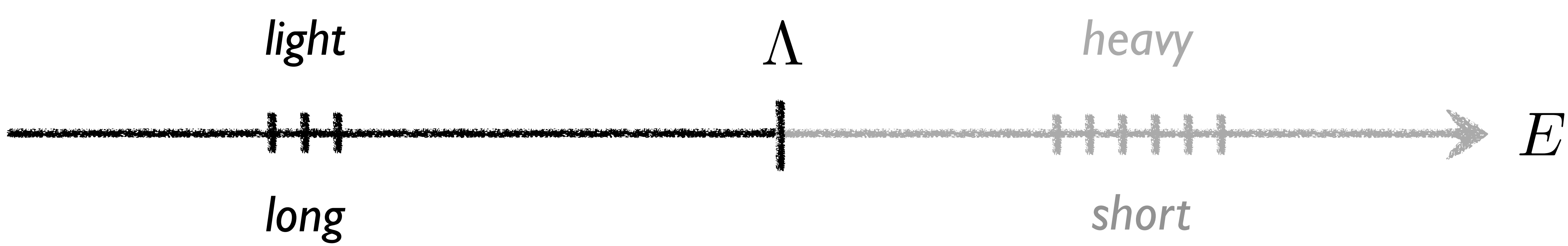
Lecture 2

Inflation in Effective Field Theory



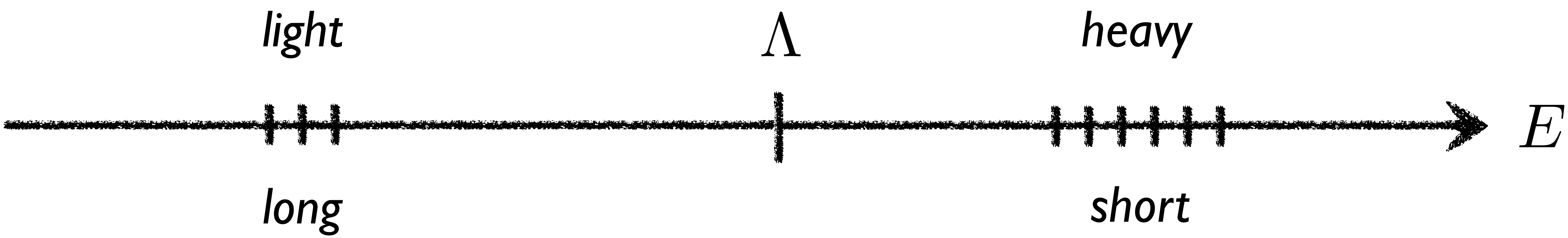
Outline

- A Review of Effective Field Theory
- Inflation in Effective Field Theory
- UV-Sensitivity of Inflation

Effective Field Theory

Effective Field Theory

Nature comes with many scales :



