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Formation of primordial black holes

- A novel formulation of PBH mass function-

Ref. TS and S.Yokoyama, arXiv:1912.04687 (*PTEP* 2020 (2020) 2, 023E03)

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PBHs=BHs that formed in the very early Universe

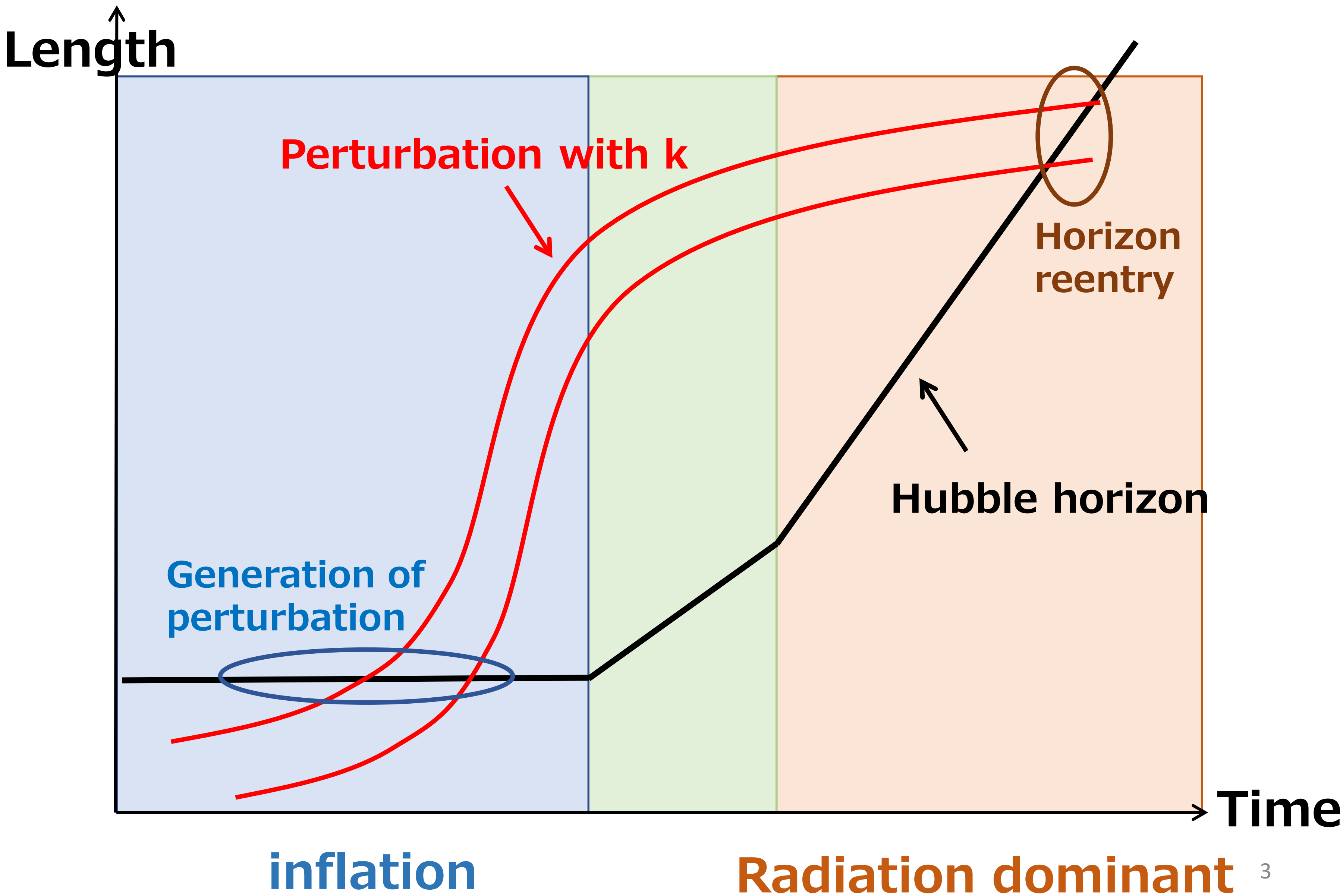
A renewed interest in PBH

An interesting possibility that BHs detected by LIGO/Virgo are PBHs.

PBHs (in some mass range) may explain dark matter.

