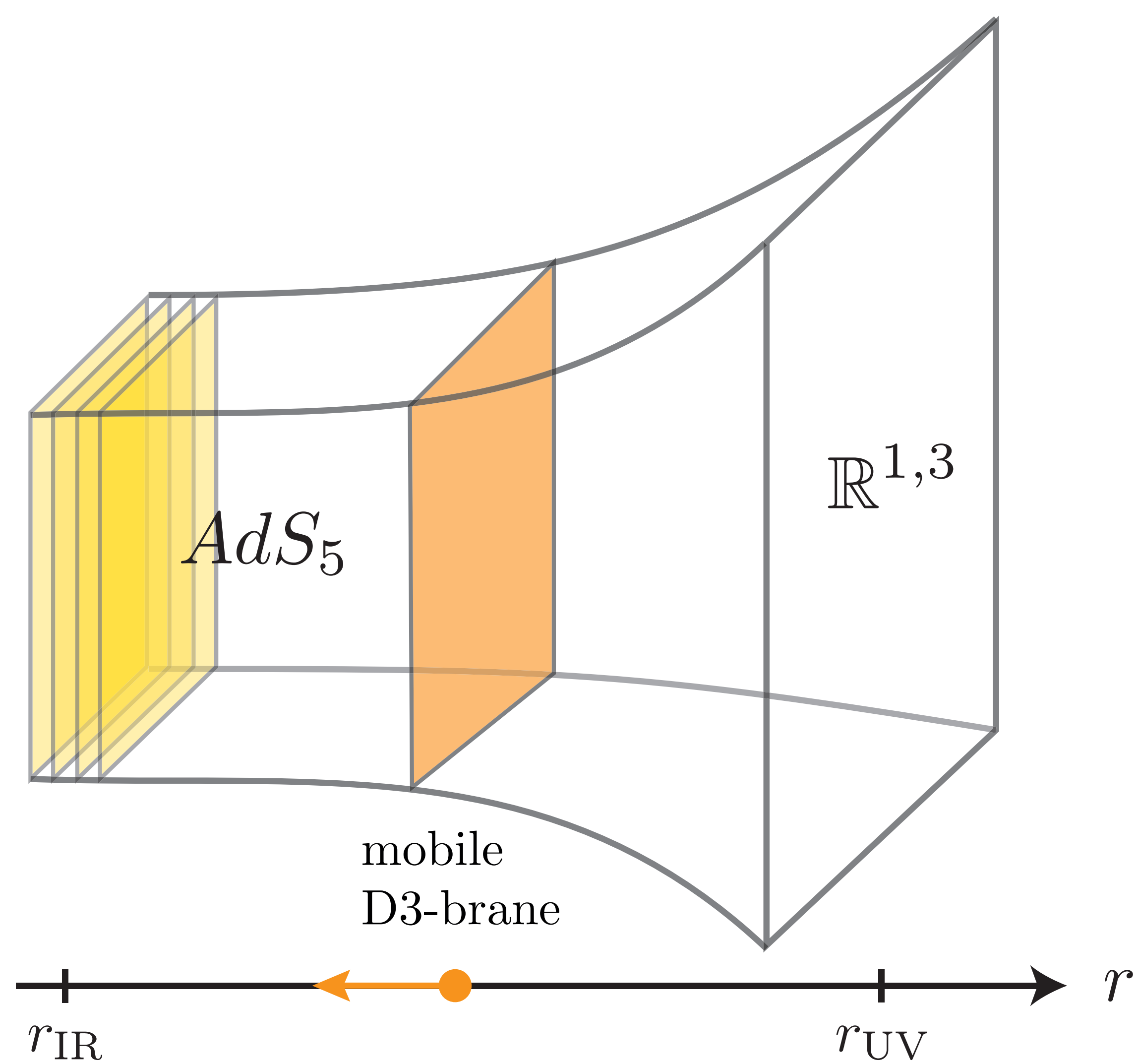
Lecture 3

Inflation in String Theory



Outline

- String Inflation as an EFT
- Moduli Stabilization
- Examples of String Inflation
  - Brane Inflation
  - Axion Monodromy Inflation

Reference: DB and Liam McAllister, *Inflation and String Theory*

String Inflation as an Effective Theory

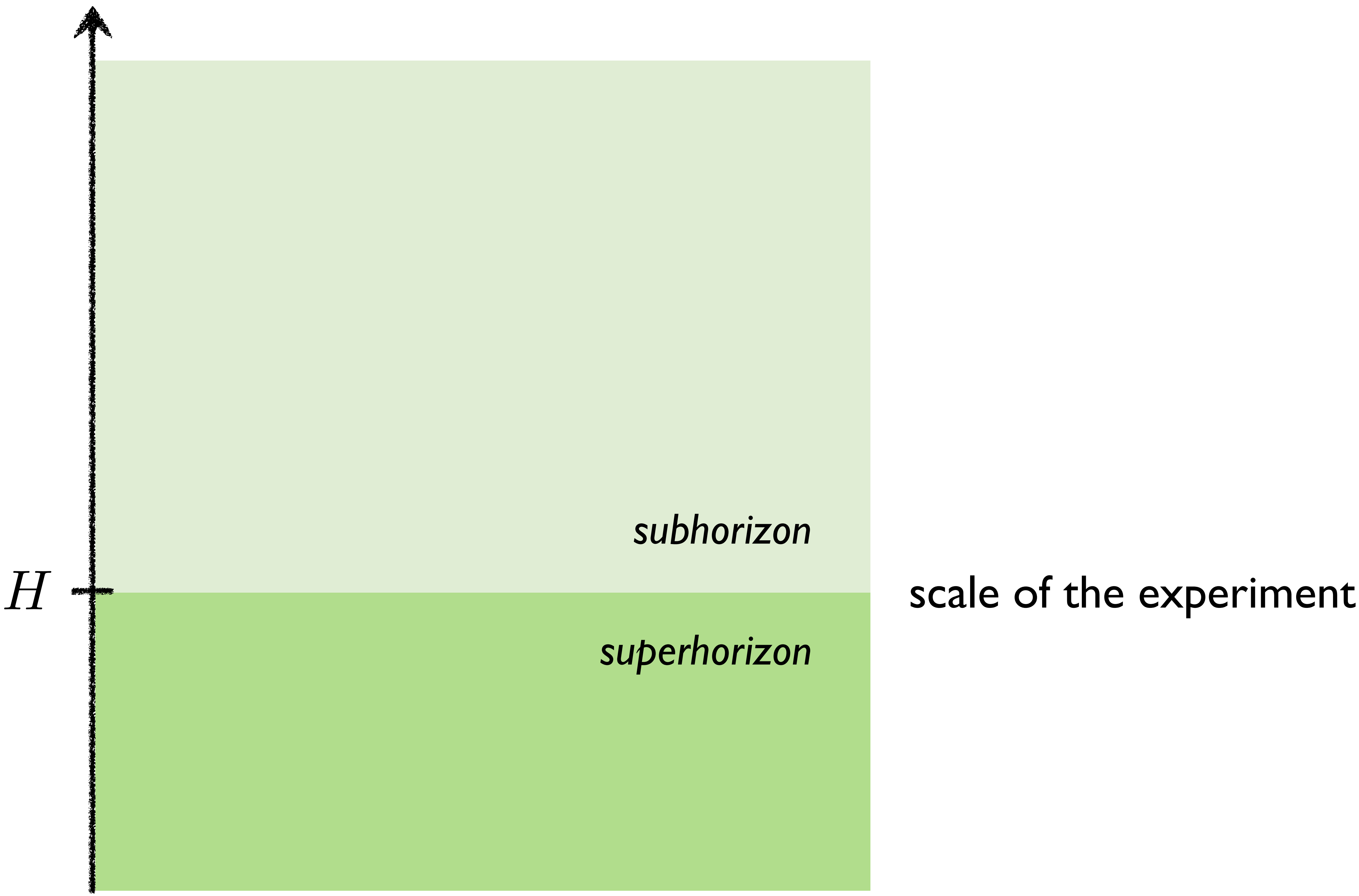
moduli

$$S_{10}[\mathcal{C}] \xrightarrow[\text{M}_4 \times \text{X}_6]{\textit{compactification}} S_4[\Phi(t)]$$

