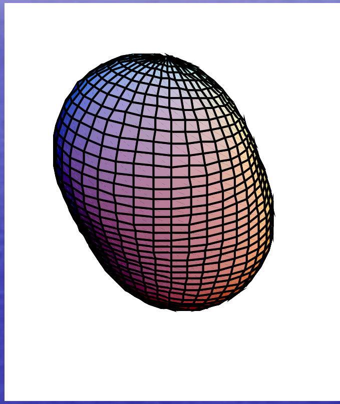
Symmetries in Nuclei - II



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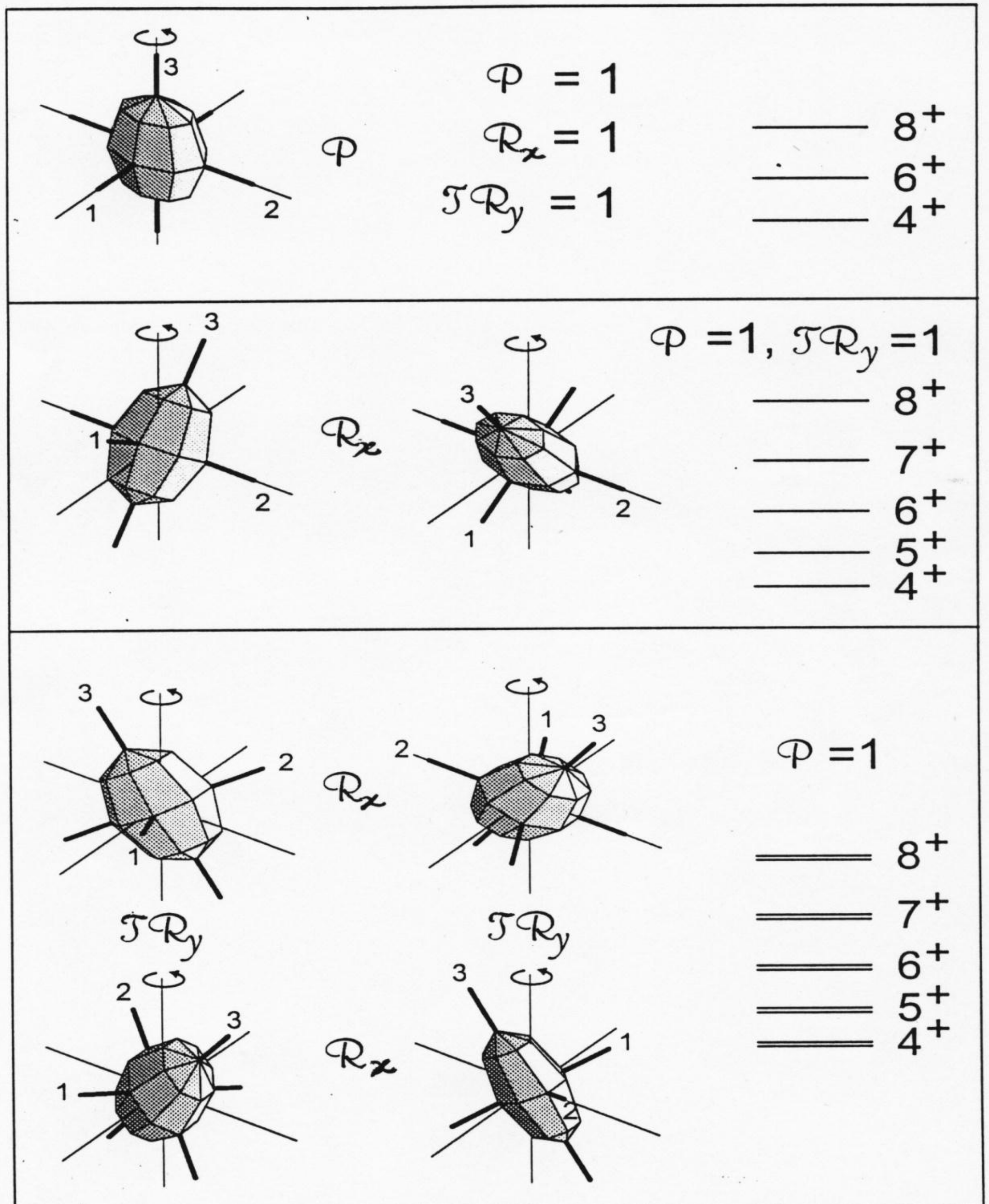
Outline

- Triaxial Ellipsoid
- Odd Multipole Shapes – Simplex
- Only two planes of symmetry
- Only one plane of symmetry
- Tetrahedral and Triangular shapes
- Tilted Axis Rotation – Planar and Aplanar
- Magnetic Top and Chiral Rotation

Ellipsoid with D2 –symmetry – Asymmetric Top

- Full *D2*-symmetry :invariance with respect to the three rotations by 180 about each of the three principal axes.
- Finite gamma deformation – K not a good quantum number
- $$\Phi^I_M(\gamma,\theta_1,\theta_2,\theta_3) = \sum_K g^I_K \left[D^I_{MK} + (-1)^I D^I_{M-K}\right].$$
- K=0 not allowed. Only even integers K=2,4,…etc. allowed.
- A typical rotational band may have I=2,4,6,….etc. as signature is still a good quantum number.

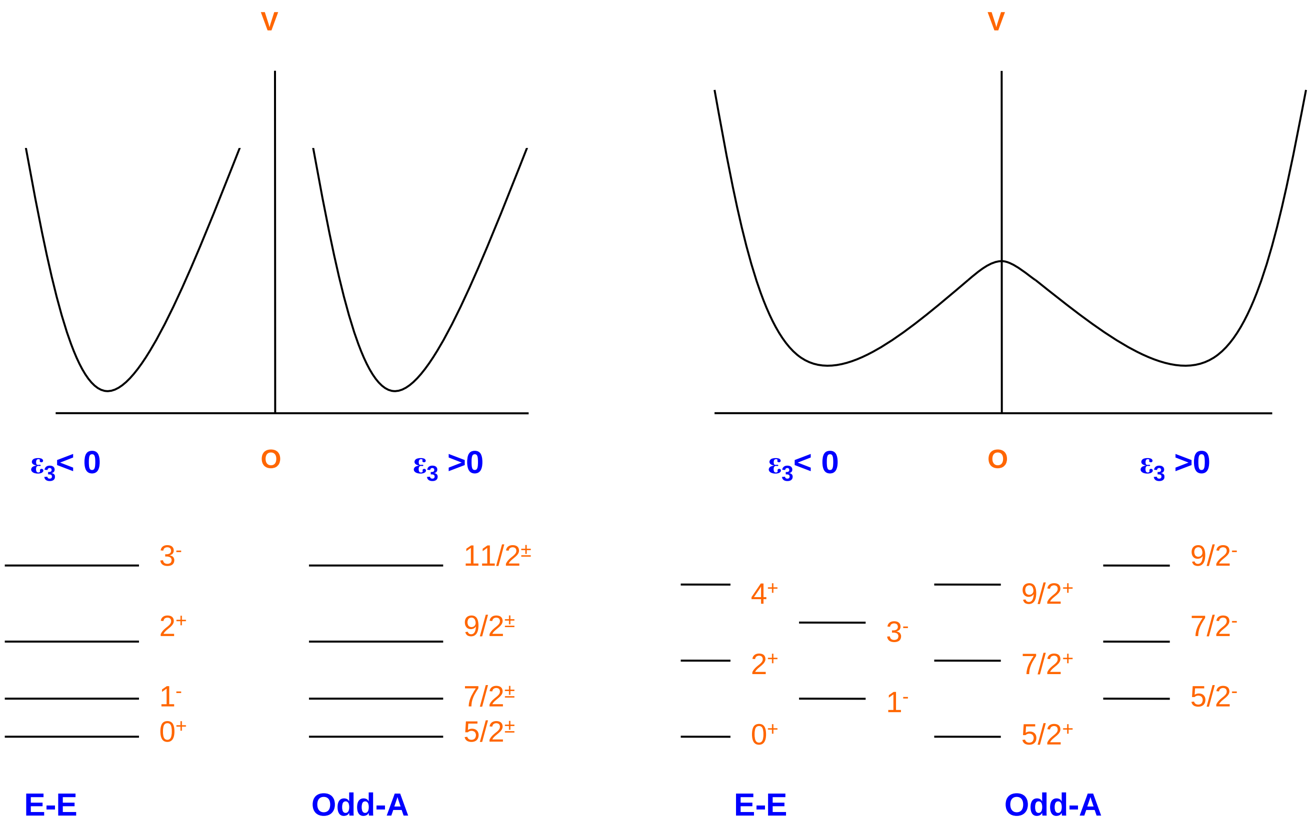

3
1
2
P
P = 1
Rx = 1
TRy = 1
8+
6+
4+
3
1
2
Rx
3
2
1
P =1, TRy =1
8+
7+
6+
5+
4+
3
2
1
Rx
2
1
3
P =1
TRy
TRy
2
3
1
Rx
3
1
8+
7+
6+
5+
4+