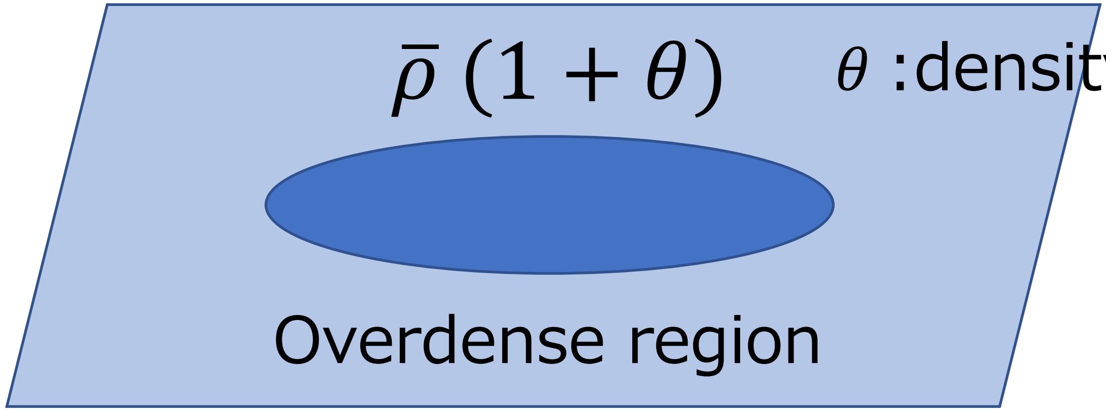
PBHs provide a unique probe of small-scale primordial perturbations.

Formation of a PBH



PBH formation

e.g. Carr 1975



θ :density constrast

Formation criterion

Gravity > Radiation pressure

$$\theta > \theta_{th} \quad (\theta_{th} \simeq 0.8)$$

PBH mass

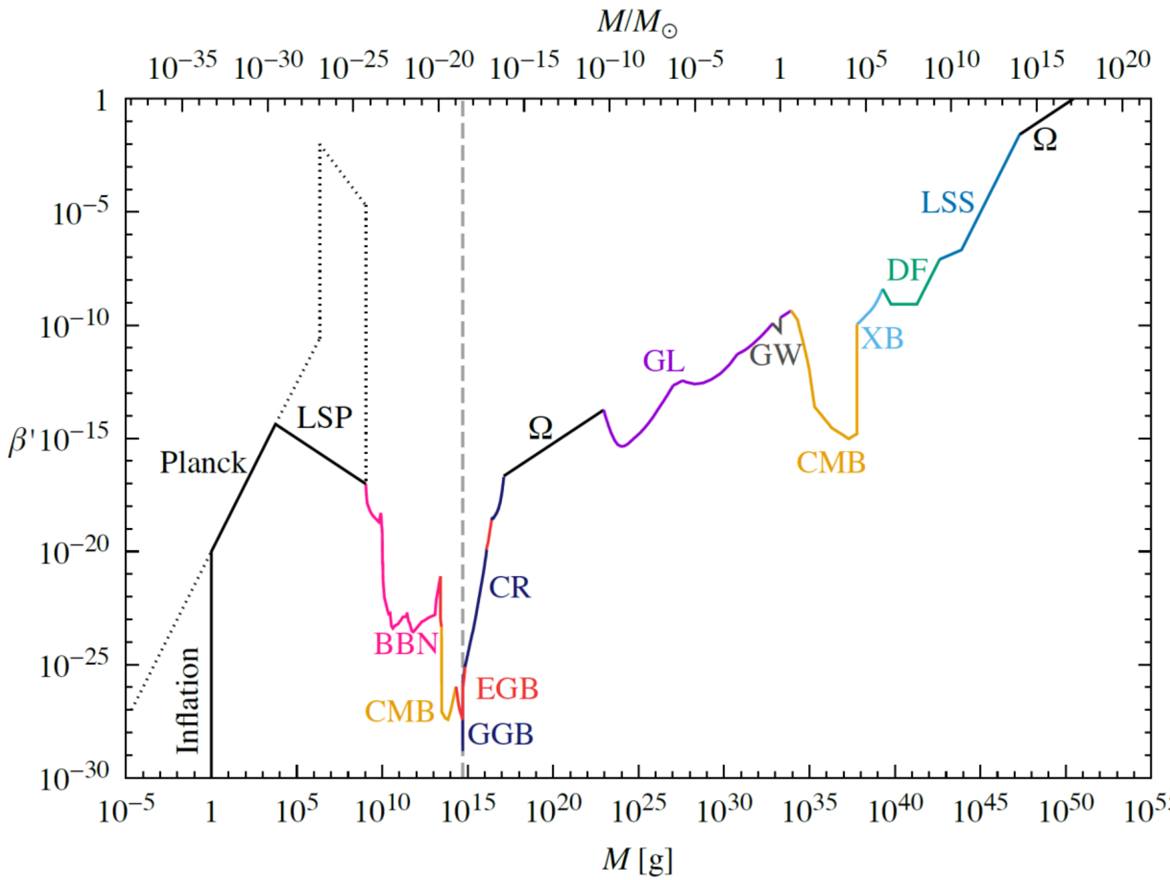
$$M_{\mathrm{PBH}} \sim \frac{1}{GH} \sim 10 M_{\odot}\left(\frac{t}{0.1ms}\right) \sim 10 M_{\odot}\left(\frac{k}{1\mathrm{pc}^{-1}}\right)^{-2}$$

