The bouncing scenario

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Framework

Homogeneous & isotropic metric (FLRW): $\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)\right]$

$\mathcal{K}$ → spatial curvature

Hubble rate $H \equiv \frac{\dot{a}}{a}$

Matter component: perfect fluid $T_{\mu\nu} = p g_{\mu\nu} + (\rho + p)\, u_\mu u_\nu$

equation of state $p = w\rho$ → $\begin{cases} w = 0 & \text{dust} \\ w = \frac{1}{3} & \text{radiation} \end{cases}$

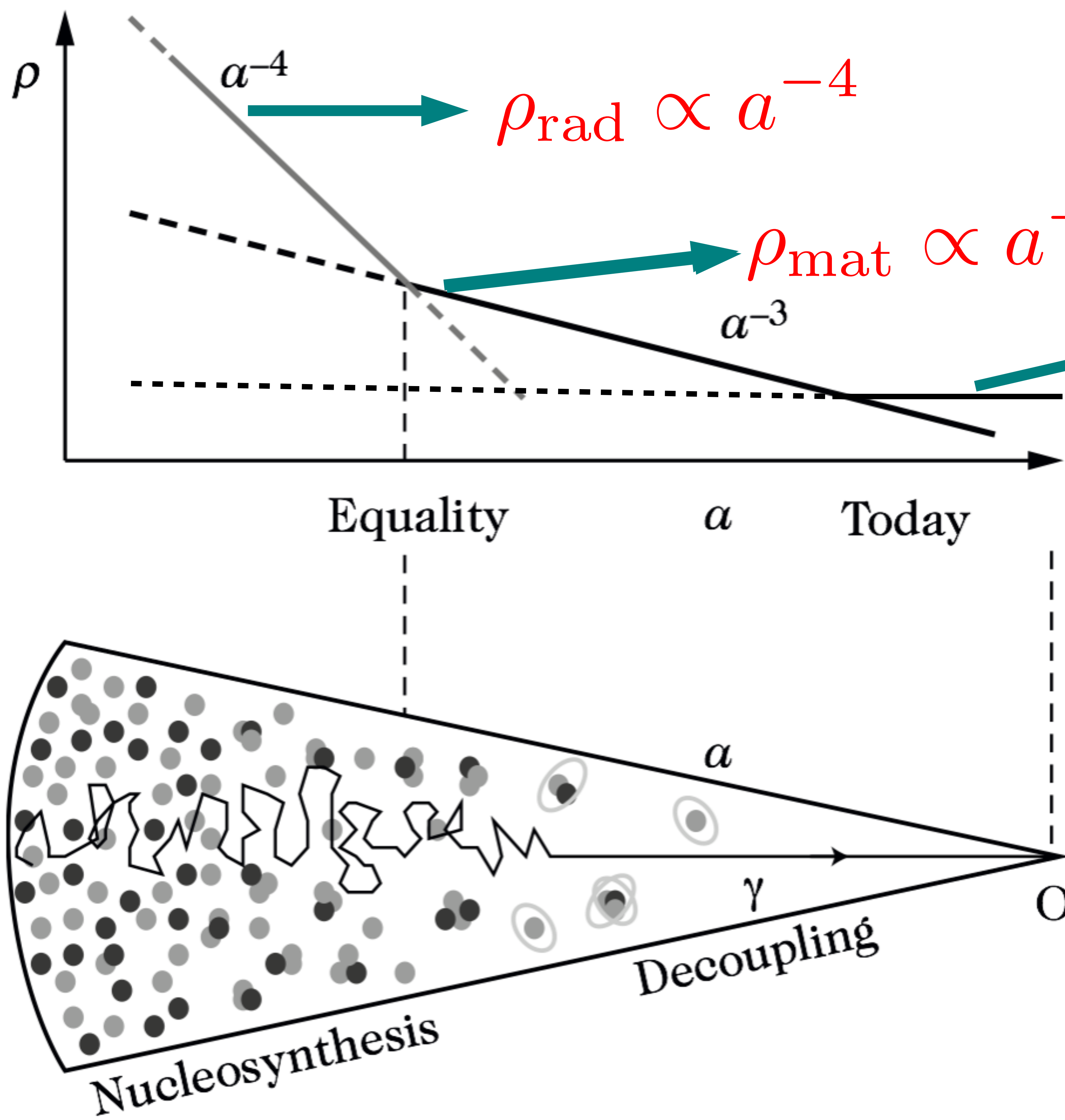
+ cosmological constant = Einstein equations

$$\begin{cases} H^2 + \dfrac{\mathcal{K}}{a^2} = \dfrac{1}{3}\left(8\pi G_{\mathrm{N}}\rho + \Lambda\right) \\ \dfrac{\ddot{a}}{a} = \dfrac{1}{3}\left[\Lambda - 4\pi G_{\mathrm{N}}\left(\rho + 3p\right)\right] \end{cases}$$

Particular solution: dust and radiation

integrate conservation equation

$$\rho\left[a\left(t\right)\right] = \rho_{\text{ini}} \exp\left\{-3\int\left[1+w(a)\right]\mathrm{d}\,\ln a\right\} \underset{w\to\text{cst}}{=} \rho_{\text{ini}}\left(\frac{a}{a_{\text{ini}}}\right)^{-3(1+w)}$$



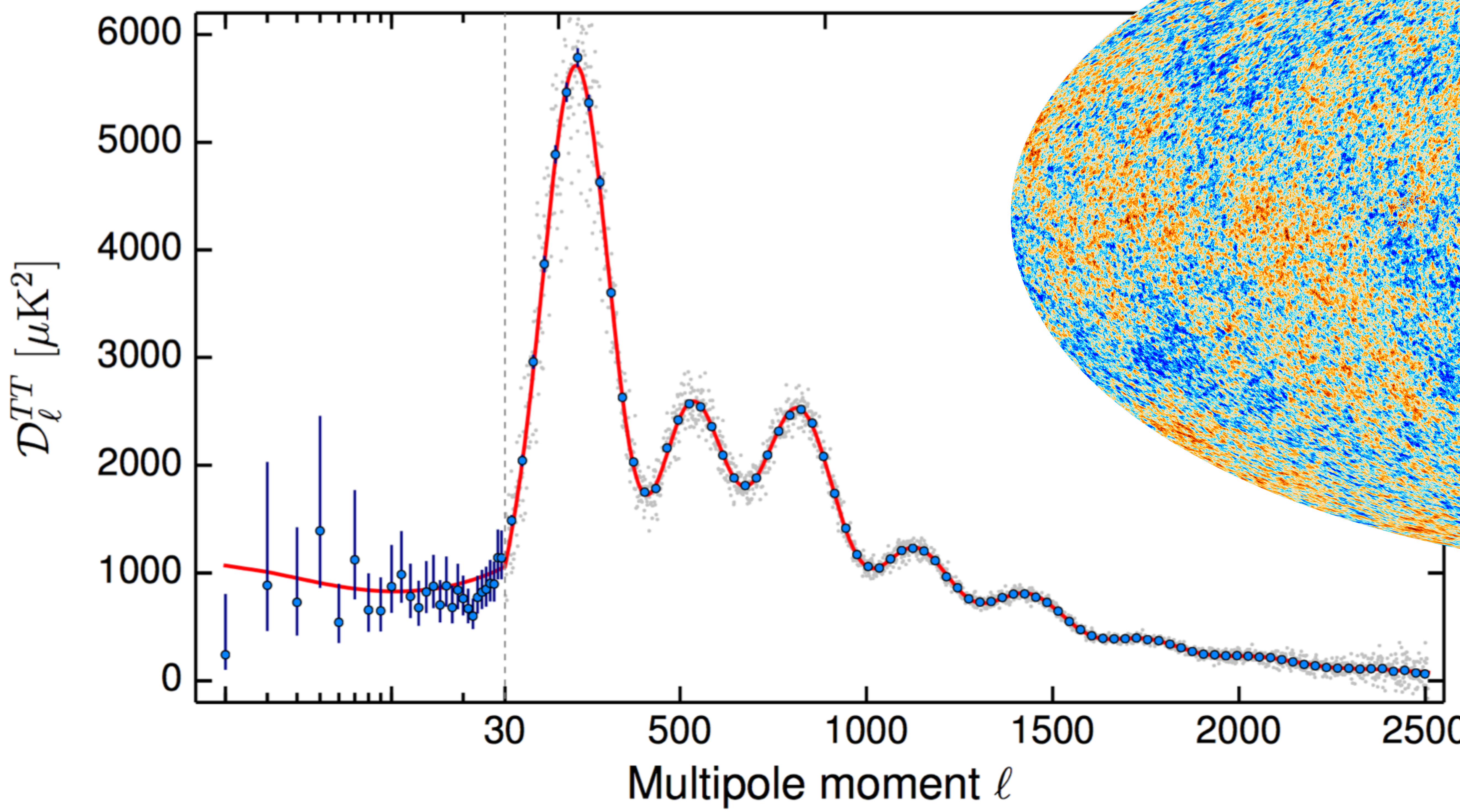
$\rho_{\text{rad}} \propto a^{-4}$

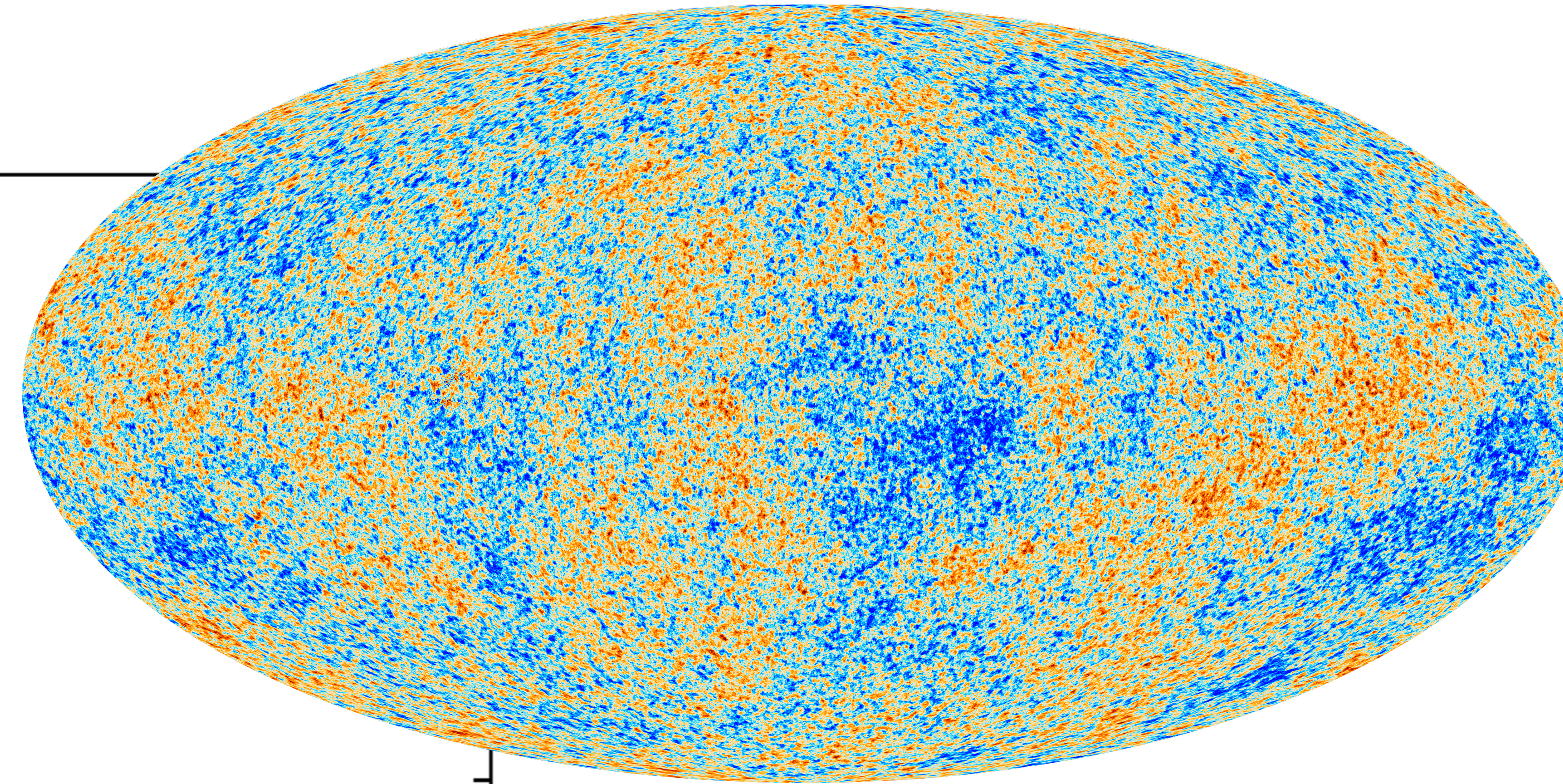
$\rho_{\text{mat}} \propto a^{-3}$

$\rho_{\Lambda} \propto a^{0}$

Phenomenologically valid description for 14 Gyrs!!!!

