The Big Bang and the Quantum

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A Ballad on the ongoing saga on the eternal themes of

the ‘Beginning’ and the ‘Origin’.

Understanding emerged from the work of many researchers, especially:

Agullo, Bojowald, Campiglia, Corichi, Chiou, Henderson, Kaminski, Lewandowski,

Nelson, Pawlowski, Rovelli, Singh, Sloan, Taveras, Vidotto, Wilson-Ewing ....

ICGC - Goa; December 14-19, 2011

1. Big-Bang: An Historical Introduction

- General Relativity: Gravity encoded in Geometry. Space-time geometry became a physical and dynamical entity. Spectacular consequences: Black holes, Gravitational Waves.

- But this fusion comes with a price: Now space-time itself ends at singularities. Big Bang: Absolute Beginning.

Friedmann (1921-1924) Lemaître (1926-1965)

The assumption of spatial homogeneity & isotropy implies that the metric has the FLRW form: $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\, \mathrm{d}\vec{x}^2$

$a(t)$: Scale Factor; Volume $v(t) \sim [a(t)]^3$ Curvature $\sim [a(t)]^{-n}$

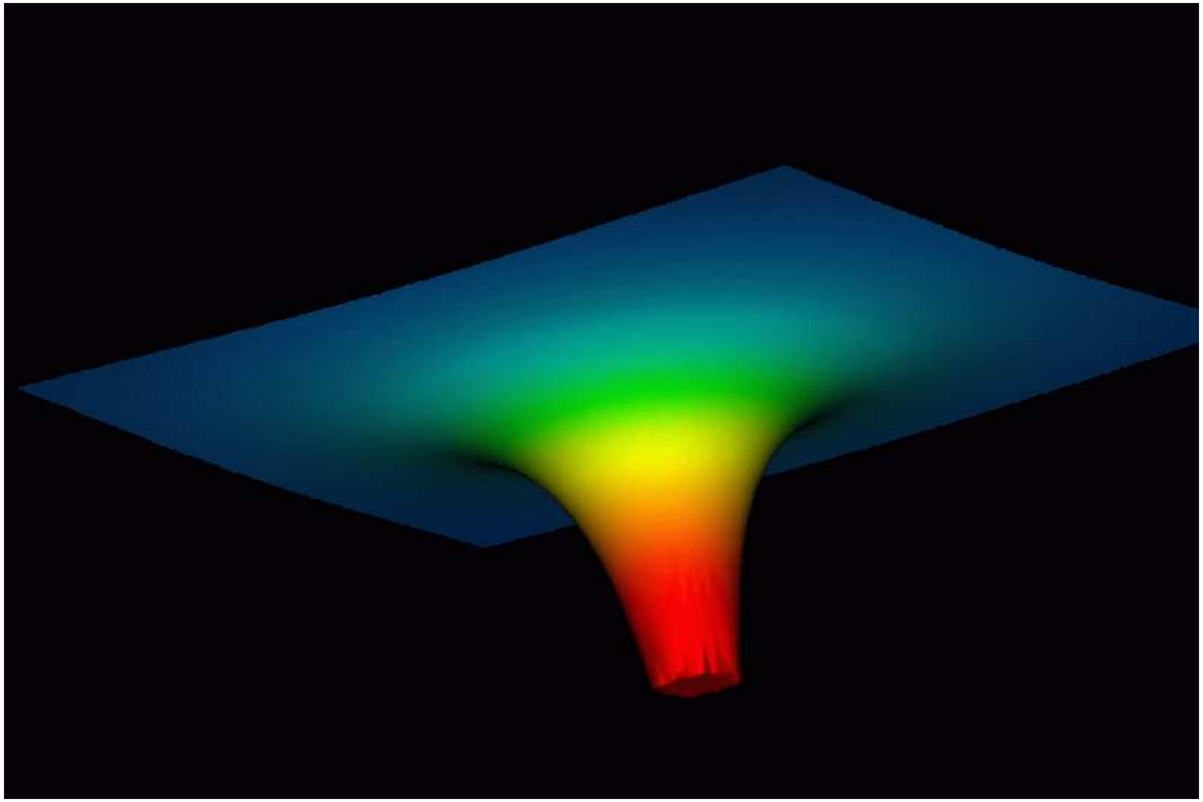
Einstein Equations $\Rightarrow$ volume $\to 0$ and Curvature $\to \infty$:

BIG BANG!! CLASSICAL PHYSICS STOPS!!

- Gamow, Alpher, Herman (1948-1967) (Detailed Nucleosynthesis).

  Gamow strongly disliked the emphasis on Big-Bang/Beginning.
- Dicke, Peebles, Roll, Wilkinson (1965 $\to$) (CMB Background)

The Big Bang in classical GR



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

The Big Bang Singularity: Twists and Turns

- Friedmann was delighted to prove Einstein wrong but not very interested in the physics of the solutions. It is Lemaître who understood the implications and took their physical significance seriously.

- Even afterwards, Einstein did not take the Big-Bang/Beginning seriously. Suggested inhomogeneities may wash it away. This view persisted.

- The Khalatnikov-Lifshitz program: "General Solution" to Einstein's equation will be singularity free (late 50's - early 60's). Gamow disliked the term *'big-bang'* and preferred to emphasize *'dynamical universe.'* Preferred to think the universe had a pre-big-bang branch.

- Paradigm Shift:

Penrose-Hawking Singularity Theorems (mid 60s): If matter satisfies 'energy conditions' then according to general relativity, cosmological space-times will necessarily have a singularity! (Lemaître's views realized.)

Excellent Historical Reference: Helge Kragh: Cosmology & Controversy

Beyond General Relativity: The Quantum

- But general expectation: theory is pushed beyond its domain of applicability. Must incorporate Quantum Physics. (Example: Instability of the Hydrogen atom in classical electrodynamics and $E_o = -me^4/2\hbar^2$ in quantum theory.)

- Big-bang is the prediction of General Relativity precisely in a domain in which it is inapplicable! Classical singularities are the gates to Physics Beyond Einstein.

- Any viable quantum gravity theory should answer the questions: What *really* happened in the Planck regime?

In the standard model, CMB occurs 380,000 years after the Big Bang.
Structure formation: Inflation is the leading paradigm. Very early. But still, matter density is less than $10^{-11}\,\rho_{\rm Pl}$ at the onset of inflation.
Far from 'proofs' that Big Bang occurred! Does quantum physics really stop if we went further back? Is there a finite Beginning? If not, what was really there before the GR era?

The Quantum

How does this paradigm of the 'Beginning' and the 'Origin' change if both matter **and** geometry are treated quantum mechanically from the start?

Will see that:

i) In cosmological models *Quantum Physics* does not stop at singularities. Quantum geometry extends its life.

ii) Loop Quantum Gravity offers a physical completion of the standard inflationary scenario. Leads to a Quantum Field Theory on cosmological quantum space-times to face the Planck regime squarely.

Idea in Loop Quantum Gravity: Retain the gravity $\leftrightarrow$ geometry duality by encoding new physics in Quantum Riemannian Geometry which was developed rigorously in the mid 1990's (AA, Baez, Lewandowski, Rovelli, Smolin,Thiemann,...). Quantum Geometry effects crucial to the Planck regime.

Organization:

$\surd$ 1. Big-Bang: An Historical Introduction $\surd$

2. A Quick Summary of the Inflationary Paradigm

3. Singularity Resolution in Loop Quantum Cosmology

4. A Quantum Gravity Completion of the Inflationary Paradigm

5. Summary.

2. Inflationary Paradigm: Quick Summary

- Major success: Prediction of inhomogeneities in CMB which serve as seeds for structure formation. Observationally relevant wave numbers in the range $\sim (k_\star, 200k_\star)$ (radius of the observable CMB surface $\sim 10\lambda_\star$).

- Rather minimal assumptions:

1. Some time in its early history, the universe underwent a phase of accelerated expansion during which the Hubble parameter H was nearly constant.