Radiation (quarks, leptons, photons..) forms a BH.

PBHs can be much lighter than stars

Relevant scales << scales of CMB and LSS

Upper limits on the PBH abundance



Carr+ 2020

y-axis: energy fraction of PBHs at their formation time

Monochromatic mass function is assumed.

Upper limits on extended mass function is possible. Carr+ 2017

How do we compute the PBH mass function?

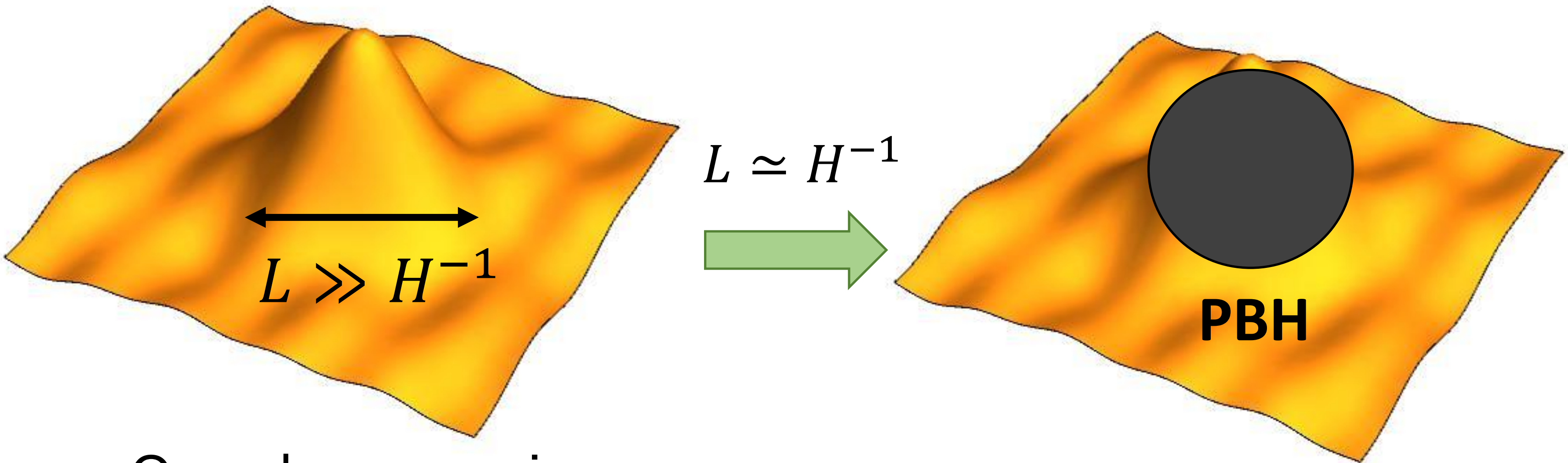
Indispensable in comparing your theory with the PBH constraints