Effective Field Theory

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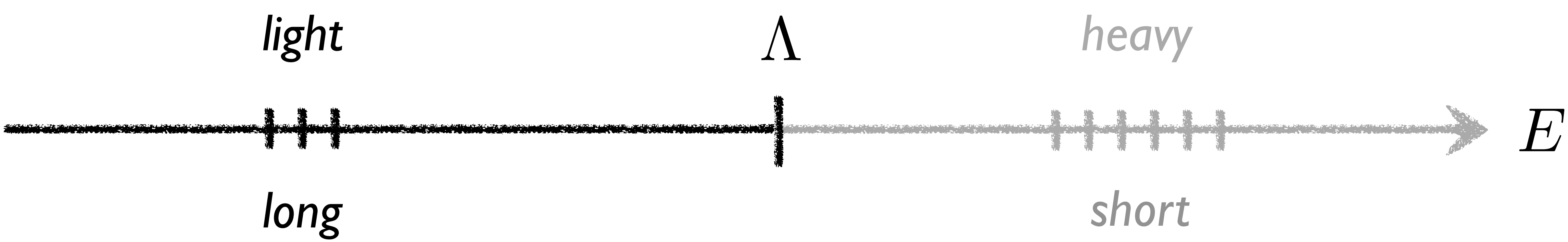
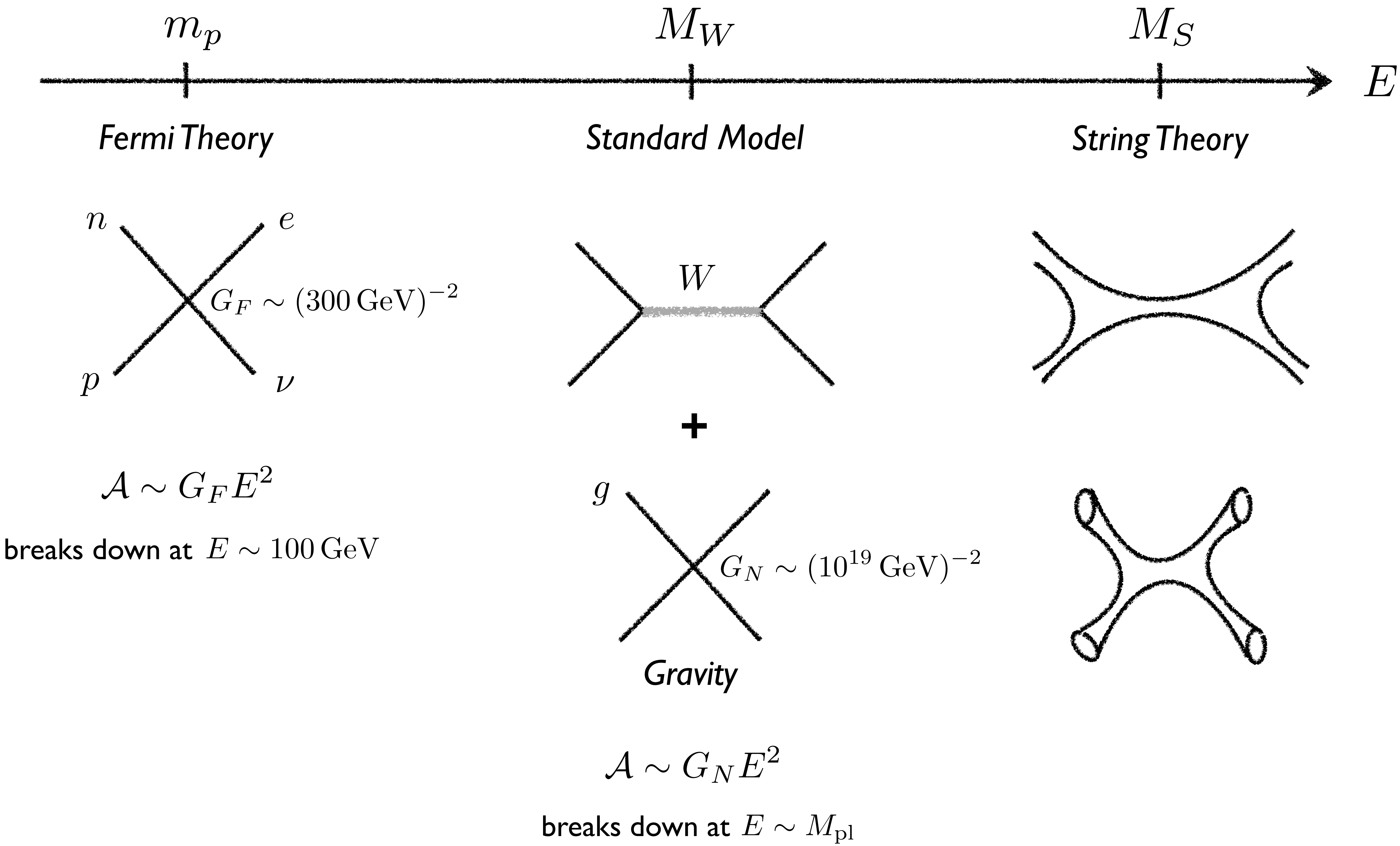


Coarse-graining over the short-distance behaviour leads to an effective theory at long distances.

We can parameterize the effective theory even if we don't know the microscopic theory.

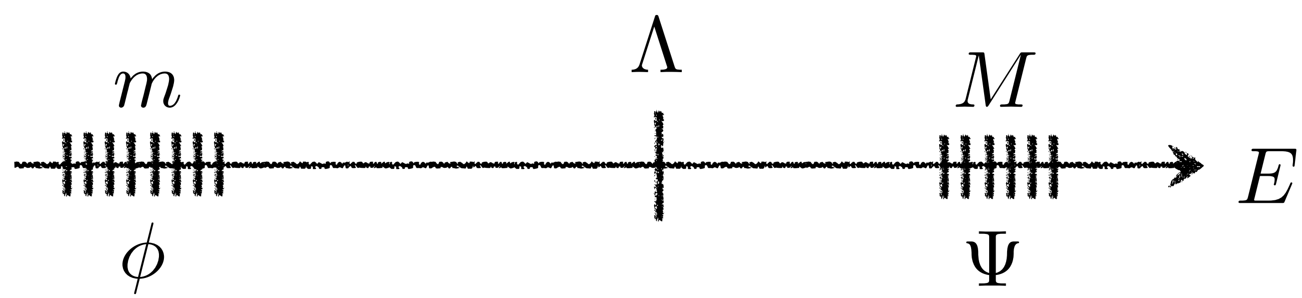
(e.g. we do fluid dynamics without speaking about atoms)

