Going beyond Mahabaleshwar: Search for CPT Violation

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PCPV 2013, Mahabaleshwar

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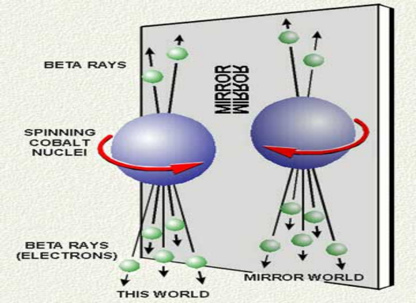
Part I: Introduction to CPT violation

Parity violation can be incorporated through the current (Lee and Yang)

P : $\bar{\psi}\gamma^\mu(1-\gamma_5)\psi \Longrightarrow \bar{\psi}\gamma^\mu(1+\gamma_5)\psi$



Maximal P violation for weak interaction

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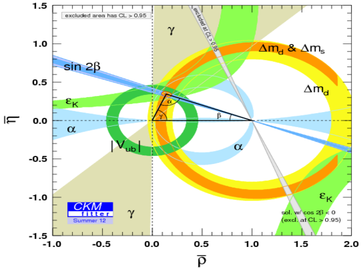
This is not enough for CP violation, you need the coupling to be complex too (Kobayashi and Maskawa)

$$\textbf{CP}: \qquad g\bar{\psi}_1\gamma^\mu(1-\gamma_5)\psi_2 + \text{h.c.} \Longrightarrow g\bar{\psi}_2\gamma^\mu(1-\gamma_5)\psi_1 + \text{h.c.}$$



H.c. involves g^*, but gauge couplings are real. Introduce quark mixing.

Large CP violation for B systems, but too small to explain n_b/n_γ

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CPT, taken in any order, is the *only* combination of C,P,T that is still conserved.

- Pauli (1940): Spin-statistics theorem, requires Lorentz invariance
- Schwinger (1951): Spin-statistics theorem, implicit use of CPT theorem
- Lüders, Pauli, Bell (1954-55): Proof of CPT theorem
- Jost (1958): General proof for axiomatic QFT

Why, then, should one look for CPT violation?

Motivation 1: George Mallory about Mt. Everest: Because it is there.

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CPT transformation on Fermion current:

$$\bar{\psi}_a^{CPT}(t,\mathbf{x})\Gamma_i\psi_b^{CPT}(t,\mathbf{x}) = \bar{\psi}_b(-t,-\mathbf{x})\Gamma_i^{CPT}\psi_a(-t,-\mathbf{x})$$

$$\Gamma_i:\ \{1,\gamma_5,\gamma_\mu,\gamma_\mu\gamma_5,\sigma_{\mu\nu}\} \Longrightarrow \{1,-\gamma_5,-\gamma_\mu,-\gamma_\mu\gamma_5,\sigma_{\mu\nu}\}$$

CPT is a good symmetry if

$$(CPT)\mathcal{L}(t,\mathbf{x})(CPT)^{-1} = \mathcal{L}(-t,-\mathbf{x})$$

That gives you an idea of what terms can potentially violate CPT.

Theorem

CPT is a good symmetry of any local Lorentz-invariant axiomatic quantum field theory with a unique vacuum state.

You can never construct a Lorentz-invariant QFT with a hermitian Hamiltonian that violates CPT.

Proof of CPT theorem is not straightforward

(see, e.g., Streater and Wightman)

Proof.

Consider real scalar field $\longrightarrow$ C is conserved

PT is $x^\mu \to -x^\mu$, proper LT, continuously connected to identity

In Euclidean space, just like a 4-d rotation — must be conserved

PT is always a good symmetry for real scalar field □

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Consequences of CPT conservation

- Particle and antiparticle must have same mass and opposite electric charge
- Particle and antiparticle, if unstable, must have same decay width
  Not true if stationary states are particle-antiparticle combinations

$$K_L \approx \frac{1}{\sqrt{2}}(K^0 + \overline{K}^0)\,, K_S \approx \frac{1}{\sqrt{2}}(K^0 - \overline{K}^0)\,,$$

$$M_{K_L} \neq M_{K_S}\,, \quad \Gamma_{K_L} \neq \Gamma_{K_S}$$

- Particle and antiparticle must have equal and opposite mag. moment
- Hydrogen and antihydrogen must have identical spectra ....

