

# QCD for the LHC

J. C. Bose Memorial Lecture

Keith Ellis  
Fermilab

April 2011

# Large Hadron Collider

12/16/1994

-Project approval

2008

- Accelerator complete

- Ring cold and under vacuum

• 9/10/2008

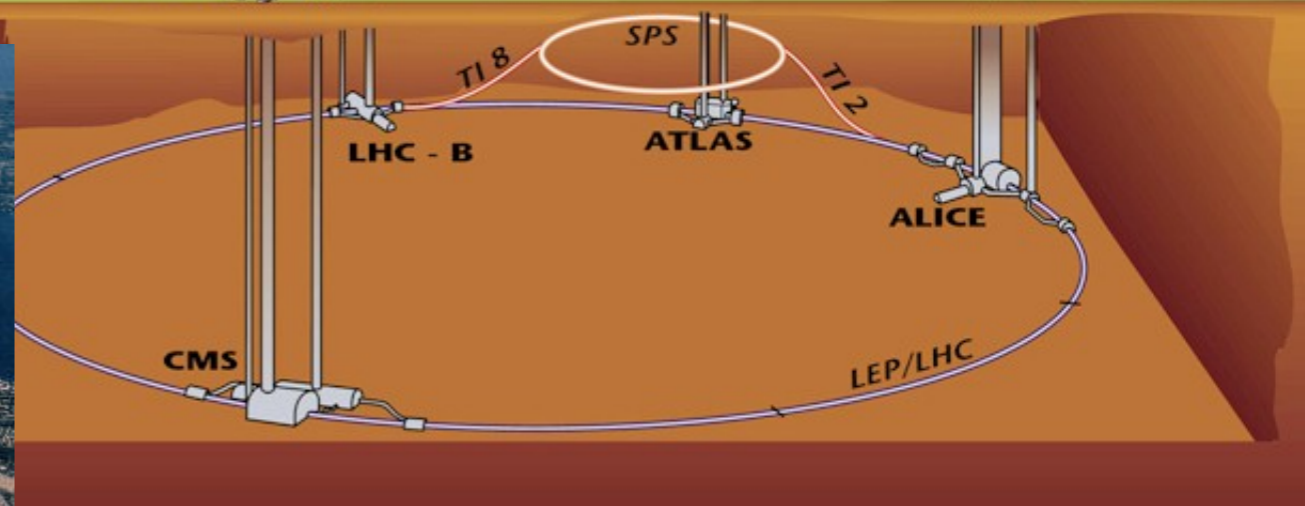
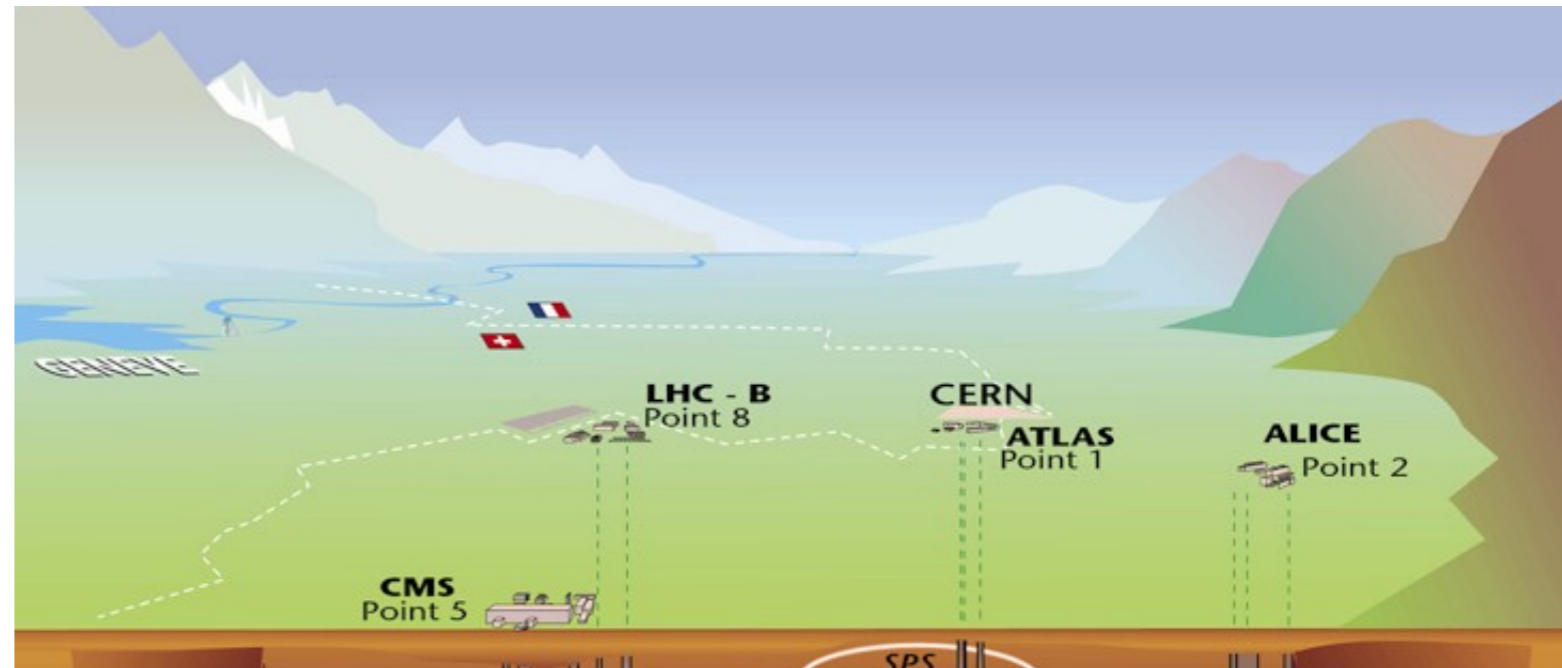
- First beams around

• 9/19/2008

- Machine damaged in incident

• 2008 – 2009

- 12 months of major repairs and consolidation



9/20/2009

- First beams around again

• 9/29/2009

- Both beams accelerated to 1.18 TeV simultaneously

• 10/8/2009

- First collisions at 2.36 TeV cm!

• 2010/2011

-  $\sim 50+26\text{pb}^{-1}$  accumulated luminosity at  $\sqrt{s}=7\text{ TeV}$

# LHC in numbers

	Tevatron (2011)	LHC 2011(2012)	LHC (2014<t<2030)
Beams	p- $\bar{p}$	p-p and Pb-Pb	p-p and Pb-Pb
Circumference [km]	6	26	26
c.o.m. Energy [TeV]	1.96	7(8)	~14
Luminosity [cm <sup>-2</sup> s <sup>-1</sup> ]	1.00E+32	2.00E+32	5.00E+34
Projected Accumulated Luminosity [fb <sup>-1</sup> ]	10	1(5)	3000
Bunch spacing [ns]	392	75/50	25
Collisions/crossing	6	1-10	20
Number of collaborators (General purpose detectors)	~300	~3000	??

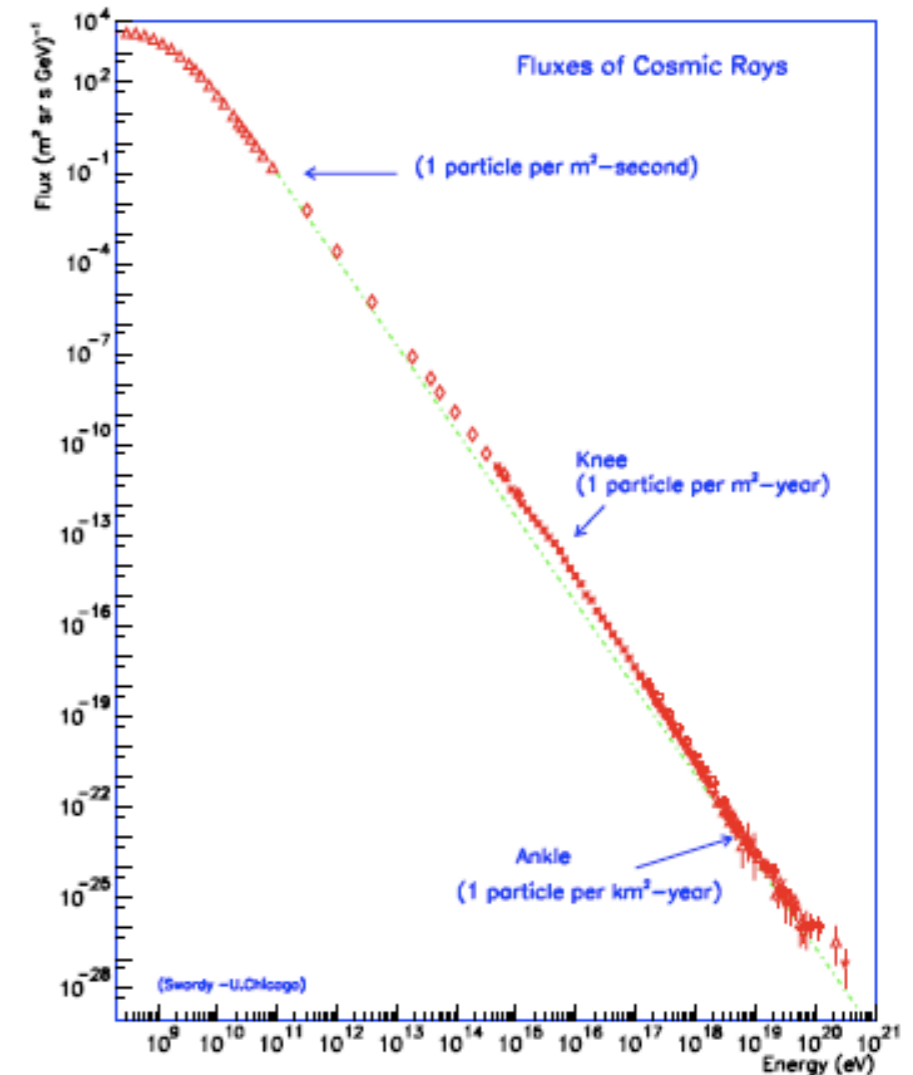
# LHC collisions .. not new

$$E_{1,2} = 7 \text{ TeV} \rightarrow s = (E_1 + E_2)^2 \sim 2 \times 10^8 \text{ GeV}^2$$

The equivalent fixed target energy  $E_{FT}$  is

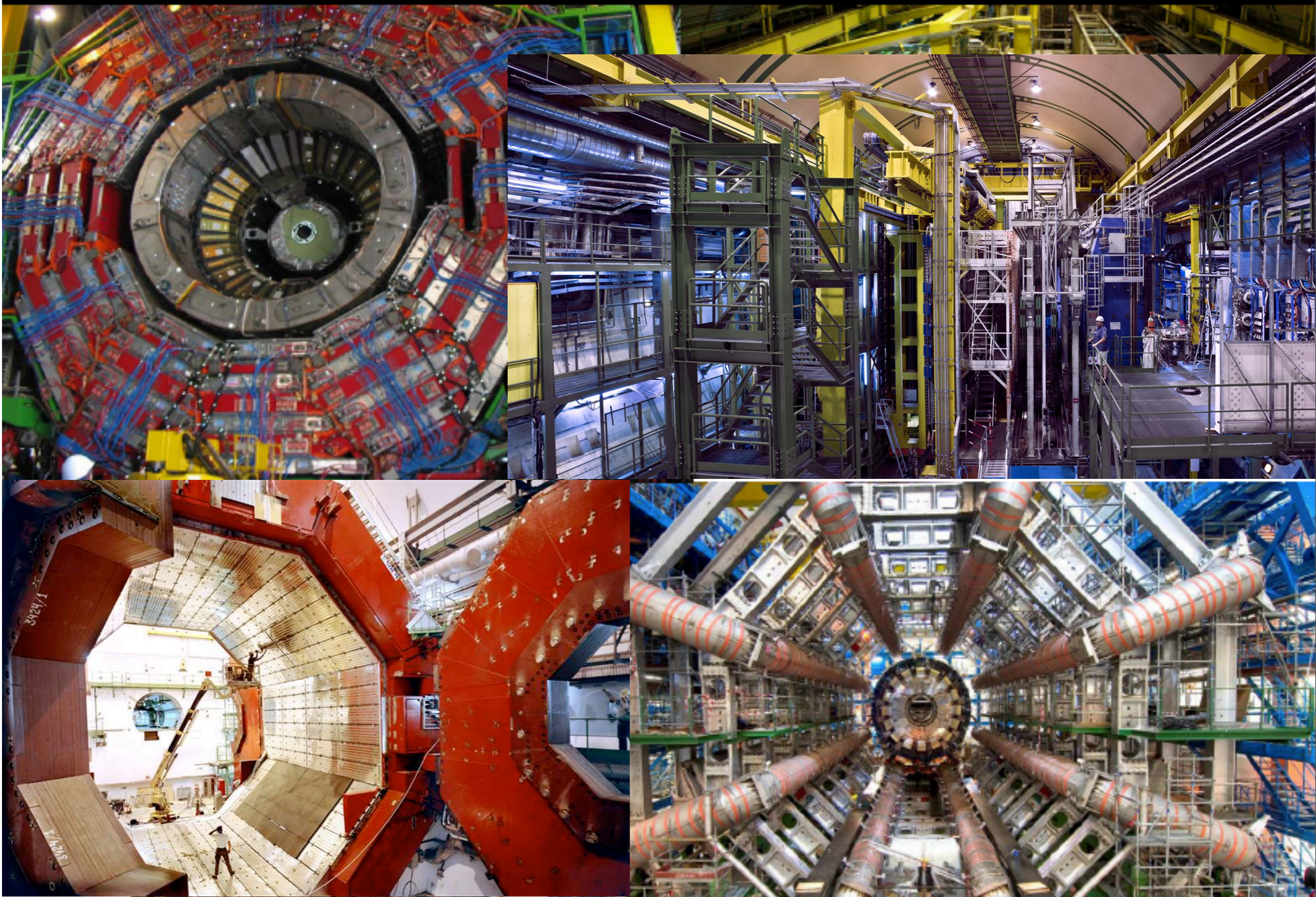
$$s = 2m_P E_{FT} \rightarrow E_{FT} = 10^8 \text{ GeV} \equiv 10^{17} \text{ eV}$$

Energies above  $10^{17}$  have been observed in cosmic rays.



What is new is the observation of these events in a controlled way by 4 sophisticated detectors.

# Four large detectors

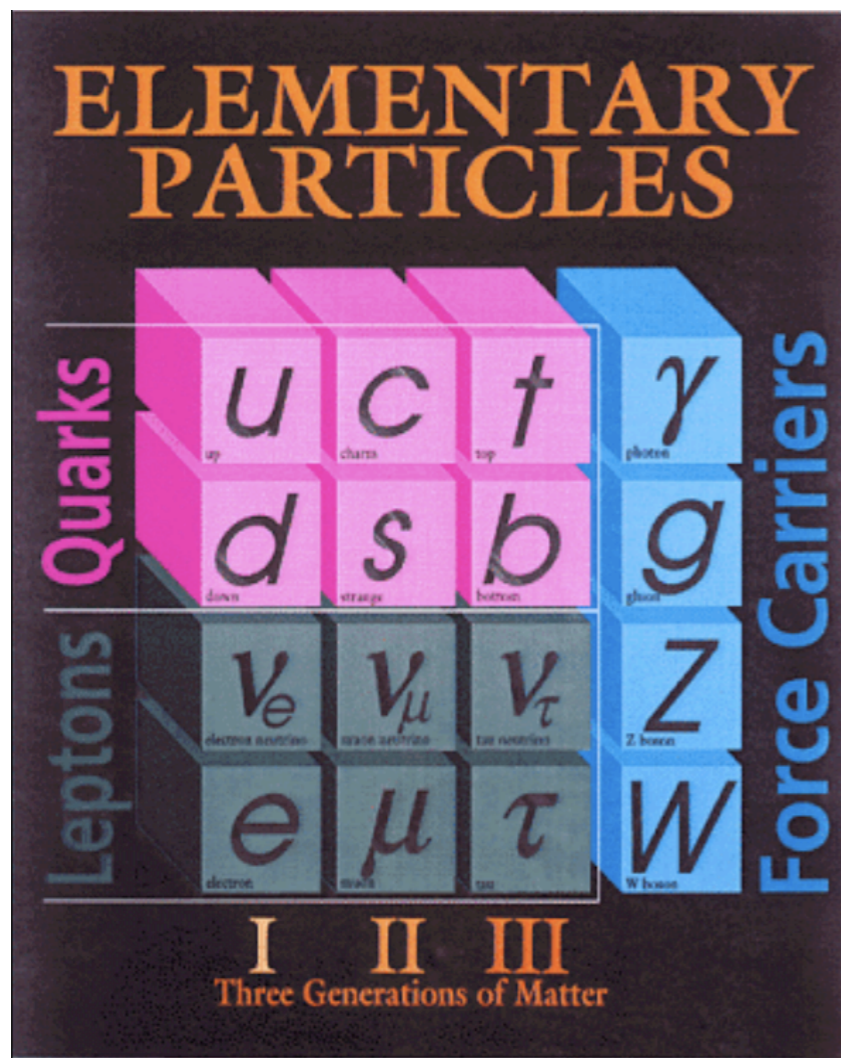


# The detectors compared

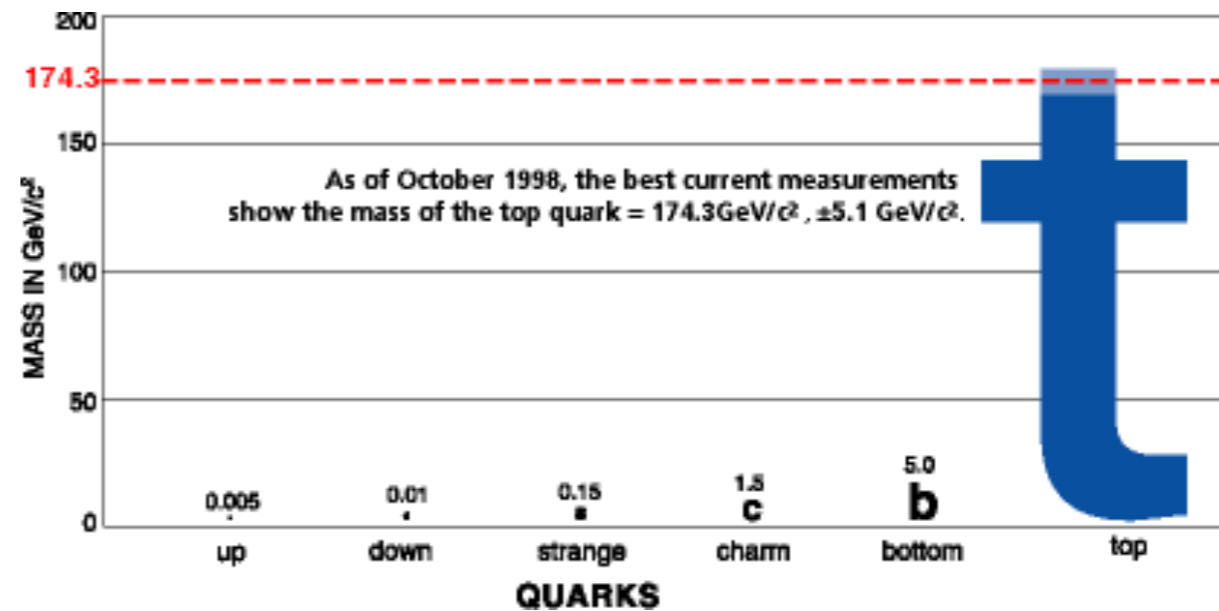
- CMS is 12,000 tons (2 x ATLAS)
- ATLAS has 8 times the volume of CMS.
- ALICE is bigger than CMS and heavier than ATLAS, specialized for heavy ion physics.
- LHCb has the best instrumented coverage of the forward region, specialized for b-physics.



# Bestiary of elementary particles



Masses of the six quarks



Mass of the Z-boson =  $91.1876 \pm 0.0021 \text{ GeV}$

Mass of the W-boson =  $80.413 \pm 0.048 \text{ GeV}$

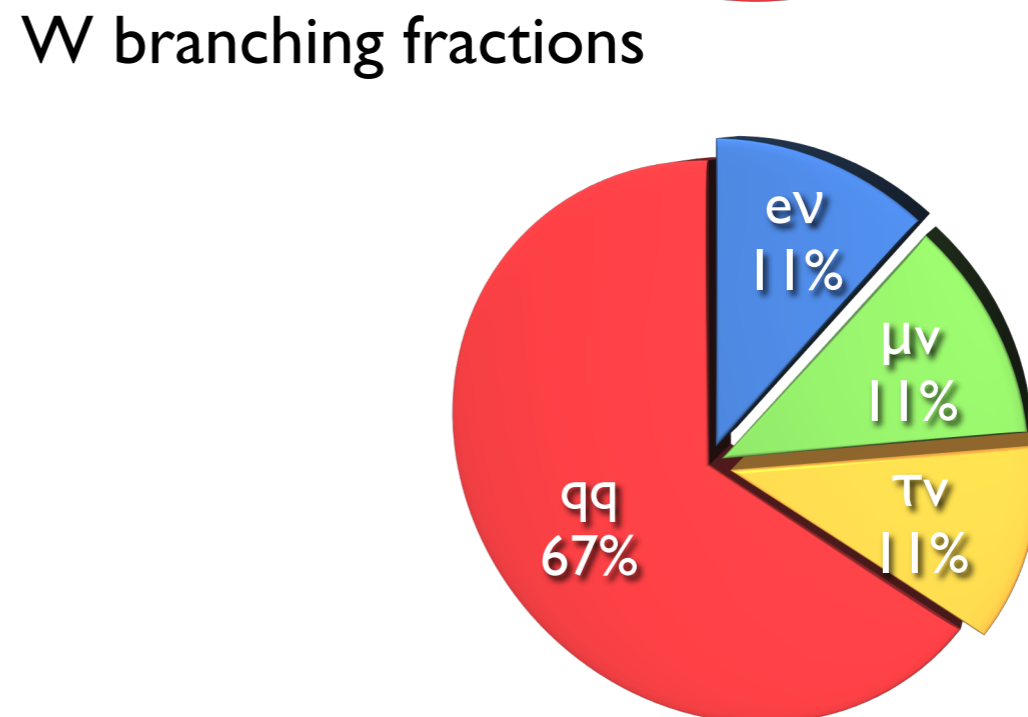
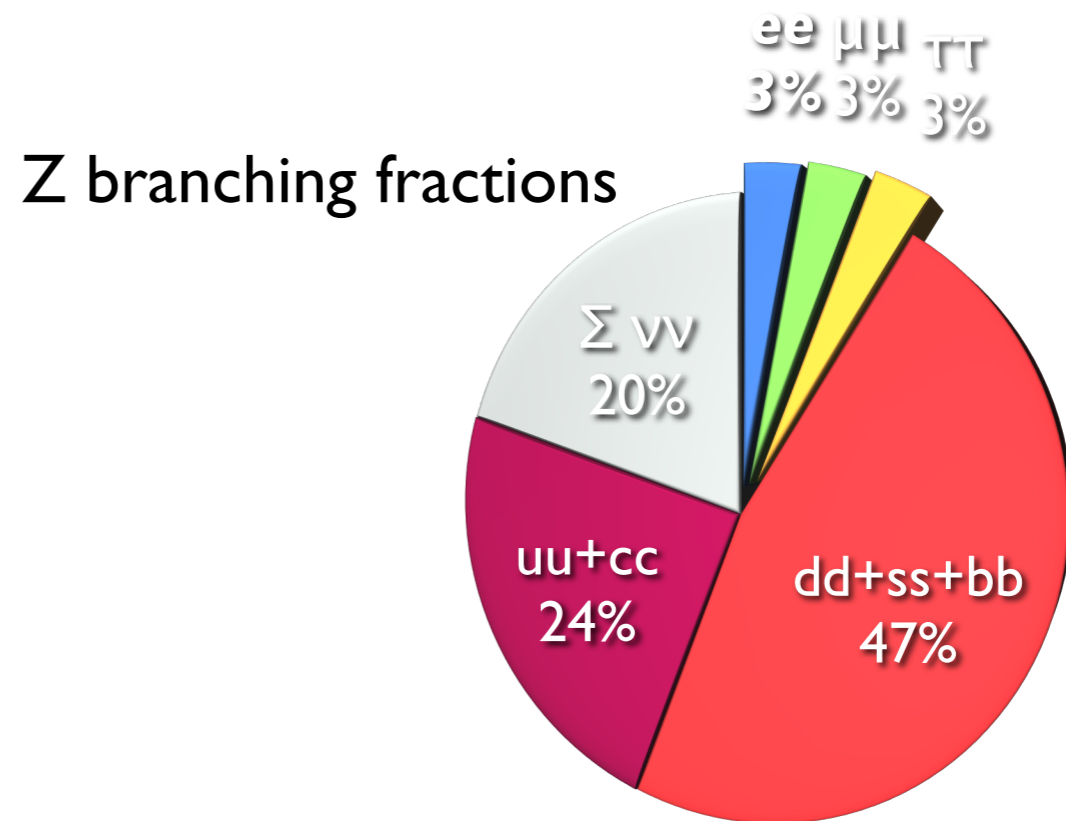
Mass of the photon = 0

(Mass of the proton =  $0.938 \text{ GeV}$ )

Primary problem of particle physics - mechanism of EW symmetry breaking.

# Decays of the Z and W

- W and Z decay by weak interaction.
- Z decays 20% of the time to neutrinos - the invisible width.
- Z has a 3.4% branching ratio into each type of charged lepton.
- W has an 11% branching ratio into each flavour of lepton neutrino.
- The remaining decays are to quarks.





# A triumph of 20th century physics: Standard model

$$SU(3) \otimes SU(2)_L \otimes U(1)$$

QCD                  QEW

Three couplings  $g_s, g_w, g_w'$

Three families of quarks  
Each quark comes in three colours

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix}$$

Eight colours of self-coupled gluons **A**

$$F_{\alpha\beta}^A = [\partial_\alpha \mathcal{A}_\beta^A - \partial_\beta \mathcal{A}_\alpha^A - g f^{ABC} \mathcal{A}_\alpha^B \mathcal{A}_\beta^C]$$

The QCD Lagrangian has a familiar form, similar to QED, but the self coupling of the gluons, leads to a different behaviour of the coupling.

