Anomalous thermal transport in irradiated tilted Weyl Semimetals

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Outline of the Talk

- Weyl Semimetals
- Irradiated WSM: Floquet Theory
- Modified Kubo Formula for the Floquet states
- Photoinduced Thermal Responses for Type I and Type II WSM phase
- Computation of thermal responses in irradiated multi-WSMs
- Discussion

Weyl Semimetals

- WSMs: certainly one of the most stunning representatives of topological material classes
- Electronic band structure is predicted to host Weyl points
- 3-dimensional linear band crossings represent massless Weyl fermions of defined chirality
- Systems exhibiting Weyl points have either time-reversal or inversion symmetry broken
- Two Weyl points always form a pair of opposite chirality

[Annu. Rev. Condens. Matter Phys. **8**, 289 (2017),
Rev. Mod. Phys. **90**, 015001 (2018)]

Symmetry Breaking Model

- Time symmetry breaking minimal model: a single pair of Weyl node, Weyl nodes are located at different momenta
- Inversion symmetry breaking model: a minimum of four Weyl points is required; leads to the energy separated Weyl points
- Small perturbations do not gap out individual Weyl points
- Large perturbations can cause Weyl points of opposite chirality to overlap and annihilate

Berry curvature in WSM

- In WSM, linearly dispersing Weyl nodes are monopoles of Berry curvature- acts like a momentum-space magnetic field
- WPs act as a source/sink of Berry curvature
- Berry curvature- responsible for the topological nature of these materials
- WSMs exhibit novel transport properties: emergent monopoles are at the heart of this property

Chiral Fermions

- Weyl fermions: massless chiral fermions
- Chirality: *helicity* or *handedness* of Weyl Fermions
- Weyl fermions- not observed as a fundamental particle in nature
- Theoretical Prediction- Weyl fermions may be realized as emergent quasiparticles in a low-energy condensed matter system
  [Herring, C. PRB 52 (1937), Murakami, S. New J Phys. (2007)]

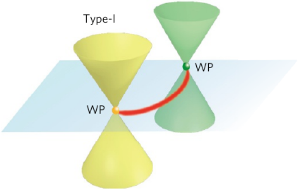
- WSM enables realization of Weyl fermions in electronic systems
  S.-Y. Xu et al. ”Discovery of a Weyl Fermion semimetal and topological Fermi arcs” Science **349**, 613 (2015)
  E. Haubold et al. Phys. Rev. B **95**, 241108 (R) (2017)

Two types of WSM

- Type-I WSMs: density of states of the Weyl fermions goes to zero at the Weyl points
- In some sense, we can think of these as the limiting case of a direct gap semiconductor where the conduction and valence bands meet at the Weyl nodes
- Type-II WSMs: bands comprising the Weyl nodes have a finite density of states at the Weyl energy
- Difference:
  Type-I WSMs host quasiparticles described by the Weyl equation
  Energy dispersion of quasiparticles in type-II WSMs violates Lorentz invariance and the Weyl cones in the momentum space are tilted

Type II WSM

- Tilting of the Weyl cone: Fermi surface surrounding the Weyl node transforms into two Fermi surfaces of electron and holes
- Signatures of a type-II WSMs have been reported in some materials
- Opens the door for further experimental study of the type-II WSMs
  [Nature 527 (2015), PRL 115 (2015), Sci Adv 3 (2017), PRB 96 (2017)]



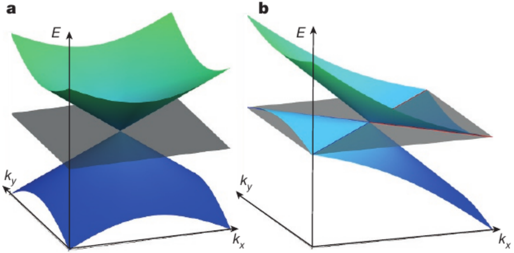


Figure: a. Type I, b. Type II

[Courtesey: Nature Physics, **12**, 1105(2016), Nature, **527** 495 (2015)]

WSM Characteristics

- WSMs: 3 defining and inter-related characteristics
- linearly dispersing Weyl nodes (or Weyl points)
- Monopoles of Berry curvature originates at the location of the Weyl points in momentum space
- Unique boundary modes known as Fermi arcs connect projections of these Weyl nodes in the surface Brillouin zone