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In a local QFT with well-defined asymptotic states, CPT violation needs Lorentz violation

[Greenberg, PRL 2002]

The reverse is not true.

Motivation 2: Strings are extended objects, so nonlocal. Critical dimensionality $d > 4$, Higher dimensional breaking of Lorentz covariance incorporated in a 4-d world?

Lorentz symmetry can be broken in noncommutative FT too.

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Lorentz violation

Lorentz breaking may be spontaneous (like SSB, but VEV to, say, a vector field) or explicit

The low-energy effective theory (SME) contains operators whose coefficients are Lorentz breaking

$$\mathcal{L} = -(a_L)_{\mu ij}\bar{L}_i\gamma^\mu L_j - (a_R)_{\mu ij}\bar{R}_i\gamma^\mu R_j$$

Physics depends on direction ! [Colladay and Kostelecky, PRD 1998]

- Lorentz transformations on the frame (observer) [passive] or on the fields (particle) [active]
- Should be inverse of each other if Lorentz symmetry is respected
- With LV, they are no longer so. Particle transformations are physically important.

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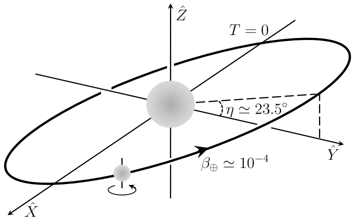
Observables that depend on sidereal time are smoking gun signals of Lorentz violation



LV operators might be CPT-even or CPT-odd

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For each Dirac fermions, there are 44 possible *observable* LV terms in nonrelativistic limit. 20 among them are CPT-odd.

[Kostelecky and Russell, RMP 2008, 0801.0287]

$$\begin{aligned} \mathcal{L} &= \bar{\psi}_i (i\Gamma_\mu D^\mu - M_{ij})\psi_j \\ \Gamma_\mu &= \gamma_\mu \delta_{ij} + c^{ij}_{\mu\nu}\gamma^\nu + d^{ij}_{\mu\nu}\gamma^\mu\gamma_5 + e^{ij}_\mu + if^{ij}_\mu\gamma_5 + \frac{1}{2}g^{ij}_{\mu\kappa\rho}\sigma^{\kappa\rho} \\ M_{ij} &= m_{ij} + im_{5ij}\gamma_5 + a^{ij}_\mu\gamma^\mu + b^{ij}_\mu\gamma^\mu\gamma_5 + \frac{1}{2}h^{ij}_{\mu\nu}\sigma^{\mu\nu} \end{aligned}$$

Blue terms are CPT-odd. Red terms are LV but CPT-even

CPT (and LV) tests have been carried out in gravity, photon, charged lepton, neutrino, proton, neutron, and meson sectors.

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CPT and LV tests

- Astrophysics
  - — Pulsar rates
  - — CMB polarization
  - — Birefringence
- Atomic physics
  - — K/He magnetometer
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Several particle physics experiments too.

1. Neutrino oscillation
2. $(g-2)_{e,\mu}$
3. e^+e^- annihilation
4. Particle and antiparicle mass measurement
5. Neutral meson oscillation
6. and others ...

Sidereal time variation for observables $\Longrightarrow$ LV
Difference between particle and antiparticle $\Longrightarrow$ LV + CPTV

Not all measurements are equally precise.

Define $p = [\mathrm{Obs(particle)} - \mathrm{Obs(antiparticle)}]/\mathrm{Obs(average)}$

$p(m_W)$	-0.002 ± 0.007	$p(m_\pi)$	$(2 \pm 5) \times 10^{-4}$
$p(m_p)$	$< 2 \times 10^{-9}$	$p(m_n)$	$(9 \pm 6) \times 10^{-5}$
$p(g_{e^+})$	$(-0.5 \pm 2.1) \times 10^{-12}$	$p(g_{\mu^+})$	$(-0.11 \pm 0.12) \times 10^{-8}$
$m_t - m_{\bar{t}}$	(-1.4 ± 2.0) GeV		