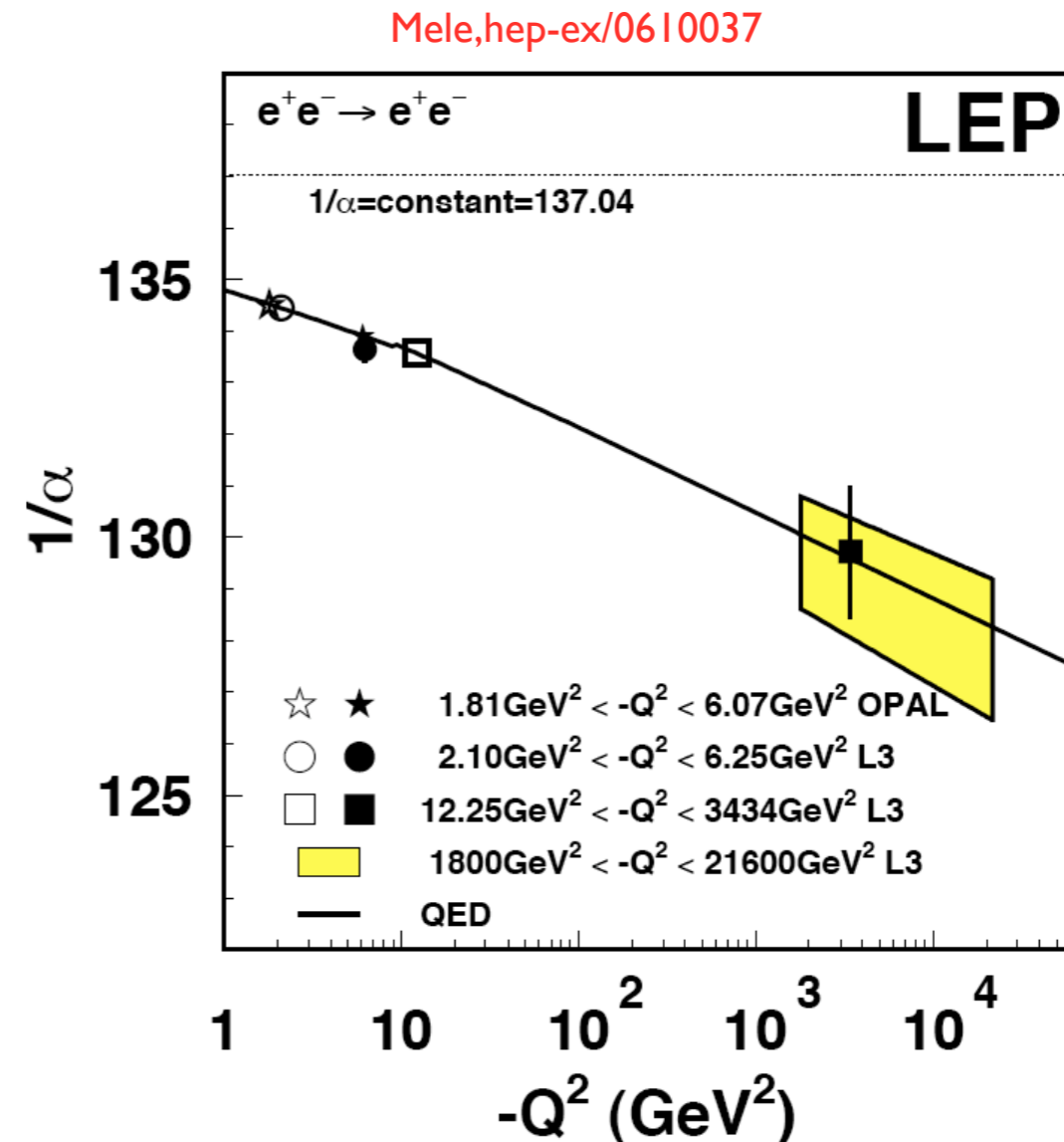
$$\mathcal{L} = -\frac{1}{4} F_{\alpha\beta}^A F_A^{\alpha\beta} + \sum_{\text{flavours}} \bar{q}_a (i \not{D} - m)_{ab} q_b$$

# Charge screening in QED

The expected behaviour of the electromagnetic coupling is confirmed by experiments on Bhabha scattering at LEP.

$$\alpha = \frac{e^2}{4\pi}$$

$$\frac{1}{\alpha(Q)} = \frac{1}{\alpha_0} - \frac{2Q_f^2}{3\pi} \ln\left(\frac{Q}{m_f}\right)$$



# Charge anti-screening in QCD

QCD has the property of anti-screening because of the self-interaction of the gluons

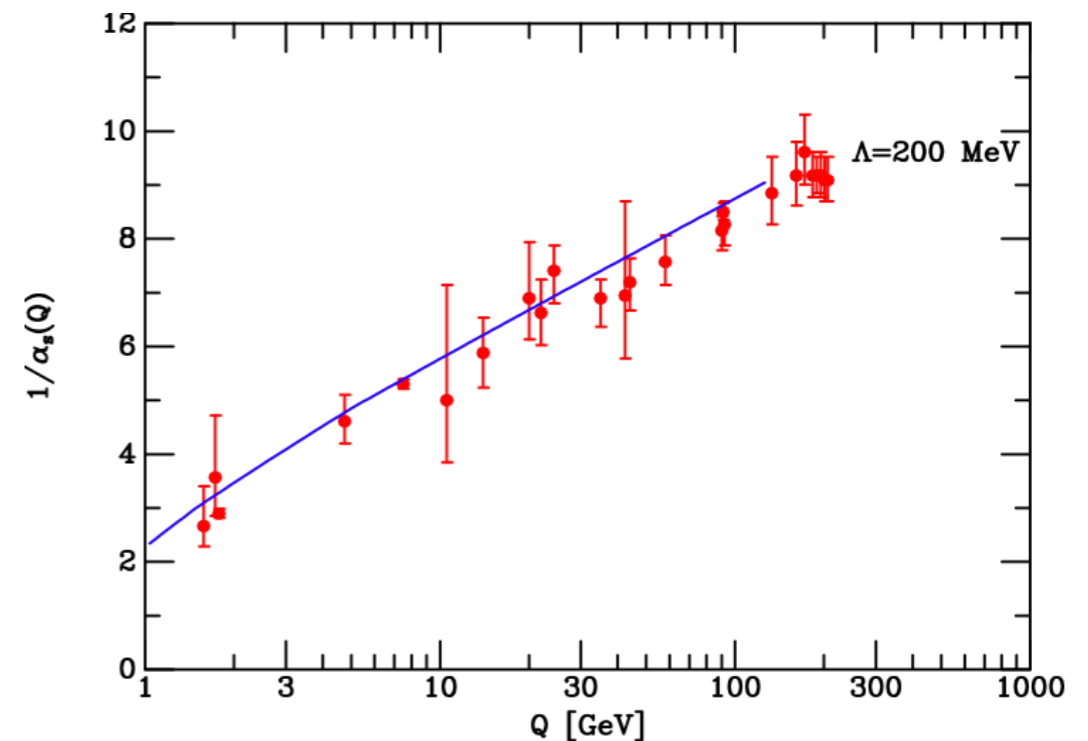
Quark anti-quark pairs screen as in QED, but gluons lead to anti-screening.

$$\alpha_s = \frac{g_s^2}{4\pi}$$

$$\frac{1}{\alpha_s(Q)} = \frac{(33 - 2n_f)}{6\pi} \ln \frac{Q}{\Lambda} + \dots$$

This is the phenomenon of **asymptotic freedom**.

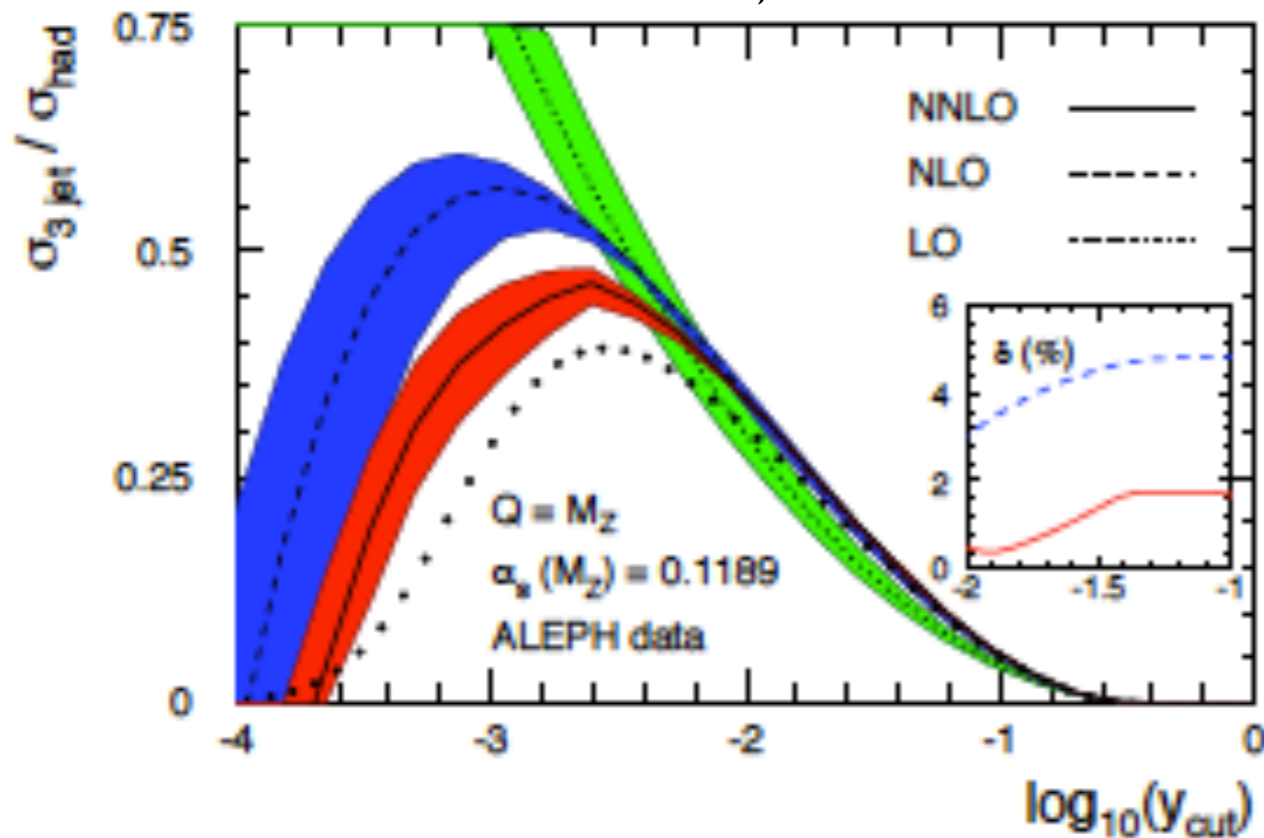
Since the coupling is small at high energy we may use the methods of perturbation theory.



# Measurement of $\alpha_s$ at high energy

e.g. from three-jet rate

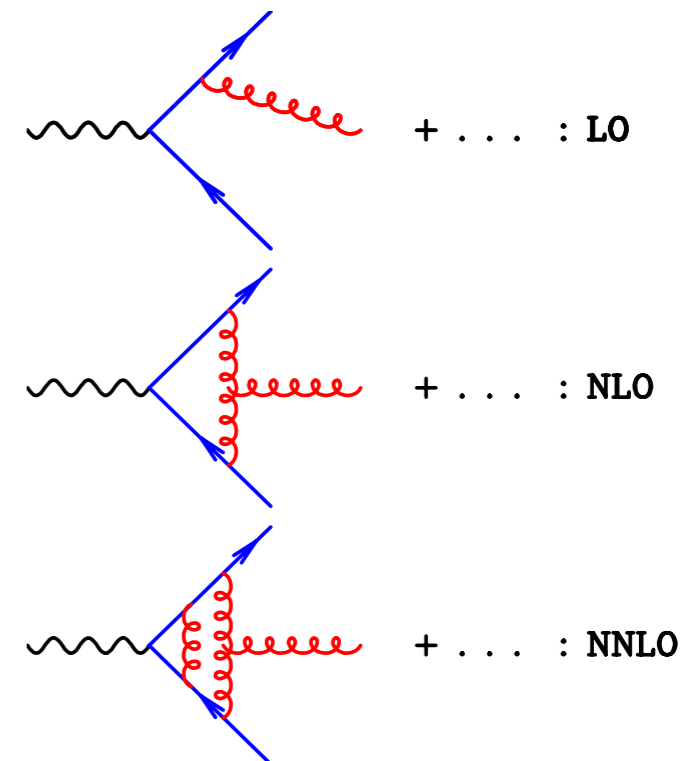
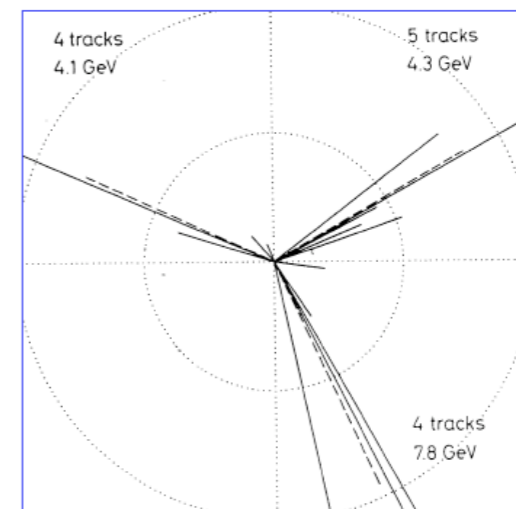
Dissertori et al, 0910.4283



requires a jet definition  
fit at  $y_{\text{cut}} = 0.02$

$$\alpha_s = 0.1175 \pm 0.0020(\text{exp}) \pm 0.0015(\text{th})$$

An early three jet event from Tasso (1979)



# Results for $\alpha_s$

The strong coupling at the scale  $M_Z$  is known to about 1%.

2009 World average

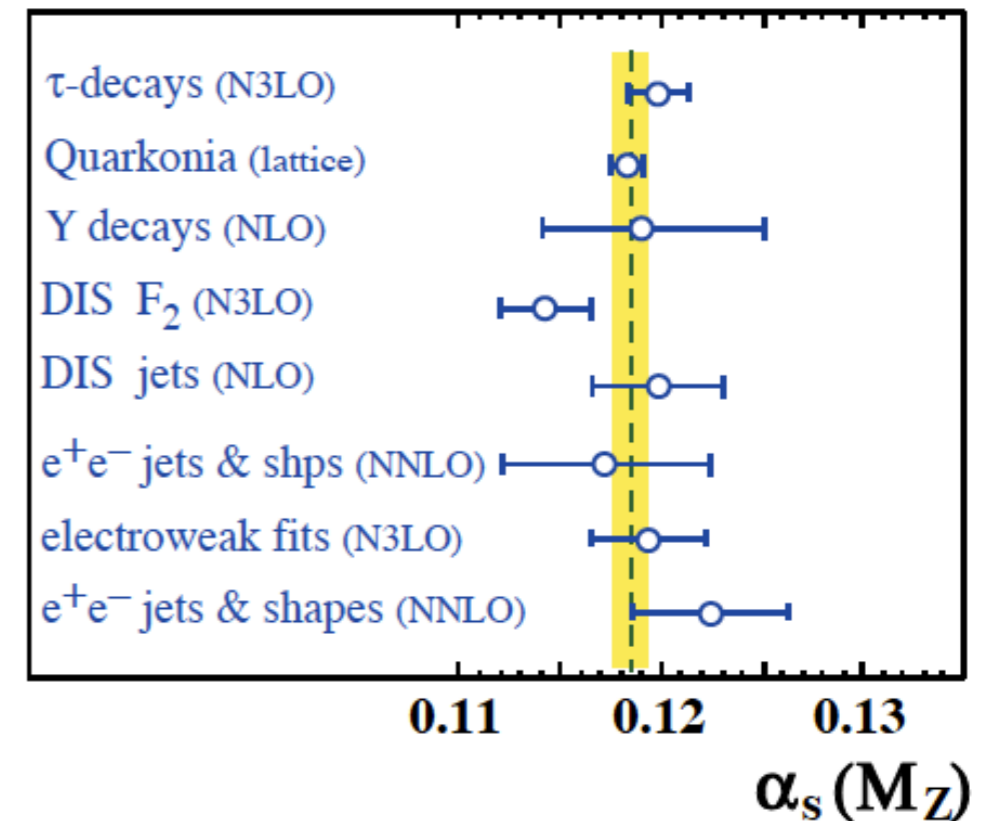
$$\alpha_s(M_Z) = 0.1184 \pm 0.0007$$

Currently the most precise value of  $\alpha_s$  comes from interpreting low energy measurements using lattice QCD.

The most precise high energy value is from jets and event shapes in  $e^+e^-$  annihilation.

The error on  $\alpha_s$  will not be the dominant error in making predictions for LHC.

Bethke, arXiv:0908.1135



$\alpha_s(M_Z)$  is smallish, but 16 times bigger than  $\alpha_{\text{QED}}(M_Z)$

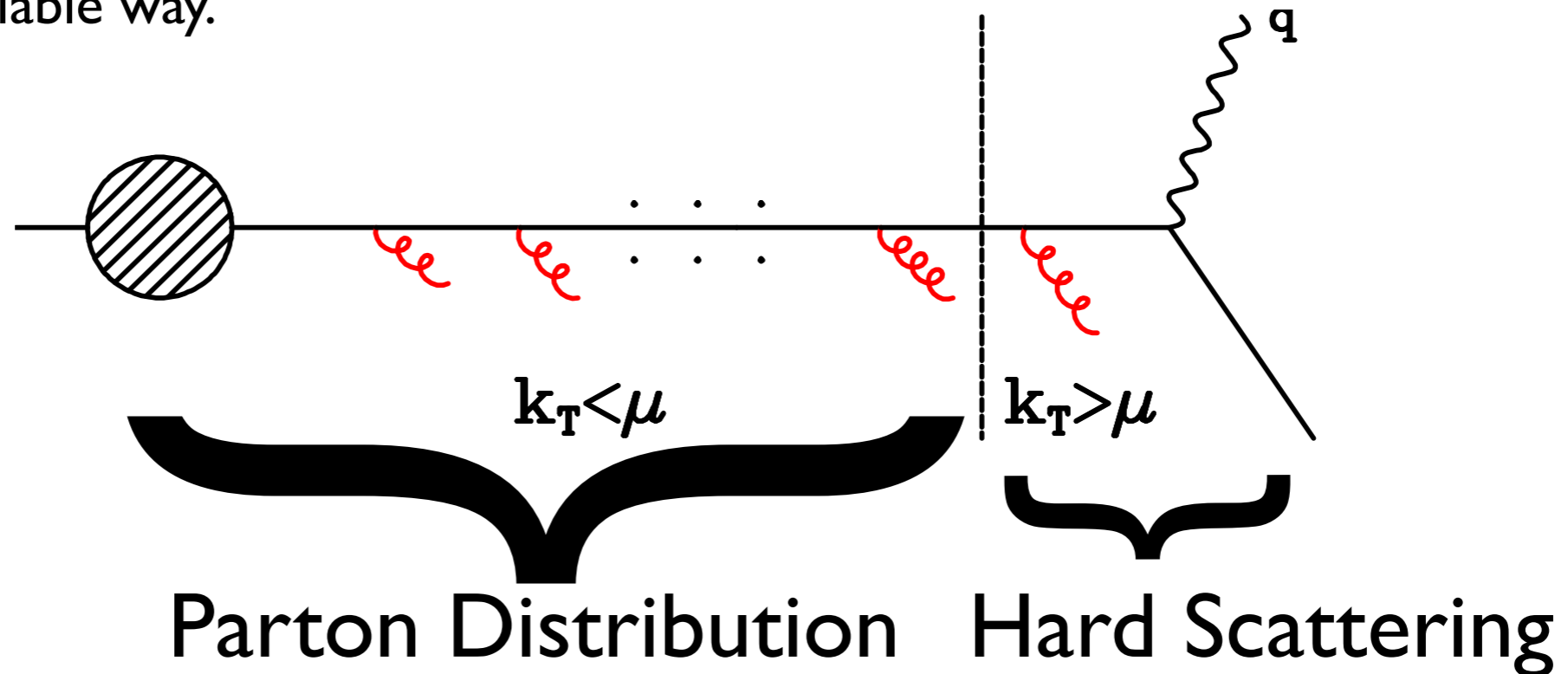
⇒ radiative corrections will be even more important than they are for QED

# The standard model and the proton

- We wish to use the standard model to describe the physical world composed of protons, pions, and kaons etc. Quarks and gluons are not observed as asymptotic states.
- The binding of quarks and gluons into protons is a non-perturbative problem, most systematically described by lattice gauge theory.
- Surprisingly, a large class of processes at high energy can be described by perturbative methods, akin to those used with such success in QED.
- a) IR safe quantities such as jet cross sections, which are insensitive to soft and collinear radiation (such as  $e^+e^-$  3-jet cross section)
- b) Quantities in which we can separate the high and low energy parts (factorizable quantities).

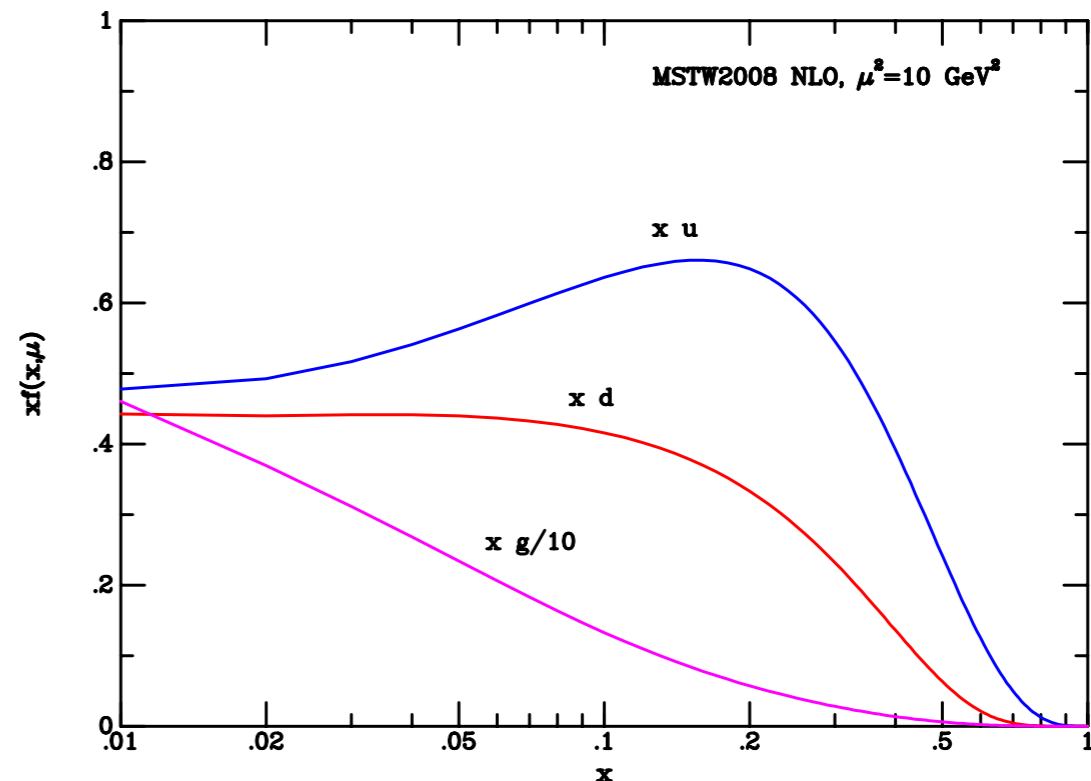
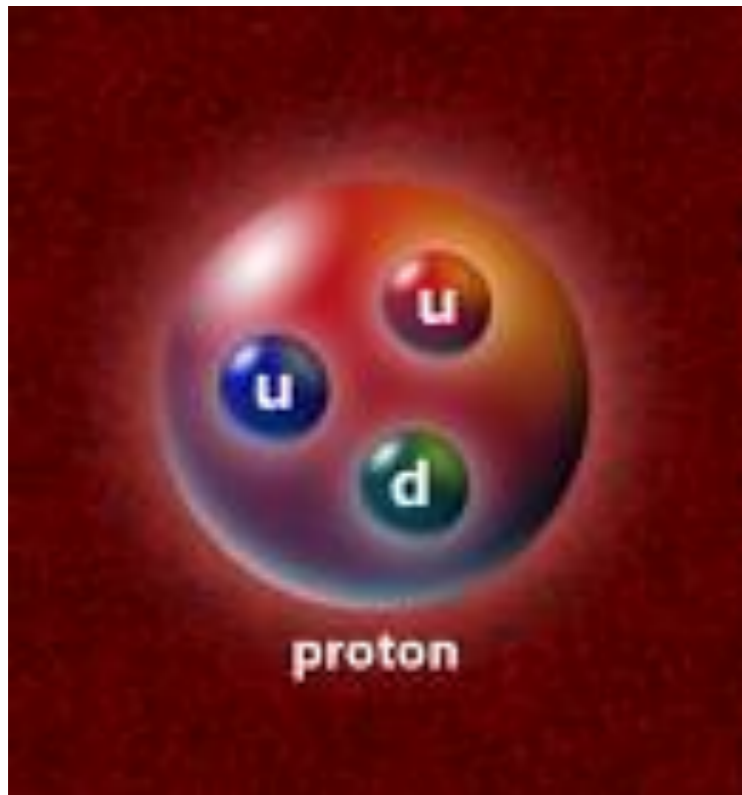
# Factorization

- Approach to deal with quantities which are IR sensitive.
- General idea is to factorize process independent low energy pieces into a lumped parameter which can be measured in one process and applied in another
- We have to introduce a factorization scale  $\mu$  to separate the low energy and high energy parts.
- The lumped parameter (the parton distribution function) depends on  $\mu$  in a calculable way.



# Proton structure

The lumped parameter is the parton distribution function.  $x$  is the fraction of the longitudinal momentum of the proton carried by a quark.

