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# Black holes: Beacons in our search for a quantum theory of space-time

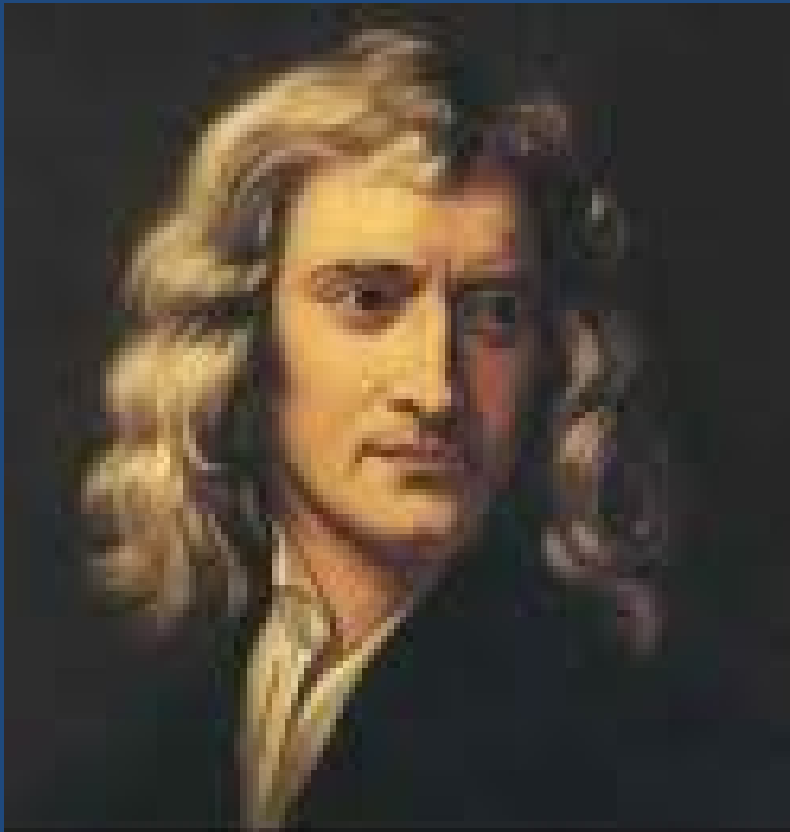
India Science Festival  
IISER Pune,  
11 January 2020

Spenta R. Wadia



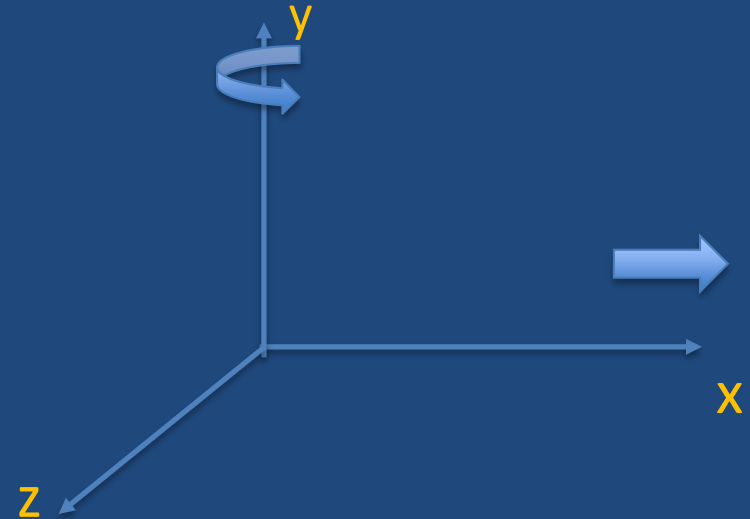
Isaac Newton (Principia  
Mathematica 1687)

Establishes a framework of  
mechanics



Newton's law of motion:  
 $\text{Force} = m_{\text{inertial}} \times \text{acceleration}$

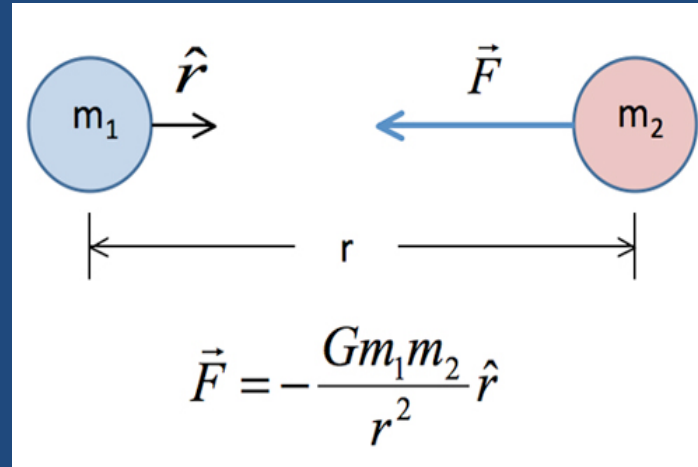
Newton formulated the laws of motion in terms of the flow in time of the position of a particle in 3-dims.



$(x(t), y(t), z(t))$  is a curve; velocity and acceleration are given by one and two time derivatives. Coordinates may be rotated or moved with constant velocity

Time is absolute and the same for all observers.

# Newton's law of Universal Gravitation



## Force acts instantaneously at a distance

*Newton (1692):* "That one body may act upon another at a distance through a vacuum without the mediation of anything else, by and through which their action and force may be conveyed from one another, is to me so great an absurdity that, I believe, no man who has in philosophic matters a competent faculty of thinking could ever fall into it."

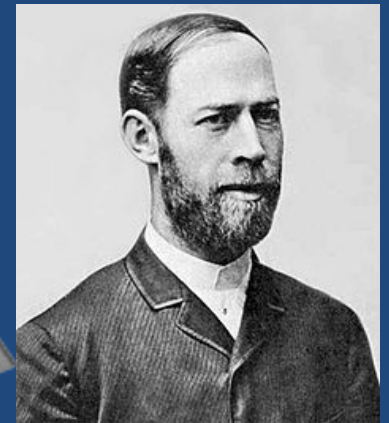
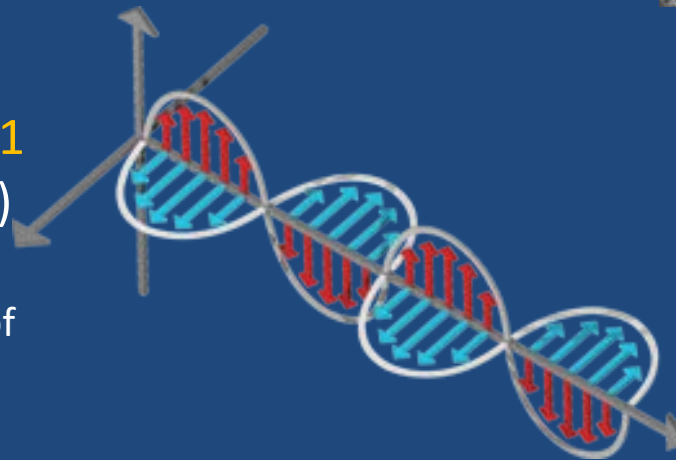
*Newton (1713)* "I have not yet been able to discover the cause of these properties of gravity from phenomena and I feign no hypothesis. It is enough that gravity does really exist and acts according to the laws I have explained, and that it abundantly serves to account for all the motions of celestial bodies."