2. During this phase the universe is well-described by a FLRW background with linear perturbations.

3. A few e-foldings before the mode $k_\star$ exited the Hubble radius during inflation, Fourier modes of quantum fields describing perturbations were in the Bunch-Davis vacuum for co-moving wave numbers in the range $\sim\ (k_\star,\ 200k_\star)$; and,

4. Soon after a mode exited the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

Then QFT on FLRW space-times (and classical GR) imply the existence of tiny inhomogeneities in CMB seen by the 7 year WMAP data. **All large scale structure emerged from vacuum fluctuations!**

Inflationary Paradigm: Incompleteness

Quantum Gravity Issues:

- Fate of the initial singularity: Is the infinite curvature really attained? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

- In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed?

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

- Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

Particle Physics Issues:

- Where from the inflaton? A single inflaton or multi-inflatons? Interactions between inflatons? How are particles/fields of the standard model created during 'reheating' at the end of inflation? ...

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3. Singularity Resolution

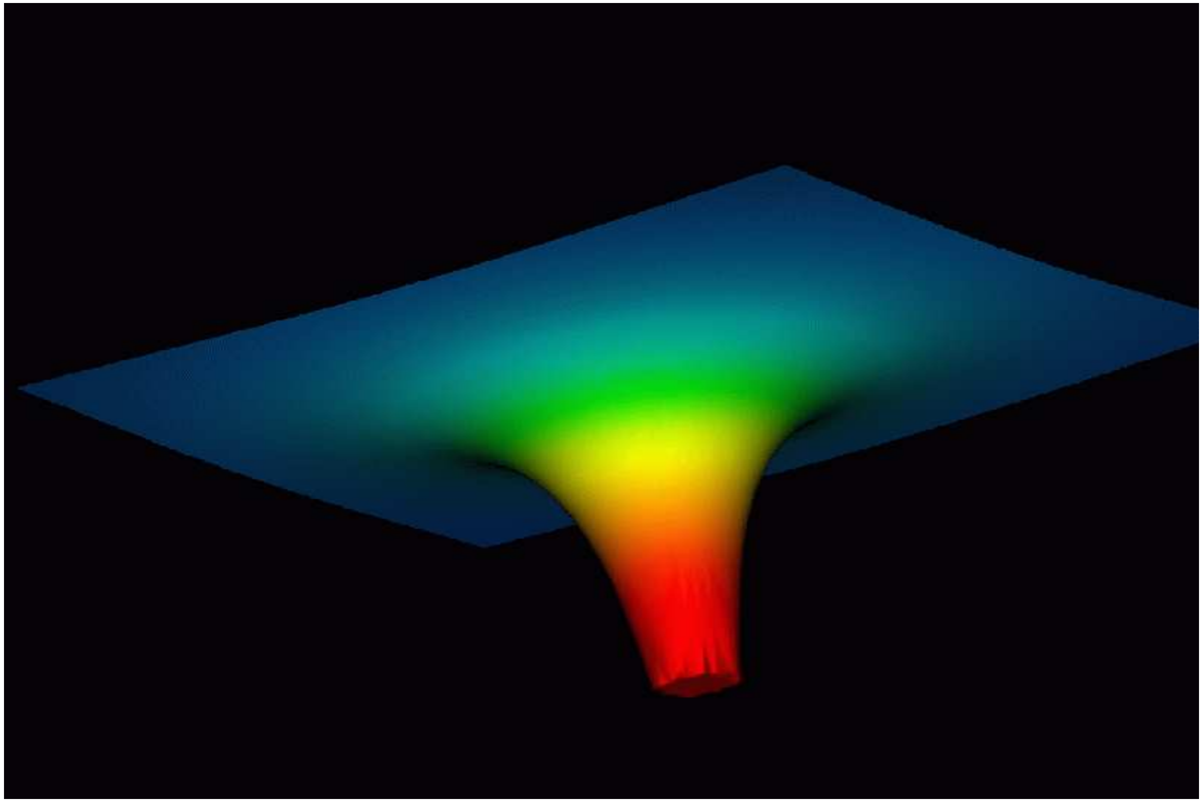
- Difficulty: UV - IR Tension. Can one have singularity resolution with ordinary matter and agreement with GR at low curvatures?(Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second.) (Green and Unruh)

- This questions have been with us for 30-40 years since the pioneering work of DeWitt, Misner and Wheeler. WDW quantum cosmology is fine in the IR but not in the UV.

- In LQC, this issue have been resolved for general homogeneous cosmologies as well as Gowdy Models. Physical observables which are classically singular (eg matter density) at the big bang have a dynamically induced upper bound on the physical Hilbert space. Mathematically rigorous and conceptually complete framework.
(AA, Bojowald, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

- Emerging Scenario:
In simplest models: Vast classical regions bridged deterministically by quantum geometry. No new principle needed to join the pre-big bang and post-big-bang branches.