Planck 2015





$\Omega_\mathcal{K} = 0.000 \pm 0.005$

$n_\mathrm{s} = 0.9639 \pm 0.0047$ almost scale invariant

excluded

$f_\mathrm{NL}^\mathrm{loc} = 0.8 \pm 5$
$f_\mathrm{NL}^\mathrm{eq} = -4 \pm 43$
$f_\mathrm{NL}^\mathrm{ort} = -26 \pm 21$

gaussian signal

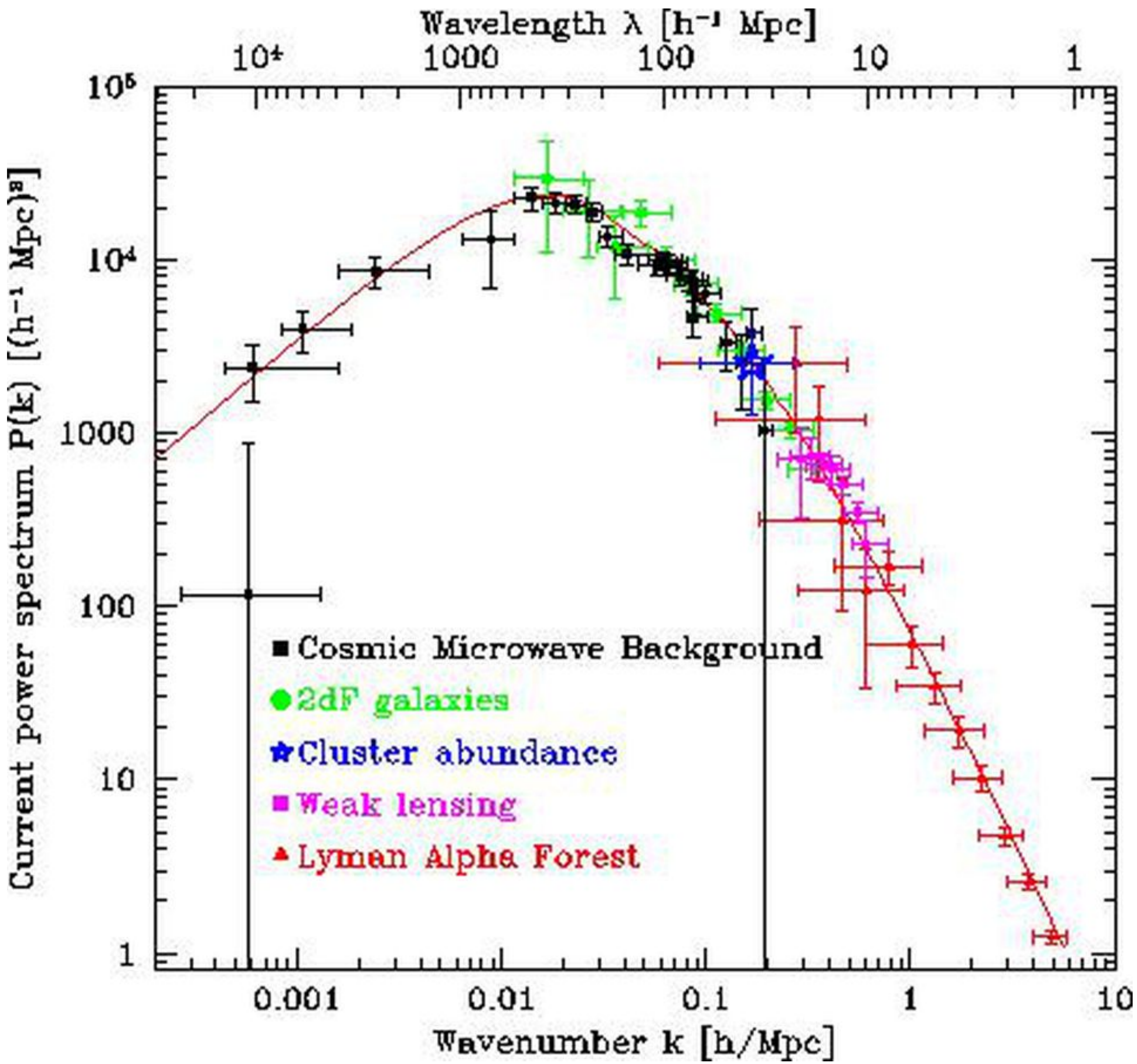
isocurvature $\lesssim 1\%$

$r < 0.08$

quantum vacuum fluctuations of a single scalar d.o.f

compatible with
INFLATION

Numerical simulation for large scale structure formation...



Standard Failures and inflationary solutions

Singularity

Not solved... actually not addressed!

Horizon

$$d_{\rm H} \equiv a(t) \int_{t_{\rm i}}^{t} \frac{{\rm d}\tau}{a(\tau)}$$

can be made as big as one wishes

Flatness

$$\frac{\rm d}{{\rm d}t} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}$$

$\ddot{a} > 0 \;\;\&\;\; \dot{a} > 0$

accelerated expansion (inflation)

Homogeneity & Isotropy

Initial Universe = very small patch
Accelerated expansion drives the shear to zero...
+ attractor

$\Longrightarrow$ vacuum state!

Perturbations

Bonus of the theory: predictions!!!

Others

dark matter/energy, baryogenesis, ...

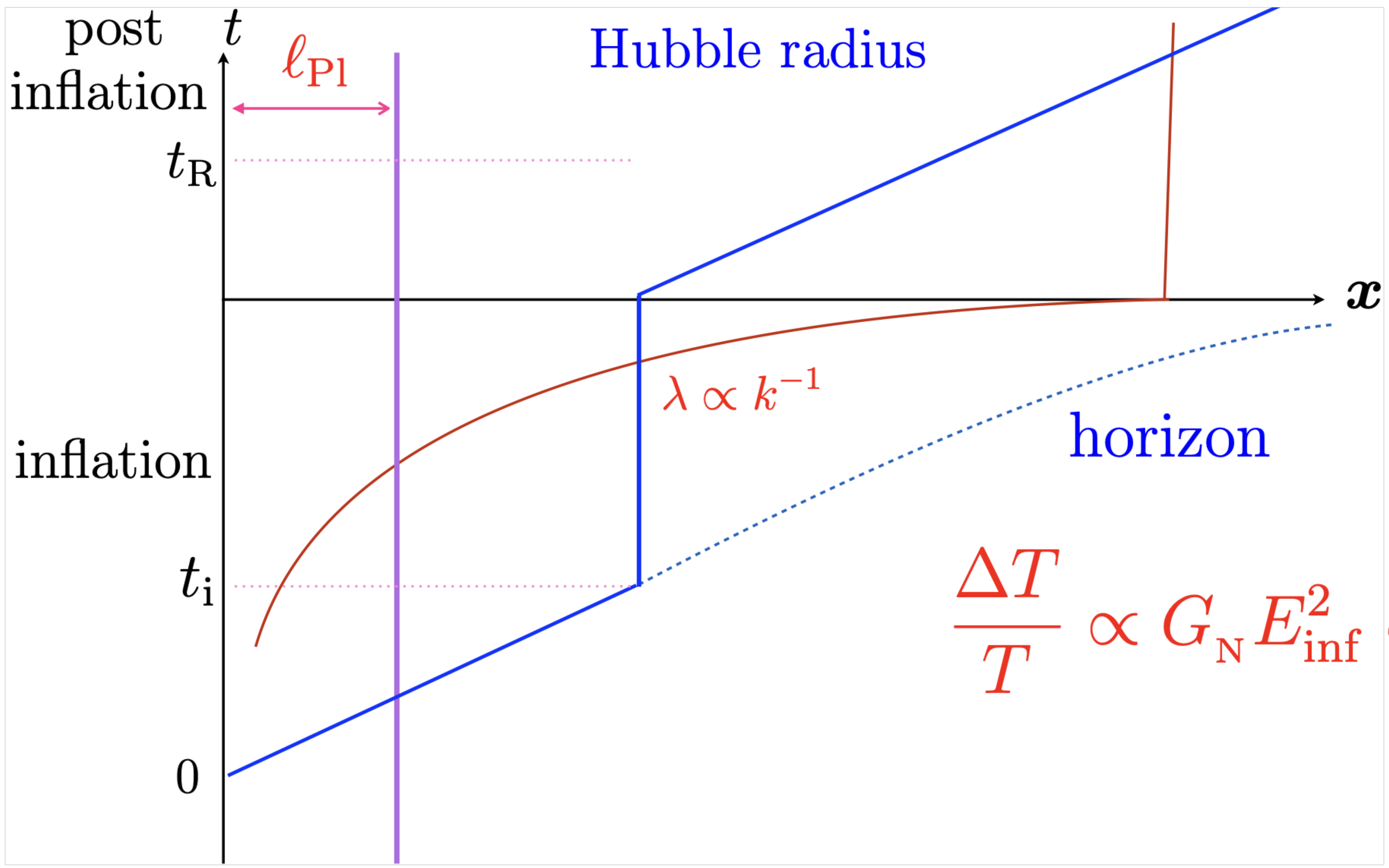
Inflation:

- ☺ solves cosmological puzzles
- ☺ uses GR + scalar fields [(semi-)classical]
- ☺ can be implemented in high energy theories
- ☺ string implementation (brane inflation, ...)

- ☺ makes falsifiable predictions ...
- ☺ ... consistent with all known observations

why bother with alternatives?

From R. Brandenberger, *in* M. Lemoine, J. Martin & PP (Eds.), "Inflationary cosmology", Lect. Notes Phys. **738** (Springer, Berlin, 2007).