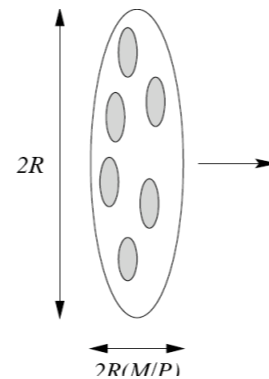


If we consider the proton to be made up of 3 quarks which carry half the momentum of the proton, the average momentum fraction per quark is  $\langle x \rangle = 1/6$ . Therefore (on average):-  
The Tevatron gives us  $q$ - $q$ bar collisions at the mass scale  $\sim 330$  GeV and the LHC@14TeV will give us  $q$ - $q$  collisions at the mass scale  $\sim 2.3$  TeV.

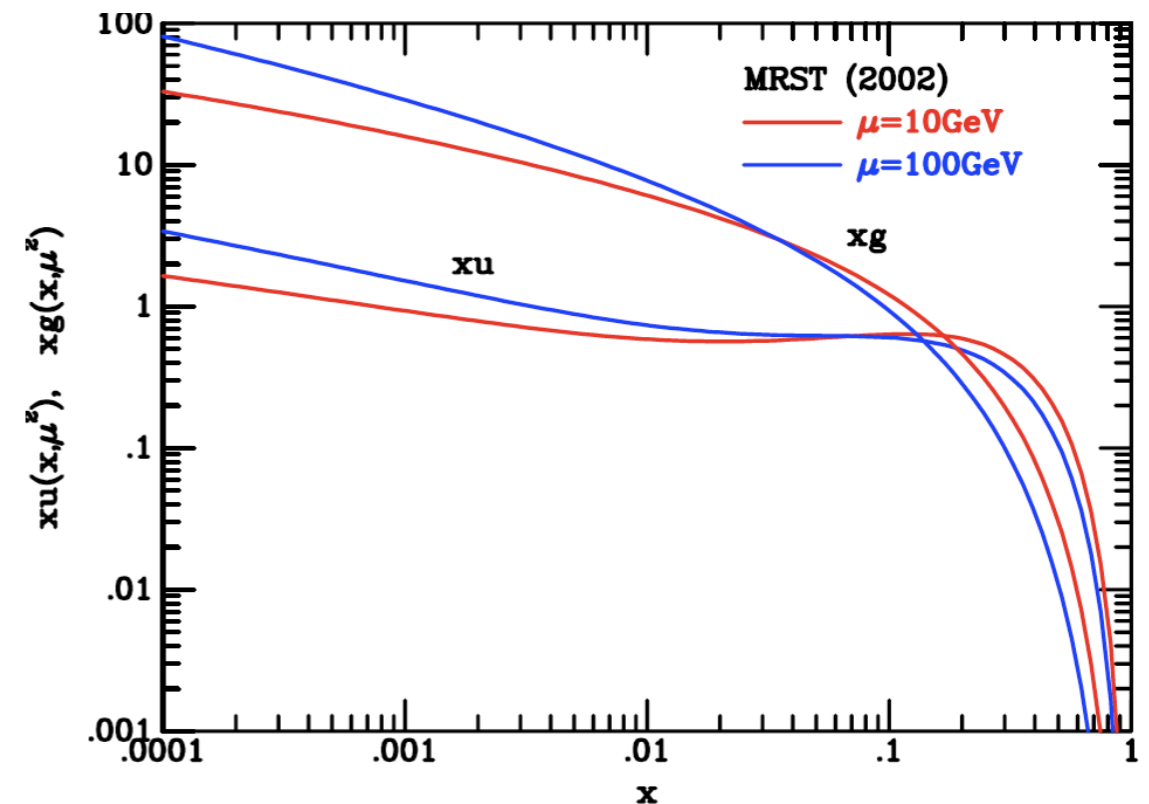
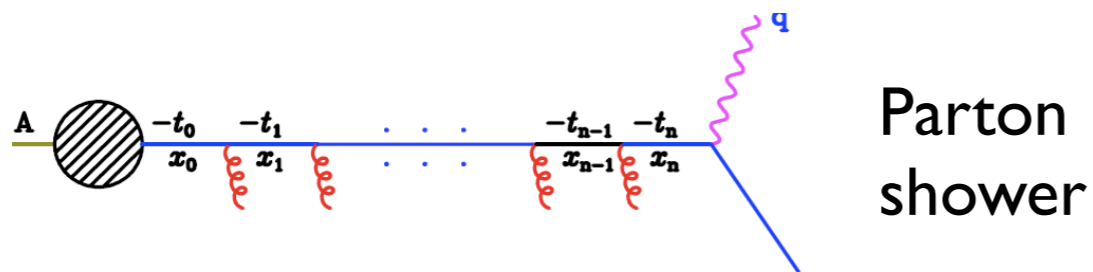


# The evolution of partons within the proton

At low resolution (small  $\mu$ ) a high energy proton can be viewed as a dilute system of partons.

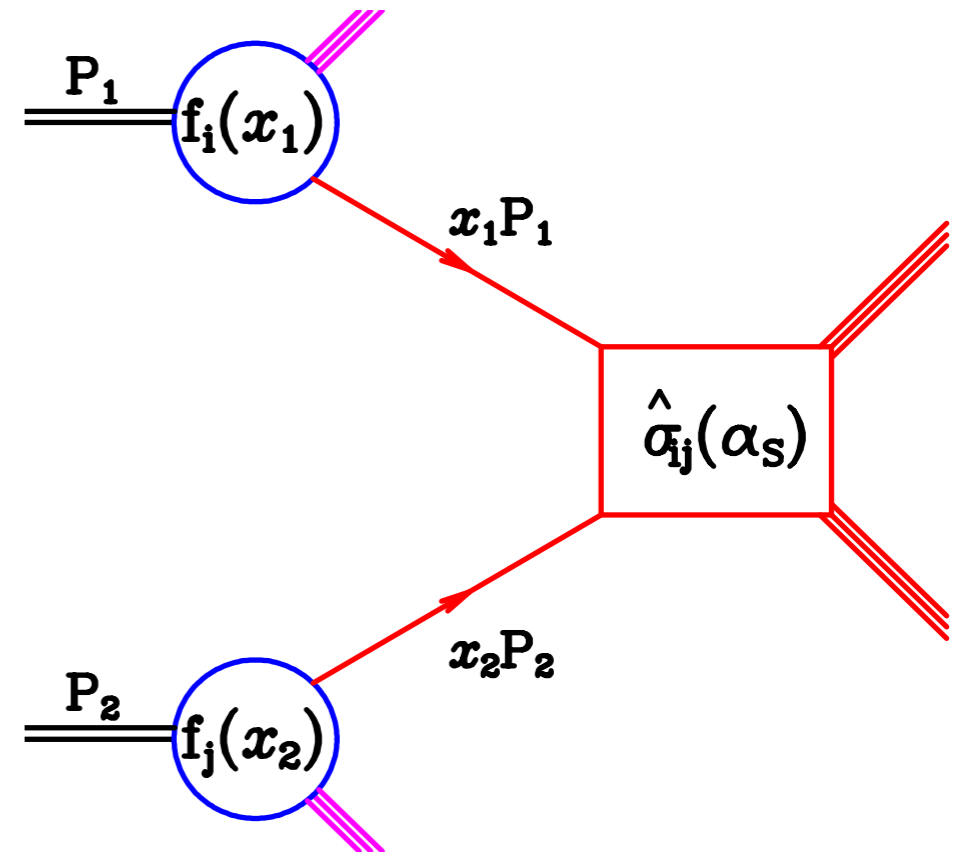


As we observe the proton with higher resolution (increasing  $\mu$ ) the number of partons grows, in a calculable way.



# QCD improved parton model

Hard QCD cross section is represented as the convolution of a short distance cross-section and non-perturbative parton distribution functions. Physical cross section is formally independent of  $\mu_F$  and  $\mu_R$



$$\sigma(P_1, P_2) = \sum_{i,j} \int dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_2, \mu_F) \hat{\sigma}_{ij}(p_1, p_2, \alpha_S(\mu_R), Q^2, \mu_R, \mu_F).$$

Physical cross section

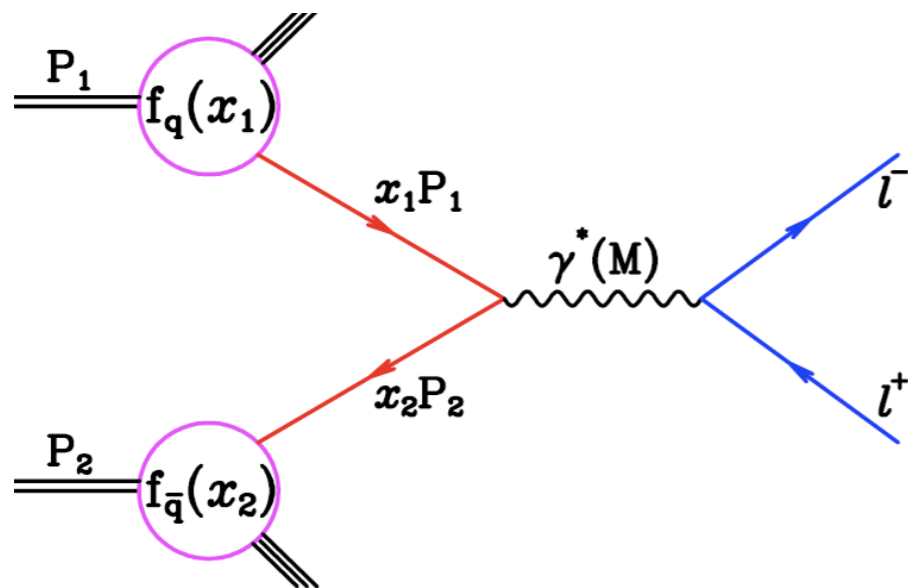
Parton distributions

Strong Coupling

short-distance cross section  $\sigma$  in LO, NLO, ...

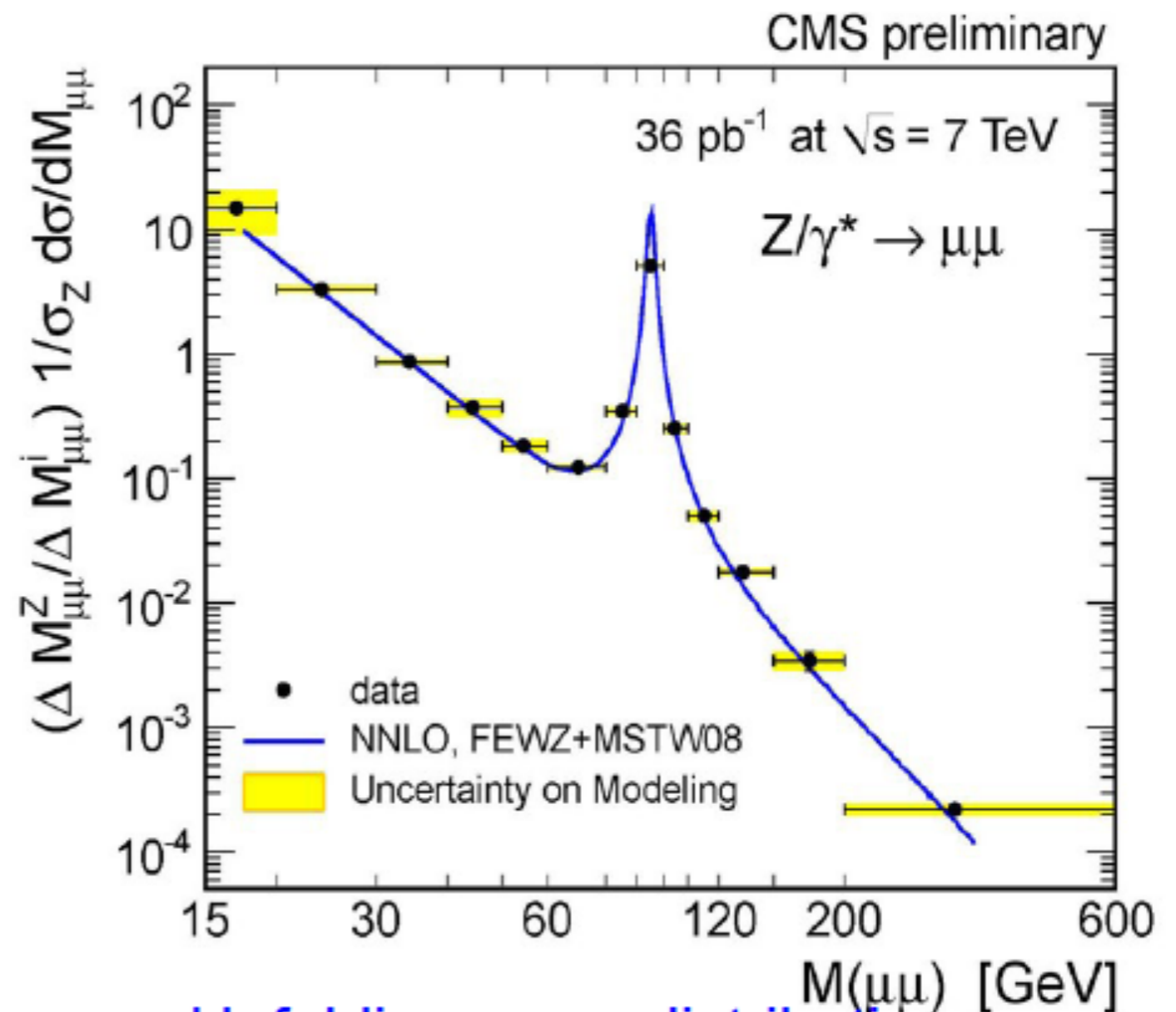
# Lepton pair production

This is the simplest process we can consider with two incoming hadrons.



$$(x_1 P_1 + x_2 P_2)^2 \approx x_1 x_2 S = M^2$$

$$z = M^2/s$$



# Drell-Yan type processes ( $\gamma^*$ , W, Z)

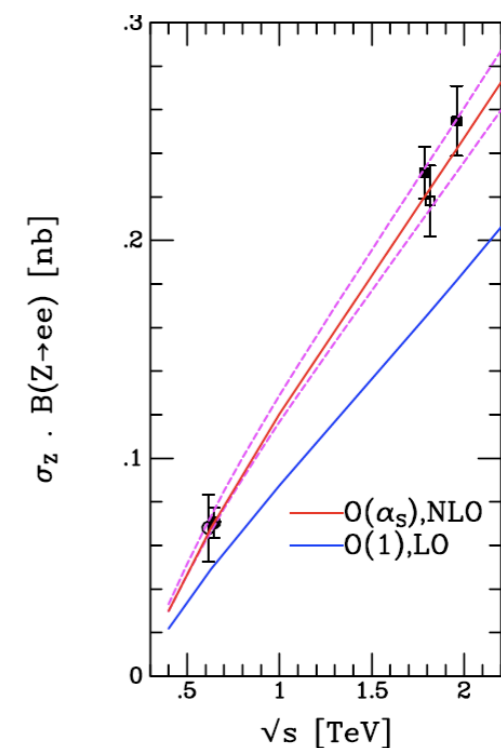
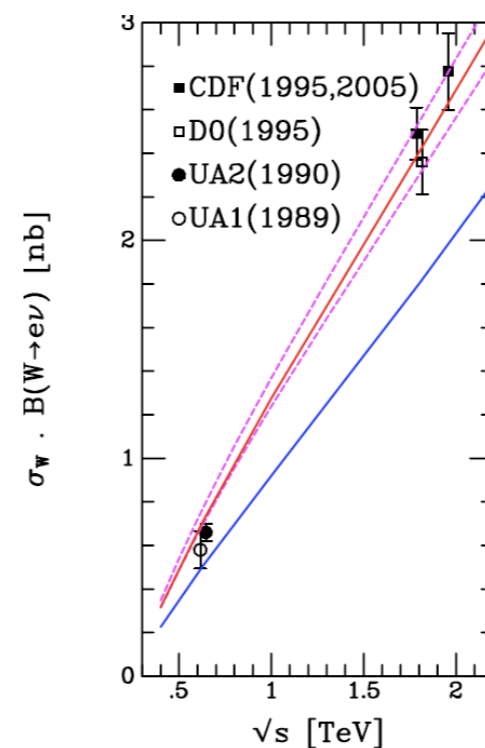
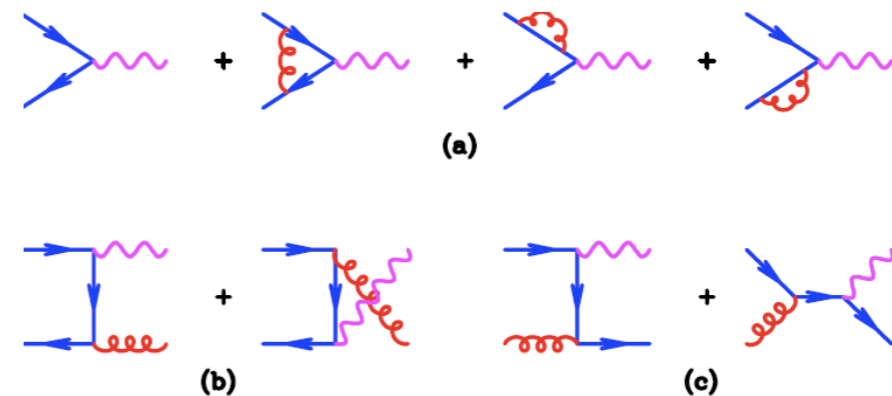
QCD provides a systematic way of improving the calculations of cross sections

$$\hat{\sigma}(z) = \delta(1-z) + \frac{\alpha_s}{2\pi} f_1(z) + \left(\frac{\alpha_s}{2\pi}\right)^2 f_2(z) + \dots$$

by expanding in the small coupling  $\alpha_s$ . Corrections are large at  $O(\alpha_s)$  but needed to achieve agreement with data.

( $\alpha_s^2$  corrections also known and lead to a further modest increase.)

## Diagrams for NLO prediction

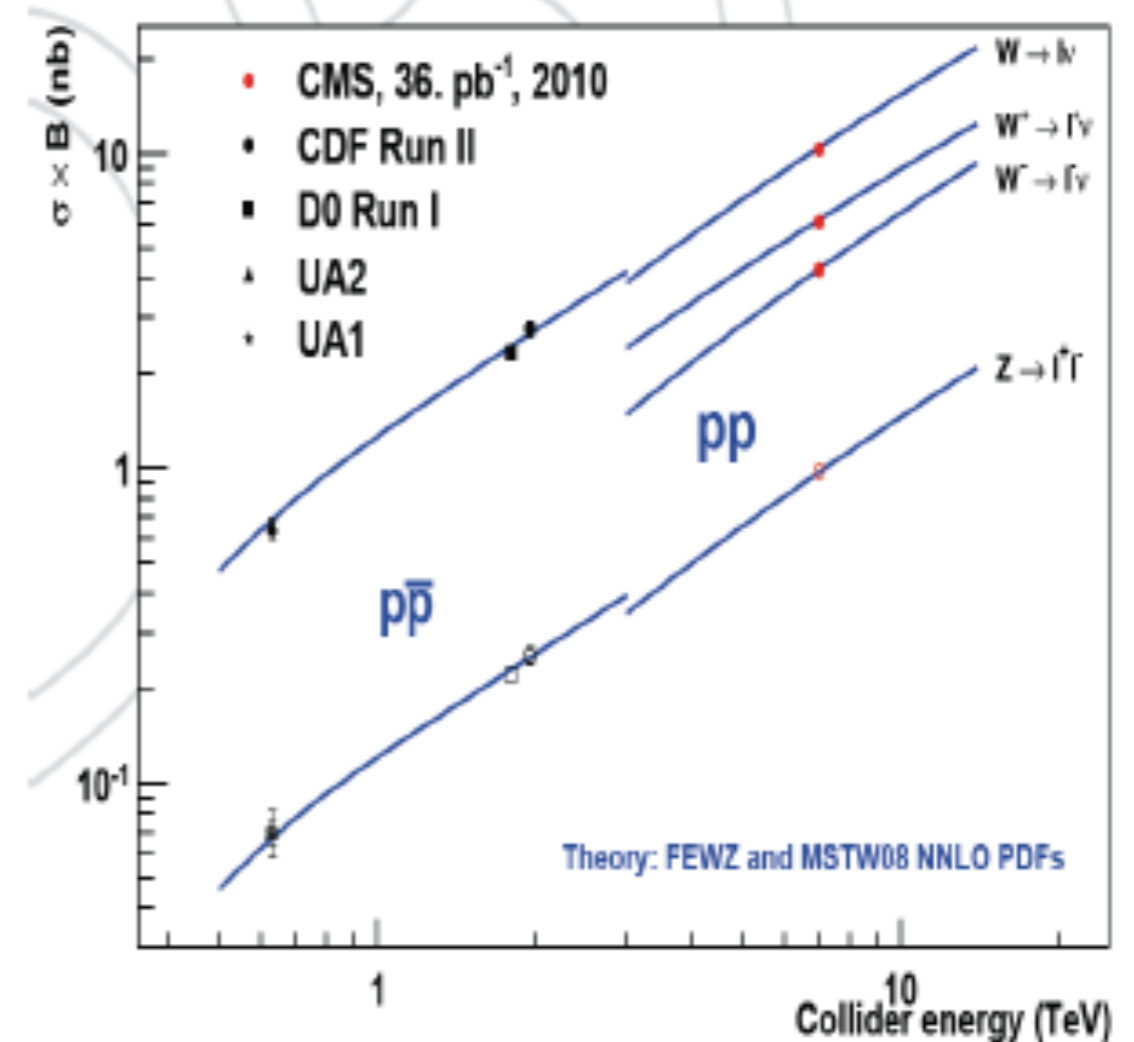


**Moral: at least NLO corrections are needed.**

# W production

New

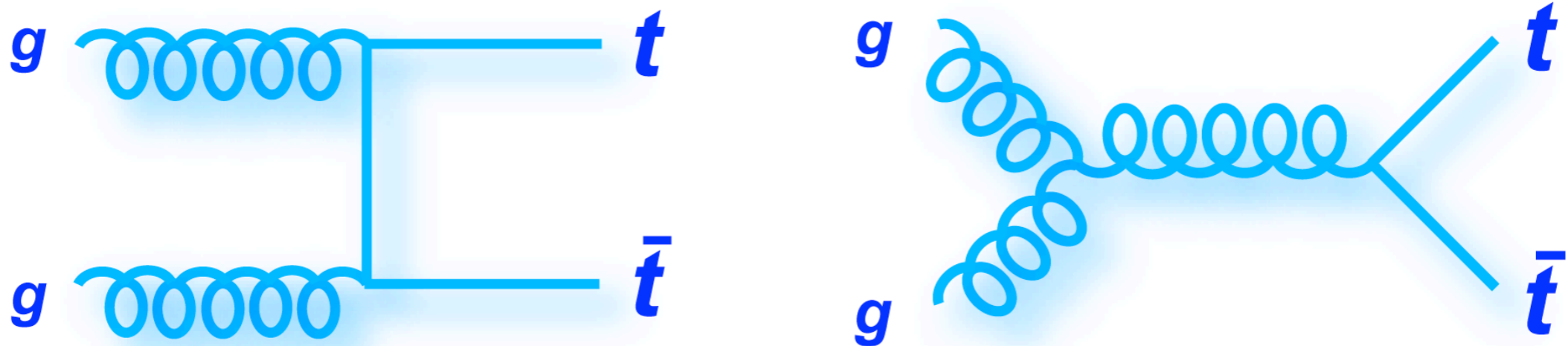
Results from  
 $\sqrt{s}=0.63 - 7 \text{ TeV}$



Stability of the perturbation series at  $\sqrt{s}=7 \text{ TeV}$

MSTW2008 (FEWZ)	LO	NLO	NNLO	Predominant Parton Process
$\sigma(W^+) \cdot B(W^+ \rightarrow \nu e^+) \text{ [nb]}$	5.06	6.20	6.28	$u \bar{d} + \bar{d} u$
$\sigma(W^-) \cdot B(W^- \rightarrow \nu e^-) \text{ [nb]}$	3.45	4.30	4.38	$d \bar{u} + \bar{u} d$

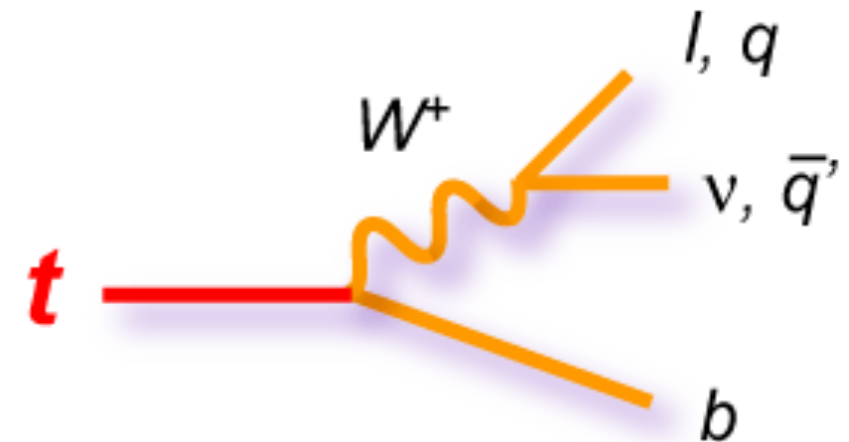
# Top quark production



- At the LHC the top quark is produced mainly by gluon-gluon fusion (80% at 7TeV).
- The top quark has a mass of  $\sim 172$  GeV, about the same as an atom of tungsten. QCD should work well for such a high mass scale.

