Black Hole Bound State Metamorphosis

Ashoke Sen

Harish-Chandra Research Institute, Allahabad, India

Bangalore, September 2012

Plan

1. The problem

A.S., arXiv:1104.1498

2. The solution

Abhishek Chowdhury, Shailesh Lal, Arunabha Saha and A.S., to appear

The problem

Consider heterotic string theory compactified on T^6.

– an $\mathcal{N} = 4$ supersymmetric string theory with 28 U(1) gauge fields at a generic point in the moduli space.

A general state in this theory is characterized by a 28 dimensional electric charge vector Q and a 28 dimensional magnetic charge vector P.

Our object of study will be quarter BPS states in this theory carrying (electric, magnetic) charge vectors (Q, P).

We shall normalize the charges so that each element is an integer.

T-duality symmetry

$$\mathbf{Q} \to \mathbf{W}\,\mathbf{Q}, \quad \mathbf{P} \to \mathbf{W}\,\mathbf{P}, \quad \mathbf{W} \in \mathbf{SO}(6,22;\mathbb{Z})$$

S-duality symmetry

$$\mathbf{Q} \to \mathbf{aQ} + \mathbf{bP}, \quad \mathbf{P} \to \mathbf{cQ} + \mathbf{dP}, \quad \begin{pmatrix} \mathbf{a} & \mathbf{b} \\ \mathbf{c} & \mathbf{d} \end{pmatrix} \in \mathbf{SL}(2,\mathbb{Z})$$

$\mathbf{Q}^2, \mathbf{P}^2, \mathbf{Q}\cdot\mathbf{P}$: **T-duality invariant bilinears**

S and T-duality invariant 'discriminant'

$$\mathbf{D}(\mathbf{Q},\mathbf{P}) = \mathbf{Q}^2\mathbf{P}^2 - (\mathbf{Q}\cdot\mathbf{P})^2$$

Bekenstein-Hawking entropy of a quarter BPS black hole carrying charges (Q,P) is

$$\pi\sqrt{\mathbf{D}(\mathbf{Q},\mathbf{P})}$$

Microscopic results

Exact microscopic results for the index counting quarter BPS states has been derived from first principles for all (Q,P) for which

$$\mathrm{gcd}\{Q_iP_j - Q_jP_i, \quad 1 \le i, j \le 28\} = 1$$

at all points in the moduli space.

Dijkgraaf, Verlinde, Verlinde; Cardoso, de Wit, Kappeli, Mohaupt; Shih, Strominger, Yin; Jatkar, A.S; David, A.S.; A.S.; Dabholkar, Gaiotto, Nampuri; Cheng, Verlinde; $\cdots$

A long term goal is to account for these microscopic results from the macroscopic viewpoint where these states are described as supersymmetric black hole solutions.

Problem with negative discriminant states

These microscopic results show that in some region of the moduli space we get non-vanishing result for the index for states with

$$D(Q, P) < 0$$

i.e. states with negative discriminant.

However there are no classical BPS black hole solution of charge (Q,P) if $D(Q, P) < 0$.

What accounts for these states in the macroscopic description?

First attempt

Dabholkar, Gaiotto, Nampuri

Considered a special class of states carrying charges (Q,P) with

$$Q^2 = -2, \quad P^2 = -2, \quad Q.P = n$$

$$D(Q,P) = 4 - n^2 < 0 \quad \text{for} \quad n > 2$$

Microscopic results: In certain region of the moduli space the index carried by such states is given by

$$(-1)^{n+1}|n|$$

DGN showed that in the same region of the moduli space there is a 2-centered black hole configuration with charges (Q,0) and (0, P) carrying the same index.

Problem

In the same region of the moduli space there are other 2-centered configuration carrying same total charge and the same index, e.g.

$$(\mathbf{Q}+\mathbf{u}\mathbf{P},\mathbf{0}) \quad \text{and} \quad (-\mathbf{u}\mathbf{P},\mathbf{P}), \quad \mathbf{u}\equiv\mathbf{Q}\cdot\mathbf{P}$$

This will spoil the agreement between the microscopic and the macroscopic result.

Before we describe the resolution we shall examine the most general situation.

Consider a general negative discriminant state carrying charges (Q,P) at some point in the moduli space.

From the microscopic counting formula we have an exact formula for the index of this state.