Effective Field Theory



Constructing EFTs from the Top Down

"integrating out"



If the full theory is known (and computable), we can integrate out the heavy fields to get an effective theory for the light fields :

$$e^{iS_{\text{eff}}[\phi]} = \int \mathcal{D}\Psi \; e^{iS[\phi,\Psi]}$$

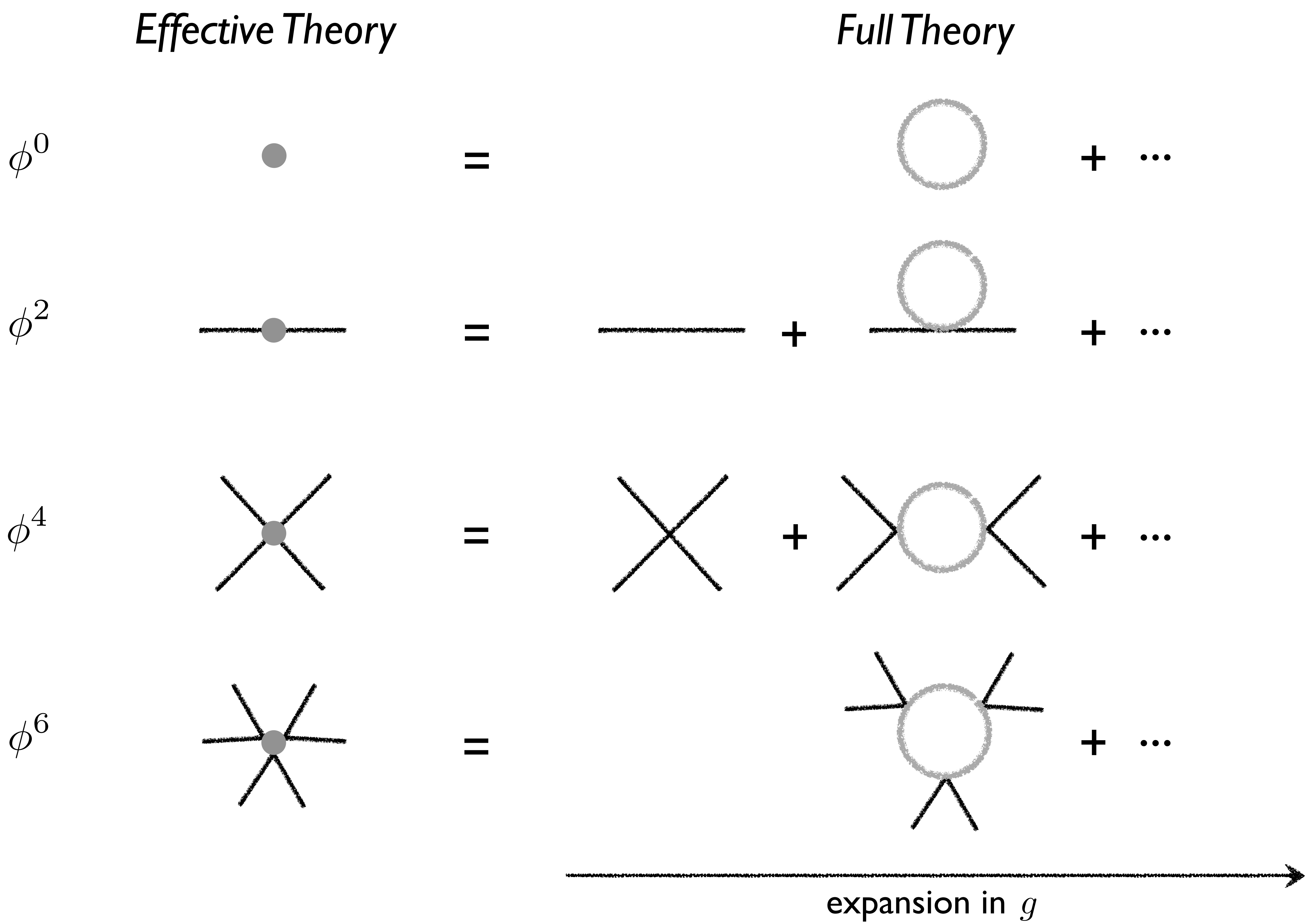
Let me illustrate this in a toy model:

$$\mathcal{L}[\phi,\Psi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4!}\lambda\phi^4$$
$$-\frac{1}{2}(\partial\Psi)^2 - \frac{1}{2}M^2\Psi^2 - \frac{1}{4}g\,\phi^2\Psi^2$$