Odd-Multipole Shapes: Simplex quantum number

- Axial Symmetry – Y30 shapes only.
- violates the $\hat{R}_x(\pi)$ and $\hat{P}$ symmetry, but preserves $\hat{R}_x \hat{P}$.
- Two minima in octupole def energy, two degenerate states arise
- Denote $\hat{S} = \hat{P}\hat{R}_x^{-1}$ which is conserved
- K=0 band satisfy: $\pi = s(-1)^I$

$$I^{\pi} = 0^+, 1^-, 2^+, 3^-, ..., s = +1$$

$$I^{\pi} = 0^-, 1^+, 2^-, 3^+, ..., s = -1$$


V
ε3< 0
O
ε3 >0
3-
2+
1-
0+
11/2±
9/2±
7/2±
5/2±
E-E
Odd-A
V
ε3< 0
O
ε3 >0
4+
2+
0+
3-
1-
9/2+
7/2+
5/2+
9/2-
7/2-
5/2-
E-E
Odd-A

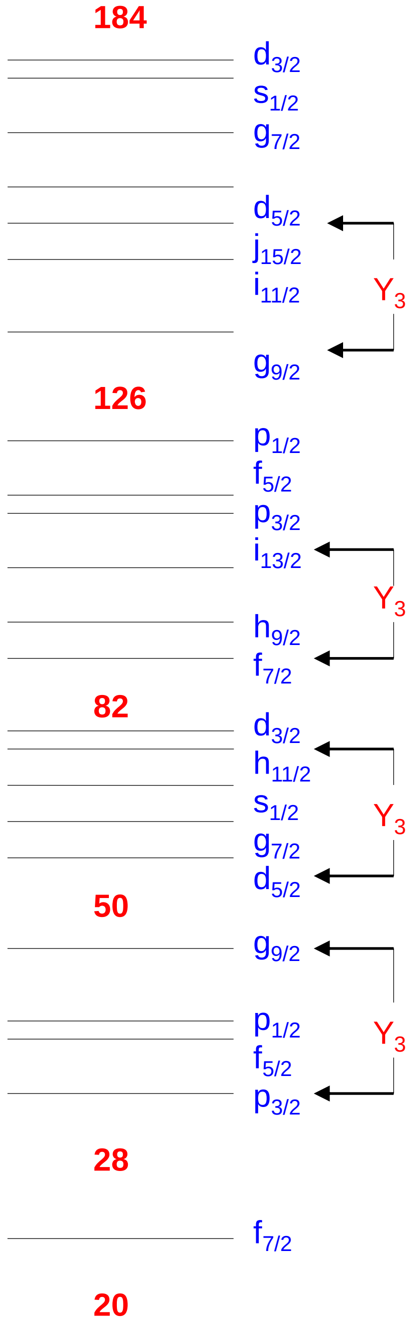
For K≠ 0, the intrinsic states have a 2-fold Kramer's degeneracy and

We obtain,

$$I = \frac{1}{2}^-, \frac{3}{2}^+, \frac{5}{2}^-, \ldots\ldots\ldots\ldots s = -i$$

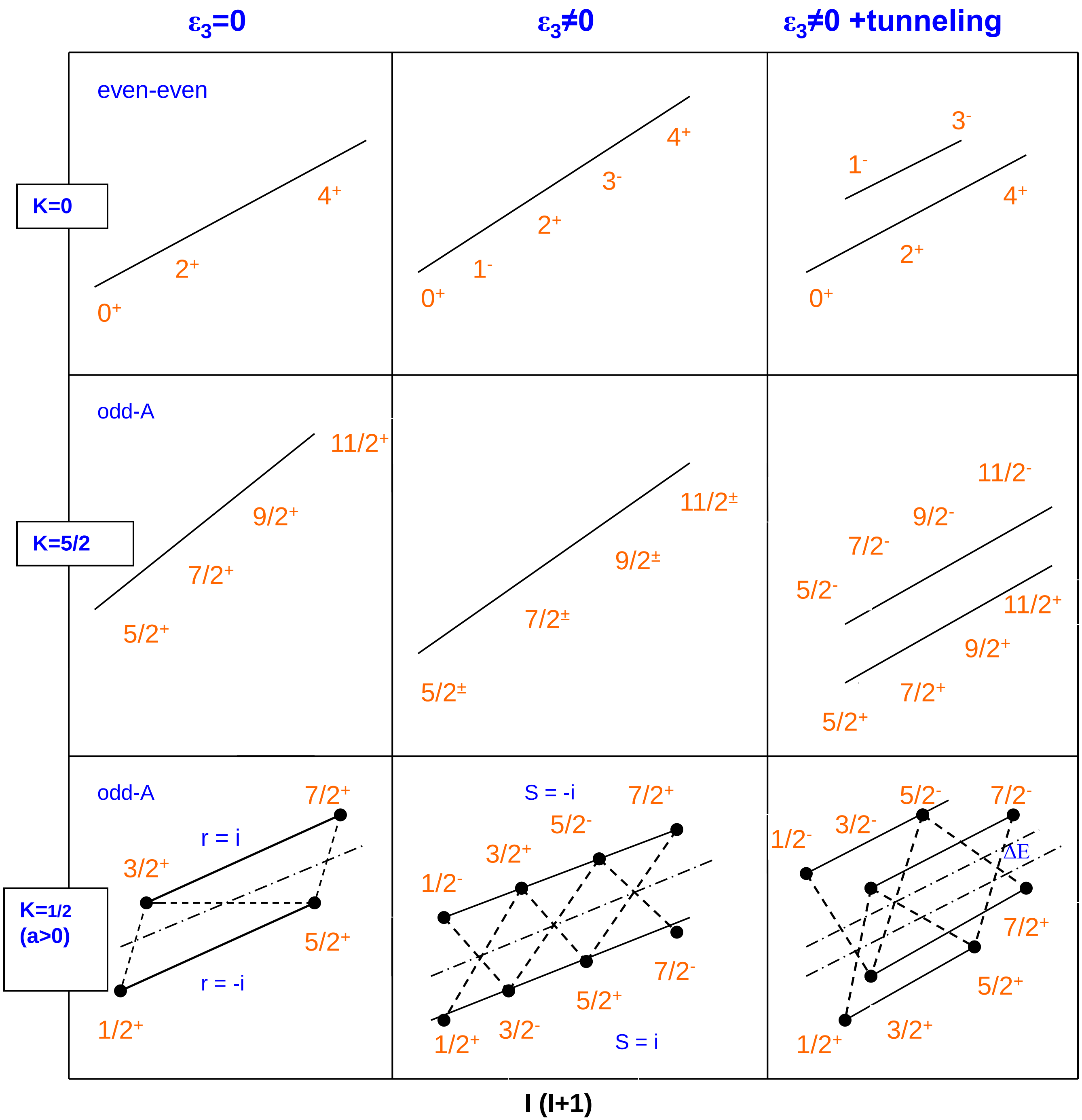
$$I = \frac{1}{2}^+, \frac{3}{2}^-, \frac{5}{2}^+, \ldots\ldots\ldots\ldots\ldots, s = +i$$