# Electric and Magnetic **Fields** and **Waves**

Michael Faraday had demonstrated that light is polarized by a magnetic field.

James Clerk Maxwell unified electricity and magnetism, predicted the existence of electromagnetic **waves** and identified light as an electromagnetic wave of oscillating electric and magnetic fields moving with a speed  $c = 3.1 \times 10^8$  kms/sec (in vacuum) (1865)

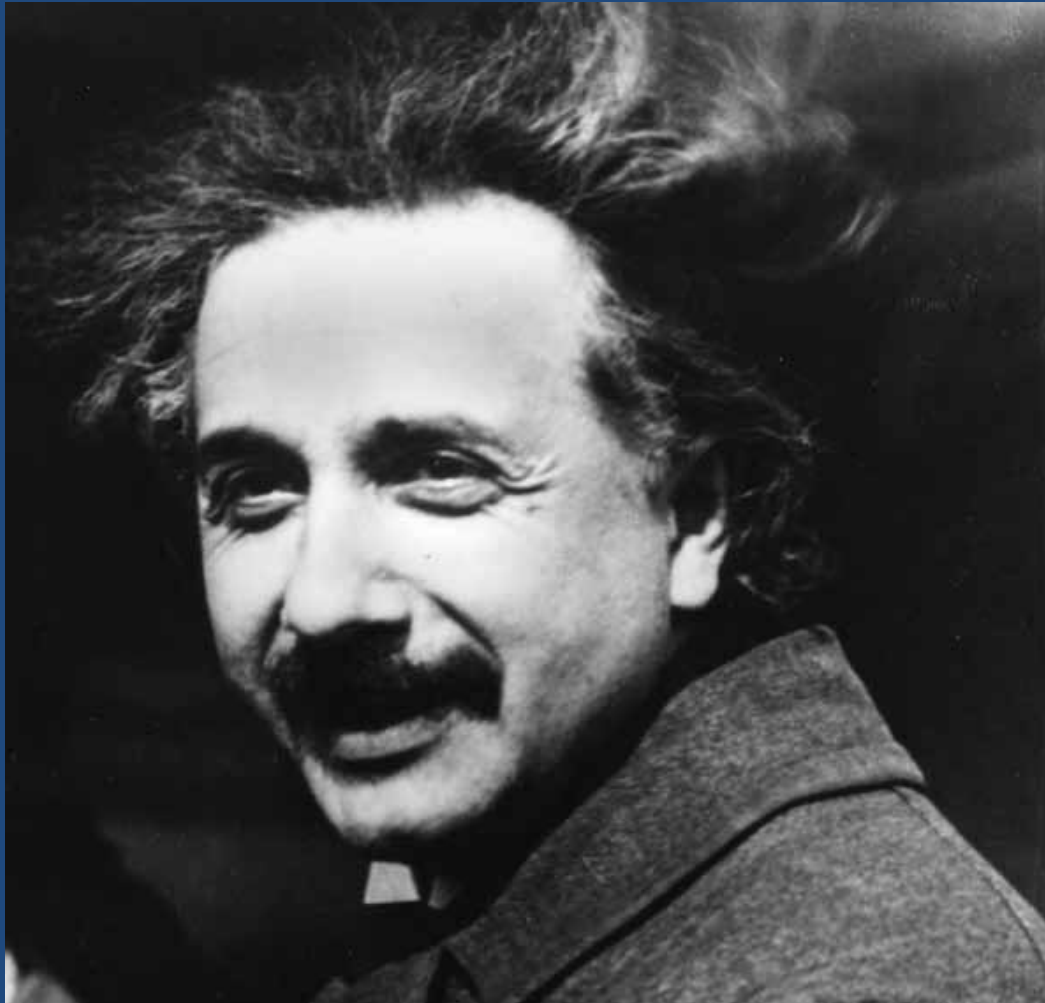
Heinrich Hertz demonstrated existence of radio waves that were predicted by Maxwell's theory with properties exactly the same as visible light (1887) except wavelength is  $10^4$  times longer.