UV-coupling

Constructing EFTs from the Top Down

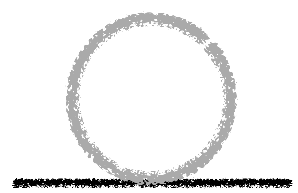
matching

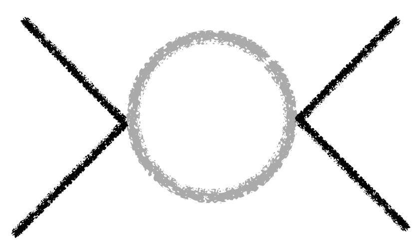


Constructing EFTs from the Top Down

Heavy fields have two effects:

1) They renormalize the IR couplings

$$\Delta m^2 \quad = \quad$$  $$\quad = \quad \frac{g}{32\pi^2}\left(\Lambda^2 - M^2 \log\left(\frac{\Lambda^2}{\mu^2}\right)\right)$$

$$\Delta\lambda \quad = \quad$$  $$\quad = \quad -\frac{3g^2}{32\pi^2}\log\left(\frac{\Lambda^2}{\mu^2}\right)$$

renormalization scale

Light scalars are unnatural:

$$m^2_{\rm phys} \;=\; m^2_{\rm bare} \;+\; \Delta m^2 \;\sim\; \mathcal{O}(M^2)$$

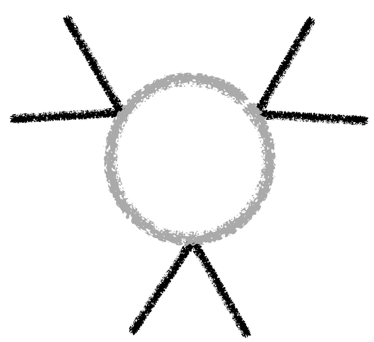
unless we tune the bare mass.

Constructing EFTs from the Top Down

Heavy fields have two effects:

1) They renormalize the IR couplings

2) They add new non-renormalizable interactions



$$\sim \quad g^3 \frac{\phi^6}{M^2}$$

These terms decouple for $M \to \infty$.

Constructing EFTs from the Top Down

"Everything that is allowed, is compulsory"

For the toy model, we generate all terms that are consistent with $\phi \to -\phi$:

$$\mathcal{L}_{\rm eff}[\phi] = -\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m_{\rm R}^2\phi^2 - \frac{1}{4!}\lambda_{\rm R}\phi^4 \qquad \textit{renormalizable}$$

$$- \sum_{i=1}^{\infty}\left(\frac{c_i}{M^{2i}}\phi^{4+2i} + \frac{d_i}{M^{2i}}(\partial\phi)^2\phi^{2i} + \cdots\right) \qquad \textit{non-renormalizable}$$

expansion in $(E/M)^{2i}$

Only a finite number of operators are relevant to describe observations with finite precision.

Constructing EFTs from the Bottom Up

"parameterizing your ingnorance"

renormalizable

operator

$$\mathcal{L}_{\rm eff}[\phi] \;=\; \mathcal{L}_0[\phi] \;+\; \sum_i c_i \,\frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i - 4}}$$

Wilson coefficient

dimension

cutoff

EFT approach:

- identify the relevant degrees of freedom (at a given energy scale)
- identify the symmetries at play
- write the lowest dimension operators compatible with the symmetries

Inflation in Effective Field Theory

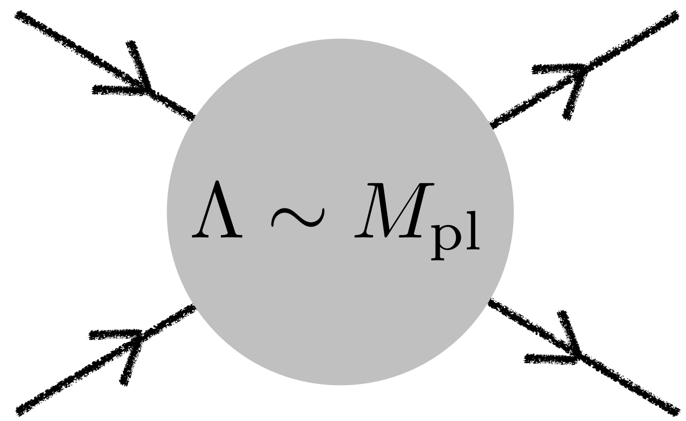
inflaton

$$S_{\rm eff}[\phi, g] \;=\; \int \mathrm{d}^4x\sqrt{-g}\left[\,\frac{M_{\rm pl}^2}{2}R \,-\, \frac{1}{2}(\partial\phi)^2 \,-\, V_0(\phi) \,+\, \sum_i c_i\,\frac{\mathcal{O}_i[\phi]}{\Lambda^{\delta_i-4}}\right]$$