Weyl Fermions

- Fermions in 3+1 spacetime dimensions obey the Dirac equation

$$(\gamma^\mu \partial_\mu - m)\Psi = 0$$

- In massless limit, Dirac equation decouples to two Weyl equations

$$\boldsymbol{\sigma}^\mu \partial_\mu \Psi_L = 0$$

$$\boldsymbol{\sigma}^\mu \partial_\mu \Psi_R = 0$$

- Ψ_L and Ψ_R - two component Weyl spinors

Weyl Hamiltonian

- Weyl Hamiltonian for massless electron with chirality $s = \pm$

$$H_s = s\hbar v_F \mathbf{k} \cdot \boldsymbol{\sigma}$$

v_F:speed of the fermion, fermion momentum $\mathbf{p} = \hbar\mathbf{k}$

- In the band structures of metals and semimetals, kinetic energy of electrons much less than their rest mass: Schrodinger equation is adequate

Lattice Models and Linearized Band Structure

- Two-band lattice model for a WSM:

$$\boldsymbol{H}(k) = a(k)\sigma_x + b(k)\sigma_y + c(k)\sigma_z$$
$$a(k) = t[\cos(k_x) + \cos(k_y) + \cos(k_z) - \cos(k_0) - 2],$$
$$b(k) = t\sin(k_y), \quad c(k) = t\sin(k_z)$$

- At $k_W = (\pm k_0, 0, 0)$, all three functions vanish
- Hamiltonian is expanded around **k**-space singularities in the low-energy theory

$$\boxed{H_{eff}(k) = v_F\,\vec{\sigma}\cdot\vec{k}}$$

Tilt in Type II WSM

- In HEP, Weyl fermions are assumed to be Lorentz invariant
- In CMP, a Lorentz non-symmetric term may be added to the conical dispersion relation of WSM
- Tilting of Weyl cone in such a way that for large enough tilts, a Lifshitz phase transition occurs
- Tilted Hamiltonian:

$$H(\mathbf{k}) = C_s \hbar k_z + s \hbar v_F \vec{\sigma} \cdot \vec{k}$$

- C_s-tilt parameter

Minimal model for tilted WSM

- Linearized Hamiltonian for a time-reversal symmetry-breaking tilted Weyl semimetal

$$H_s(k) = \hbar C_s(k_z - sQ) + s\hbar v\sigma.(\mathbf{k} - sQe_z)$$

- Two Weyl nodes ($s = \pm$) of opposite chirality separated in momentum space
- $2Q$: Distance between the Weyl points along e_z
  v-Fermi velocity when the tilt parameter $C_s = 0$
- Type I WSM: $|C_s| < v$, for Type II $|C_s| > v$

Irradiation of WSM

- Periodic circularly polarized laser beam

$$E(t) = E_0(\cos\omega t, -\sin\omega t)$$

- E_0 and ω - amplitude and frequency of the optical field
- Induce a driving term in the Hamiltonian

$$V_s(t) = s\hbar v A_0(\sigma_x \sin\omega t + \sigma_y \cos\omega t)$$

- $U(1)$ gauge field is introduced via Pierel's substitution
- $A_0 = E_0/\omega$

Time dependent Hamiltonian

- Time dependent Hamiltonian

$$H_s(k,t) = H_s(k) + V_s(t)$$

$$H_s(k) = \hbar C_s(k_z - sQ) + s\hbar v\sigma.(\mathbf{k} - sQe_z)$$

$$V_s(t) = s\hbar v A_0(\sigma_x \sin\omega t + \sigma_y \cos\omega t)$$

- H_s represent the unperturbed static Hamiltonian
- V_s represent the perturbative potential originated due to driving
- High Frequency expansion : Floquet effective Hamiltonian
- For high ω describe the driven system over a period of T

Floquet Theory

- Periodically driven quantum systems: attracts intensive interests from a viewpoint of dynamically controlling quantum phases of matter by external drives
- Effect of a periodic drive: usually interpreted in terms of a change of the Hamiltonian to an effective static one derived from the Floquet theory and high frequency expansion
- Imperative to understand the detailed behaviour of the effective Hamiltonian for varied frequency and amplitude of the periodic drive
- Mechanism of light matter interaction in quantum materials challenges the knowledge of material physics in a new direction

Floquet Hamiltonian

- Floquet theory- great advantage as the effective static Hamiltonian is much easier to treat than the original time dependent one
- A price to pay: time dependent Schroedinger equation in Hilbert space H
- Floquet Hamiltonian in an extended Hilbert space $H \otimes T$, T represent multi-photon dressed states
- To interpret analysis in terms of the static Hamiltonain in H, a mapping is required
- Possible in high frequency regime, $\hbar\omega >> \Delta_E$, Δ_E: maximal energy associated with real electronic inter and intra band transitions
- System can be effectively regarded as a time-averaged one