# Albert Einstein

## Special Relativity 1905

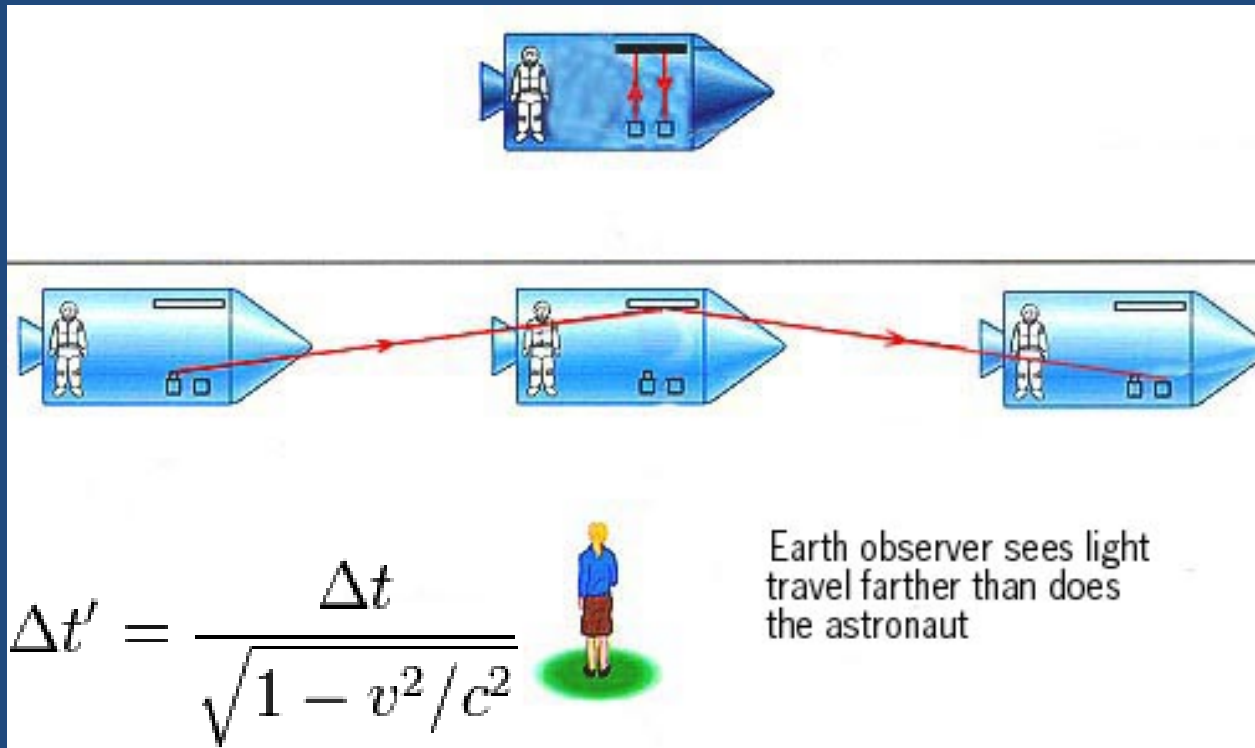


# Lorentz, Poincare, Einstein: Special Relativity (1905)

## Implications of Maxwell's theory:

Speed of light is the same whether you run towards it or away from it. Space and time have to adjust themselves to ensure this!

*Time intervals between events depend on your state of motion; things happen (according to us) more slowly for a moving observer than for us.*



Minkowski:  
space-time  
geometry

$$ds^2 = (c\Delta t)^2 - (\Delta x)^2$$



# Einstein's Two Puzzles

Newton's law of gravity in conflict with special relativity, and  
why only special relativity?

- That the force of gravity acts instantaneously is not consistent with Special Relativity! Einstein would like to have the force of gravity communicated at the speed of light by a field analogous to the electro-magnetic field of Faraday and Maxwell.
- Special Relativity is restricted to frames with relative constant velocity, but the laws of physics must be valid in any spacetime reference frame including those which are accelerating...

# Special Relativity, Gravity → General Relativity

The resolution of Einstein's puzzles lead to the General Theory of Relativity

- 1) Where the gravitational force felt in a small neighborhood of space-time is understood in terms of acceleration of the frame...the “Principle of Equivalence” ...the fall when a car suddenly brakes
- 2) Where the linear mixing of space and time in special relativity is replaced by a non-linear mixing, called a general coordinate transformation.



Einstein 1915

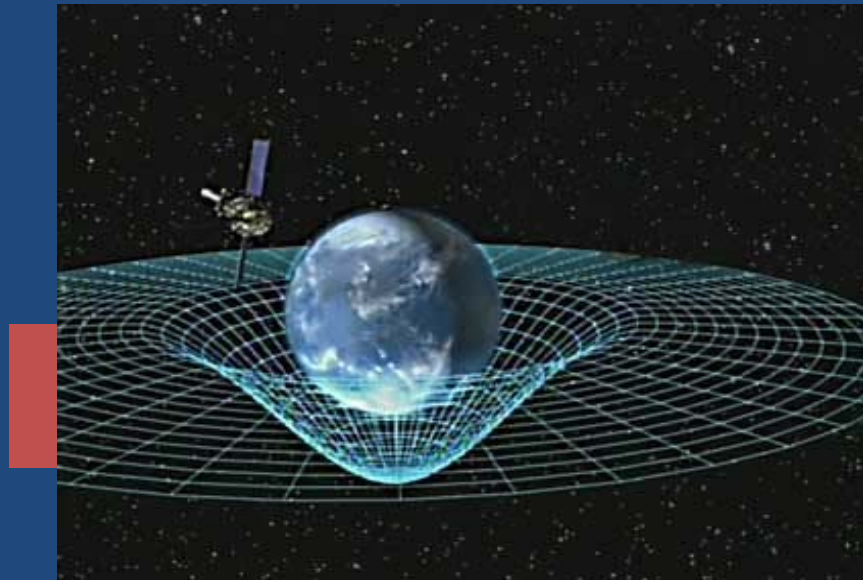
# General Relativity: Gravity as Geometry

The equations of GR describe the shape changes of the geometry of space-time caused by massive objects to which other objects respond.

In a curved space-time an object follows a path that maximizes the time in the frame of the object (proper time).

In GR the space-time grid is 'elastic', communicative and causal...but very very stiff!

**GR is a good theory for physics on large scales**



# Einstein's equations have surprising and remarkable solutions describing different space-times

1. Expanding and accelerating space-time (+ve cosmological constant) (Friedman 1922, LeMaitre 1927)
2. Gravitational waves (Einstein 1916)
3. Black Holes (Schwarzschild 1916; Kerr 1963)