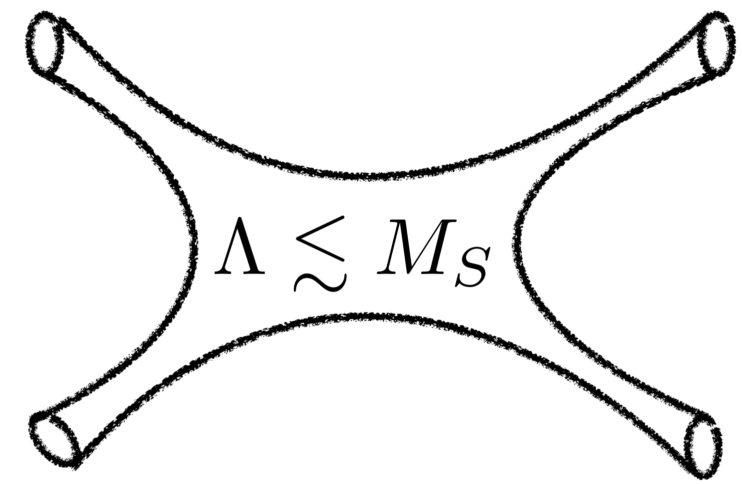
graviton

What is Λ?

graviton scattering:

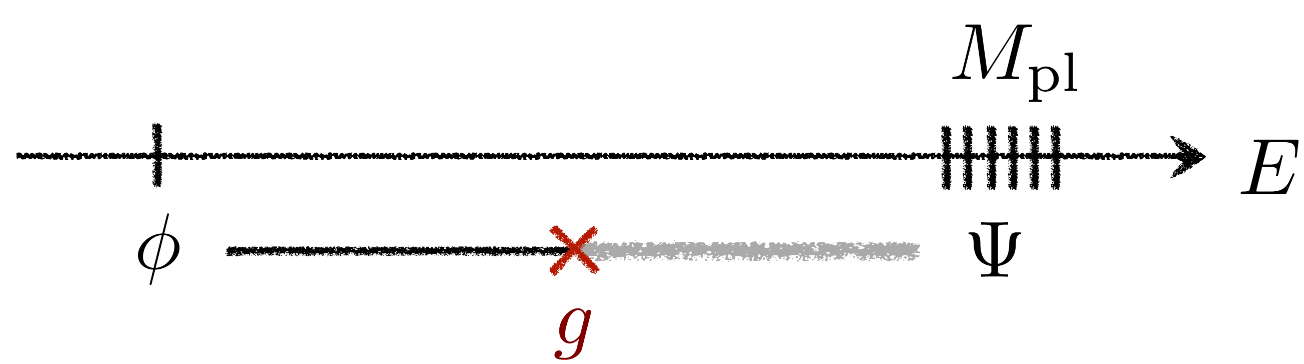


string theory:



What is c_i?

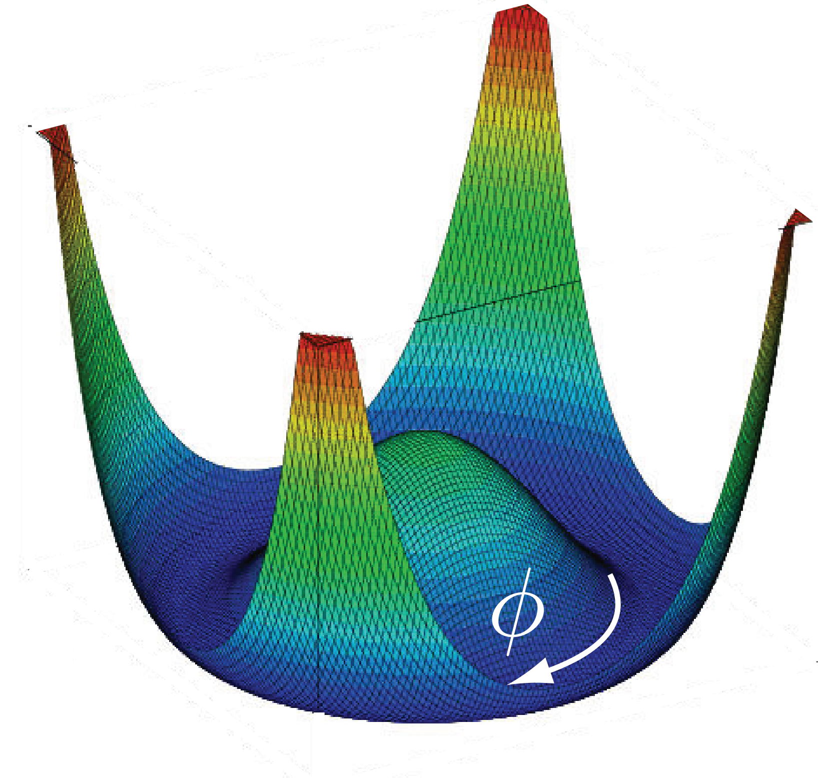
We know very little about the couplings to Planck-scale degrees of freedom:



- without a good reason: $g \sim 1 \longrightarrow c_i \sim 1$ Wilsonian naturalness
- with a good reason: $g \ll 1 \longrightarrow c_i \ll 1$
  - $\hookrightarrow$ symmetry

Global internal symmetries are a good reason to forbid couplings to UV degrees of freedom:

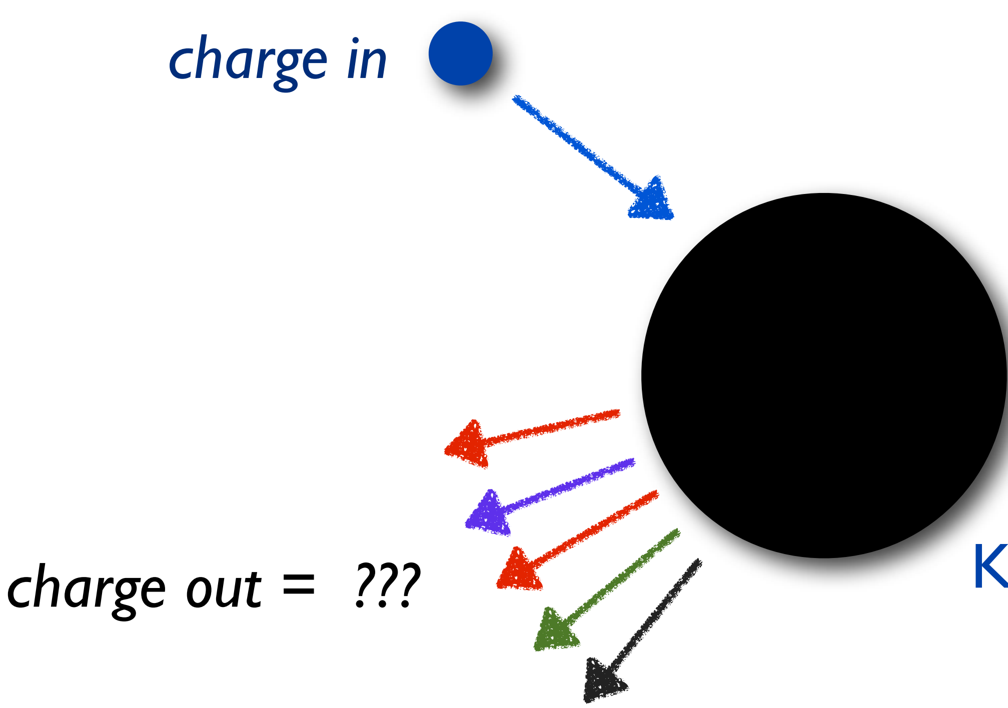
$$\phi \mapsto \phi + const.$$



But, quantum gravity breaks continuous global symmetries:

Banks et al. (1999)
Seiberg and Banks (2010)

e.g. black hole evaporation



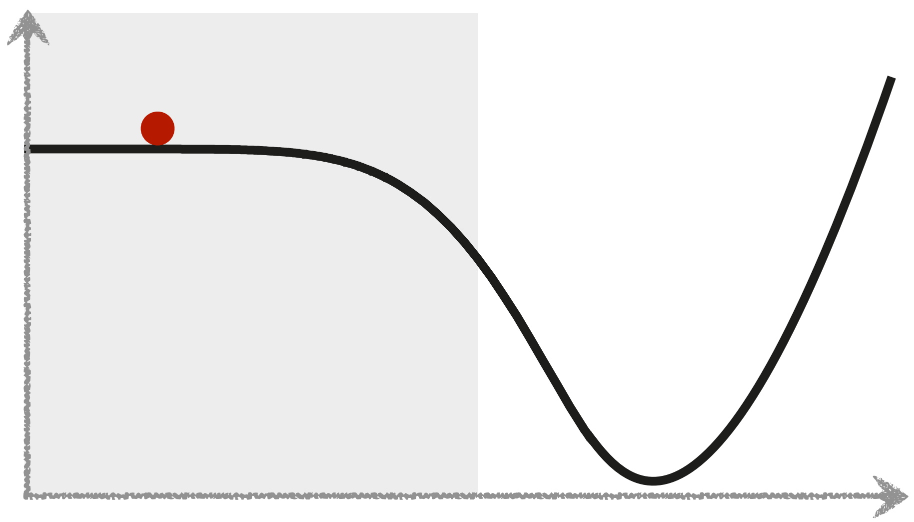
Kallosh et al. (1995)

Cannot assume that Planck-suppressed operators respect the symmetry.