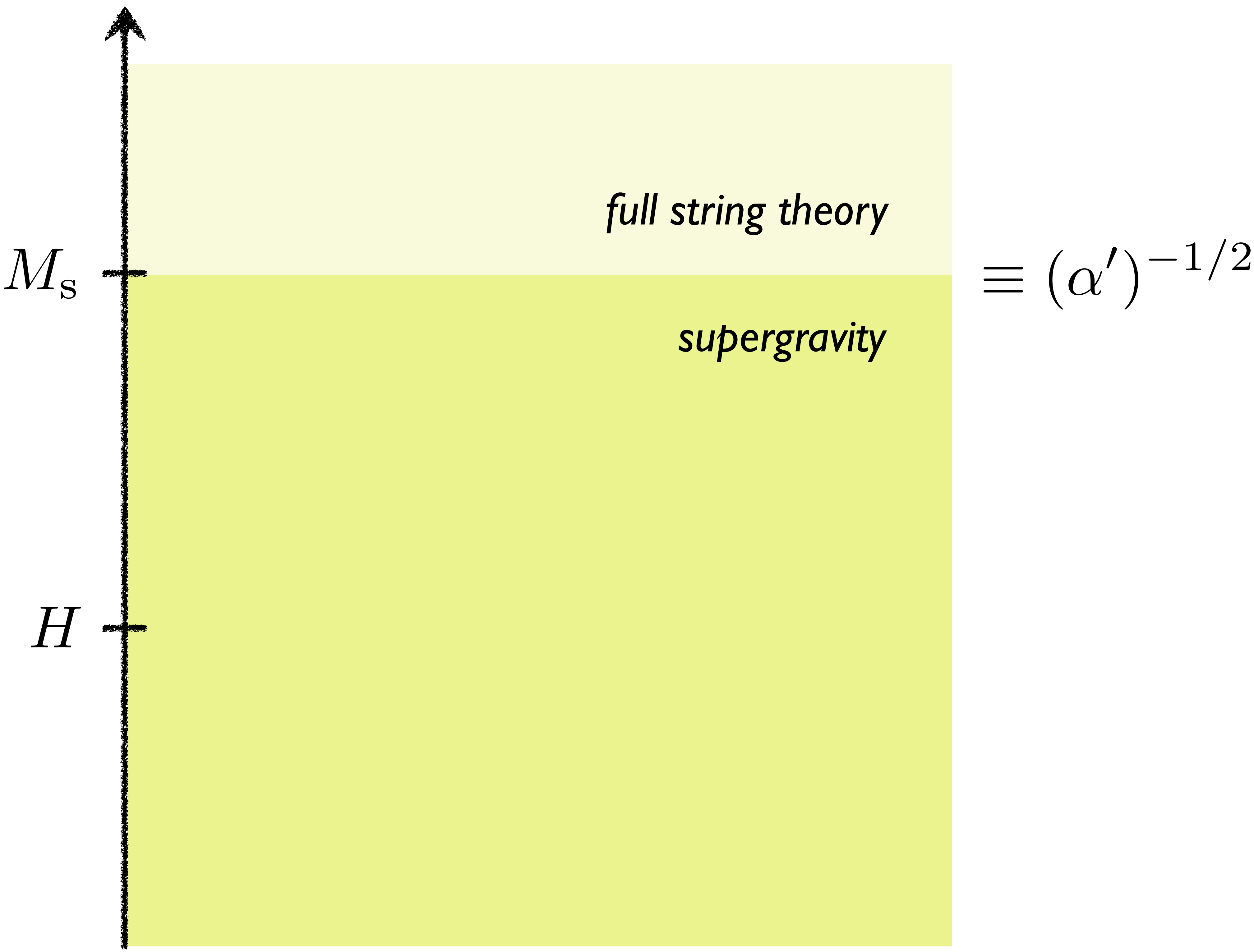
4D Effective Action

- topology, geometry
- fluxes
- local sources
  (D-branes, O-planes)