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Take, as an example, the charge equality of electron and positron

$e^+ + e^- \to \gamma + \gamma$

Direct PDG: $(Q_{e^+} + Q_{e^-})/|Q_{e^-}| < 4 \times 10^{-8}$

Much better bound assuming charge conservation
$(Q_{e^+} + Q_{e^-})/|Q_{e^-}| \sim Q_\gamma/|Q_{e^-}| < 10^{-33}$

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- Is $\omega_+ = \omega_-$?
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Both the answers are consistent with zero — one of the most precise measurements. [Muon $g-2$ Collab., PRL 2008]

However, this does not say anything, for example, about the CPT violating parameters in the τ sector — CPT violation can be a flavour dependent thing.

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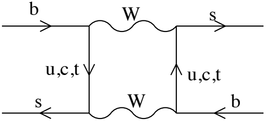
Part II: CPT violation in the K and B systems

$K^0 - \overline{K}^0$ mixing and CPT violation

Beam of neutral Kaon in its rest frame

$$|K(t)\rangle = a_1(t)|K^0\rangle + a_2(t)|\overline{K}^0\rangle\,, \qquad \langle K^0|\overline{K}^0\rangle = 0$$



The evolution is given by

$$i\frac{\partial}{\partial t}\begin{pmatrix} a_1(t) \\ a_2(t)\end{pmatrix} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M_{22} - \frac{i}{2}\Gamma_{22}\end{pmatrix}\begin{pmatrix} a_1(t) \\ a_2(t)\end{pmatrix}$$

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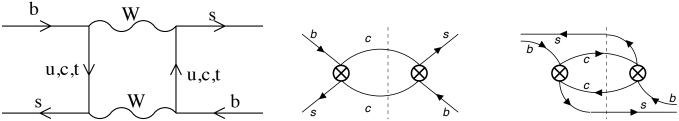

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The Hamiltonian matrix H can be parametrized by 7 parameters as relative phase between H_{12} and H_{21} is meaningless.
Diagonalize with eigenvalues λ_1 and λ_2:

$$\lambda_1 - \lambda_2 = \Delta M + \frac{i}{2}\Delta\Gamma\,, \quad \Delta M = M_1 - M_2\,, \quad \Delta\Gamma = \Gamma_2 - \Gamma_1\,.$$

$$M = (M_{11} + M_{22})/2\,, \quad \Gamma = (\Gamma_{11} + \Gamma_{22})/2\,, \quad \Delta M\,, \quad \Delta\Gamma$$

$$\theta = \frac{H_{22} - H_{11}}{\Delta M - \frac{i}{2}\Delta\Gamma} \qquad \chi = \frac{|H_{12}|^2 - |H_{21}|^2}{|H_{12}|^2 + |H_{21}|^2}$$

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Mass eigenstates: K_L and K_S

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The three important parameters for the Kaon sector:

$$\omega = \frac{A(K_S \to 2\pi)_{I=2}}{A(K_S \to 2\pi)_{I=0}} \quad |\Delta I| \neq \frac{1}{2}$$

$$\epsilon = \frac{A(K_L \to 2\pi)_{I=0}}{A(K_S \to 2\pi)_{I=0}} \quad \not{C}\!P$$

$$\epsilon' = \frac{A(K_L \to 2\pi)_{I=2}A(K_S \to 2\pi)_{I=0} - A(K_L \to 2\pi)_{I=0}A(K_S \to 2\pi)_{I=2}}{\sqrt{2}[A(K_S \to 2\pi)_{I=0}]^2}$$

Related parameters:

$$\eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \epsilon + \frac{\epsilon'}{1+\omega/\sqrt{2}} \approx \epsilon + \epsilon'$$

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \epsilon - \frac{2\epsilon'}{1-\sqrt{2}\omega} \approx \epsilon - 2\epsilon'$$

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$$K_{S,L} = \frac{1}{\sqrt{2(1+|\epsilon_{S,L}|^2)}} \left((1+\epsilon_{S,L})K^0 \pm (1-\epsilon_{S,L})\overline{K}^0\right)$$

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$$= \epsilon \pm \bar{\delta}$$

If CPT is conserved, $\bar{\delta} = 0$. The reverse is *not* true!