High frequency limit

- A conventional approach is based on Floquet-Magnus expansion
- Effective Hamiltonian explicitly depends on the phase of the drive
- van Vleck degenerate perturbation theory is free from any phase dependence of the drive
- not easy to derive higher order terms in $1/\omega$

- High frequency limit of irradiation
- Zero-photon dressed states are decoupled from other dressed states
- van-Vleck expansion: include virtual processes for electrons absorbing and emitting one photon with a second order perturbation theory
- Explicit form of $\mathcal{V}_s = \frac{[V_{-1}, V_{+1}]}{\hbar\omega}$

$$H_{eff} \sim H_0 + \frac{[H_{-1}, H_1]}{\omega}$$

- Floquet expansion is done upto second order in $\frac{1}{\omega}$
- All higher order contributions to the first order vanish
- Use of a high frequency laser field leads to a plethora of interesting effects

Effective light induced Hamiltonian

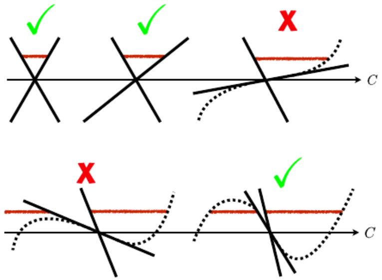
- High Frequency limit: Effective time independent Hamiltonian

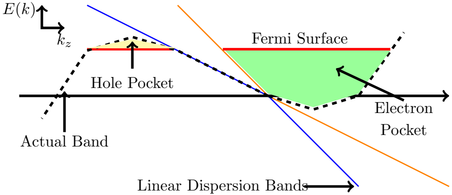
$$H_s^F = \hbar C_s[k_z - s(Q+\Delta)] + s\hbar v\sigma.[k - s(Q+\Delta)e_z] + s\hbar C_s\Delta$$

- Floquet parameter $\Delta = \frac{\hbar v A_0^2}{2\omega}$ - contribution of the radiation field.
- Hamiltonian with radiation and without radiation - similar
- Weyl nodes further displaced by a distance 2Δ in momentum space

Exclusion of the $|C| \sim v$ regime

- Linearized model of irradiated WSM: does not work well around the Lifshitz transition between the type I and type II phases
- Go on increasing the tilt in the type I phase, Fermi surface of each cone will grow
- Tilt is further increased: depending on the position of the Fermi level, above a certain value of tilt, electron and hole pocket will again co-exist
- Qualitatively correct description of the system is obtained by adding a physical momentum cut off
- Study is restricted to deep in the type I and type II phase excluding $|C| \sim v$ region





[PRB 96, 115202 (2017), PRB 98,2015109 (2018)]

Floquet states

- Hamiltoninan is influenced by a periodic potential
- Floquet states: time analogue of Bloch states $|\Phi(t)\rangle$
- Satisfy

$$|\Phi(t+T)\rangle = |\Phi(t)\rangle$$

- Schroendinger equation

$$H(t)|\Phi(t)\rangle = \epsilon_\alpha|\Phi(t)\rangle$$

- ϵ_α : quasienergies
- Floquet states- orthonormal under a time averaged inner product

$$\langle\Phi_\alpha(t)|\Phi_\beta(t)\rangle := \frac{1}{T}\int_0^T dt\,\langle\Phi_\alpha(t)\rangle\,\Phi_\beta(t) = \delta_{\alpha\beta}.$$

Floquet-Kubo Formula

$$\sigma_{ab}^{F} = i \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha \neq \beta} \frac{f_\beta(\mathbf{k}) - f_\alpha(\mathbf{k})}{\epsilon_\beta(\mathbf{k}) - \epsilon_\alpha(\mathbf{k})} \times \frac{\langle\langle \Phi_\alpha(\mathbf{k}) | J_b | \Phi_\beta(\mathbf{k}) \rangle\rangle \langle\langle \Phi_\beta(\mathbf{k}) | J_a | \Phi_\alpha(\mathbf{k}) \rangle\rangle}{\epsilon_\beta(\mathbf{k}) - \epsilon_\alpha(\mathbf{k}) + i\eta}$$

- $|\Phi_\alpha(\mathbf{k})\rangle$:states of the effective Floquet Hamiltonian
- $J_{a(b)}$: current operator
- Floquet-Kubo formula, conductivity tensor is time averaged
- Energies have been replaced with the Floquet quasi-energies ϵ_α

[PRB 79, 169901 (2009), PRB 98,2015109 (2018)]