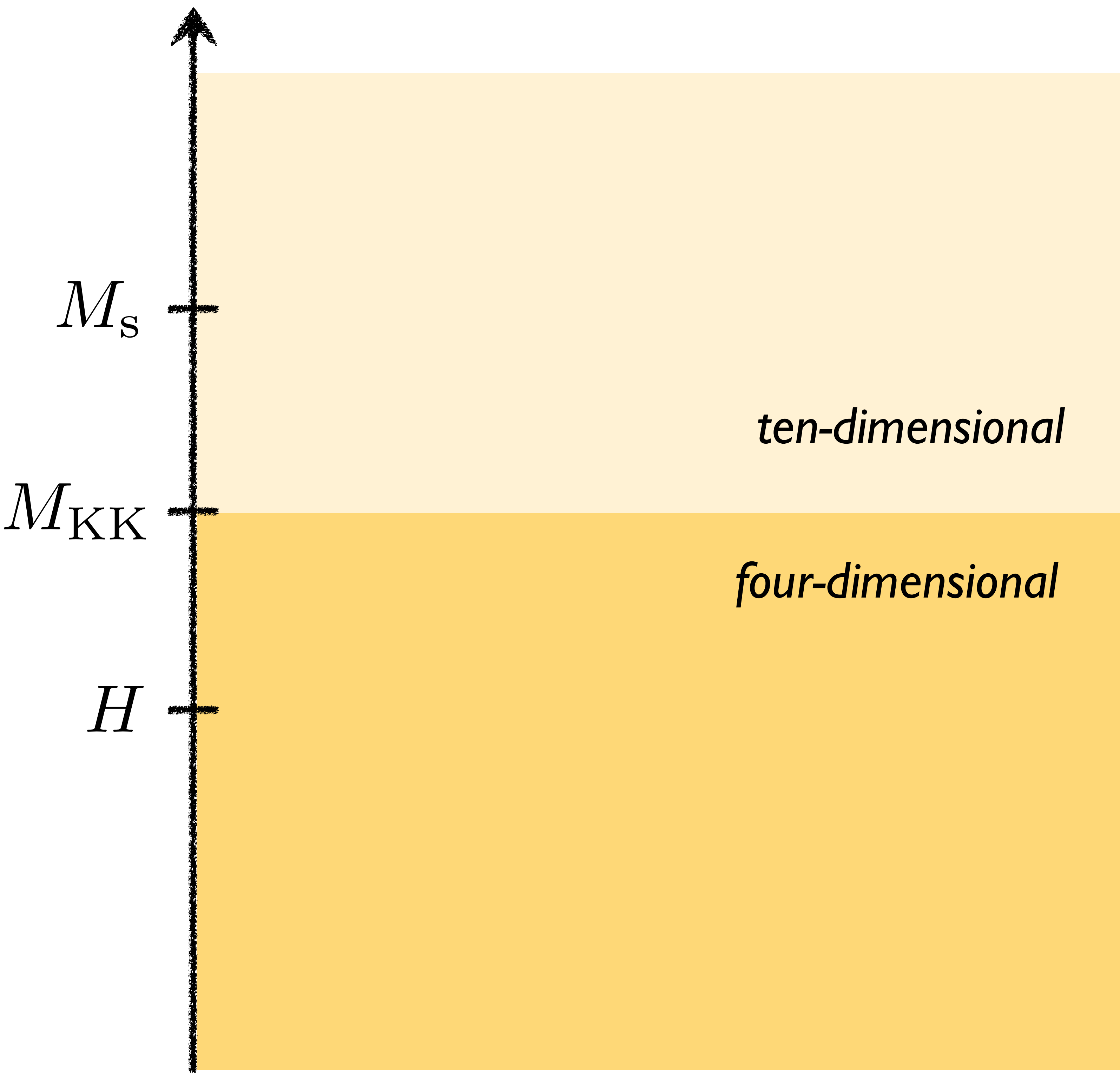
Energy Scales



Energy Scales



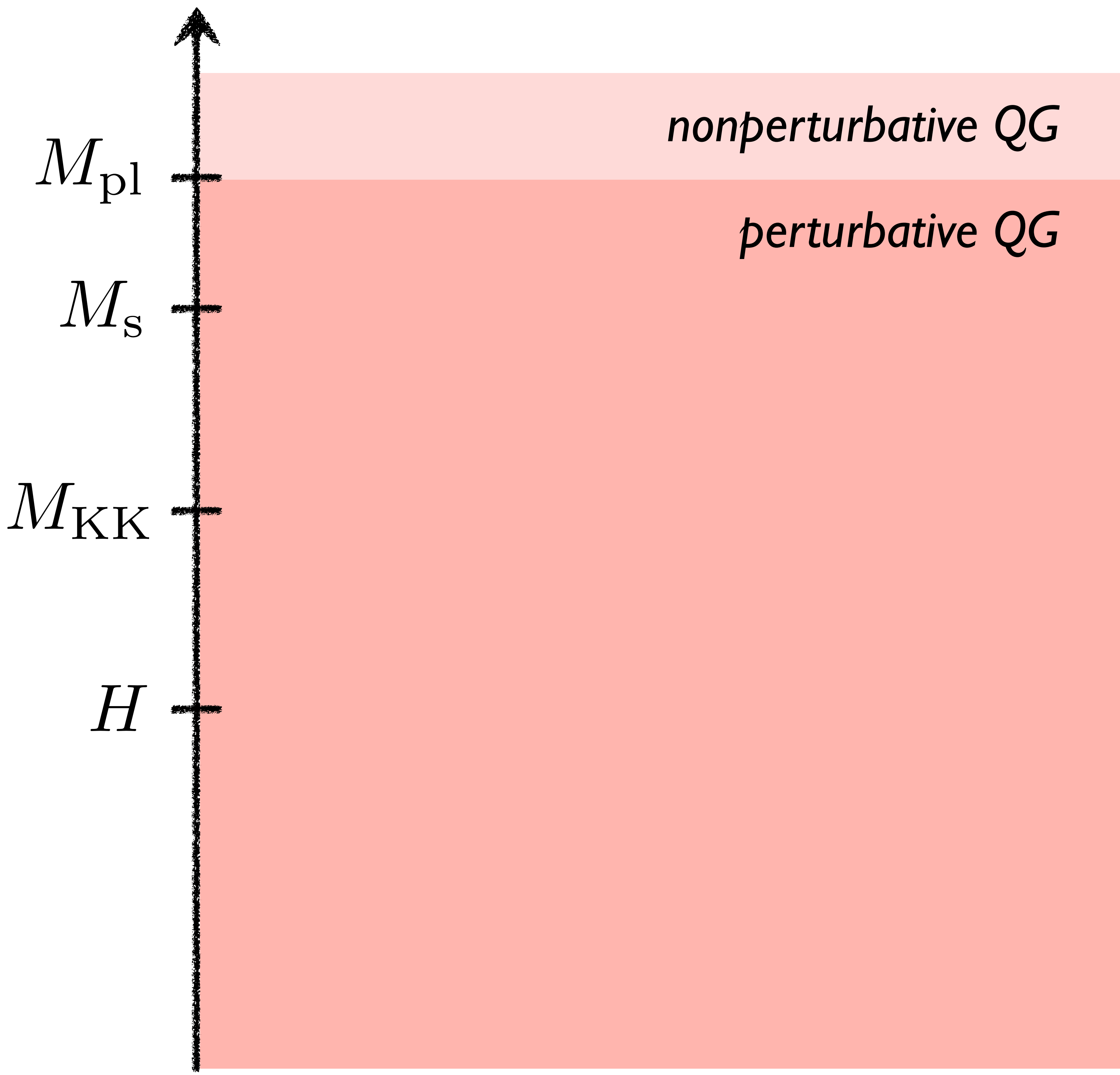
Energy Scales



$\sim M_{\mathrm{s}} \mathcal{V}^{-1/6}$

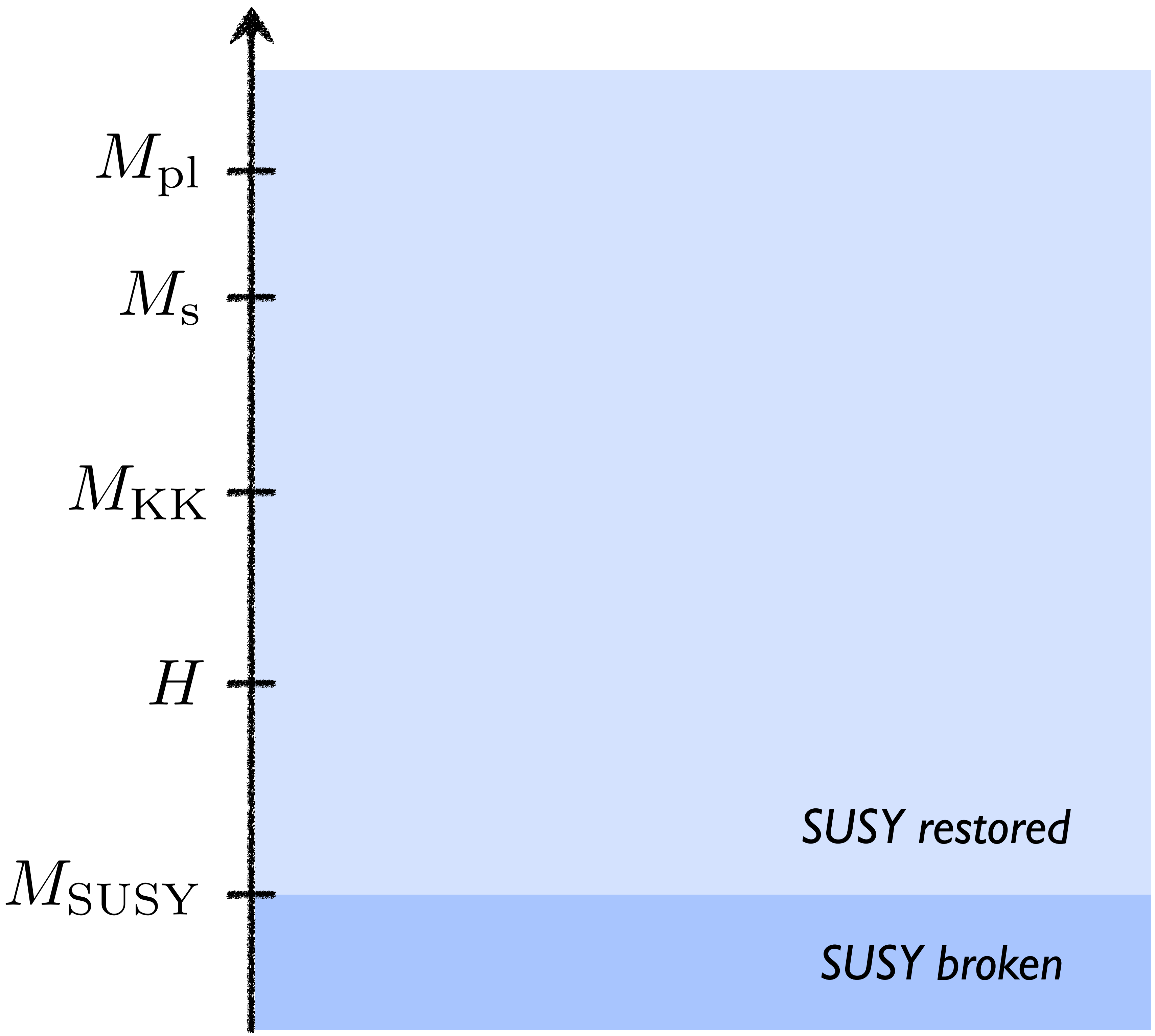
volume in string units

Energy Scales

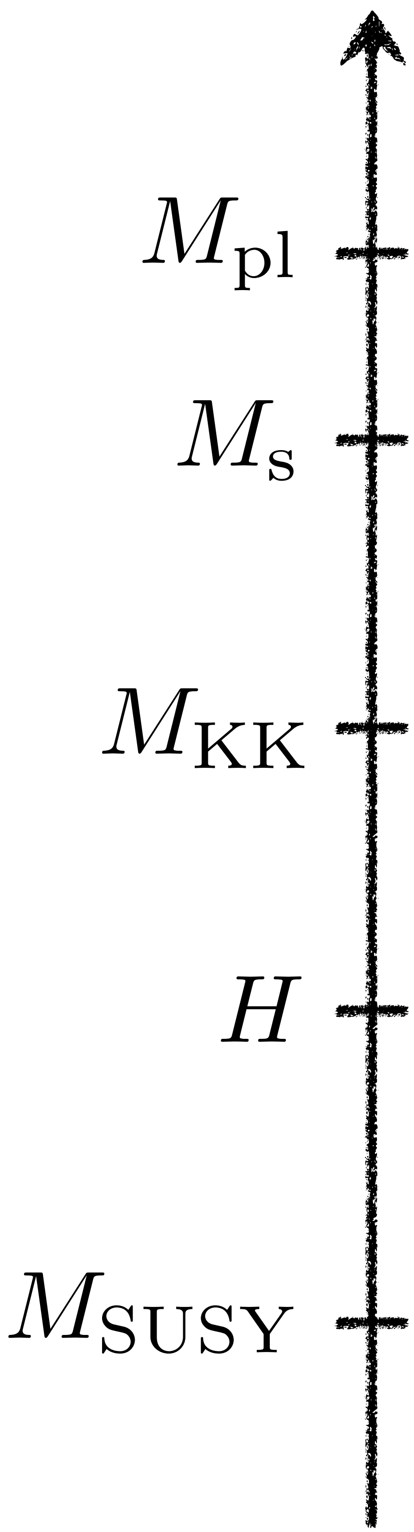


$$\sim \frac{M_{\rm s}}{g_{\rm s}}\left(\frac{M_{\rm s}}{M_{\rm KK}}\right)^3$$

Energy Scales



Energy Scales



All controlled models of string inflation work with this hierarchy of scales.