- Singularity $\exists t_{(\pm\infty)}; a(t) \to 0$
- Trans-Planckian

$$\exists t; \ell(t) = \ell_0 \frac{a(t)}{a_0} \leq \ell_{\rm Pl}$$

- Hierarchy (amplitude)?

$$\frac{V(\varphi)}{\Delta\varphi^4} \leq 10^{-12}$$

- Classical GR?

$$\frac{\Delta T}{T} \propto G_{\rm N} E_{\rm inf}^2 \sim \left(\frac{E_{\rm inf}}{M_{\rm Pl}}\right)^2 \longrightarrow E_{\rm inf} \simeq 10^{-3} M_{\rm Pl}$$

- η problem & Lyth bound
- Initial condition & entropy
- Eternal inflation & measure (anthropic)

A brief history of bouncing cosmology

- R. C. Tolman, "*On the Theoretical Requirements for a Periodic Behaviour of the Universe*", PRD 38, 1758 (1931)
- G. Lemaître, "*L'Univers en expansion*", Ann. Soc. Sci. Bruxelles (1933)

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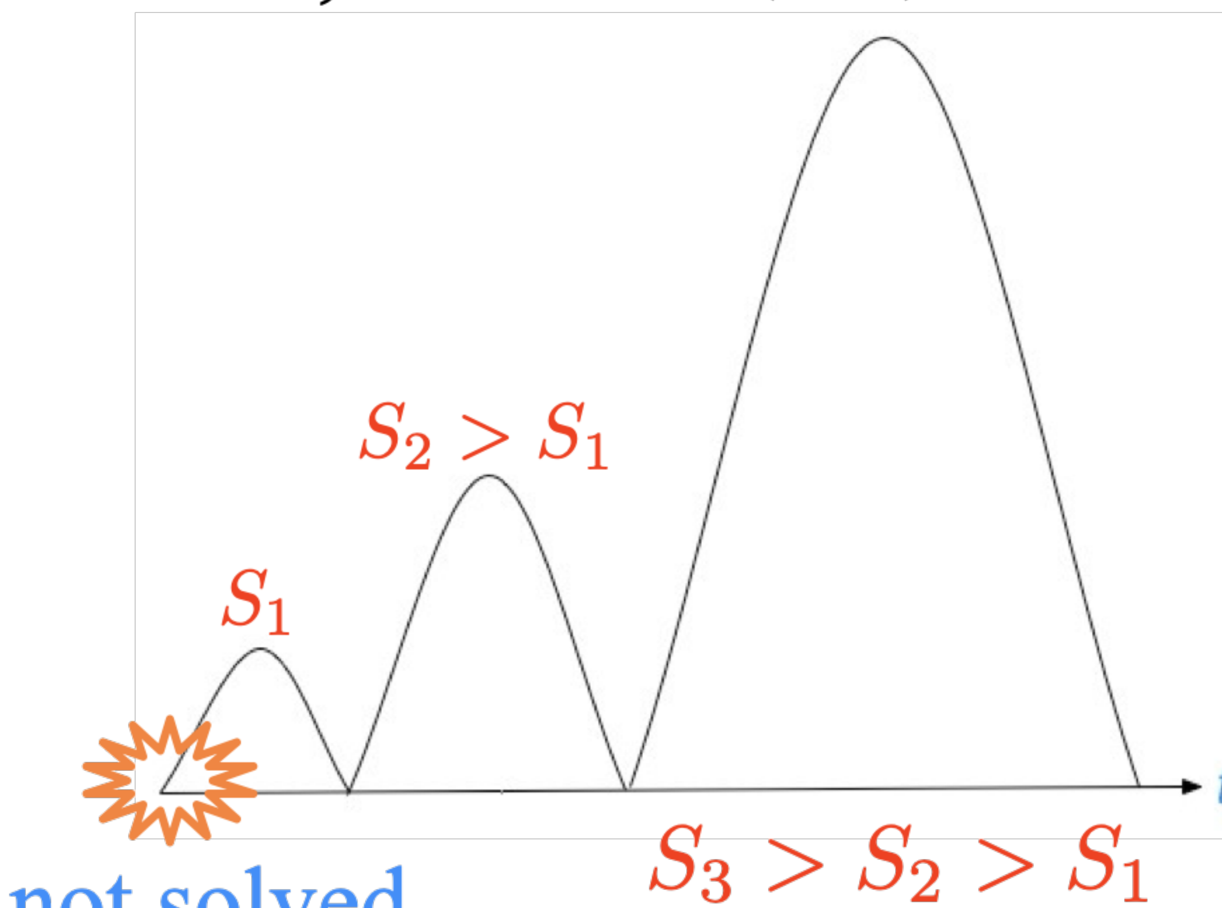


…

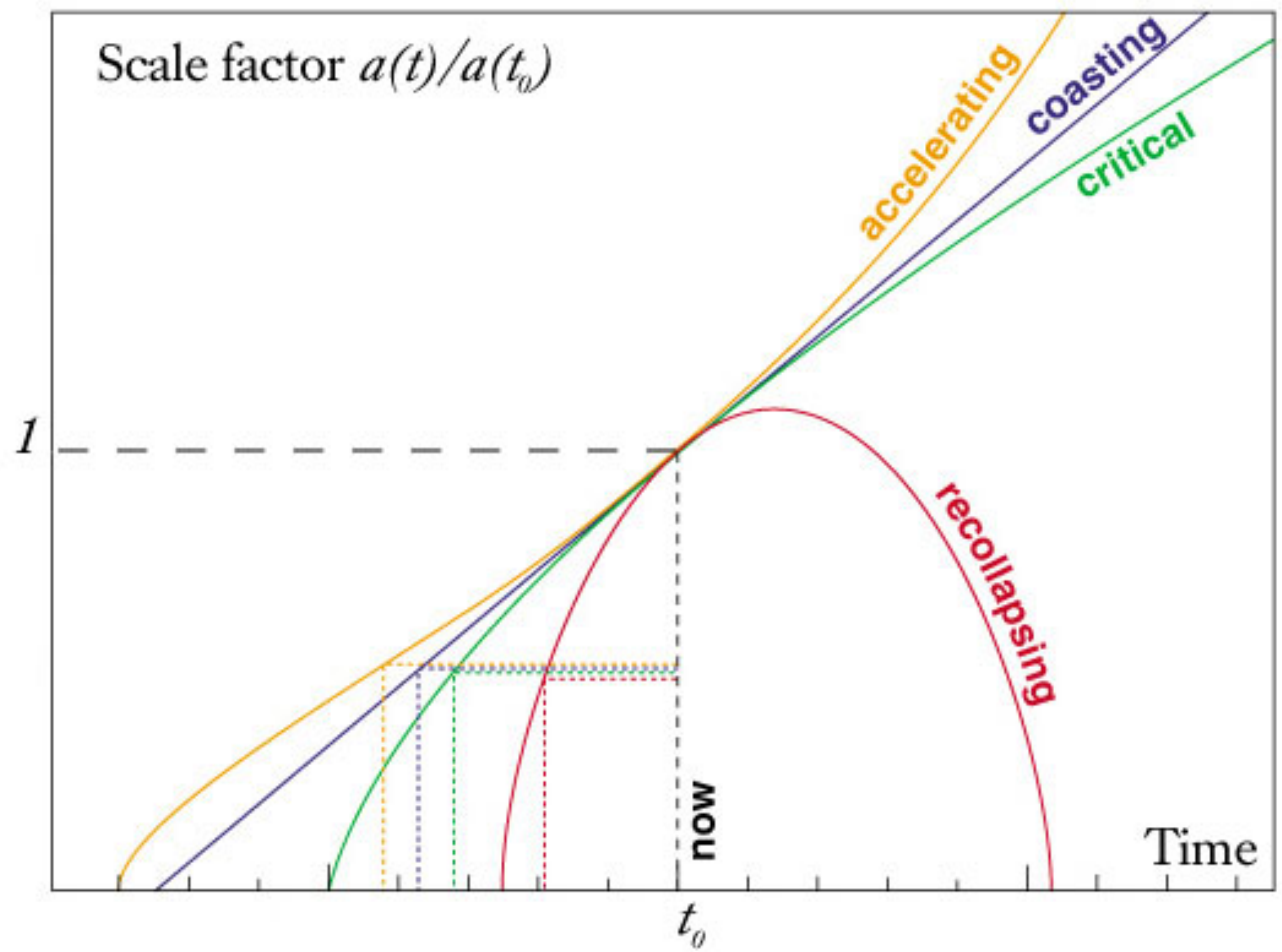
Singularity pb not solved

- A. A. Starobinsky, "*On one non-singular isotropic cosmological model*", Sov. Astron. Lett. 4, 82 (1978)
- V. N. Melnikov, S.V. Orlov, Phys. Lett. A 70, 263 (1979).
- R. Durrer & J. Laukerman, "*The oscillating Universe: an alternative to inflation*", Class. Quantum Grav. 13, 1069 (1996)

- Many new ideas, models…

- M. Novello & S.E. Perez Bergliaffa, "*Bouncing cosmologies*", Phys. Rep. **463**, 127 (2008)
- D. Battefeld & PP, "*A Critical Review of Classical Bouncing Cosmologies*", Phys. Rep. **571**, 1 (2015)
- R. Brandenberger & PP, "*Bouncing cosmologies: Progress and problems*", Found. Phys. (2017)

The issue we are interested in: the singularity



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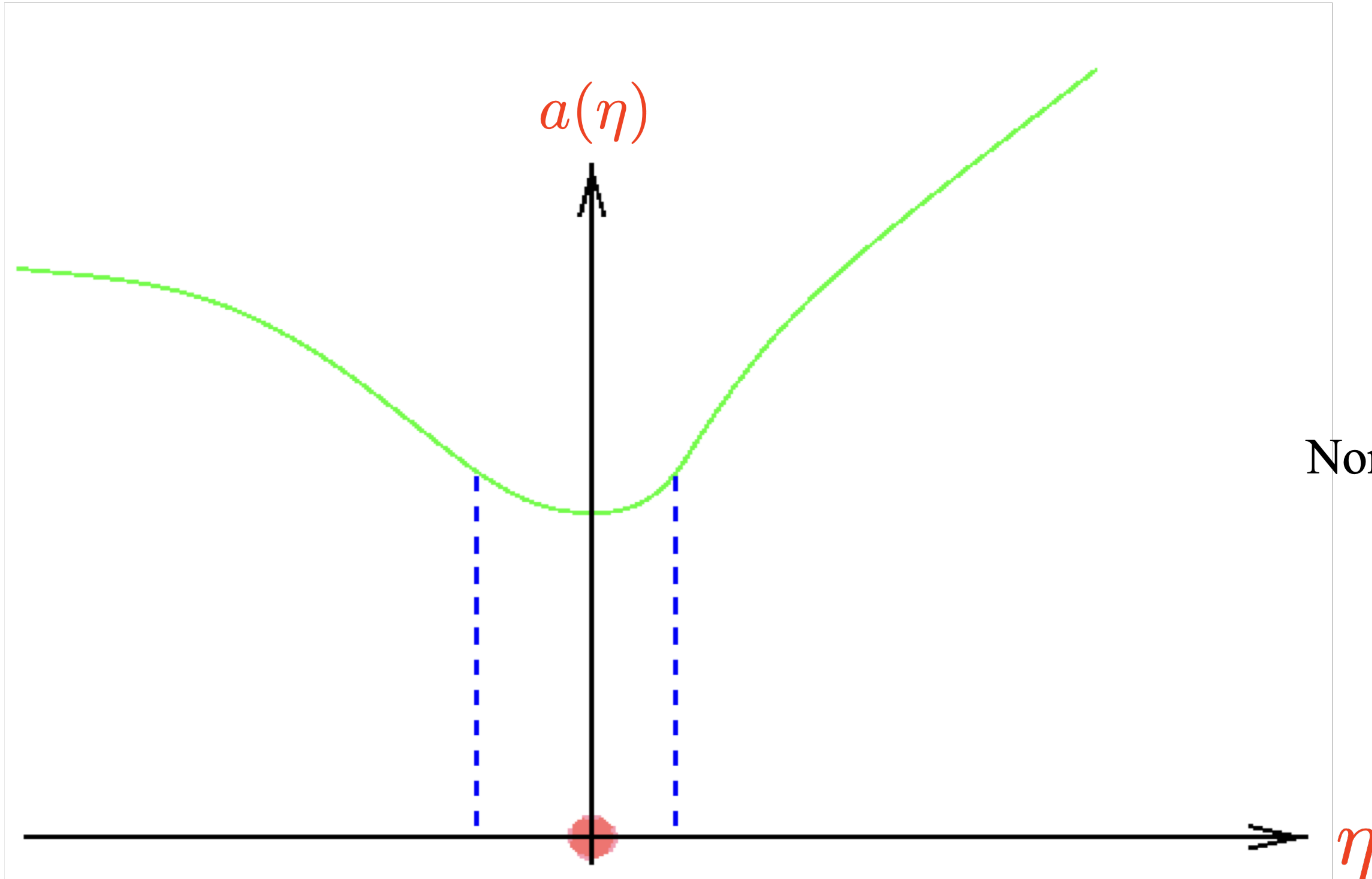