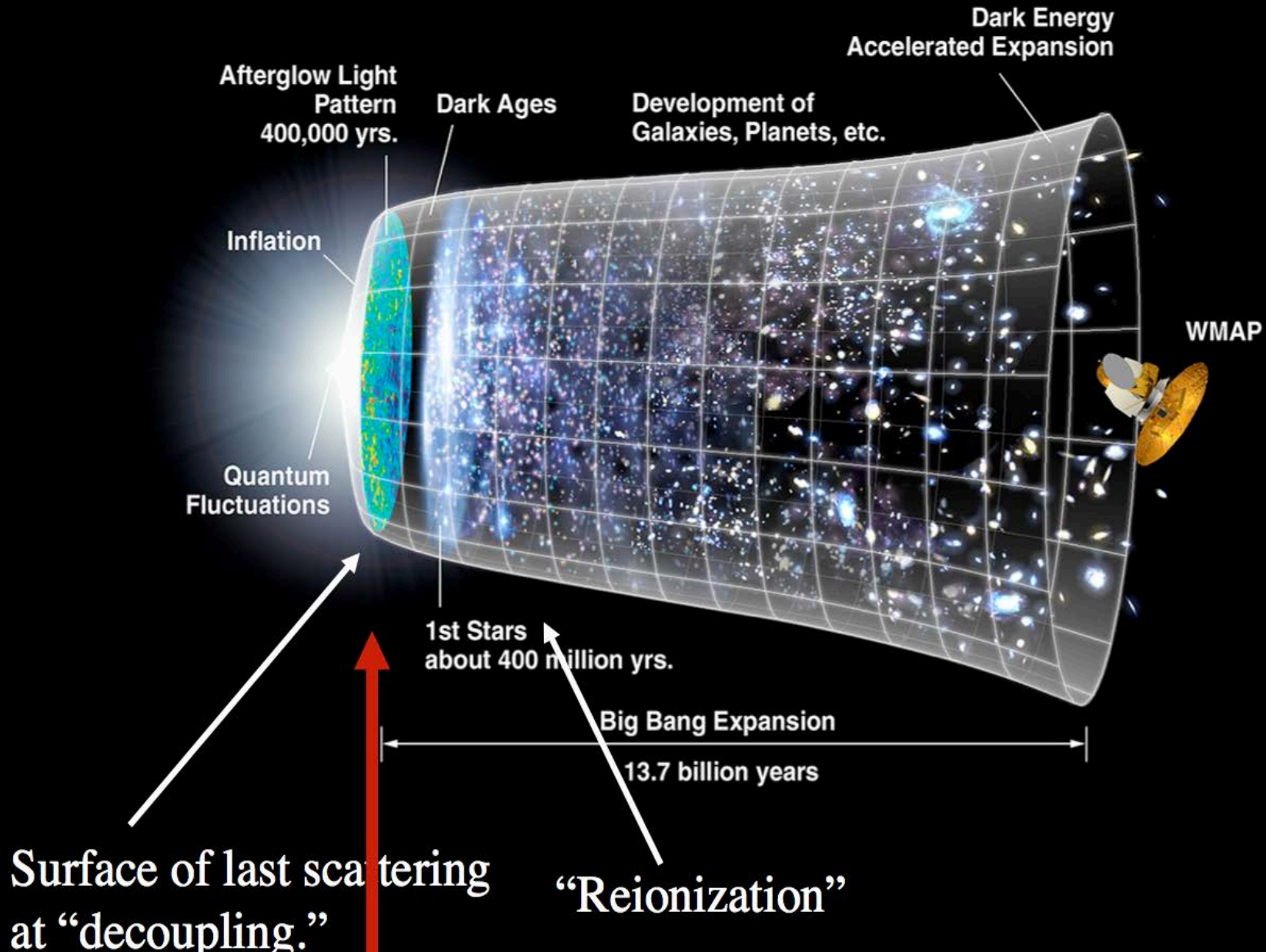
GR becomes a framework to discuss

black holes, gravitational waves and cosmology...

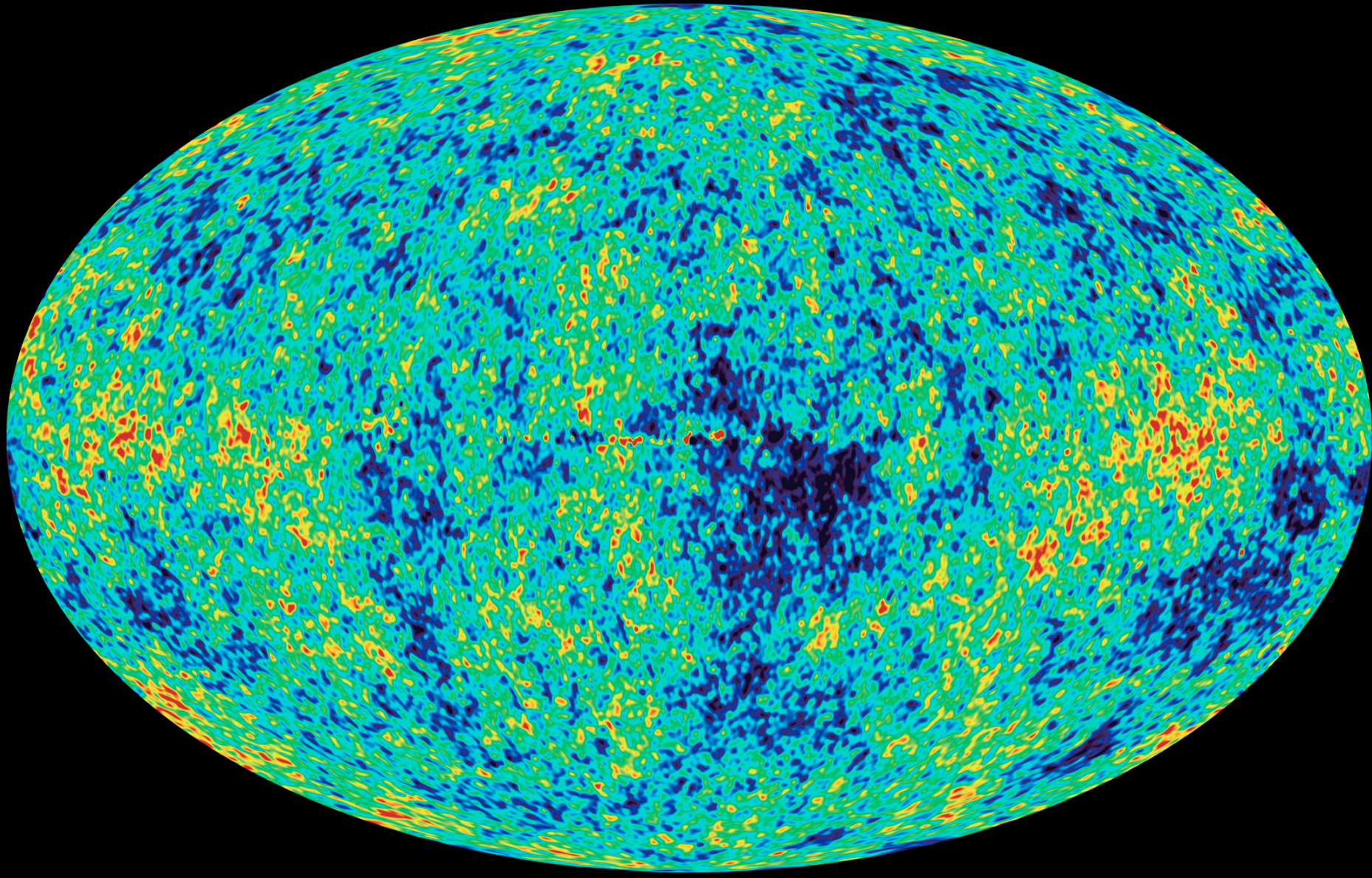
by now these applications are experimentally confirmed!



