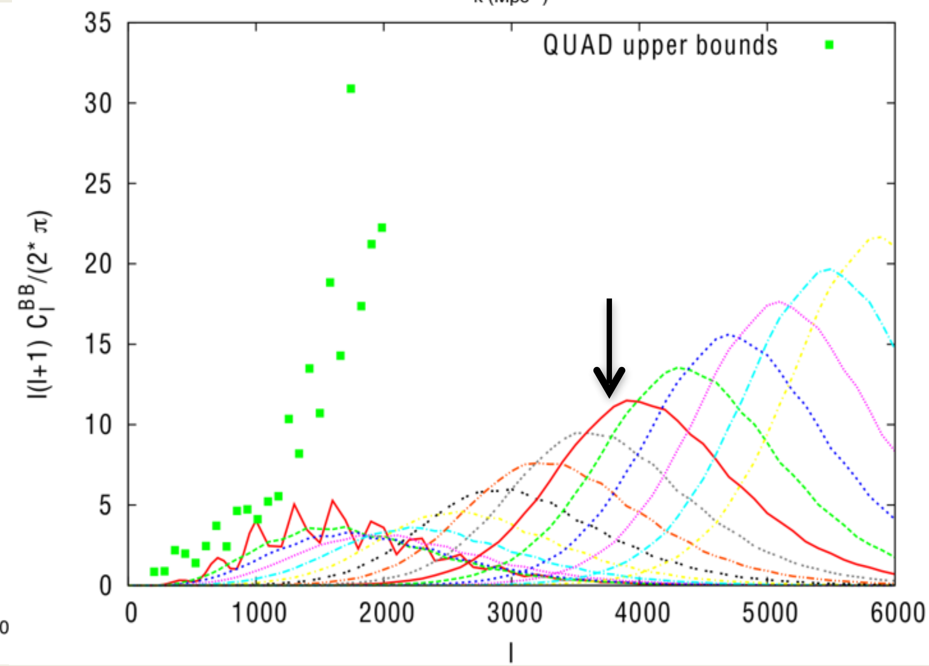
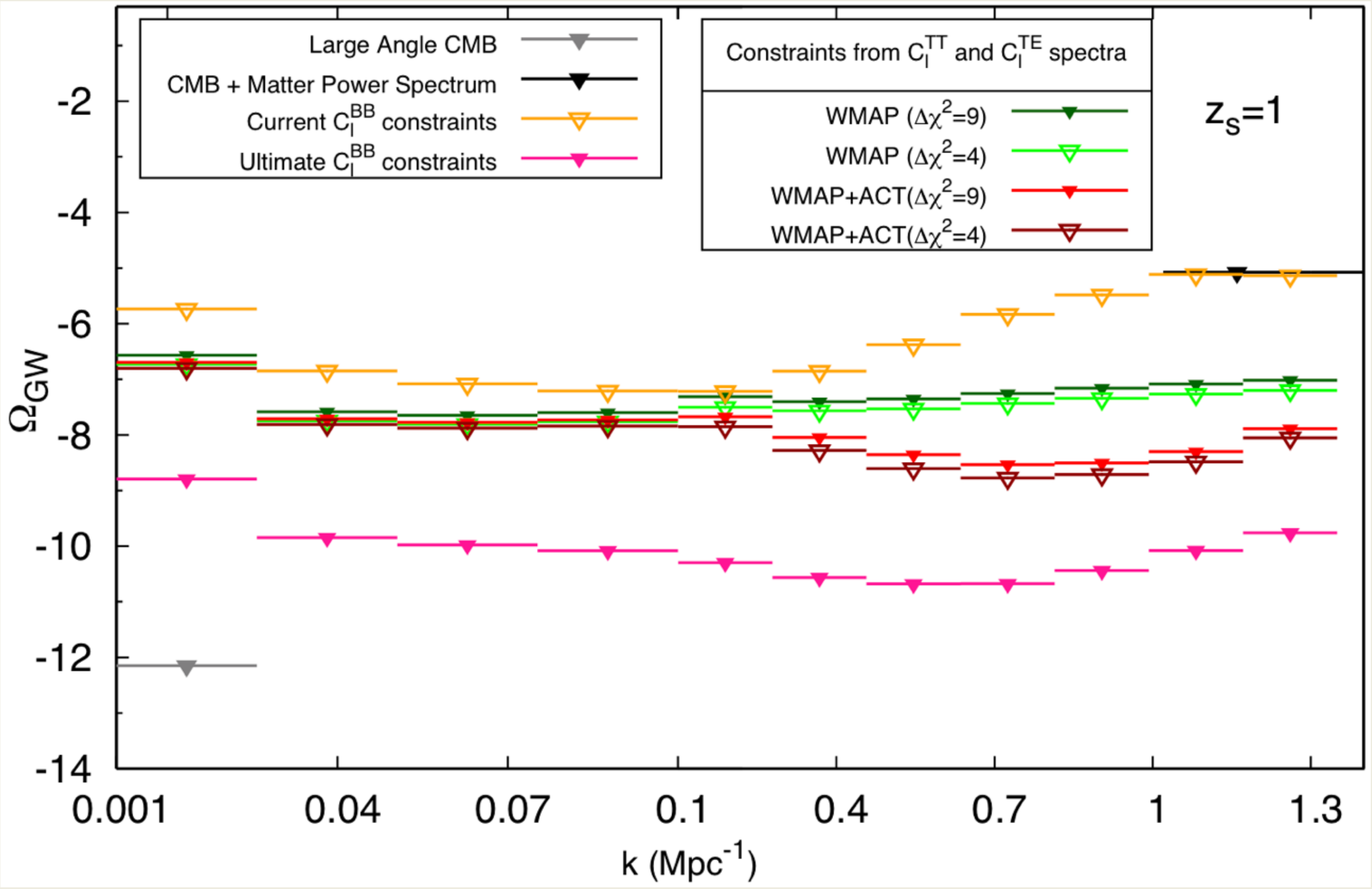