UV Sensitivity

Slow-Roll Inflation

Slow-roll inflation occurs on a flat potential:



$$\epsilon \equiv \frac{M_{\rm pl}^2}{2}\left(\frac{V'}{V}\right)^2 < 1$$

$$\eta \equiv M_{\rm pl}^2 \frac{V''}{V} < 1$$

Scale-invariant, superhorizon fluctuations require

$$\boxed{\eta \approx \frac{m^2}{3H^2} \sim \mathcal{O}(0.01)}$$

The Eta Problem

Consider the following dimension-six operator: $\Delta V = V_0 \frac{\phi^2}{\Lambda^2}$

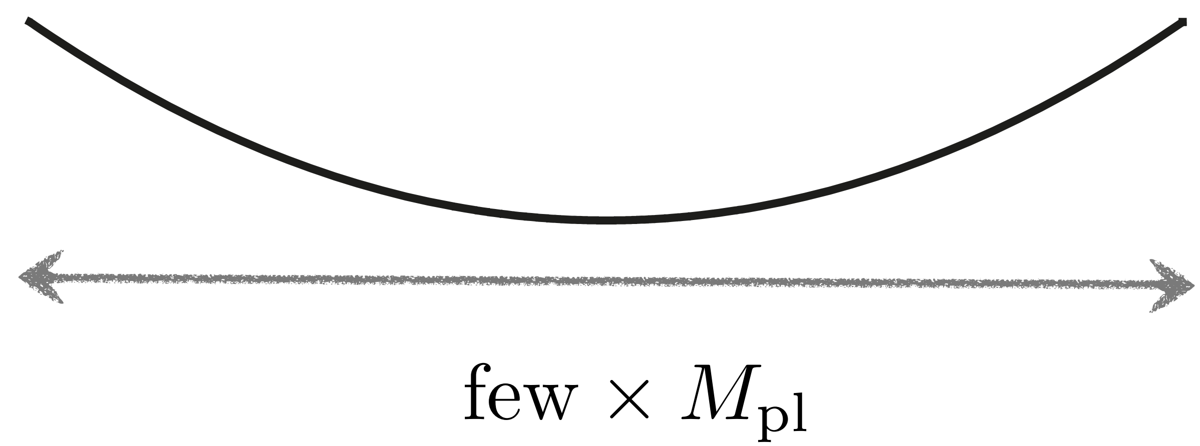
Its effect on the eta parameter is

$$\Delta\eta \approx \frac{M_{\rm pl}^2}{\Lambda^2} \geq 1$$

Even Planck-suppressed operators are dangerous.

Large-Field Inflation

Inflationary models with an observable level of gravitational waves have super-Planckian field excursions:

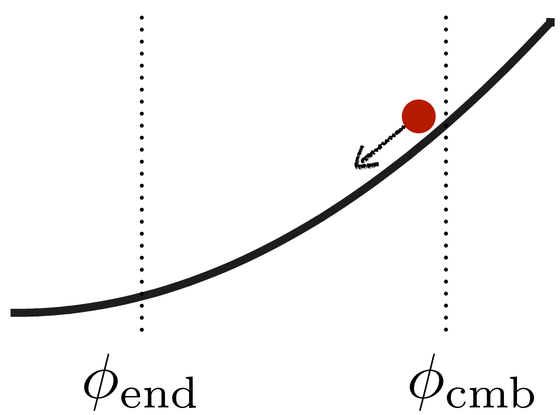


Digression: The Lyth Bound

Recall that $\Delta_s^2 = \left(\frac{H}{\dot\phi}\right)^2\left(\frac{H}{2\pi}\right)^2$ and $\Delta_t^2 = \frac{8}{M_{\rm pl}^2}\left(\frac{H}{2\pi}\right)^2$.