Definition of the PBH mass function: $f(M)$

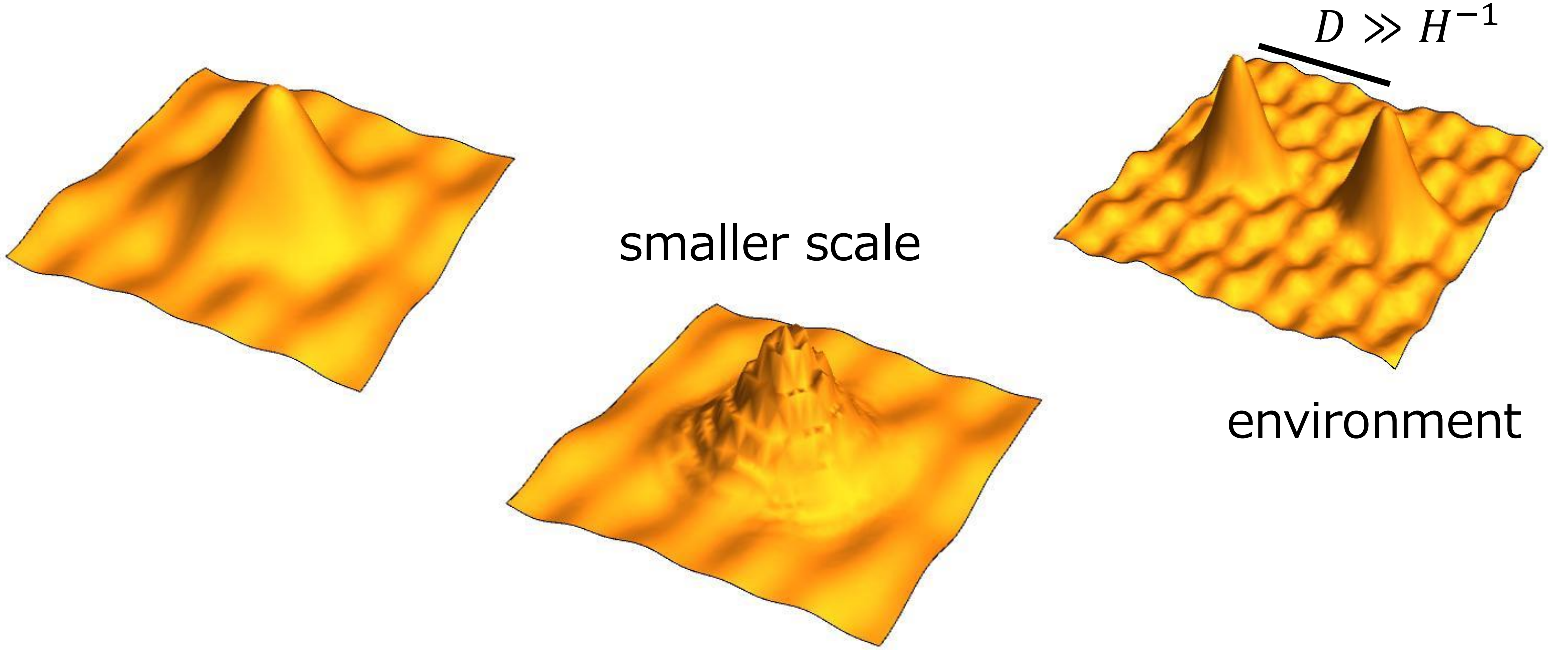
$$\int f(M) d \ln M = 1$$

represents a probability in infinitesimal mass bin $(M, M + dM)$.

$\theta > \theta_{th}$



Overdense region



Environment and smaller scale are not relevant to PBH formation

PBH formation

window function

$$\theta_R(\vec{x}) = \int W(R, \vec{x}-\vec{y})\theta(\vec{y})d^3y$$

$$\theta_R > \theta_{th} \qquad M(R) = 1/H(R)$$

Computations of the PBH mass function

1. Press-Schechter-like approach

   Kim&Lee 1996, Carr+ 2009, Carr+ 2017, Carr+ 2018, Byrnes+ 2018,
   Kawasaki+ 2019, Young+ 2019, Wang+ 2019

2. Peak theory

   Green 2004, Young+ 2014, Yoo+ 2018, Germani+ 2018, Kalaja+ 2019

There remains a conceptually unclear point.