Constraints for GW sourced at different z



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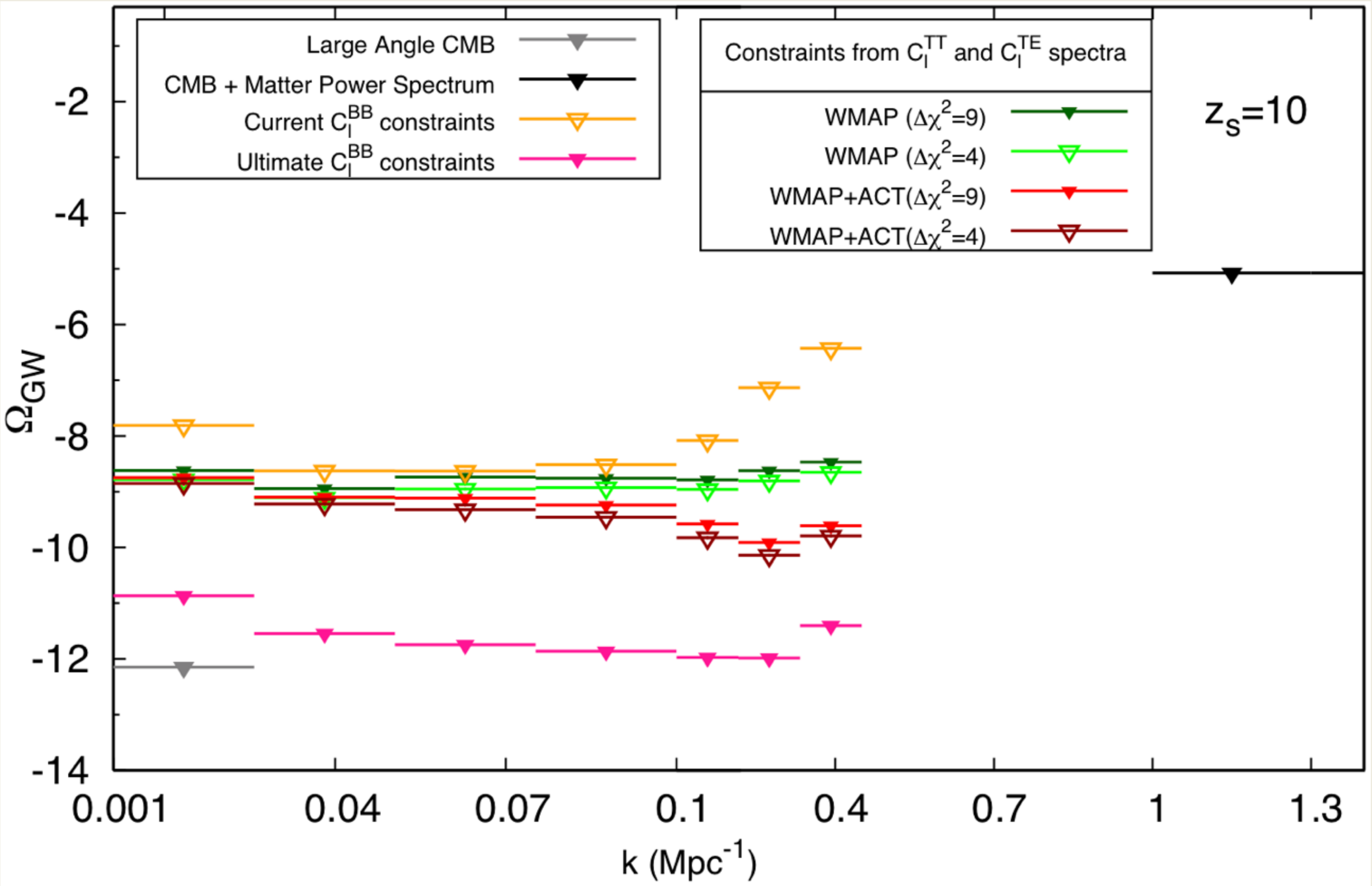