The tensor-to-scalar ratio is $r \equiv \frac{\Delta_t^2}{\Delta_s^2} = \frac{8}{M_{\rm pl}^2}\left(\frac{\dot\phi}{H}\right)^2 = \frac{8}{M_{\rm pl}^2}\left(\frac{d\phi}{dN}\right)^2$

$\mathrm{d}N \equiv H\mathrm{d}t = \mathrm{d}\ln a$



$$\frac{\Delta\phi}{M_{\rm pl}} = \int_{N_{\rm end}\equiv 0}^{N_{\rm cmb}\sim 60} \mathrm{d}N \sqrt{\frac{r(N)}{8}} \approx \left(\frac{r}{0.01}\right)^{1/2}$$

Observable g-waves ⟷ ***super-Planckian vev's***

$r > 0.01$ $\Delta\phi > M_{\rm pl}$

↑ comes from ***astrophysics***

↑ fundamental scale of ***quantum gravity***

Large-Field Inflation

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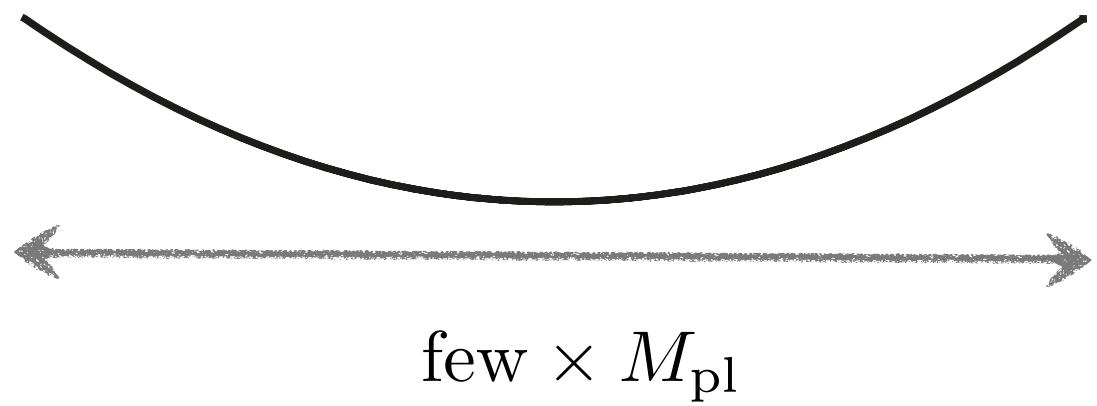
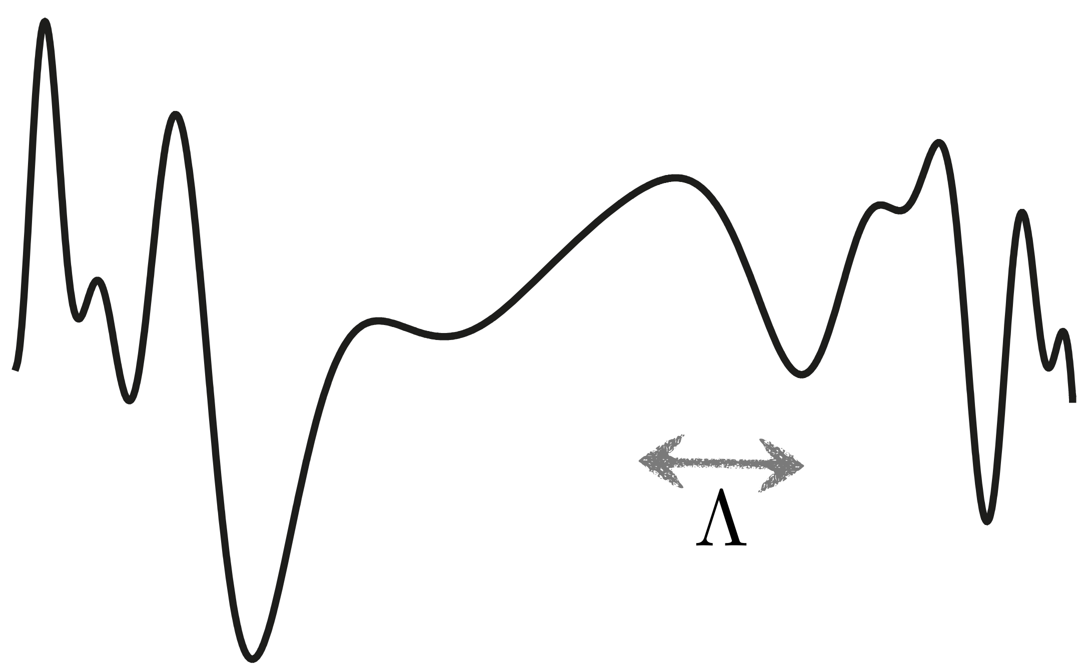


while the "generic" EFT "expectation" is



$$\frac{\Delta V}{V} = \sum_n c_n \left(\frac{\phi}{\Lambda}\right)^n$$

Fine-tuning is not an option. A shift symmetry becomes compulsory.

Notice that the UV-sensitivity of inflation persists even if $\Lambda \mapsto M_{\rm pl}$.

To address these challenges requires a theory of quantum gravity, which leads us to:

Inflation in String Theory