CPT violating parameters enter into the definition of the states and hence affect the observables.

$\mathrm{Re}(\bar{\delta}) = (2.3 \pm 2.7) \times 10^{-4}, (2.51 \pm 2.25) \times 10^{-4}$
$\mathrm{Im}(\bar{\delta}) = (0.4 \pm 2.1) \times 10^{-5}, (-1.5 \pm 1.6) \times 10^{-5}$

[KLOE 2006, KTeV 2011]

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$B^0 - \overline{B}^0$ mixing and CPT violation

Formalism is almost identical [Lavoura, Ann. Phys. (1991)]
The constraints can be completely different — CPT violation may be flavour-dependent. Also, ΔM large but $\Delta\Gamma$ small.

- Lifetime difference can be significant
- CPT violation may affect direct CP-violating asymmetries, including semileptonic and dileptonic
- For semileptonic decays $B, \bar{B} \to \ell^{\pm} X^{\mp} f$, the time-ordering of leptonic and hadronic decays may change due to CPT violation

[Datta, Paschos, Singh (PLB 2002), Balaji, Horn, Paschos (PRD 2003), Xing (PRD 1994, PLB 1999)]

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CPT violation in Mixing

Introduce CPT violation in the Hamiltonian matrix through the parameter δ, can be complex:

$$\delta = \frac{H_{22} - H_{11}}{\sqrt{H_{12} H_{21}}} ,$$

Solutions:

$$\lambda = \left[H_{11} + H_{12}\alpha \left(y + \frac{\delta}{2} \right) \right] , \quad \left[H_{22} - H_{12}\alpha \left(y + \frac{\delta}{2} \right) \right] ,$$

where $y = \sqrt{1 + \frac{\delta^2}{4}}$ and $\alpha = \sqrt{H_{21}/H_{12}}$.

Eigenstates

$$\begin{aligned} |B_H\rangle &= p_1|B^0\rangle + q_1|\overline{B}^0\rangle\,, \\ |B_L\rangle &= p_2|B^0\rangle - q_2|\overline{B}^0\rangle\,. \end{aligned}$$

Normalisation

$$|p_1|^2 + |q_1|^2 = |p_2|^2 + |q_2|^2 = 1\,.$$

Define

$$\eta_1 = \frac{q_1}{p_1} = \left(y + \frac{\delta}{2}\right)\alpha\,; \quad \eta_2 = \frac{q_2}{p_2} = \left(y - \frac{\delta}{2}\right)\alpha\,; \quad \omega = \frac{\eta_1}{\eta_2}\,.$$

δ and hence y are CPT violating. If $|\delta| \ll 1$, $y \approx 1$.

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$\Delta a_\mu = r_1 a^1_\mu - r_2 a^2_\mu$, $\beta^\mu = (1, \vec{\beta})$

$$\delta = -\frac{1}{2} \frac{\beta^\mu \Delta a_\mu}{\Delta M - i\Delta\Gamma/2}$$

Numerator varies with time as $\vec{\beta}$ rotates with $\Delta\vec{a}$.
BaBar (0711.2713) got δ consistent with zero (first two spectral powers) from OS dilepton events
Belle(1203.0930):

$$\mathrm{Re}(\delta_d) = (-3.8 \pm 9,9) \times 10^{-2}$$
$$\mathrm{Im}(\delta_d) = (1.14 \pm 0.93) \times 10^{-2}$$

Similar results in K (KTeV) and D (FOCUS) systems. Time for B_s.

Consider decay to a CP eigenstate f.

$$A_f = \langle f|H|B_q\rangle\,, \quad \bar{A}_f = \langle f|H|\overline{B_q}\rangle\,.$$

$$\xi_{f_1} = \eta_1 \frac{\bar{A}_f}{A_f}\,, \quad \xi_{f_2} = \eta_2 \frac{\bar{A}_f}{A_f}\,.$$

In the SM, both are equal and $\xi_{f_1} = \xi_{f_2} = \xi_f$. For single-channel processes, $|\xi_f| = 1$.