Moduli

= zero energy deformations of X_6

= massless particles on M_4

- volume

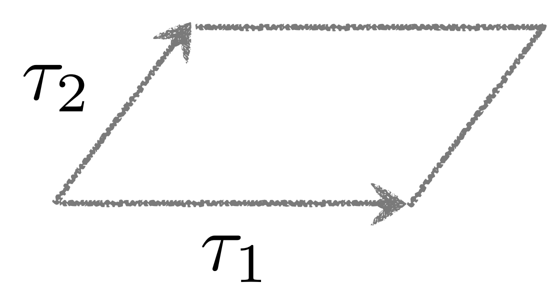
  vacuum Einstein equations are scale-invariant

- dilaton

  even a modulus in 10D

- metric moduli

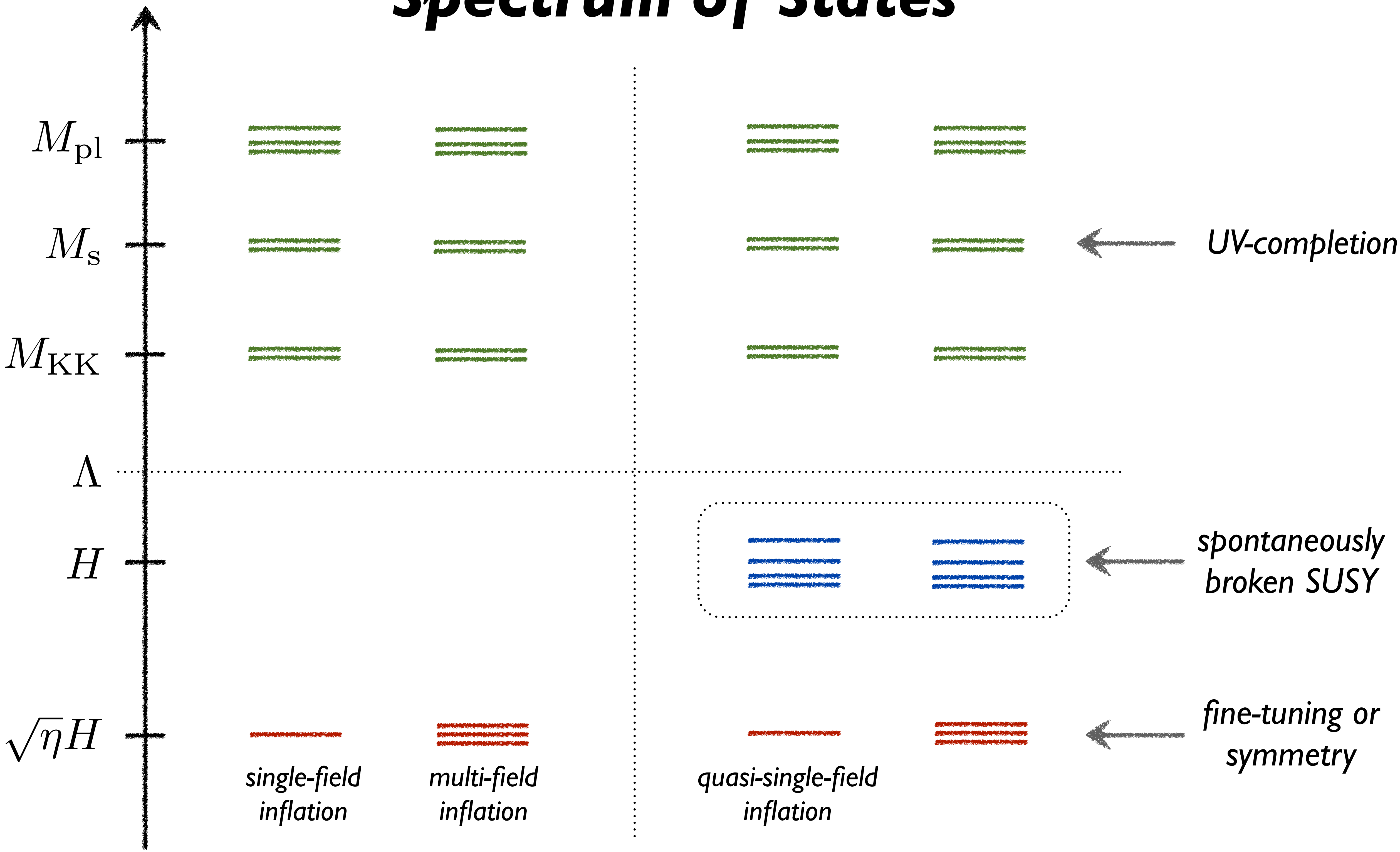
  - ☐ *Kähler moduli*
  - ☐ *complex structure moduli*



- axions

- brane moduli

Spectrum of States



To derive the spectrum of states and their interactions, we need to study ***compactification*** and ***moduli stabilization***.

Common Challenges

- Moduli stabilization does ***not*** decouple from the inflationary dynamics.

  It is hard to keep the inflaton light.

- If one field is light, why not many?

  Fields with $m < \frac{3}{2}H$ are quantum-mechanically active and determine the inflationary phenomenology.

Why string vacua are dirty.

Denef, *Les Houches Lectures on Constructing String Vacua*

Dine-Seiberg Problem

Let ρ be a modulus and $\rho \to \infty$ correspond to the weakly-coupled region.

compactification volume

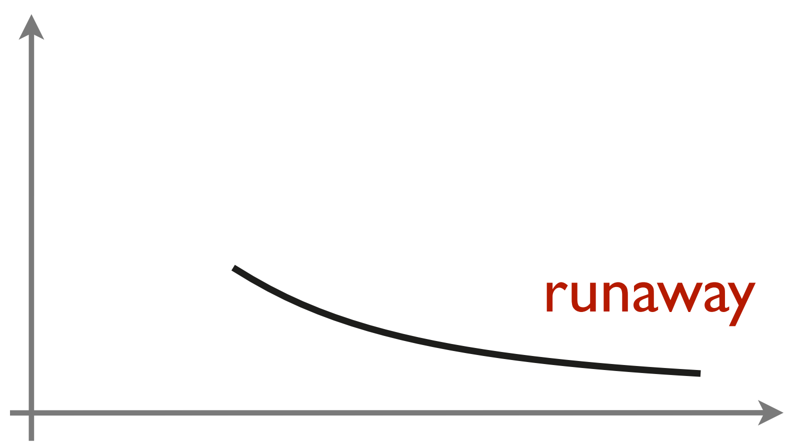
(inverse) string coupling

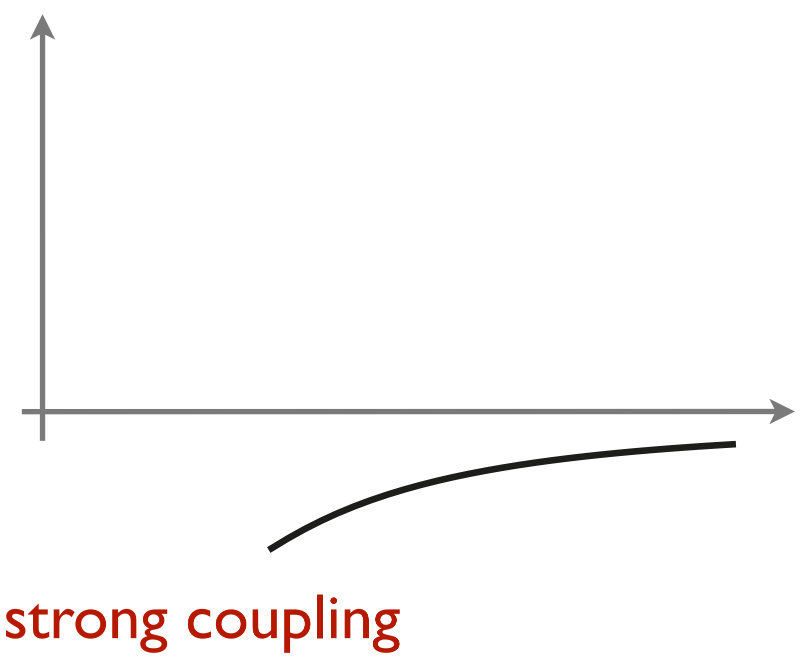
Dine-Seiberg Problem

Let ρ be a modulus and $\rho \to \infty$ correspond to the weakly-coupled region.