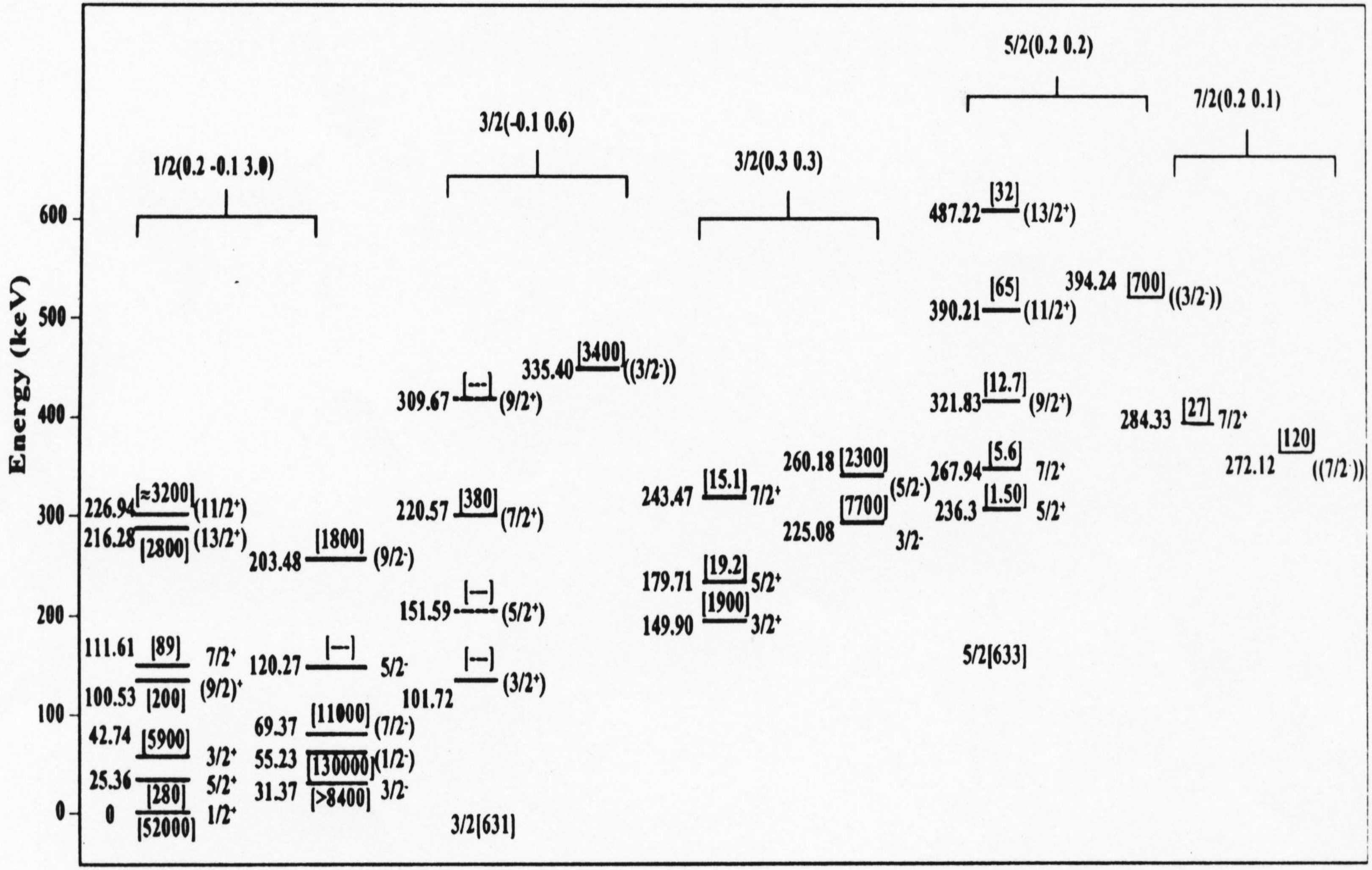
with the levels having I ≥ K allowed.



Regions of Octupole Deformation

For example, *Δl=3 orbitals for both neutrons and protons come close together just beyond Pb-208*


ε3=0
ε3≠0
ε3≠0 +tunneling
K=0
K=5/2
K=1/2 (a>0)
even-even
odd-A
odd-A
0+ 2+ 4+
0+ 1- 2+ 3- 4+
0+ 2+ 4+ 1- 3-
5/2+ 7/2+ 9/2+ 11/2+
5/2± 7/2± 9/2± 11/2±
5/2+ 7/2+ 9/2+ 11/2+ 5/2- 7/2- 9/2- 11/2-
1/2+ 3/2+ 5/2+ 7/2+
r = i
r = -i
S = -i
S = i
1/2+ 1/2- 3/2+ 3/2- 5/2- 5/2+ 7/2+ 7/2-
1/2- 3/2- 5/2- 7/2- 1/2+ 3/2+ 5/2+ 7/2+
ΔE
I (I+1)


Energy (keV)
0
100
200
300
400
500
600
1/2(0.2 -0.1 3.0)
3/2(-0.1 0.6)
3/2(0.3 0.3)
5/2(0.2 0.2)
7/2(0.2 0.1)
226.94 [≈3200] (11/2+)
216.28 [2800] (13/2+)
111.61 [89] 7/2+
100.53 [200] (9/2)+
42.74 [5900] 3/2+
25.36 [280] 5/2+
0 [52000] 1/2+
203.48 [1800] (9/2-)
120.27 [--] 5/2-
69.37 [11000] (7/2-)
55.23 [130000] (1/2-)
31.37 [>8400] 3/2-
309.67 [--] (9/2+)
335.40 [3400] ((3/2-))
220.57 [380] (7/2+)
151.59 [--] (5/2+)
101.72 [--] (3/2+)
3/2[631]
243.47 [15.1] 7/2+
179.71 [19.2] 5/2+
149.90 [1900] 3/2+
260.18 [2300] (5/2-)
225.08 [7700] 3/2-
487.22 [32] (13/2+)
390.21 [65] (11/2+)
321.83 [12.7] (9/2+)
267.94 [5.6] 7/2+
236.3 [1.50] 5/2+
5/2[633]
394.24 [700] ((3/2-))
284.33 [27] 7/2+
272.12 [120] ((7/2-))

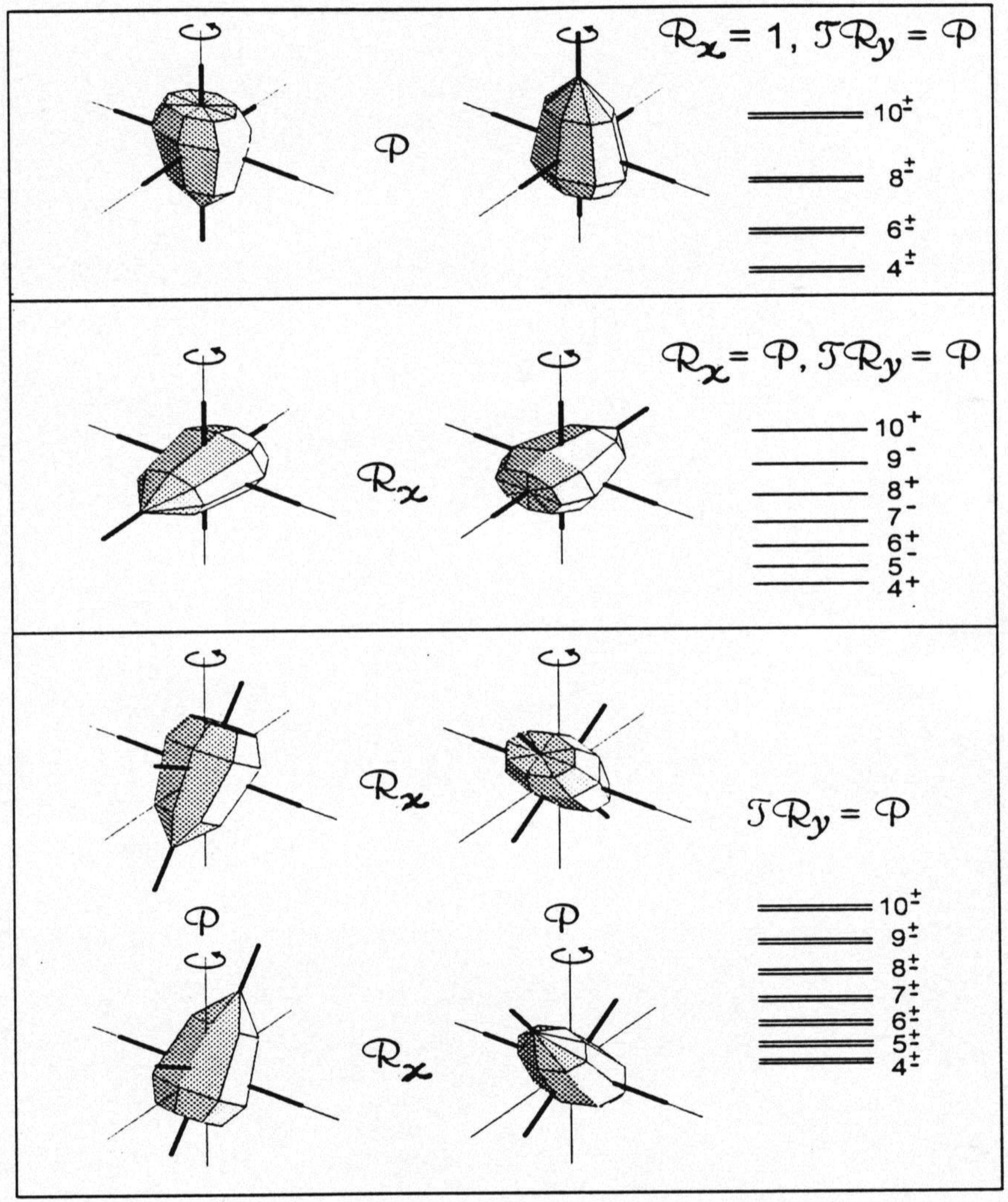
Density Distribution has two planes of symmetry