# The Standard Model of Cosmology





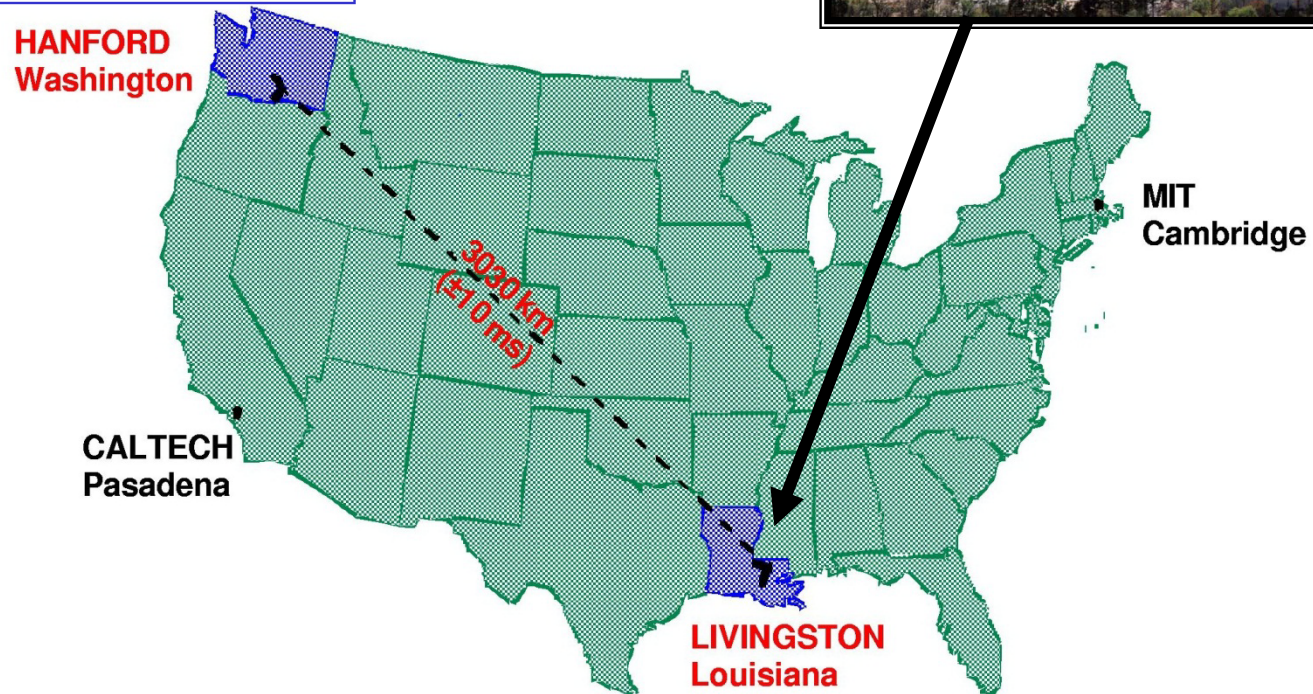


A Picture of the infant universe revealed in micro-wave radiation. Mean temperature 2.71 deg K. Temperature fluctuations are between -200 to +200 micro-Kelvin

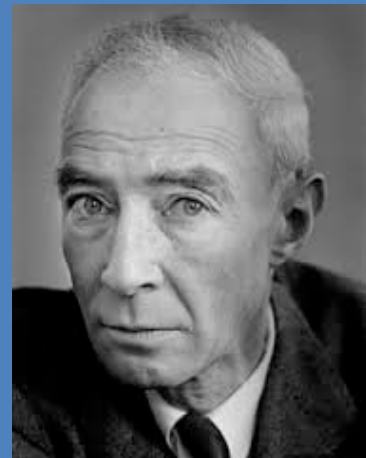
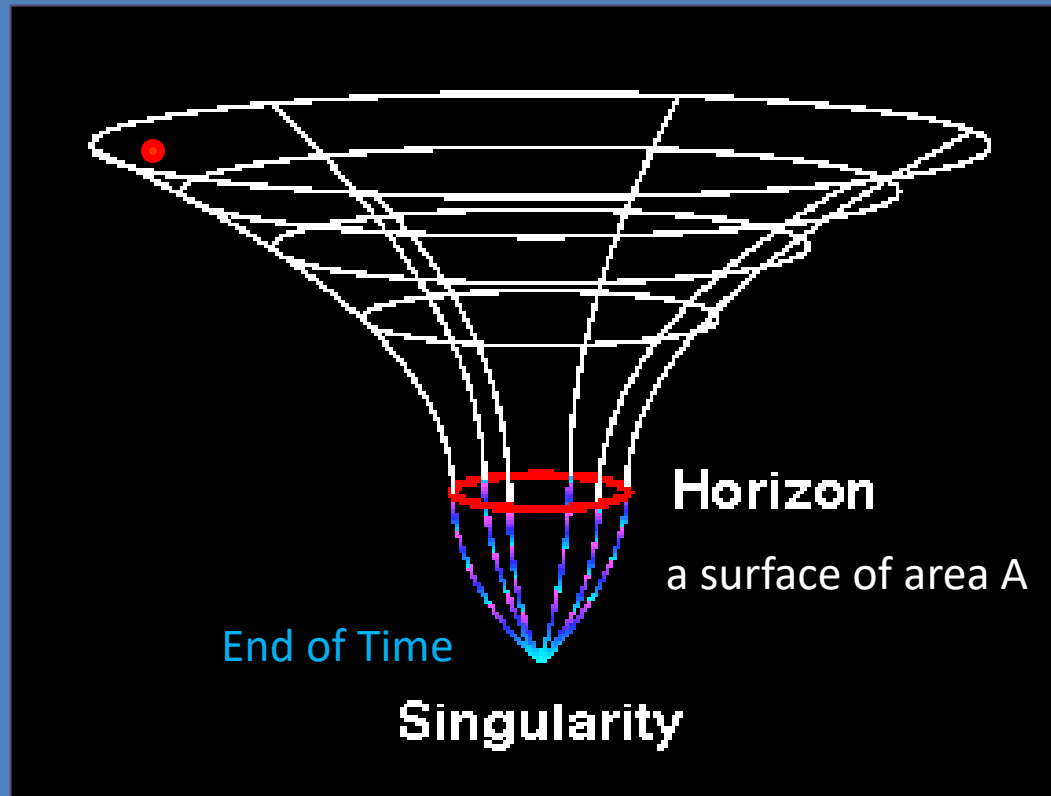




# On 14 September 2015, at the LIGO sites gravitational waves were detected



# General Relativity predicts Black Holes



Black holes are characterized by a horizon and a singularity. They exist in Nature!

Schwarzschild

Chandrasekhar & Oppenheimer





The first image of a supermassive black hole and its shadow in the Messier 87 galaxy , in the Virgo cluster. (Event Horizon Telescope collaboration)

# Black holes have entropy and temperature

## Jacob Bekenstein's thought experiment (1973):

Throw a bucket of hot water into a black hole.  
The water, which has some mass, disappears from the observable universe, and the final state has less 'entropy' than we began, violating the 2<sup>nd</sup> law of thermodynamics!