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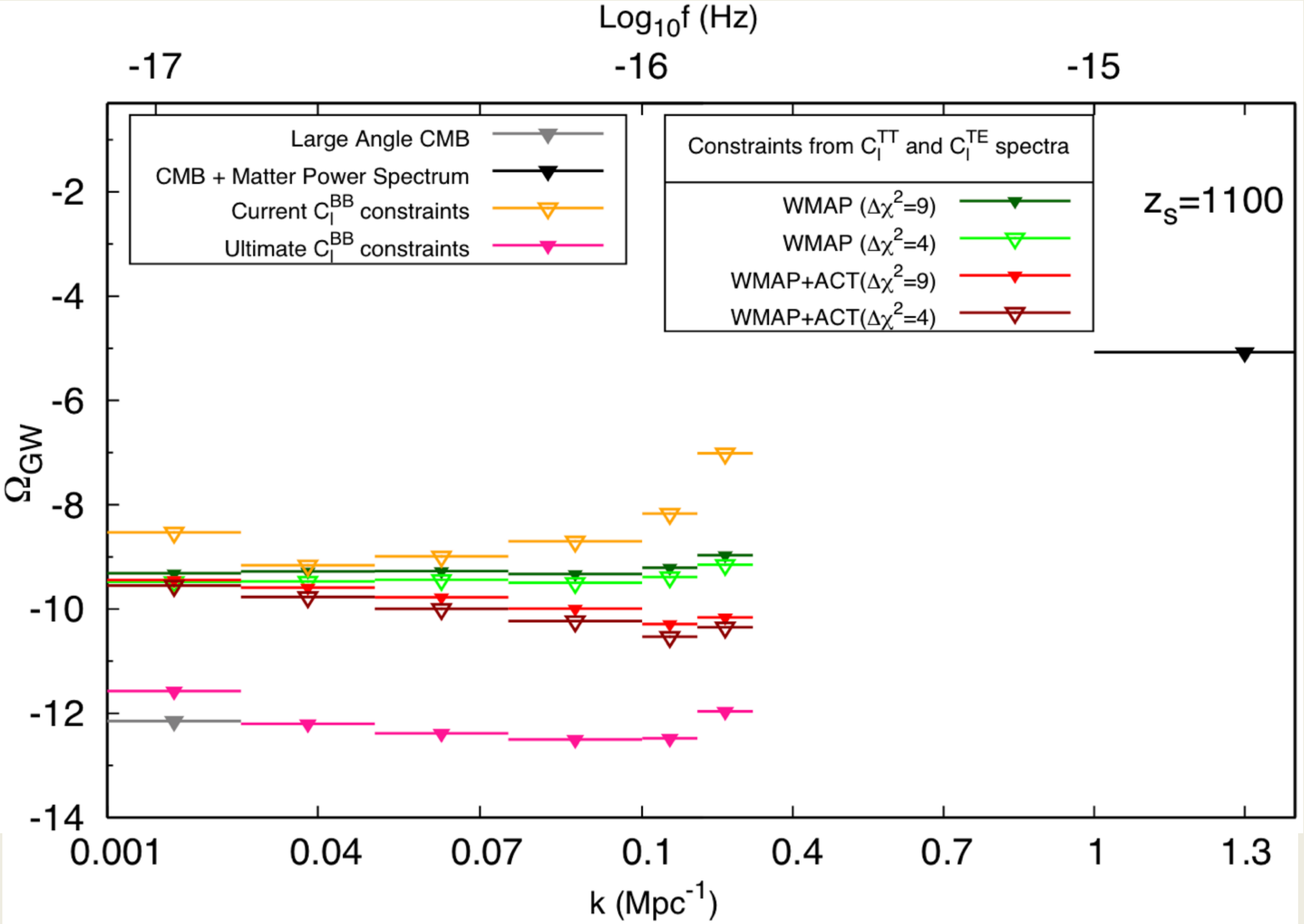
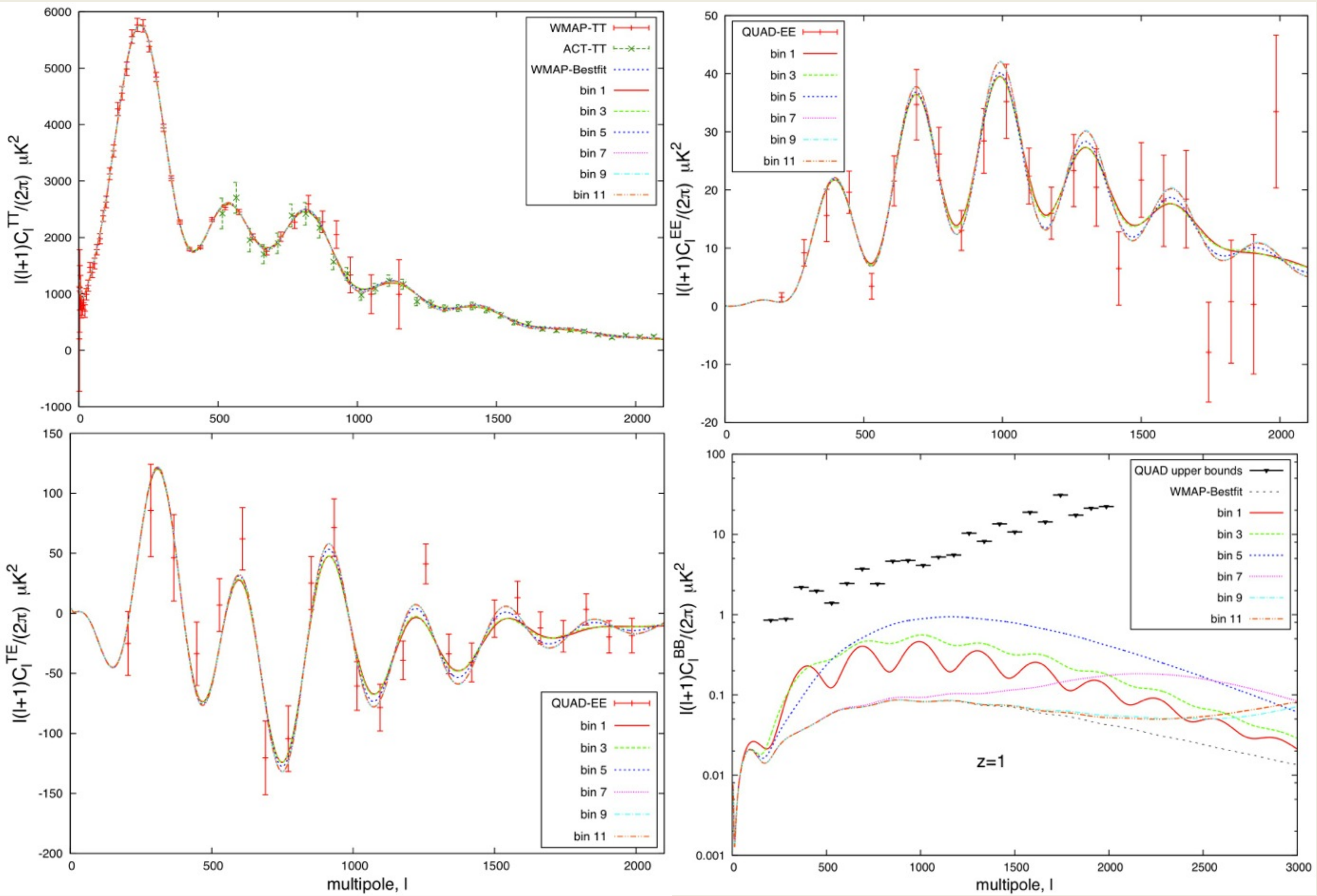