- Axial symmetry is lost for μ≠ 0
- Two independent planes of symmetry for μ even.
- Rotation about the long axis is possible.
- For

$$Y_{3\mu}\ \mu \neq 0,\ \ R_x(\pi) = 1,\quad R_y(\pi)T = P$$

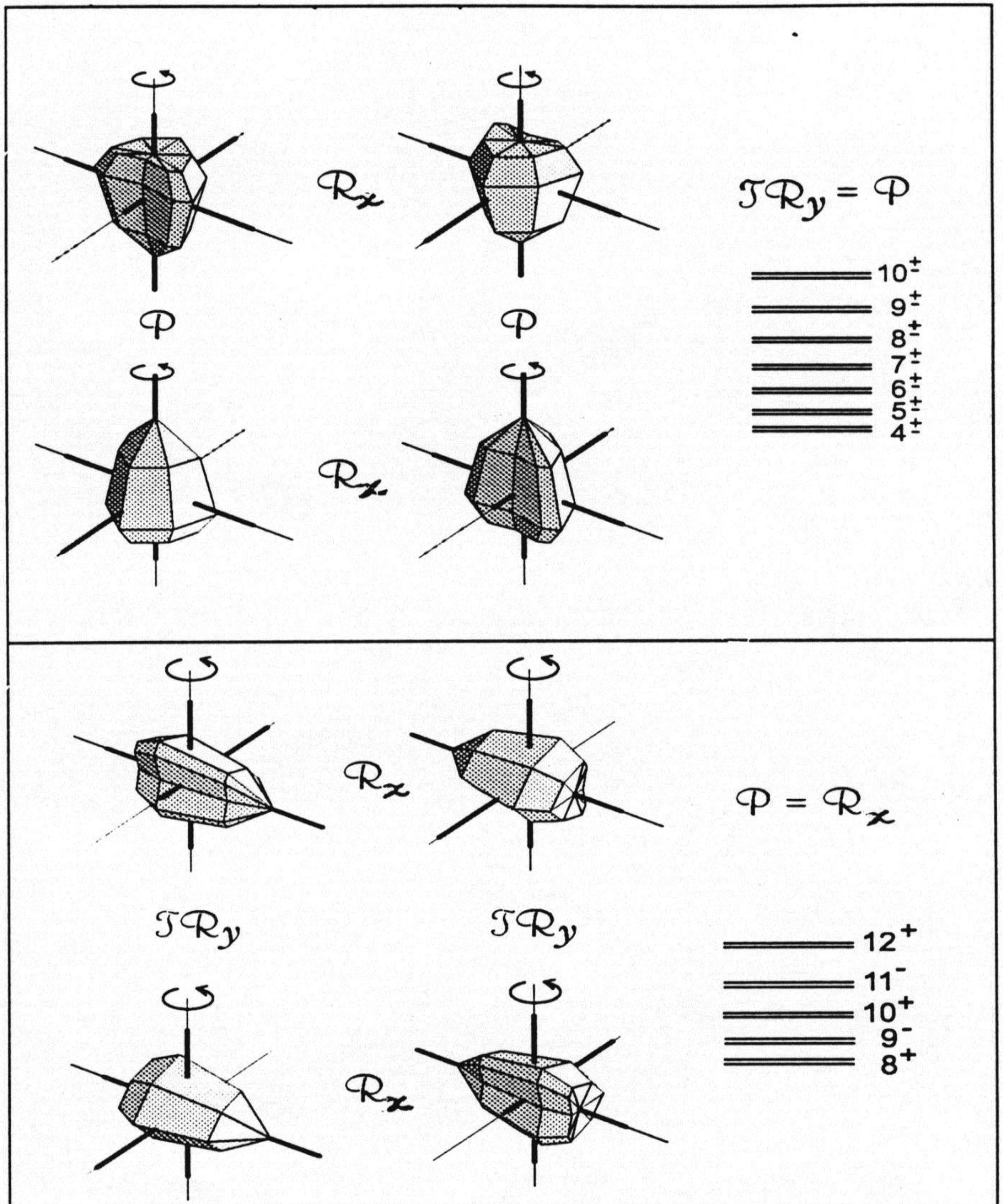
Parity doublets of odd- or, even-angular momenta arise

$$I = 2^{\pm}, 4^{\pm}, ...\qquad I = 1^{\pm}, 3^{\pm}, 5^{\pm}, ...$$


$\mathcal{R}_x = 1,\ \mathcal{T}\mathcal{R}_y = \mathcal{P}$
$\mathcal{P}$
$10^{\pm}$
$8^{\pm}$
$6^{\pm}$
$4^{\pm}$
$\mathcal{R}_x = \mathcal{P},\ \mathcal{T}\mathcal{R}_y = \mathcal{P}$
$\mathcal{R}_x$
10^{+}
9^{-}
8^{+}
7^{-}
6^{+}
5^{-}
4^{+}
$\mathcal{R}_x$
$\mathcal{T}\mathcal{R}_y = \mathcal{P}$
$\mathcal{P}$
$\mathcal{P}$
$\mathcal{R}_x$
$10^{\pm}$
$9^{\pm}$
$8^{\pm}$
$7^{\pm}$
$6^{\pm}$
$5^{\pm}$
$4^{\pm}$

Density distribution has one plane of symmetry

- Odd-μ components allowed
- Rotation is possible along the long axis as well as any one of the short axes
- Signature is not a good quantum number. Both even and odd-spins will occur
- Invariance under space inversion lost; both parities will occur.
- Rotation about long axis $I^{\pi} = 4^{\pm}, 5^{\pm}, 6^{\pm}, ...$
- Rotation about short axis $I^{\pi} = (8^{+})^2, (9^{-})^2, (10^{+})^2, ...$
- Chiral partners obtained



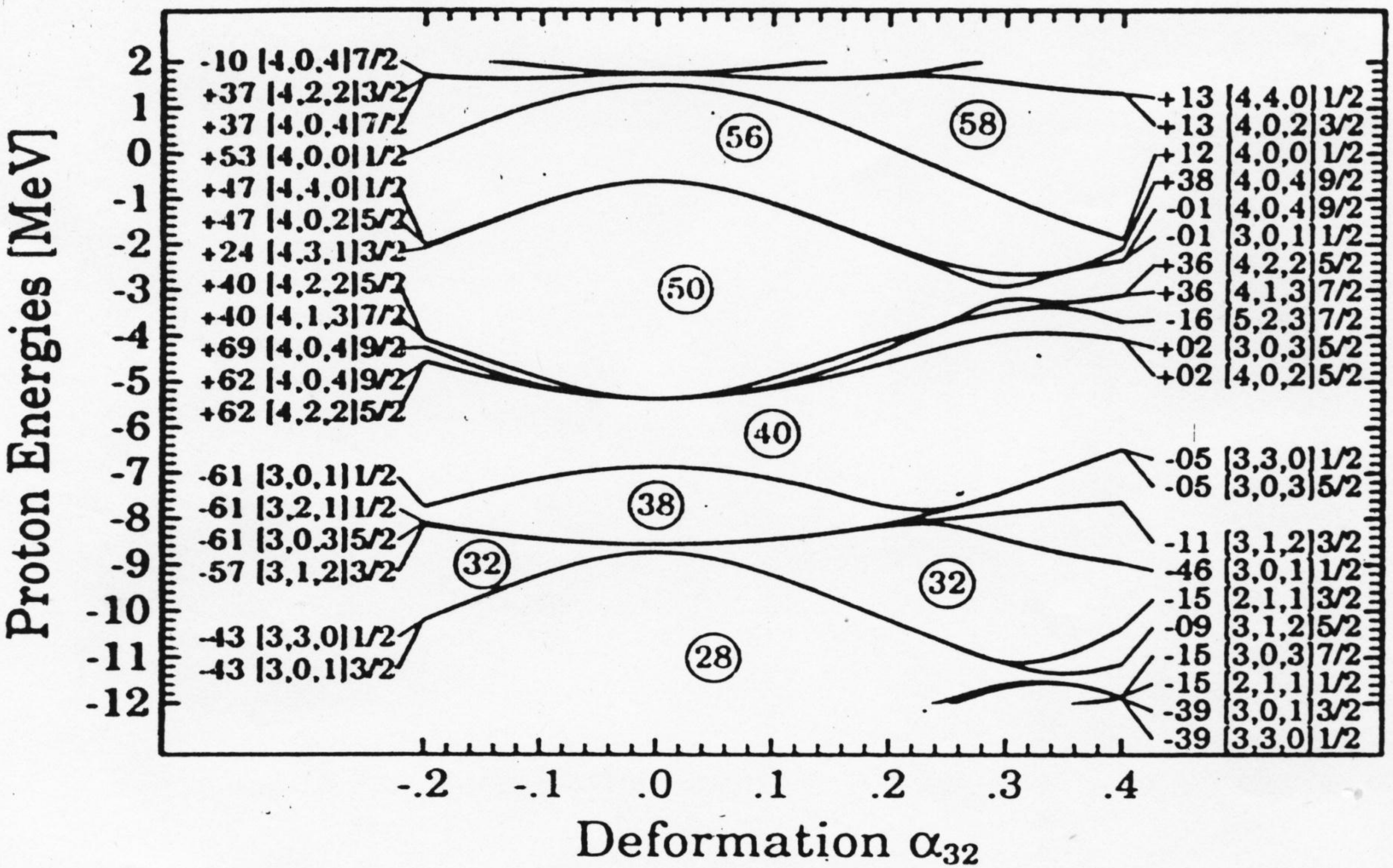
Rx
TRy = P
10±
9±
8±
7±
6±
5±
4±
P
P
Rx
Rx
P = Rx
TRy
TRy
12+
11−
10+
9−
8+
Rx