Goal: Identify all multi-centered black hole solutions which could contribute to the index of this state, calculate the index of each of them, and see if the sum agrees with the microscopic result.

General result: The only contributions come from 2-centered configurations, with charges

$$(a\widetilde{Q}, c\widetilde{Q}) \quad \text{and} \quad (b\widetilde{P}, d\widetilde{P}), \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z})$$

$$a\widetilde{Q} + b\widetilde{P} = Q, \quad c\widetilde{Q} + d\widetilde{P} = P$$

The index carried by this configuration

Denef, Moore

$$(-1)^{\widetilde{Q}\cdot\widetilde{P}+1} |\widetilde{Q}\cdot\widetilde{P}| \, d_h(\widetilde{Q}^2/2) \, d_h(\widetilde{P}^2/2)$$

where

$$\sum_n d_h(n) e^{2\pi i n\tau} = e^{-2\pi i\tau} \prod_{n=1}^{\infty} \left(1 - e^{2\pi i n\tau}\right)^{-24}$$

Note: $d_h(\widetilde{Q}^2/2)$ vanishes unless $\widetilde{Q}^2 \geq -2$

At any given point in the moduli space, which of these two centered contributions exist can be found using Denef's rules.

Add up all the contributions.

Does this agree with the microscopic result?

It does, provided we use some additional rules.

The additional rules are needed when either $\widetilde{Q}^2 = -2$ or $\widetilde{P}^2 = -2$ or both.

Suppose $\widetilde{P}^2 = -2$.

Consider a different 2-centered configuration carrying same total charge

$$(a'\widetilde{Q}', c'\widetilde{Q}') \quad \text{and} \quad (b'\widetilde{P}', d'\widetilde{P}')$$

$$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} a & b-au \\ c & d-au \end{pmatrix}, \quad \widetilde{Q}' \equiv \widetilde{Q} + u\widetilde{P}, \quad \widetilde{P}' \equiv \widetilde{P}, \quad u \equiv \widetilde{Q}\cdot\widetilde{P}$$

Then

$$\widetilde{Q}'^2 = \widetilde{Q}^2, \quad \widetilde{P}'^2 = \widetilde{P}^2, \quad \widetilde{Q}'\cdot\widetilde{P}' = -\widetilde{Q}\cdot\widetilde{P}$$

Thus this has the same index as the earlier 2-centered configuration.

Summary: We have a pair of 2-centered configurations each carrying the same index I (say).

Suppose the first configuration exists in the subspace R_1 of the moduli space and the second configurations exists in the subspace R_2 of the moduli space.

Naive expectation:

The contribution to the index from this pair of configurations is 2 I in $R_1 \cap R_2$, and I in the rest of $R_1 \cup R_2$.

However what we need for agreement with the microscopic result is that the index is I in $R_1 \cap R_2$ and vanishes outside this region!

Similar effects are present in gauge theories.

The situation is best described by using S-duality to map the pair of configurations to

$$(\mathbf{Q}, \mathbf{0}) \quad \text{and} \quad (\mathbf{0}, \mathbf{P}), \quad \mathbf{P}^2 = -2$$

and

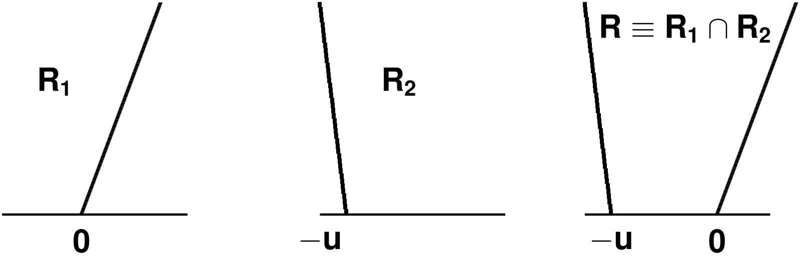
$$(\mathbf{Q} + \mathbf{uP}, \mathbf{0}) \quad \text{and} \quad (-\mathbf{uP}, \mathbf{P}), \quad \mathbf{u} \equiv \mathbf{Q}.\mathbf{P}$$

Both have index

$$\mathbf{I} = (-1)^{\mathbf{u}+1} \, |\mathbf{u}| \, \mathbf{d_h}(-2) \, \mathbf{d_h}(\mathbf{Q}^2/2)$$

We shall now describe the region of existence of these pair of solutions in the axion-dilaton moduli (τ) space for fixed values of the other moduli.