- Current correlation functions: time averaged
- Time averaged current correlation function :

$$\langle\langle\Phi_\beta(\mathbf{k})|J|\Phi_\alpha(\mathbf{k})\rangle\rangle = \frac{1}{T}\int_0^T dt \sum_m \sum_n e^{-i\Omega(n-m)t}\langle u_\alpha^m|J|u_\beta^n\rangle$$
$$= \sum_m \sum_n \delta_{nm}\langle u_\alpha^m|J|u_\beta^n\rangle$$
$$= \sum_n \langle u_\alpha^n|J|u_\beta^n\rangle$$

- $|u_\alpha\rangle$: Fourier counterpart- eigenstates of the effective Floquet Hamiltonian

High Frequency expansion

- HFE: $|u_\alpha^n\rangle \sim \mathcal{O}(\omega^{-n})$
- In leading order calculations, only the zeroth level Floquet states $|u_\alpha^0\rangle$ contribute
- Current correlator in terms of $|u_\alpha^0\rangle$

$$\langle\langle\Phi_\beta(\mathbf{k})|\, J\, |\Phi_\alpha(\mathbf{k})\rangle = \sum_n \langle u_\alpha^n|\, J\, |u_\beta^n\rangle = \langle u_\alpha^0|\, J\, |u_\beta^0\rangle$$

Conductivity Tensor

- Computed using the eigenstates of the effective Hamiltonian to leading order in perturbation theory
- Conductivity tensor

$$\sigma^F_{ab} = i \int \frac{d\mathbf{k}}{(2\pi)^d} \sum_{\alpha \neq \beta} \frac{f_\beta(\mathbf{k}) - f_\alpha(\mathbf{k})}{\epsilon_\beta(\mathbf{k}) - \epsilon_\alpha(\mathbf{k})} \times \frac{\langle e_\alpha(\mathbf{k})|J_b|e_\beta(\mathbf{k})\rangle \langle e_\beta(\mathbf{k})|J_a|e_\alpha(\mathbf{k})\rangle}{\epsilon_\beta(\mathbf{k}) - \epsilon_\alpha(\mathbf{k}) + i\eta}$$

- Resembles the Kubo form for the undriven case
- In terms of effective Floquet states $|e_{\alpha(\beta)}\rangle = \sum_n |u^n_{\alpha(\beta)}\rangle$ and quasi-energies $\epsilon_{\alpha(\beta)}$

Matsubara Green's function method

- Use Matsubara Green's function method (with $\hbar = 1$) for the Floquet-Kubo formula
- Current-current correlation function

$$\Pi_{ij}(\Omega, \mathbf{q}) = T \sum_{\omega_n} \sum_{s=\pm} \int \frac{d^3k}{(2\pi)^3} J_i^{(s)} \times G_s(i\omega_n, \mathbf{k}) J_j^{(s)} G_s(i\omega_n - i\Omega_m, \mathbf{k} - \mathbf{q}) \Big|_{i\Omega_m \to \Omega + i\delta}$$

- $G_s(i\omega_n, \mathbf{k})$ - single particle Green's function of the electron

Current operator

- Current operator: $J_i^{(s)} = e\,(C_s\delta_{iz} + sv\sigma_i)$
- $i, j = \{x, y, z\}$, T : temperature (setting the Boltzmann constant as unity)
- $\omega_n(\Omega_m)$: Fermionic (Bosonic) Matsubara frequencies
- One particle Green's function

$$G_s(i\omega_n, \mathbf{k}) = \frac{1}{2}\sum_{t=\pm 1} \frac{1 - st\boldsymbol{\sigma}\cdot\frac{\mathbf{k}-s(Q+\Delta)\mathbf{e}_z}{|\mathbf{k}-s(Q+\Delta)\mathbf{e}_z|}}{i\omega_n + \mu - C_s(k_z - s(Q+\Delta) + tv|\mathbf{k} - s(Q+\Delta)\mathbf{e}_z| - sC_s\Delta},$$

Π_{xy}

- Hall conductivity related to the current-current correlation function

$$\sigma_{xy} = -\lim_{\Omega\to 0} \frac{\Pi_{xy}(\Omega, 0)}{i\Omega}$$

- Summing over the Matsubara frequencies and taking trace over Pauli matrices,

$$\Pi_{xy}(\Omega, 0) = \Pi_{xy}^{(+)}(\Omega, 0) + \Pi_{xy}^{(-)}(\Omega, 0)$$

- Contributions from the two Weyl cones are separated
- Further,

$$\Pi_{xy}^{(s)}(\Omega, 0) = \Pi_{0}^{(s)}(\Omega, 0) + \Pi_{\mathrm{FS}}^{(s)}(\Omega, 0)$$