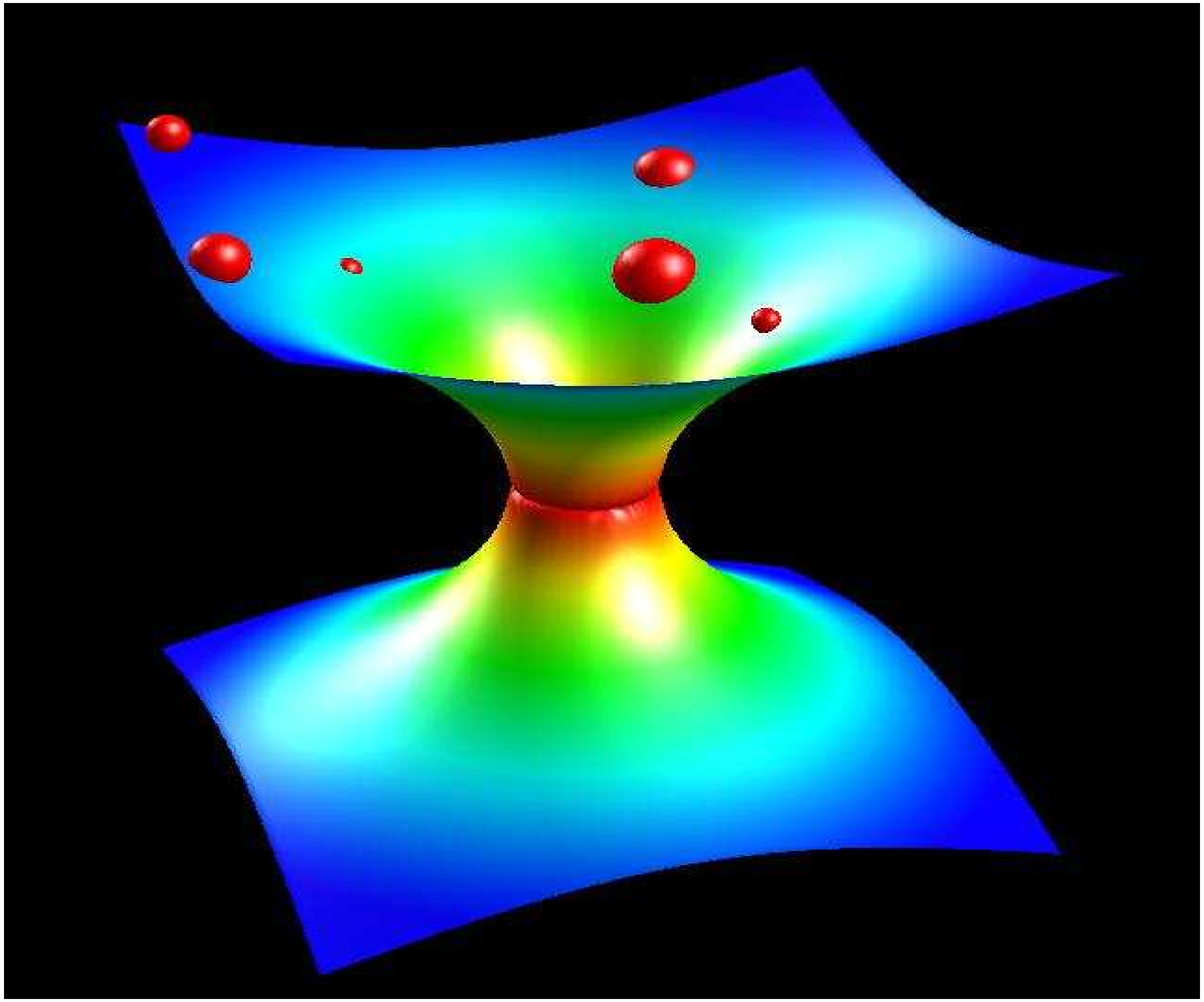
The Big Bang in classical GR: k=0 Model



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

The Big Bang in LQC: k= 0 Model

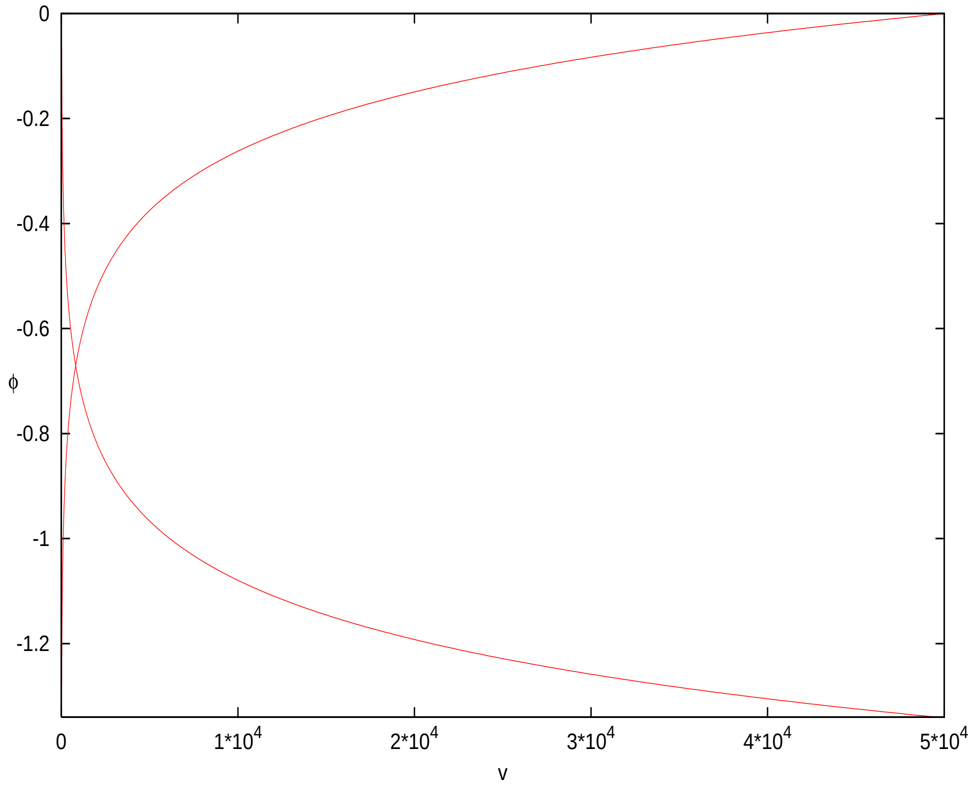


Artist's depiction of the Quantum Bounce Credits: Dr. Cliff Pickover.

In loop quantum cosmology, our post-big-bang branch of the universe is joined to a pre-big-bang branch by a quantum bridge: Gamow's bounce