1. Press-Schechter-like approach

$$\beta(R) = \int_{\theta_{\rm th}}^{\infty} P(\theta_R) d\theta_R,$$

$P(\theta_R)$: pdf of θ_R

A. *Interpret* $\beta(R)$ as an energy fraction of PBHs with their mass $> M(R)$.

$$f(M) \propto \frac{d}{dR}\beta(R)$$

Kim&Lee 1996

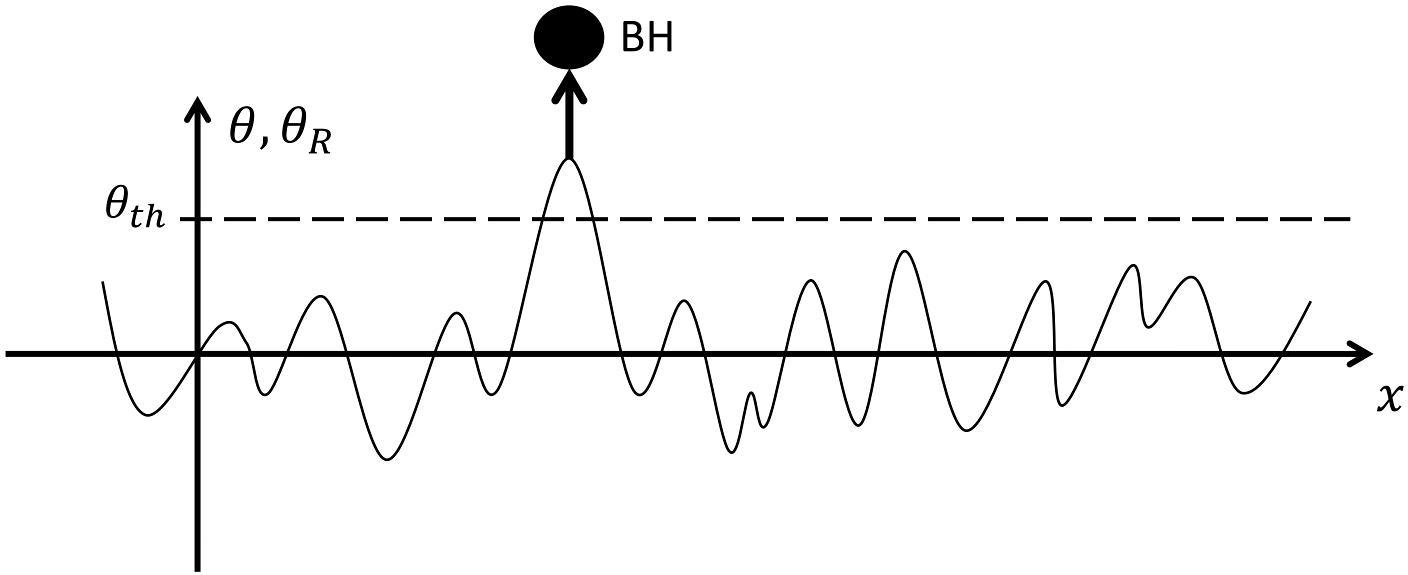
B. *Interpret* $\beta(R)$ as an energy fraction of PBHs per logarithmic bin.

$$f(M) \propto \beta(R)$$

Carr+ 2009

These prescriptions are not conceptually clear.

2. Peak theory



A. Peak number density based on θ Yoo+2018, Germani+ 2018

Mass function is automatically derived.
But no smoothing procedure.

B. Peak number density based on θ_R Kalaja+ 2019

Same issue as in the previous slide

PTEP

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A novel formulation of the primordial black hole mass function