Vacuum and FS contribution

$$\sigma_{xy}^{(s)} = \sigma_0^{(s)} + \sigma_{\rm FS}^{(s)},$$

$$\sigma_0^{(s)} = -e^2 \int_{-\Lambda_0 - s(Q+\Delta)}^{\Lambda_0 - s(Q+\Delta)} \frac{dk_z}{2\pi} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi} \frac{sk_z}{2k^3}, \qquad (1)$$

$$\sigma_{\rm FS}^{(s)} = e^2 \int_{-\Lambda - s(Q+\Delta)}^{\Lambda - s(Q+\Delta)} \frac{dk_z}{2\pi} \int_0^\infty \frac{k_\perp dk_\perp}{2\pi}$$
$$\times \frac{sk_z}{2k^3} \Big[n_F(C_s k_z + vk - \mu + sC_s\Delta)$$
$$- n_F(C_s k_z - vk - \mu + sC_s\Delta) + 1 \Big]$$

Salient Features

- $sk_z/2k^3$ - z-component of the Berry curvature of the Weyl cone with chirality s
- Berry curvature features in both the vacuum and FS contribution
- Interestingly, both the tilt C_s and the Floquet parameter Δ affect the Fermi-Dirac distribution function
- Cut-off Λ_0, introduced in the k_z integral, does not affect the vacuum contribution to the Hall conductivity
- Cutoff in σ^s_{FS}, denoted as Λ, is crucial for finite Fermi surface effects in both the type-I and type-II regime

- For $T \to 0$, and performing the $k_\perp$ integration,

$$\sigma_0^{(s)} = -\frac{se^2}{8\pi^2} \int_{-\Lambda_0 - s(Q+\Delta)}^{\Lambda_0 - s(Q+\Delta)} dk_z \, \mathrm{sign}(k_z),$$

$$\sigma_{\mathrm{FS}}^{(s)} = -\frac{se^2}{8\pi^2} \int_{-\Lambda - s(Q+\Delta)}^{\Lambda - s(Q+\Delta)} dk_z \left[\mathrm{sign}(k_z) - \frac{vk_z}{|C_s k_z - \mu + sC_s|} \right] \times [(\Theta(v^2 k_z^2 - C_s k_z + sC_s\Delta - \mu)^2) - 1]$$

- $\Theta(x)$ - Heaviside function
- Vacuum contribution

$$\sigma_0 = \frac{e^2}{2\pi^2}(Q + \Delta)$$

- Integral for the FS contribution is evaluated for both types of WSMs

Berry Curvature associated with irradiated WSM

- Berry potential

$$\mathcal{A}_\alpha(\mathbf{k}) = -i << \phi_\alpha(\mathbf{k})|\nabla_{\mathbf{k}}|\phi_\alpha(\mathbf{k}) >>$$

- Berry curvature in terms of the Floquet states

$$\nabla_{\mathbf{k}} \times \mathcal{A}_\alpha(\mathbf{k})$$

- As the leading order contribution come from the zeroth level Floquet states

$$\mathcal{A}_\alpha(\mathbf{k}) = -i\langle u_\alpha^0|\nabla_{\mathbf{k}}|u_\alpha^0\rangle$$

Vacuum Hall conductivity for light induced WSM

- Hall conductivity

$$\sigma_{xy} = e^2 \int \frac{d^3k}{(2\pi)^3} \sum_{\alpha} f_{\alpha}(k)[\nabla_k \times \mathcal{A}]_z$$

- $f(k) = 1$ for $\mu = 0$
- For tilt symmetric case,

$$\sigma_{xy} = \frac{e^2(Q + \Delta)}{2\pi^2}$$

- Without light

$$\sigma_{xy} = \frac{e^2 Q}{2\pi^2}$$

- Vacuum conductivity depends linearly on the distance between the Weyl nodes in the momentum space

Thermoelectric properties in irradiated WSM

- Tilted time reversal breaking model possess a non-zero anomalous Hall conductivity in $x - y$ plane : σ_{xy}
- Mott rule and Wiedemann-Franz law hold good
- Assume that the Luttinger phenomenological transport equations valid for effective time independent Floquet Hamiltonian
- Use thermal transport equations

[JETP Lett. 103, 717 (2016), Phys. Rev. Lett 119,036601 (2017), Phys. Rev. B 96, 121116 (2017), J Phys. C 10, 2153 (1977), Phys. Rev. B 96, 115202 (2017)]

Transport Current

- Anomalous Nernst Current

$$J_a = -\alpha_{xy}\epsilon_{ab}\partial_b T$$

- Anomalous thermal Hall current

$$J_a^Q = -K_{xy}\epsilon_{ab}\partial_b T$$

[PRB 96, 115202 (2017), PRB 98, 205109 (2018)]