### Bekenstein-Hawking

$$S_{\text{BH}} = \alpha A_{\text{H}}$$

To avoid this conclusion a black hole must have entropy, so that  $\Delta S = \Delta M/T$  according to the laws of thermodynamics.

Area theorems of GR:

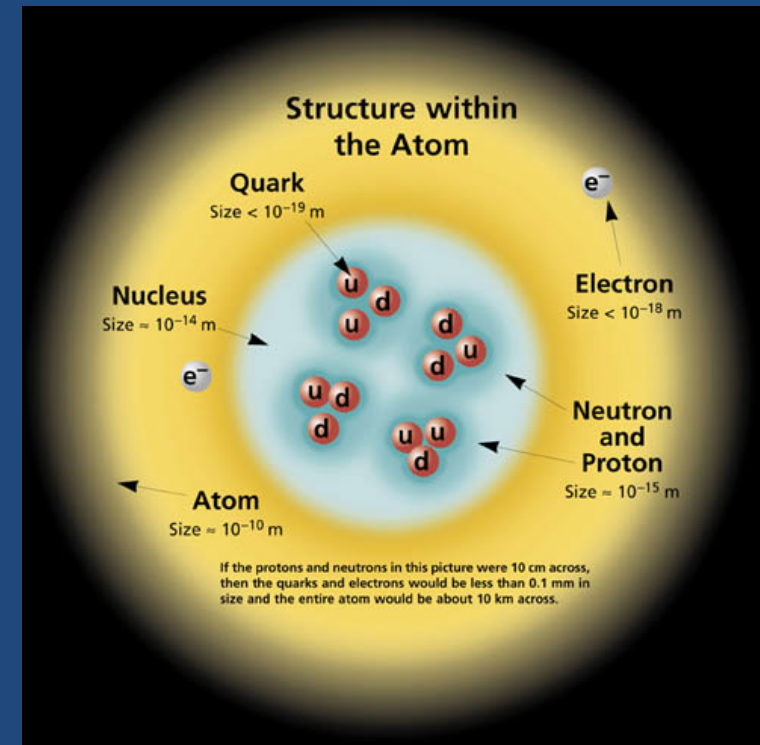
$Bh_1 + Bh_2 \rightarrow Bh_{(12)}$

$$A_{12} \geq A_1 + A_2$$

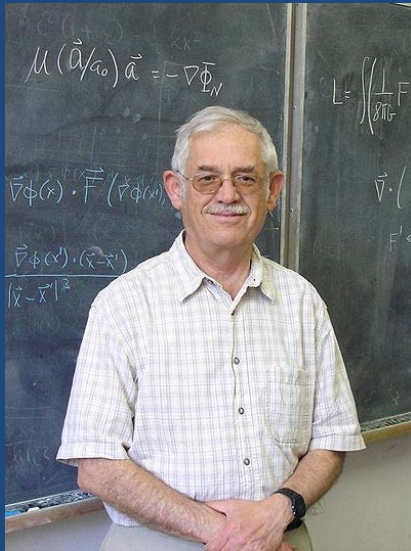
Bekenstein, using area theorems of GR, proposed his famous formula that the BH entropy is proportional to the area of the horizon of the BH.

# Enter Quantum Mechanics

- A 20<sup>th</sup> century scientific revolution:  
(Planck, Einstein, Bohr, Heisenberg, Schrodinger, Born, Dirac, Feynman and many many others)
- New laws for all particles especially below atomic scales.
- Electronic devices, computers, lasers, superconductors, superfluids, quantum computing...
- Colliding elementary particles in the LHC in Geneva all follow the laws of quantum mechanics ...  
tested to  $10^{-16}$  cms



# Jacob Bekenstein and Stephen Hawking





# Quantum Mechanics and Black Holes

In QM if a particle falls into a BH with a certain *probability*, then there is an equal probability for it to be emitted.

Hence quantum mechanics requires that  
**black holes must radiate!**

Hawking calculated the temperature of the black hole (1974); and argued that the end point of BH evaporation is purely thermal radiation.

$$T_H = \frac{\hbar c^3}{8\pi G k_B M}$$

- $T_{\text{sun}} = 3.6 \times 10^{-7} \text{ K}$
- $T_{\text{earth}} = 0.1 \text{ K}$
- $T_{M=10^{18} \text{ kg}} = 7000 \text{ K}$  (white light)

Temperature formula + 1<sup>st</sup> law of thermodynamics =>

$$S_{BH} = \frac{kc^3}{4\hbar G} A$$

One of the important formulas of 20<sup>th</sup> century physics that contains all three fundamental constants!

Bekenstein's constant of proportionality  $\alpha=1/(4A_{Pl})$

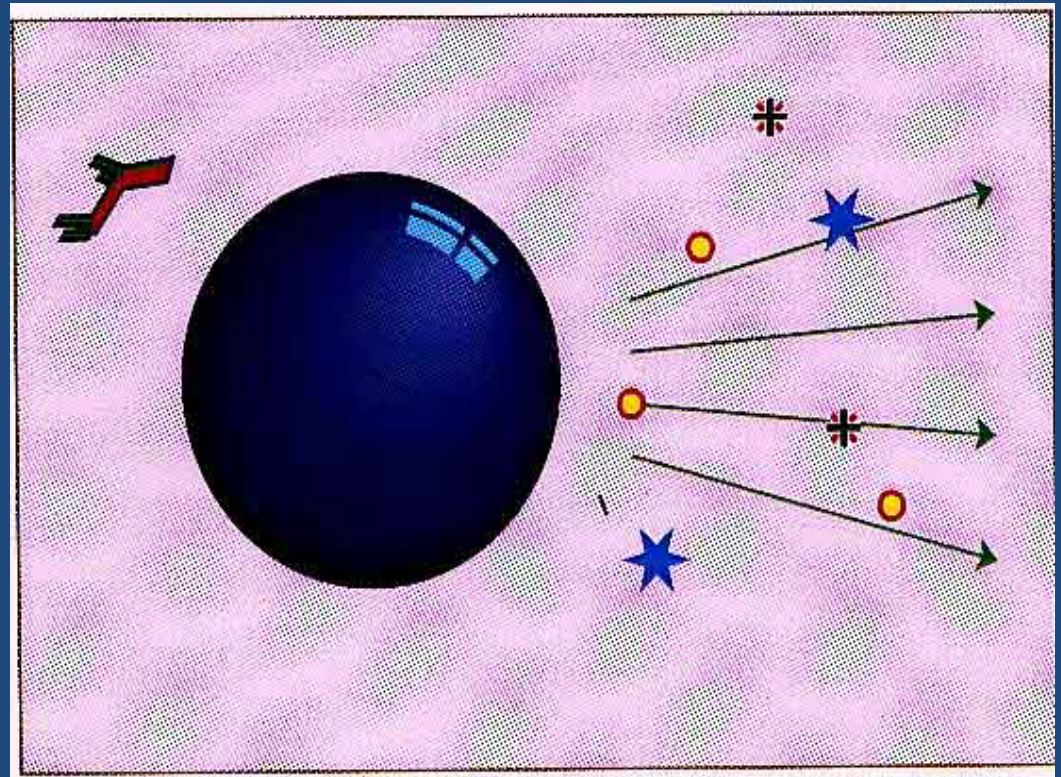
$A_{Pl} = (\hbar G/c^3) = (10^{-66}) \text{ cm}^2$ , is the 'Planck area' in 3-dims.