The untagged rate $\Gamma_U[f,t] = \Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)$

$$Br[f] = \frac{1}{2}\int_0^\infty dt\ \Gamma[f,t]\,.$$

Consider decay to a CP eigenstate f.

$$A_f = \langle f|H|B_q\rangle\,, \quad \bar{A}_f = \langle f|H|\overline{B_q}\rangle\,.$$

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$$+(\ldots)\cos(\Delta M_q t) + (\ldots)\sin(\Delta M_q t)\Bigg]$$

Simplification:
For B_d system, $\Delta\Gamma_d \ll 1$, $\cosh \to 1$, $\sinh \to 0$, easier fit to decay profile
For $|\delta| \ll 1$, keep only the linear terms. For B_s, keep Γ_s too

$$Br[f] = \frac{|A_f|^2}{2}\Bigg[\frac{1}{\Gamma_s}\{2 - \mathrm{Im}(\delta)\mathrm{Im}(\xi_f)\}$$
$$+\frac{\Gamma_s}{(\Delta m)^2 + (\Gamma_s)^2}\mathrm{Im}(\delta)\mathrm{Im}(\xi_f) + \frac{\Delta\Gamma_s}{(\Gamma_s)^2}\mathrm{Re}(\xi_f)\Bigg].$$

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The B mesons can be tagged:
$\Gamma_T[f,t] = \Gamma(B_q(t) \to f) - \Gamma(\bar{B}_q(t) \to f)$

- Fit both untagged and tagged profiles
- $\mathrm{Re}(\delta)$ from cos and sinh terms, $\mathrm{Im}(\delta)$ from sin and cos terms

$$A_{CPT}(f,t) = \frac{\Gamma_T[f,t]}{\Gamma_U[f,t]} = \frac{\Gamma(B_q(t) \to f) - \Gamma(\bar{B}_q(t) \to f)}{\Gamma(B_q(t) \to f) + \Gamma(\bar{B}_q(t) \to f)},$$

Goes to usual CP asymmetry A_{CP} if $\delta = 0$.
No change in semileptonic CP asymmetry if only new physics is CPT violation.

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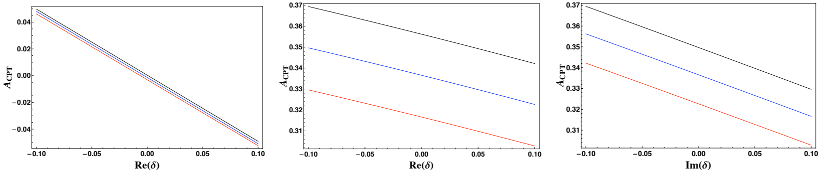
$$A_{CPT}(f) = \frac{\int_0^\infty dt\ \Gamma_T[f,t]}{\int_0^\infty dt\ \Gamma_U[f,t]} = \frac{\int_0^\infty dt\ [\Gamma(B_q(t)\to f) - \Gamma(\bar{B}_q(t)\to f)]}{\int_0^\infty dt\ [\Gamma(B_q(t)\to f) + \Gamma(\bar{B}_q(t)\to f)]}\,.$$



Top to bottom: $\mathrm{Im}/\mathrm{Re}(\delta) = -0.1, 0.0.1$

[AK, Nandi, Patra, PRD 2010]

- Consider a specific example

$$B_s, \overline{B_s} \to D_s^{\pm} K^{\mp}$$

- Can proceed through
  $b \to c\bar{u}s$ $[\propto V_{cb}V_{us}^*]$ and
  $b \to u\bar{c}s$ $[\propto V_{ub}V_{cs}^* = \exp(-i\gamma)]$.
- Only tree-level in SM and $\propto \lambda^3$. Comparable rates.