CMB spectra lensed by GW
z=1



Summary : CMB, A New probe of SGWB.

- The lensing kernels for scalar and tensor lensing have been shown to have a very similar form.
- The correct lensing kernel has been derived for lensing due to GW.
- Tensors are seen to be equally efficient at transferring power between E-mode and B–mode of CMB polarization.
- This probe provides a new window into Gravitational Waves which has not been previously explored.
- If the current measurements of B-modes are to be believed, then we have already saturated the best constraints !!

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Thank you for your attention

GW Power Spectra ⟷ GW Energy Density

$$\Omega_{GW} = \frac{\rho_{GW}}{\rho_0}$$

$$\rho_{GW} = \frac{c^2}{32\pi G}\langle \dot{h}_{ij}\dot{h}^{ij}\rangle$$

$$\rho_0 = \frac{3c^2H_0^2}{8\pi G}$$

Critical energy density of the Universe

$$\Omega_{GW}(k) = \frac{4\pi}{3}(\frac{c}{H_0})^2 k^3 P_T(k)[k\mathcal{T}']^2$$

$$\Omega_{GW}\Big|_{bin} = \int_{bin} dlnk\ \Omega_{GW}(k)$$

Weak Lensing.

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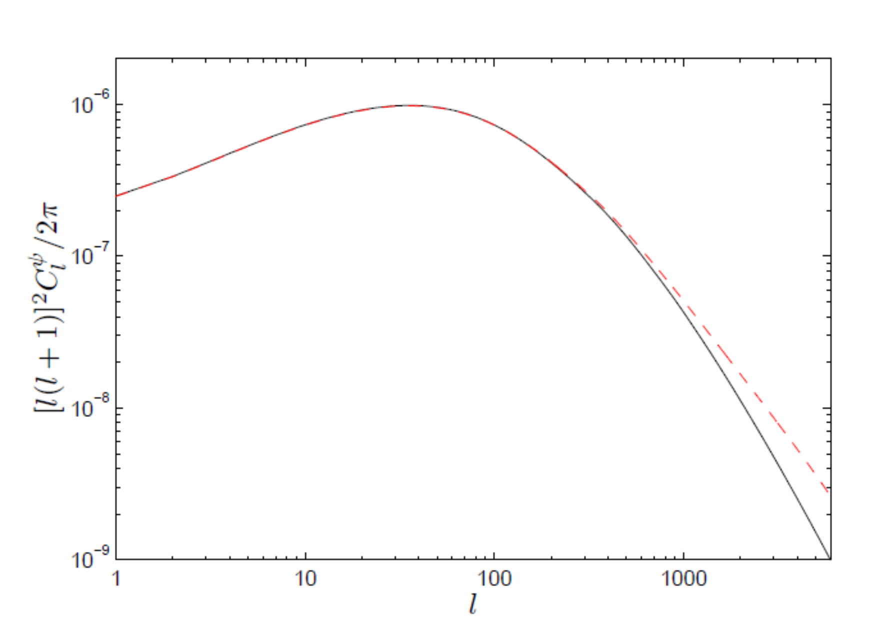


Deflection field on the sphere,

$$\vec{\Delta} = \nabla\psi + \nabla \times \Omega$$

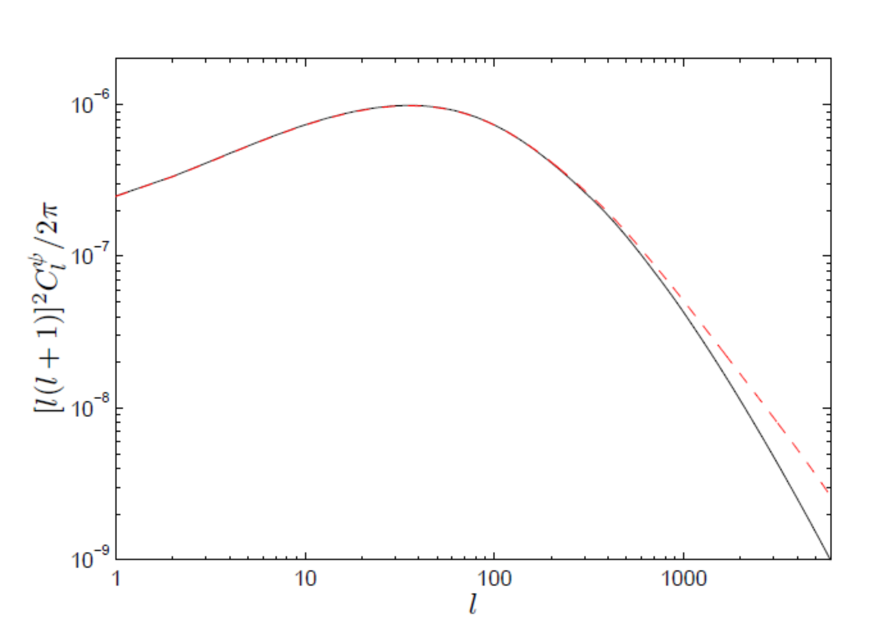
Projected Lensing potential :

$$\psi(\hat{n}) = -2\int_0^{\eta_0} d\eta \frac{\eta_0 - \eta}{\eta_0 \eta} \Psi(\hat{n}, \eta_0 - \eta)$$

Weak Lensing.

Lensing Remaps the CMB anisotropies

$$\tilde{T}(\hat{n}) = T(\hat{n} + \vec{\Delta})$$



Deflection field on the sphere,

$$\vec{\Delta} = \nabla\psi + \nabla \times \Omega$$

Projected Lensing potential :

$$\psi(\hat{n}) = -2\int_0^{\eta_0} d\eta \frac{\eta_0 - \eta}{\eta_0 \eta} \Psi(\hat{n}, \eta_0 - \eta)$$

Similarity of lensing kernels !

$$\tilde{C}_l^{TT} = \quad C_l^{TT} - l(l+1)RC_l^{TT} + \sum_{l_1 l_2} \frac{C_{l_1}^{TT}}{2l+1}\left[C_{l_2}^{\oplus}(F_{ll_1l_2}^{\oplus})^2 + C_{l_2}^{\otimes}(F_{ll_1l_2}^{\otimes})^2\right]$$

$$|F_{ll_1l_2}^{\oplus}| = \frac{1}{2}\left[l_1(l_1+1) + l_2(l_2+1) - l(l+1)\right]\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}}\begin{pmatrix} l & l_1 & l_2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$|F_{ll_1l_2}^{\otimes}| = \frac{1}{2}\sqrt{l_1(l_1+1)l_2(l_2+1)}\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}}\begin{pmatrix} l & l_1 & l_2 \\ 0 & -1 & 1 \end{pmatrix}\left[1 - (-1)^{l+l_1+l_2}\right]$$

H. Padmanabhan, A. Rotti and T. Souradeep, Phys. Rev. D 88, 063507 (2013)

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Similarity of lensing kernels !