# Decay of the top quark

The top quark decays by a semi-weak process



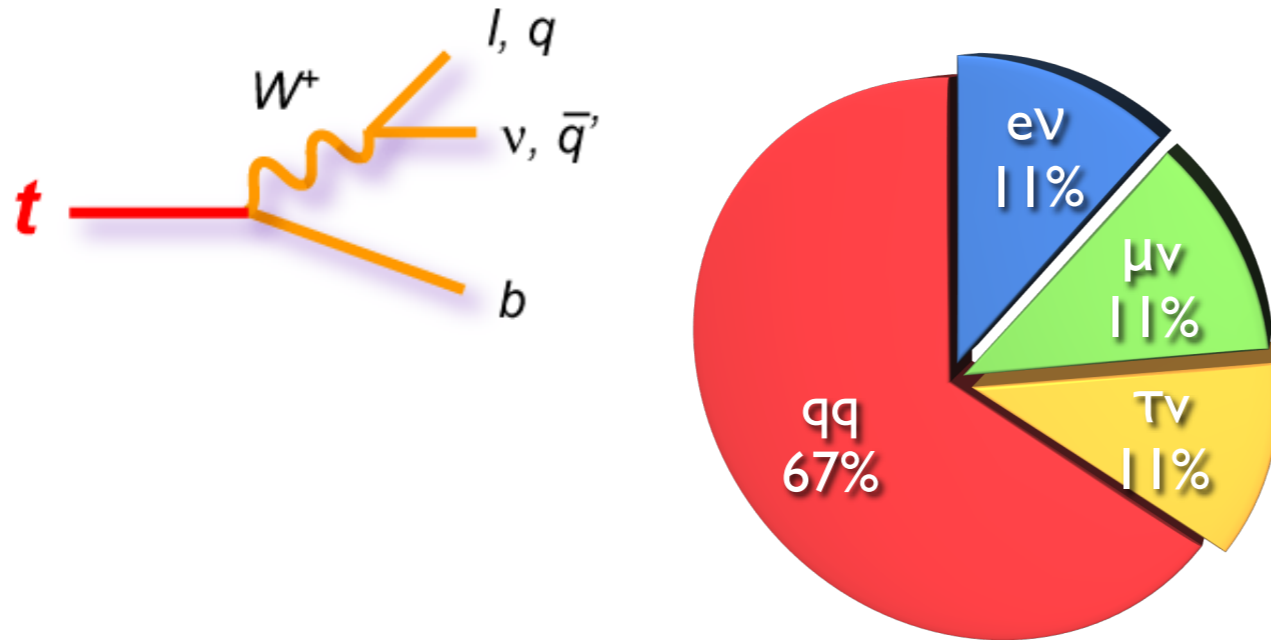
$$\Gamma(t \rightarrow bW^+) = \frac{G_F m_t^3}{8\pi\sqrt{2}} |V_{tb}|^2 \left(1 - \frac{M_W^2}{m_t^2}\right)^2 \left(1 + \frac{2M_W^2}{m_t^2}\right)$$

With three generations of quarks we can constrain the Cabibbo-like angle  $V_{tb} = 0.9991 \pm 0.0003$ , so that for  $m_t = 172$  GeV and  $M_W = 80.41$  GeV

$$\Gamma(t \rightarrow bW^+) = 1.46 \text{ GeV} \implies \tau_t = 0.45 \times 10^{-24} \text{ s}$$

The time scale of top decay is shorter than a typical hadronic scale.  
The top quark decays before it has time to form hadrons.

# Top pair decay



Top quark branching fractions are fixed by the decay modes of the  $W$ .

At the LHC (and Tevatron) the  $t\bar{t}$  process is most easily observed in the lepton+jets, or dilepton channel.

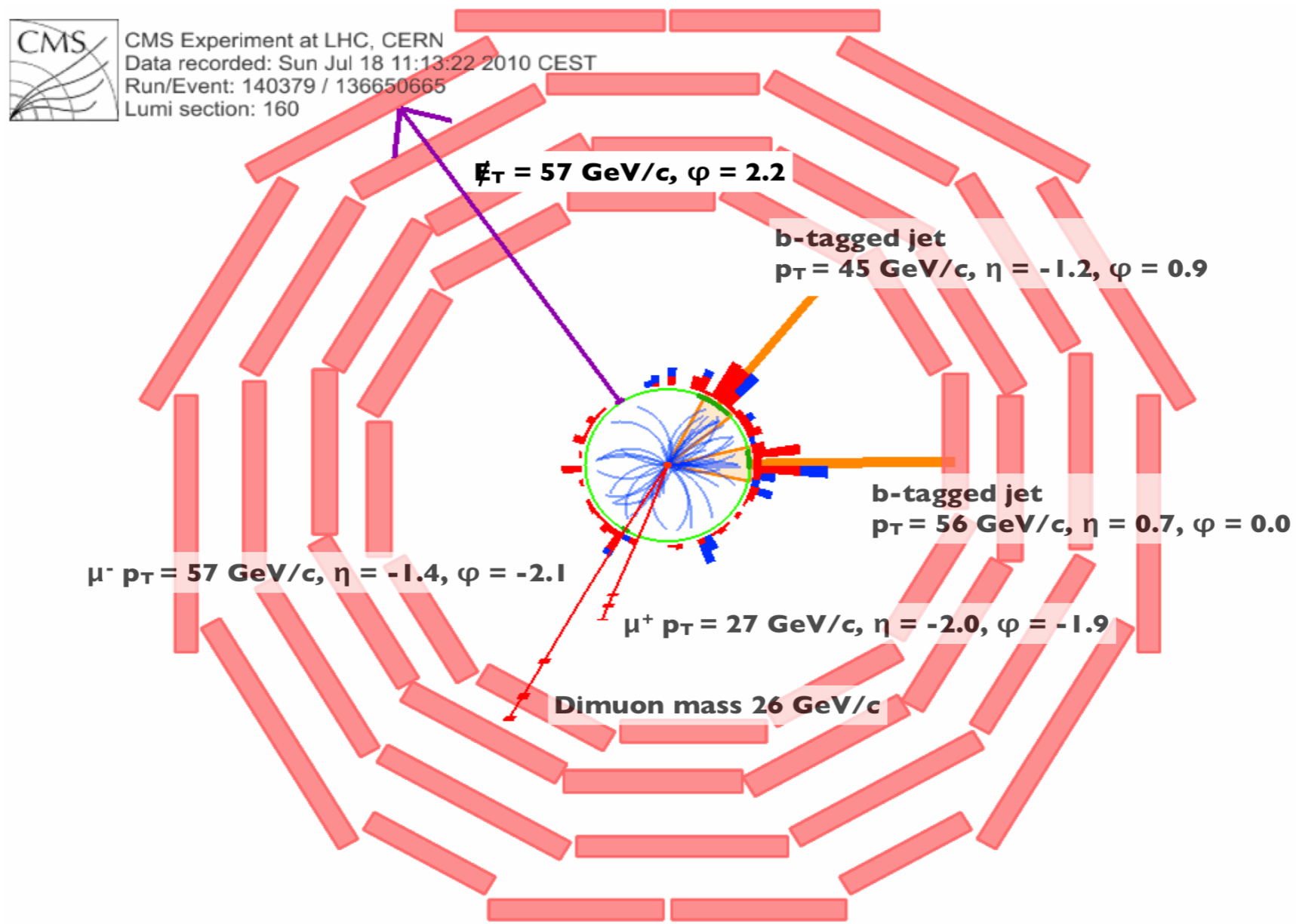
Top Pair Decay Channels

$\bar{c}s$	electron+jets	muon+jets	tau+jets	all-hadronic	
$\bar{u}d$					
$\tau^-$	$e\tau$	$\mu\tau$	$\tau\tau$	tau+jets	
$\mu^-$	$e\mu$	$\mu\mu$	$\mu\tau$	muon+jets	
$e^-$	$e\bar{e}$	$e\mu$	$e\tau$	electron+jets	
$W$ decay	$e^+$	$\mu^+$	$\tau^+$	$u\bar{d}$	$c\bar{s}$





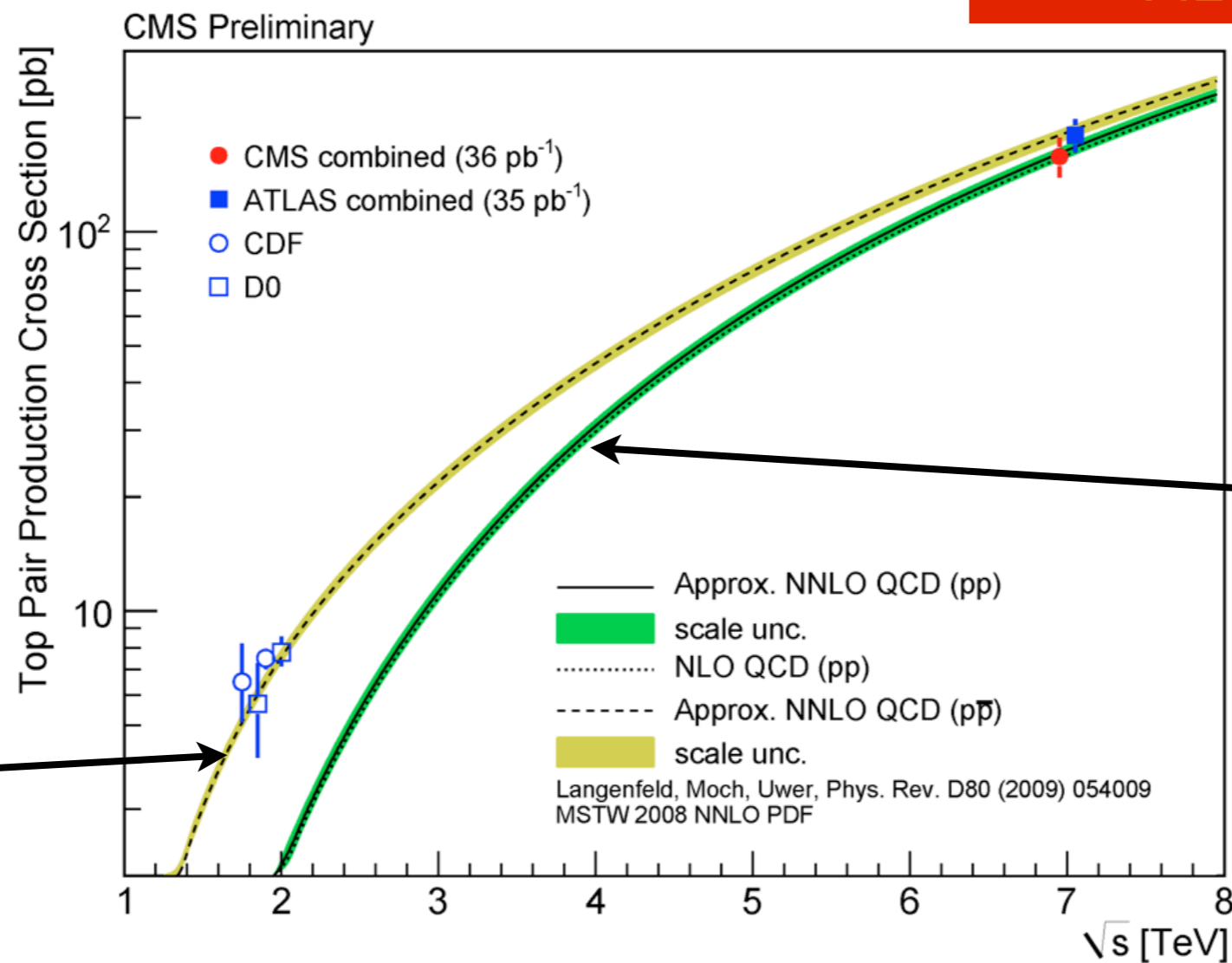
# Alternative view of this event



Reconstructed mass is in the range  $160\text{--}220 \text{ GeV}/c^2$  (consistent with  $m_{\text{top}}$ )

# Top quark cross section measurements at LHC

Theory curves similar to NLO prediction



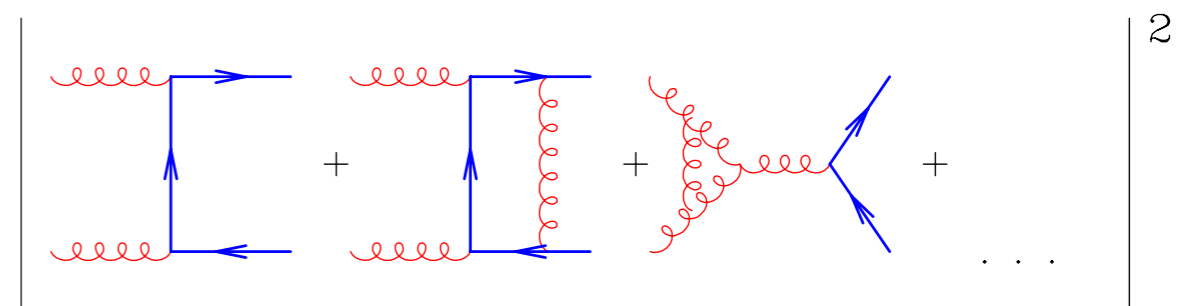
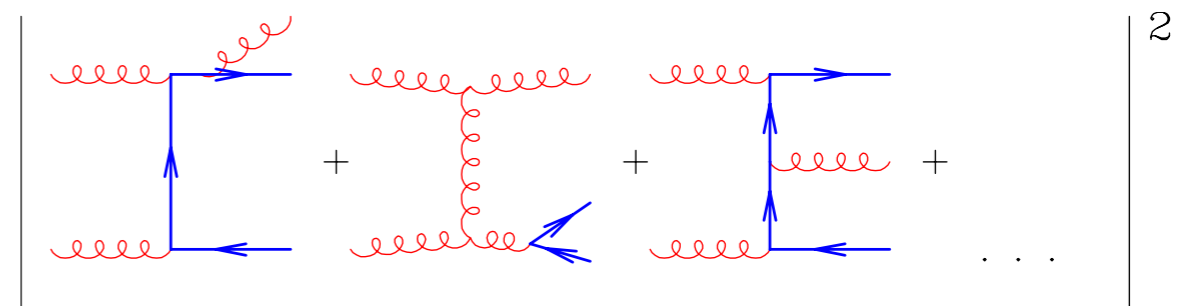
proton  
anti-proton

proton proton

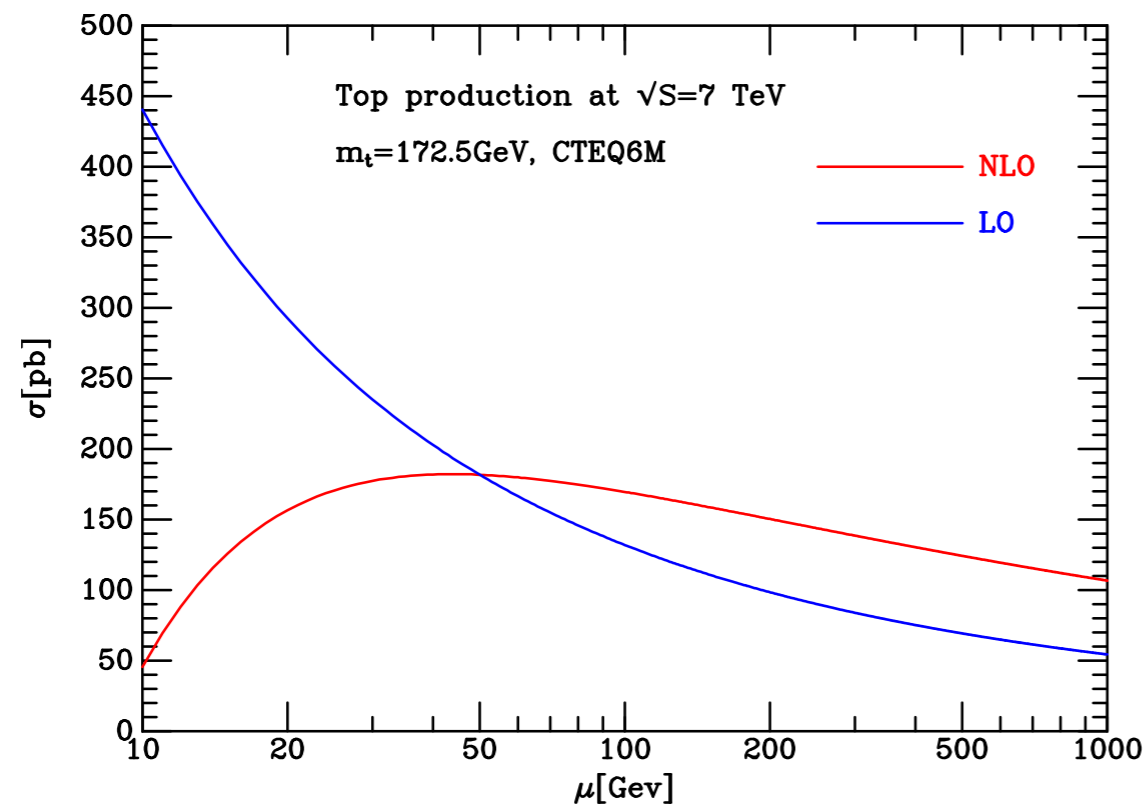
# Ingredients of a NLO calculation

- Born process (LO).
- Interference of one-loop with LO
- Real radiation
- Theoretical issues are efficient calculation of phase space and calculation of loop diagrams.

Example  $gg \rightarrow t\bar{t}$



# 10% predictions from perturbative QCD?



At high energy the short distance cross sections are calculable in the

$$= c_0 \alpha_S^n + c_1 \alpha_S^{n+1} + c_2 \alpha_S^{n+2} \dots$$

LO      NLO      NNLO

Although  $\alpha_S$  is of order 0.1, leading order is not sufficient for 10% corrections. This problem becomes more acute for large  $n$ , ie, as the number of jets increases.

$\mu$  is the renorm/factorization scale-an unphysical parameter

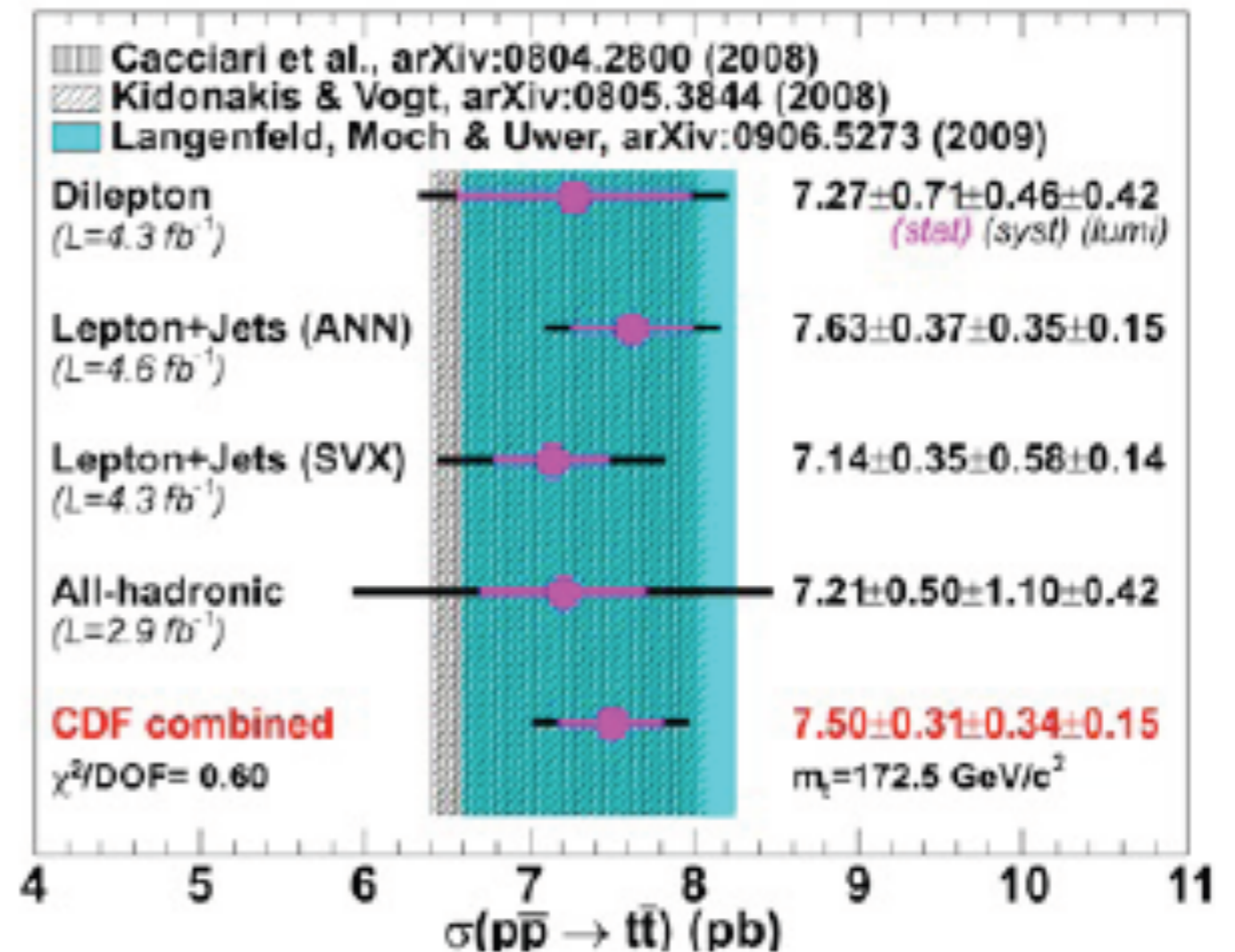
$$\mu \frac{d\sigma}{d\mu} = 0$$

$$\sigma_{tt}^{NLO} = c_2 \alpha_S^2 + c_3 \alpha_S^3 \text{ therefore } \mu \frac{d\sigma_{tt}^{NLO}}{d\mu} = O(\alpha_S^4)$$

In order to get ~10% accuracy we need to include at least NLO.

# At the Tevatron

Precision of the NLO theory is challenged by experiment.

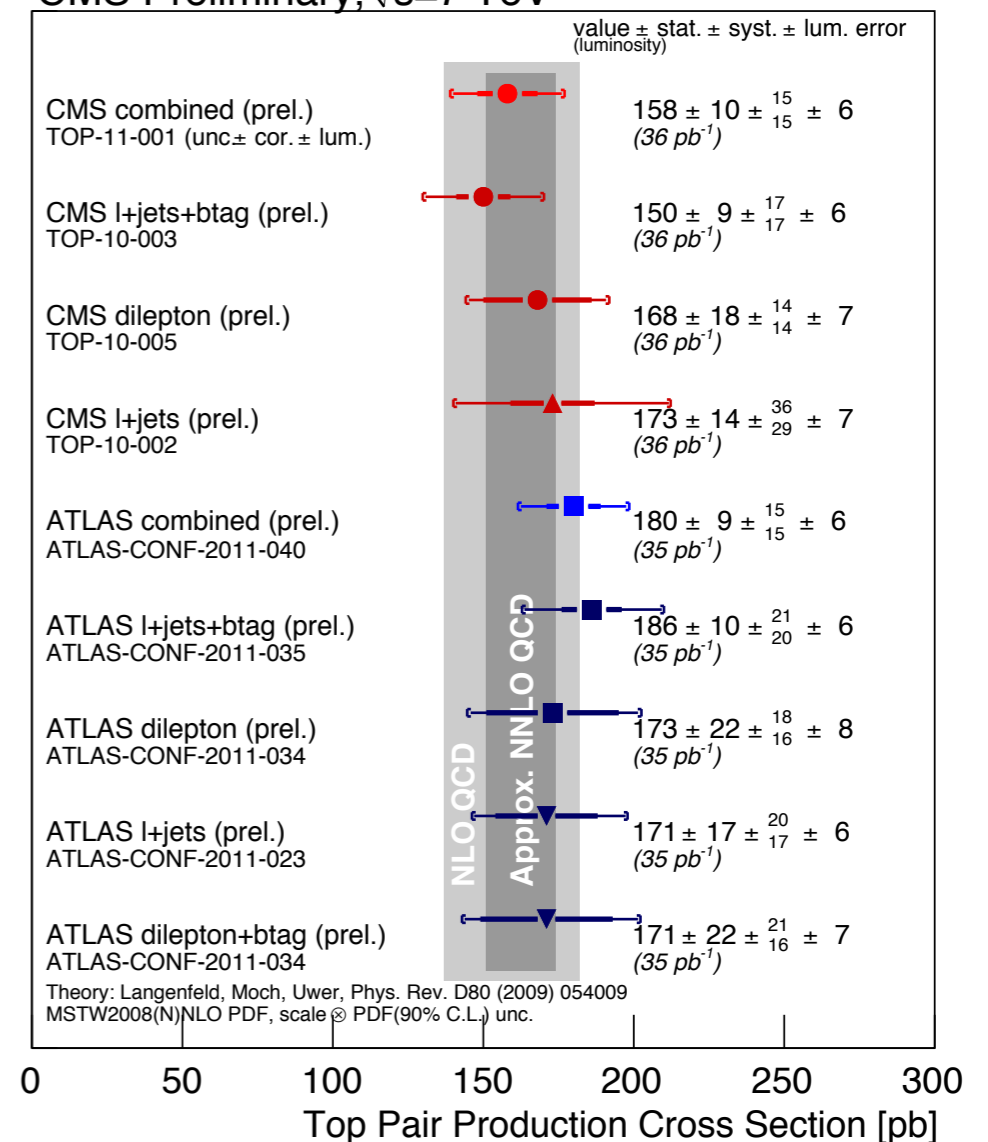


Studies ongoing to extend theory prediction to NNLO ...