$\mathrm{Br}(B_s \to D_s K) = (1.90 \pm 0.23) \times 10^{-4}$ [LHCb 2012]

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$$
\begin{aligned}
\Gamma(B_s(t) \to f) - \Gamma(\overline{B_s}(t) \to f) \quad &= \quad [P_1 \sinh(\Delta\Gamma_s t/2) + Q_1 \cosh(\Delta\Gamma_s t/2) \\
f \equiv D_s^+ K^-, \quad \textbf{Tagged} \quad & \qquad +R_1 \cos(\Delta M_s t) + S_1 \sin(\Delta M_s t)] \\
& \qquad \times e^{-\Gamma_s t} |A_f|^2 \,, \\
\Gamma(B_s(t) \to f) + \Gamma(\overline{B_s}(t) \to f) \quad &= \quad [P_2 \sinh(\Delta\Gamma_s t/2) + Q_2 \cosh(\Delta\Gamma_s t/2) \\
\textbf{Untagged} \quad & \qquad +R_2 \cos(\Delta M_s t) + S_2 \sin(\Delta M_s t)] \\
& \qquad \times e^{-\Gamma_s t} |A_f|^2 \,,
\end{aligned}
$$

Absence of CPT violation means $P_1 = Q_1 = R_2 = S_2 = 0$

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Similar observables $\bar{P}_1 - \bar{S}_2$ for $B_s \to \bar{f}(= D_s^- K^+)$

$$\frac{R_1 + \bar{R}_1}{P_2 + \bar{P}_2} = \frac{\mathrm{Re}(\delta)}{2}, \qquad \frac{Q_2 - \bar{Q}_2}{S_1 - \bar{S}_1} = \frac{\mathrm{Im}(\delta)}{2}.$$

Hadronic uncertainties and BSM effects in mixing cancel out in the ratio!

One can refine the analysis. LHCb with 200 fb^{-1} can reach up to $\mathrm{Re}(\delta) \sim 0.1$

[AK, Nandi, Patra, Soni, PRD 2013]

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CP violation can be present in decay only and not mixing. Parametrize by some complex parameter y_f.

$$
\begin{aligned}
A(B_s \to D_s^+ K^-) &= T_1 e^{i\gamma} (1 - y_f) \\
A(B_s \to D_s^- K^+) &= T_2 (1 + y_f^*) \\
A(\overline{B_s} \to D_s^+ K^-) &= T_2 (1 - y_f) \\
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\end{aligned}
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\begin{aligned}
A_{br}^{CPT} &= \frac{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle - \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle}{\langle \mathrm{Br}(B_s \to D_s^+ K^-)\rangle + \langle \mathrm{Br}(B_s \to D_s^- K^+)\rangle} \\
&= -2\,\frac{\mathrm{Re}(y_f)}{1 + |y_f|^2} \approx -2\,\mathrm{Re}(y_f)
\end{aligned}
$$

LHCb at 200 fb^{-1}: $\mathrm{Re}(y_f) \sim 0.003$

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Triple-product asymmetries

- Consider $B \to V_1 V_2$

$$B(p) \to V_1(k_1, \varepsilon_1) + V_2(k_2, \varepsilon_2)$$

- Construct $\alpha \equiv \vec{k}_1.(\vec{\varepsilon}_1 \times \vec{\varepsilon}_2)$
- The asymmetry

$$\frac{\Gamma(\alpha > 0) - \Gamma(\alpha < 0)}{\Gamma(\alpha > 0) + \Gamma(\alpha < 0)}$$

  is odd under the time-reversal operator T. If CPT holds, this is a signal of CP violation.
- TP asymmetries should be observables in other systems too.

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- A lot of TPs are zero in SM but nonzero in BSM with a second amplitude. TPs can be nonzero even if the strong phase difference is zero.
- One can also relate the s,p,d wave amplitudes with the so-called transversity amplitudes $A_0, A_{||}, A_\perp$
- Final state decay distributions probe the interference terms of these amplitudes — probe for T violation.
- Some asymmetries are zero in SM and CPT conserving BSM but become nonzero in SM + CPTV.

[AK and Patra 2013]

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Conclusions

- CPT is supposed to be a good symmetry in any local Lorentz-invariant QFT.
- However, CPT may be violated if LI is broken. LI can be broken by string interactions, noncommutative coordinates, strong gravity ....
- CPTV needs LV, the reverse is not true.
- Various particle physics tests for CPTV: neutrinos, $(g-2)_{e,\mu}$, etc.
- Strong bounds in K and B_d systems.
- Time to look in B_s, LHC can uncover such signals. We should stay tuned.

Thank you. Bon appetit.

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