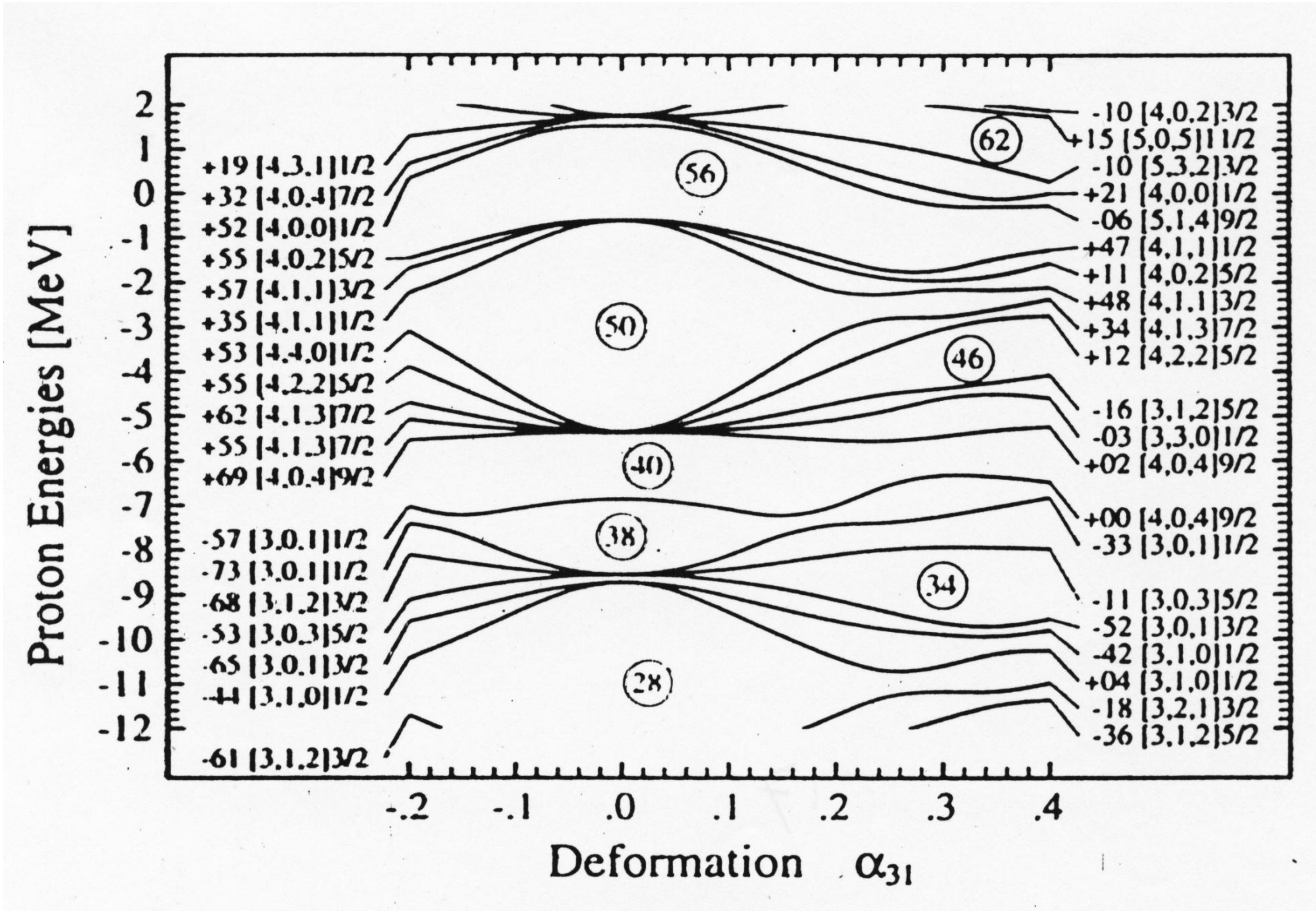
Tetrahedral and Triangle Symmetries

- Tetrahedral symmetry is related to Y-32 term
- Large shell gaps at N, Z=16, 20, 32, 40, 56,58,70,90-94
- and at N=136/142
- Tetrahedral Equilibrium shapes of the order of 0.13 for

$$^{80}_{40}Zr_{40}, ^{108}_{40}Zr_{68}, ^{160}_{70}Yb_{40}, \quad ^{242}_{100}Fm_{142}$$

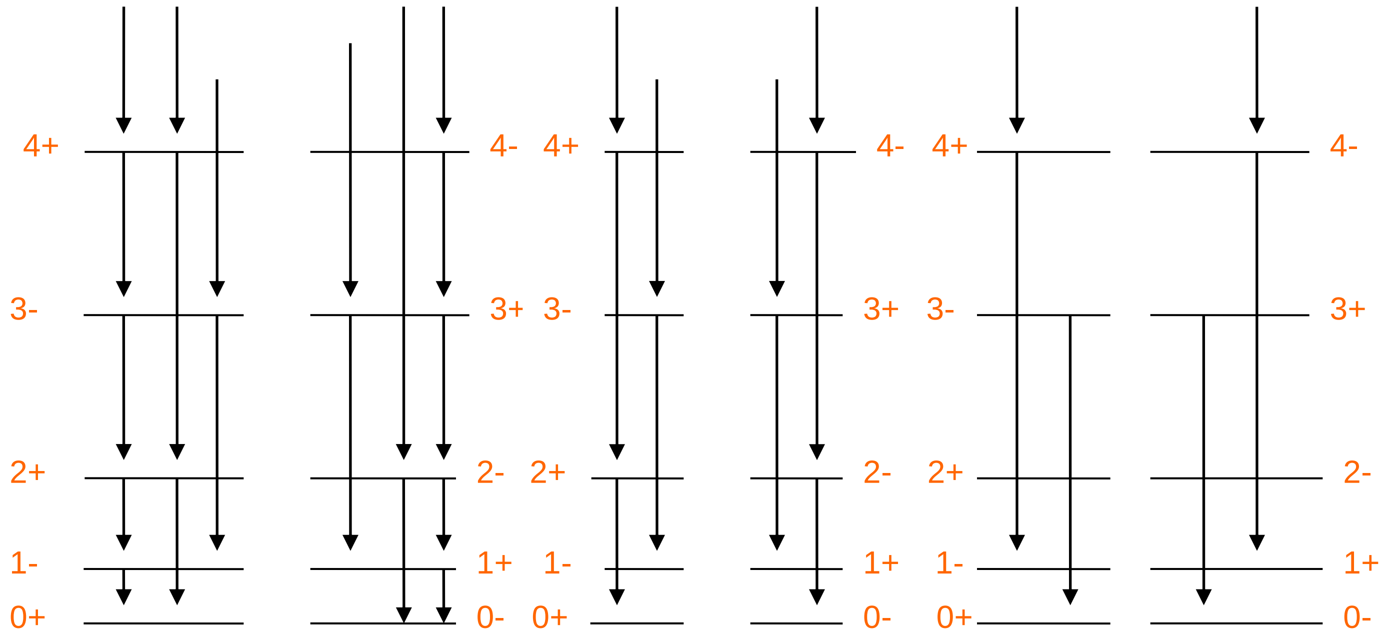
- Obey simplex symmetry, parity doublets formed


Proton Energies [MeV]
Deformation α_{32}
-10 [4,0,4]7/2
+37 [4,2,2]3/2
+37 [4,0,4]7/2
+53 [4,0,0]1/2
+47 [4,4,0]1/2
+47 [4,0,2]5/2
+24 [4,3,1]3/2
+40 [4,2,2]5/2
+40 [4,1,3]7/2
+69 [4,0,4]9/2
+62 [4,0,4]9/2
+62 [4,2,2]5/2
-61 [3,0,1]1/2
-61 [3,2,1]1/2
-61 [3,0,3]5/2
-57 [3,1,2]3/2
-43 [3,3,0]1/2
-43 [3,0,1]3/2
+13 [4,4,0]1/2
+13 [4,0,2]3/2
+12 [4,0,0]1/2
+38 [4,0,4]9/2
-01 [4,0,4]9/2
-01 [3,0,1]1/2
+36 [4,2,2]5/2
+36 [4,1,3]7/2
-16 [5,2,3]7/2
+02 [3,0,3]5/2
+02 [4,0,2]5/2
-05 [3,3,0]1/2
-05 [3,0,3]5/2
-11 [3,1,2]3/2
-46 [3,0,1]1/2
-15 [2,1,1]3/2
-09 [3,1,2]5/2
-15 [3,0,3]7/2
-15 [2,1,1]1/2
-39 [3,0,1]3/2
-39 [3,3,0]1/2
56
58
50
40
38
32
32
28



