ICTS Summer School on Numerical Relativity 2013 – Mathematical formulation
Course information

Outline

- A brief review of GR (roughly Day 1)
  - Tensors and curvature
  - Einstein's field equations
  - Some important solutions
- The 3+1 decomposition (roughly Days 2 and 3)
  - Foliations of spacetime
  - Intrinsic and extrinsic curvature
  - The Lie derivative
  - The Gauss, Codazzi and Ricci equations
  - The ADM equations
- Solving the constraint equations (roughly Day 4)
  - Conformal Transformations
  - Elementary solutions to the Hamiltonian constraint
  - The conformal transverse-traceless decomposition
  - Bowen-York solutions
- Solving the evolution equations (roughly Day 5)
  - Comparison with Maxwell's equations
  - The "generalized harmonic" approach
  - The BSSN equations
  - Choosing the lapse and shift

Some References

Any textbook on General Relativity will be useful for a review of GR, but I particularly recommend

- T. Moore, *A General Relativity Workbook*, University Science Books
- S. Carroll, *Spacetime and Geometry*, Addison-Wesley

My lectures will mostly be based on

- T. W. Baumgarte & S. L. Shapiro, *Numerical Relativity: Solving Einsteins Equations on the Computer*, Cambridge University Press

but I also recommend

- M. Alcubierre, *Introduction to 3+1 Numerical Relativity*, Oxford University Press
- E. Gourgoulhon, *3+1 Formalism in General Relativity*, Springer
- C. Bona, C. Palenzuela-Luque & C. Bona-Casas, *Elements of Numerical Relativity and Relativistic Hydrodynamics*, Springer

Some Conventions

- We will use *geometrized units*, in which $c = G = 1$
- Indices $a, b, c, \ldots, h$ and $o, p, q, \ldots$ run over spacetime indices, while $i, j, k, \ldots, n$ run over spatial indices only ("Fortran" convention)
- We use the Einstein summation convention, by which we sum over repeated indices
- The flat spacetime (or space) metric is denoted by η_{ab} (or η_{ij}) in *any* coordinate system. Only in Cartesian (inertial) coordinates do we have $\eta_{ab} = \text{diag}(-1, 1, 1, 1)$.
- In general (but not always) will we refer to objects associated with
  - the spacetime M as g_{ab}, ${}^{(4)}\Gamma^a_{bc}$, ∇_a, ${}^{(4)}R_{ab}$, etc.
  - a spatial slice Σ as γ_{ij}, Γ^i_{jk}, D_i, R_{ij}, etc.
  - a conformally related space as $\bar{\gamma}_{ij}$, $\bar{\Gamma}^i_{jk}$, $\bar{D}_i$, $\bar{R}_{ij}$, etc.
- The symmetric and antisymmetric parts of a tensor are defined in the usual way, e.g.

$$T_{(ab)} \equiv \frac{1}{2}(T_{ab} + T_{ba}) \quad \text{and} \quad T_{[ab]} \equiv \frac{1}{2}(T_{ab} - T_{ba}).$$