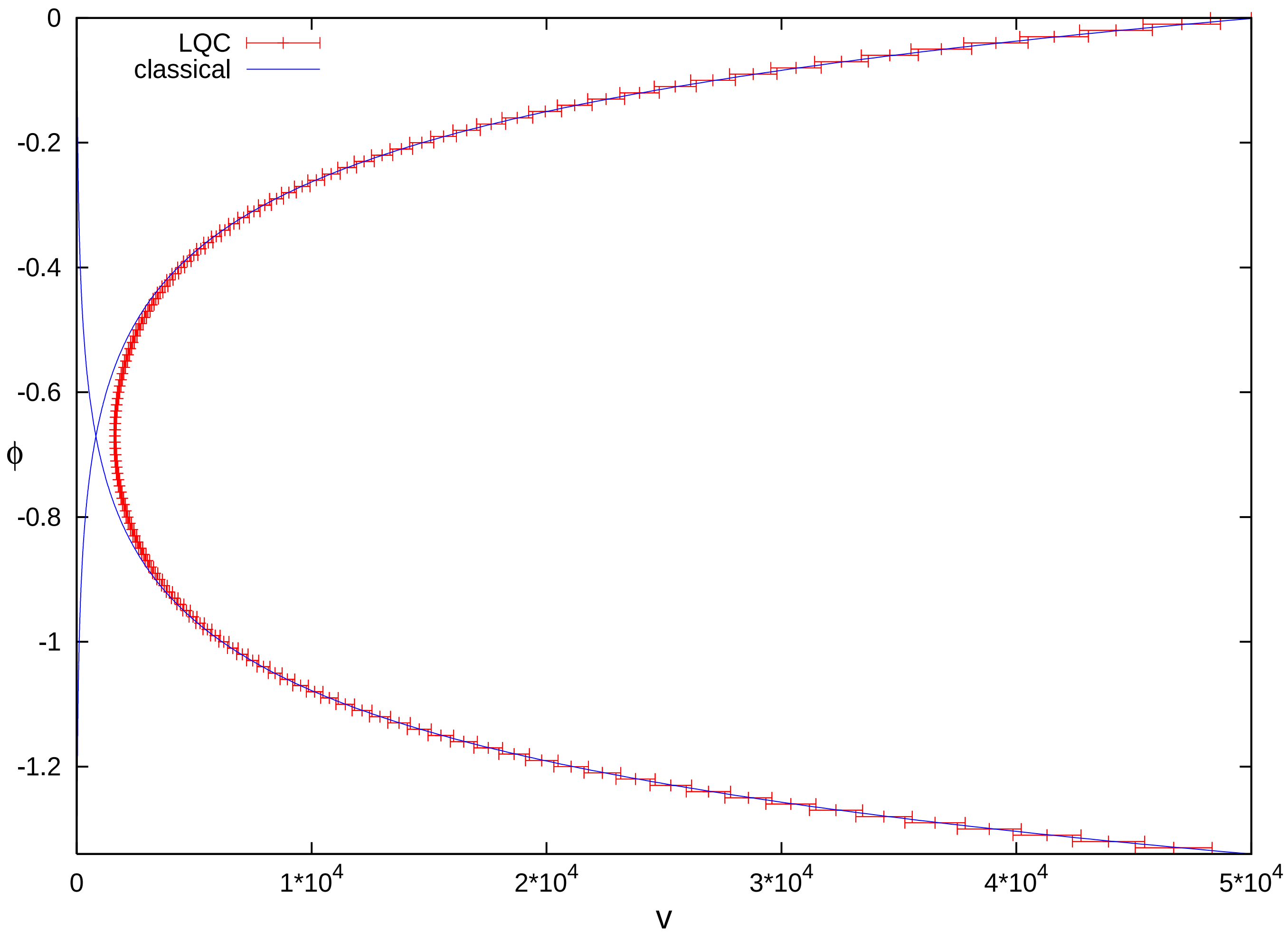
The k=0 FLRW Model

FLRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical Solutions

k=0 LQC

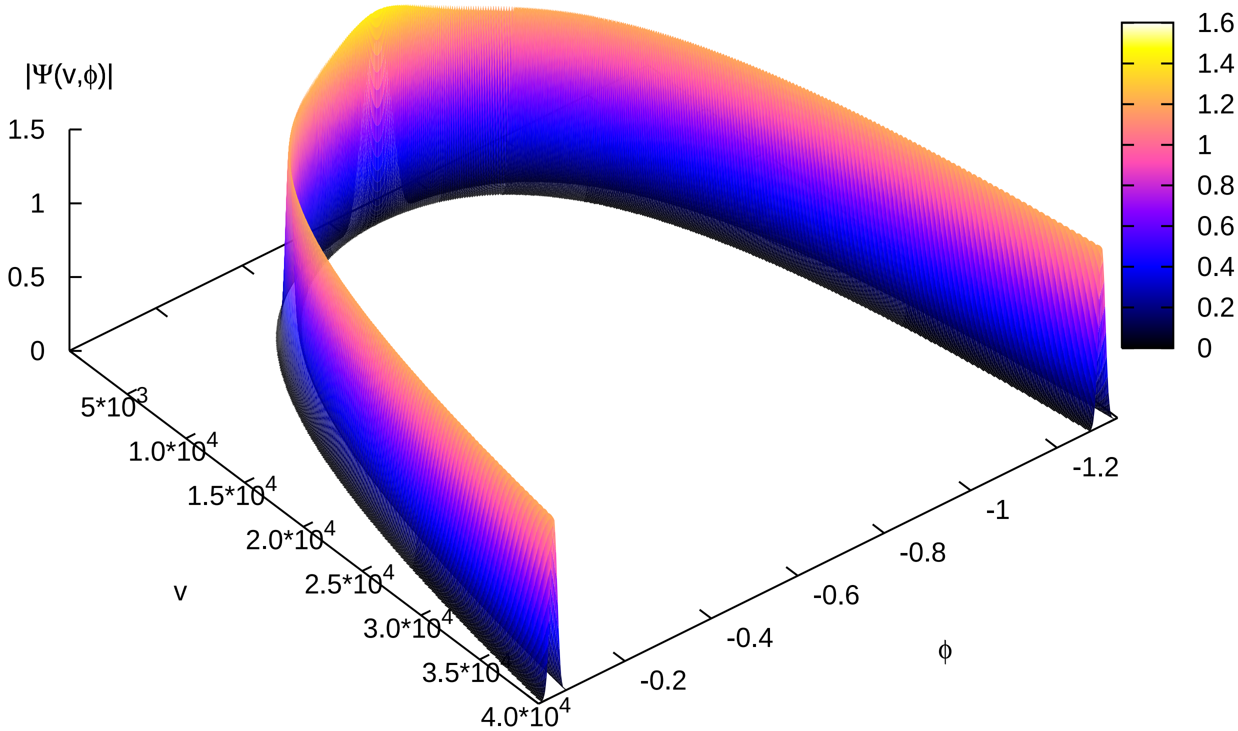


(AA, Pawlowski, Singh)

Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 1/lp^2$ or $\rho \approx 10^{-3}\rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical at the GUT scale.
(since $\rho < 10^{-11}\rho_{\rm Pl}$ at the onset of the GUT era).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects (discreteness of eigenvalues of geometric operators.): Main difference from WDW theory.

k=0 Results

• To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where } \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

• The matter density operator $\hat{\rho}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.

• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Physics does not end at singularities. A robust super-inflation phase immediately after the bounce.

• Surprise: Effective equations and their solutions provide an excellent approximation to the full quantum evolution even near and at the bounce.

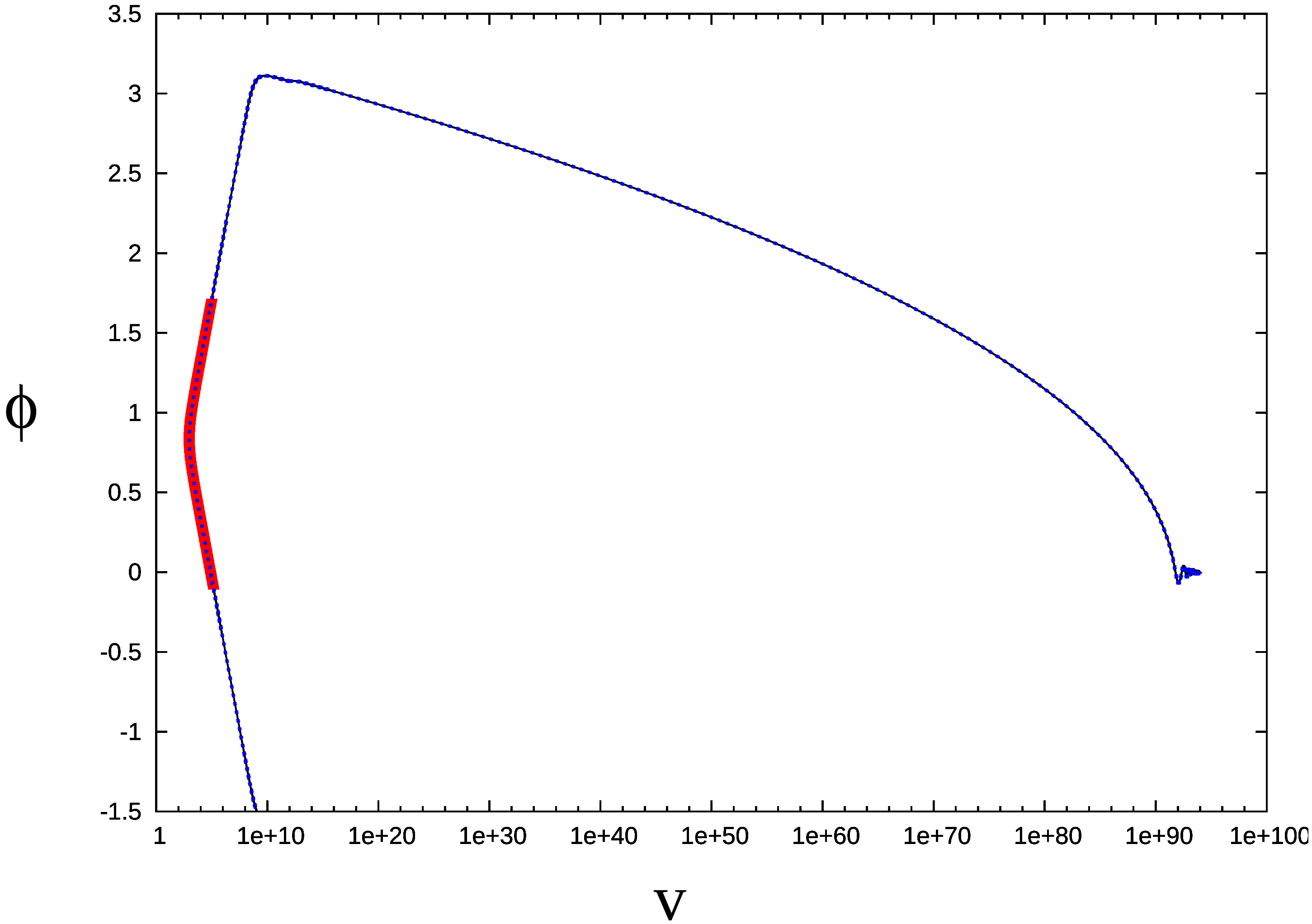
Generalizations

• More general singularities: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip) or curvature or their derivatives diverge at finite values of scale factor (sudden death). Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh).

• Beyond Isotropy and Homogeneity: Inclusion of a cosmological constant and the standard $m^2\phi^2$ inflationary potential. Inclusion of anisotropies. $k = 1$ closed cosmologies. The Gowdy model with inhomogeneities and gravitational waves. Singularities are resolved and Planck scale physics explored in all these cases.
(AA, Bentevigna, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

• **Summary: Cumulative evidence strongly suggests that all strong curvature, space-like singularities resolved by quantum geometry effects. For the k=0, FLRW background used in inflation, big bang replaced by a quantum bounce.**

Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

4. An LQG Completion of the Inflationary Paradigm

• **Strategy:** Continue to use the same successful strategy that led to singularity resolution. Focus on the appropriate sector of classical GR and pass to quantum gravity using LQG techniques.

• **Sector of interest for inflation:** Linear Perturbations off FLRW background with an inflaton ϕ in a suitable potential as the matter source. Includes inhomogeneities, but as perturbations.