Quantum corrections generate a potential, which satisfies $\lim_{\rho \to \infty} V(\rho) = 0$.

At first order, there are two possibilities:



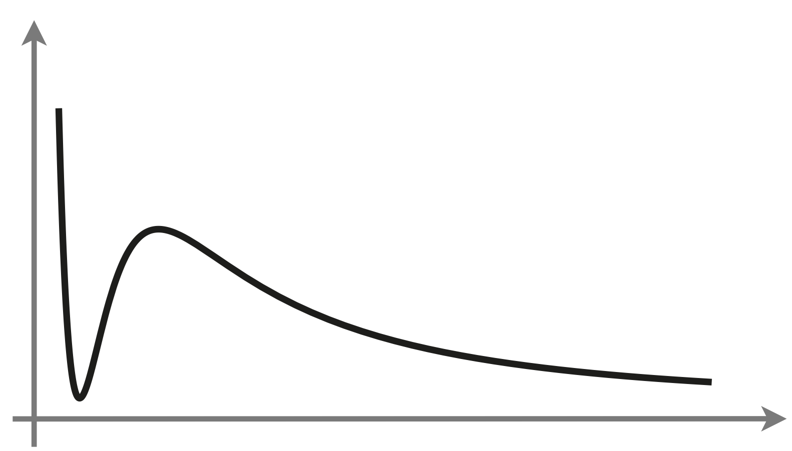


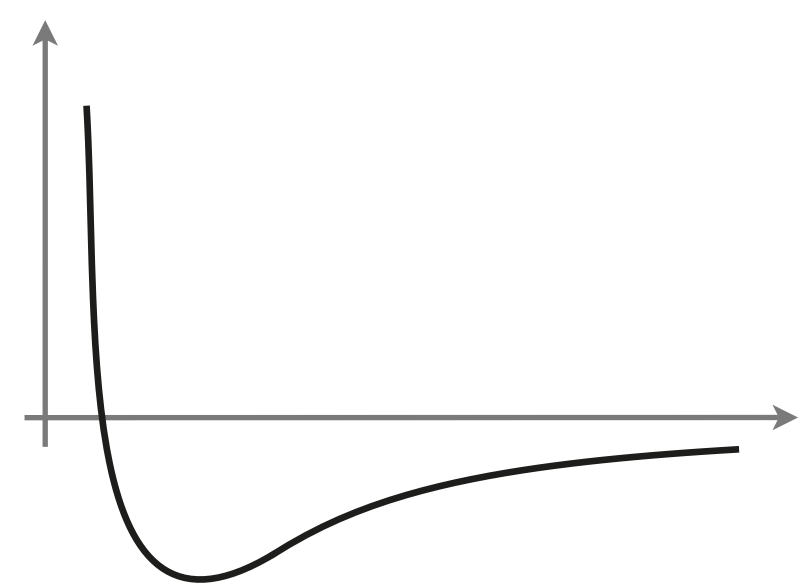
Dine-Seiberg Problem

Let ρ be a modulus and $\rho \to \infty$ correspond to the weakly-coupled region.

Quantum corrections generate a potential, which satisfies $\lim_{\rho\to\infty} V(\rho) = 0$.

Need to include higher-order corrections to generate a local minimum:





"The string vacuum we live in is probably strongly coupled."

Dine and Seiberg

"When corrections can be computed, they are not important, and when they are important, they cannot be computed." Denef

Maldacena-Nunez No-Go

Einstein-Hilbert

scalars, p-forms, ...

Consider D-dimensional gravity coupled to arbitrary massless fields with positive kinetic terms, and zero or negative potential

Theorem:

There do not exist non-singular warped compactifications to de Sitter.

Maldacena and Nunez

String theory evades the theorem by having higher-order curvature corrections and singular negative tension O-planes.

Case Study: Type IIB Vacua

Douglas and Kachru, *Flux Compactifications*

Kachru, Kallosh, Linde and Trivedi, *De Sitter Vacua in String Theory*

Giddings, Kachru, and Polchinski, *Hierarchies from Fluxes in String Compactifications*

Type IIB Supergravity

The bosonic fields of type IIB supergravity are:

- metric G_{MN}
- dilaton Φ
- NS 3-form $H_3 = \mathrm{d}B_2$
- RR p-forms $F_p = \mathrm{d}C_{p-1}$

- metric $G^{(E)}_{MN} \equiv e^{-\frac{1}{2}\Phi} G_{MN}$
- axiodilaton $\tau \equiv C_0 - ie^{-\Phi}$
- 3-form flux $G_3 \equiv F_3 - \tau H_3$
- 5-form flux $\tilde{F}_5 \equiv F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$

The type IIB action is

$$\boxed{S_{\mathrm{IIB}} = \frac{1}{2\kappa^2} \int \mathrm{d}^{10}X \sqrt{-G} \left[R - \frac{|\partial\tau|^2}{2(\mathrm{Im}\,\tau)^2} - \frac{|G_3|^2}{12\mathrm{Im}\,\tau} - \frac{1}{4}|\tilde{F}_5|^2 \right] + \text{CS}}$$

$+\,\alpha'$ corrections
$+\,g_s$ corrections

where $\kappa^2 \equiv \frac{1}{2}(2\pi)^7(\alpha')^4$

Type IIB Supergravity

In addition, we have localized sources: D-brane and O-planes.

The action of a Dp-brane is

DBI CS

$$\boxed{S_{\mathrm{Dp}} = -T_p \int \mathrm{d}^{p+1}\sigma \sqrt{-\det(G_{ab} + \cdots)} + \mu_p \int C_{p+1}}$$

tension

charge

worldvolume flux

$\mu_p = g_s T_p$

$$T_p \equiv \frac{1}{(2\pi)^p g_s (\alpha')^{(p+1)/2}}$$

induced metric

$$G_{ab} \equiv \frac{\partial X^M}{\partial \sigma^a} \frac{\partial X^N}{\partial \sigma^b} G_{MN}$$

Dimensional Reduction

Consider the following ansatz for the metric:

$$\mathrm{d}s^2 = e^{-6u(x)} g_{\mu\nu} \mathrm{d}x^\mu \mathrm{d}x^\nu + e^{2u(x)} \hat{g}_{mn} \mathrm{d}y^m \mathrm{d}y^n$$

