Quantum Anomalies

Classical anomaly: symmetry not restored even if the symmetry breaking parameter vanishes

Example: dissipative anomaly in turbulence — time reversibility remains broken even in the limit of vanishing viscosity.

Quantum anomaly: Classical symmetry (invariance) of a theory is not preserved at the quantum level

Example: Axial anomaly — non-conservation of 'chiral' particle density in massless Dirac Fermions in an Electromagnetic field.

Am. J. Phys. 61 (2), February 1993

Anomalies for pedestrians

Barry R. Holstein

Scholarpedia article (2008): *Axial anomaly*

Roman W Jackiew

Axial or Chiral Anomaly in massless Dirac Fermions

Massless Dirac fermions $i\gamma^\mu \frac{\partial}{\partial x_\mu}\psi = 0 \Rrightarrow i\gamma^\mu \frac{\partial}{\partial x_\mu}\psi_\pm = 0$

$$\psi_\pm = P_\pm \psi \qquad P_\pm = \frac{1}{2}(I \pm \gamma_5) \qquad P_\pm \psi_\pm = \pm\psi_\pm$$

Additional symmetry: axial gauge symmetry => time ind. axial charge

Chiral current: $j_5^\mu = j_+^\mu - j_-^\mu \Rrightarrow \partial_\mu j_5^\mu = \frac{1}{8\pi^2} {}^*F^{\mu\nu} F_{\mu\nu}$

Anomalous source Term

Non conservation of "chiral charge" upon quantization = **axial/chiral anomaly**

Scholarpedia article (2008): *Axial anomaly*
Roman W Jackiew

Massless Fermions in condensed matter systems

937 PHYSICAL REVIEW

Accidental Degeneracy in the Energy Bands of Crystals

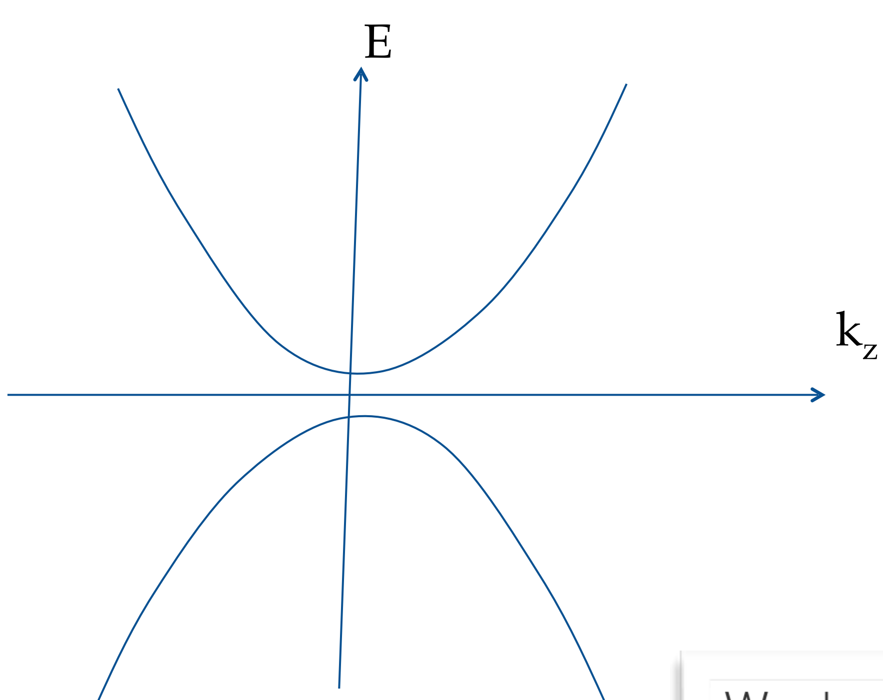
CONYERS HERRING
Princeton University, Princeton, New Jersey

(Received June 16, 1937)

For a crystal without an inversion center, the energy separation $\delta E(\mathbf{k}+\boldsymbol{\kappa})$ in the neigborhood of a point $\mathbf{k}$ where contact of equivalent manifolds occurs may be expected to be of the order of κ as $\kappa \rightarrow 0$, for all directions of $\boldsymbol{\kappa}$.

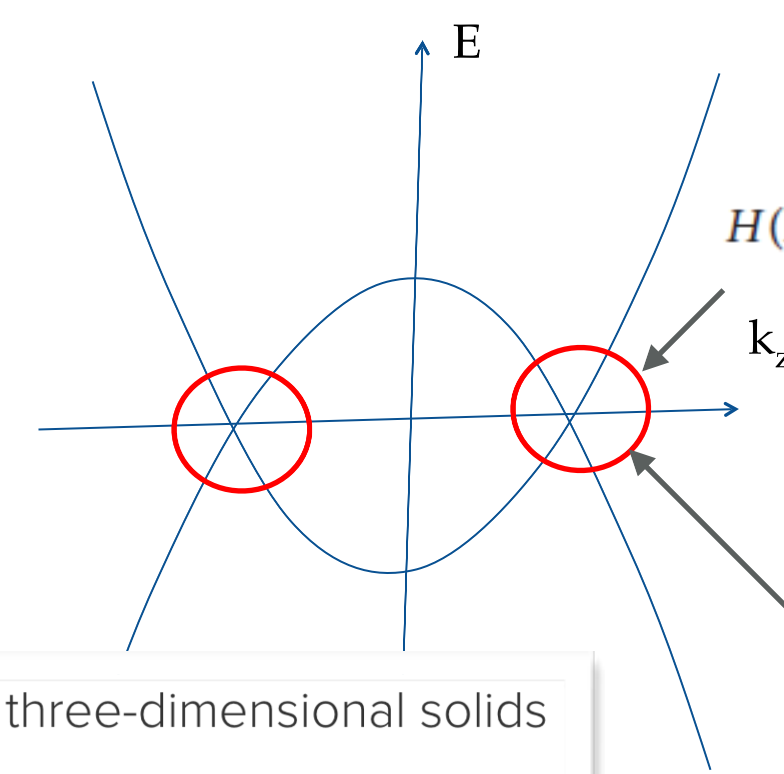
An Example

$$H(\mathbf{k}) = t_z(2+\gamma-\cos k_x - \cos k_y - \cos k_z)\sigma_z + t_x(\sin k_x)\sigma_x + t_y(\sin k_y)\sigma_y$$



Band inversion

$-1 < \gamma < 1$



$H(\mathbf{k}) = \varepsilon_0\sigma_0 \pm \hbar v_F(\mathbf{k}-\mathbf{k}_0)\cdot\boldsymbol{\sigma}$

Linear dispersion around nodes at $(0, 0, \pm k_0)$, where $\cos k_0 = \gamma$

Weyl and Dirac semimetals in three-dimensional solids

N. P. Armitage, E. J. Mele, and Ashvin Vishwanath
Rev. Mod. Phys. **90**, 015001 – Published 22 January 2018

Weyl Fermions

massless Dirac Fermions with a definite handedness

In non-centrosymmetric or magnetic materials, the non-degenerate conduction and valance band can form accidental band crossing at generic momenta.

In the vicinity of such crossing point, the Hamiltonian will be,

$$H(\delta\mathbf{k}) = f_0(\mathbf{k}_0)\sigma_0 + v_0\sigma_0 + \sum_{a=x,y,z} (\mathbf{v}_a \cdot \delta\mathbf{k})\ \sigma^a.$$

For $\mathbf{v}_a = v\hat{a}$, where $\hat{a} = \hat{x}, \hat{y}, \hat{z}$, $H(\delta\mathbf{k})$ describe the equation of Weyl fermion.

Weyl Fermions

$$H(\delta\mathbf{k}) = f_0(\mathbf{k}_0)\sigma_0 + v_0\sigma_0 + \sum_{a=x,y,z} (\mathbf{v}_a \cdot \delta\mathbf{k})\ \sigma^a$$

absence of time reversal symmetry or the inversion symmetry is essential

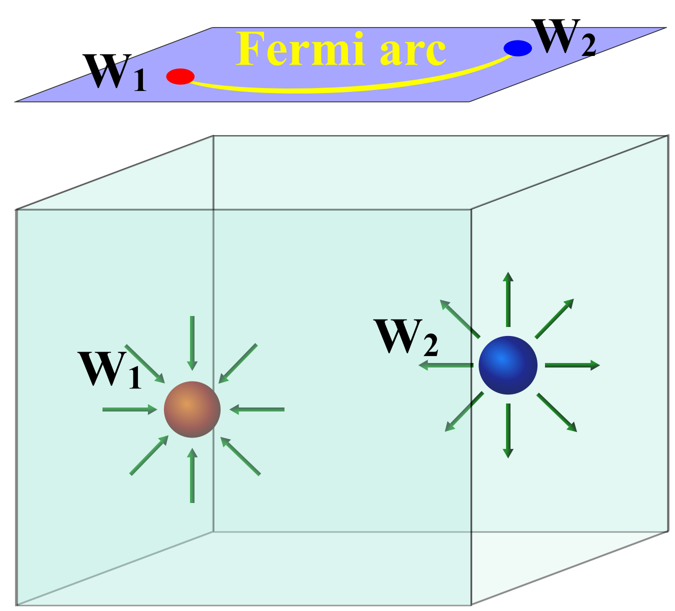
massless two component Dirac Fermions with a definite handedness

Chiral Charge

$$C = sign(\mathbf{v}_x \cdot \mathbf{v}_y \times \mathbf{v}_z)$$

Chirality

Weyl points are also monopoles of berry curvature in k-space



Interesting transport phonemena

Chiral Anomaly (Addler, Bell and Jackiw)

anomalous non-conservation of chiral charge in high energy physics

Analogue in crystals

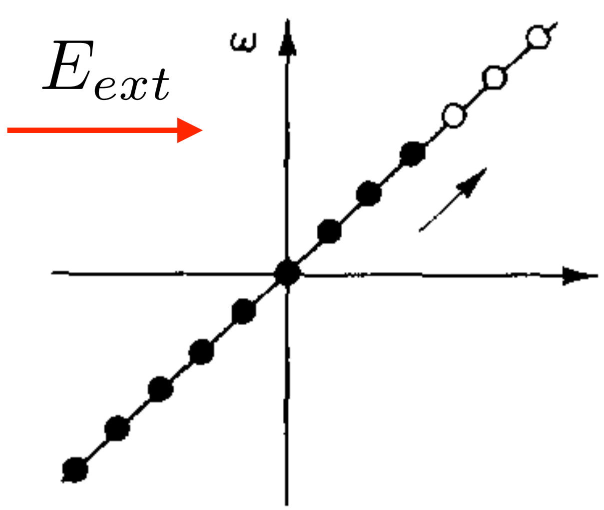
THE ADLER–BELL–JACKIW ANOMALY AND WEYL FERMIONS IN A CRYSTAL

H.B. NIELSEN
Niels Bohr Institute and Nordita, 17 Blegdamsvej, DK2100, Copenhagen Ø, Denmark

and

Masao NINOMIYA [1]
Department of Physics, Brown University, Providence, RI 02912, USA

Example in 1D:



$$\dot{N}_R = \text{ Rate change of Fermi Surface } = \frac{1}{2\pi}\dot{k}$$

$$\text{and} \quad \hbar\dot{k} = eE_{ext} \quad \Longrightarrow \quad \dot{N}_R = \frac{eE_{ext}}{2\pi}$$

$$\Longrightarrow \quad \dot{N}_R + \dot{\bar{N}}_L = \frac{eE_{ext}}{\pi}$$

Chiral Charge Pumping in 1D WSM

Electrical Chiral Anomaly in Weyl Semimetals

$$H = k^{(a)} v\, \mathbf{p} \cdot \boldsymbol{\sigma} \qquad k^{(a)} = \pm 1$$

Landau Levels in 3D WSM:

$$\epsilon_n(p_z) = \begin{cases} \pm v\sqrt{2n\frac{\hbar e}{c}B + p_z^2}, & n = 1,2,\ldots, \\ k^{(a)} v p_z, & n = 0. \end{cases}$$

momentum along B
(effectively the 1D problem)

$$\dot{N}_R \propto E \times \text{degeneracy of Landau levels}$$

$$\dot{N}_R \propto \mathbf{E}.\mathbf{B}$$

$$e\partial_t n_{L/R} + \nabla \cdot \mathbf{j}_{L/R} = \mp \frac{e^3}{4\pi^2\hbar^2}\mathbf{E}\cdot\mathbf{B} \pm \frac{e}{2\tau_v}(n_R - n_L),$$

Chiral Charge Pumping
in 3D WSM Stabilized
by inter-node scattering

Quantum anomalies in Chiral fluids: Gravitational Chiral Anomaly

Gravitational: 'general covariance of the theory is destroyed'

In curved space, chiral fluids violate both chiral charge and chiral energy conservation:

$$\nabla_\mu T^{\mu\nu} = F^{\nu\mu} J_\mu - \frac{G}{16\pi^2} \nabla_\mu \left[\varepsilon^{\rho\sigma\alpha\beta} F_{\rho\sigma} R^{\nu\mu}_{\alpha\beta} \right],$$

$$\nabla_\mu J^\mu = -\frac{C}{8} \varepsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} - \frac{G}{32\pi^2} \varepsilon^{\mu\nu\rho\sigma} R^{\alpha}_{\beta\mu\nu} R^{\beta}_{\alpha\rho\sigma},$$

C: chiral/axial anomaly
G: chiral/axial gravitational anomaly

GRAVITATIONAL ANOMALIES

Luis ALVAREZ-GAUMÉ[1]
Lyman Laboratory of Physics, Harvard University, Cambridge, MA 02138, USA
Edward WITTEN[2]
Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA
Received 7 October 1983
Nuclear Physics B234 (1983) 269–330

PRL **107,** 021601 (2011)
Gravitational Anomaly and Transport Phenomena
Karl Landsteiner, Eugenio Megías, and Francisco Pena-Benitez

PHYSICAL REVIEW B **89**, 075124 (2014)
Anomalous transport of Weyl fermions in Weyl semimetals
Karl Landsteiner

Hydrodynamic theory of thermoelectric transport and negative magnetoresistance in Weyl semimetals
Andrew Lucas[a,1], Richard A. Davison[a,1], and Subir Sachdev[a,b,1]
PNAS 2016

Temperature gradient = Gravitational Potential (!)

Q: How do we calculate thermal response from Kubo formulas

A:

PHYSICAL REVIEW VOLUME 135, NUMBER 6A 1964

Theory of Thermal Transport Coefficients*

J. M. LUTTINGER

Varying EM field produces electric current + density fluctuations

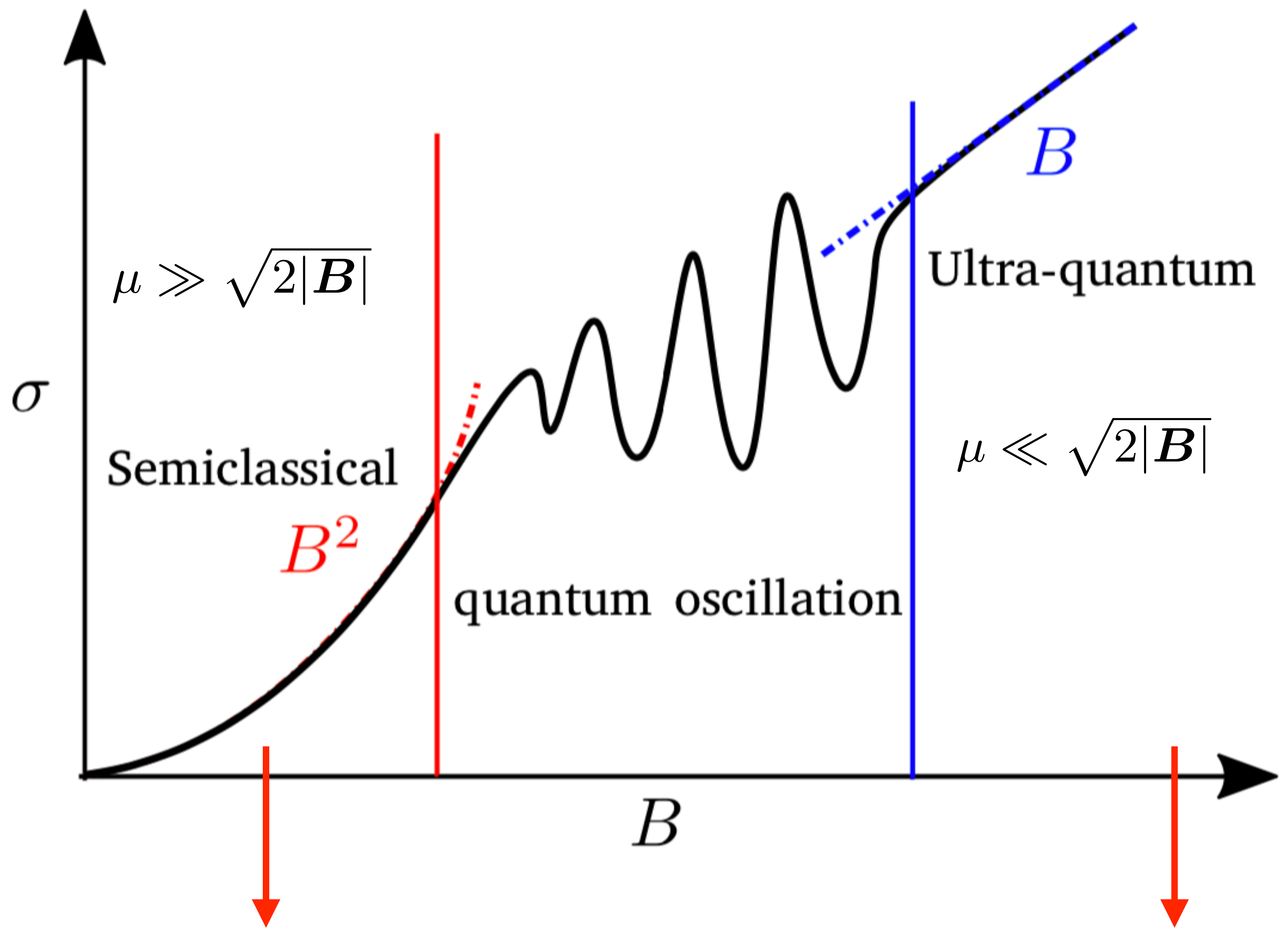
Varying gravitational field produces energy current + temperature fluctuations
(Energy density behaves as mass density, couples to gravitational potential)

$$\frac{\nabla T}{T} = \nabla \Phi$$

Φ ⟶ Gravitational potential

One route to probe gravitational anomalies is thermal transport experiments

Three regimes of Magneto-transport in WSM



Berry Curvature
(no Landau Levels)

Only lowest Landau
Level is occupied

Berry Curvature = magnetic field in momentum space

Evolution of Bloch bands leads to extra phase: (Berry Phase)

$$\gamma_n = \oint_C d\boldsymbol{q} \cdot \langle u_n(\boldsymbol{q})|i\boldsymbol{\nabla}_{\boldsymbol{q}}|u_n(\boldsymbol{q})\rangle$$

Recall Aharanov Bohm phase:

$$\delta_{\mathrm{AB}} = \oint_C d\mathbf{l} \cdot \mathbf{A}$$

Vector Potential: $\mathbf{A}$ ⟷ Berry Connection: $\langle u_n(\boldsymbol{q})|i\boldsymbol{\nabla}_{\boldsymbol{q}}|u_n(\boldsymbol{q})\rangle$

↓ ↓

Magnetic Field: ⟷ Berry Curvature:

$$\mathbf{B} = \nabla_{\mathbf{r}} \times \mathbf{A}$$

$$\boldsymbol{\Omega}_n(\boldsymbol{q}) = \boldsymbol{\nabla}_{\boldsymbol{q}} \times \langle u_n(\boldsymbol{q})|i\boldsymbol{\nabla}_{\boldsymbol{q}}|u_n(\boldsymbol{q})\rangle$$

↓

Magnetic field in momentum space!
Electrons "ko **ghuma** dega"

Interesting transport phonemena

Magnetic field in momentum space !

Finite $\Omega_n(\mathbf{k})$ can occur when either is broken

Time-reversal symmetry	Inversion symmetry
$\Omega_n(-\mathbf{k}) = -\Omega_n(\mathbf{k})$	$\Omega_n(-\mathbf{k}) = \Omega_n(\mathbf{k})$

Changes the dynamics of electrons in crystals

PHYSICAL REVIEW B VOLUME 59, NUMBER 23 15 JUNE 1999-I

ARTICLES

Wave-packet dynamics in slowly perturbed crystals: Gradient corrections and Berry-phase effects

Ganesh Sundaram and Qian Niu

$$\dot{\mathbf{r}} = \frac{1}{\hbar}\nabla_{\mathbf{k}}\epsilon_{\mathbf{k}} - \dot{\mathbf{k}} \times \mathbf{\Omega}_{\mathbf{k}}$$

→ Band velocity

$$\hbar\dot{\mathbf{k}} = -e\mathbf{E} - e\dot{\mathbf{r}} \times \mathbf{B}$$

EOM in presence of Berry Curvature

Anomalous Hall effect

Chiral magnetic effect

$$\dot{\mathbf{r}} = D_{\mathbf{k}} \left[\mathbf{v}_{\mathbf{k}} + \frac{e}{\hbar}(\mathbf{E} \times \boldsymbol{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar}(\mathbf{v}_{\mathbf{k}} \cdot \boldsymbol{\Omega}_{\mathbf{k}})\mathbf{B} \right]$$

$$\hbar\dot{\mathbf{k}} = D_{\mathbf{k}} \left[-e\mathbf{E} - e(\mathbf{v}_{\mathbf{k}} \times \mathbf{B}) - \frac{e^2}{\hbar}(\mathbf{E} \cdot \mathbf{B})\boldsymbol{\Omega}_{\mathbf{k}} \right]$$

$\hbar\mathbf{v}_{\mathbf{k}} = \nabla_{\mathbf{k}}\epsilon_{\mathbf{k}}$ and $D_{\mathbf{k}} = D(\mathbf{B}, \boldsymbol{\Omega}_{\mathbf{k}}) \equiv [1 + \frac{e}{\hbar}(\mathbf{B} \cdot \boldsymbol{\Omega}_{\mathbf{k}})]^{-1}$

Lorentz Force (Hall effect + Planar Hall effect)

Planar Hall effect

Berry Phase Effects on Electronic Properties RMP, 2010

Di Xiao

Materials Science & Technology Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, USA

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Qian Niu

Department of Physics, The University of Texas at Austin, Austin, TX 78712, USA

Origin of Quantum Anomalies in Semiclassical transport

$$\dot{\mathbf{r}} = D_{\mathbf{k}}\left[\mathbf{v}_{\mathbf{k}} + \frac{e}{\hbar}(\mathbf{E}\times\mathbf{\Omega}_{\mathbf{k}}) + \frac{e}{\hbar}(\mathbf{v}_{\mathbf{k}}\cdot\mathbf{\Omega}_{\mathbf{k}})\mathbf{B}\right]$$

$$\hbar\dot{\mathbf{k}} = D_{\mathbf{k}}\left[-e\mathbf{E} - e(\mathbf{v}_{\mathbf{k}}\times\mathbf{B}) - \frac{e^2}{\hbar}(\mathbf{E}\cdot\mathbf{B})\mathbf{\Omega}_{\mathbf{k}}\right]$$

$$\mathbf{j}^e = -e\int[d\mathbf{k}]D^{-1}\,\dot{\mathbf{r}}\,g_{\mathbf{k}}$$

$g_{\mathbf{k}} \rightarrow f(\epsilon_{\mathbf{k}}, \mu, T)$ **in equilibrium**

Do a Sommerfeld expansion $(\mu > k_B T)$

Chiral magnetic velocity

$$\mathbf{j}^s_e = e\left(\mathcal{C}^s_0\mu\mathbf{B} + T\mathcal{C}^s_1\mathbf{B}\right) \quad \text{Charge}$$

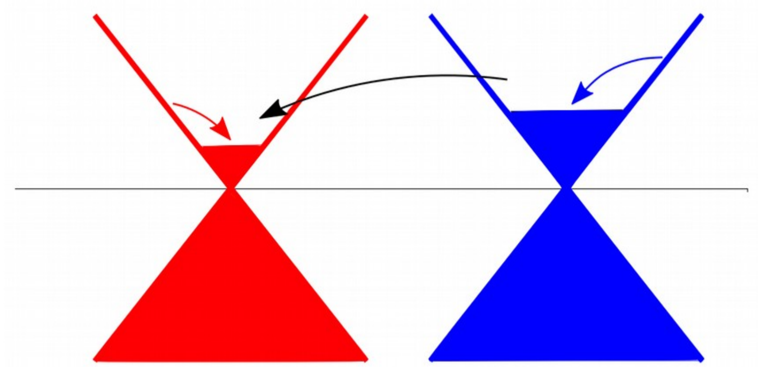
$$\mathbf{j}^s_Q = \mu^2\frac{\mathcal{C}^s_0}{2}\mathbf{B} + \mu T\mathcal{C}^s_1\mathbf{B} + T^2\mathcal{C}^s_2\mathbf{B} \quad \text{Energy}$$

$\mathcal{C}^s_0$ = Electrical CA, $\mathcal{C}^s_1$ = Thermal CA, $\mathcal{C}^s_2$ = Gravitational CA

Non-equilibrium: Boltzmann transport

$$\frac{\partial g^s_{\mathbf{r},\mathbf{k}}}{\partial t} + \dot{\mathbf{r}}^s \cdot \nabla_{\mathbf{r}}\, g^s_{\mathbf{r},\mathbf{k}} + \dot{\mathbf{k}}^s \cdot \nabla_{\mathbf{k}}\, g^s_{\mathbf{r},\mathbf{k}} = I_{\rm coll}\{g^s_{\mathbf{r},\mathbf{k}}\}$$

Local equilibrium approximation



$$I^s_{\rm coll} = -\frac{g^s_{\mathbf{r},\mathbf{k}} - f\left(\tilde{\epsilon}^s, \mu^s, T^s\right)}{\tau_0} - \frac{g^s_{\mathbf{r},\mathbf{k}} - f(\tilde{\epsilon}^s, \mu^{\bar{s}}, T^{\bar{s}})}{\tau_v}$$

— Intravalley relaxation + Intervalley relaxation

— Each node has its own chemical potential and Temperature

Generally $\tau_v \gg \tau_0$ "Chiral Limit"

Quantum Anomalies in Semiclassical transport

Integrating the Boltzmann equation over the BZ gives

$$\frac{\partial \mathcal{N}^s}{\partial t} = \mathcal{C}_0^s \ \mathbf{E}\cdot\mathbf{B} + \mathcal{C}_1^s \ \nabla T \cdot \mathbf{B} - \frac{\mathcal{N}^s - \mathcal{N}^{\bar{s}}}{\tau_v}$$

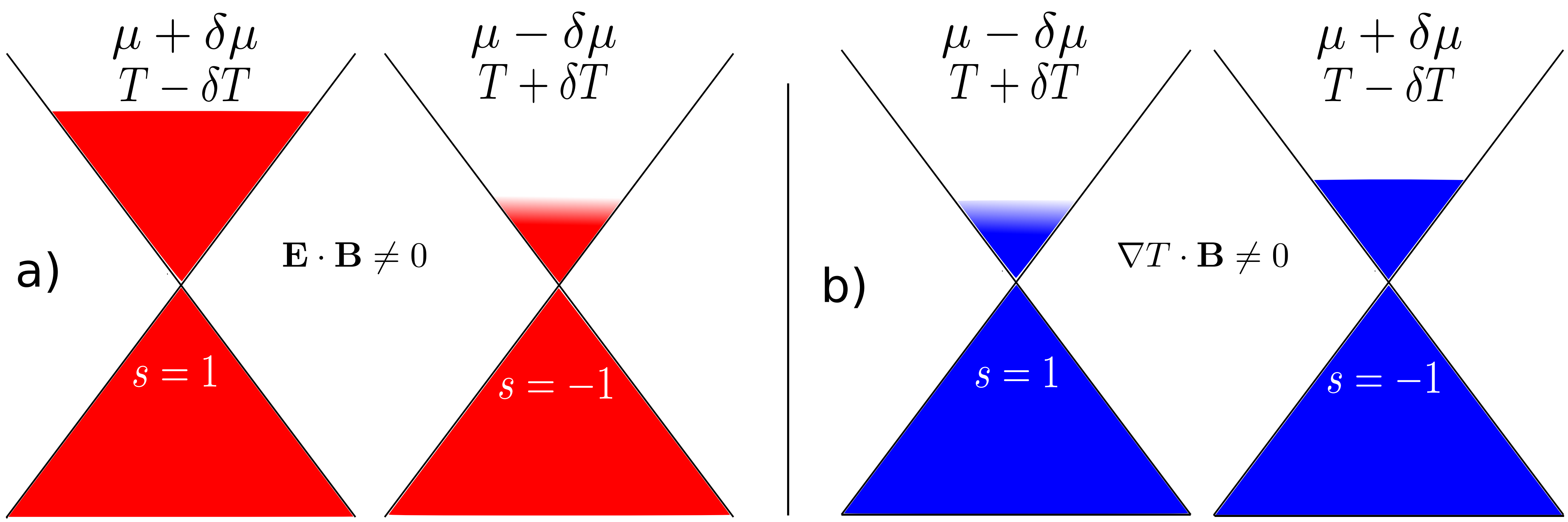
Chiral Charge pumping

$$\frac{\partial \mathcal{E}^s}{\partial t} = (eT\mathcal{C}_1^s + \mu\mathcal{C}_0^s)\ \mathbf{E}\cdot\mathbf{B} + (\mu\mathcal{C}_1^s + \mathcal{C}_2^s)\ \nabla T\cdot\mathbf{B} - \frac{\mathcal{E}^s - \mathcal{E}^{\bar{s}}}{\tau_v}$$

Chiral Energy pumping

- Chiral energy and charge pumping are stabilised by the inter-valley scattering to reach a steady state.
- This chiral transfer leads to chemical potential and temperature imbalance between the two Weyl nodes in the steady state.

Quantum Anomalies in Semiclassical transport



Manifest as chiral chemical potential imbalance and chiral temperature imbalance

Lead to a different mechanism for charge and energy transfer in WSM

This has interesting consequences for megneto-transport phenomena for WSM

Local chemical potential and local temperature

Working in the linear response regime in ∇T and $\mathbf{E}$

we have $\delta\mu^s < \mu$, and $\delta T^s < T$

$$\begin{pmatrix} \delta\mu^s \\ \delta T^s/T \end{pmatrix} = -\frac{\tau_v}{2} \begin{pmatrix} \mathcal{D}_0^s & \mathcal{D}_1^s \\ \mathcal{D}_1^s & \mathcal{D}_2^s \end{pmatrix}^{-1} \begin{pmatrix} \Lambda_0^s & \Lambda_1^s \\ \Lambda_1^s & \Lambda_2^s \end{pmatrix} \begin{pmatrix} e\mathbf{B}\cdot\mathbf{E} \\ \mathbf{B}\cdot\nabla T/T \end{pmatrix}$$

where

Generalized energy densities

$$\begin{pmatrix} \mathcal{D}_n^s \\ \Lambda_n^s \end{pmatrix} = \int \frac{d\mathbf{k}}{(2\pi)^3} (\tilde{\epsilon}^s - \mu)^n \left(-\partial_{\tilde{\epsilon}^s} f\right) \times \begin{pmatrix} 1 + e\mathbf{\Omega}^s\mathbf{B}/\hbar \\ \frac{e}{\hbar}(\tilde{\mathbf{v}}^s \cdot \mathbf{\Omega}^s) \end{pmatrix}$$

Generalized energy velocities

The Anomaly coefficients

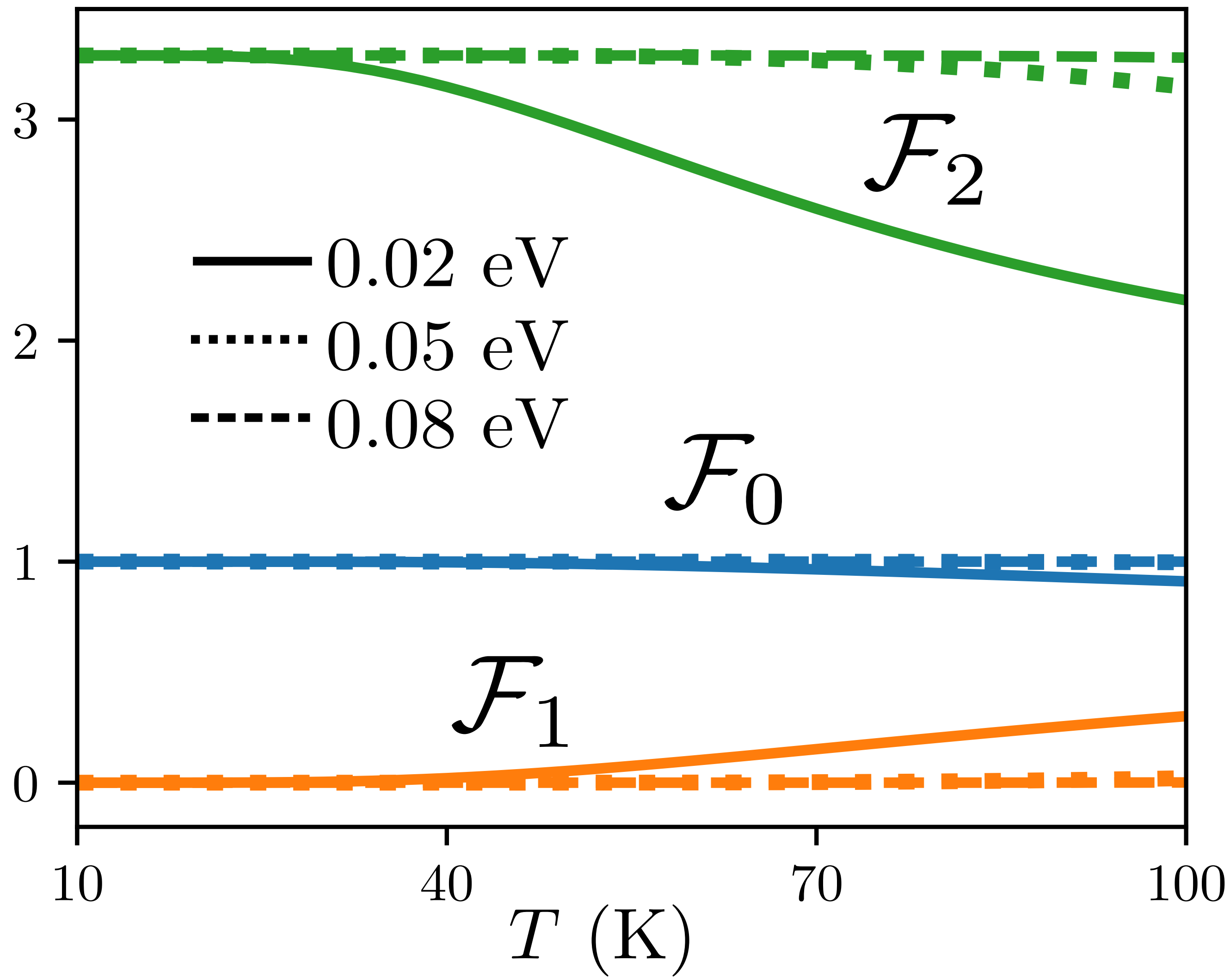
$$\{\mathcal{C}_0^s, \mathcal{C}_1^s, \mathcal{C}_2^s\} = \left\{\frac{\Lambda_0^s}{e}, \frac{\Lambda_1^s}{T}, \frac{\Lambda_2^s}{T}\right\} = -s\frac{e}{4\pi^2\hbar^2}\left\{\frac{1}{e}, 0, \frac{\pi^2}{3}k_B^2 T\right\}$$

$\mu \gg k_B T$

Different Chiral anomaly coefficients

$$\{\mathcal{C}_0^s, \mathcal{C}_1^s, \mathcal{C}_2^s\} = \left\{\frac{\Lambda_0^s}{e}, \frac{\Lambda_1^s}{T}, \frac{\Lambda_2^s}{T}\right\} = -s\frac{e}{4\pi^2\hbar^2}\left\{\frac{1}{e}, 0, \frac{\pi^2}{3}k_B^2 T\right\}$$

$\mu \gg k_B T$



$$\{\Lambda_0^s, \Lambda_1^s, \Lambda_2^s\} \approx -s\frac{e}{4\pi^2\hbar^2}\left\{\mathcal{F}_0, \frac{1}{\beta}\mathcal{F}_1, \frac{1}{\beta^2}\mathcal{F}_2\right\}$$

Local chemical potential and local temperature

$$\begin{pmatrix} \delta\mu^s \\ \delta T^s/T \end{pmatrix} = -\frac{\tau_v}{2} \begin{pmatrix} \mathcal{D}_0^s & \mathcal{D}_1^s \\ \mathcal{D}_1^s & \mathcal{D}_2^s \end{pmatrix}^{-1} \begin{pmatrix} \Lambda_0^s & \Lambda_1^s \\ \Lambda_1^s & \Lambda_2^s \end{pmatrix} \begin{pmatrix} e\mathbf{B}\cdot\mathbf{E} \\ \mathbf{B}\cdot\nabla T/T \end{pmatrix}$$

$$\begin{pmatrix} \mathcal{D}_n^s \\ \Lambda_n^s \end{pmatrix} = \int \frac{d\mathbf{k}}{(2\pi)^3} (\tilde{\epsilon}^s - \mu)^n \left(-\partial_{\tilde{\epsilon}^s} f\right) \times \begin{pmatrix} 1 + e\mathbf{\Omega}^s\mathbf{B}/\hbar \\ \frac{e}{\hbar}(\tilde{\mathbf{v}}^s\cdot\mathbf{\Omega}^s) \end{pmatrix}$$

$\mu \gg k_B T$

$$\begin{pmatrix} \delta\mu^s \\ \delta T^s/T \end{pmatrix} = s\frac{\tau_v \hbar^2 v_F^3}{\mu^2}\frac{1}{2\hbar} \begin{pmatrix} \frac{e^2}{2} & -\frac{e k_B}{\beta\mu}\frac{\pi^2}{3} \\ -\frac{e^2}{\mu} & \frac{e}{T} \end{pmatrix} \begin{pmatrix} \mathbf{E}\cdot\mathbf{B} \\ \nabla T\cdot\mathbf{B} \end{pmatrix}$$

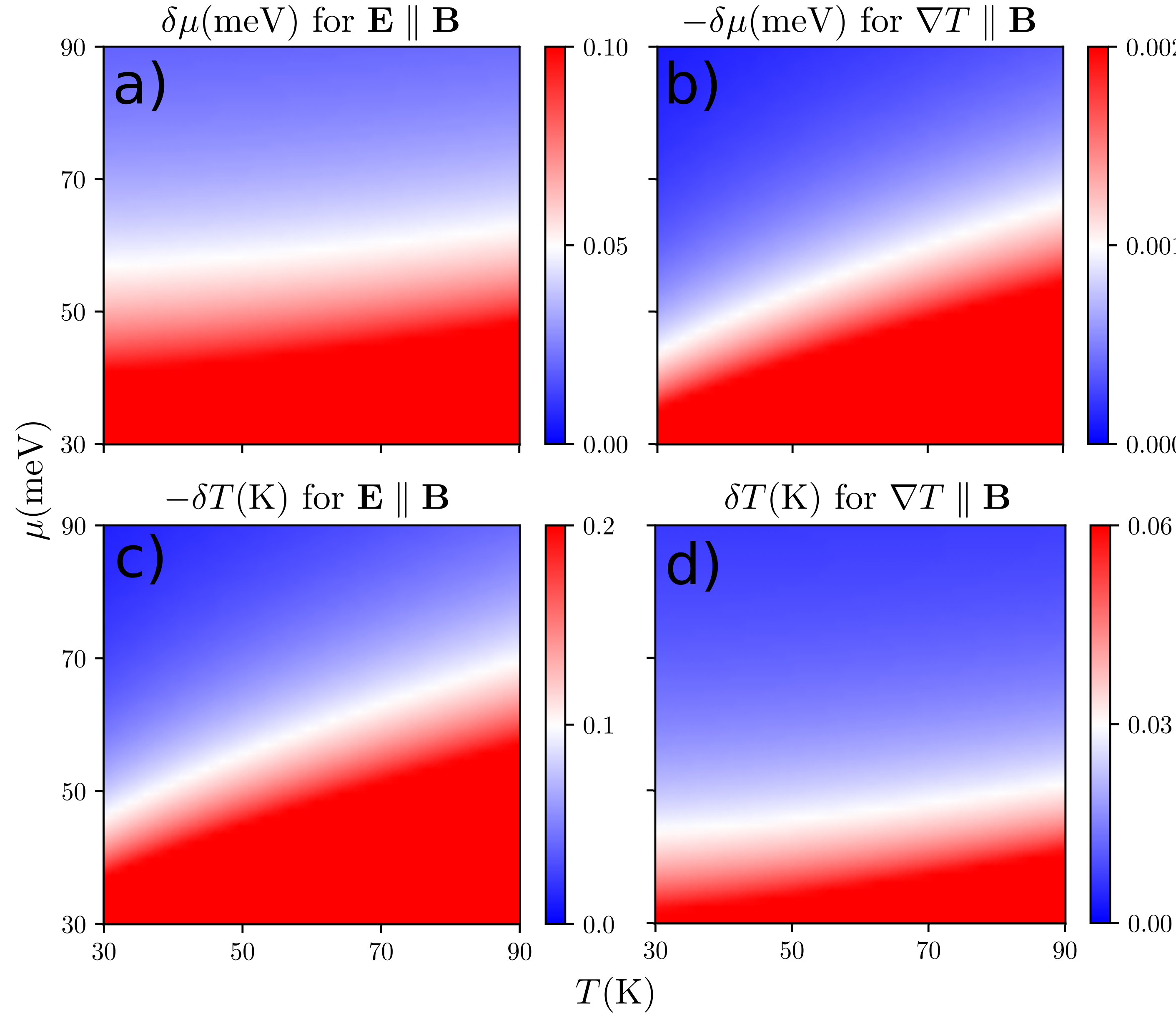


FIG. 2. The temperature and chemical potential dependence of the chiral chemical potential $\delta\mu^s$ and chiral temperature $\delta\mu^s$ for $s = 1$ node. Panel (a) and (b) shows the electric field and temperature induced $\delta\mu$ respectively. Surprisingly they contribute in opposite fashion. Panel (c) and (d) shows electric field and temperature induced δT respectively. Here we have chosen $v_F = 2\times 10^5$ m/s, $\tau_v = 10^{-9}$ s, $B = 6$ T, sample length $l = 50$ μm, $|\mathbf{E}| = 1$ mV/l, and $|\nabla T| = 350$ mK/l.

The anomaly induced transport coefficients in WSM

$$\begin{pmatrix} \mathbf{J}^s_e \\ \mathbf{J}^s_Q \end{pmatrix} = \frac{\mathbf{B}\tau_v}{2} \begin{pmatrix} e\Lambda^s_0 & e\Lambda^s_1 \\ -\Lambda^s_1 & -\Lambda^s_2 \end{pmatrix} \mathcal{D}_s{}^{-1} \begin{pmatrix} \Lambda^s_0 & \Lambda^s_1 \\ \Lambda^s_1 & \Lambda^s_2 \end{pmatrix} \begin{pmatrix} e\mathbf{B}\cdot\mathbf{E} \\ \mathbf{B}\cdot\nabla\mathbf{T}/T \end{pmatrix}$$

Lowest order in B $\quad \mu \gg k_B T$

polar angles of B

T dependence

$$\begin{pmatrix} \sigma & \alpha \\ \bar{\alpha} & \bar{\kappa} \end{pmatrix}_{ij} = \frac{\tau_v}{2} \frac{e^2 v_F^3}{8\pi^2\hbar} \frac{B^2}{\mu^2} \begin{pmatrix} e^2 & \frac{2\pi^2}{3}\frac{ek_B}{\beta\mu} \\ \frac{2\pi^2}{3}\frac{ek_B T}{\beta\mu} & \frac{\pi^2}{3}k_B^2 T \end{pmatrix} \mathcal{L}_{ij}(\theta,\phi)$$

Angular dependence

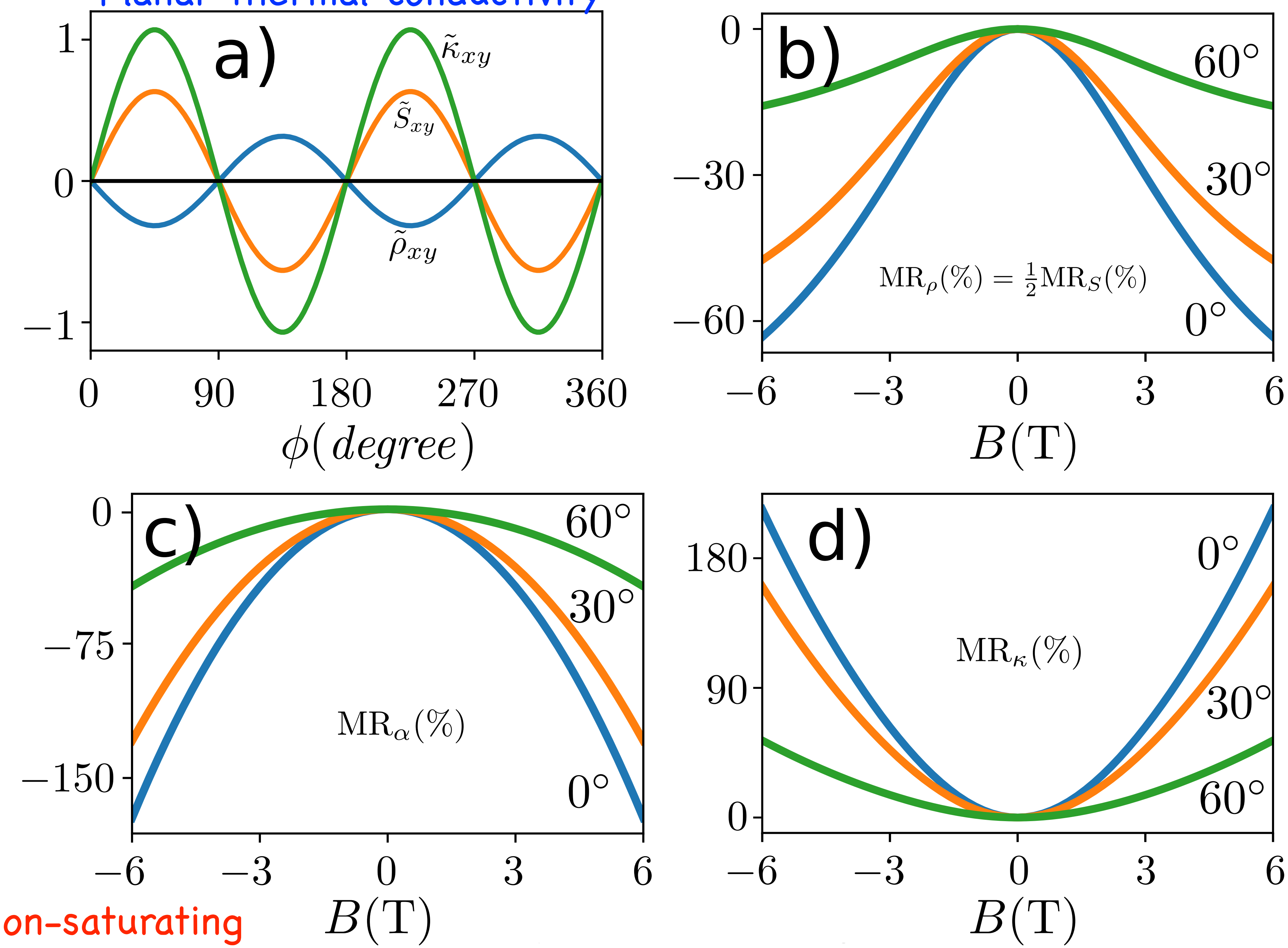
$$\mathcal{L}(\theta,\phi) \equiv \begin{pmatrix} \sin^2\theta\cos^2\phi & \frac{1}{2}\sin^2\theta\sin 2\phi & \frac{1}{2}\sin 2\theta\cos\phi \\ \frac{1}{2}\sin^2\theta\sin 2\phi & \sin^2\theta\sin^2\phi & \frac{1}{2}\sin 2\theta\sin\phi \\ \frac{1}{2}\sin 2\theta\cos\phi & \frac{1}{2}\sin 2\theta\sin\phi & \cos^2\theta \end{pmatrix}$$

The magneto-transport coefficients in WSM: B dependence

Planar Hall resistivity,
Planar Seebeck effect
Planar thermal conductivity

Negative saturating resistivity MR,
Negative saturating Seebeck MR

$\mu > 0$



Negative non-saturating
electro-thermal conductivity MR

$$\mathrm{MR}_\gamma \equiv \gamma(B)/\gamma(0) - 1$$

Negative non-saturating
thermal conductivity MR

LETTER

doi:10.1038/nature23005

Experimental signatures of the mixed axial–gravitational anomaly in the Weyl semimetal NbP

Johannes Gooth[1,2], Anna C. Niemann[1,3], Tobias Meng[4], Adolfo G. Grushin[5], Karl Landsteiner[6], Bernd Gotsmann[2], Fabian Menges[2], Marcus Schmidt[7], Chandra Shekhar[7], Vicky Süß[7], Ruben Hühne[3], Bernd Rellinghaus[3], Claudia Felser[7], Binghai Yan[7,8] & Kornelius Nielsch[1,3]

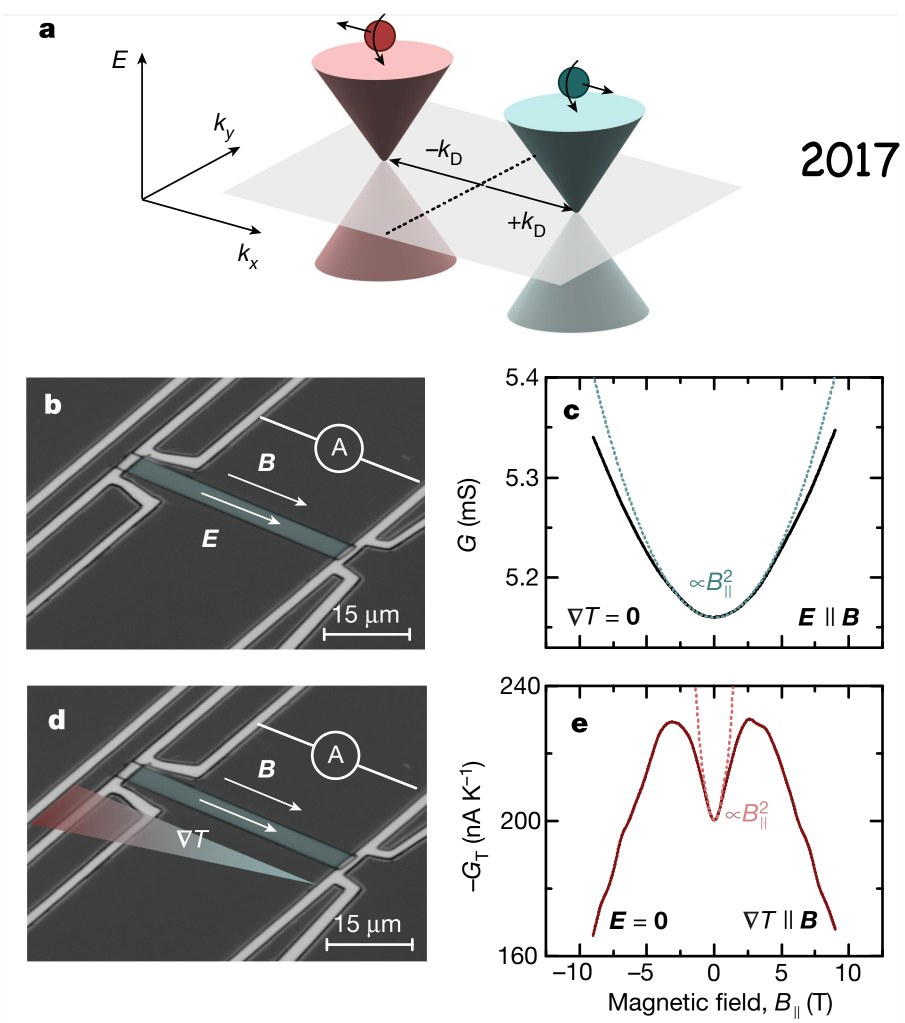
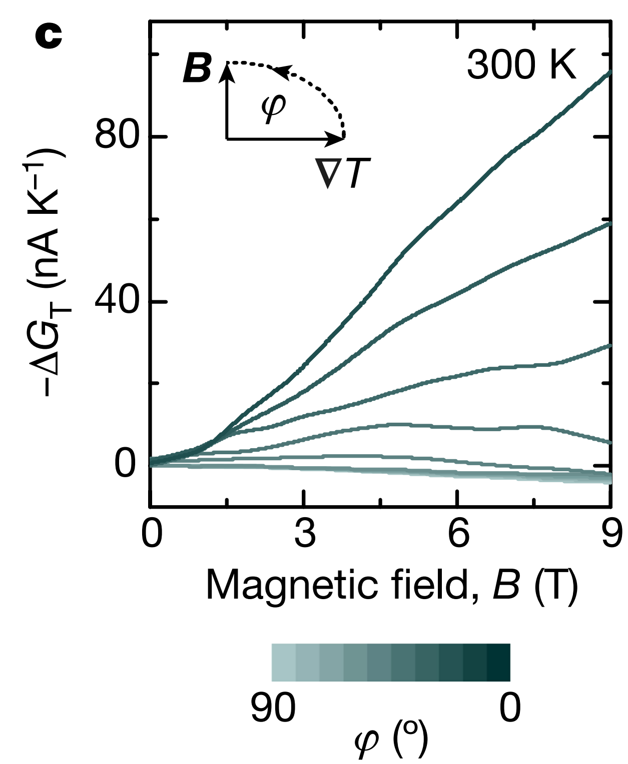


Figure 1 | Positive magneto-conductance $G(B_{\parallel})$ and magneto-thermoelectric conductance $G_T(B_{\parallel})$ in the Weyl semimetal NbP.

Observe positive magneto-thermal conductance for $\mathbf{B} \parallel \nabla \mathbf{T}$

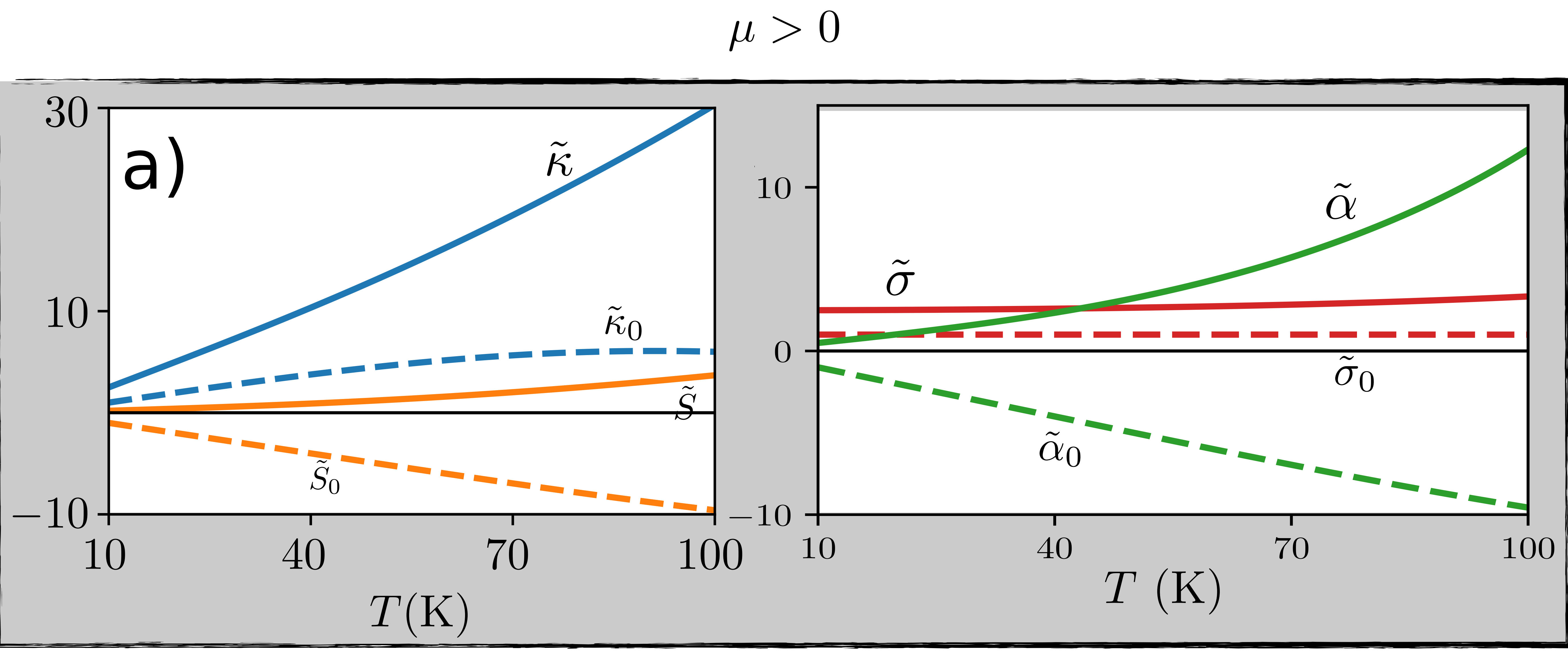


arXiv:1810.02300

Observation of an anomalous heat current in a Weyl fermion semimetal

Clemens Schindler,[1] Satya N. Guin,[1] Walter Schnelle,[1] Nitesh Kumar,[1] Chenguang Fu,[1] Horst Borrmann,[1] Chandra Shekhar,[1] Yang Zhang,[1] Yan Sun,[1] Claudia Felser,[1] Tobias Meng,[2] Adolfo G. Grushin,[3] and Johannes Gooth[1,*]

The magneto-transport coefficients in WSM



Temp. Dependence of longitudinal component of transport coefficients

- **Huge enhancement of thermal conductivity (6x for 100 K)**
- **Seebeck coefficient reverses sign with B!**
- **Huge enhancement in thermo-electric conductivity + sign change with B!**
- **All these are a manifestation of chiral gravitational or mixed anomaly in WSM**

Thermal chiral anomaly in the magnetic-field induced ideal Weyl phase of $Bi_{89}Sb_{11}$

Dung Vu (1), Wenjuan Zhang (2), Cüneyt Şahin (3,4), Michael Flatté (3,4), Nandini Trivedi (2), Joseph P. Heremans (1,2,5)

arXiv:1906.02248

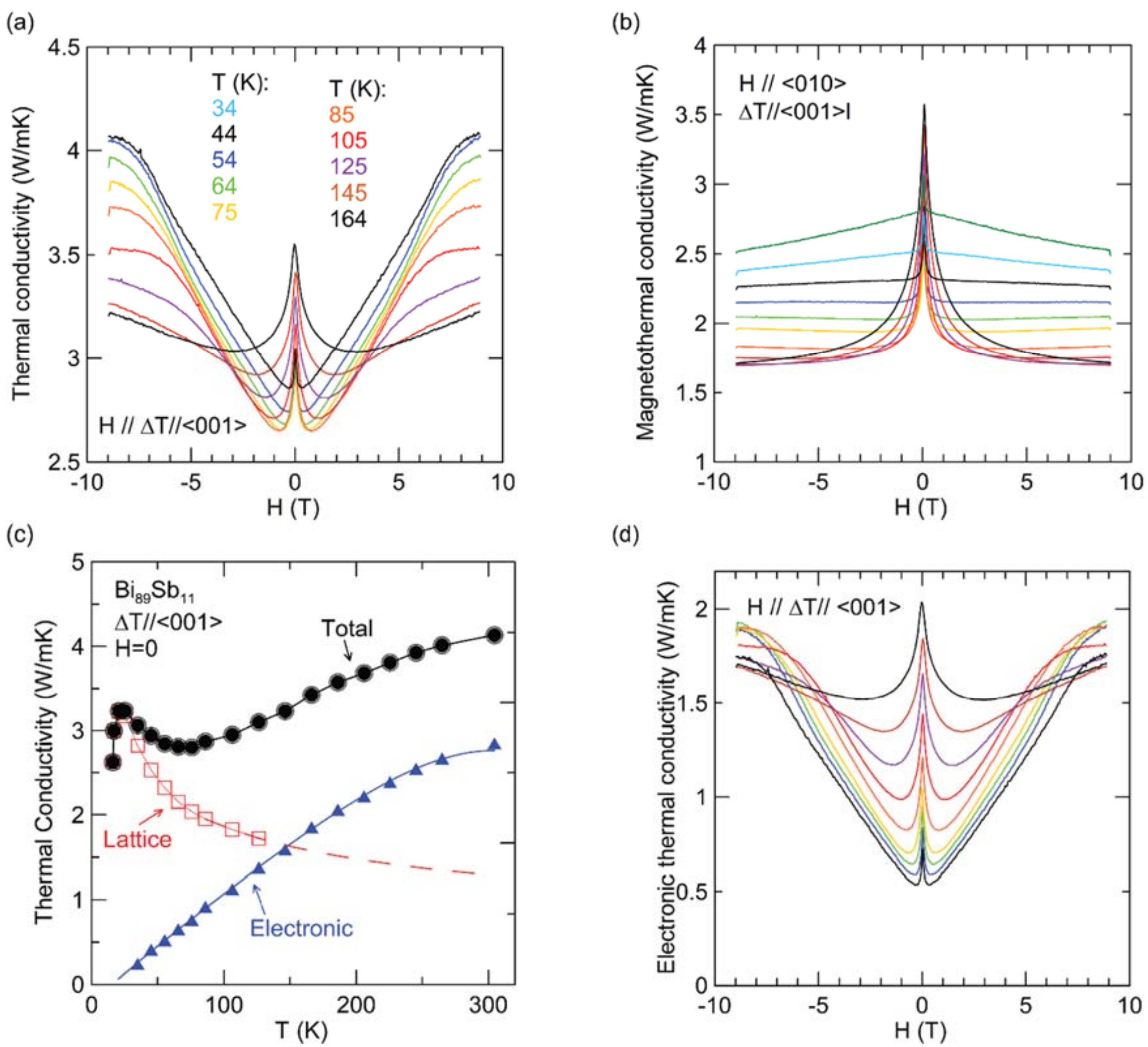


Figure 3: Thermal conductivity κ_{zz} of $Bi_{89}Sb_{11}$ along the trigonal (z=<001>) direction.

The signature of thermal Chiral anomaly is large enhancement of the thermal conductivity for $\mathbf{B} \parallel \nabla \mathbf{T}$

Magneto-transport in the Landau quantization regime

$$\hat{H}^s = s v_F \boldsymbol{\sigma} \cdot (\hat{\mathbf{p}} + e\mathbf{A})$$

$$\mathbf{A} = (-By, 0, 0)$$

$$\epsilon_n^s = \begin{cases} -s\hbar v_F k_z & n = 0 \\ \pm\sqrt{(\hbar v_F k_z)^2 + 2n(\hbar\omega_c)^2} & n \geq 1 \end{cases}$$

$$v_{nz}^s = \frac{\partial \epsilon_n^s}{\hbar \partial k_z} = \begin{cases} -s v_F & n = 0 \\ \hbar v_F^2 k_z / \epsilon_n^s & n \geq 1 \end{cases}$$

Equilibrium Currents

$$j_{e,\mathrm{eq}}^s = -e \sum_n \mathfrak{D} \int \frac{dk_z}{2\pi} v_{nz}^s f_n^s \qquad \mathfrak{D} = 1/2\pi l_B^2$$

$$j_s^e = -e\left(\mu \mathcal{C}_0^s + k_B T \mathcal{C}_1^0\right) B \ ,$$

$$j_s^{\mathcal{E}} = \left(\mu^2 \frac{\mathcal{C}_0^s}{2} + \mu k_B T \mathcal{C}_1^s + k_B^2 T^2 \mathcal{C}_2^s\right) B$$

$$\mathcal{C}_\nu^s = \frac{e}{2\pi\hbar} \sum_n \int \frac{dk_z}{2\pi} v_{nz}^s \left(\frac{\epsilon_n^s - \mu}{k_B T}\right)^\nu \left(-\frac{\partial f_n^s}{\partial \epsilon_n^s}\right)$$

Magneto-transport in the Landau quantization regime - II

Local equilibrium + relaxation time approximation

$$\partial_t g_n^s + \dot{\mathbf{k}}_n^s \cdot \boldsymbol{\nabla}_{\mathbf{k}} g_n^s + \dot{\mathbf{r}}_n^s \cdot \boldsymbol{\nabla}_{\mathbf{r}} g_n^s = -\frac{g_n^s - \bar{g}_n^s}{\tau} - \frac{\bar{g}_n^s - f_n^s}{\tau_v}$$

Particle density:

$\mathcal{N}^s = \mathfrak{D} \sum_n \int \frac{dk_z}{2\pi} g_n^s$

$$\frac{\partial \mathcal{N}^s}{\partial t} + \boldsymbol{\nabla}_{\mathbf{r}} \cdot \mathbf{J}_n^s + eEB\mathcal{C}_0^s = -\frac{\mathcal{N}^s - \mathcal{N}_0^s}{\tau_v}$$

$$\boldsymbol{\nabla}_{\mathbf{r}} \cdot \mathbf{J}_n^s = k_B B \nabla T \mathcal{C}_1^s$$

Heat density:

$\mathcal{Q}^s = \mathfrak{D} \sum_n \int \frac{dk_z}{2\pi} \tilde{\epsilon}_n^s g_n^s$

$$\frac{\partial \mathcal{Q}^s}{\partial t} + \boldsymbol{\nabla}_{\mathbf{r}} \cdot \mathbf{J}_Q^s + eEB\mathcal{C}_1^s k_B T = -\frac{\mathcal{Q}^s - \mathcal{Q}_0^s}{\tau_v}$$

$$\boldsymbol{\nabla}_{\mathbf{r}} \cdot \mathbf{J}_Q^s = k_B^2 T B \nabla T \mathcal{C}_2^s$$

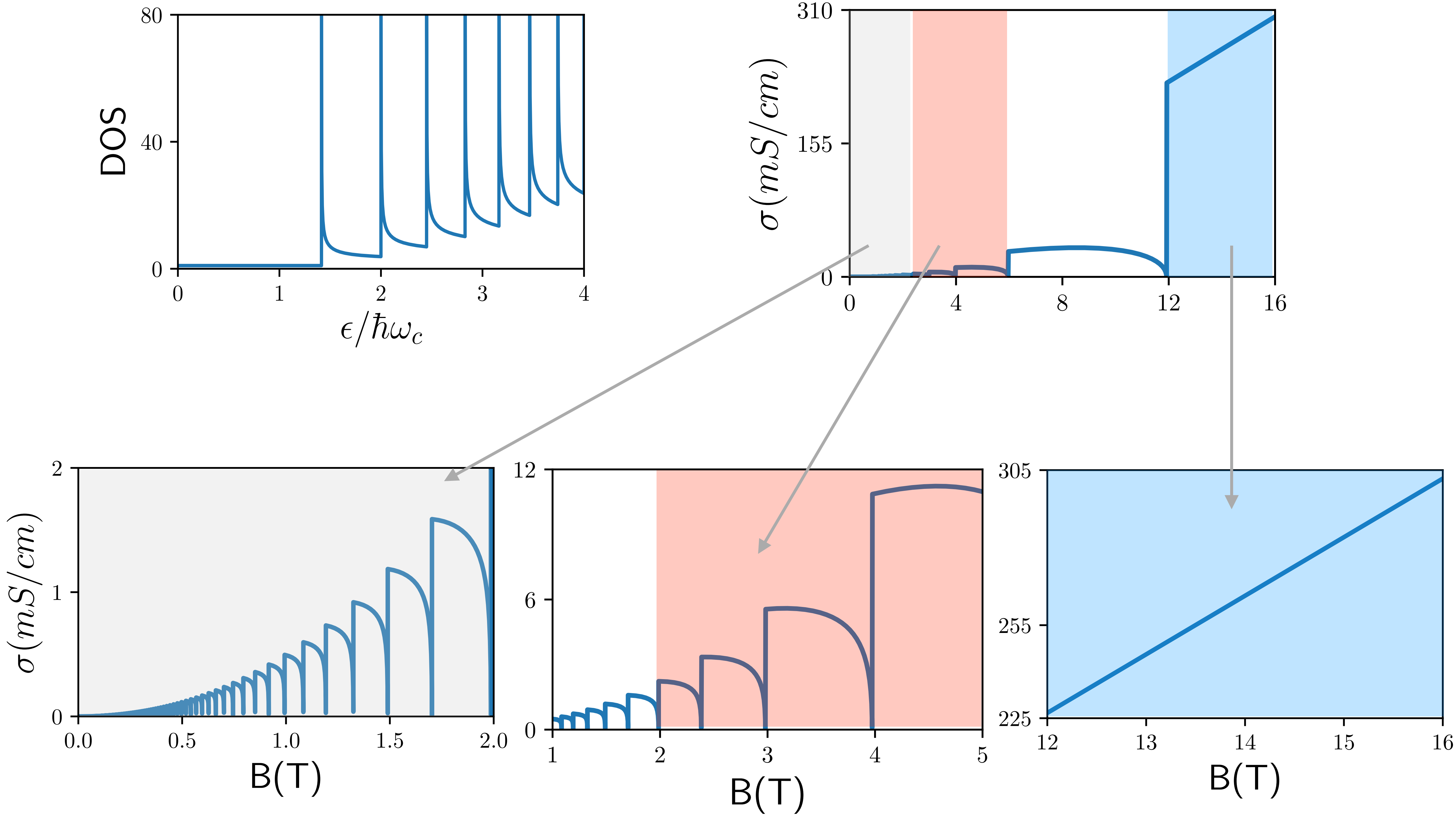
Longitudinal Magneto-transport

$$j_e^s = -e \sum_n \mathfrak{D} \int \frac{dk_z}{2\pi} v_{nz}^s g_n^s \qquad j_Q^s = \sum_n \mathfrak{D} \int \frac{dk_z}{2\pi} \tilde{\epsilon}_n^s v_{nz}^s g_n^s$$

$$\begin{pmatrix} j_e^s \\ j_Q^s \end{pmatrix} = \tau_v B^2 \begin{pmatrix} -e\mathcal{C}_0^s & -e\mathcal{C}_1^s \\ k_B T \mathcal{C}_1^s & k_B T \mathcal{C}_2^s \end{pmatrix} [\mathcal{D}^s]^{-1} \begin{pmatrix} \mathcal{C}_0^s & \mathcal{C}_1^s \\ \mathcal{C}_1^s & \mathcal{C}_2^s \end{pmatrix} \begin{pmatrix} eE \\ k_B \nabla T \end{pmatrix}$$

$$\mathcal{D}^s = \begin{pmatrix} \mathcal{D}_0^s & \mathcal{D}_1^s \\ \mathcal{D}_1^s & \mathcal{D}_2^s \end{pmatrix} \qquad \mathcal{D}_\nu^s = \sum_n \mathfrak{D} \int \frac{dk_z}{2\pi} \left(\frac{\epsilon_n^s - \mu}{k_B T} \right)^\nu \left(-\frac{\partial f_n^s}{\partial \epsilon_n^s} \right)$$

Longitudinal Magneto-transport



Quantum anomaly + magneto-transport in Weyl semimetals

- Explored chiral electrical- thermal- and gravitational- anomaly in WSM
- Origin: chiral magnetic velocity in the semiclassical picture or chiral Landau levels (lowest) in the Landau quantization regime.
- Chiral charge and energy pumping leads to chemical potential and temperature imbalance in the Weyl nodes.
- This leads to distinct anomaly induced magneto-transport features (negative MR, huge enhancement of thermal conductivity, sign reversal of S, etc.)

arXiv:1909.07711

Thermal and gravitational chiral anomaly induced magneto-transport in Weyl semimetals

Kamal Das[1,*] and Amit Agarwal[1,†]

Longitudinal magneto-transport in Weyl semimetals due to chiral anomalies in quantizing magnetic field

Kamal Das,[1,*] Sahil Kumar Singh,[1,†] and Amit Agarwal[1,‡]

Magneto-transport induced by chiral anomalies in WSM

The answer motivates other questions:

- Is there similar physics if the Weyl nodes are away from "local equilibrium" ? What role does B dependence of tau_v play ?

- Is there similar/different physics near the charge neutrality point in WSM ?

- How will these quantum anomalies coefficients manifest in optical experiments: photoconductivity, polarization rotation, pump-probe spectroscopy etc. Other unambiguous experimental signatures ?