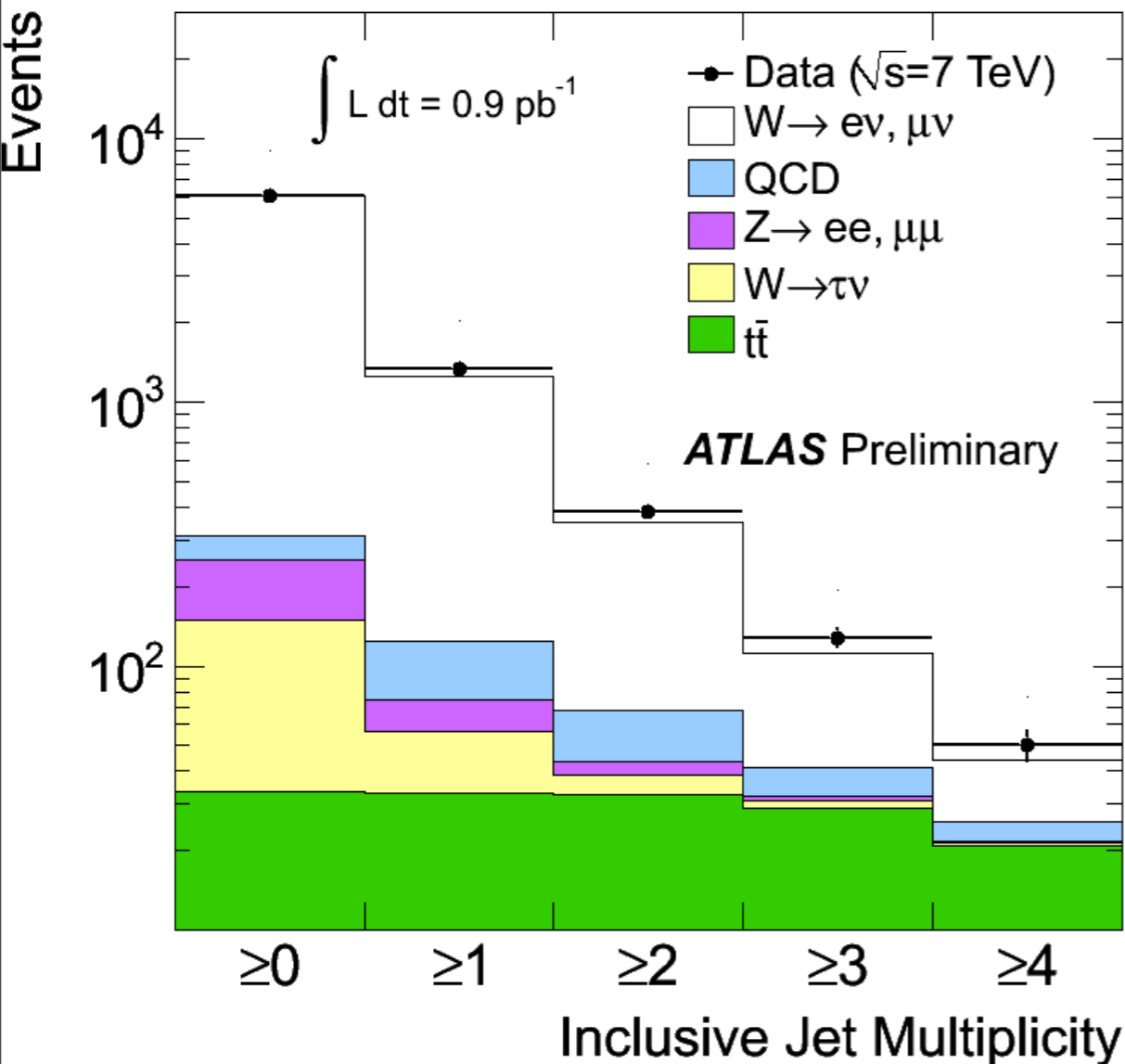
# Comparison of top production cross section with theory at $\sqrt{s}=7$ TeV

Data is already so precise with only  $35\text{pb}^{-1}$  that theory calculated in NLO has an estimated error larger than experimental error.  
(NNLO calculations under way).

CMS Preliminary,  $\sqrt{s}=7$  TeV



# W+jets data at 7 TeV



So W + 4 jet events are here and we need to have (at least) NLO theoretical predictions for them.

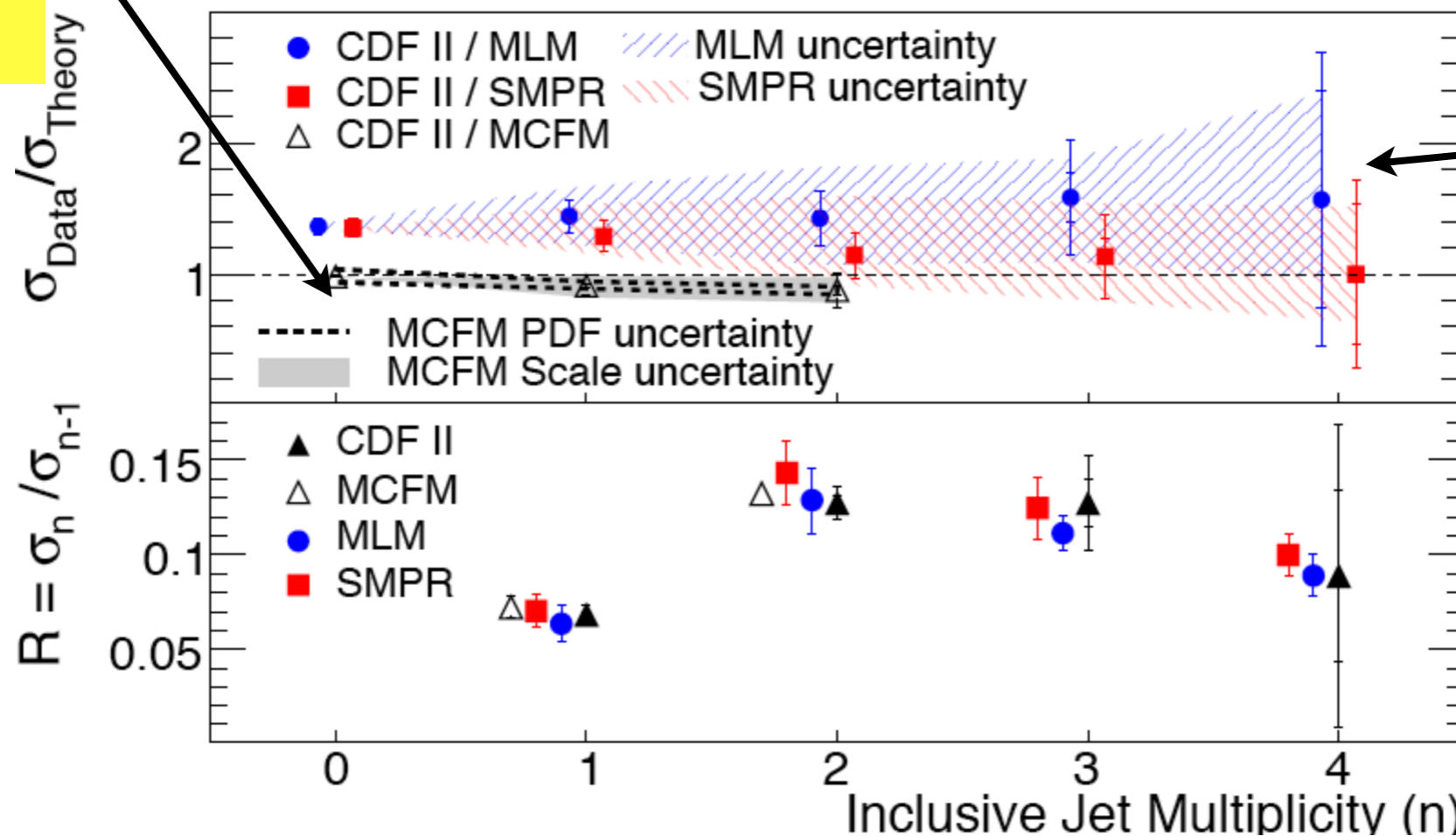
No published measurements to compare with yet at LHC, but at the Tevatron....



# W + n-jet rates from CDF

Aaltonen et al., arXiv: 0711.4044

NLO results

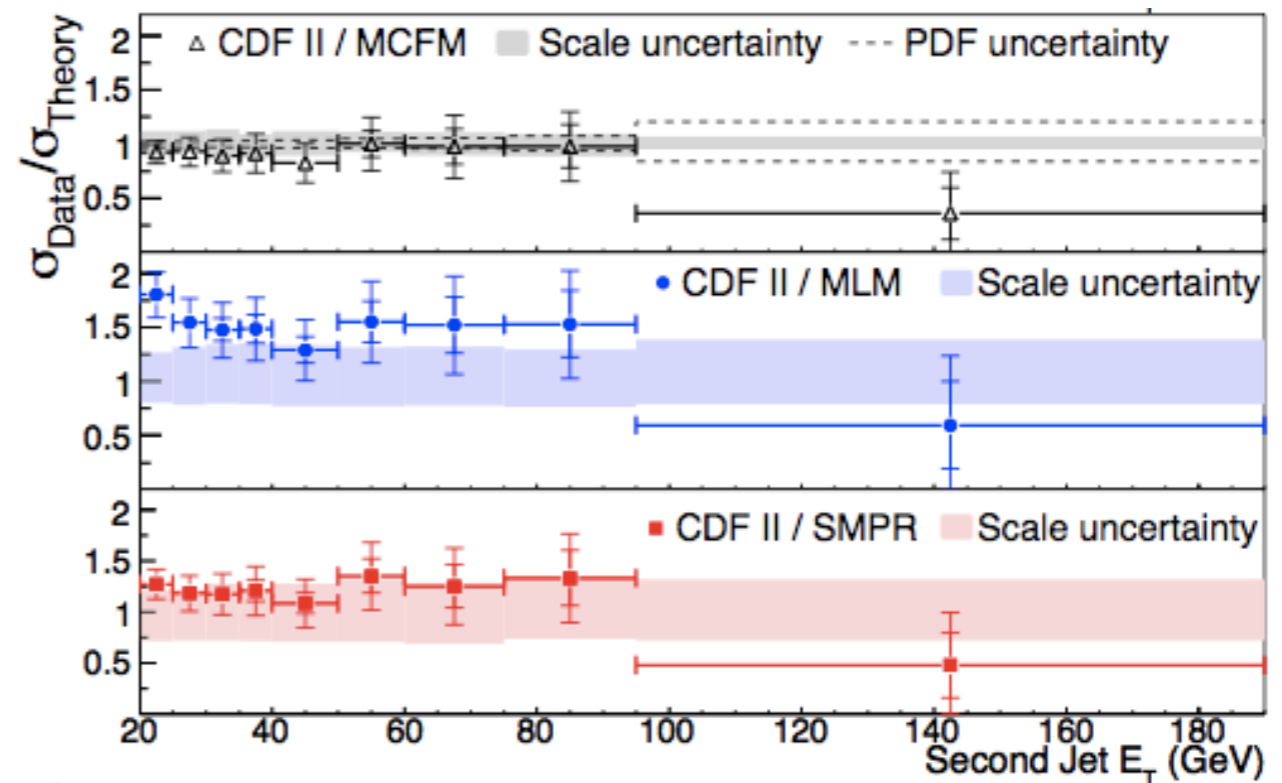


Tree graphs

Both uncertainty on rates and deviation of Data/Theory from 1 are smaller in MCFM (NLO) than in other calculations. The ratio R agrees well for all theory calculations, but only available from MCFM for  $n < 3$  in 2007. Note the increasing uncertainty with the number of jets in tree graph calculations.

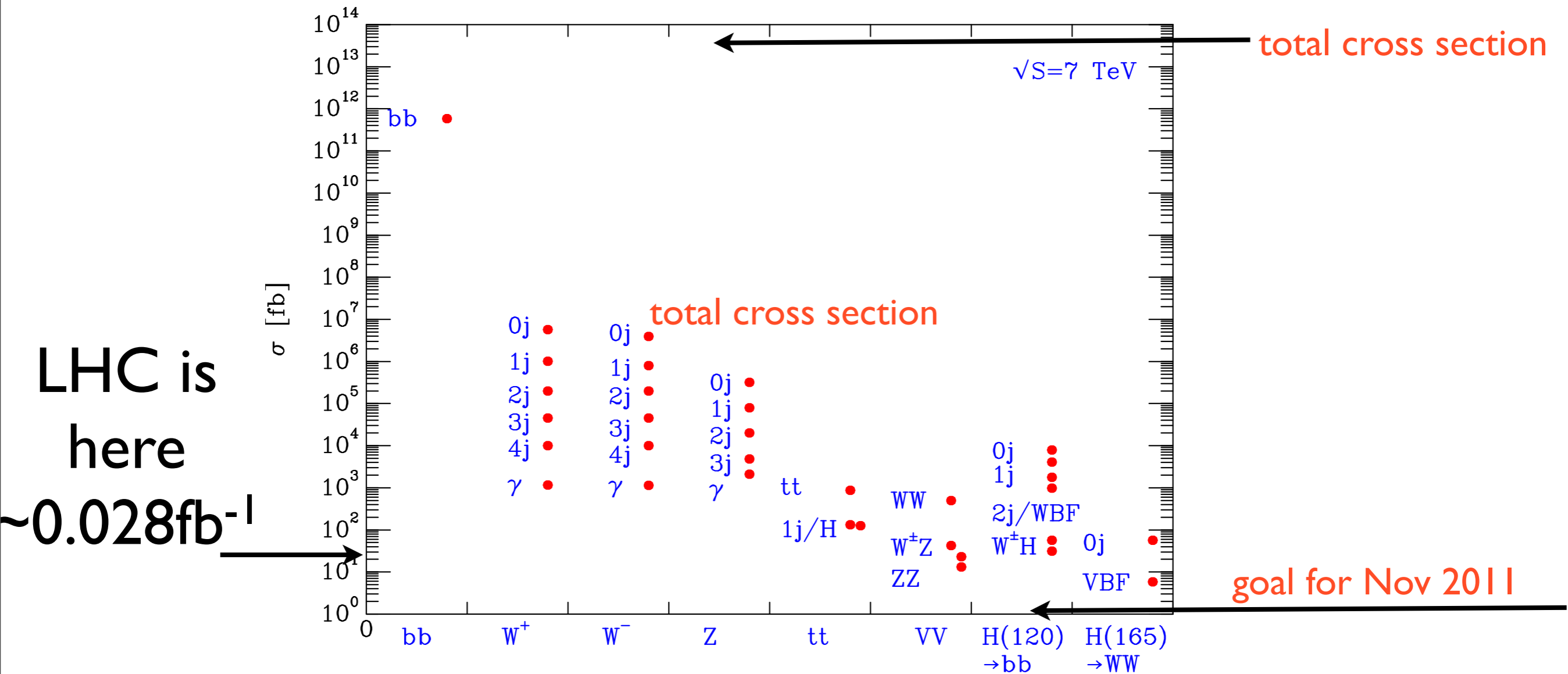
# W+n jet results at the Tevatron

W+0, 1 and 2 jet rates from NLO (MCFM) have been compared successfully (including shapes in  $E_T$ ) with data at the Tevatron.



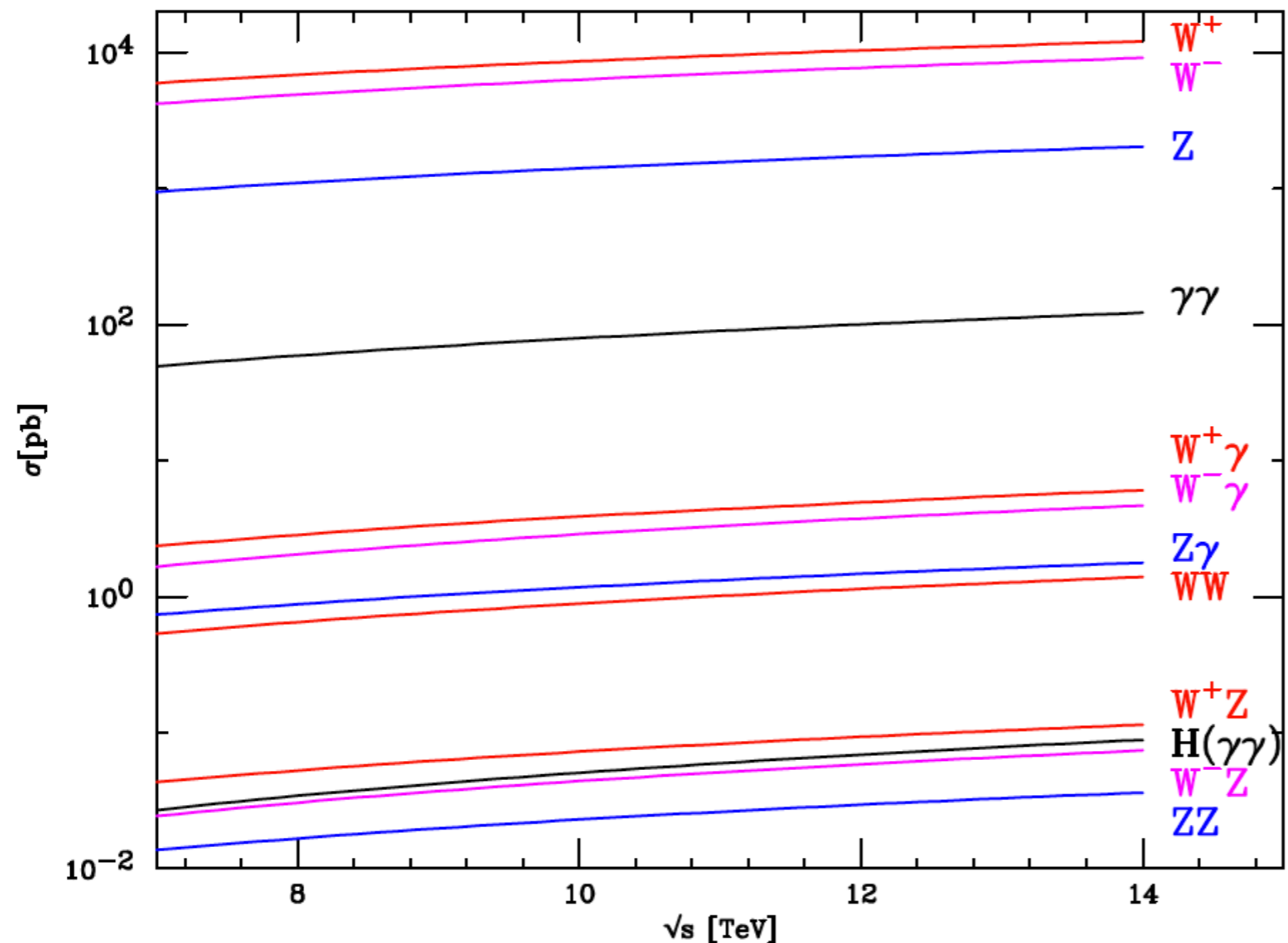
Success of W+ 1,2 jets predictions and the fact that LO uncertainties become larger as the number of jets increases -- strong motivation to calculate vector boson + 3,4.... jets

# SM Ladder at 7 TeV



Includes decay of W/Z to one species of charged lepton and semi-leptonic decay of top ( $t \rightarrow b l \nu$ ) (where applicable) and jets,  $E_t > 25$  GeV.

# Ladder of EW processes at the LHC 7 TeV



Cross sections including decay of vector bosons to one species of charged lepton (where appropriate)

10 events at end of 2011

All of these processes will be examined in 2011.

# Industrial approach to NLO

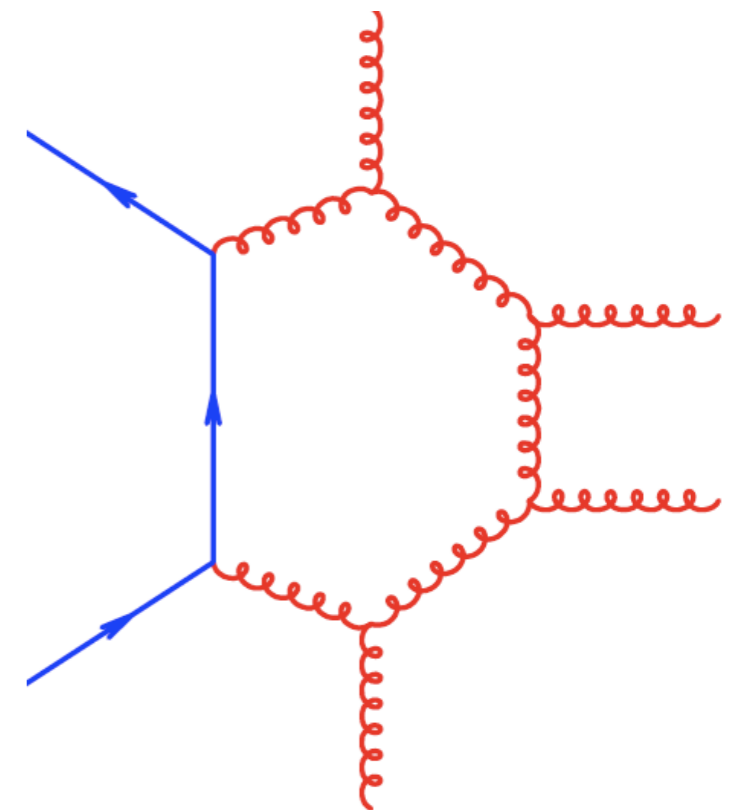
- Preceding examples show us that NLO calculations can really improve the quality of the predictions.
- This will be important as we rediscover the SM at the LHC and also for the estimation of backgrounds to BSM physics.
- Backgrounds are best estimated from data, but in many circumstances it is helpful to have corroborating theoretical estimates.
- Hence an industrial style approach to NLO QCD is needed.
- Automatic generation of NLO corrections is the wave of the future.

# Calculation of one-loop amplitudes

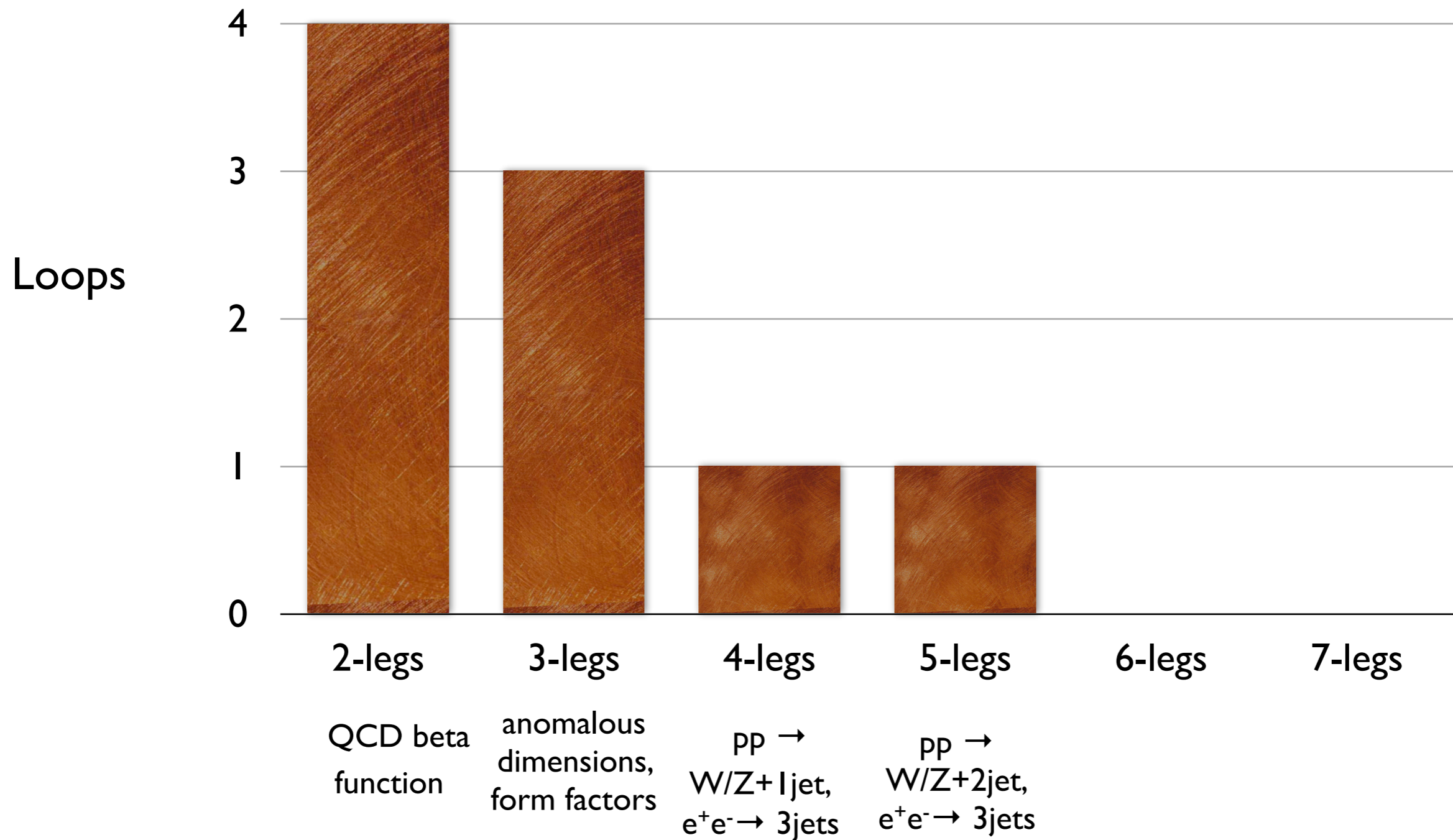
Feynman diagrams can be the answer for a moderate number of external legs, but will not be the answer as the number of legs increases. There are too many diagrams with cancellations between them.