Non singular bounce

Model listing:

Quantum gravity

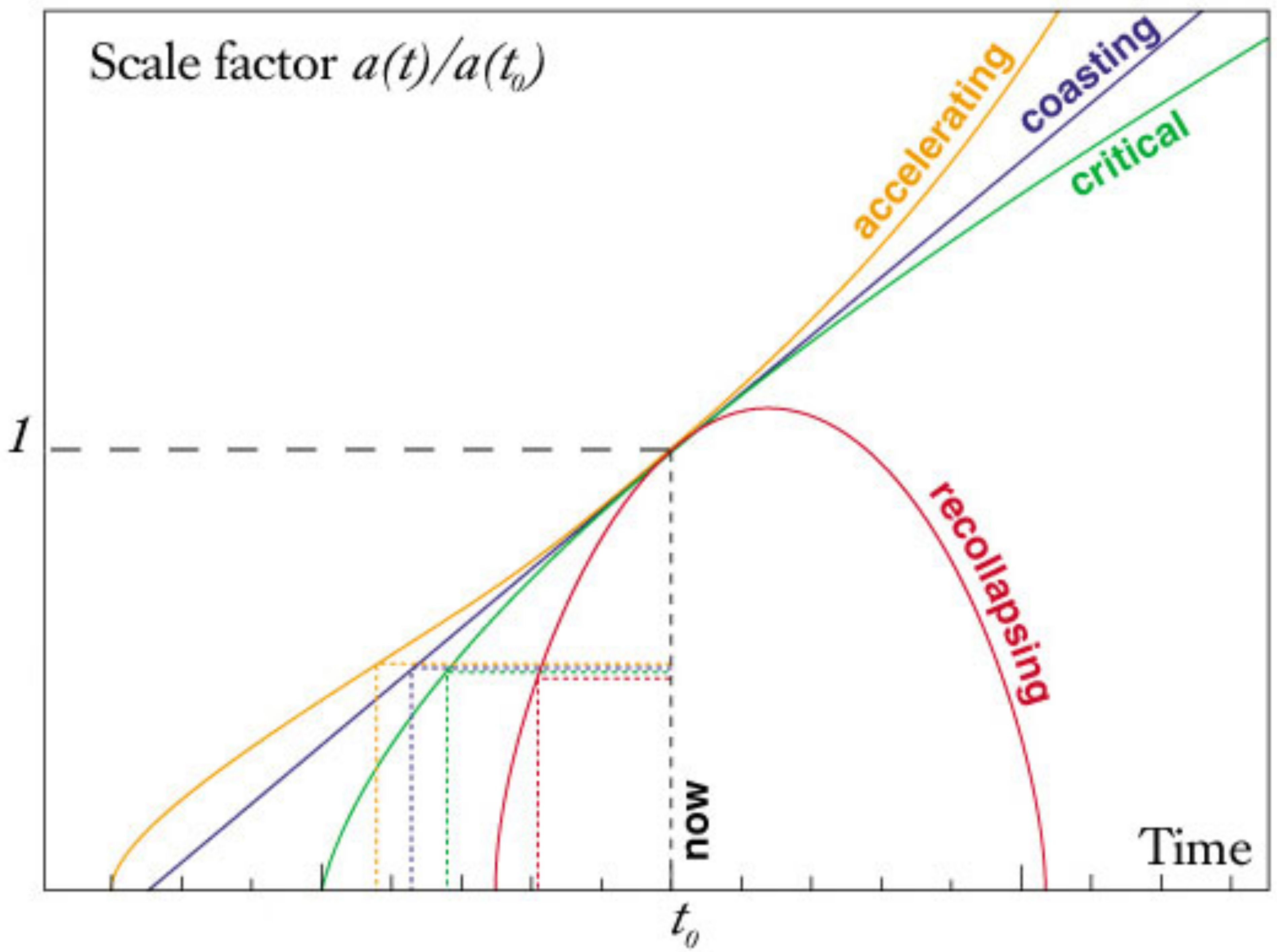
LQG & LQC

Canonical quantum gravity (WdW)

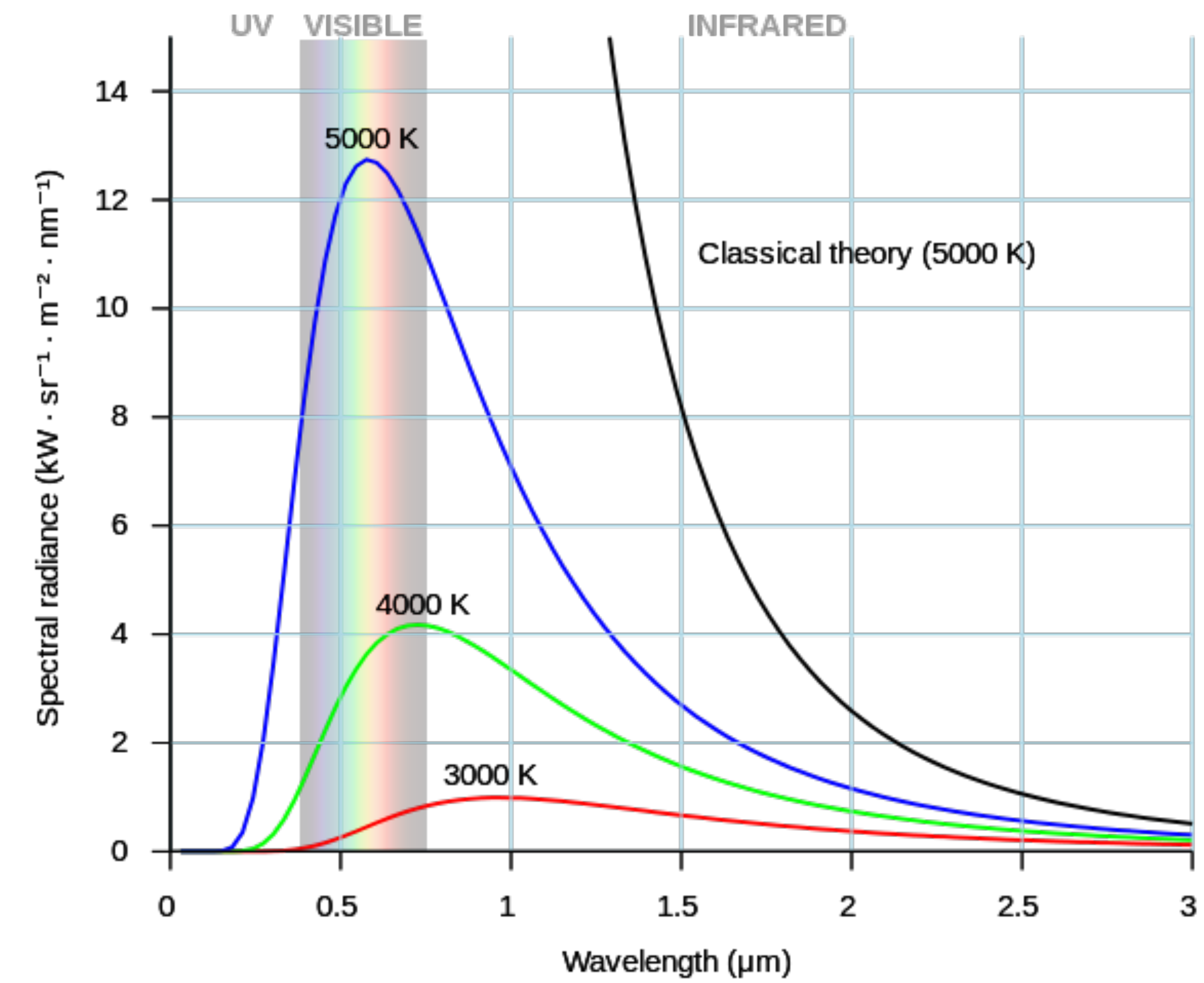
String theory

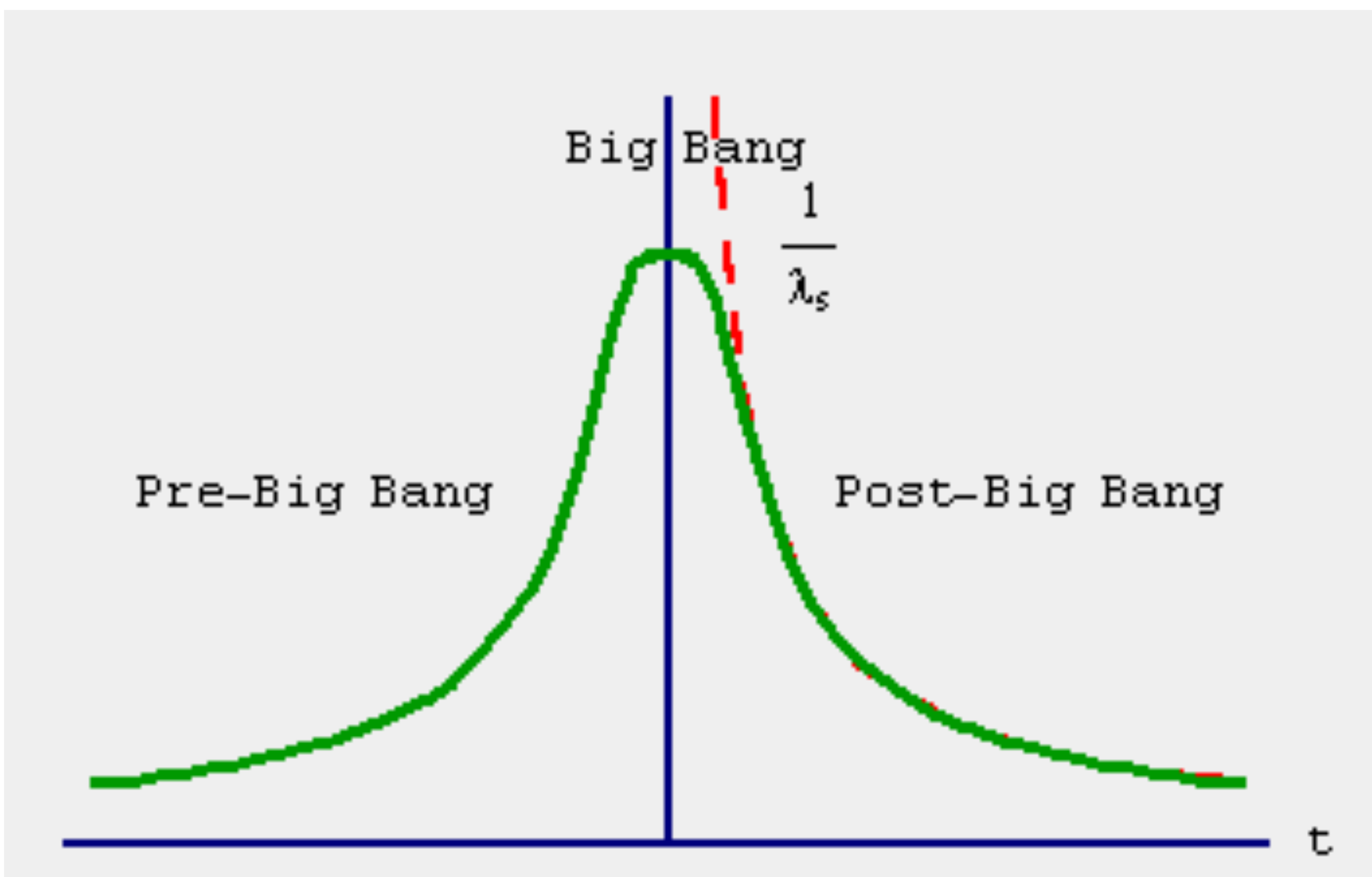
Non relativistic quantum gravity

Singularity problem...