Truncated Phase Space $\ni$ $\{(a, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and their conjugate momenta$\}$
Quantum Theory: Start with $\Psi(a, \phi;\, \delta h_{ab}(x), \delta\phi(x))$ and impose appropriately truncated constraints.

Then $\mathbf{\Psi} = \mathbf{\Psi}(a, \phi;\, S(x), T^{(I)}(x)) \;\equiv\; \mathbf{\Psi}(a, \phi;\, S_{\bar{k}}, T_{\bar{k}}^{(I)})$; with $I = 1, 2$.
i.e., states depend only on 3 gauge invariant DOF of perturbations.
They 'evolve' via: $-i\partial_\phi \mathbf{\Psi} = \left(\hat{H}_{\rm BG} + \hat{H}_{\rm Pert}\right)\mathbf{\Psi}$

• **Final Picture:** Test Perturbations $\Rightarrow$ Factorization.

$$\mathbf{\Psi}(a, \phi;\, S(x), T^{(I)}(x)) = \Psi(a, \phi) \otimes \psi(S_{\bar{k}}, T_{\bar{k}}^{(I)}, \phi)$$

Solutions: $\Psi(a, \phi)$ can be taken to be sharply peaked at an effective LQC trajectory of part 3. $\psi(S_{\bar{k}}, T_{\bar{k}}^{(I)}, \phi)$ propagates on the quantum geometry determined by $\Psi(a, \phi)$.

Inflationary Paradigm: Incompleteness

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- In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed?

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

- In the more complete theory, is the Bunch-Davis vacuum at the onset of the slow roll compatible with WMAP generic or does it need enormous fine tuning?

4.A Background Geometry

How generic is the necessary slow roll inflationary phase?

• Even if a theory allows for inflation, a sufficiently long slow roll may need extreme fine tuning. To test this, we need a measure on the space $\mathcal{S}$ of solutions to the equations. Elegant solution: Use the Liouville measure to calculate *a priori* probabilities (Gibbons, Hawking, Page, ...). They are useful, if extremely low or extremely high.

• Controversy in the literature. For the $m^2\phi^2$ potential, answers from probability close to 1 (Kauffman, Linde, Mukhanov) to e^{-165} (Gibbons, Turok) ! Main Reason: The question is ill posed in general relativity.

• Problem: The Liouville volume of $\mathcal{S}$ is infinite! But the infinity is a gauge artifact (associated with the $a \to \lambda\, a$ rescaling freedom). But because of the Big Bang singularity, no natural way to factor out the gauge freedom. Vastly different answers stem from different gauge fixing.

(AA, Sloan; Corichi, Karami)

Probability of the WMAP slow roll in LQC

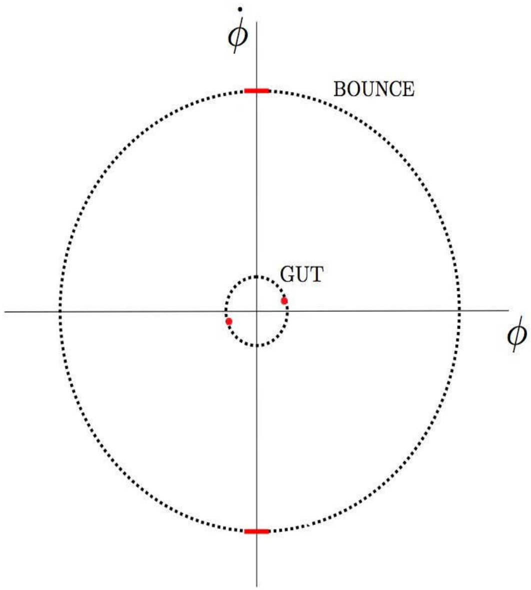
• In LQC, the Big Bang is replaced by the Big Bounce where the effective geometry and matter fields are all smooth. So, it is natural to use that surface to carry out the required gauge fixing.

• Start with generic data at the bounce. Evolve. Will it enter slow roll at the $\sim$ GUT energy scale determined by the 7 year WMAP data ($\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4$) ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

• Answer:

YES. Probability of NOT achieving the slow roll compatible with WMAP data, in particular with~ 63 e-foldings $< 3 \times 10^{-6}$

In LQC,
'almost every' initial data at the bounce evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future.
Result much stronger than the 'attractor' idea.



• This is only an a priori probability.
But because it is so high, it would be heavy burden on additional inputs to change them significantly.

Inflationary Paradigm: Incompleteness

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4.B Perturbations

- For $\Psi(a, \phi)$ want a state peaked at an effective dynamical trajectory.

(The trajectory will eventually encounter the slow roll compatible with WMAP.)

For $\psi(S_{\bar{k}}, T^{(I)}_{(\bar{k})}; \phi_B)$: Need energy density in perturbations at the bounce $\lesssim$ the quantum fluctuation of energy density in the background.

- **Key Questions:**

1. Is $\rho_{\rm Pert} \ll \rho_{\rm BG}$ from the bounce to the onset of the slow roll compatible with WMAP (so that our truncation strategy is justified by self-consistency)?

2. At the end of the WMAP compatible slow roll, do we recover the inflationary power spectrum: $\Delta^2_{R}(k, t_k) \approx \frac{H^2(t_k)}{\pi m^2_{\rm Pl} \epsilon(t_k)}$? ($t_k$ is the time the mode k exits the Hubble horizon during slow roll)

3. Does $\psi(S_{\bar{k}}, T^{(I)}_{(\bar{k})}; \phi_B)$ evolve to a state which is indistinguishable from the Bunch Davis vacuum at the onset of slow roll (for WMAP observations) ?

If so, we will have a quantum gravity completion of the inflationary paradigm.

An LQG Completion of the Inflationary Paradigm

- Analysis involves several conceptual and technical subtleties. In the end, the **answers to all three questions is in the affirmative.** (Agullo, AA, Nelson)

- Trans-Planckian Frequencies: Problem of principle in QFT on classical FRW space-times. Need a quantum gravity completion.