Proton Energies [MeV]
Deformation α_{31}
+19 [4,3,1]1/2
+32 [4,0,4]7/2
+52 [4,0,0]1/2
+55 [4,0,2]5/2
+57 [4,1,1]3/2
+35 [4,1,1]1/2
+53 [4,4,0]1/2
+55 [4,2,2]5/2
+62 [4,1,3]7/2
+55 [4,1,3]7/2
+69 [4,0,4]9/2
-57 [3,0,1]1/2
-73 [3,0,1]1/2
-68 [3,1,2]3/2
-53 [3,0,3]5/2
-65 [3,0,1]3/2
-44 [3,1,0]1/2
-61 [3,1,2]3/2
-10 [4,0,2]3/2
+15 [5,0,5]11/2
-10 [5,3,2]3/2
+21 [4,0,0]1/2
-06 [5,1,4]9/2
+47 [4,1,1]1/2
+11 [4,0,2]5/2
+48 [4,1,1]3/2
+34 [4,1,3]7/2
+12 [4,2,2]5/2
-16 [3,1,2]5/2
-03 [3,3,0]1/2
+02 [4,0,4]9/2
+00 [4,0,4]9/2
-33 [3,0,1]1/2
-11 [3,0,3]5/2
-52 [3,0,1]3/2
-42 [3,1,0]1/2
+04 [3,1,0]1/2
-18 [3,2,1]3/2
-36 [3,1,2]5/2
62
56
50
46
40
38
34
28

Dipole Mom. ≠0 **Dipole Mom. =0** **Dipole Mom.=0 Quadrupole Mom.=0**



Pear-shape-like Octupole **Tetraheda** **Pure Tetrahedra**

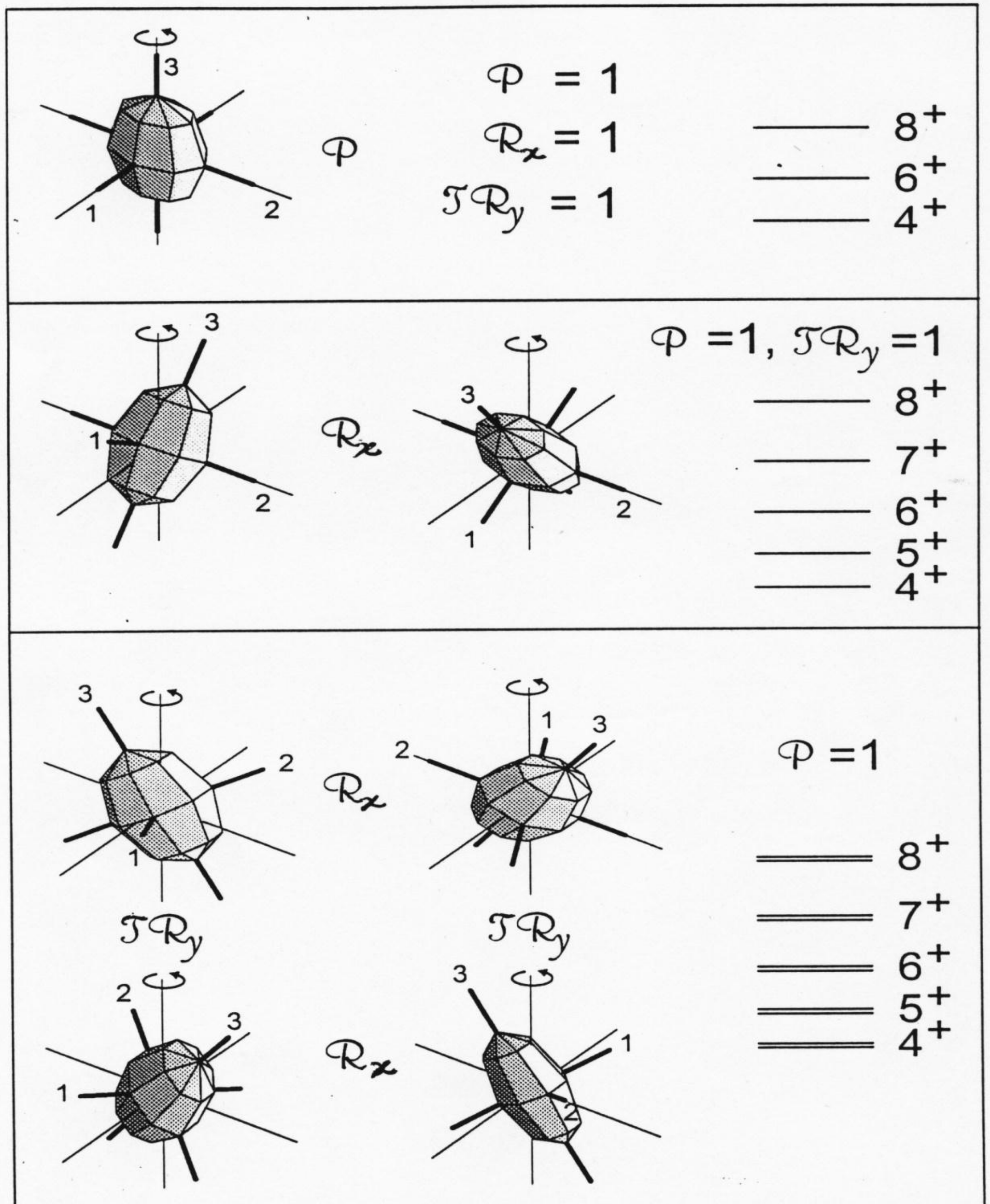
Triangular Shape corresponds to Y-33 def.

- Has only one plane of symmetry
- Possible in proton rich N=Z nuclei

$$^{64}Ge, ^{68}Se, ^{72}Kr, ^{76}Sr, ^{80}Zr, ^{84}Mo$$

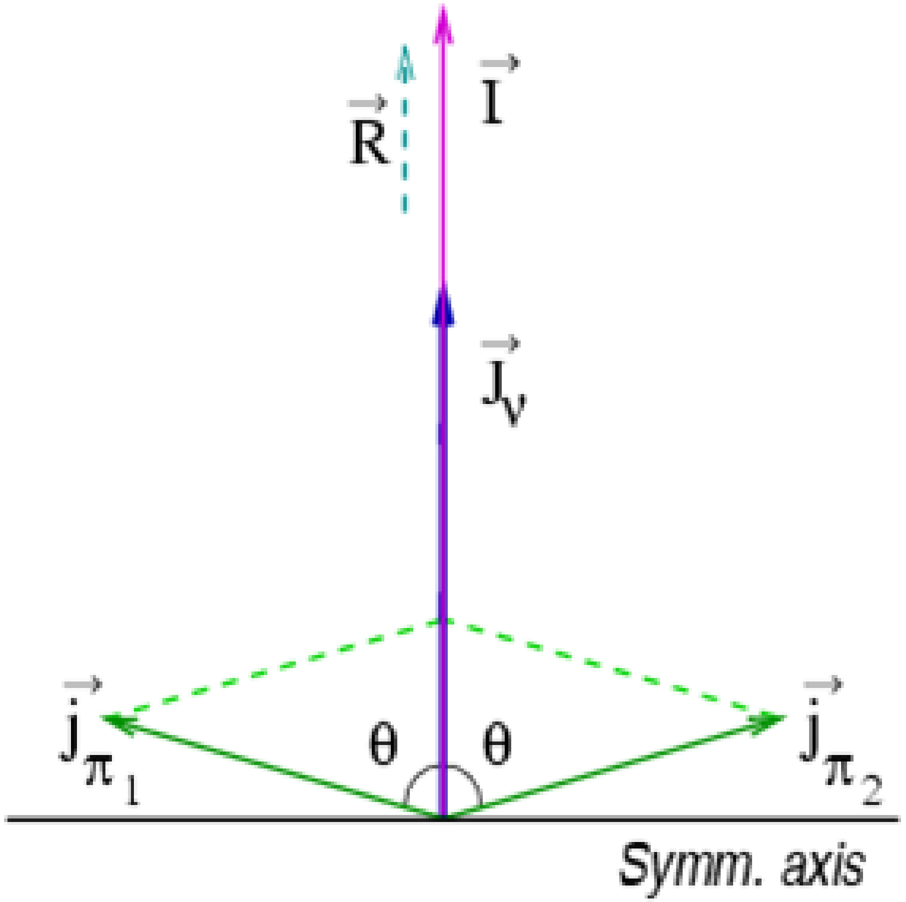
Tilted Axis Rotation

- Riemann - classical rotation about an axis different than principal axes possible in ellipsoids
- Planar Tilt – Axis of rotation in a principal plane
- P and $R_y(\pi)T$ are conserved .
- Signature not conserved
- A band like $I^{\pi} = 4^+, 5^+, 6^+, \ldots\ldots$ is observed
- Observed in Magnetic Rotation Bands