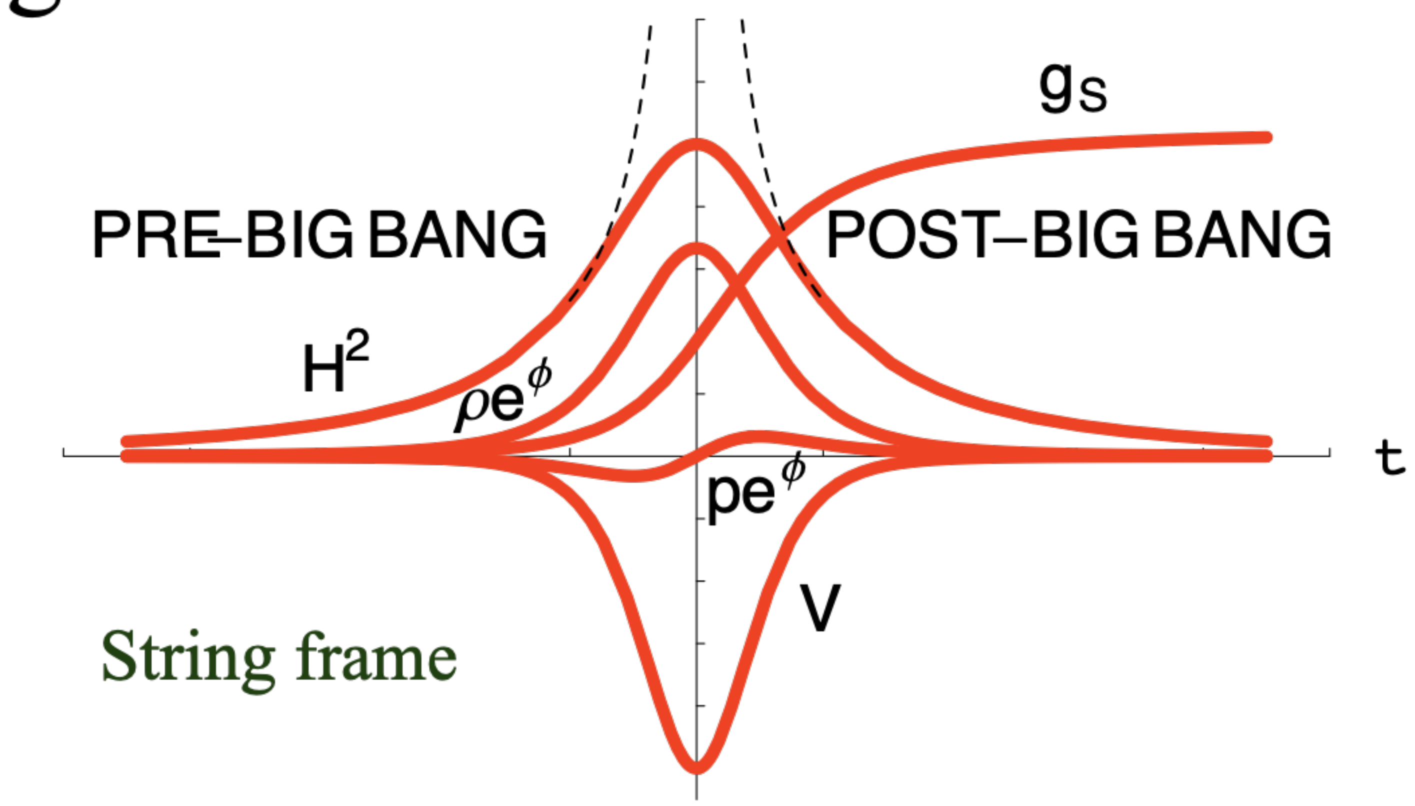
a quantum effect?

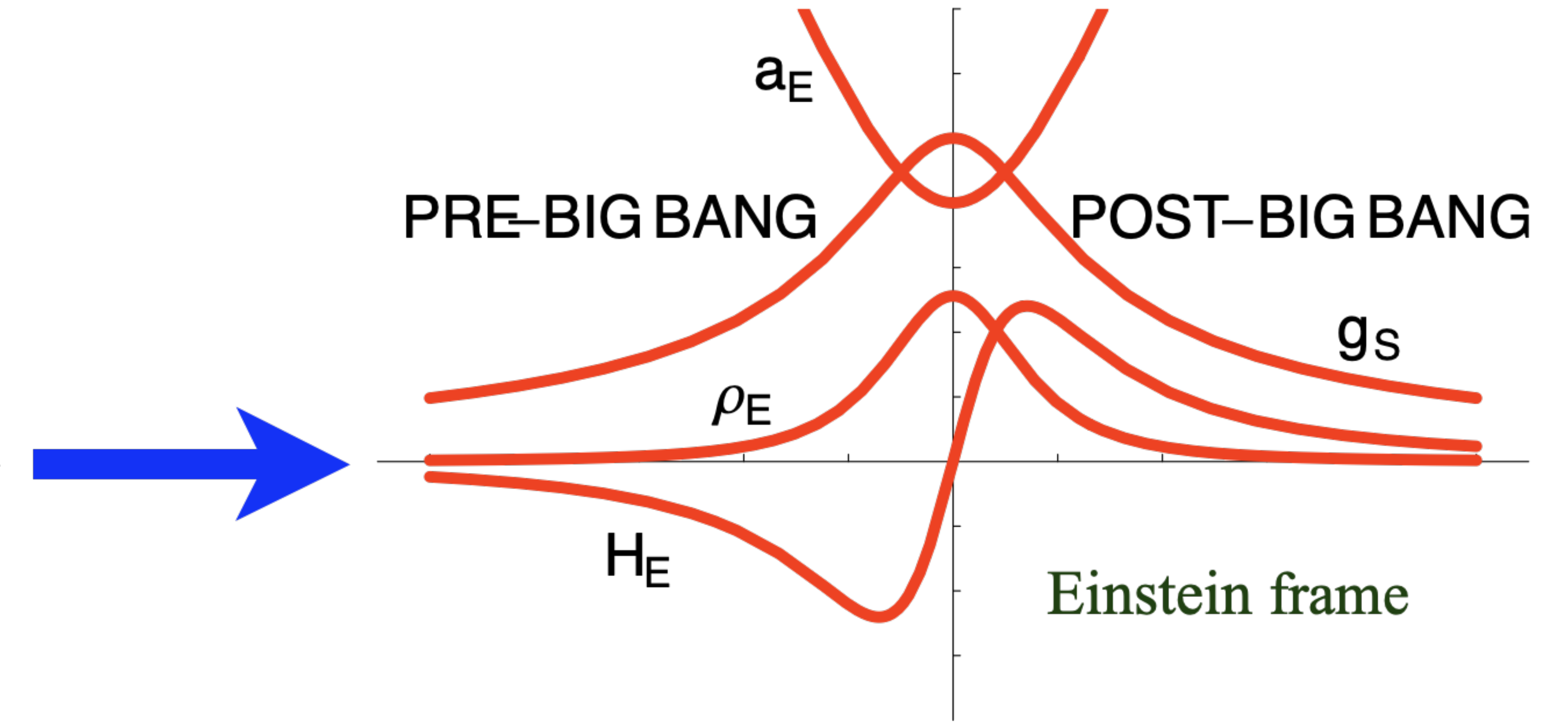




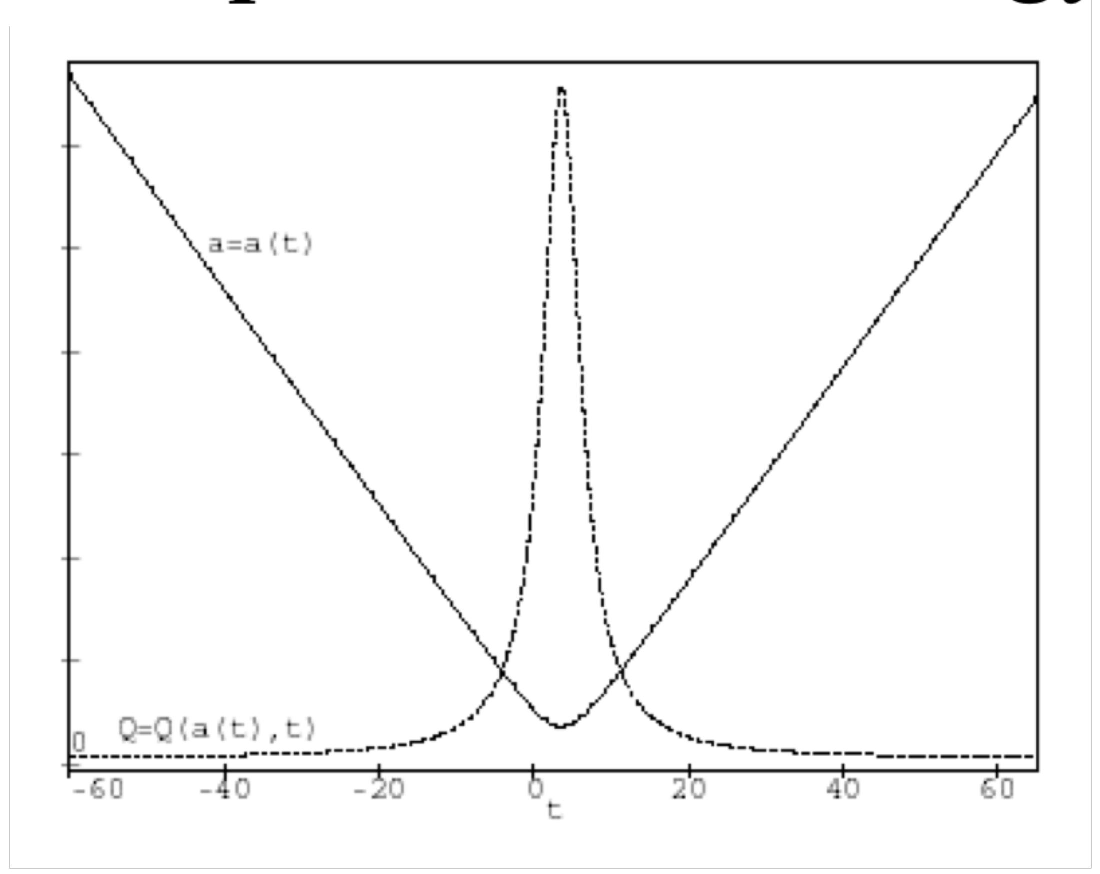
M.Gasperini & G. Veneziano, *Phys. Rep.* **373**, 1 (2003), hep-th/0207130 & hep-th/0703055

Pre Big Bang scenario:

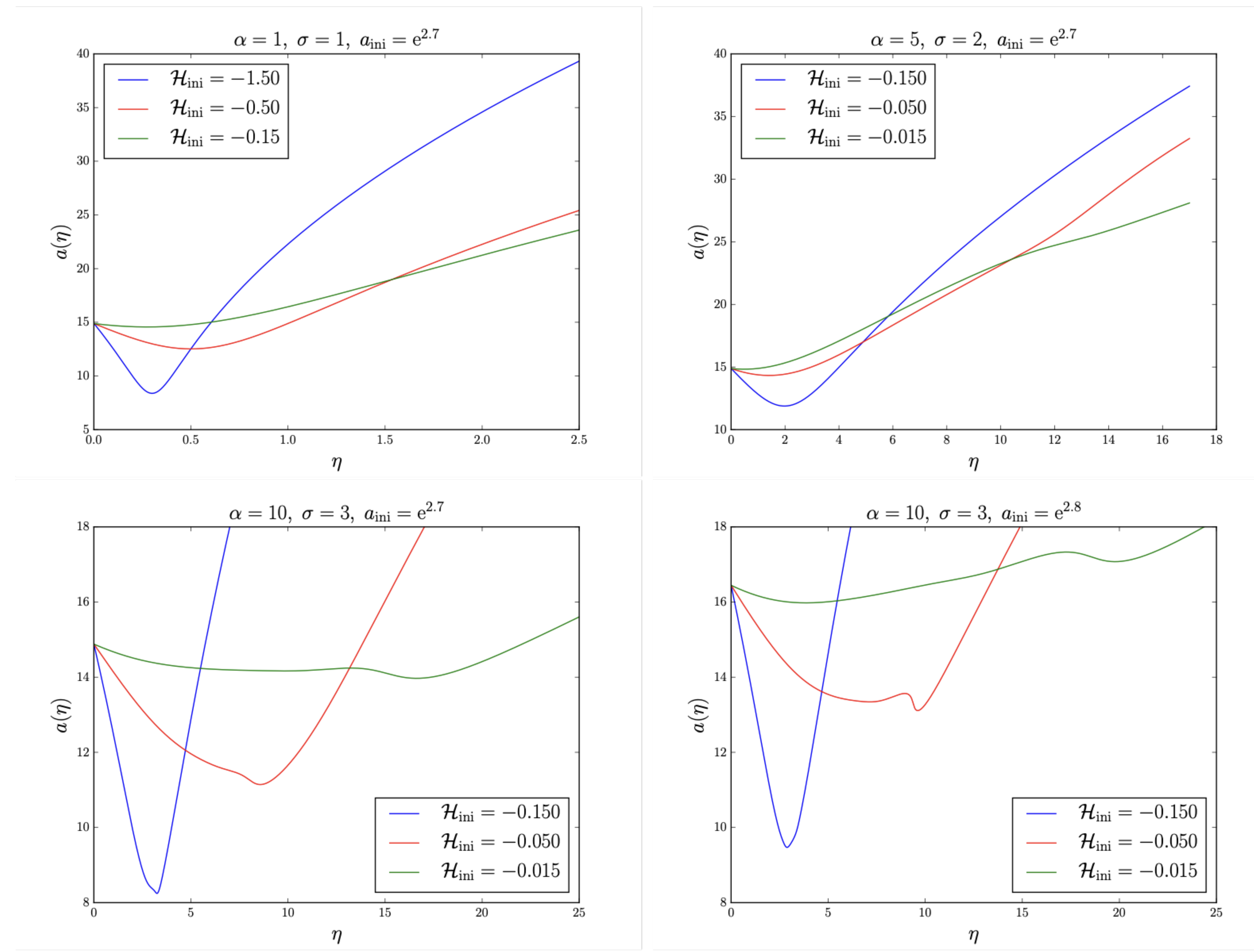




quantum cosmology:



J. Acacio de Barros, N. Pinto-Neto & M. Sagorio-Leal
Phys. Lett. A**241**, 229 (1998)



S. Vitenti & PP
Mod.Phys.Lett. A**31**, 1640006 (2016).

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LQG & LQC

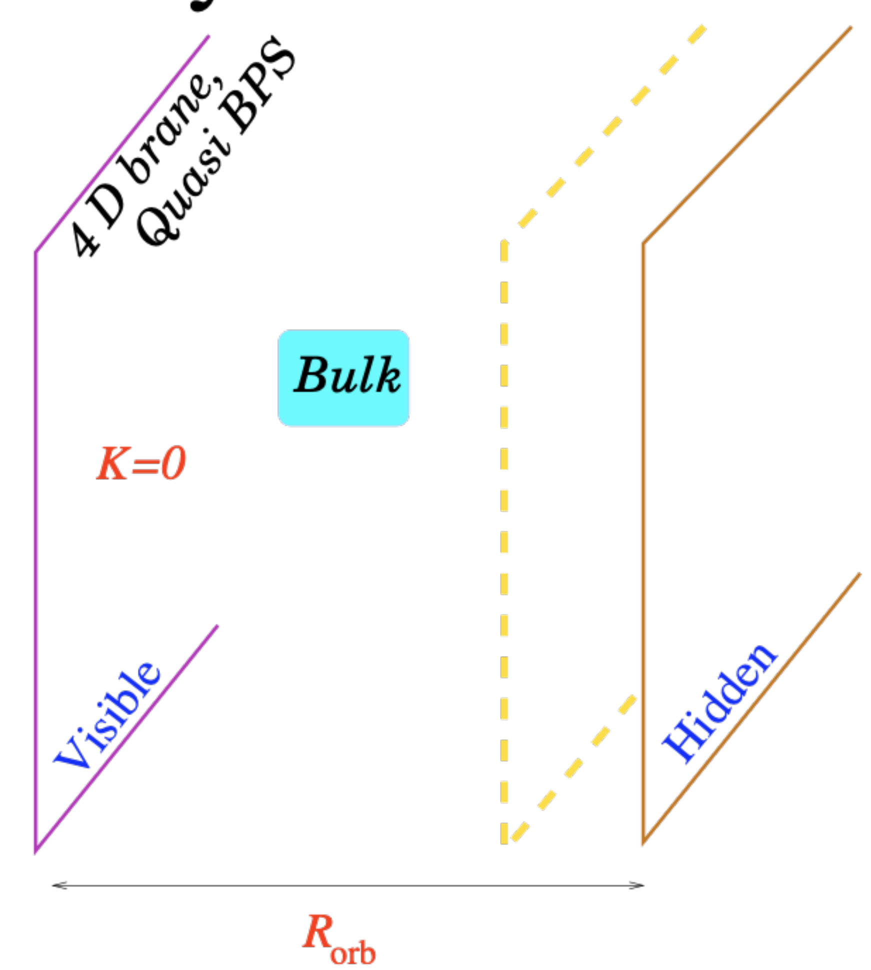
Non relativistic quantum gravity

Canonical quantum gravity (WdW)