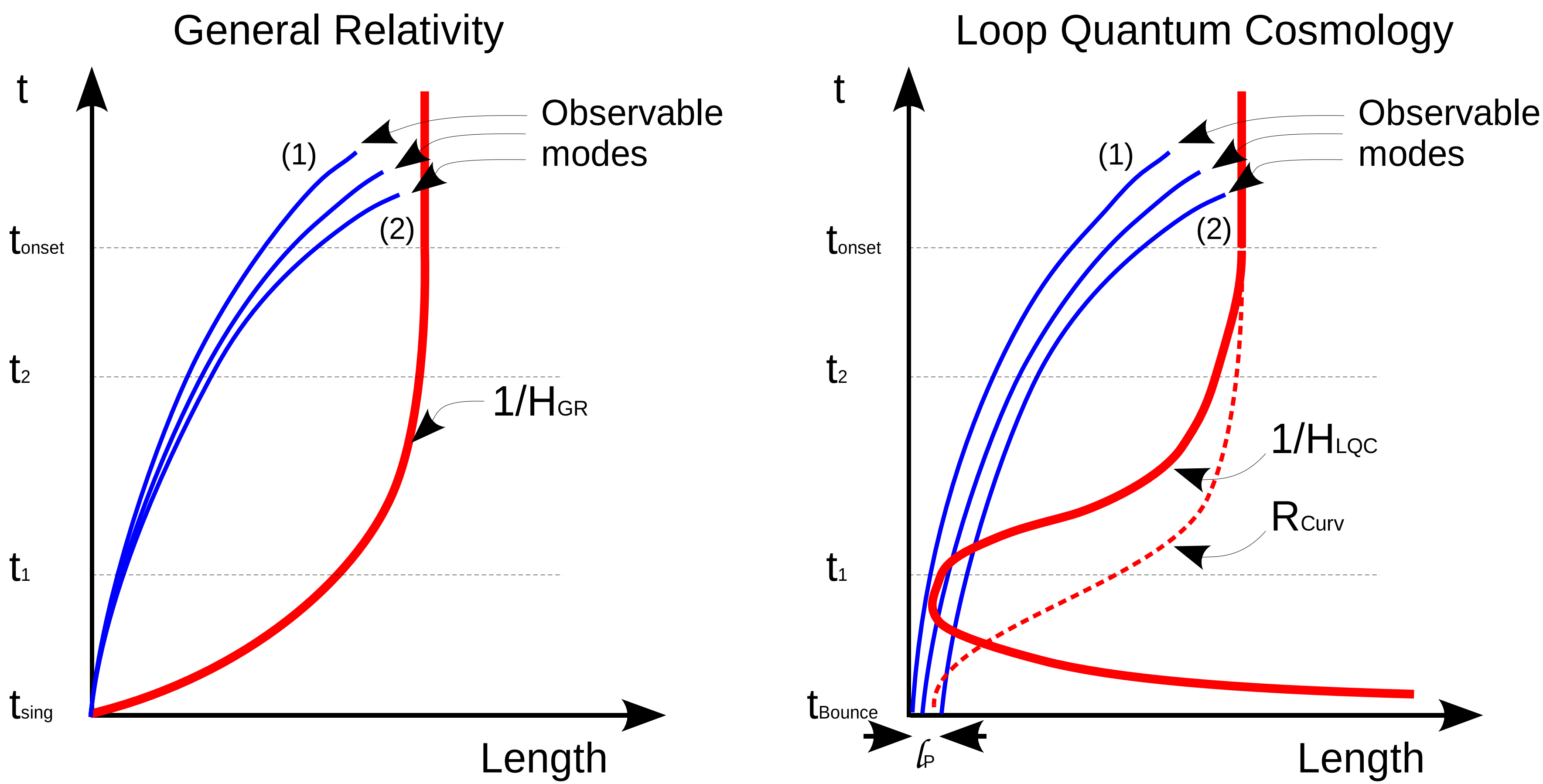
In the LQG completion, frequency by itself is not relevant. For example: In the treatment of the background quantum geometry, p_ϕ is highly trans-Planckian (typical realistic values: $> 10^{120}$ Planck units!!). But $\hat{\rho}$ is bounded above by $\sim 0.41\rho_{\rm Pl}$.

Background quantum geometry has no problem with trans-Planckian modes per say. They can be readily incorporated provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$. And this is the case.

- **All four quantum gravity related features have been addressed. LQG strategy provides a quantum gravity completion of the standard inflationary paradigm.**

An LQG Completion of the Inflationary Paradigm

Non-triviality of the result: Trans-Planckian problem in GR; Superinflation and the subtle behavior of the Hubble radius in LQG.



5. Summary

• Can one provide a quantum gravity completion of the inflationary paradigm?

Background geometry:

• Big Bang singularity: In LQG, quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (AA, Pawlowski, Singh, ...)
New paradigm: Quantum space-times may be vastly larger than Einstein's. The horizon problem dissolves.

• UV and IR Challenge: Singularity resolution and the detailed recovery of classical GR at low curvatures/densities? Met in cosmological models. Singularities analyzed are of direct cosmological interest.

• How generic is inflation? Question has a well posed formulation because of singularity resolution. Probability that initial data at the big bounce evolves to a solution that admits a slow roll phase compatible with the the 7 year WMAP data is closer to one than 3×10^{-6}! (AA, Sloan)

Summary (contd)

Perturbations:

• Since they propagate on quantum geometry, the trans-Planckian issues can be systematically analyzed.
(AA, Kaminski, Lewandowski)

For modes relevant to observations, the natural 'vacuum' at the bounce evolves to the Bunch Davis state as the onset of the WMAP slow roll $\Rightarrow$ Predictions of the standard inflationary scenario for the power spectra, spectral indices & ratio of tensor to scalar modes recovered starting from the deep Planck era.(Agullo, AA, Nelson)

• Robustness: Here we considered a truncation of GR adapted to the inflationary paradigm and used LQG techniques to construct the quantum gravity theory. There exists another strategy in LQG (Barreau, Grain, et al). It is not as sharp, e.g., w.r.t. the WMAP data, systematically addressing trans-Planckian issues, etc. But key results are qualitatively the same.

Final Remarks

• Deeper, physical understanding of the boundary conditions at the bounce from physics of the contracting branch? Semi-heuristic considerations suggest that a proper formulation of a quantum version of Penrose's Weyl curvature hypothesis naturally leads to these boundary conditions. Can these considerations be made hard and precise?

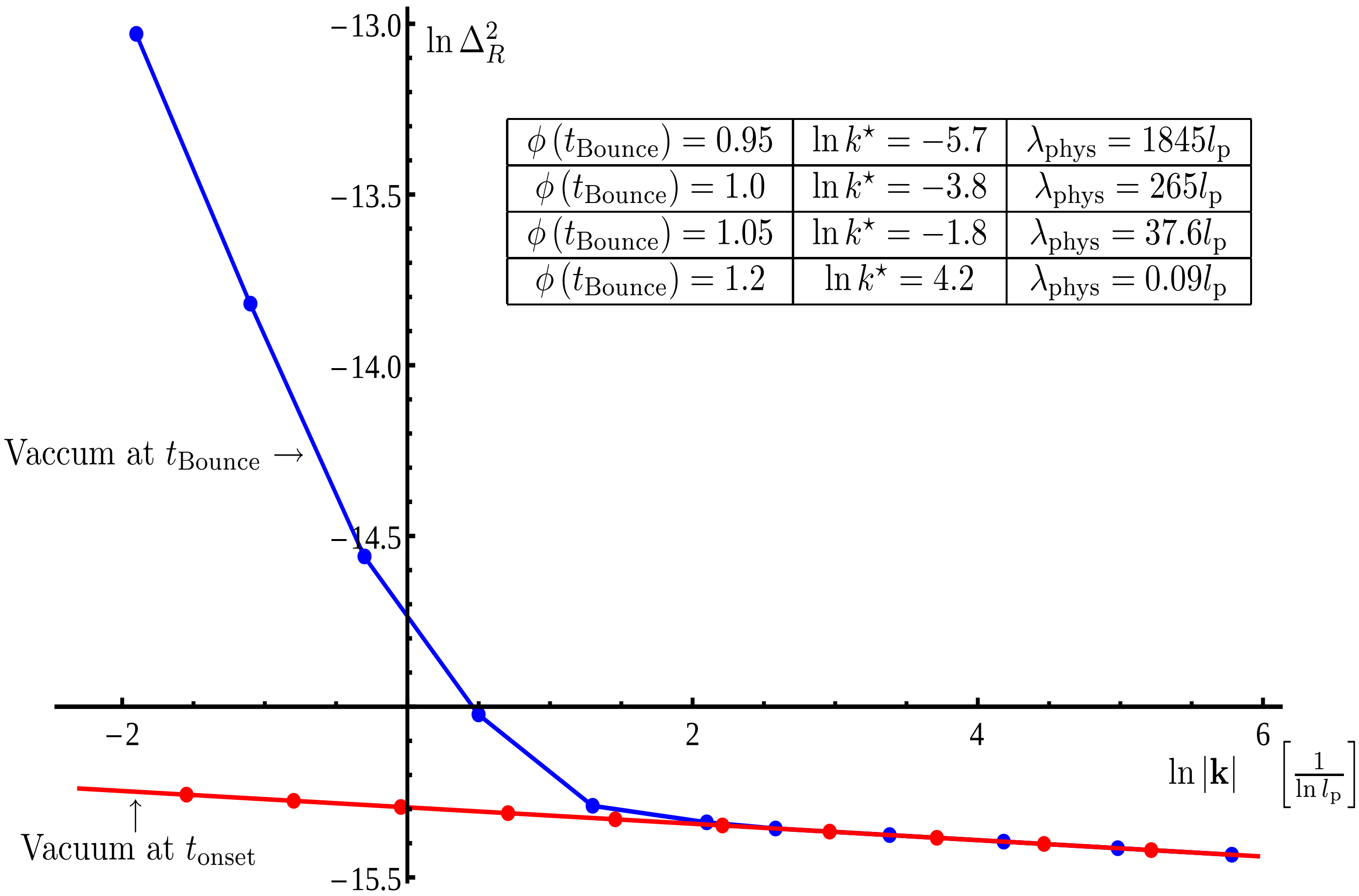
• LQG does not imply that inflation must have occurred because it does not address particle physics issues. The analysis simply assumed that there is an inflaton with a suitable potential. But it does show concretely that many of the standard criticisms (eloquently voiced by Brandenberger in particular) of inflation can be addressed by facing the Planck regime squarely. Since these quantum gravity issues are very general, LQG may well provide quantum gravity completions also of other scenarios.