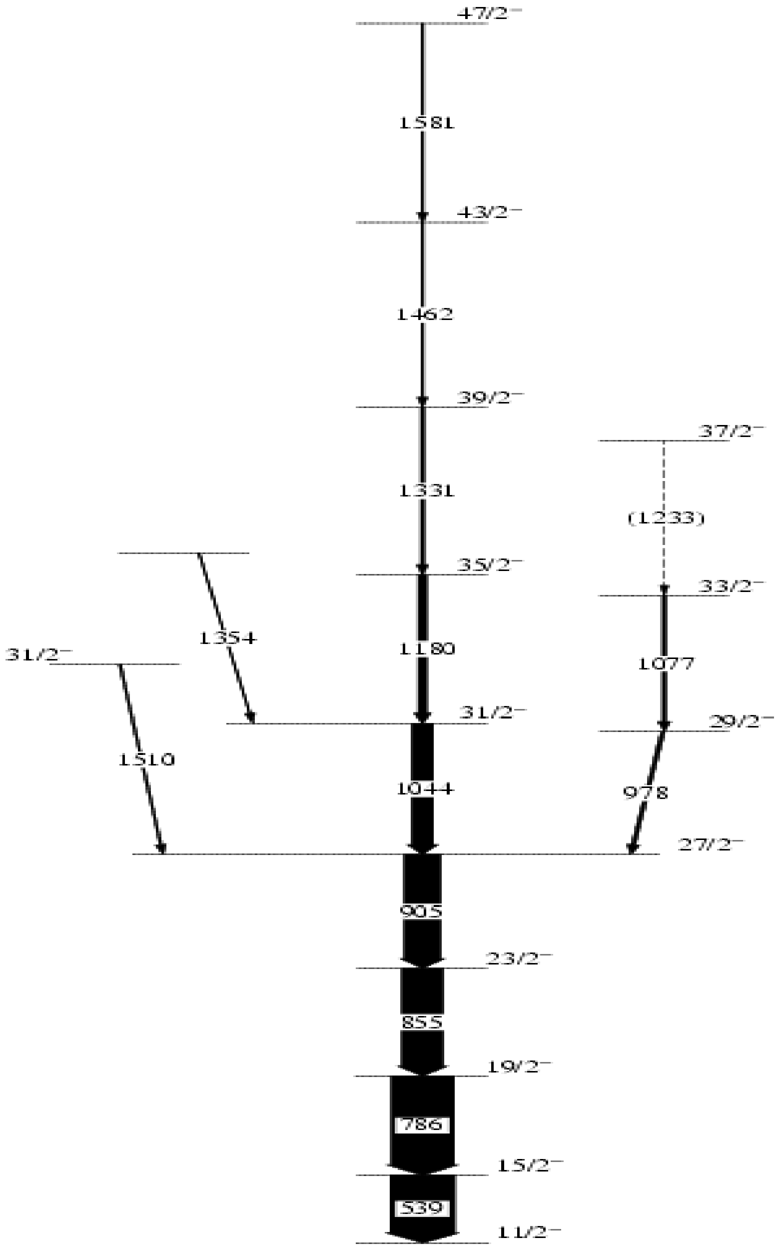


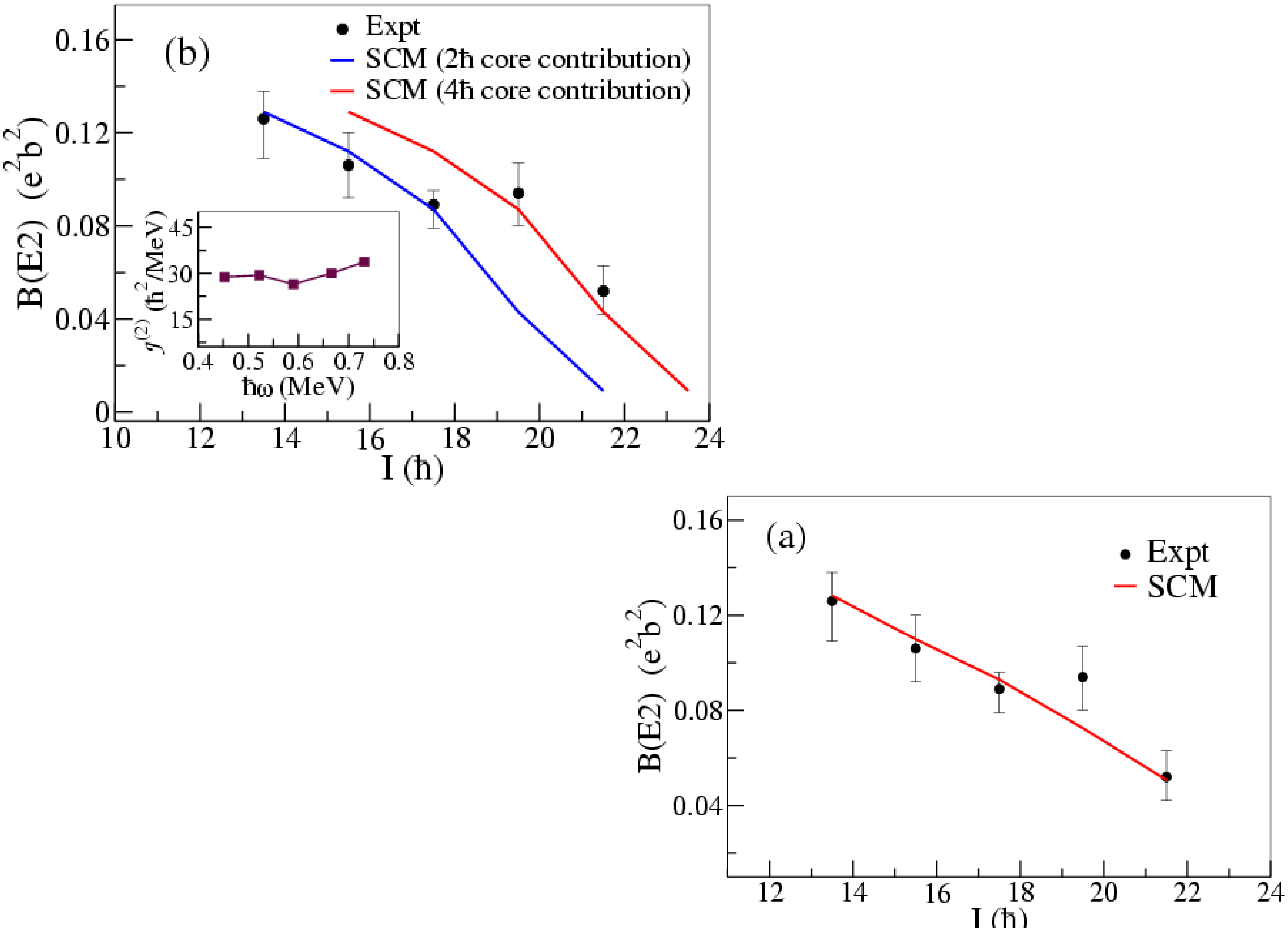
3
1
2
P
P = 1
Rx = 1
TRy = 1
8+
6+
4+
3
1
2
Rx
3
2
1
P =1, TRy =1
8+
7+
6+
5+
4+
3
2
1
Rx
1
3
2
P =1
TRy
TRy
2
3
1
Rx
3
1
8+
7+
6+
5+
4+