Likewise for lensing of polarization power spectra

$$|{}_2F^{\oplus}_{ll_1l_2}| = \frac{1}{2}[l_1(l_1+1) + l_2(l_2+1) - l(l+1)]\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -2 & 0 \end{pmatrix}$$

$$|{}_2F^{\otimes}_{ll_1l_2}| = \sqrt{\frac{l_2(l_2+1)(2l+1)(2l_1+1)(2l_2+1)}{8\pi}} \times \left(\sqrt{\frac{(l_1+2)(l_1-1)}{2}}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -1 & -1 \end{pmatrix} - \sqrt{\frac{(l_1-2)(l_1+3)}{2}}\begin{pmatrix} l & l_1 & l_2 \\ 2 & -3 & 1 \end{pmatrix}\right)$$

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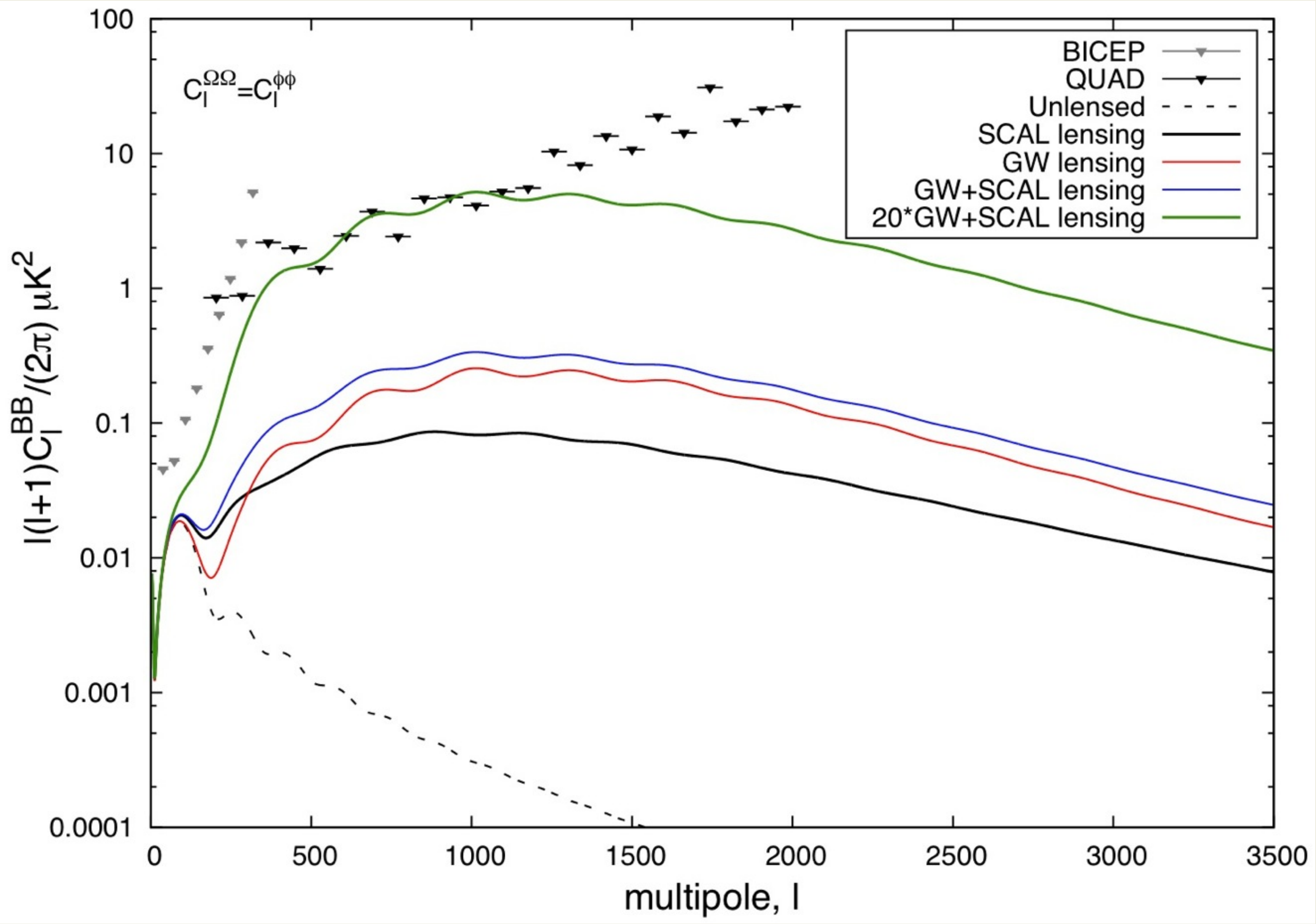
H. Padmanabhan, A. Rotti and T. Souradeep, Phys. Rev. D 88, 063507 (2013)