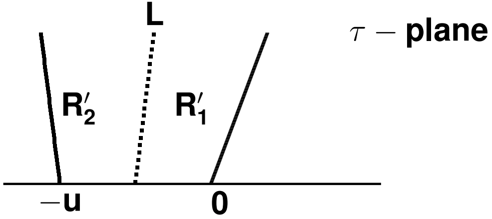
$\tau = \tau_1 + i\tau_2$ - plane



R_1: The subspace of the UHP in which the first configuration (Q,0) + (0,P) exists

R_2: The subspace of the UHP in which the second configuration (Q + u P)+ (-u P, P) exists

A pictorial representation of what we need for agreement with the microscopic result



L: A hypothetical line such that the region of existence of the first configuration is restricted to the right of L, and the region of existence of the second configuration is restricted to the left of L

The first configuration metamorphoses into the second configuration across L.

Question: What is the physical origin of the boundary L?

The solution

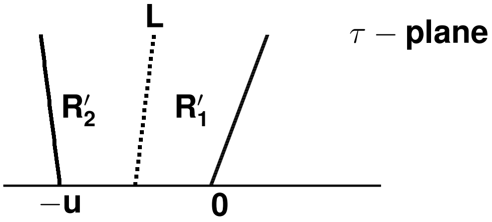
Chowdhury, Lal, Saha, A.S., to appear

Let $\tau = \tau_1 + i\tau_2$ be the value of the axion-dilaton modulus far away from both centers.

We work in the limit of large τ_2, i.e. weak coupling, so that magnetic charges are much heavier than the electric charges.

– we can treat the (Q,0) center of the first configuration and the (Q+uP,0) center of the second configuration as probe, which does not affect the background.

The background is produced by the charge $(0, P)$ for the first configuration and the charge $(-uP, P)$ for the second configuration, but we can replace $(-uP, P)$ by $(0, P)$ since the effect of the electric charges on the background is being ignored.



Question: In the presence of a background produced by the charge $(0, P)$ how does a test charge $(Q, 0)$ metamorphose into a test charge $(Q + uP, 0)$ across the boundary L?

Strategy: Find the restriction on the moduli by requiring the existence of an equilibrium configuration of the test charge in the presence of the background.

The fake story

We represent the field produced by the charge (0,P) as a single centered black hole solution. Bates, Denef; Denef

The equilibrium position of the test charge is obtained by balancing the gravitational force against the electrostatic force on the charge.

Note: Even though the charge (0,P) is purely magnetic, in the presence of non-zero τ_1 (theta-angle) the system actually represents a dyon, and for large enough τ_1 the induced electric charge cannot be ignored.

The players in the force balance condition

Gravitational force: g_{00} and the scalar fields (which determine the mass of the test charge)

Electrostatic force: The electric potential produced by the background.

By solving the force balance condition we can find the equilibrium postion of the test charges for the two systems:

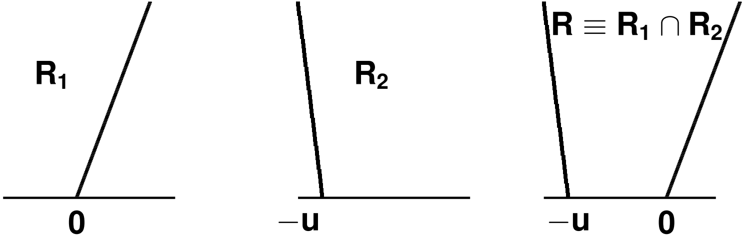
$$\mathbf{r_1} = \mathbf{f}(\mathbf{Q}; \tau_1, \tau_2, \cdots), \quad \mathbf{r_2} = \mathbf{f}(\mathbf{Q} + \mathbf{uP}; \tau_1, \tau_2, \cdots)$$

$\cdots$: other moduli at infinity

Denef

$$\mathbf{r_1} = \mathbf{f}(\mathbf{Q}; \tau_1, \tau_2, \cdots), \quad \mathbf{r_2} = \mathbf{f}(\mathbf{Q} + \mathbf{uP}; \tau_1, \tau_2, \cdots)$$

Requiring r_1 to be positive gives the region R_1 and requiring r_2 to be postive gives the region R_2.



At the boundaries of R_1 and R_2, r_1 and r_2 go to infinity.