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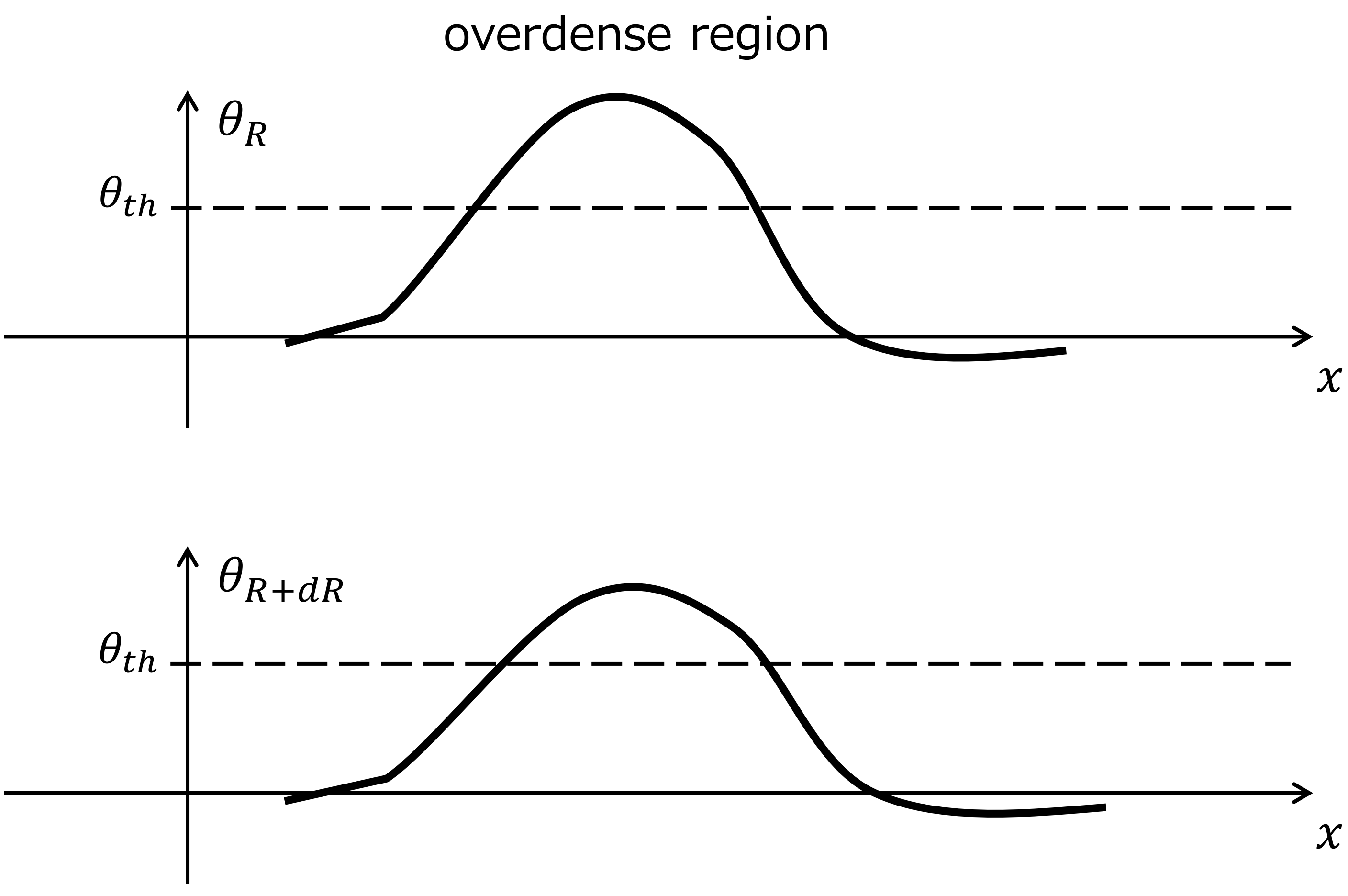
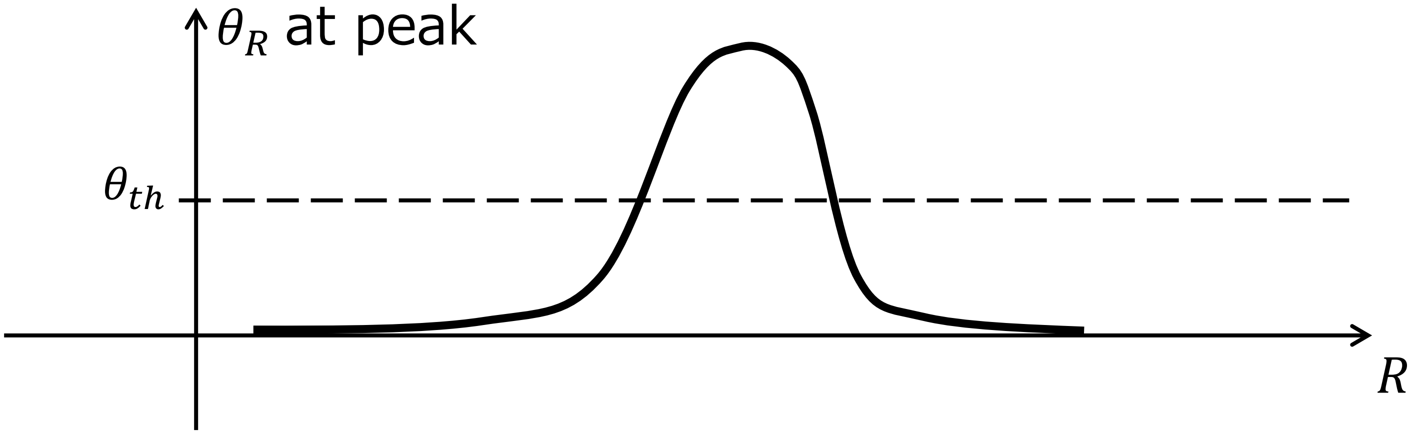
Computations of the primordial black hole (PBH) mass function discussed in the literature have conceptual issues. They stem from the fact that the mass function is a differential quantity and the standard criterion of the PBH formation from the seed primordial fluctuations cannot be directly applied to the computation of the differential quantities. We propose a new criterion of