Process	Amplitude	# of diagrams at 1 loop
$t\bar{t}$	$t\bar{t}gg$	30
$t\bar{t}+1$ jet	$t\bar{t}ggg$	341
$t\bar{t}+2$ jets	$t\bar{t}gggg$	4341
$t\bar{t}+3$ jets	$t\bar{t}ggggg$	63800

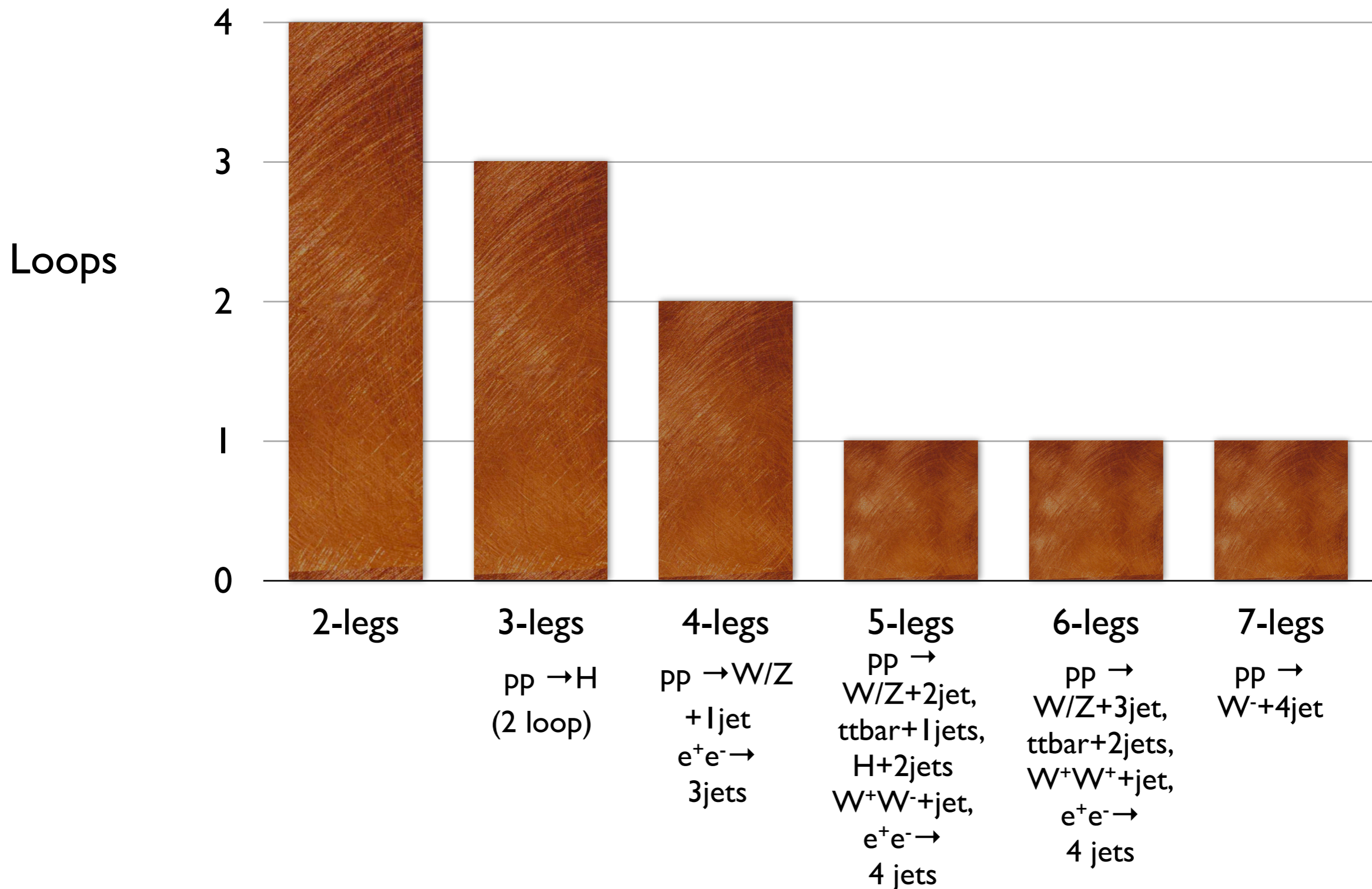
One of the 4,341 Feynman diagrams



# Loops and legs (circa 2007)



# Loops and legs - 2011





# Ingredients in a one-loop calculation

- For NLO calculations, any one-loop amplitude (no matter how many legs) can be written as a sum of sums of scalar boxes, triangles, bubbles and tadpoles (+ so-called rational piece).

$$A = \sum d_j \text{ (box) } + \sum c_j \text{ (triangle) } + \sum b_j \text{ (bubble) } + \sum a_j \text{ (tadpole)}$$

- The determination of the coefficients,  $d_j, c_j, b_j, a_j$  can be determined by semi-numerical methods, especially D-dimensional unitarity.
- The scalar integrals are all known analytically, see e.g. [QCDLoop.fnal.gov](http://QCDLoop.fnal.gov), (RKE,Zanderighi)

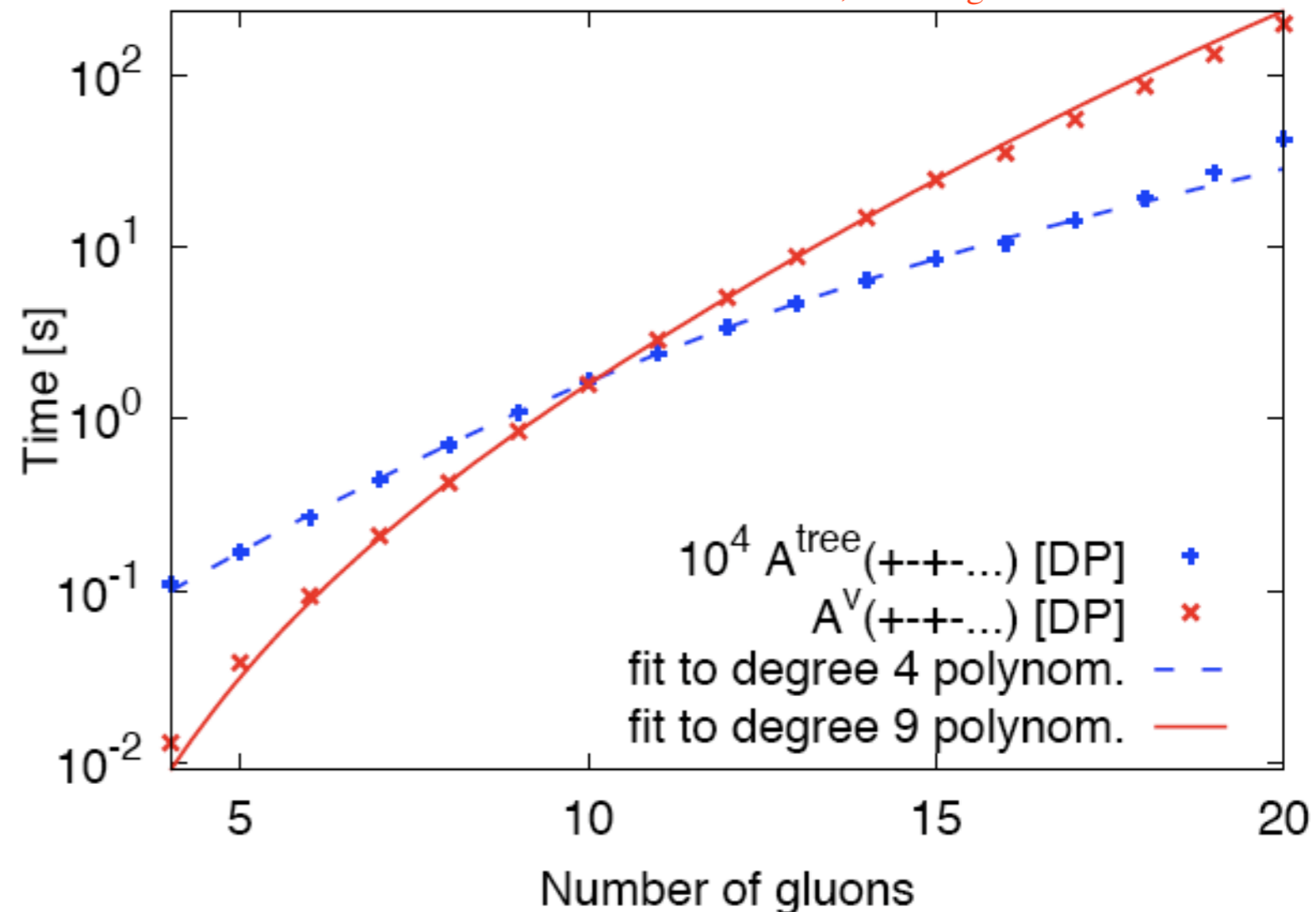
$$I_4^D(p_1^2, p_2^2, p_3^2, p_4^2; s_{12}, s_{23}; m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\mu^{4-D}}{i\pi^{\frac{D}{2}} \Gamma} \times \int d^D l \frac{1}{(l^2 - m_1^2 + i\epsilon)((l + q_1)^2 - m_2^2 + i\epsilon)((l + q_2)^2 - m_3^2 + i\epsilon)((l + q_3)^2 - m_4^2 + i\epsilon)},$$

# One loop calculation of pure gluon amplitudes

Giele, Zanderighi arXiv:0805.2152

Time to calculate one-loop amplitude scales as  $N^9$  as expected. For small numbers of legs  $N=4,5,6$  the times are of the order of 10's of milliseconds

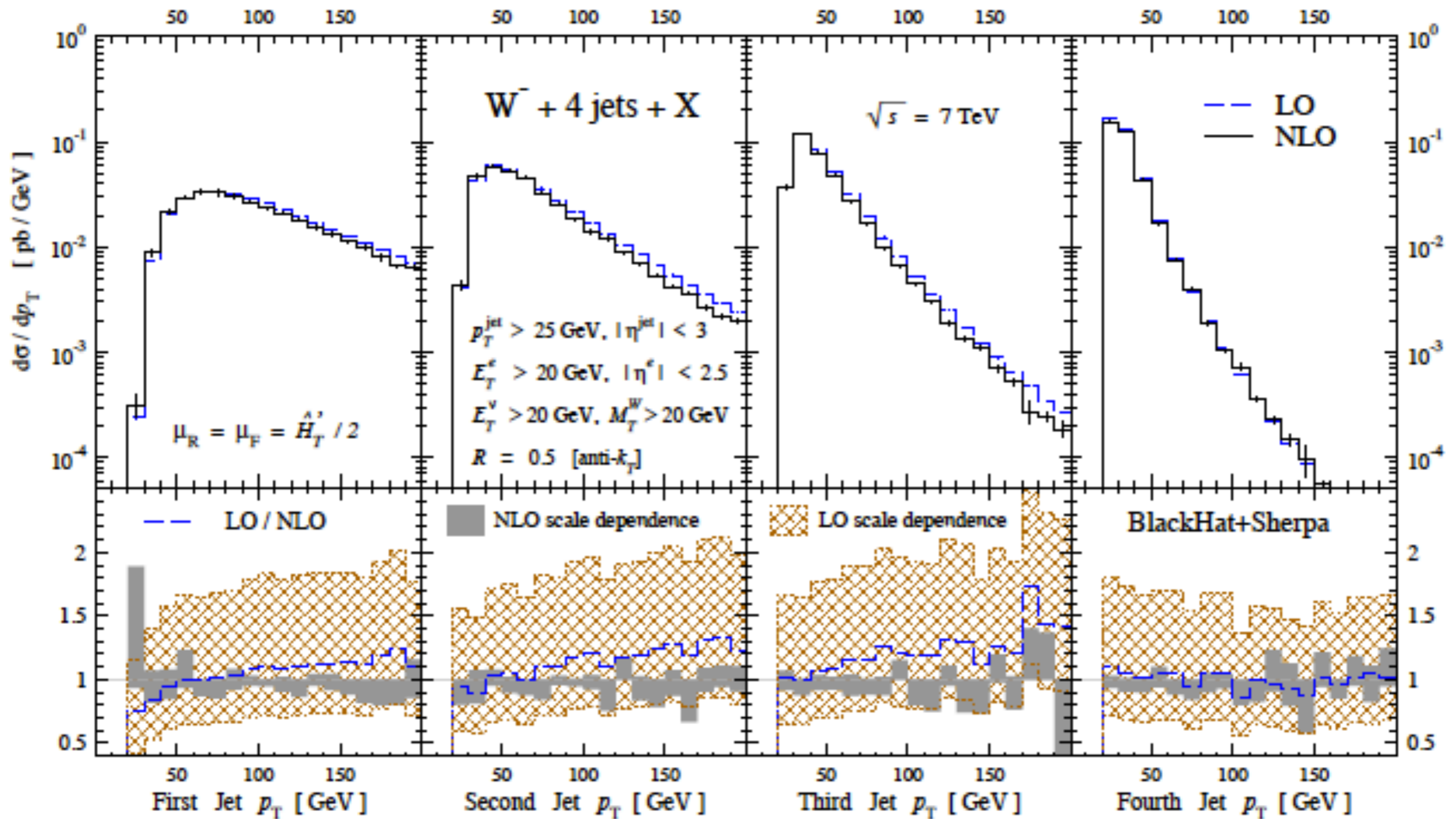
4g: Ellis-Sexton(1985)  
5g: Bern-Dixon-Kosower(1993)  
6g: Ellis-Giele-Zanderighi(2006)



**D-dimensional unitarity is a disruptive technology!**

# $W^- + 4\text{jets}$ at NLO at 7 TeV

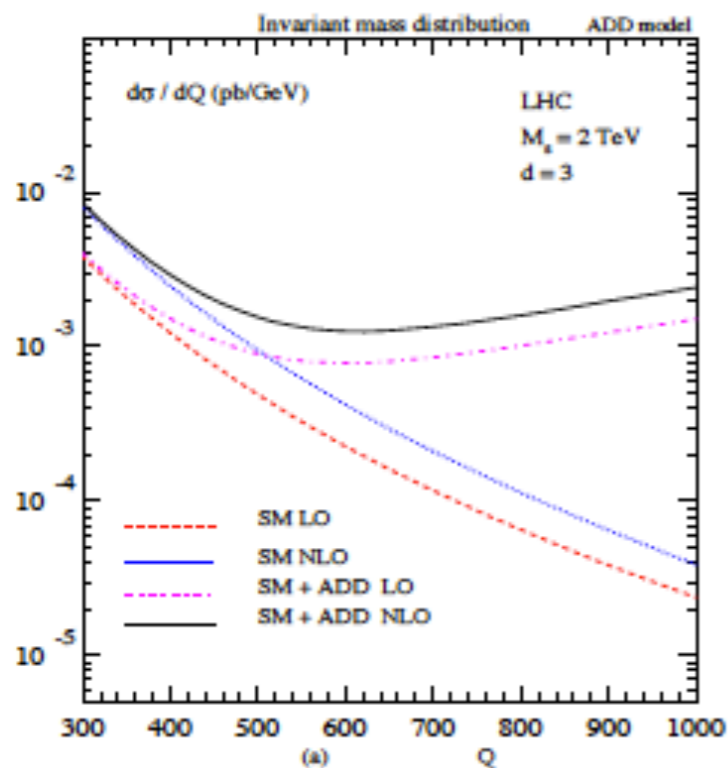
Blackhat-Sherpa, arXiv:1009.2338v2 (Feb 2011)



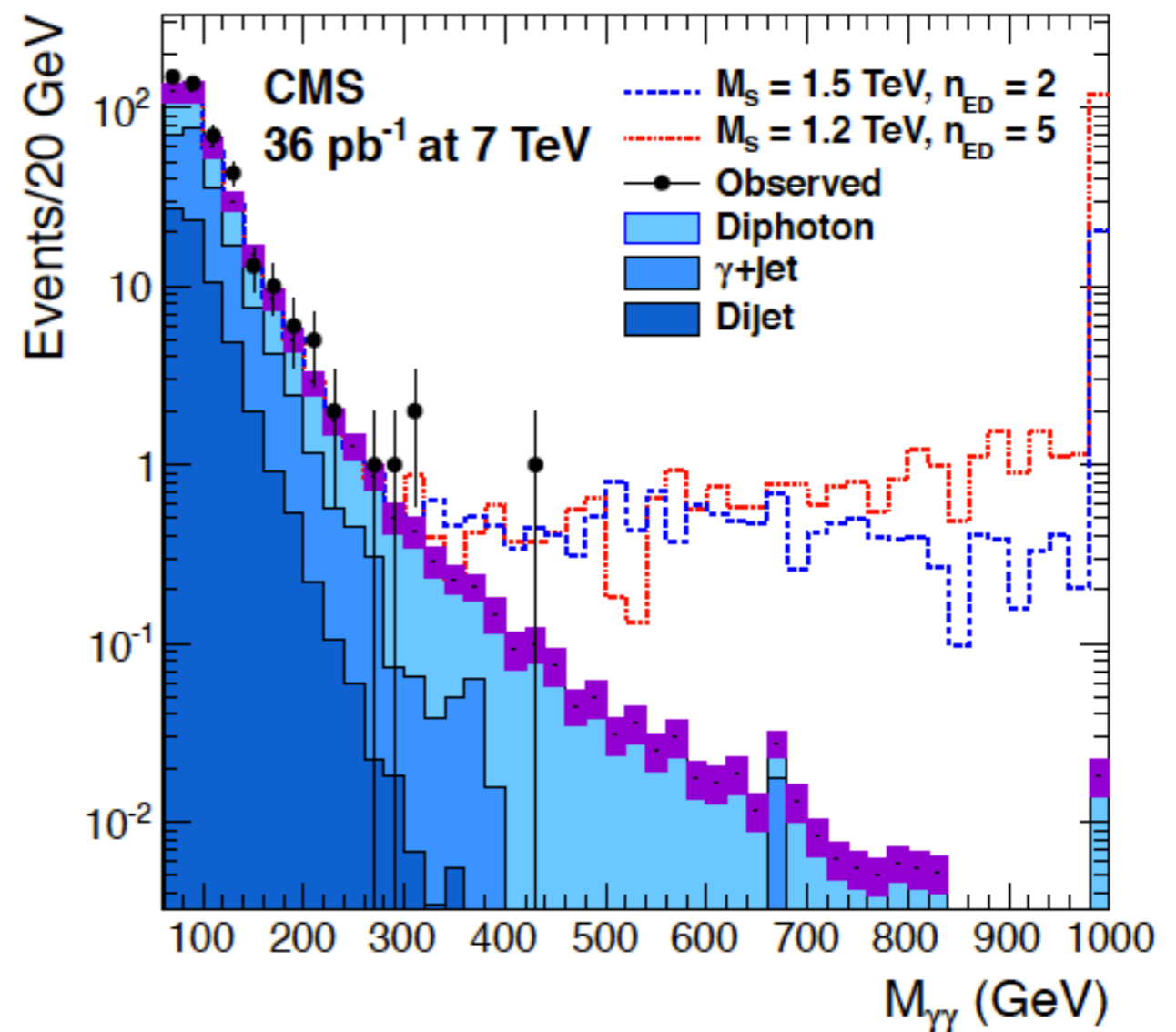
Currently in leading colour approximation for virtual amplitudes

# NLO corrections for new physics processes

For example in a ADD model in which gravity extends into extra dimensions.



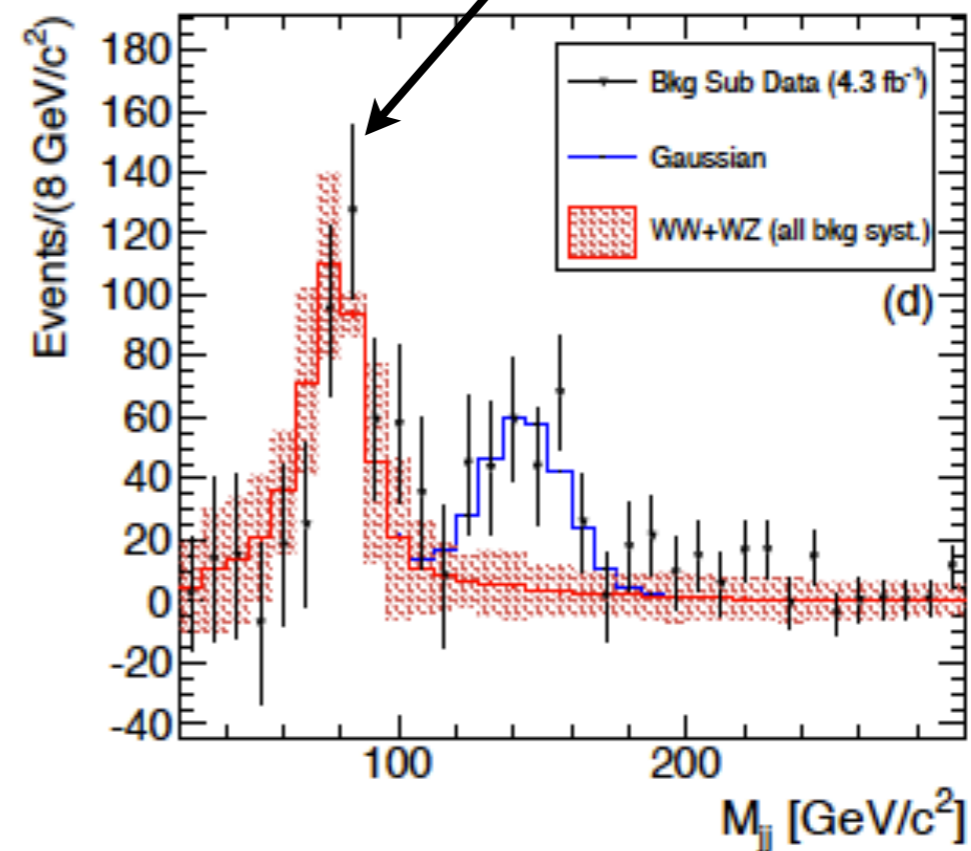
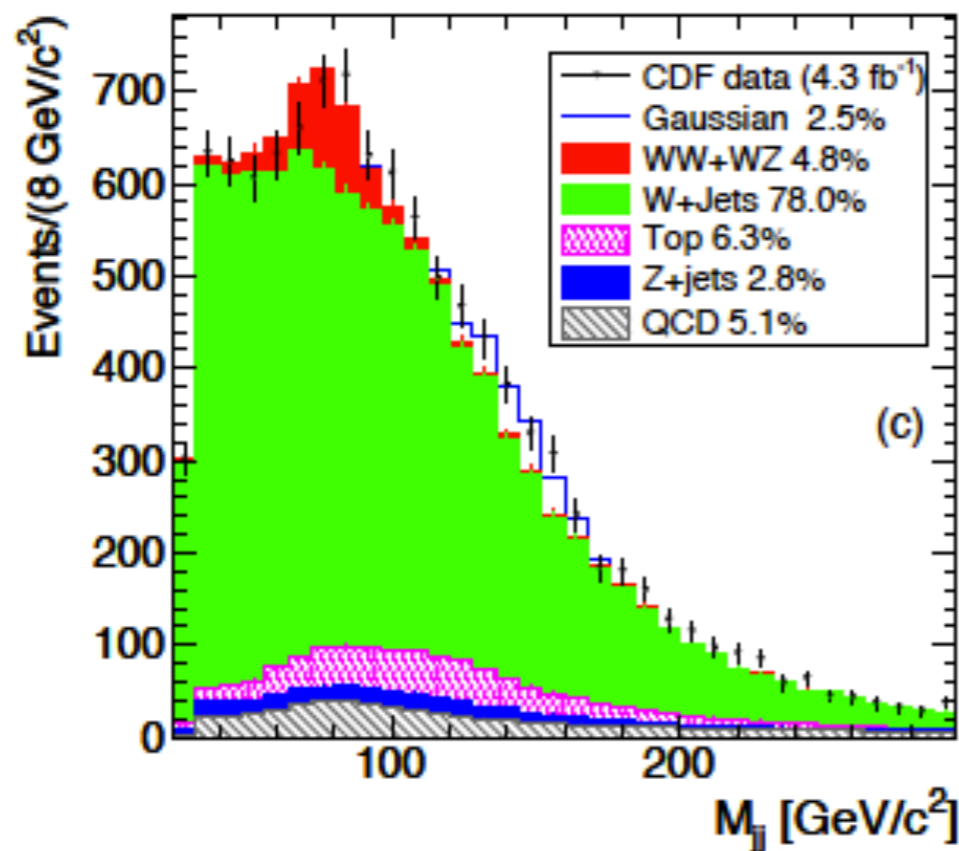
Kumar, Mathews, Ravindran & Tripathi hep-ph/0902.4894



Limit on model would be degraded if NLO corrections were not included

# Backgrounds to new physics processes

CDF: New April 4th, 2011



“Invariant mass distribution of jet pairs produced in association with a W boson in ppbar collisions at  $\sqrt{s}=1.96 \text{ TeV}$ ”, arXiv:1104.0699

Cross section  $WX$  times branching ratio  $X(\rightarrow \text{two jets})$  is estimated to be  $4 \text{ pb}$

# Background

- Significance of the signal clearly depends on validity of estimation of background, especially  $W+2$  jet background.
- Current significance of signal is  $3.2 \sigma$  if LO order estimates are used and increases to  $3.4 \sigma$  if NLO estimate is used.

# NLO Wishlist

2010

	process wanted at NLO	background to
✓	1. $pp \rightarrow VV + \text{jet}$	$t\bar{t}H$ , new physics Dittmaier, Kallweit, Uwer; Campbell, Ellis, Zanderighi
✓	2. $pp \rightarrow H + 2 \text{ jets}$	$H$ in VBF Campbell, Ellis, Zanderighi; Ciccolini, Denner Dittmaier
✓	3. $pp \rightarrow t\bar{t}b\bar{b}$	$t\bar{t}H$ Bredenstein, Denner Dittmaier, Pozzorini; Bevilacqua, Czakon, Papadopoulos, Pittau, Worek
✓	4. $pp \rightarrow t\bar{t} + 2 \text{ jets}$	$t\bar{t}H$ Bevilacqua, Czakon, Papadopoulos, Worek
✗	5. $pp \rightarrow VV b\bar{b}$	VBF $\rightarrow H \rightarrow VV$ , $t\bar{t}H$ , new physics
✓	6. $pp \rightarrow VV + 2 \text{ jets}$	VBF $\rightarrow H \rightarrow VV$ VBF: Bozzi, Jäger, Oleari, Zeppenfeld
✓	7. $pp \rightarrow V + 3 \text{ jets}$	new physics CFB, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre; Ellis, Melnikov, Zanderighi
✓	8. $pp \rightarrow VVV$	SUSY trilepton Lazopoulos, Melnikov, Petriello; Hankele, Zeppenfeld; Binoth, Ossola, Papadopoulos, Pittau
✓	9. $pp \rightarrow b\bar{b}b\bar{b}$	Higgs, new physics <span style="float: right;">GOLEM</span>

Table from Carola Berger

# Madloop - Automatic generation of NLO corrections

- For many years there has been an automatic generator of LO matrix elements, (Madgraph) and events (Madevent)
- A similar development for NLO has just appeared, (Rikkert Frederix et al), (Madloop).
- Open issues are the computation time for this Feynman graph-based method and its scaling with the number of legs.