Thermal Responses

- Anomalous Nernst conductivity :

$$\alpha_{xy} = eLT\frac{d\sigma_{xy}}{d\mu}$$

- Anomalous thermal Hall conductivity :

$$K_{xy} = LT\sigma_{xy}$$

- $L = \pi^2 k_b 2/3e$- Lorentz number, e is the electronic charge and k_B is the Boltzmann constant
- Computation of σ_{xy} as a function of μ

WSM-I

- Anomalous Nernst and thermal Hall conductivities in $T \to 0$ limit for a type-I WSM,

$$\alpha_{xy} = \frac{-ek_B^2 TC}{18\hbar^2 v^2}$$
$$K_{xy} \approx \frac{k_B^2 T}{6\hbar}\left[\left(Q + \Delta\right) - \frac{C(\mu - C\Delta)}{3\hbar v^2}\right]$$

- $sC_s = C$ for $s = \pm$, $C_+ = -C_- = C$

Thermal Hall conductivity- Type 1

- K_{xy} varies smoothly around the point $\mu = C\Delta$
- Rewrite the Hall conductivity

$$K_{xy} = \frac{k_B^2 T}{6\hbar}[Q - \frac{\mu}{3\hbar v^2}C] + \frac{k_B^2 T}{6\hbar}[1 + \frac{C^2}{3\hbar v^2}]\Delta = K_{xy}^0 + K_{xy}^{\Delta}$$

- $\Delta = 0$ gives us back the results without radiation
- K_{xy}^0 -Hall conductivity in the absence of irradiation
- K_{xy}^{Δ}- positive contribution of the laser field

Dependence on the radiation field

- Dominant contribution to Hall conductivity comes from the shift in node spacing
- As the amplitude of the radiation field is increased, the effective chemical potential $(\mu - C\Delta)$ decreases
- Anomalous thermal Hall conductivity grows with the amplitude of the irradiation field
- Linearized model predicts a linear dependence of K_{xy} on μ in the type-I regime
- Nernst conductivity is constant and remains unchanged by the optical field

Type II WSM

- Nernst conductivity and thermal Hall conductivity :

$$\alpha_{xy} = -\frac{ek_B^2 T v}{6\hbar^2 C^2}\left[\ln\left(\frac{C^2\Lambda}{v(C\Delta - \mu)}\right) - 1\right]$$

$$K_{xy} = \frac{ek_B^2 T v}{6C\hbar}\left[\left(Q + \Delta\right) - \frac{(\mu - C\Delta)}{\hbar C}\ln\left(\frac{C^2\Lambda}{v(C\Delta - \mu)}\right)\right]$$

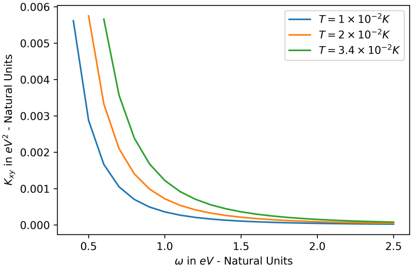
- K_{xy} depends nonlinearly on the chemical potential
- Decreases logarithmically for increasing Δ
- Changing the amplitude of the photon field affects the Nernst conductivity

- Physical momentum cutoff is difficult to estimate without using a non-linear model
- Eliminate the Λ dependence

$$[-\frac{6\hbar C}{k_B^2 T v}K_{xy} + Q + \Delta]\frac{\hbar C}{C\Delta - \mu} = \frac{6\hbar^2 C^2}{e k_B^2 T v}\alpha_{xy} + 1$$

- Provides a way to experimentally verify our findings independent of the cutoff

Type1 WSM



0.006
0.005
0.004
0.003
0.002
0.001
0.000
K_{xy} in eV^2 - Natural Units
0.5
1.0
1.5
2.0
2.5
ω in eV - Natural Units
$T = 1 \times 10^{-2}K$
$T = 2 \times 10^{-2}K$
$T = 3.4 \times 10^{-2}K$