Basic idea

To add one extra condition to the existing ones on the PBH formation





There is a narrow finite range of R where $\theta_R > \theta_{th}$.

Outcome: just a formation of a BH.

We identify the scale of the produced PBH at a point where

$$\frac{\partial}{\partial R}\theta_R = 0$$

Conditions for the PBH formation

1. $$\frac{\partial}{\partial x^i}\theta_R = 0, \qquad \frac{\partial}{\partial R}\theta_R = 0$$

Our new ingredient

2. $$\theta_R > \theta_{th}$$

3. $$\lambda_a\left(\theta_{R,ij}\right) < 0$$ **Peak is maximum**

The new ingredient enables us to formulate the mass function without introducing artificial interpretation.

PBH mass function (**main result**)

$$f(M) = \frac{M}{n_{\rm PBH}} \int dR \, \langle J \delta(M - m(R,\theta_R)) \Theta(\theta_R - \theta_{\rm th}) \prod_{a=1}^{4} \delta(\theta_{R,a}) \Theta(-\lambda_a) \rangle$$

$$J = \begin{vmatrix} \theta_{R,xx} & \theta_{R,xy} & \theta_{R,xz} & \theta_{R,xR} \\ \theta_{R,yx} & \theta_{R,yy} & \theta_{R,yz} & \theta_{R,yR} \\ \theta_{R,zx} & \theta_{R,zy} & \theta_{R,zz} & \theta_{R,zR} \\ \theta_{R,Rx} & \theta_{R,Ry} & \theta_{R,Rz} & \theta_{R,RR} \end{vmatrix}$$

$$\langle \mathcal{F} \rangle = \int [d\theta] \, \mathcal{F}(\theta) P[\theta]$$

$P[\theta]$: Probability density of θ

1. Valid for non-Gaussian θ (encoded in $P[\theta]$)
2. Effect of the critical collapse is included ($m(R, \theta_R)$).

Application: Gaussian case

Tokeshi+, arXiv:2005.07153

We ignore the critical collapse.