breathing mode

reference metric with fixed volume

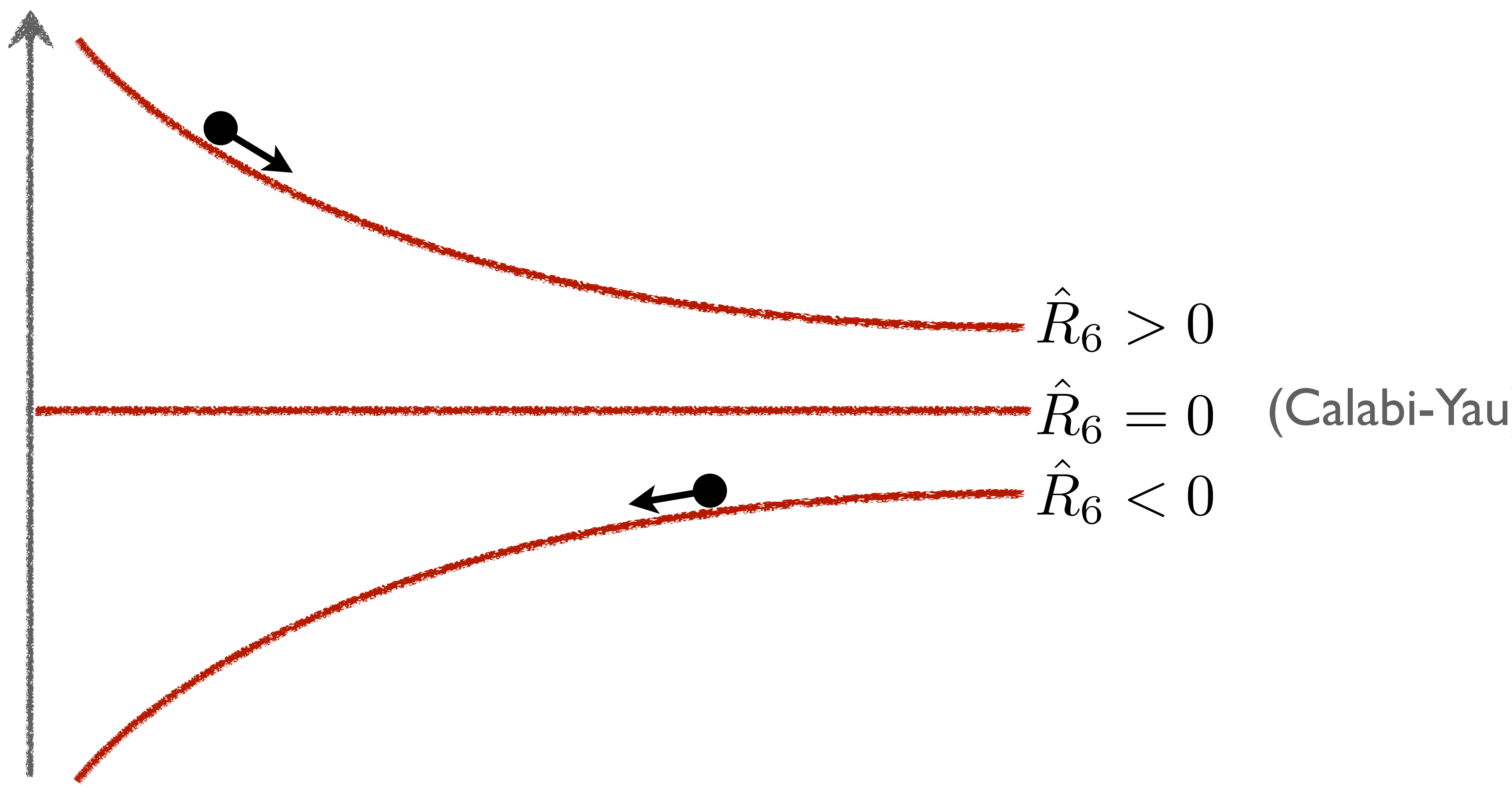
and substitute it into the gravity part of the IIB action

$$\begin{aligned} S_{\mathrm{EH}}^{(10)} &= \frac{1}{2\kappa^2} \int \mathrm{d}^{10} X \sqrt{-G}\, R_{10} \\ &= \frac{M_{\mathrm{pl}}^2}{2} \int \mathrm{d}^4 x \sqrt{-g} \left[R_4 + e^{-8u} \hat{R}_6 + \frac{12}{u^2} \partial_\mu u \, \partial^\mu u \right] \end{aligned}$$

where $M_{\mathrm{pl}}^2 \equiv \dfrac{\mathrm{Vol}(X_6)}{g_s^2 \kappa^2}$.

Dimensional Reduction

$$S_4 = \frac{M_{\rm pl}^2}{2}\int \mathrm{d}^4x\sqrt{-g}\left[R_4 + e^{-8u}\hat{R}_6 + \frac{12}{u^2}\partial_\mu u\,\partial^\mu u\right]$$



At least the breathing mode is unfixed in vacuum compactifications.

Therefore consider compactifications with ***branes*** and ***fluxes***.

Moduli Stabilization I: Classical Effects

Consider the ansatz:

warp factor

$$\mathrm{d}s^2 = e^{2A(y)}\mathrm{d}x^\mu \mathrm{d}x_\mu + e^{2u(x)}e^{-2A(y)}\hat{g}_{mn}\mathrm{d}y^m\mathrm{d}y^n$$

$$\tilde{F}_5 = (1+\star)\,\mathrm{d}\alpha(y) \wedge \mathrm{d}x^0 \wedge \mathrm{d}x^1 \wedge \mathrm{d}x^2 \wedge \mathrm{d}x^3$$

GKP

This satisfies the supergravity equations of motion if

$$\boxed{\star G_3 = i G_3} \quad \text{and} \quad \boxed{e^{4A} = \alpha}$$

imaginary self-dual (ISD)

Ex: *Show that a D3-brane feels no force in an ISD compactification, i.e. show that the potential for a D3-brane is*

$$V_{\mathrm{D3}} = T_3(e^{4A} - \alpha)$$

Moduli Stabilization I: Classical Effects

Turning on $G_3 = F_3 - \tau H_3$ creates a potential for the complex structure moduli and the dilaton.

recall: $\mathcal{L} \subset G_3 \wedge \star G_3$

metric

It is convenient to describe this in 4D N=1 supergravity:

- The flux superpotential is

holomorphic (3,0) form

$$\boxed{W = \int G_3 \wedge \Omega}$$

Gukov, Vafa, and Witten

- The Kähler potential is

$$\boxed{K = -3\ln(T + \bar{T}) + \cdots}$$

where $T \equiv \underbrace{\int_{\Sigma_4} \sqrt{G}}_{\propto\, e^{4u}} + i \int_{\Sigma_4} C_4$ is the complexified Kähler modulus.

Moduli Stabilization I: Classical Effects

The scalar potential is $V_F = e^K \left[K^{A\bar{B}} D_A W \overline{D_B W} - 3|W|^2 \right]$

Ex: *Show that* $K^{T\bar{T}} \partial_T K \partial_{\bar{T}} K = 3$ *, so that*

$$V_F = e^K \sum_{A,B \neq T} K^{A\bar{B}} D_A W \bar{(} D_B W)$$

no-scale potential

This is positive semi-definite and has a minimum $V_F = 0$

when $D_A W = 0$ for all $A \neq T$.

The volume is still unfixed.

Moduli Stabilization I: Classical Effects

In perturbation theory, T does not appear in W.

Reason: $T = \cdots + i \underbrace{\int_{\Sigma_4} C_4}_{\equiv \theta}$

axion
with shift symmetry $\theta \mapsto \theta + const.$

So, $W = T^\alpha$ is forbidden to all orders in α' and g_s .

There are then two options for generating a potential for T :

1) Perturbative corrections to K. $\longrightarrow$ LVS

2) Nonperturbative corrections to W. $\longrightarrow$ KKLT

We will look at the second.

Moduli Stabilization II: Quantum Corrections

Consider N_c D7-branes wrapping a 4-cycle Σ_4 in X_6 .

Ex: *Show that the dimensional reduction of the D7-brane action gives*

$$S = \frac{1}{4g^2} \int \mathrm{d}^4 x \sqrt{-g}\ \mathrm{Tr}[F_{\mu\nu} F^{\mu\nu}]$$

gauge coupling

$$\boxed{\frac{1}{g^2} = \frac{T_3 \mathrm{Vol}(\Sigma_4)}{8\pi^2}}$$

Gaugino condensation in the gauge theory leads to

$$|W_{\langle\lambda\lambda\rangle}| = 16\pi^2 M_{\mathrm{UV}}^3 \exp\left(-\frac{1}{N_c}\frac{8\pi^2}{g^2}\right) \propto \exp\left(-\frac{T_3 \mathrm{Vol}(\Sigma_4)}{N_c}\right)$$

Moduli Stabilization II: Quantum Corrections

Since $\mathrm{Vol}(\Sigma_4) \propto T + \bar{T}$, we have

$$\boxed{W_{\langle\lambda\lambda\rangle} = \mathcal{A} e^{-aT}} \qquad \text{where} \quad a \equiv \frac{2\pi}{N_c}$$

This break no-scale and stabilizes the volume.