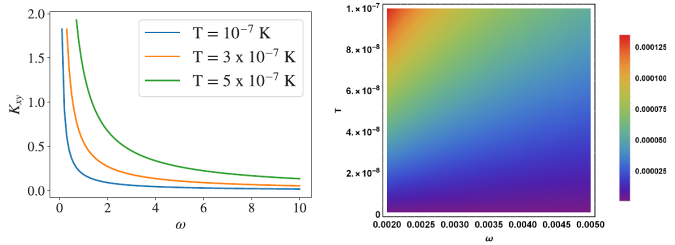


Figure: Type I

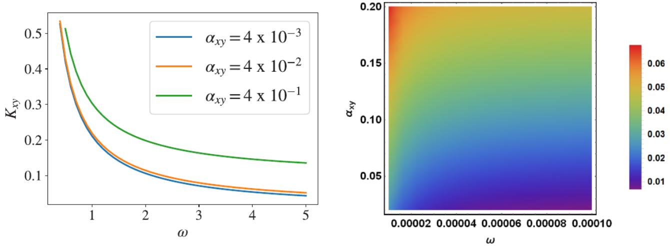


Figure: Type II

Multi-WSM

- Multi-Weyl semimetals (mWSMs) have an anisotropic non-linear dispersion along a 2-D plane
- A linear dispersion in an orthogonal direction
- topological charge *n* can be greater than one with the crystalline symmetries bounding its maximum value to three
- Interesting transport properties

[PRL **107**, 186806 (2011), PRL **108**, 266802 (2012), Nature Comm. **5**, 4898 (2014), PRB **95**, 161113 (2017), PRB **96**, 155138 (2017), JHEP **12**, 069 (2018)]

mWSM Hamiltonian

- Low energy Hamiltonian for a mWSM near each Weyl point

$$H^s_{\mathbf{k}} = \hbar C_s(k_z - sQ) + s\hbar\alpha_n \boldsymbol{\sigma} \cdot (\mathbf{n}_{\mathbf{k}} - s\mathbf{e})$$

- $\mathbf{n}_{\mathbf{k}} = [k^n_\perp \cos(n\phi_{\mathbf{k}}), k^n_\perp \sin(n\phi_{\mathbf{k}}), \frac{vk_z}{\alpha_n}]$
- $\mathbf{e} = (0, 0, Q)$, Q is the separation between two Weyl nodes.
- $\phi_{\mathbf{k}} = \mathrm{Arctan}(\frac{k_y}{k_x})$ and $k_\perp = \sqrt{k_x^2 + k_y^2}$
- C_s: tilt parameter associated with s Weyl node.
- For type-I WSM- $|C_s|/v \ll 1$ while for type-II $|C_s|/v \gg 1$.
- We restrict to the inversion symmetric case, $sC_s = C$ for $s = \pm$, $C_+ = -C_- = C$

[JHEP 12, 069 (2018)]

Effective Floquet mWSM Hamiltonian

$$
\begin{aligned}
H^F_{\mathbf{k}} &= H^s_{\mathbf{k}} + V^s_{\mathbf{k}} \\
&\quad = C_s(k_z - sQ) + s\alpha_n \boldsymbol{\sigma} \cdot (\mathbf{n_k} - sQ\hat{e}_z) \\
&\quad + \frac{\alpha_n^2}{\omega} \sum_{p=1}^{n} \frac{1}{p} ({}^nC_p A_0^2)^p k_\perp^{2n-2p} \sigma_z \\
&= C_s(k_z - sQ) + s\alpha_n \mathbf{n}'_{\mathbf{k}} \cdot \boldsymbol{\sigma}
\end{aligned}
$$

- $\mathbf{n}'_{\mathbf{k}} = (k_\perp^n \cos(n\phi_k), k_\perp^n \sin(n\phi_k), T_{\mathbf{k}}/\alpha_n)$.
- $T_{\mathbf{k}} = vk_z + \frac{\alpha_n^2}{\omega} \sum_{p=1}^{n} \beta_p^n k_\perp^{2(n-p)} = \Delta_n + T'_{\mathbf{k}}$
- $T'_{\mathbf{k}} = vk_z + \frac{\alpha_n^2}{\omega} \sum_{p=1}^{n-1} \beta_p^n k_\perp^{2(n-p)}$

- $\Delta_n = \frac{\alpha_n^2 A_0^{2n}}{n\omega}$- a momentum dependent contribution
- n'_z-component of the effective Hamiltonian acquires $k_\perp$ contribution coupled with the external electromagnetic field A_0 and ω.