String theory

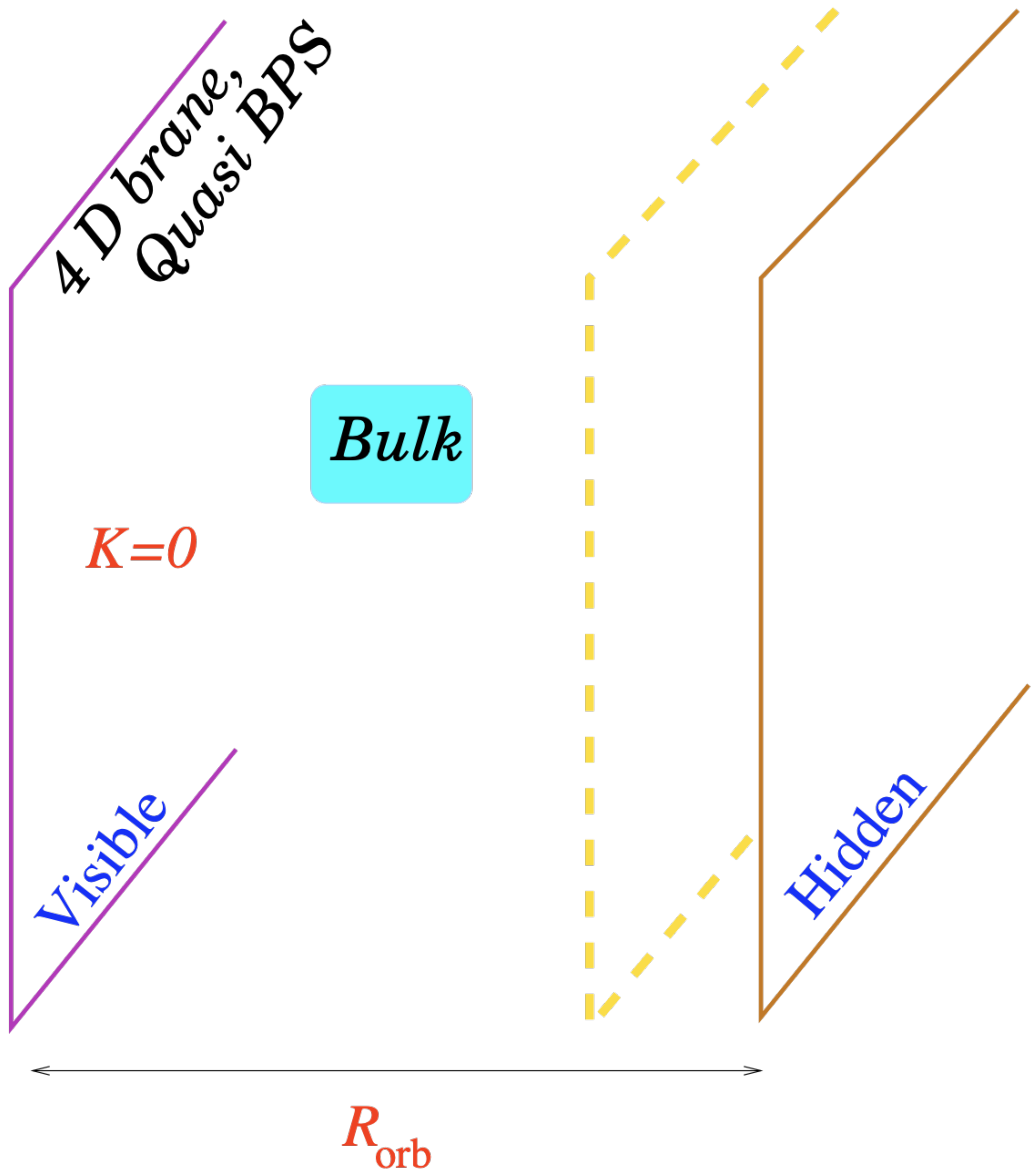
Ekpyrotic & cyclic

Branes



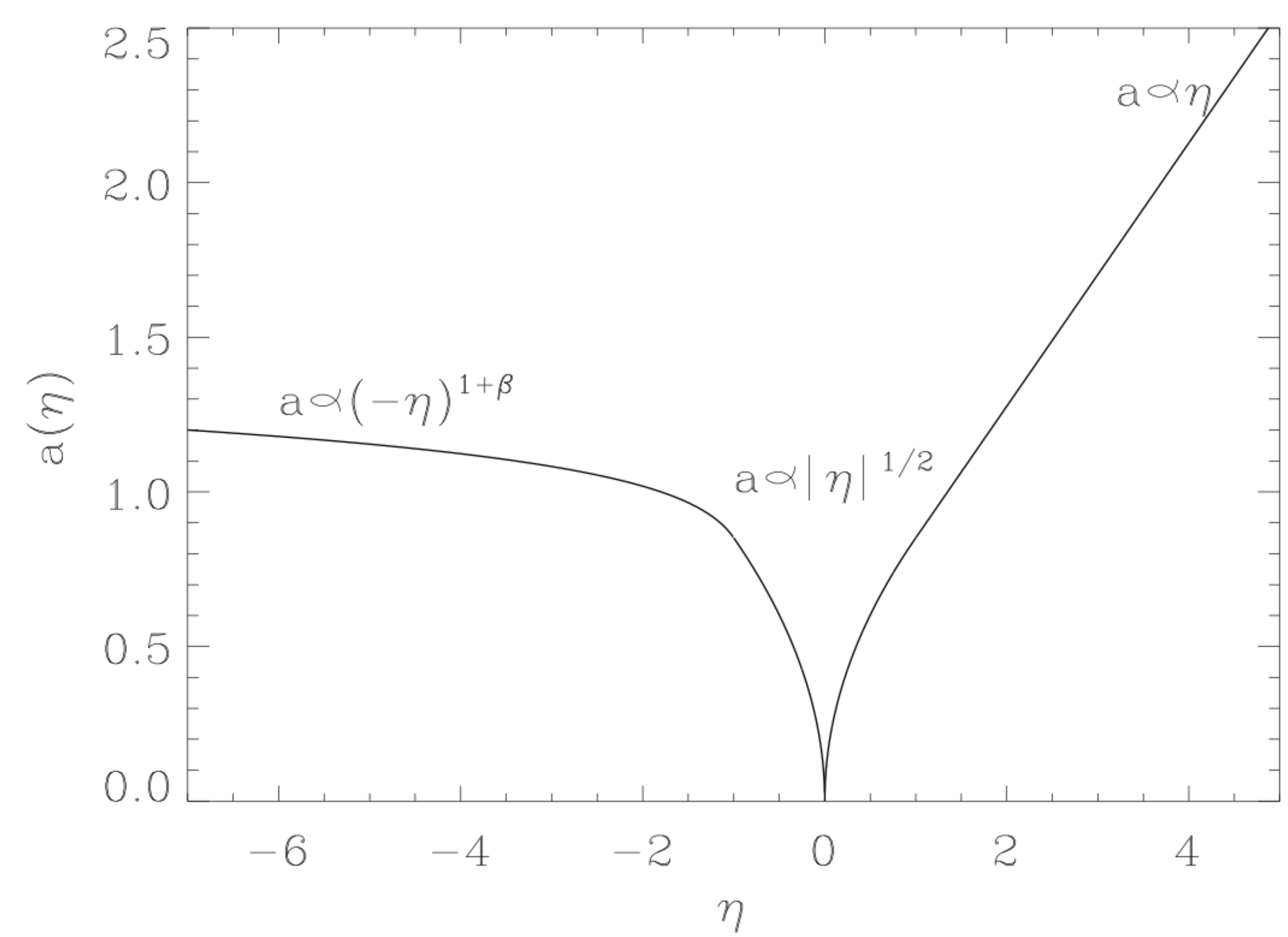
Ekpyrotic scenario:

$$\mathcal{S}_5 \propto \int_{\mathcal{M}_5} \mathrm{d}^5 x \sqrt{-g_5}\left[R_{(5)} - \frac{1}{2}(\partial\varphi)^2 - \frac{3}{2}\frac{\mathrm{e}^{2\varphi}\mathcal{F}^2}{5\,!}\right],$$



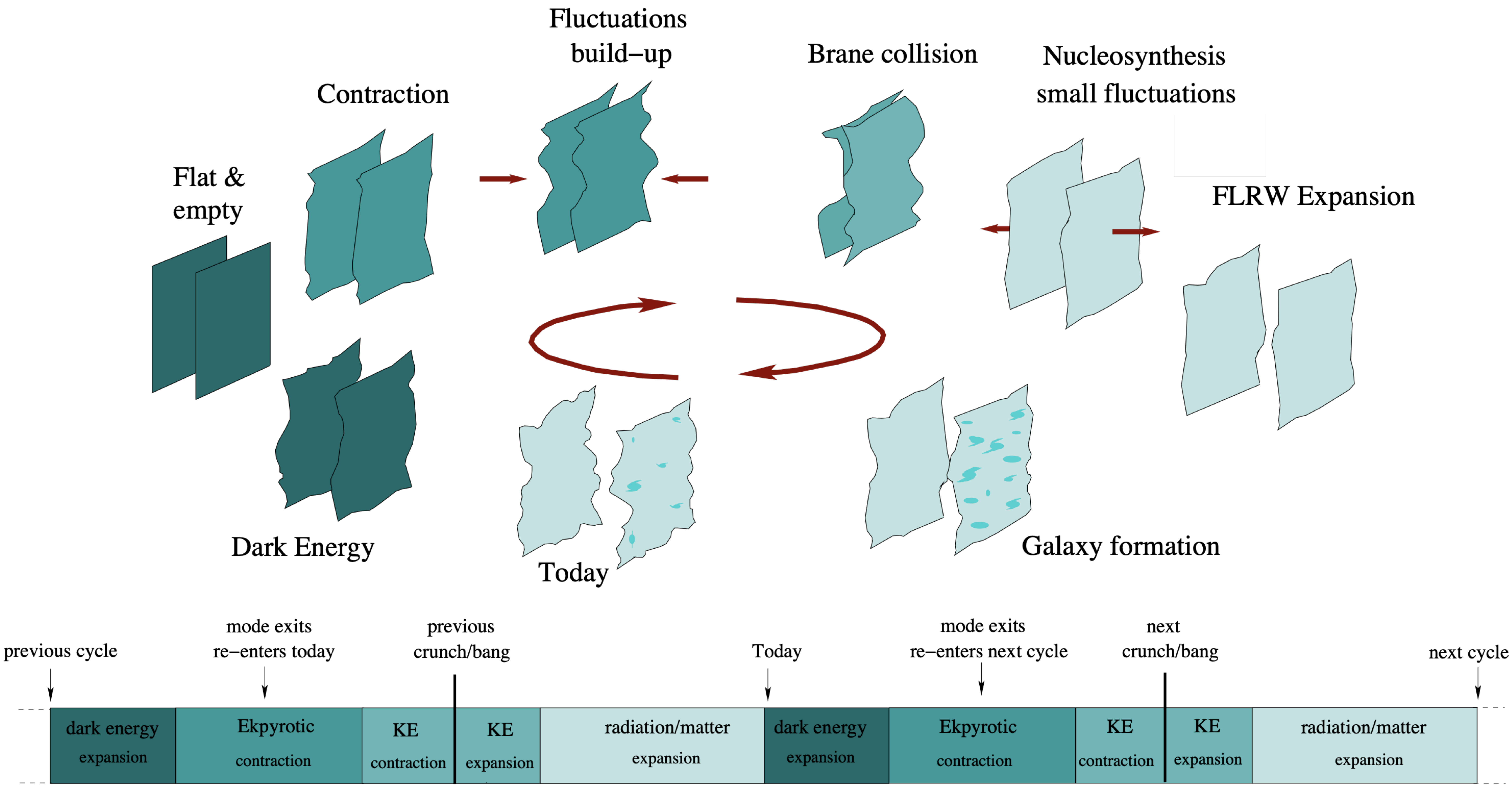
$$\mathcal{S}_4 = \int_{\mathcal{M}_4} \mathrm{d}^4 x \sqrt{-g_4}\left[\frac{R_{(4)}}{2\kappa} - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$V(\varphi) = -V_{\rm i}\exp\left[-\frac{4\sqrt{\pi\gamma}}{m_{\rm Pl}}(\varphi - \varphi_{\rm i})\right]$$



Singular…

Cyclic extension



Model listing:

Quantum gravity

Ekpyrotic & cyclic

Branes

String gas cosmology

Antigravity

Galileon

Massive gravity

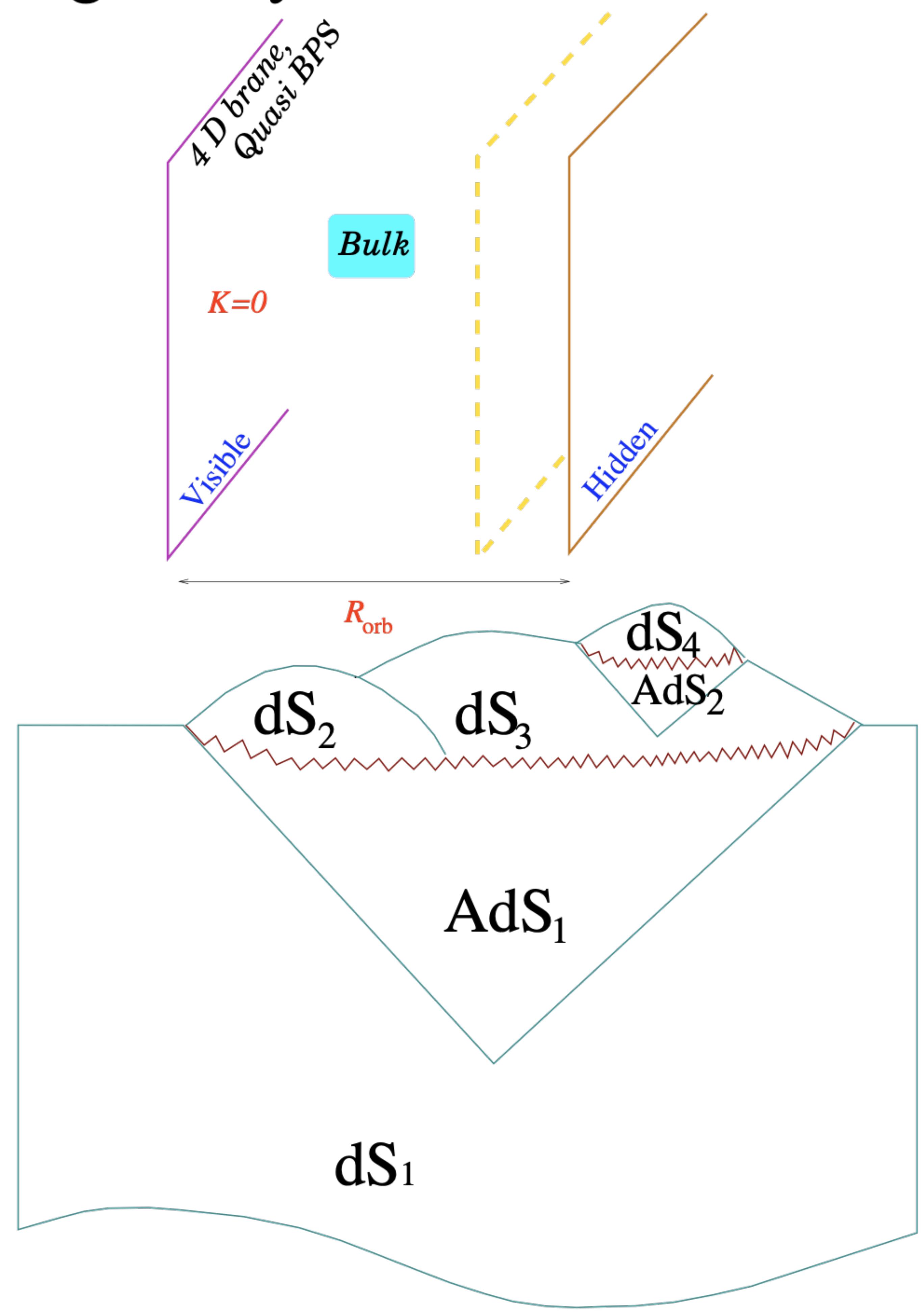
Multiverse models

Strings & AdS/CFT

LQG & LQC

Canonical quantum gravity (WdW)