At low energy, complex structure and dilaton can be integrated out and the EFT for T is:

$$K = -3\ln(T+\bar{T})$$

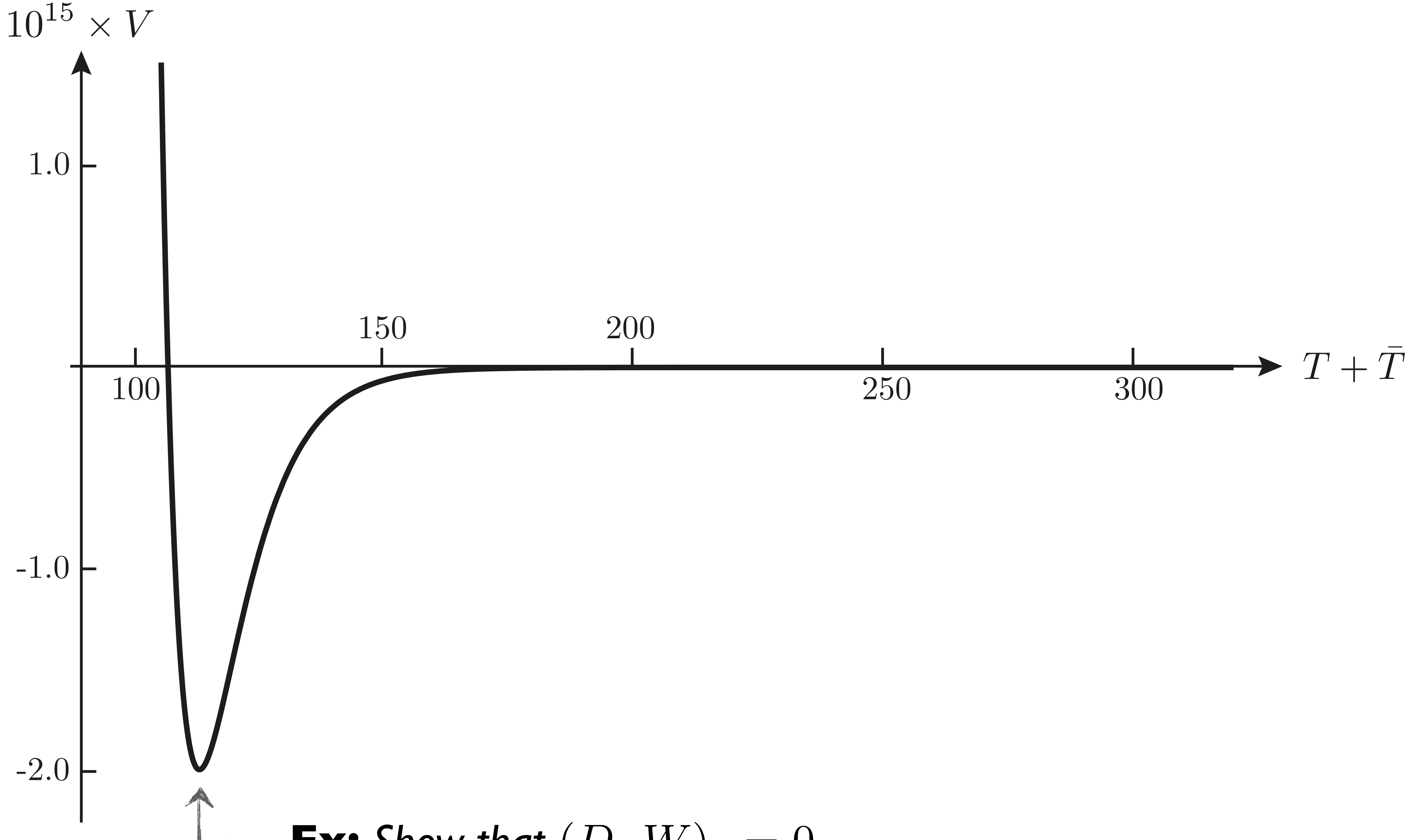
$$W = W_0 + \mathcal{A} e^{-aT}$$

Ex: *Show that*

$$V_F = \frac{a\mathcal{A} e^{-a(T+\bar{T})}}{2(T+\bar{T})^2} \left[\left(1 + \frac{T+\bar{T}}{3}\right) a\mathcal{A} e^{-a(T+\bar{T})} + W_0 \right]$$

Plot this for $\mathcal{A} = 1$, $a = 0.1$ *and* $W_0 = -10^{-4}$.

AdS Vacua



Ex: *Show that* $(D_T W)_\star = 0$

i.e. the vacuum is ***SUSY AdS***.

De Sitter Vacua

There are various proposals for ***uplifting*** to de Sitter:

For example, we may add an anti-D3-brane:

Recall $\mathcal{L}_{\mathrm{D3}} = \mathcal{L}_{\mathrm{DBI}} + \mathcal{L}_{\mathrm{CS}} = 0$ in ISD compactifications.

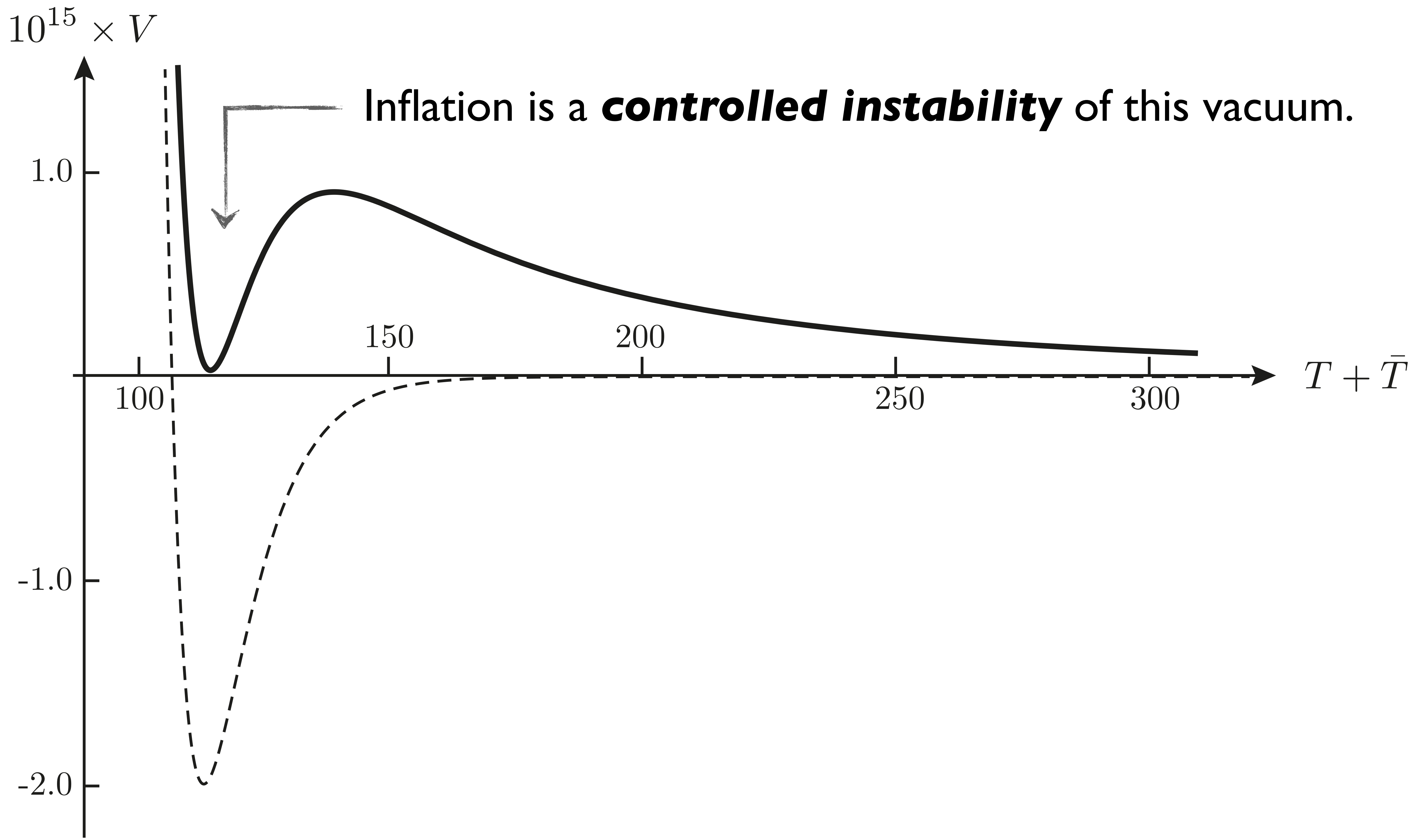
So, $\mathcal{L}_{\overline{\mathrm{D3}}} = \mathcal{L}_{\mathrm{DBI}} - \mathcal{L}_{\mathrm{CS}} \neq 0$

In Einstein frame, the anti-D3 contribution is

$$\boxed{\delta V_{\overline{\mathrm{D3}}} = e^{-12u} D}$$

This leads to metastable dS vacua.

De Sitter Vacua



Brane Inflation

KKLMMT, *Towards Inflation in String Theory*

- ***Setup:***

  Consider a spacetime-filling D3-brane (pointlike in the extra dimension).