k is Boltzmann's constant

# Hawking Radiation and Information Loss

Hawking:

A black hole forms in various ways, but it always evaporates in the same way. The final state is thermal radiation leading to **information loss** as there is no memory of the initial state! This called the “Information Paradox”



# Information Loss by Hawking radiation is not allowed by Quantum Mechanics!

Since the black hole radiation is thermal the initial information about the details of the formation of a black hole is lost.

This violates a fundamental principle of Quantum Mechanics where information is never lost, except if we do statistical averages, as in quantum statistical mechanics.

When a block of wood burns, there is no conflict with quantum mechanics!



# Many Questions and Puzzles

1. Is there a theory of quantum gravity with degrees of freedom (states) that would account for black hole entropy:

$$S_{\text{BH}} = k \log(N)?$$

(Bekenstein-Hawking formula = Boltzmann formula)

2. Can information loss for black holes be understood as due to an averaging of a very large number of internal states, like in quantum statistical mechanics?
3. Can one calculate Hawking radiation in such a theory?
4. Is there an exact formulation of a non-perturbative string theory (quantum gravity)?
5. Where is the error in Hawking's calculation that creates a conflict with QM?

# A new framework of physics to answer these questions?

The most promising framework is 'String Theory'. It is a framework where one can do calculations and obtain finite answers. It contains general relativity at long distances.

It provides new degrees of freedom besides the graviton that can account for the entropy of black holes using Boltzmann's formula; and one can calculate Hawking radiation. (This is demonstrated for a class of black holes)

In the special case of anti-deSitter spacetimes it provides an exact holographic formulation of quantum gravity on the boundary of AdS (More later)

Most recently in lower dimensions one can point to the error in Hawking's calculation. Information theoretic ideas and holography have played an important role in this understanding.

# Partial answers

Q1. Is there a theory of quantum gravity with degrees of freedom (states) that would account for black hole entropy:

$$S_{\text{BH}} = k \log(N)?$$

(Bekenstein-Hawking formula = Boltzmann formula)

A1: In 1996 Strominger and Vafa calculated BH entropy and provided the first concrete evidence in a calculable model that the black hole space-time is a sort of a hydrodynamic description of more basic underlying constituents (of string theory) called D-branes discovered earlier by Polchinski.



Q 2. Can information loss for black holes be understood as due to an averaging of a very large number of internal states, like in quantum statistical mechanics?

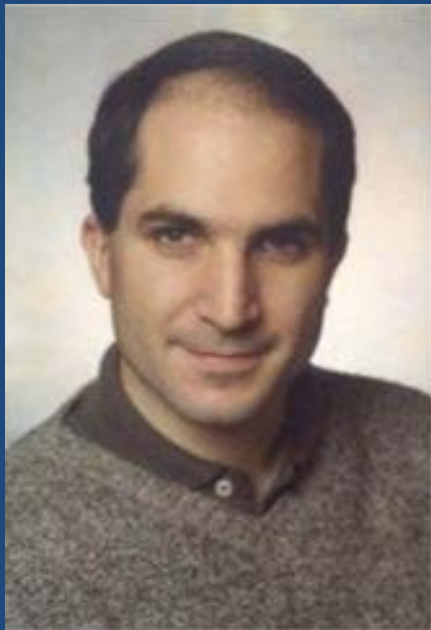
Q3. Can one calculate Hawking radiation in such a theory?

A2 and A3:

Hawking radiation and BH thermodynamics can be calculated in the framework of statistical mechanics in this constituent model of the black hole! (Dhar, Mandal, David, Wadia (1996; 2002); Das, Mathur; Callan, Maldacena)

# Holography and a precise formulation of Quantum Gravity...the AdS/CFT correspondence

-Juan Maldacena (1997)



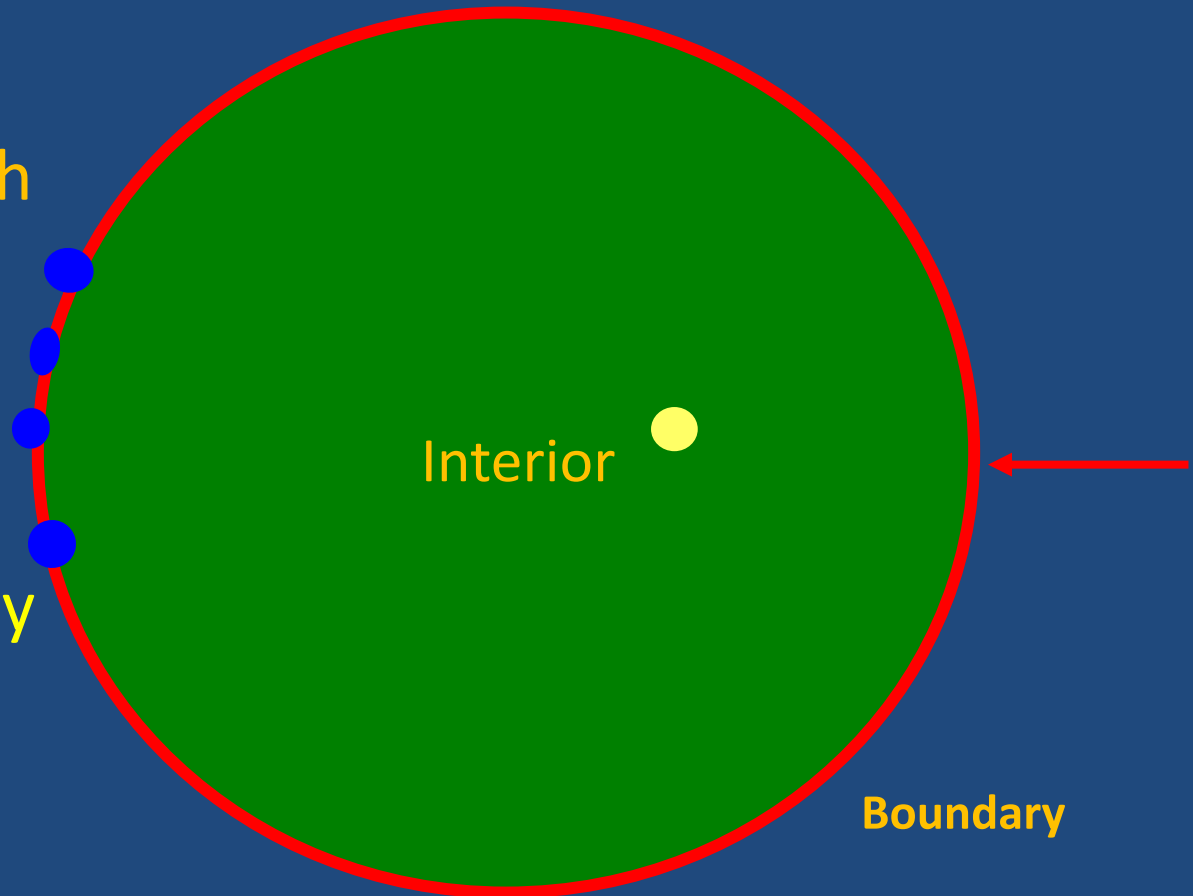
This development has led to enormous activity over the last 20 years...brought together people working in string theory, condensed matter physics, quantum information, chaotic systems...