Anti-Magnetic Rotation



AMR band in ^{105}Cd



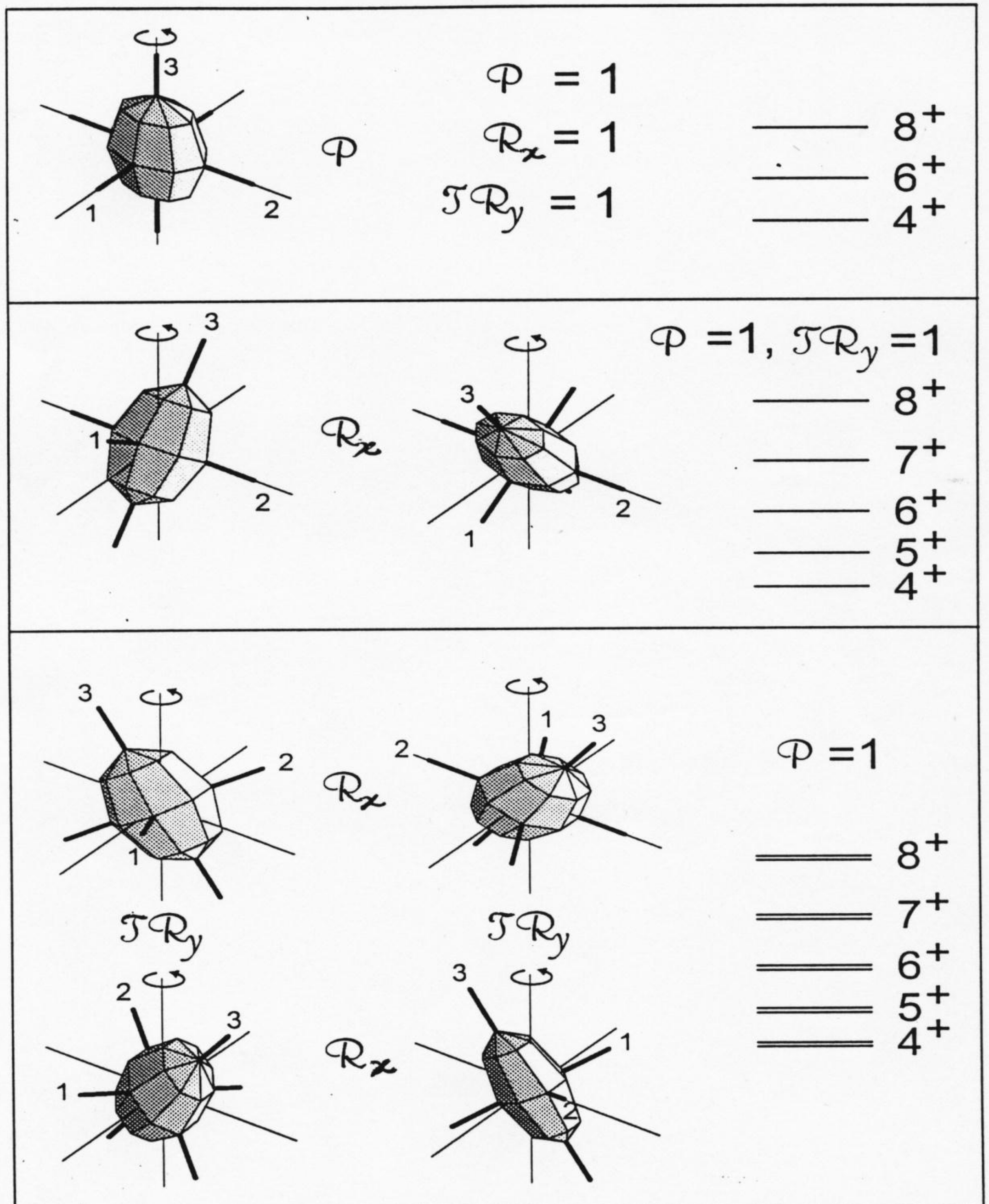

(b)
Expt
SCM (2ħ core contribution)
SCM (4ħ core contribution)
B(E2) (e²b²)
0.16
0.12
0.08
0.04
0
10
12
14
16
18
20
22
24
I (ħ)
$\mathcal{J}^{(2)}$ (ħ²/MeV)
45
30
15
0.4
0.5
0.6
0.7
0.8
ħω (MeV)
(a)
Expt
SCM
B(E2) (e²b²)
0.16
0.12
0.08
0.04
12
14
16
18
20
22
24
I (ħ)

Aplanar Tilt:

- Only parity is conserved
- Four distinct situations, related by $R_x(\pi)$ *and* $R_y(\pi)T$ exist
- Band obtained:

$$I^{\pi} = (4^{+})^{2}, (5^{+})^{2}, (6^{+})^{2}, ...$$

- Observed in Chiral Bands



$\mathcal{P} = 1$
$\mathcal{R}_x = 1$
$\mathcal{T}\mathcal{R}_y = 1$
$\mathcal{P}$
8^+
6^+
4^+
$\mathcal{P} = 1$, $\mathcal{T}\mathcal{R}_y = 1$
$\mathcal{R}_x$
8^+
7^+
6^+
5^+
4^+
$\mathcal{P} = 1$
$\mathcal{R}_x$
$\mathcal{T}\mathcal{R}_y$
$\mathcal{T}\mathcal{R}_y$
$\mathcal{R}_x$
8^+
7^+
6^+
5^+
4^+

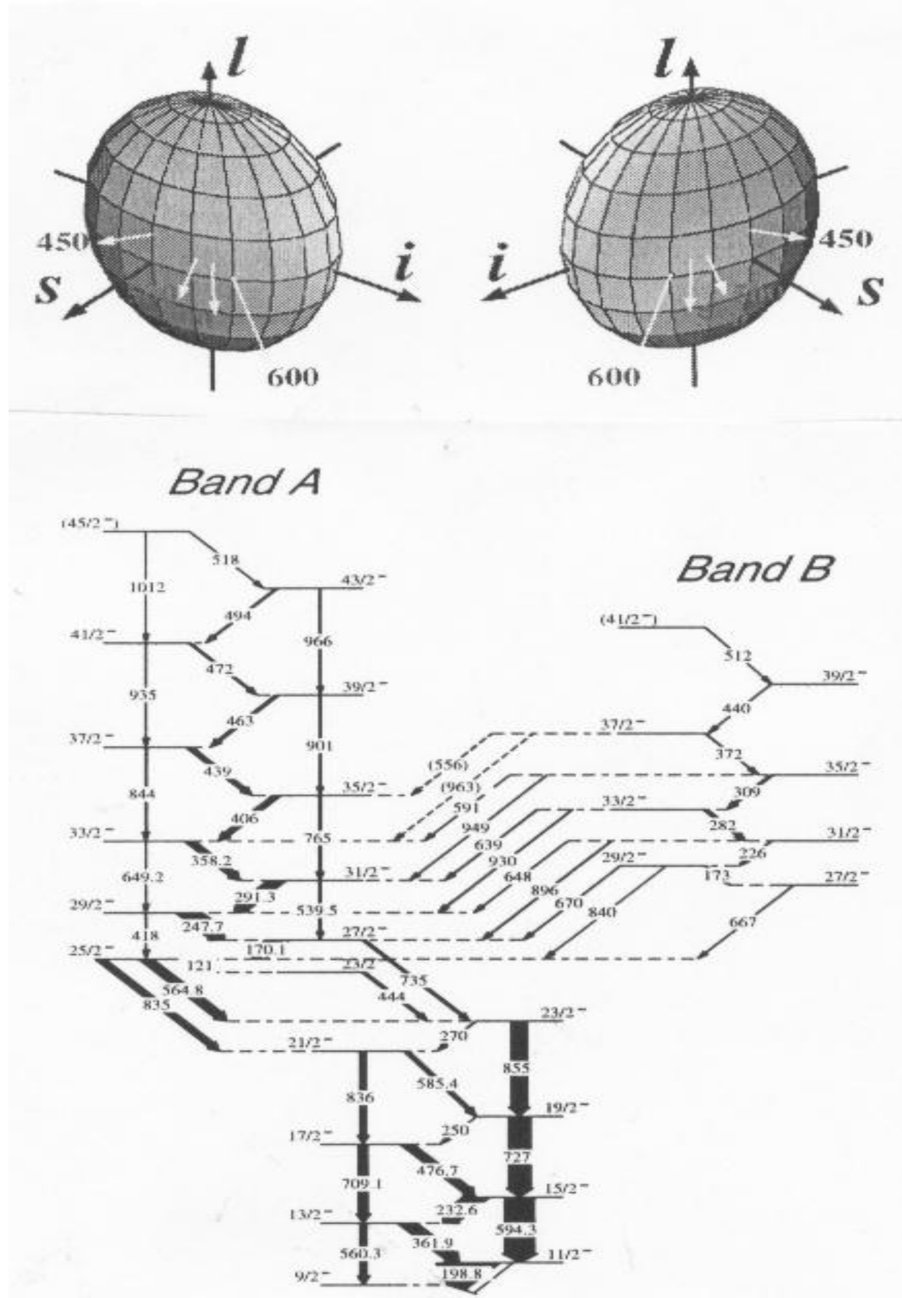
Chiral Bands

- Aplanar Tilt in Tri-axial shape
- First visualised in Odd-Odd nuclei
- Odd-proton aligned along the short axis
- Odd-neutron aligned along the long axis
- Rotational contribution along the intermediate axis
- Resultant of the three ang mom is out of the three planes
- Parity is conserved but $R_y(\pi)T$ is broken
- Two pairs of identical $\Delta I=1$ bands having the same parity
- Tunneling between the right- and the left-handed system gives rise to a splitting between the levels.

Nd-135, Z=60, N=75

$2\pi(h11/2)$, 1v(h11/2)

Zhu et al, 2003,
PRL
91,2003,132501



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