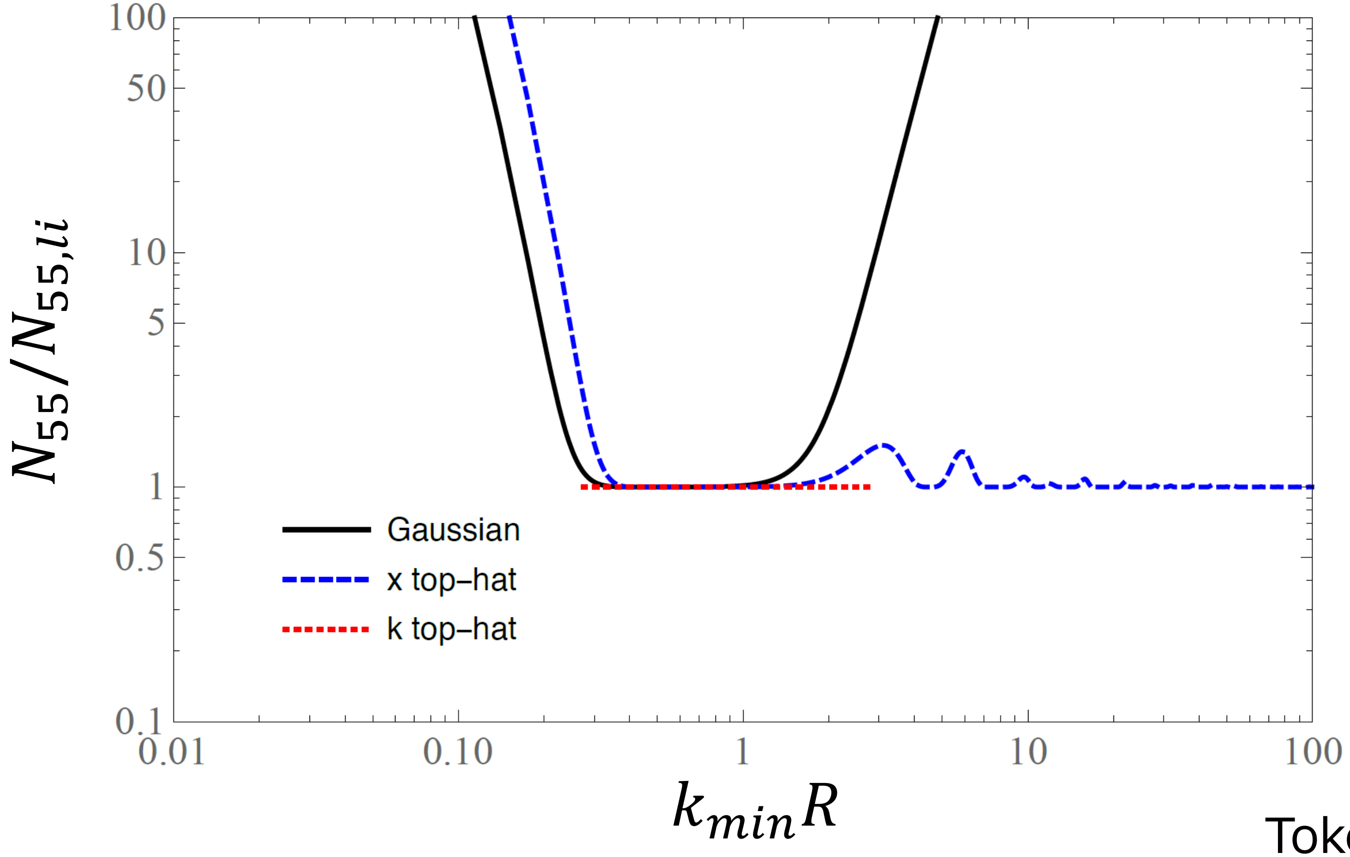
$$f(M) = \frac{M}{n_{\rm PBH} m'(R)} I(M)$$

$$\begin{aligned} I(M) = &\frac{1}{\sqrt{\det(2\pi L)}}\sqrt{\frac{2}{N_{55}}} \\ &\times \left[\left(A + B_{ij}N_{ij} + 3C_{ijkl}N_{ij}N_{kl} - \frac{(B_{ij}+6C_{ijkl}N_{kl})N_{i5}N_{j5}}{N_{55}} + \frac{3C_{ijkl}N_{i5}N_{j5}N_{k5}N_{l5}}{N_{55}^2}\right)\right. \\ &\quad \times \frac{\sqrt{\pi}}{2}\mathrm{erfc}\left(\sqrt{\frac{N_{55}}{2}}\,\delta_{\rm th}\right) \\ &\quad + \frac{\delta_{\rm th}}{\sqrt{2N_{55}}}\left((B_{ij}+6C_{ijkl}N_{kl})N_{i5}N_{j5} + \frac{\delta_{\rm th}(3+N_{55}\delta_{\rm th}^2)C_{ijkl}N_{i5}N_{j5}N_{k5}N_{l5}}{N_{55}}\right) \\ &\quad \left.\times \exp\left(-\frac{N_{55}}{2}\delta_{\rm th}^2\right)\right] , \end{aligned} \tag{2.9}$$

$$N_{55} = \frac{\langle {\theta_{R,R}}^2\rangle}{\langle {\theta_R}^2\rangle\langle {\theta_{R,R}}^2\rangle - \langle \theta_R\theta_{R,R}\rangle^2}$$

($N_{55,li} = \frac{1}{\langle {\theta_R}^2\rangle}$ in the literature)

$$P_{\mathcal{R}}(k) = A\,\Theta(k - k_{min})\Theta(k_{max} - k) \qquad k_{max} = 10k_{min}$$



Tokeshi+ 2020

New formulation yields narrower PBH mass function than the conventional one.

Summary

Computations of the PBH mass function in the literature have conceptual issues.

We proposed a new criterion of the PBH formation and provided a new formulation of the PBH mass function.