Many important contributions from various groups in India.

Gravity in the interior →  
described by interacting particles on the boundary.

Interior is a  
Gravity/String Th  
In AdS  
space-time

On the boundary  
lives a unitary  
QFT

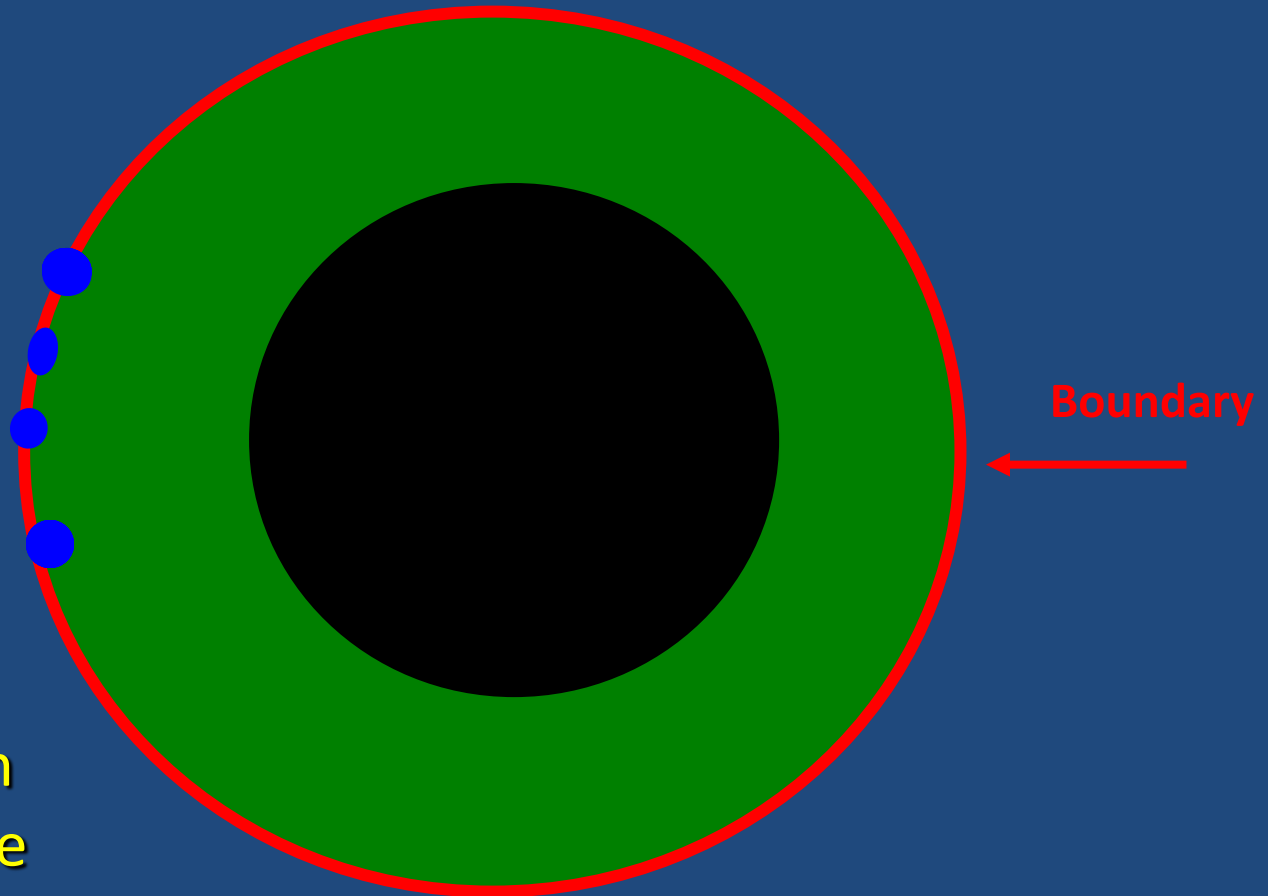




BH in AdS is dual to a unitary QFT at finite Temperature.

Temperature  
and entropy  
→ motion of  
particles  
on the  
boundary.

In principle  
solves information  
Puzzle because the  
boundary that tracks events in  
the bulk is 'unitary'...no information loss.



# Missing piece in Hawking's calculation

Using holography and ideas of quantum information, (very recently) there has been an understanding in the context of low dim models of the time dependence of the quantum entanglement entropy of the Hawking radiation...the so called 'Page Curve'.

Hawking had missed out an important piece in this formula, which crucially reflects the unitarity of the evaporation process.

Also an exact calculation of BH formation in a low dim. was done: Dhar, Gaikwad, Mandal and SRW (2019).

# Epilouge

Pursuing the resolution of the 'information paradox' presented by the work of Bekenstein and Hawking has led us to unearth deep facets of the quantum theory of space-time and gravity...and led to the discovery of the AdS/CFT holographic correspondence.

Many deep questions remain, e.g. an explicit understanding of the singularity of a black hole, where time ends. This could enable an understanding of the beginning of time...how the universe began...

# Acknowledgement

- The Infosys Foundation Homi Bhabha Chair
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