Vacuum conductivity from Berry curvature

- Considering cylindrical polar coordinates for convenience of integration

$$\begin{aligned}
\sigma^a_{xy}(n) &= e^2 \int \frac{d\boldsymbol{k}}{4\pi^2} \sum_s \Omega^s_F(\boldsymbol{k}) \\
&\simeq \frac{n\alpha_n^{2-2/n} e^2}{4\pi^2} \int_{z'_l}^{z'_u} \int_0^\infty dk_\perp dk_z \frac{k_z k_\perp}{(k_z^2 + k_\perp^2)^{3/2}} \\
&\simeq \frac{n\alpha_n^{2-2/n} e^2}{2\pi^2} (-Q + \Delta_n)
\end{aligned}$$

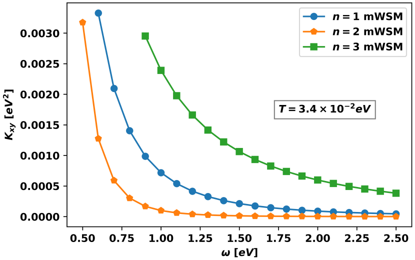
- Without light $\sigma^a_{xy}(n) = -\frac{n\alpha_n^{2-2/n} e^2}{2\pi^2} Q$

Floquet-Kubo formula and thermal responses

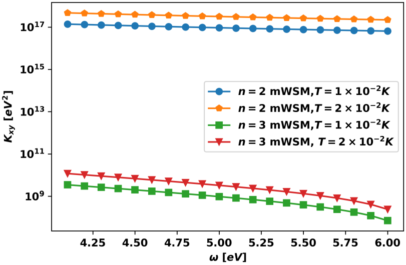
$$
\begin{aligned}
K^{l}_{xy}(n) &= \frac{\pi^2}{3e^2}k_B^2 T\sigma^{l}_{xy} \\
&= n\frac{Tk_B^2}{12}\frac{\alpha_n^{2-2/n}}{v}\{(Q+\Delta_n) - C\frac{\mu - C\Delta_n}{6v^2} \\
&+ 4\beta_2'' a(M)\cdot\mu^{2/n-2} + 4\beta_3'' a(2M)\cdot\mu^{4/n-2}\}
\end{aligned}
$$

$$
\begin{aligned}
\alpha^{l}_{xy}(n) &= e\frac{\pi^2}{3e^2}k_B^2 T\frac{d\sigma^{l}_{xy}}{d\mu} \\
&= n\frac{ek_B^2}{12}\cdot\frac{\alpha_n^{2-2/n}}{v}\{-\frac{C}{6v^2} + 4\beta_2'' a(M)\cdot(\frac{2}{n}-2)\mu^{2/n-3} \\
&+ 4\beta_3'' a(2M)\cdot(\frac{4}{n}-2)\mu^{4/n-3}\}
\end{aligned}
$$

Variation of K_{xy} with ω

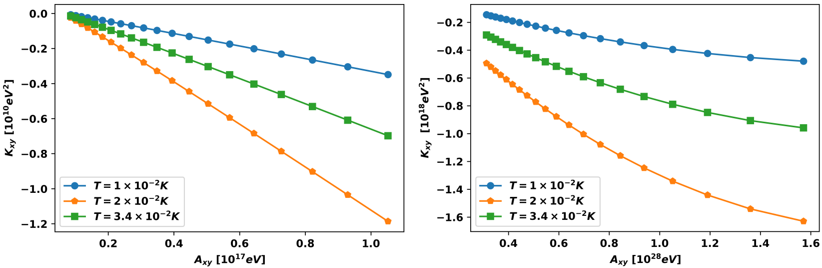


Type I



Type II

Type II variation of K_{xy} with α_{xy}



$n = 2$ $n = 3$

Discussion

- Effect of a periodically driving circularly polarised laser beam on tilted WSM
- Analytic study in the high frequency limit
- Employ the Kubo formula for the effective Floquet states
- For low tilt, i.e for type I WSM, the anomalous thermal Hall conductivity grows quadratically with the amplitude E_0 of the optical field
- Nernst conductivity remains unaffected in type I phase
- For type II, the Hall conductivity decreases non-linearly with E_0, while the Nernst conductivity falls off logarithmically with E_0^2

mWSM

- Analytical study of the photoinduced thermal transport in mWSM using Floquet-Kubo formalism
- Anisotropic nature of the dispersion can lead to an extra momentum dependent Floquet term along with the momentum independent Δ_n term
- Δ_n term gives us the n times single Weyl results in the conductivity tensor in its leading order
- Momentum dependent term leads to the sub-leading correction in σ_{xy}

- Thermal Hall conductivity for $n \geq 2$ contains a perturbative correction which varies non-linearly with μ, leading to a chemical potential dependent Nernst conductivity, unlike the type-I $n = 1$ WSM
- For type-II mWSMs, the transport coefficients for $n \geq 2$ exhibits algebraic dependence on the momentum cutoff as compared to the weak logarithmic dependence present in the $n = 1$ WSM case

Collaborators

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- Based on :
  PRB 98, 205109(2018) (with AM and DC)
  arXiv:1901.06716 (with AM)-under Review with PRB
  mWSM- paper (with TN and AM)- in preparation

THANK YOU