Process	$\mu$	$n_{lf}$	Cross section (pb)	
			LO	NLO
a.1 $pp \rightarrow t\bar{t}$	$m_{top}$	5	$123.76 \pm 0.05$	$162.08 \pm 0.12$
a.2 $pp \rightarrow tj$	$m_{top}$	5	$34.78 \pm 0.03$	$41.03 \pm 0.07$
a.3 $pp \rightarrow tjj$	$m_{top}$	5	$11.851 \pm 0.006$	$13.71 \pm 0.02$
a.4 $pp \rightarrow t\bar{b}j$	$m_{top}/4$	4	$25.62 \pm 0.01$	$30.96 \pm 0.06$
a.5 $pp \rightarrow t\bar{b}jj$	$m_{top}/4$	4	$8.195 \pm 0.002$	$8.91 \pm 0.01$
b.1 $pp \rightarrow (W^+ \rightarrow) e^+ \nu_e$	$m_W$	5	$5072.5 \pm 2.9$	$6146.2 \pm 9.8$
b.2 $pp \rightarrow (W^+ \rightarrow) e^+ \nu_e j$	$m_W$	5	$828.4 \pm 0.8$	$1065.3 \pm 1.8$
b.3 $pp \rightarrow (W^+ \rightarrow) e^+ \nu_e jj$	$m_W$	5	$298.8 \pm 0.4$	$300.3 \pm 0.6$
b.4 $pp \rightarrow (\gamma^*/Z \rightarrow) e^+ e^-$	$m_Z$	5	$1007.0 \pm 0.1$	$1170.0 \pm 2.4$
b.5 $pp \rightarrow (\gamma^*/Z \rightarrow) e^+ e^- j$	$m_Z$	5	$156.11 \pm 0.03$	$203.0 \pm 0.2$
b.6 $pp \rightarrow (\gamma^*/Z \rightarrow) e^+ e^- jj$	$m_Z$	5	$54.24 \pm 0.02$	$56.69 \pm 0.07$
c.1 $pp \rightarrow (W^+ \rightarrow) e^+ \nu_e b\bar{b}$	$m_W + 2m_b$	4	$11.557 \pm 0.005$	$22.95 \pm 0.07$
c.2 $pp \rightarrow (W^+ \rightarrow) e^+ \nu_e t\bar{t}$	$m_W + 2m_{top}$	5	$0.009415 \pm 0.000003$	$0.01159 \pm 0.00001$
c.3 $pp \rightarrow (\gamma^*/Z \rightarrow) e^+ e^- b\bar{b}$	$m_Z + 2m_b$	4	$9.459 \pm 0.004$	$15.31 \pm 0.03$
c.4 $pp \rightarrow (\gamma^*/Z \rightarrow) e^+ e^- t\bar{t}$	$m_Z + 2m_{top}$	5	$0.0035131 \pm 0.0000004$	$0.004876 \pm 0.000002$
c.5 $pp \rightarrow \gamma t\bar{t}$	$2m_{top}$	5	$0.2906 \pm 0.0001$	$0.4169 \pm 0.0003$
d.1 $pp \rightarrow W^+ W^-$	$2m_W$	4	$29.976 \pm 0.004$	$43.92 \pm 0.03$
d.2 $pp \rightarrow W^+ W^- j$	$2m_W$	4	$11.613 \pm 0.002$	$15.174 \pm 0.008$
d.3 $pp \rightarrow W^+ W^+ jj$	$2m_W$	4	$0.07048 \pm 0.00004$	$0.1377 \pm 0.0005$
e.1 $pp \rightarrow HW^+$	$m_W + m_H$	5	$0.3428 \pm 0.0003$	$0.4455 \pm 0.0003$
e.2 $pp \rightarrow HW^+ j$	$m_W + m_H$	5	$0.1223 \pm 0.0001$	$0.1501 \pm 0.0002$
e.3 $pp \rightarrow HZ$	$m_Z + m_H$	5	$0.2781 \pm 0.0001$	$0.3659 \pm 0.0002$
e.4 $pp \rightarrow HZ j$	$m_Z + m_H$	5	$0.0988 \pm 0.0001$	$0.1237 \pm 0.0001$
e.5 $pp \rightarrow Ht\bar{t}$	$m_{top} + m_H$	5	$0.08896 \pm 0.00001$	$0.09869 \pm 0.00003$
e.6 $pp \rightarrow Hb\bar{b}$	$m_b + m_H$	4	$0.16510 \pm 0.00009$	$0.2099 \pm 0.0006$
e.7 $pp \rightarrow Hjj$	$m_H$	5	$1.104 \pm 0.002$	$1.036 \pm 0.002$

Table 2: Results for total rates, possibly within cuts, at the 7 TeV LHC, obtained with MADFKS and MADLOOP. The errors are due to the statistical uncertainty of Monte Carlo integration. See the text for details.

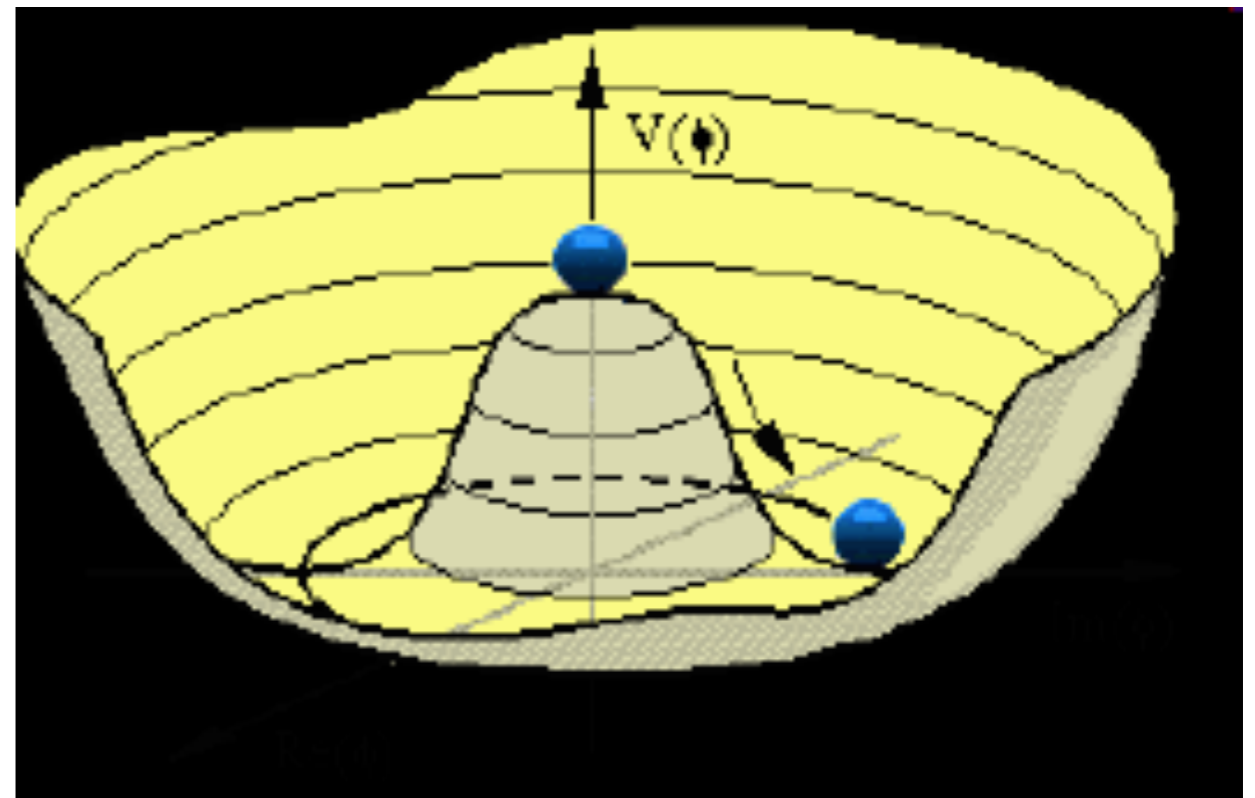
Hirschi et al , arXiv:1103.0621

NLO results available for any arbitrary process, with acceptable computing times for small enough N



# The Higgs mechanism

- We require a gauge invariant way to give mass to the W and Z, but not to the photon.
- A solution is the Higgs mechanism.
- The mechanism relies on Broken symmetry.

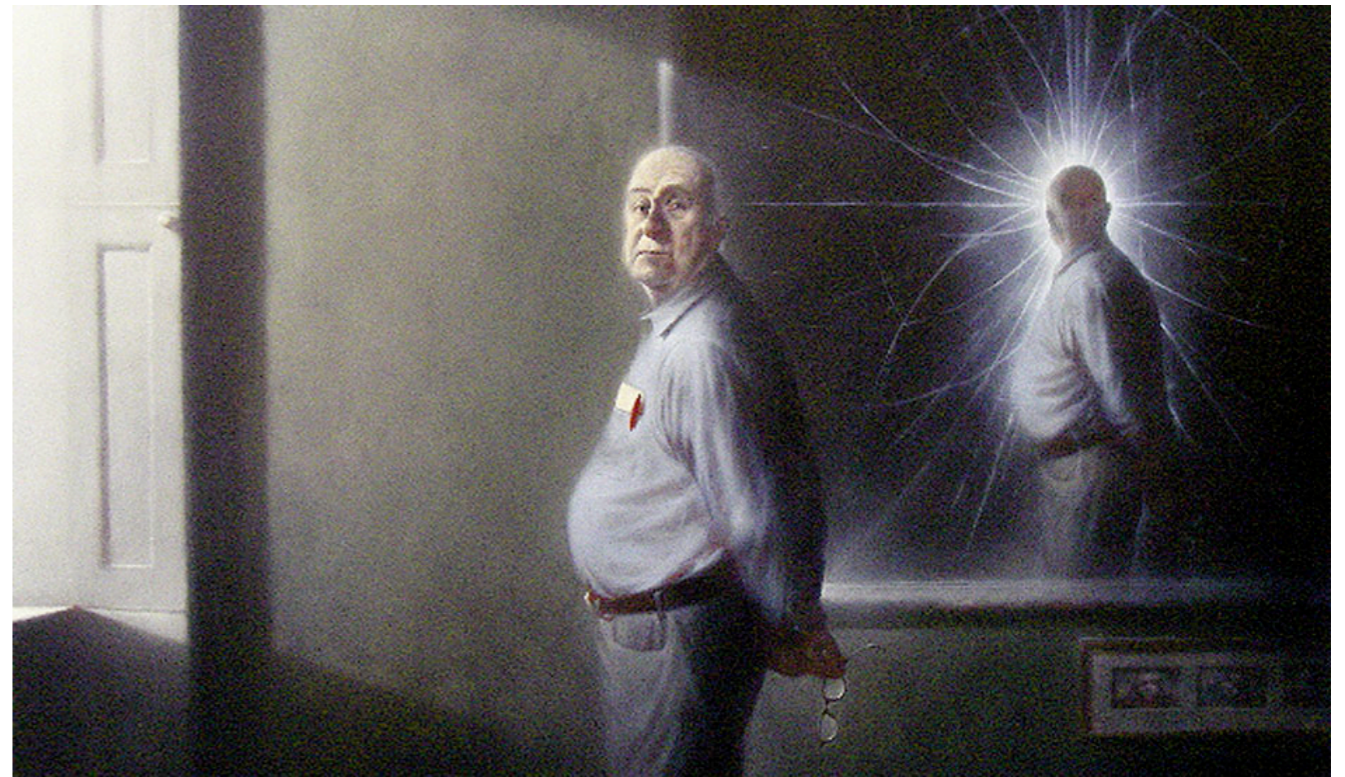


Choice of the minimum breaks the symmetry

# The Higgs boson

- Add a complex doublet of scalar fields (4 degrees of freedom)
- Couple the doublet to massless gauge fields
- 3 degrees of freedom are absorbed to give longitudinal degrees of freedom to the  $W^+, W^-, Z^0$ , (thus evading the Goldstone theorem).
- The remaining degree of freedom is the physical Higgs boson, a necessary consequence of the Higgs mechanism.

Peter Higgs: portrait by Ken Currie

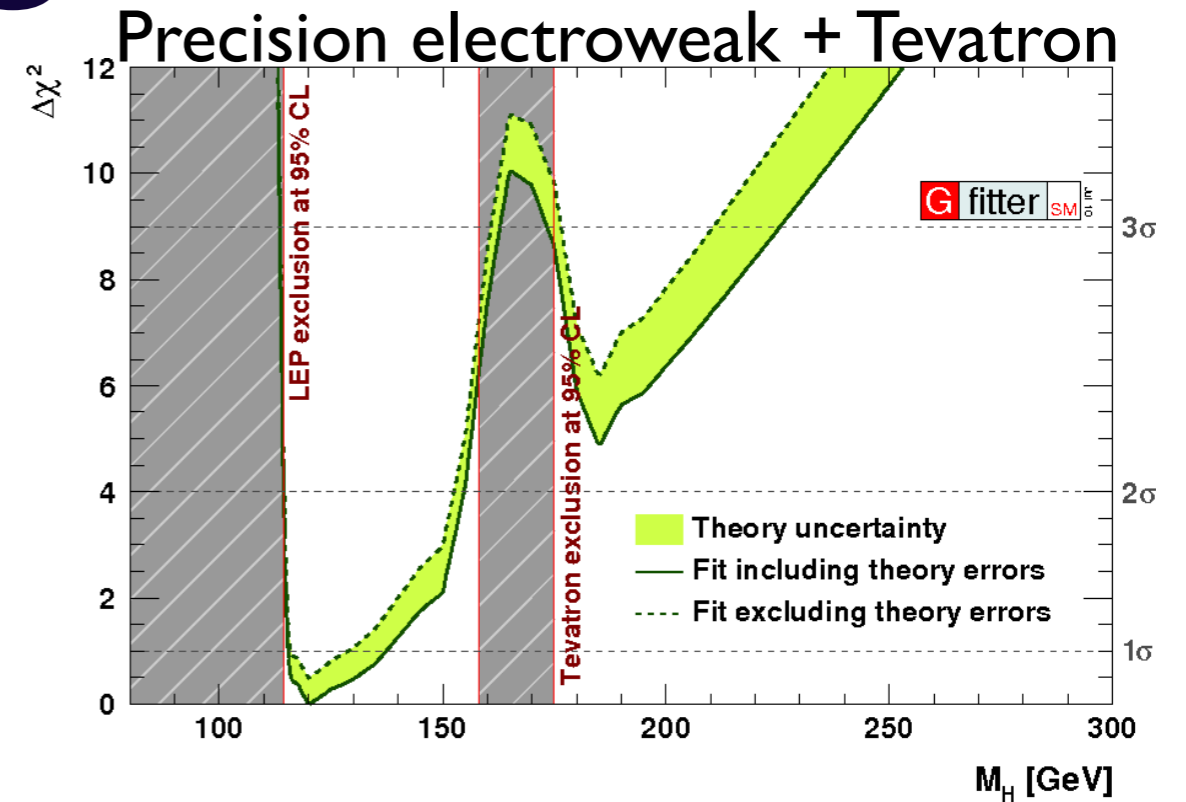
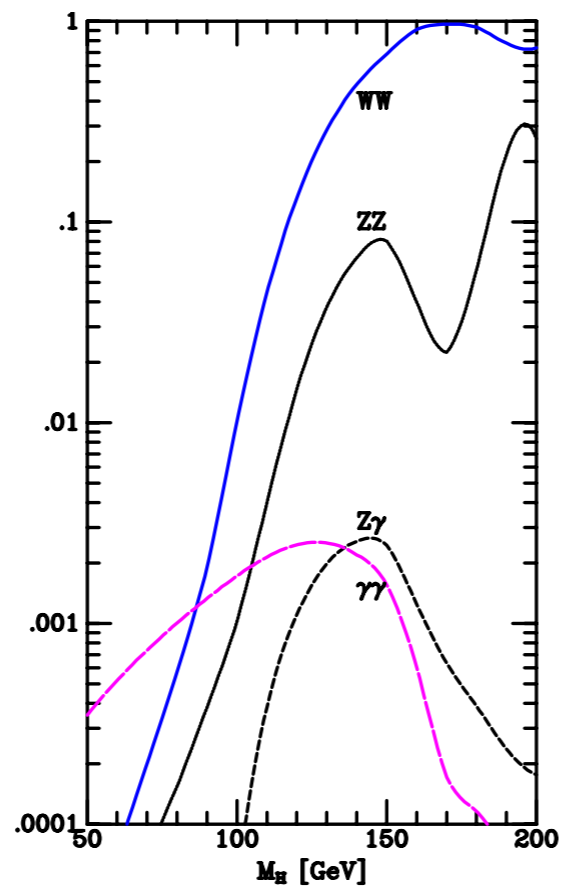
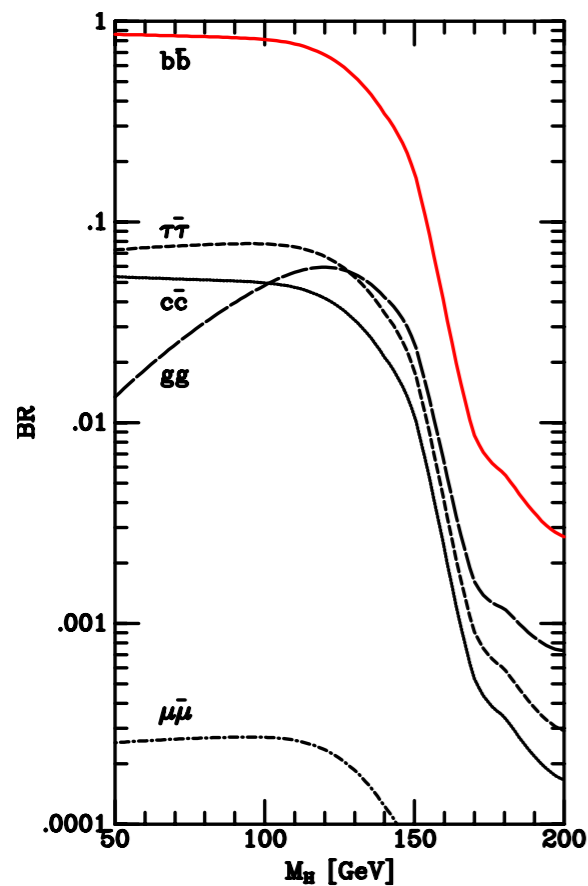


‘The next day Stanley Deser had arranged for me to talk at Harvard, where an equally skeptical audience awaited. Sidney Coleman told me (in 1989) that they “had been looking forward to tearing apart this idiot who thought he could get around the Goldstone theorem.” ‘

Peter Higgs in ‘My life as a boson: The story of “The Higgs”’

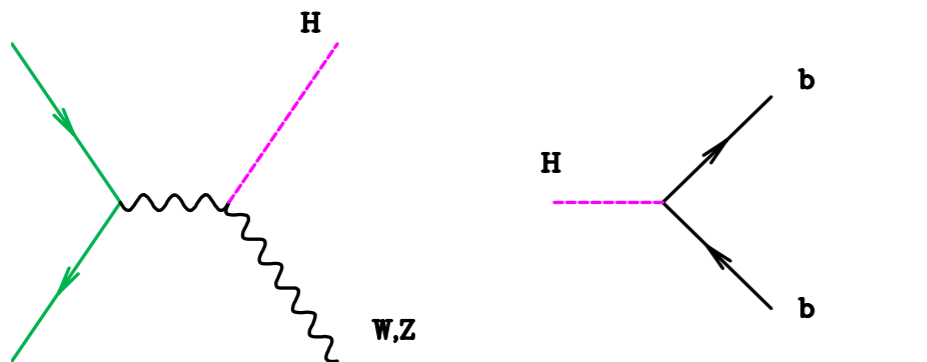
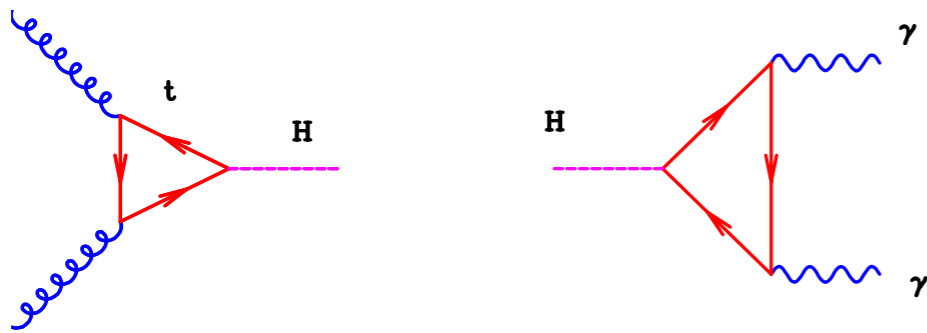
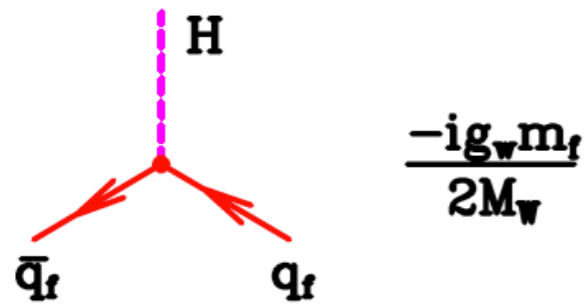
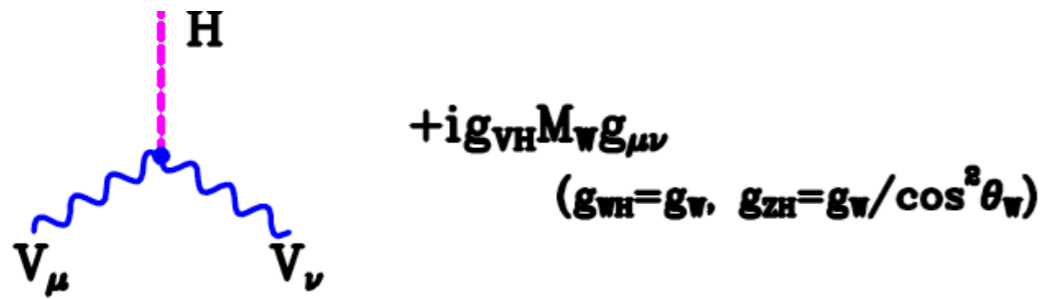
# Expectations for the Higgs

$$M_H = 121^{+17}_{-6} \text{ GeV}$$



In the low mass region the Higgs decays primarily into  $b\bar{b}$

# Higgs boson interactions



The Higgs boson couples proportional to mass

Favoured mode for detection of a light Higgs at LHC proceeds (both in production and decay) through loops

Favoured mode for the detection of a light Higgs at Tevatron proceeds (both in production and decay) through tree diagrams.

# Higgs limits at the Tevatron

The Tevatron Higgs limits depend on an accurate estimate of the cross section, including its perturbative stability.

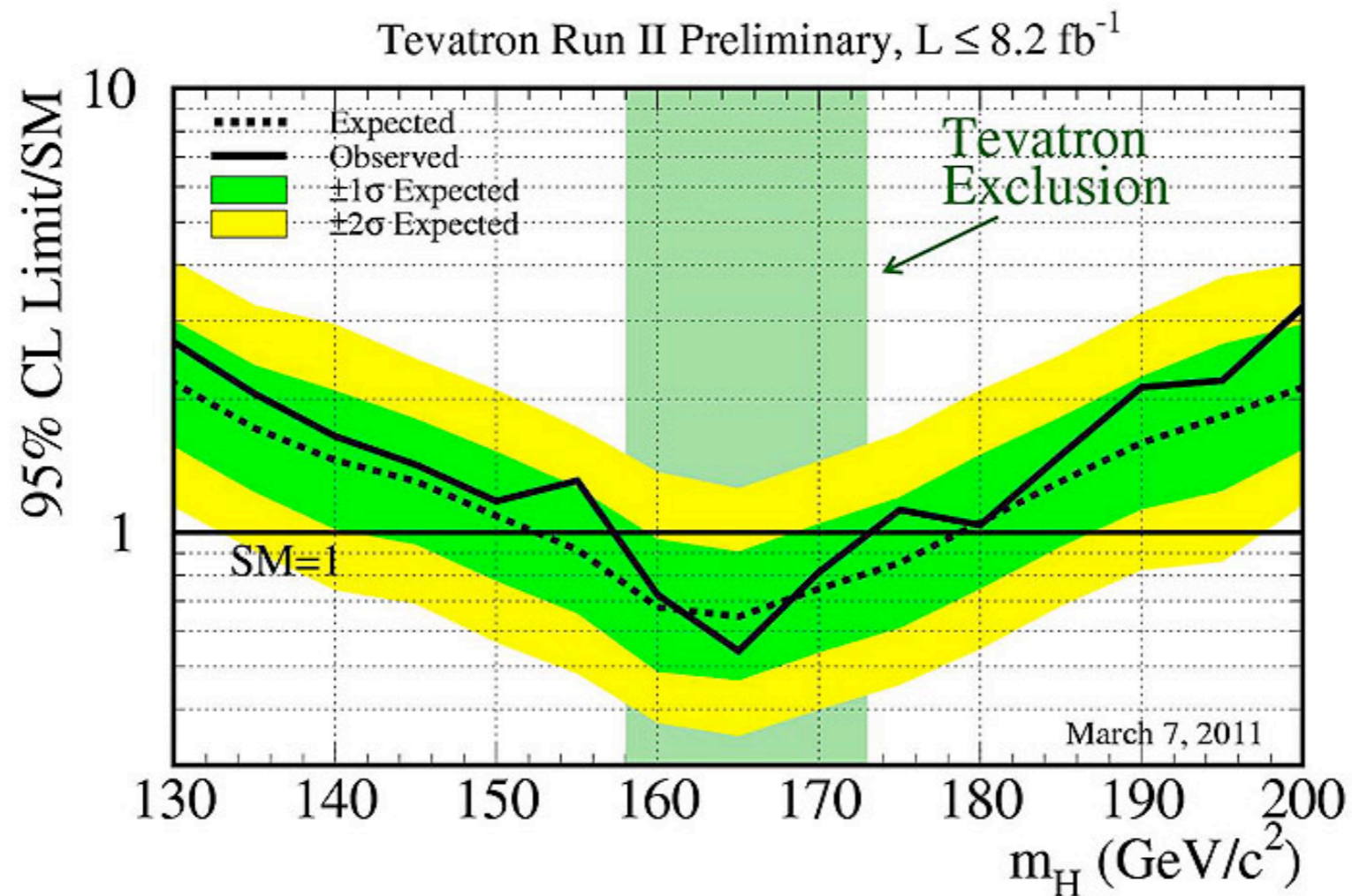
Anastasiou et al, arXiv:0905.3529

$\sigma(pp \rightarrow H \rightarrow W^+W^-)$ [fb]	LO	NLO	NNLO	$K^{\text{NLO}}$	$K^{\text{NNLO}}$
total( $\mu=m_H$ )	$1.398 \pm 0.001$	$3.366 \pm 0.003$	$4.630 \pm 0.010$	2.412	3.312
with selection cuts ( $\mu=m_H$ )	$0.525 \pm 0.001$	$1.129 \pm 0.003$	$1.383 \pm 0.004$	2.150	2.594

Selection cuts, especially veto on jet activity, increase the perturbative stability.

Moral: NNLO corrections are sometimes important; we need exclusive information so that we can apply selection cuts.