• Caution: This is **not** a review of LQC. There are $\gtrsim 500$ articles in PRL, PRD, CQG and JCAP.

(Short Review: Proceedings of the Cosmology Conference, Paris, arXiv:1005.5491
Detailed review: AA & Singh: CQG 28, 213001 (2011))

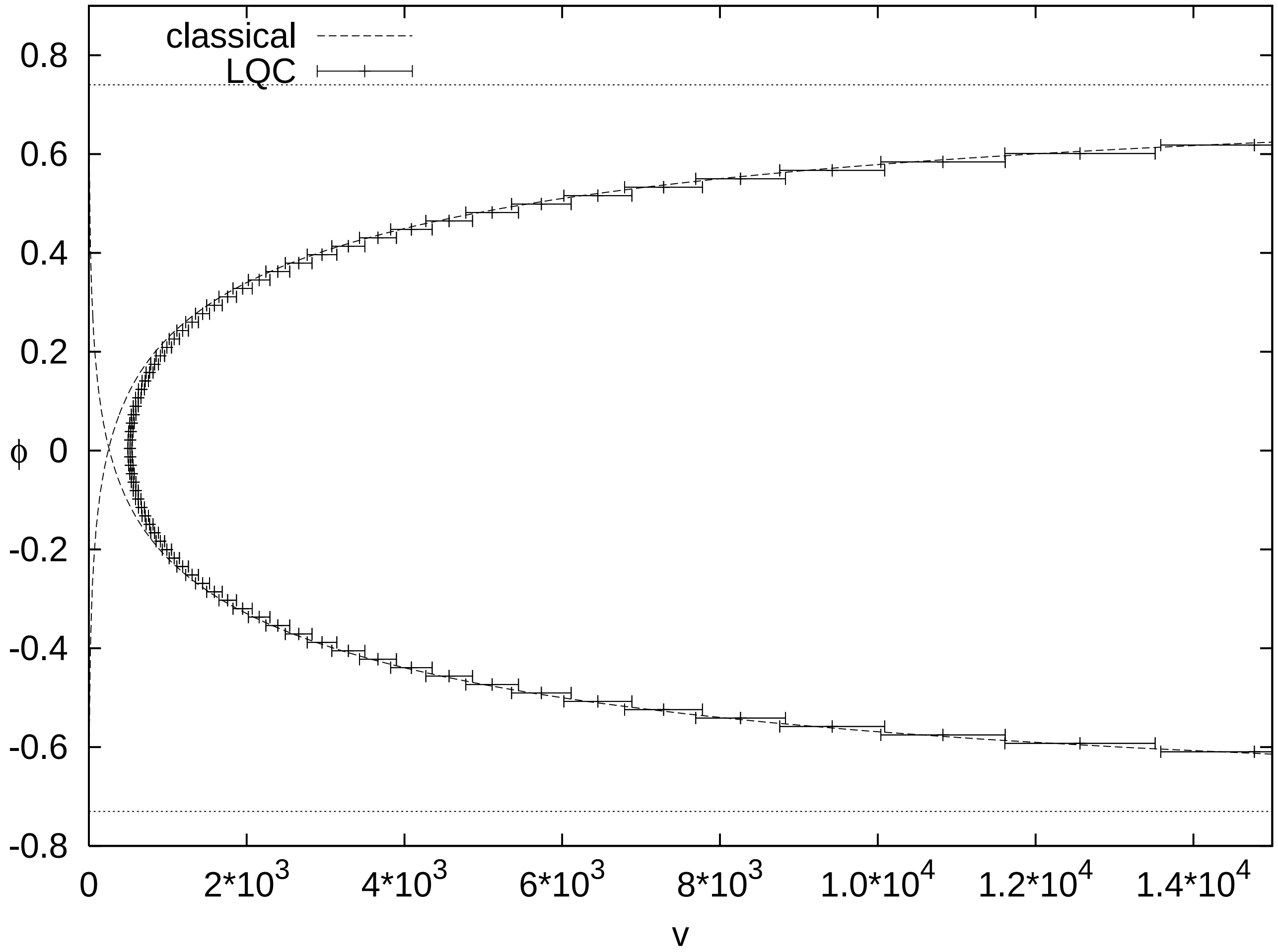
Supplementary Material

Predictions for the Power spectrum



Mathematical power spectra for the scalar mode. Red: standard inflationary scenario using the Bunch-Davis vacuum at the onset of slow roll. Blue: LQG power spectrum. For modes whose wave length at the CMB surface is less than or equal to the size of the observable universe at that time, the two are indistinguishable.

k=0 Model with Positive Λ

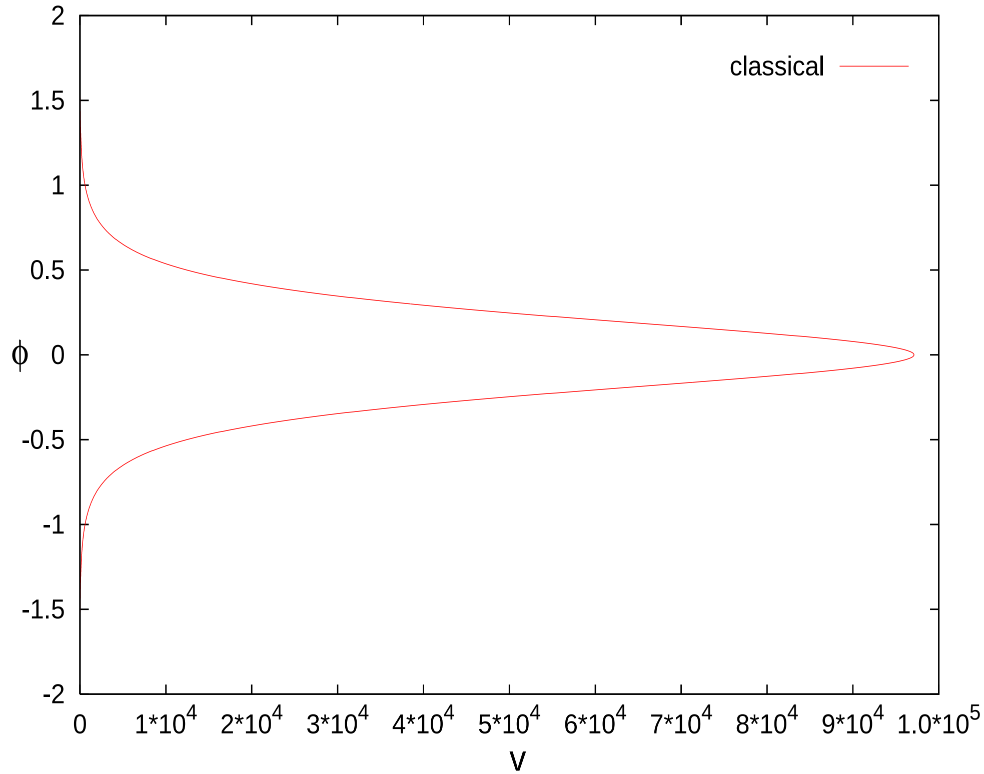


Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski)