Lensed Polarization Power Spectra



GW Power Spectra ⟷ Deflection Spectra

$$C_l^{h\otimes} = \frac{\pi}{l^2(l+1)^2}\frac{(l+2)!}{(l-2)!}\int d^3\mathbf{k} P_T(k,z)|T_{eff}|^2$$

$$T_{eff} = 2k\int_{\eta_z}^{\eta_0} d\eta' T(k,\eta')\left(\frac{j_l(x)}{x^2}\right)_{x=k(\eta_0-\eta')}$$

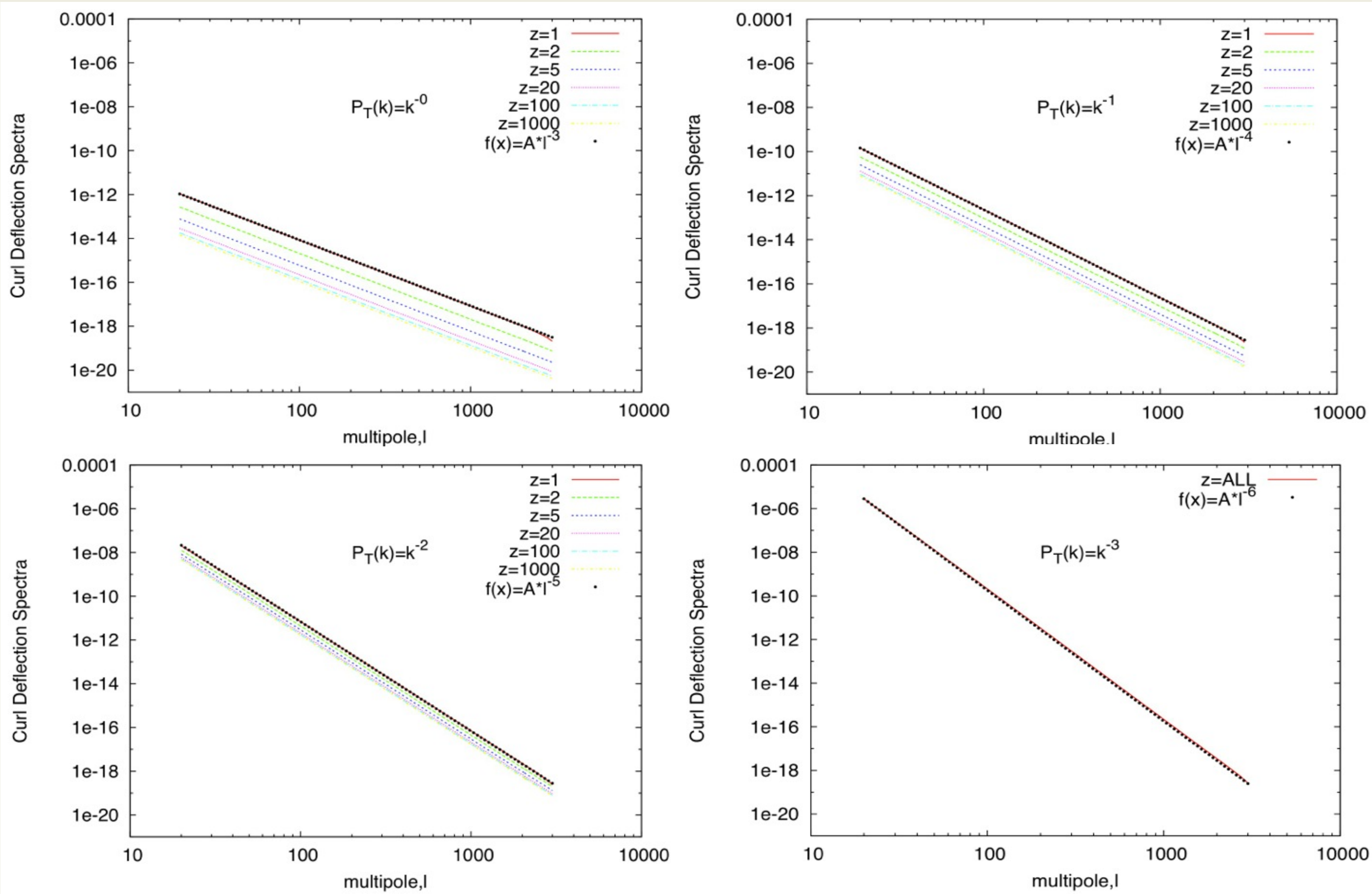
Projection on-to the sphere.