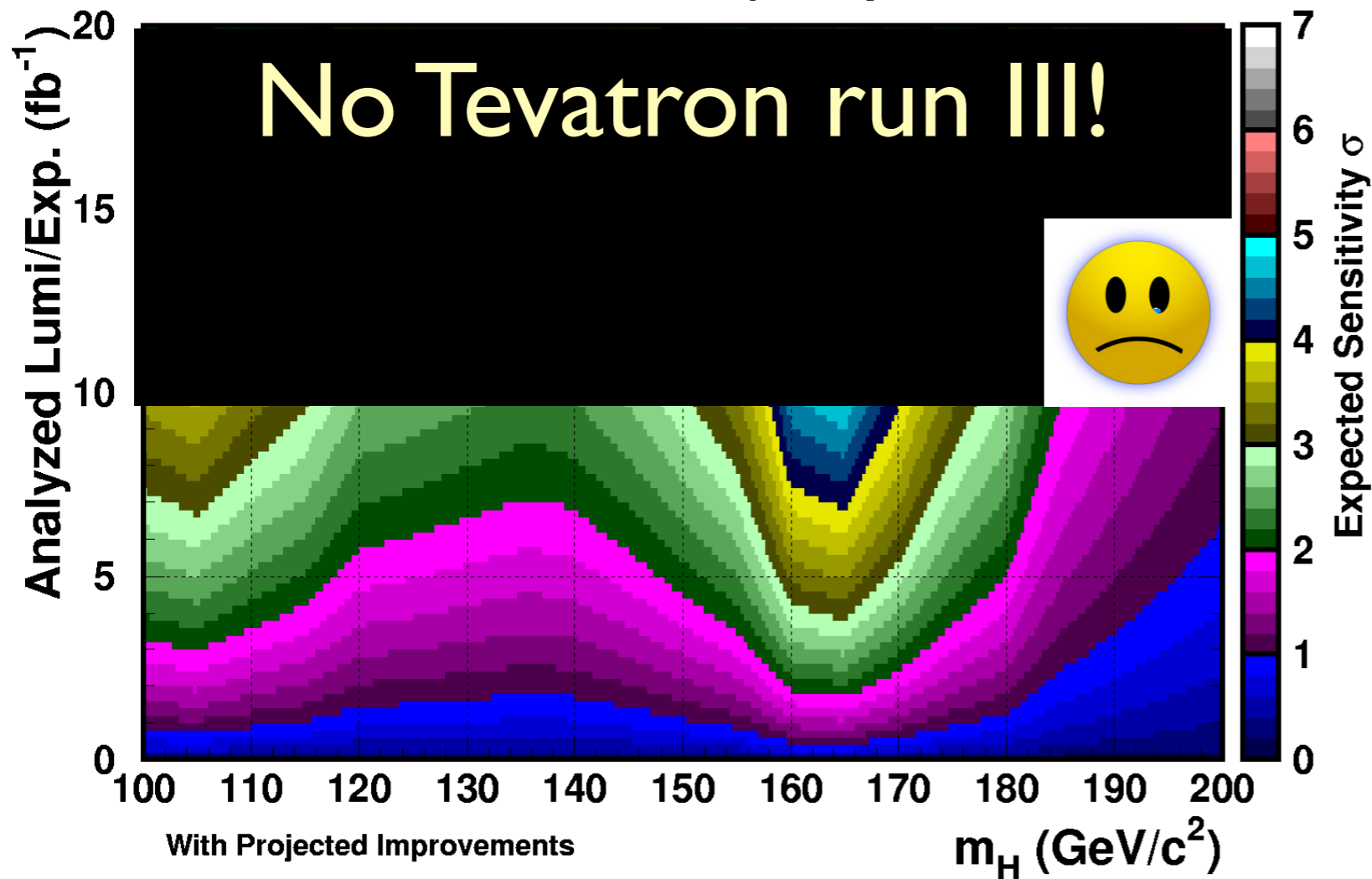
# Tevatron High Mass exclusion



Tevatron operating at  $\sqrt{s}=1.96 \text{ TeV}$  excludes Higgs between  $158 < M_H < 173 \text{ GeV}$  at 95%cl

# Higgs search projections at the Tevatron

2xCDF Preliminary Projection

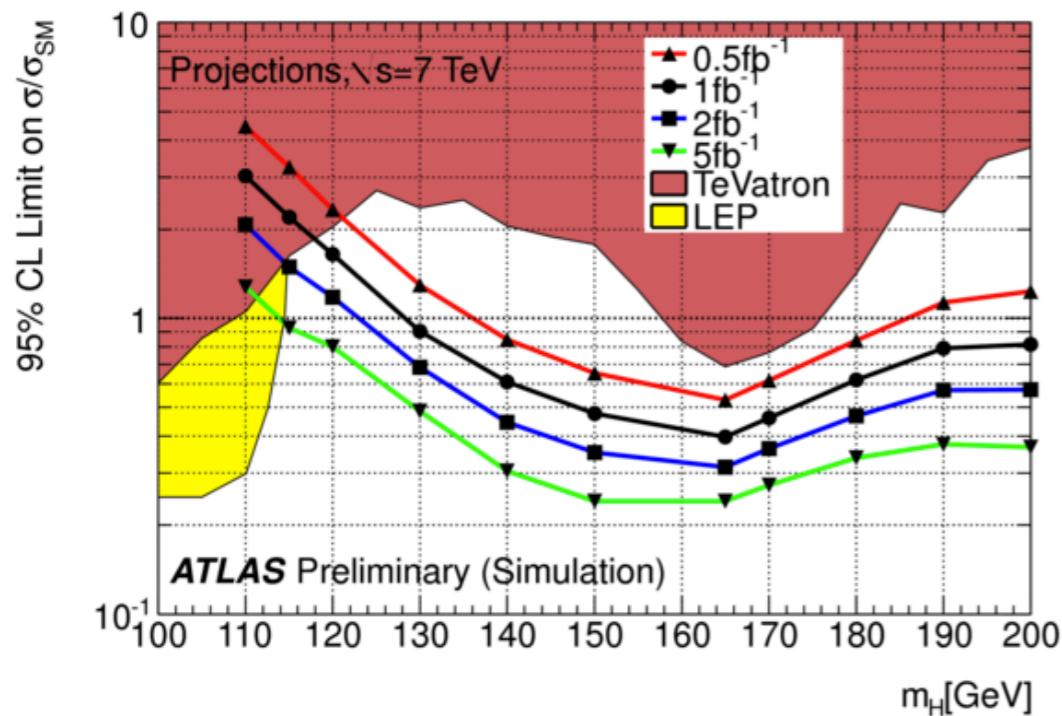


Tevatron luminosity  
10 fb<sup>-1</sup> per  
experiment by end of  
2011

By the end of 2011,  
2.4  $\sigma$  across the mass range  $114 < m_H < 180$  GeV  
3 $\sigma$  at  $m_H = 114$  GeV

# Projected LHC Higgs sensitivity at 7 TeV

State of the art cross sections: NNLO for  $gg \rightarrow H$ , NLO for VBF, VH ; Background processes at NLO (MCFM)



ATLAS + CMS $\approx 2 \times$ CMS	95% CL exclusion	3 $\sigma$ sensitivity	5 $\sigma$ sensitivity
1 fb <sup>-1</sup>	120 - 530	135 - 475	152 - 175
2 fb <sup>-1</sup>	114 - 585	120 - 545	140 - 200
5 fb <sup>-1</sup>	114 - 600	114 - 600	128 - 482
10 fb <sup>-1</sup>	114 - 600	114 - 600	117 - 535

The Higgs boson, if it exists between masses of 114-600 GeV will be discovered or ruled out in the next two years! (with a slightly worrying exception for 5  $\sigma$  for the low mass region).



# Radiative Corrections

- Precision tests of the standard model.
- Estimation of backgrounds:  
shapes for extrapolation and to subtract irreducible backgrounds, (ie cases where background and signature are not distinguishable).
- Before discovery of new physics -- to set accurate limits
- After discovery of new physics -- to determine the parameters of the model

# Conclusions

- Significant advances in the calculation of one-loop multi-leg processes in the last three years. (We are almost in a position where the majority of the processes to be explored at LHC@7TeV with one  $\text{fb}^{-1}$  are calculated).
- Dream of automatic NLO calculation is becoming a reality.
- And there are already calculations needed at NNLO.....

# Backup

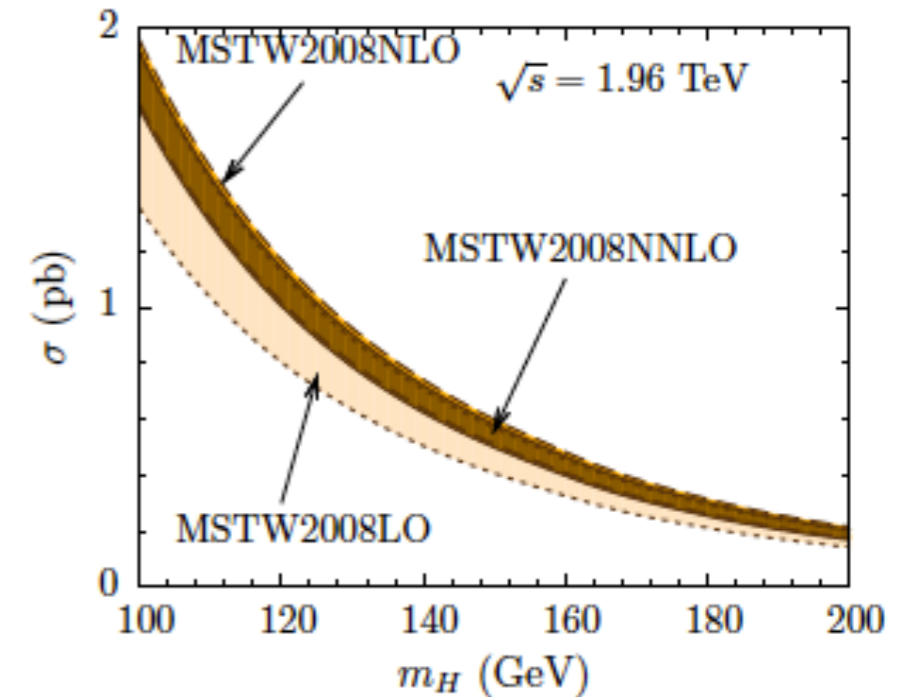
# Higgs boson at 1.96 TeV

Two contrasting views on the uncertainty on the gluon-gluon fusion Higgs cross section

Ahrens et al, (ABNY) 0808.3008,0809.4283,1008.3162  $\sigma_{\text{ABNY}}(M_H = 165) = 385^{+6}_{-2} \text{ }^{+30}_{-32} \text{ fb}$

Baglio and Djouadi, (BD)1003.4266,1009.1363  $\sigma_{\text{BD}}(M_H = 165) = 377^{+154}_{-135} \text{ fb}$

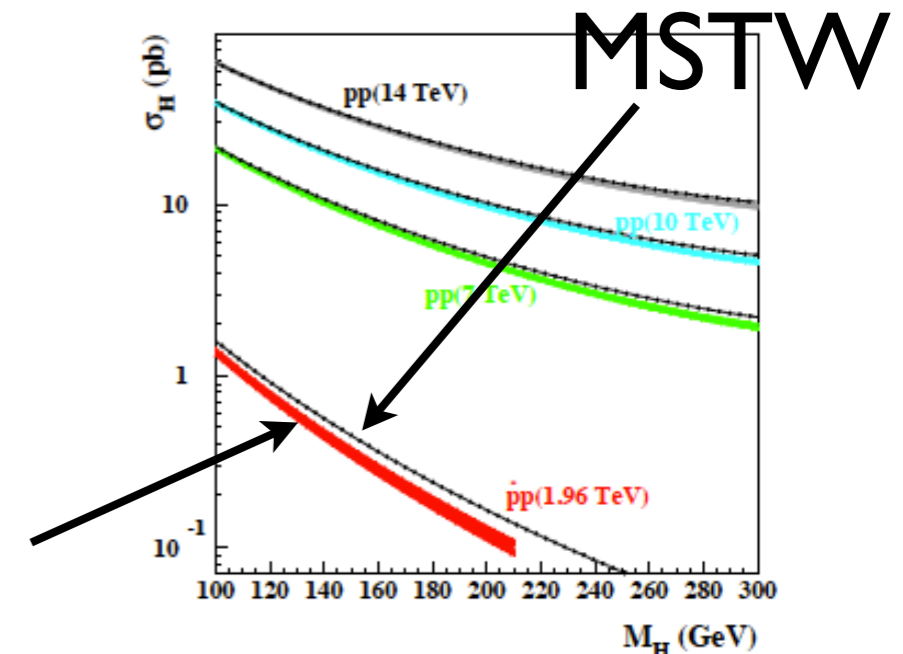
Source of uncertainty	ABNY	BD
Scale variation	3%(N <sup>3</sup> LL)	+15%/-20%(NNLO)
PDF	5-10%	25% (including $\alpha_s$ )
$\alpha_s$	6% (not strong correlation with PDF)	strongly correlated (included with PDF)



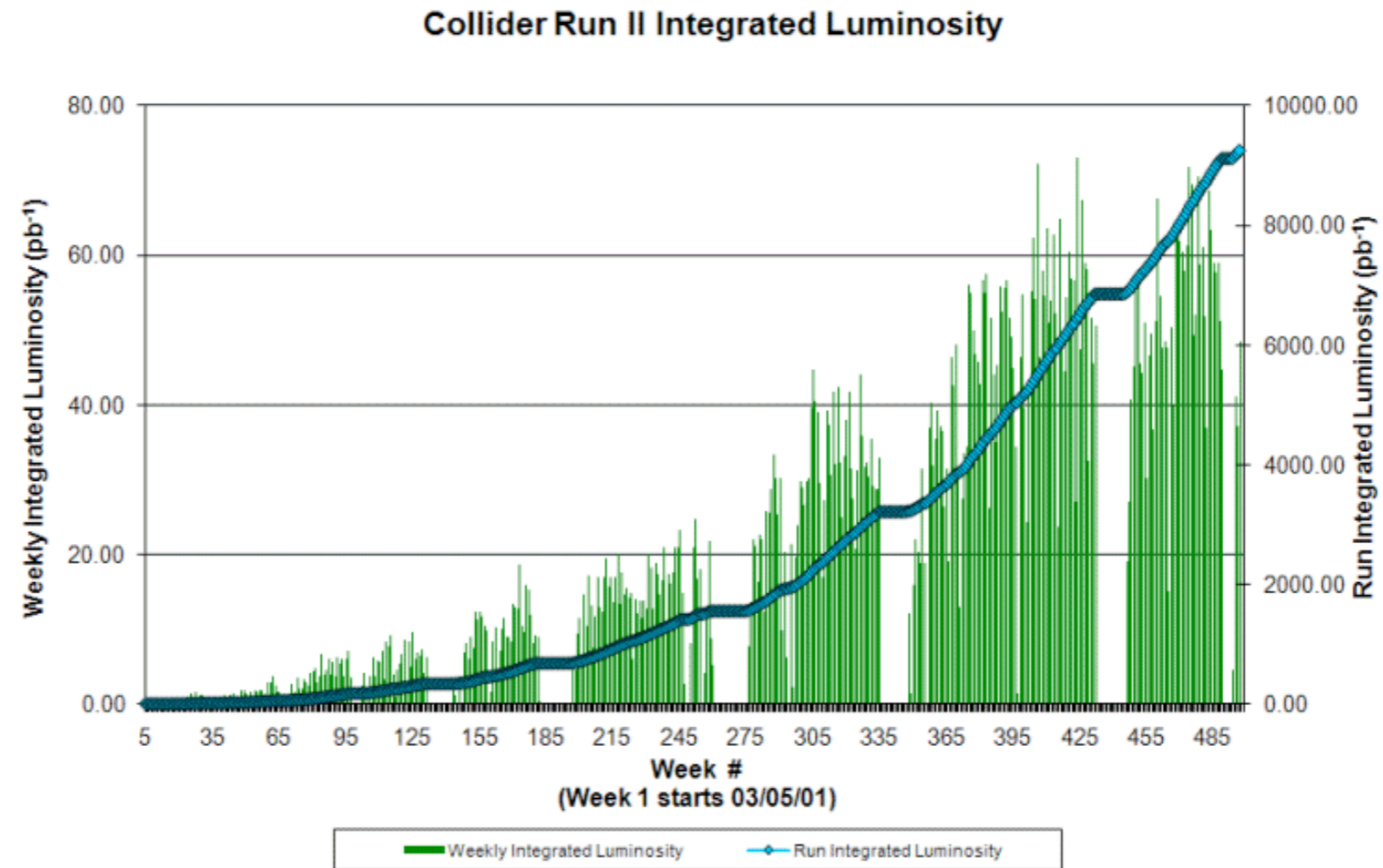
Major source of discrepancy is inclusion of ABKM parton distribution, MSTW and CTEQ give similar results.

Long(?) term solution : find the Higgs!

ABKM



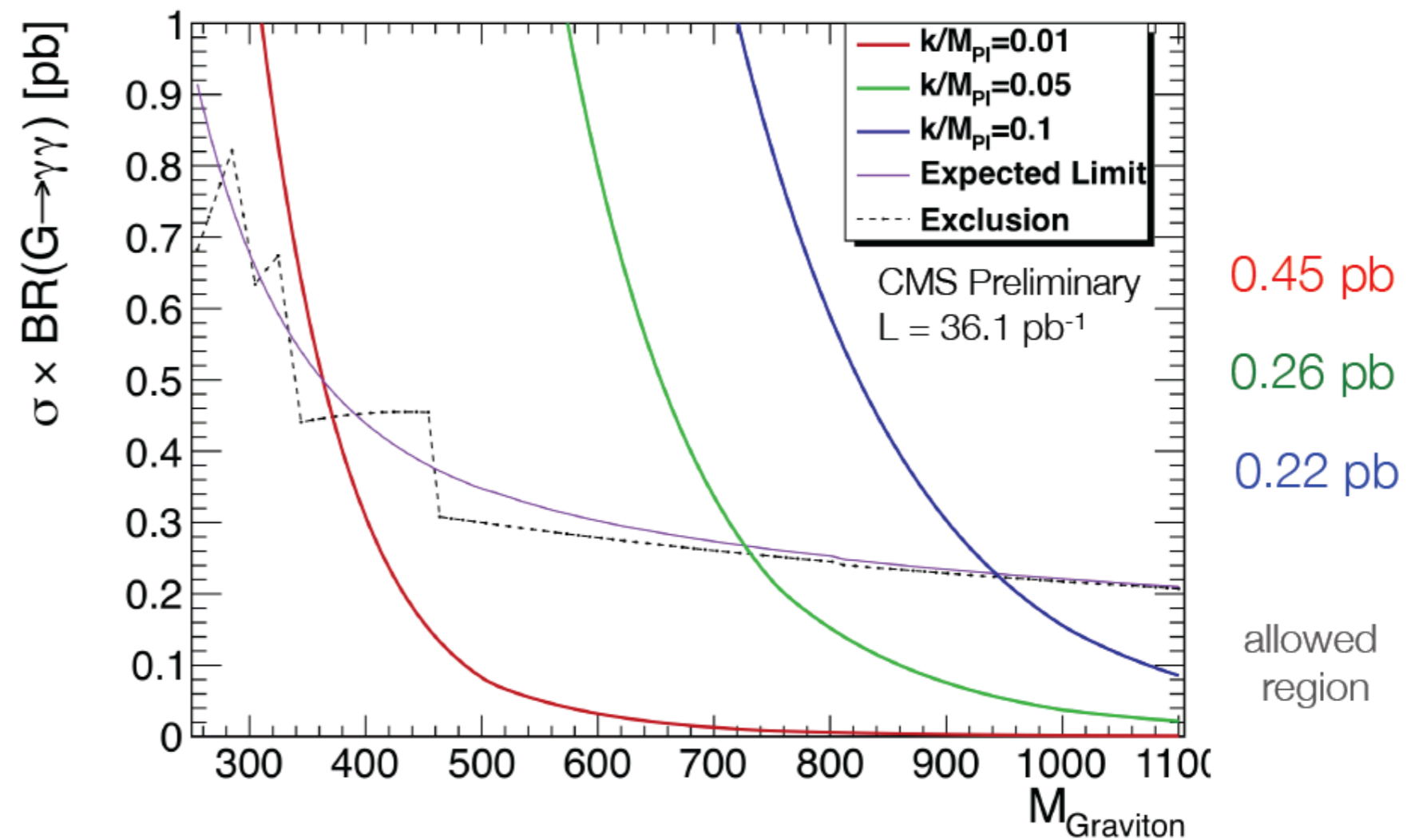
# Tevatron Luminosity



Average weekly luminosity exceeds  $50\text{pb}^{-1}$ !

Data taking efficiency CDF ( $\sim 85\%$ ) and D0 ( $\sim 92\%$ )

# RS Graviton $\rightarrow \gamma\gamma$ cross section limits



# Unitarity for one-loop diagrams

Important steps include:-

- First modern use of the idea [Bern, Dixon, Kosower](#)
- Cuts w.r.t. to loop momenta give (box) coefficients directly, complex momenta [Britto, Cachazo, Feng](#)
- OPP tensor reduction scheme, [Ossola, Pittau, Papadopoulos](#)
- Integrating the OPP procedure with unitarity [Ellis, Giele, Kunszt](#)
- D-dimensional unitarity [Giele, Kunszt, Melnikov](#)

# Semi-numerical unitarity in a nutshell

Imagine an integrand expressible in terms of bubbles (two denominators) and tadpoles (one denominator).

$$\mathcal{A}(l) = \frac{b}{d_1(l)d_2(l)} + \frac{a_1}{d_1(l)} + \frac{a_2}{d_2(l)}$$

$$d_1(l) = l^2 - m^2, \quad d_2(l) = (l+q)^2 - m^2$$

$l$ -independent coefficients  $b, a_1, a_2$  can be extracted numerically

$$b = \left\{ d_1(l)d_2(l) [\mathcal{A}(l)] \right\} \Big|_{l=l_{12}}$$

$$a_1 = \left\{ d_1(l) \left[ \mathcal{A}(l) - \frac{b}{d_1(l)d_2(l)} \right] \right\} \Big|_{l=l_1}$$

$$a_2 = \left\{ d_2(l) \left[ \mathcal{A}(l) - \frac{b}{d_1(l)d_2(l)} \right] \right\} \Big|_{l=l_2}$$

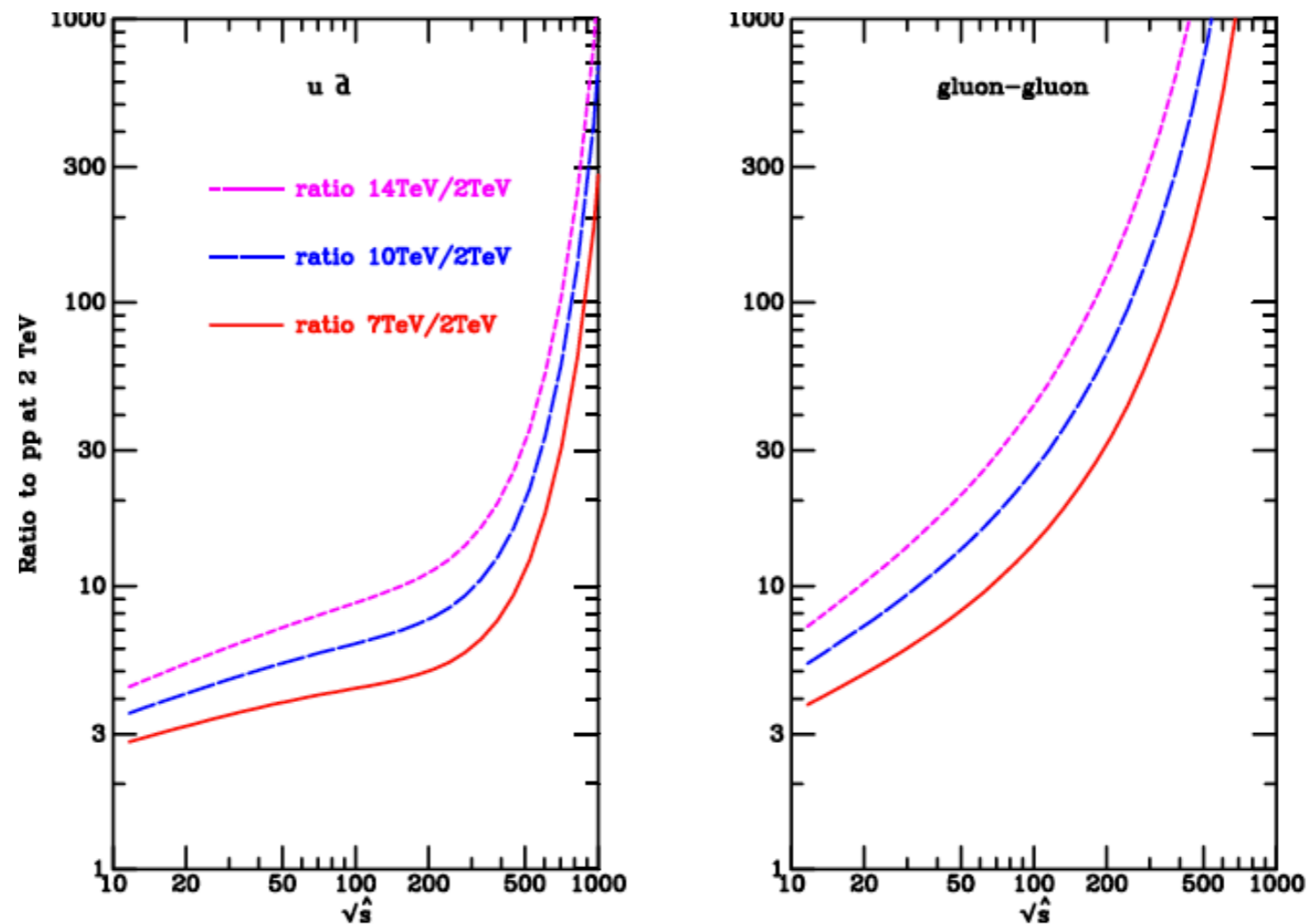
Fine print: Need complete understanding of parametric form of integrand (including terms which vanish upon integration and extension to  $d$ -dimensions)



# Parton luminosities

## Tevatron vs LHC

Not all of the beam energy is available for interaction. The available energy is determined by the parton distribution functions which can be combined into parton luminosities



- For qqbar initiated physics at a mass scale below 200 GeV, the Tevatron with  $10\text{fb}^{-1}$  is superior to the LHC at  $\sqrt{s}=7\text{TeV}$  with  $1\text{fb}^{-1}$ .