  The position in the extra dimension plays the role of the inflaton.

Recall that a D3-brane in an ISD compactification feels no force.

A good starting point for inflation?

But, ISD is broken in the stabilized compactification.

What is the D3-brane potential?

- ***Upshot:*** Brane inflation has an eta problem.

  I will show this from several different perspectives.

Eta Problem from 10D

Tr(Einstein) - Bianchi:

$$\boxed{\underbrace{\nabla^2}_{\text{Laplacian on } X_6}(e^{4A} - \alpha) \;=\; \underbrace{R_4}_{\text{4D Ricci}} \;+\; \underbrace{|G_-|^2}_{\text{IASD flux}} \;+\; \text{local}}$$

Laplacian on X_6

4D Ricci

IASD flux $G_- \equiv (\star - i)G_3$

(sourced e.g. by gaugino condensation)

In quasi-dS: $R_4 \approx 12H^2$

This induces a dangerous mass term for the inflaton:

$$V \equiv T_3(e^{4A} - \alpha) \;=\; V_0 + H^2\phi^2 + \cdots$$

and hence a large contribution to the eta parameter

$$\eta \;=\; \frac{2}{3} + \cdots$$

Eta Problem from 4D

Backreaction on the compactification volume, Vol(X_6), leads to

$$K = -2\ln\mathcal{V}(T,\phi) \quad \text{where} \quad \mathcal{V}(T,\phi) = (T+\bar{T}-\phi\bar{\phi})^{3/2}$$

de Wolfe-Giddings

This implies the eta problem

$$V_F = e^K\left[\,\ldots\,\right] = \frac{C}{\mathcal{V}^2(\phi)} + \cdots$$

$$= V_0 + H^2\phi^2 + \cdots \quad \text{as before.}$$

Backreaction on the 4-cycle volume, Vol(Σ_4), leads to

$$W = W_0 + W_{\langle\lambda\lambda\rangle}(T,\phi)$$

This implies extra terms in the inflaton potential.

Successful inflation can be achieved by fine-tuning.

Axion Monodromy Inflation

Could the inflaton be an axion?

Axions have a shift symmetry, $a \mapsto a + const.$, broken by nonperturbative effects to $a \mapsto a + 2\pi$.

The axion Lagrangian is

decay constant

$$\mathcal{L} = -\frac{1}{2} f^2 (\partial a)^2 - \Lambda^4 \cos(a)$$

$$= -\frac{1}{2}(\partial\phi)^2 - \Lambda^4 \cos\left(\frac{\phi}{f}\right)$$

Freese, Frieman, Olinto
Natural Inflation

Ex: *Show that successful inflation requires* $f \gg M_{\rm pl}$.

Is this reasonable?

Axions in String Theory

In string theory, we get axions from the dimensional reduction of p-forms:

e.g. $$B_2 = \sum_{I=1}^{h^{1,1}} b_I(x)\,\omega^I \qquad\qquad b_I \equiv \frac{1}{\alpha'}\int_{\Sigma_I} B_2$$

harmonic 2-forms

4D axions

2-cycle

What is the decay constant?

Digression: Axion Decay Constant

Svrcek and Witten

Examine the axion kinetic term:

$$S_{10} \supset \frac{1}{2(2\pi)^7 g_s^2 (\alpha')^4} \int \mathrm{d}^{10}X \, |\mathrm{d}B_2|^2$$

$$= \frac{1}{2} \int \mathrm{d}^4 x \sqrt{-g}\, \gamma^{IJ} \, \partial^\mu b_I \partial_\mu b_J \, |\mathrm{d}B_2|^2$$

$$\gamma^{IJ} \equiv \frac{1}{6(2\pi)^7 g_s^2 (\alpha')^4} \int_{X_6} \omega^I \wedge \star \omega^J$$

$$\mapsto f_I^2 \, \delta_{IJ}$$

For an isotropic compactification, with $\mathcal{V}(\alpha')^3 = L^6$, we have

$$\int \omega \wedge \star\omega \sim \sqrt{g_6}\, g_6^{\cdot\cdot}\, g_6^{\cdot\cdot} \sim L^2$$

Using $\alpha' M_{\rm pl}^2 = \dfrac{2}{(2\pi)^7} \dfrac{\mathcal{V}}{g_s^2}$, we get $\boxed{\dfrac{f^2}{M_{\rm pl}^2} \approx \dfrac{1}{6} \dfrac{(\alpha')^2}{L^4}}$.

So, $f \ll M_{\rm pl}$ in computable limits of string theory!

Axions in String Theory

In string theory, we get axions from the dimensional reduction of p-forms:

e.g. $$B_2 = \sum_{I=1}^{h^{1,1}} b_I(x)\,\omega^I \qquad\qquad b_I \equiv \frac{1}{\alpha'}\int_{\Sigma_I} B_2$$

harmonic 2-forms

4D axions

2-cycle

What is the decay constant?

$$f \ll M_{\mathrm{pl}}$$

Natural inflation with $V(\phi) = \Lambda^4 \cos\left(\frac{\phi}{f}\right)$ seems hard to achieve.

But, let's be careful about the $V(\phi)$ we assume.

Digression: The Axion Shift Symmetry

Wen and Witten
Dine and Seiberg

Consider the worldsheet coupling of B_2 :

$$S_\sigma \supset -\frac{1}{2\pi\alpha'}\int_{\Sigma_2} B_2 \equiv -\frac{b}{2\pi}$$

$$= -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\, \epsilon^{ab}\partial_a X^M \partial_b X^N B_{MN}(X)$$

Expand $B_{MN}(X)$ around $X_{(0)} \equiv 0$:

$$B_{MN}(X) = B_{MN}(0) + X^P \partial_P B_{MN}(0) + \cdots$$

leads to $\partial_\mu b$

only this term can violate the shift symmetry

$$S_\sigma \supset -\frac{1}{4\pi\alpha'}\int_{\Sigma_2} \mathrm{d}^2\sigma\; \partial_a \left(\epsilon^{ab} X^M \partial_b X^N B_{MN}(0)\right)$$

total derivative

vanishes if the worldsheet has ***no boundary***.

Digression: The Axion Shift Symmetry

Wen and Witten
Dine and Witten

In the absence of D-branes, $V(b)$ must vanish to all orders in α' and g_s .

On the other hand, the shift symmetry can be broken by ***D-branes*** and ***nonperturbative effects*** (instantons).

Axions Monodromy

McAllister, Silverstein and Westphal

- ***Idea:*** Take a compactification without D-branes ($V = 0$), then slightly lift the flat axion direction.

- ***Setup:*** IIB on CY_3 O3/O7

A D5-brane (NS5-brane) wrapping a two-cycle Σ_2 generates a potential for the b-axion (c-axion).

To satisfy Gauss' law an anti-D5-brane wraps a homologous 2-cycle Σ_2' .

Axions Monodromy

McAllister, Silverstein and Westphal

We find the axion potential from the dimensional reduction of the 5-brane actions:

$$V = 2T_5 \int_{\Sigma_2} \mathrm{d}^2\sigma \sqrt{-\det(G+B)}$$

Ex: *Show that*

$$V = 2T_5 \sqrt{\ell^2 + b^2}$$

where ℓ *is the size of* Σ_2 .

Axions Monodromy Inflation

For $b \gg \ell$, we have $V \approx 2T_5\, b \equiv \mu^3 f\, b$.

The canonically-normalized inflaton action is

$$\boxed{\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu^3\phi}$$

NB: The b-axion actually has an eta problem, but the same result arises for the c-axion.

Generalization to other axions:

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \mu^{4-p}\phi^p \qquad \text{with} \quad p < 2$$

Consistency Checks

- Symmetry breaking from nonperturbative effects
  - universal eta problem for the b-axion
  - success of the c-axion model-dependent

- Symmetry breaking from backreaction
  - induced D3-brane charge on NS5-brane

backreacts on the geometry
changes the strengths of nonperturbative effects
modifies the inflaton potential

$\Updownarrow$

geometry must be arranged to have
an *additional* approximate symmetry

Please see our review or the original papers for the details.

Phenomenology

- Large tensors

- (Maybe) resonant non-Gaussianity

Summary and Conclusions

- The Planck data is incredible!
- Inflation successfully describes the data.
- The inflationary mechanism is UV-sensitive.
- Inflationary models in string theory exist
  ... but should be explored further.

Thanks for your attention!