– walls of marginal stability.

Are there other restrictions on r_1, r_2 which could produce a left boundary of R_1 and right boundary of R_2?

By examining the solution we find that in the background produced by (0,P), a test charge (P,0) becomes massless at a radius

$$\mathbf{r_e} = \mathbf{g}(\tau_1, \tau_2, \cdots)$$

– known as the enhancon

Johnson, Peet, Polchinski

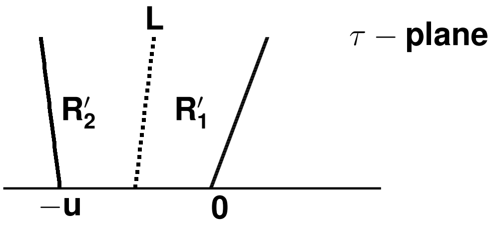
Thus the background should not be trusted for $\mathbf{r} < \mathbf{r_e}$.

Requiring $\mathbf{r_1} > \mathbf{r_e}$ and $\mathbf{r_2} > \mathbf{r_e}$ we can further restrict the allowed range of τ.

$$\mathbf{r_1 > r_e \quad \Rightarrow \quad f(Q; \tau_1, \tau_2, \cdots) > r_e}$$

$$\mathbf{r_2 > r_e \quad \Rightarrow \quad f(Q + uP; \tau_1, \tau_2, \cdots) > r_e}$$

These can be translated into restrictions on τ.



The $r_1 = r_e$ and $r_2 = r_e$ equations give identical conditions on τ.

The real story

Vanishing of the mass of the test charge (P,0)

$\Rightarrow$ enhanced SU(2) gauge symmetry at $r = r_e$

This suggests that the correct description of the magnetically charged center with charge (0,P) is not as a black hole but as a BPS monopole of the SU(2) gauge theory. Wijnholt, Zhukov

Of course we have to gravitationally dress this but exact solution in supergravity is known. Harvey, Liu

Thus we replace the original solution by the new solution (in ‘string gauge’) and repeat the analysis of the equilibrium configuration of the test charge.

We need to analyze how the background value of the players – g_{00}, the scalar fields and the electric potential – differ from the earlier solution.

Result: For all these fields the effect is to replace r by $\hat{r}$ where

$$\frac{1}{\hat{r}} = \frac{1}{r} - \frac{1}{r_e} \coth \frac{r}{r_e} + \frac{1}{r_e}$$

Thus the equilibrium position of the test charge is given by the same function of the moduli and charges as before if we replace r by $\hat{r}$.

$$\hat{r}_1 = f(Q; \tau_1, \tau_2, \cdots), \quad \hat{r}_2 = f(Q + uP; \tau_1, \tau_2, \cdots)$$

f: the same function that appeared in the fake story.

$$\frac{1}{\hat{r}} = \frac{1}{r} - \frac{1}{r_e}\coth\frac{r}{r_e} + \frac{1}{r_e}$$

Since the new solution is smooth everywhere we need to require that the location of the test charges are in the range

$$0 < r_1, r_2 < \infty$$

This is equivalent to

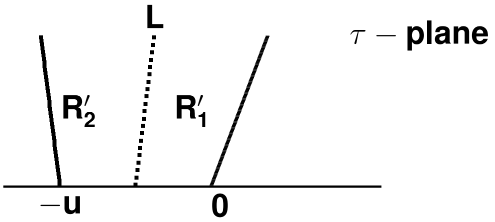
$$r_e < \hat{r}_1, \hat{r}_2 < \infty$$

This translates to a restriction on the moduli using

$$\hat{r}_1 = f(Q; \tau_1, \tau_2, \cdots), \quad \hat{r}_2 = f(Q + uP; \tau_1, \tau_2, \cdots)$$

– exactly the same restriction which appeared in the fake story.

Summary: The result of the true story is the same as that of the fake story.



New feature: On the boundary L, both test charges reach the origin of the smooth solution.

Since at the origin the SU(2) symmetry is restored and the electric field vanishes, the test charge can flip its SU(2) electric charge from Q to Q+uP at no cost in energy, and hence metamorphose to each other.

Conclusion

In the test charge approximation we have a first principle explanation of black hole bound state metamorphosis.

This shows that the macroscopic and microscopic results for the index carried by negative discriminant states are in complete agreement without any ad hoc assumption.