GW transfer function (cosmic evolution)

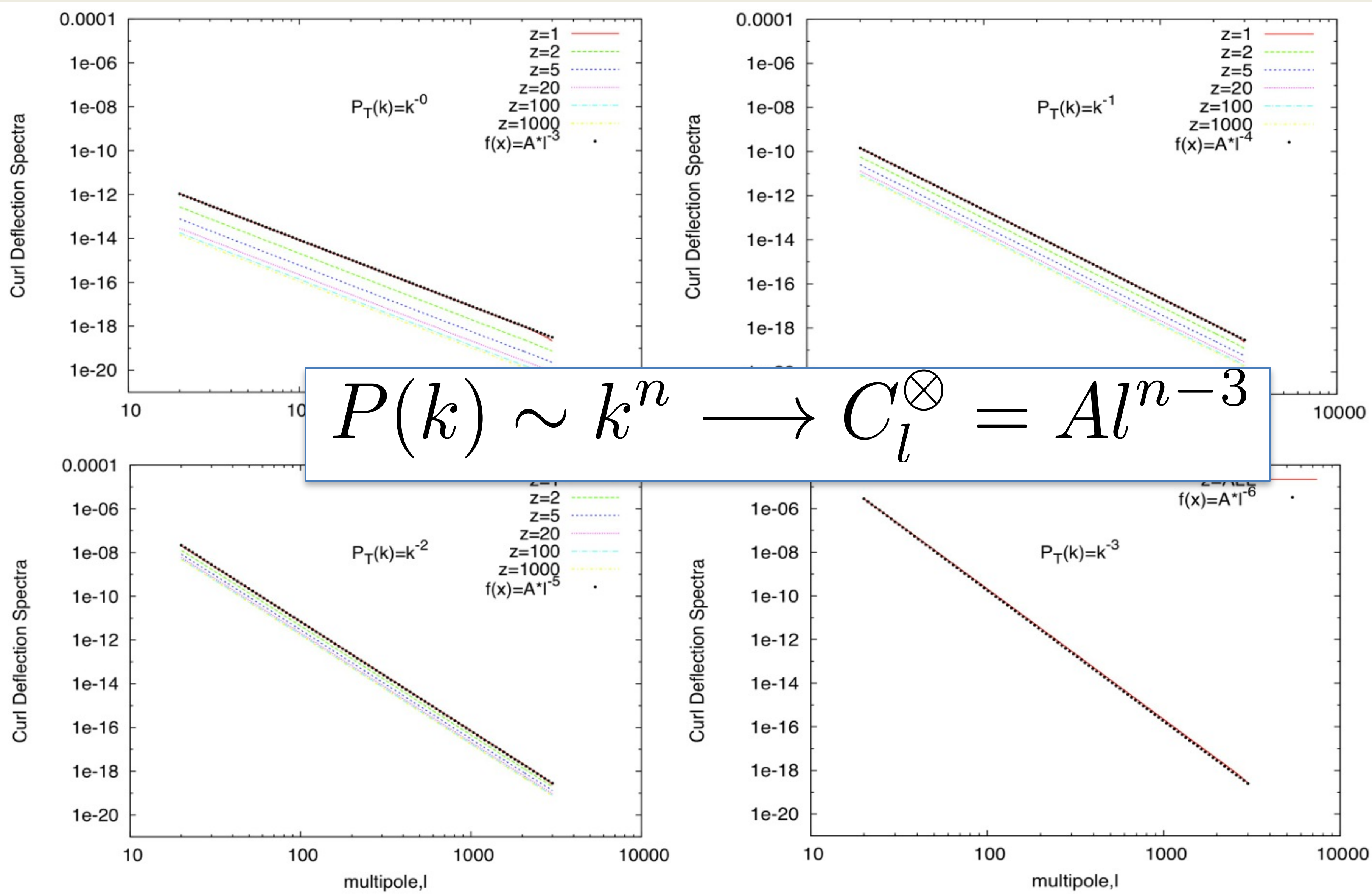
- Gravitational waves also induce gradient type displacements, but we ignore them in this study.

S. Dodelson, E. Rozo, and A. Stebbins, Phys. Rev. Lett. **91**, 021301 (2003).

Choosing the form of the tensor power spectrum



Choosing the form of the tensor power spectrum



$$P(k) \sim k^n \longrightarrow C_l^{\otimes} = A l^{n-3}$$

Choosing the form of the tensor power spectrum