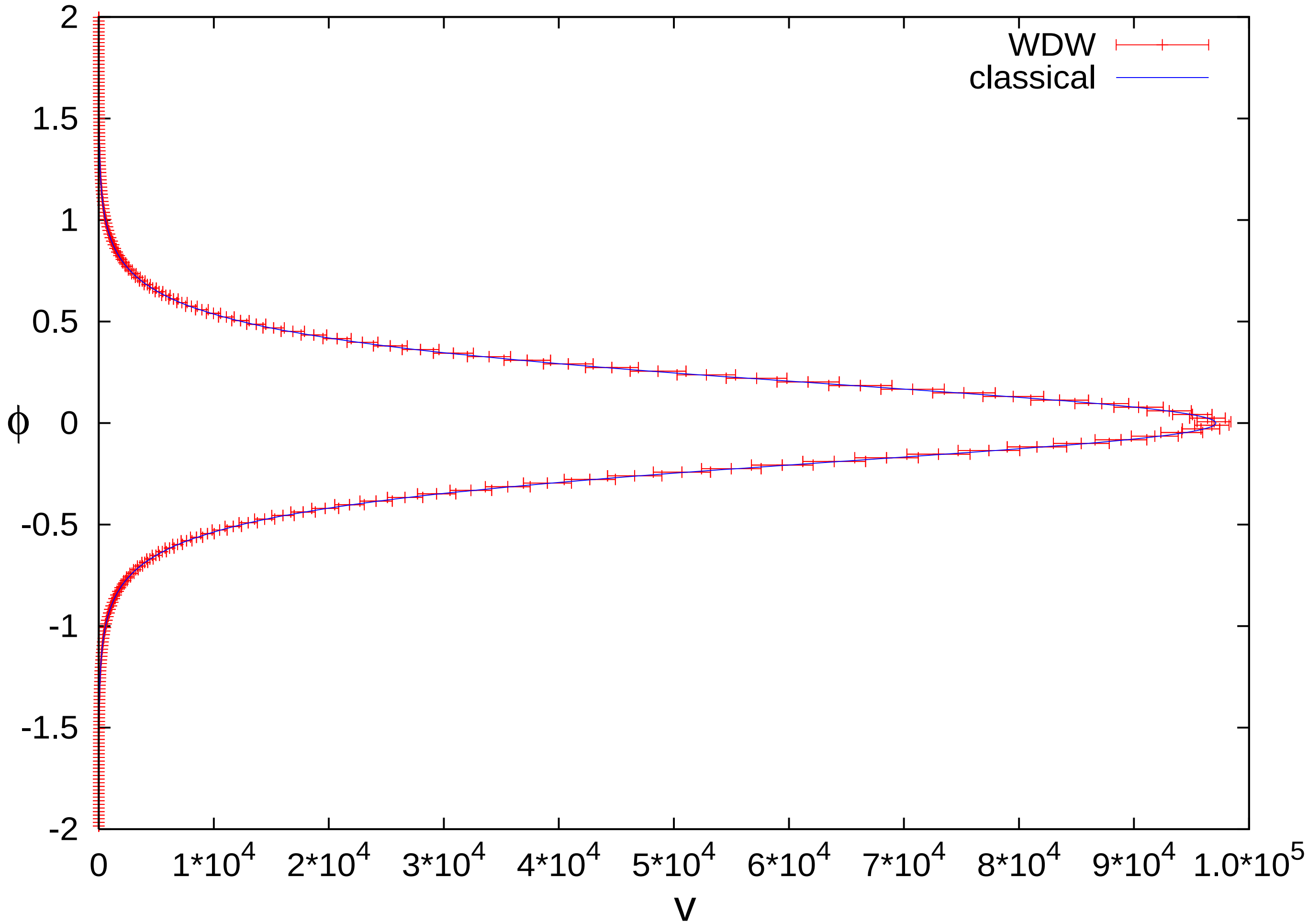
The k=1 Closed Model: Bouncing/Phoenix Universes.

Another Example: $k = 1$ FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Lemaître, Tolman, Sakharov, Dicke,...)



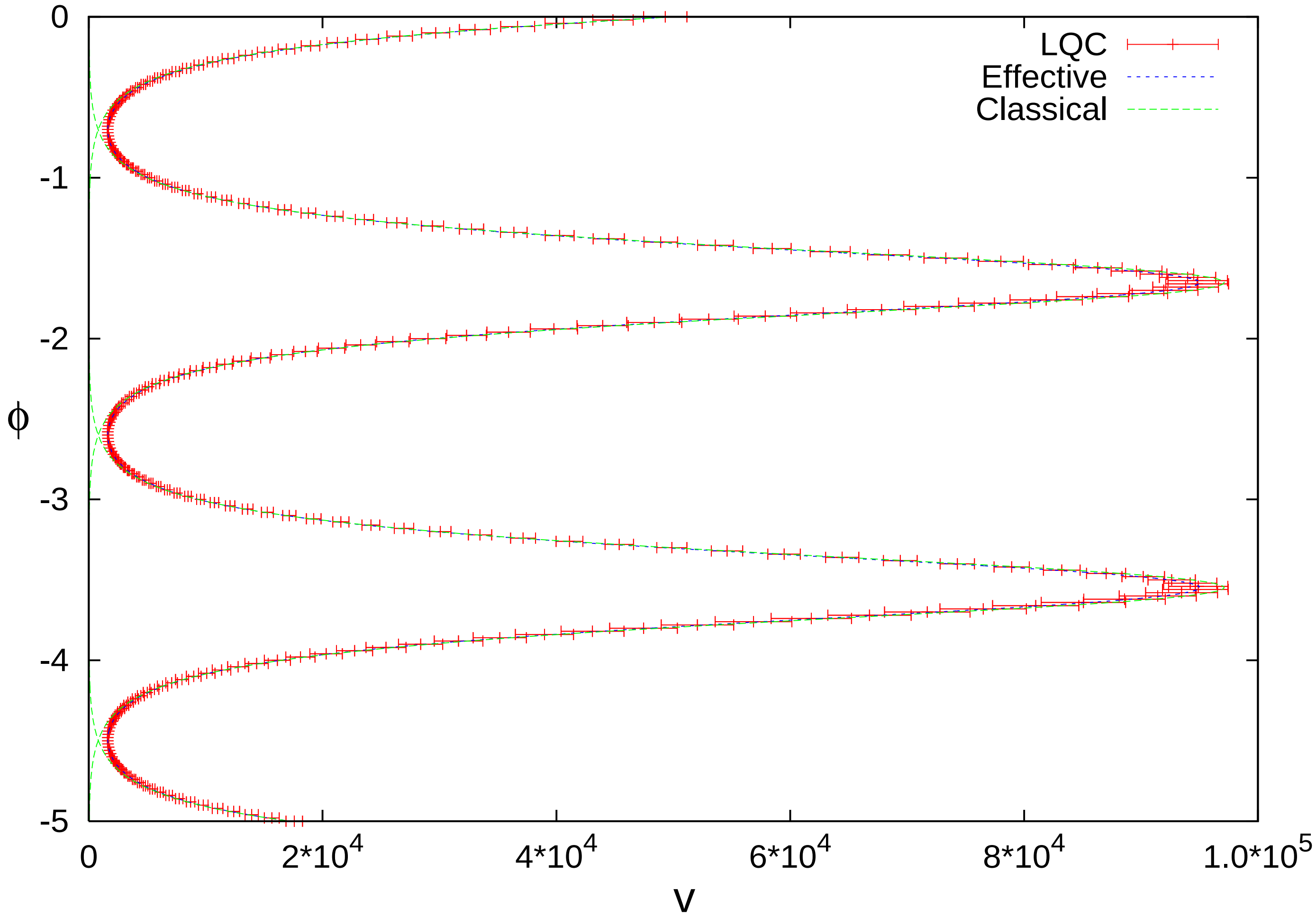
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_\phi$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

Merits and Limitations of QC

One's first reaction: Symmetry reduction gives only toy models! Full theory much richer and much more complicated. But examples can be powerful.

- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models.

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. recent work of the Madrid group on Gowdy models which have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory.