Berry phase of composite fermion Fermi liquid

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VILLUM FONDEN

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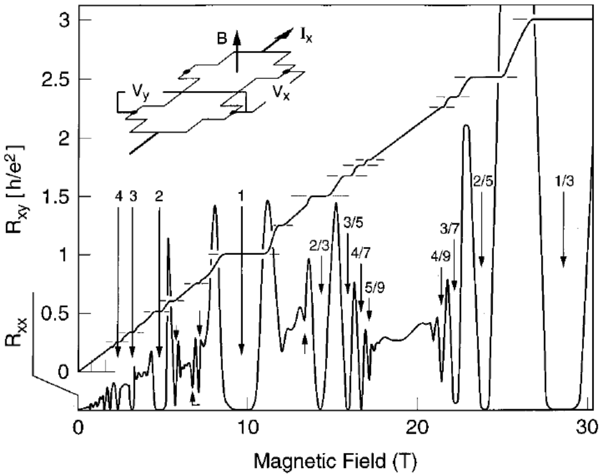
Plan of the talk

- Lowest Landau level FQHE: composite fermion theory
- Puzzle of the Fermi wave vector of composite fermions
- Berry phase of composite fermions

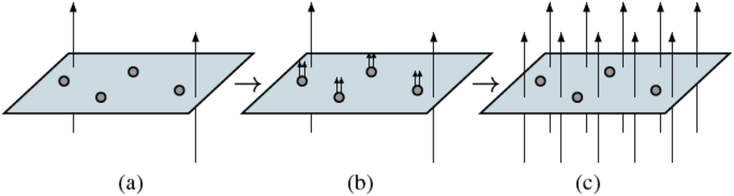
FQHE in the LLL predominantly occurs at $\nu = n/(2pn \pm 1)$



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

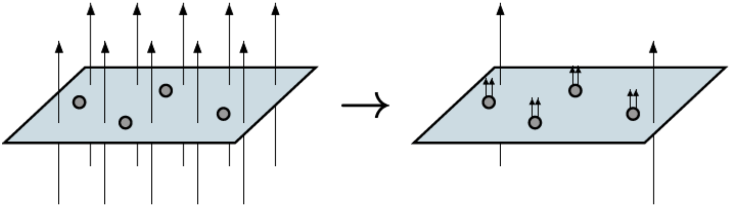
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

Composite fermions experience a reduced magnetic field

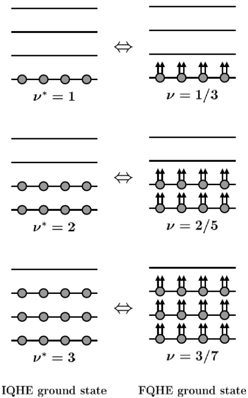


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 = h/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions are analogous to IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\mathrm{CF}}_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\mathrm{LLL}}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

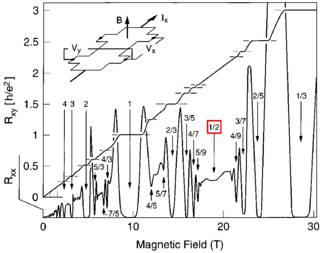
Φ_n wave function of n filled LLs.

$\mathcal{P}_{\mathrm{LLL}}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Mystery of the $\nu = 1/2$ state



- composite fermions absorb all of the magnetic flux: $B^* = 0$

  Halperin, Lee and Read, Phys. Rev. B **47**, 7312 (1993)

- In zero effective magnetic field CFs form a Fermi sea

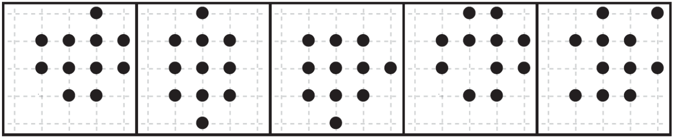
Clustered CFs have good overlap with exact eigenstates

$$\Psi^{\text{CFFS}}_{\nu=\frac{1}{2}} = \mathcal{P}_{\text{LLL}}\left(\text{Det}[e^{i\vec{k}\cdot\vec{r}}]\prod_{i<j}(z_i - z_j)^2\right)$$

Cluster composite fermion momenta into a Fermi-sea like configuration to minimize the sum of single-particle CF energies

M. Fremling *et al.*, Phys. Rev. B **97**, 035149 (2019)

$\mathbf{N = 11}$



$\left\|\left\langle \Psi^{\text{CFFS}}_{\nu=\frac{1}{2}} \middle\| \Psi^{\text{LLL}}_{\nu=\frac{1}{2}} \right\rangle\right\|$					
overlap	0.981955	0.736717	0.992142	0.226429	0.276141

Unclustered composite fermion states do not provide an accurate representation of the exact LLL Coulomb ground state.

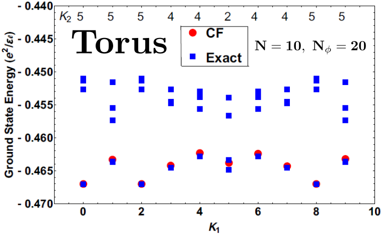
J. Wang *et al.*, Phys. Rev. B **99**, 125123 (2019)

CFFS wave function at $\nu = 1/2$ is very accurate

$$\Psi^{\rm CFFS}_{\nu=\frac{1}{2}} = \mathcal{P}_{\rm LLL}\left(\mathrm{Det}[e^{i\vec{k}\cdot\vec{r}}]\prod_{i<j}(z_i - z_j)^2\right).$$

$$\left|\left\langle \Psi^{\rm CFFS}_{\nu=\frac{1}{2}} \middle| \Psi^{\rm LLL\ \ Coulomb}_{\nu=\frac{1}{2}} \right\rangle\right| = 0.9925, \quad N = 16$$

E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000)

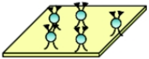


S. Pu, M. Fremling and J. K. Jain, Phys. Rev. B **98**, 075304 (2018)

CF Fermi wave vector can be experimentally measured

Courtesy: Mansour Shayegan

Composite fermion commensurability oscillations



$B^* = B - B_{1/2}$

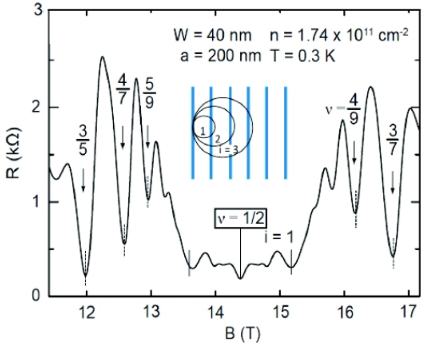
$\frac{2R_C^*}{a} = i + \frac{1}{4}$

$2R_C^* = \frac{2\hbar k_F^*}{eB^*}$

$B_i^* = \frac{\hbar\sqrt{4\pi n^*}}{ea\left(i+\frac{1}{4}\right)}$

$$\mathrm{k}_\mathrm{F}^* = \sqrt{4\pi\rho}$$

$$\mathrm{k}_\mathrm{F}^*\ell = \sqrt{2\nu}$$

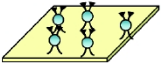


Kamburov, Phys. Rev. Lett. 113, 196801 (2014)

CF density is same as electron density for $\nu < 1/2$

Courtesy: Mansour Shayegan

Density of composite fermions?



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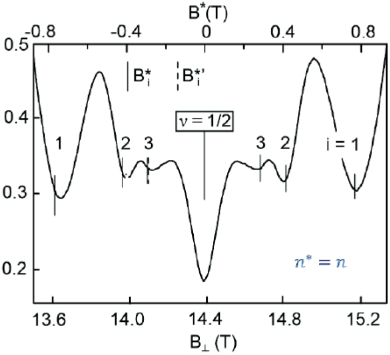
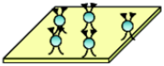


CF density is the hole density for $\nu > 1/2$

Courtesy: Mansour Shayegan

Density of composite fermions?



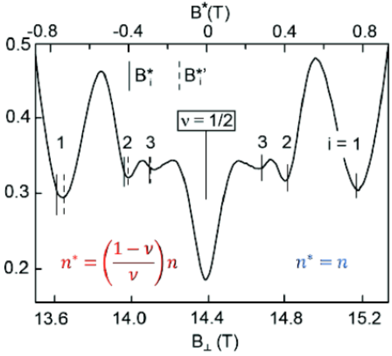
$B^* = B - B_{1/2}$

$\frac{2R_C^*}{a} = i + \frac{1}{4}$

$2R_C^* = \frac{2\hbar k_F^*}{eB^*}$

$B_i^* = \frac{\hbar\sqrt{4\pi n^*}}{ea\left(i + \frac{1}{4}\right)}$

$B_i^{*\prime} \leftrightarrow$ minority density



CF density is equal to the density of minority carriers

$$k_F^* \ell = \begin{cases} \sqrt{2\nu}, & \nu < 1/2 \text{ (electron-CFs)} \\ \sqrt{2(1-\nu)}, & \nu > 1/2 \text{ (hole-CFs)} \end{cases}$$

D. Kamburov *et al.*, Phys. Rev. Lett. **113**, 196801 (2014)

electron-CFs and hole-CFs lead to same description

$$\Psi^{\rm electron-CFs}_{\nu=\frac{n+1}{2n+1}} = \mathcal{P}_{\rm LLL}\left(\prod_{i<j}(z_i-z_j)^2[\Phi_{n+1}]^*\right)$$

$$\Psi^{\rm hole-CFs}_{\nu=\frac{n+1}{2n+1}} = \mathcal{P}_{\rm p\text{-}h}\left(\mathcal{P}_{\rm LLL}\left(\prod_{i<j}(z_i-z_j)^2\Phi_n\right)\right)$$

Explicit construction shows that these two wave functions have high overlaps with each other and represent the same state

X. G. Wu *et al.*, Phys. Rev. Lett. **71**, 153 (1993)

CFFS wave function is highly particle-hole symmetric

$$\Psi^{\mathrm{CFFS}(1)}_{\nu=\frac{1}{2}} = \mathcal{P}_{\mathsf{LLL}}\left(\mathrm{Det}[e^{i\vec{k}\cdot\vec{r}}]\prod_{i<j}(z_i - z_j)^2\right)$$

$$\Psi^{\mathrm{CFFS}(2)}_{\nu=\frac{1}{2}} = \mathcal{P}_{\mathsf{p\text{-}h}}\Psi^{\mathrm{CFFS}(1)}_{\nu=\frac{1}{2}}$$

The microscopic wave function for the composite fermion Fermi sea has an almost perfect overlap with its hole conjugate:

$$\left|\left\langle \Psi^{\mathrm{CFFS}(1)}_{\nu=\frac{1}{2}} \middle| \Psi^{\mathrm{CFFS}(2)}_{\nu=\frac{1}{2}} \right\rangle\right| = 0.9994, \quad N = 16$$

a byproduct of the fact that the wave function is a very good approximation of the Coulomb state which is exactly p-h symmetric

E. H. Rezayi and F. D. M. Haldane, Phys. Rev. Lett. **84**, 4685 (2000)

CF Fermi wave vector from the pair-correlation function

- Use the microscopic wave function to evaluate pair-correlation
- Extract Fermi wave vector from the pair-correlation $g(r)$

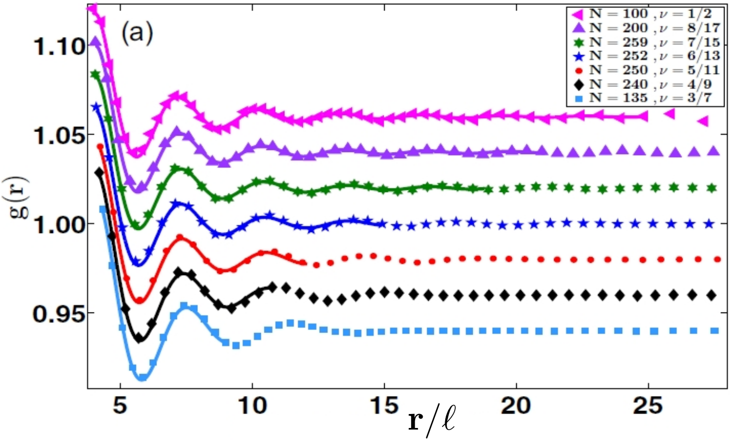
$$g(r) = 1 + A(r\sqrt{4\pi\rho_e})^{-\alpha}\sin(2k_F^* r + \theta)$$

(motivated from Fermi sea of electrons)

- Fermi wave vector, in units of ℓ^{-1}, are identical for two states related by particle-hole symmetry
- Evaluate $k_F^*\ell$ for states at $\nu = n/(2n \pm 1)$ and at $\nu = 1/2$.

Ajit C. Balram, C. Tőke, and J. K. Jain, Phys. Rev. Lett. **115**, 186805 (2015)

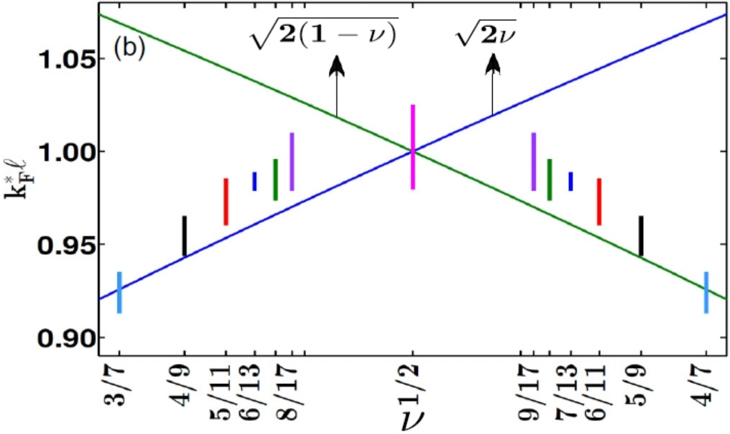
Fermi-sea like correlations in a partially filled LL



Ajit C. Balram, C. Tőke, and J. K. Jain, Phys. Rev. Lett. **115**, 186805 (2015)

Ajit C. Balram and J. K. Jain, Phys. Rev. B **96**, 235102 (2017)

CF Fermi wave vector consistent with minority carriers



Luttinger's theorem holds for composite fermions

Ajit C. Balram and J. K. Jain, Phys. Rev. B **96**, 235102 (2017)

Chern-Simons field theory is not particle-hole symmetric

The Halperin, Lee and Read (HLR) Chern-Simons theory is not projected into the LLL and therefore is not particle-hole symmetric.

$$\Psi^{\mathrm{CS-CF}}_{\nu=\frac{n}{2n+1}} = \prod_{i<j}\left(\frac{z_i - z_j}{|z_i - z_j|}\right)^2 \Phi_n$$

This wave function does not live in the Hilbert space of LLs

In the theoretical limit of infinite cyclotron energy, the HLR-CS theory cannot be the right field theory of FQHE.

B. I. Halperin, P. A. Lee, and N. Read, Phys. Rev. B **47**, 7312 (1993)

Is the composite fermion a Dirac particle?

Son's theory:

Proposal: Composite fermions are two-component Dirac fermions. Field theory based on this proposal is manifestly particle-hole symmetric.

Consequence: Composite fermions acquire a Berry phase of π when moved around the Fermi surface.

Test using microscopics: evaluate Berry phase from wave function

D. T. Son, Phys. Rev. X **5**, 031027 (2015)

Overlap as a matrix element of density operator

The overlap integral between two successive points along the Fermi circle vanishes due to momentum conservation. Circumvent this by inserting an operator that makes the overlap non-vanishing.

Bloch wave function $\Psi_{\vec{k}}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{\vec{k}}(\vec{r})$

Overlap: $\langle u_{\vec{k}_1}|u_{\vec{k}_2}\rangle = \langle\Psi_{\vec{k}_1}|\rho(\vec{k}_1 - \vec{k}_2)|\Psi_{\vec{k}_2}\rangle$

$\rho(\vec{k})$ is the Fourier-transformed density operator $e^{i\vec{k}\cdot\vec{r}}$

S. Geraedts *et al.*, Phys. Rev. Lett. **121**, 147202 (2018)

J. Wang *et al.*, Phys. Rev. B **99**, 125123 (2019)

Berry phase of a composite fermion

Assuming Ψ is a Slater determinant of N composite fermions

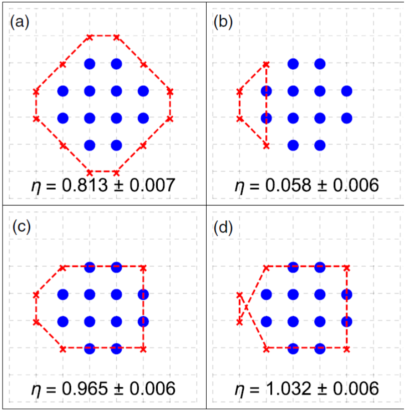
Berry phase factor $e^{i\tilde{\Phi}_{\Gamma}} = \prod_{\text{path}} \left\langle \Psi\left(\{\vec{k}_i'\}\right) \middle| \hat{\rho}(\vec{q}) \middle| \Psi\left(\{\vec{k}_i\}\right) \right\rangle$

$$\rho(\vec{q}) = \sum_{i=1}^{N} e^{i\vec{q}\cdot\vec{r}}, \quad \vec{k}_i' = \vec{k}_i + \vec{q}\delta_{i,N}$$

$\hat{\rho}$ is the LLL projected density operator
Γ is closed path in momentum space taken by k_N, the momentum of the CF which traverses the path, while others remain fixed

S. Geraedts *et al.*, Phys. Rev. Lett. **121**, 147202 (2018)

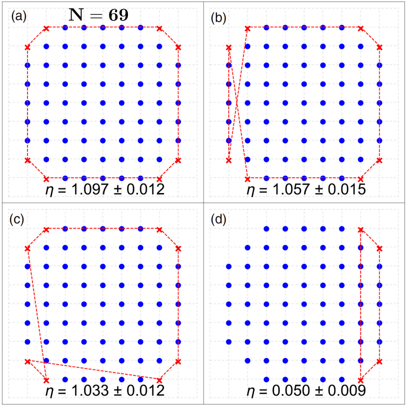
Berry phase for small system sizes $\approx \pi$



Berry phase factor $e^{i\tilde{\Phi}_\Gamma} = (-1)^\eta$

J. Wang *et al.*, Phys. Rev. B **99**, 125123 (2019)

π Berry phase when CFs encircle the Fermi center



Berry phase factor $e^{i\tilde{\Phi}_\Gamma} = (-1)^\eta$

J. Wang *et al.*, Phys. Rev. B **99**, 125123 (2019)

Outlook

- Berry phase of composite fermions is consistent with Son's field theory of FQHE.
- Connection to two-component Dirac fermions remains unclear.
- Berry phase is sensitive to particle-hole symmetry breaking perturbations.

  S. Pu, M. Fremling and J. K. Jain, Phys. Rev. B **98**, 075304 (2018)

- Is the distribution of Berry curvature uniform in momentum space or is it concentrated at the Fermi center?

Thank you for your attention!

References:

- Ajit C. Balram, C. Tőke, and J. K. Jain, Phys. Rev. Lett. **115**, 186805 (2015)
- Ajit C. Balram and J. K. Jain, Phys. Rev. B **96**, 235102 (2017)
- S. Geraedts *et al.*, Phys. Rev. Lett. **121**, 147202 (2018)
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