String theory



Non relativistic quantum gravity

Horava-Lifshitz

Lee-Wick & Quintom

F(R), *f(T)*, Gauss-Bonnet

Mimetic matter

Non-linear electromagnetic action

Spinors & torsion

[1] V. Mukhanov, Physical foundations of cosmology (Cambridge University Press, 2005).
[2] P. Peter and J.-P. Uzan, Primordial cosmology, Oxford Graduate Texts (Oxford University Press, Oxford, 2009).
[3] P. Ade et al. (BICEP2 Collaboration), Phys.Rev.Lett. **112**, 241101 (2014), 1403.3985.
[4] A. H. Guth, Phys. Rev. **D23**, 347 (1981).
[5] A. Ijjas, P. J. Steinhardt, and A. Loeb, Phys. Lett. **B723**, 261 (2013), 1304.2785.
[6] A. Ijjas, P. J. Steinhardt, and A. Loeb (2014), 1402.6980.
[7] A. D. Linde, Lect. Notes Phys. **738**, 1 (2008), 0705.0164.
[8] M. Lemoine, J. Martin, and P. Peter, Lect. Notes Phys. **738** (2008).
[9] D. Baumann (2009), 0907.5424.
[10] J. Martin, C. Ringeval, and V. Vennin, Phys.Dark Univ. (2014), 1303.3787.
[11] J. Martin, C. Ringeval, R. Trotta, and V. Vennin, JCAP **1403**, 039 (2014), 1312.3529.
[12] D. Baumann and L. McAllister (2014), 1404.2601.
[13] A. H. Guth, D. I. Kaiser, and Y. Nomura, Phys.Lett. **B733**, 112 (2014), 1312.7619.
[14] A. Linde (2014), 1402.0526.
[15] J. D. Barrow and A. R. Liddle, Gen.Rel.Grav. **29**, 1503 (1997), gr-qc/9705048.
[16] C. Ringeval, M. Sakellariadou, and F. Bouchet, JCAP **0702**, 023 (2007), astro-ph/0511646.
[17] P. Peter and C. Ringeval, JCAP **1305**, 005 (2013), 1302.0953.
[18] P. Ade et al. (Planck Collaboration) (2013), 1303.5085.
[19] T. Battefeld and S. Watson, Rev. Mod. Phys. **78**, 435 (2006), hep-th/0510022.
[20] R. H. Brandenberger, A. Nayeri, and S. P. Patil, Phys.Rev. **D90**, 067301 (2014), 1403.4927.
[21] Y.-S. Piao, B. Feng, and X.-m. Zhang, Phys.Rev. **D69**, 103520 (2004), hep-th/0310206.
[22] F. T. Falciano, M. Lilley, and P. Peter, Phys.Rev. **D77**, 083513 (2008), 0802.1196.
[23] M. Lilley, L. Lorenz, and S. Clesse, JCAP **1106**, 004 (2011), 1104.3494.
(2002), gr-qc/0109038.
[48] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. **D64**, 123522 (2001), hep-th/0103239.
[49] J. Martin and P. Peter, Phys. Rev. **D68**, 103517 (2003), hep-th/0307077.
[50] B. Xue and P. J. Steinhardt, Phys. Rev. Lett. **105**, 261301 (2010), 1007.2875.
[51] N. Deruelle and V. F. Mukhanov, Phys. Rev. **D52**, 5549 (1995), gr-qc/9503050.
[52] B. Craps, T. Hertog, and N. Turok, Phys. Rev. **D86**, 043513 (2012), 0712.4180.
[53] B. Craps, T. Hertog, and N. Turok, Phys. Rev. **D80**, 086007 (2009), 0905.0709.
[54] M. Smolkin and N. Turok (2012), 1211.1322.
[55] I. Bars, S.-H. Chen, and N. Turok, Phys. Rev. **D84**, 083513 (2011), 1105.3606.
[56] I. Bars (2011), 1109.5872.
[57] I. Bars, S.-H. Chen, P. J. Steinhardt, and N. Turok, Phys. Lett. **B715**, 278 (2012), 1112.2470.
[58] I. Bars, P. Steinhardt, and N. Turok, Phys. Rev. **D89**, 043515 (2014), 1307.1848.
[59] I. Bars, P. J. Steinhardt, and N. Turok, Phys. Lett. **B726**, 50 (2013), 1307.8106.
[60] I. Bars (2012), 1209.1068.
[61] I. Bars, P. Steinhardt, and N. Turok, Phys.Rev. **D89**, 061302 (2014), 1312.0739.
[62] P. Peter and N. Pinto-Neto, Phys. Rev. **D66**, 063509 (2002), hep-th/0203013.
[63] C. Lin, R. H. Brandenberger, and L. Perreault Levasseur, JCAP **1104**, 019 (2011), 1007.2654.
[64] T. Qiu, J. Evslin, Y.-F. Cai, M. Li, and X. Zhang, JCAP **1110**, 036 (2011), 1108.0593.
[65] Y.-F. Cai, T. Qiu, Y.-S. Piao, M. Li, and X. Zhang, JHEP **0710**, 071 (2007), 0704.1090.
[66] C. Kounnas, H. Partouche, and N. Toumbas, Nucl. Phys. **B855**, 280 (2012), 1106.0946.
[67] T. Biswas, A. Mazumdar, and W. Siegel, JCAP **0603**, 009 (2006), hep-th/0508194.
[68] T. Biswas, R. Brandenberger, A. Mazumdar, and W. Siegel, JCAP **0712**, 011 (2007), hep-th/0610274.
[69] D. Langlois and A. Naruko, Class. Quant. Grav. **30**, 205012 (2013), 1305.6346.
[70] M. Koehn, J.-L. Lehners, and B. Ovrut, Phys. Rev. **D88**, 023528 (2013), 1302.0840.
[71] M. Koehn, J.-L. Lehners, and B. A. Ovrut, Phys.Rev. **D90**, 025005 (2014), 1310.7577.
[72] L. Battarra, M. Koehn, J.-L. Lehners, and B. A. Ovrut, JCAP **1407**, 007 (2014), 1404.5067.
[73] L. Smolin, Rev. Mod. Phys. pp. 655–682 (2004), hep-th/0408048.
[74] C. Rovelli, PoS **QGQGS2011**, 003 (2011), 1102.3660.
[75] H. Nicolai, K. Peeters, and M. Zamaklar, Class. Quant. Grav. **22**, R193 (2005), hep-th/0501114.
[76] A. Henderson, A. Laddha, and C. Tomlin, Phys.Rev. **D88**, 044028 (2013), 1204.0211.
[77] A. Henderson, A. Laddha, and C. Tomlin, Phys. Rev. **D88**, 044029 (2013), 1210.3960.
[78] C. Tomlin and M. Varadarajan, Phys. Rev. **D87**, 044039 (2013), 1210.6869.
[79] W. Atwood et al. (LAT Collaboration), Astrophys. J. **697**, 1071 (2009), 0902.1089.
[80] L. Shao, Z. Xiao, and B.-Q. Ma, Astropart.Phys. **33**, 312 (2010), 0911.2276.
[81] M. Ackermann et al. (Fermi GBM/LAT Collabora-
[24] Z.-G. Liu, Z.-K. Guo, and Y.-S. Piao, Phys.Rev. **D88**, 063539 (2013), 1304.6527.
[25] T. Biswas and A. Mazumdar, Class.Quant.Grav. **31**, 025019 (2014), 1304.3648.
[26] E. Hubble, Proc. Nat. Acad. Sci. **15**, 168 (1929).
[27] S. Perlmutter et al. (Supernova Cosmology Project), Astrophys. J. **517**, 565 (1999), astro-ph/9812133.
[28] A. G. Riess et al. (Supernova Search Team), Astron. J. **116**, 1009 (1998), astro-ph/9805201.
[29] A. Coc, J.-P. Uzan, and E. Vangioni, JCAP **1410**, 050 (2014), 1403.6694.
[30] R. Teyssier, S. Pires, S. Prunet, D. Aubert, C. Pichon, et al., Astron.Astrophys. **497**, 335 (2009), 0807.3651.
[31] A. Borde and A. Vilenkin, Int.J.Mod.Phys. **D5**, 813 (1996), gr-qc/9612036.
[32] J. Martin, C. Ringeval, R. Trotta, and V. Vennin, Phys.Rev. **D90**, 063501 (2014), 1405.7272.
[33] R. Flauger, J. C. Hill, and D. N. Spergel, JCAP **1408**, 039 (2014), 1405.7351.
[34] M. J. Mortonson and U. Seljak, JCAP **1410**, 035 (2014), 1405.5857.
[35] R. H. Brandenberger, V. F. Mukhanov, and A. Sornborger, Phys. Rev. **D48**, 1629 (1993), gr-qc/9303001.
[36] M. Novello and S. P. Bergliaffa, Phys. Rept. **463**, 127 (2008), 0802.1634.
[37] G. Lemaitre, Ann. Soc. Sci. Brux. Ser. I Sci. Math. Astron. Phys. **A47**, 49 (1927).
[38] R. C. Tolman, Phys. Rev. **38**, 1758 (1931).
[39] G. Lemaître, Ann. Soc. Sci. Brux. **53**, 51 (1933).
[40] G. Lemaitre, Gen. Rel. Grav. **29**, 641 (1997).
[41] J. D. Barrow and M. P. Dabrowski, Mon.Not.Roy.Astron.Soc. **275**, 850 (1995).
[42] A. A. Starobinskii, Sov. Astron. Lett. **4**, 82 (1978).
[43] A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980).
[44] B. A. Bassett, S. Tsujikawa, and D. Wands, Rev. Mod. Phys. **78**, 537 (2006), astro-ph/0507632.
[45] M. Gasperini and G. Veneziano, Phys. Rept. **373**, 1 (2003), hep-th/0207130.
[46] R. Durrer and J. Laukenmann, Class. Quant. Grav. **13**, 1069 (1996), gr-qc/9510041.
[47] P. Peter and N. Pinto-Neto, Phys. Rev. **D65**, 023513
tions), Nature **462**, 331 (2009), 0908.1832.
[82] F. Aharonian et al. (H. E. S. S. Collaboration), Phys. Rev. Lett. **101**, 170402 (2008), 0810.3475.
[83] A. Abramowski et al. (HESS Collaboration), Astropart.Phys. **34**, 738 (2011), 1101.3650.
[84] G. Amelino-Camelia, J. R. Ellis, N. Mavromatos, D. V. Nanopoulos, and S. Sarkar, Nature **393**, 763 (1998), astro-ph/9712103.
[85] V. Vasileiou, A. Jacholkowska, F. Piron, J. Bolmont, C. Couturier, et al., Phys. Rev. **D87**, 122001 (2013), 1305.3463.
[86] M. Bojowald, Class. Quant. Grav. **29**, 213001 (2012), 1209.3403.
[87] A. Ashtekar and P. Singh, Class. Quant. Grav. **28**, 213001 (2011), 1108.0893.
[88] G. Vereshchagin, JCAP **0407**, 013 (2004), gr-qc/0406108.
[89] P. Singh, K. Vandersloot, and G. Vereshchagin, Phys.Rev. **D74**, 043510 (2006), gr-qc/0606032.
[90] T. Cailleteau, P. Singh, and K. Vandersloot, Phys.Rev. **D80**, 124013 (2009), 0907.5591.
[91] L. Linsefors and A. Barrau, Phys. Rev. **D87**, 123509 (2013), 1301.1264.
[92] J. Amorós, J. de Haro, and S. D. Odintsov, Phys.Rev. **D87**, 104037 (2013), 1305.2344.
[93] J. Mielczarek and W. Piechocki, Phys. Rev. **D86**, 083508 (2012), 1108.0005.
[94] J. P. Gazeau, J. Mielczarek, and W. Piechocki, Phys. Rev. **D87**, 123508 (2013), 1303.1687.
[95] A. Barrau, T. Cailleteau, J. Grain, and J. Mielczarek, Class.Quant.Grav. **31**, 053001 (2014), 1309.6896.
[96] E. Wilson-Ewing, JCAP **1308**, 015 (2013), 1306.6582.
[97] B. Gupt and P. Singh, Class.Quant.Grav. **30**, 145013 (2013), 1304.7686.
[98] Y.-F. Cai and E. Wilson-Ewing, JCAP **1403**, 026 (2014), 1402.3009.
[99] A. Corichi and T. Vukasinac, Phys.Rev. **D86**, 064019 (2012), 1202.1846.
[100] M. Bojowald, G. M. Hossain, M. Kagan, and S. Shankaranarayanan, Phys. Rev. **D79**, 043505 (2009), 0811.1572.
[101] M. Bojowald, G. M. Hossain, M. Kagan, and S. Shankaranarayanan, Phys. Rev. **D78**, 063547 (2008), 0806.3929.
[102] T. Cailleteau, L. Linsefors, and A. Barrau, Class.Quant.Grav. **31**, 125011 (2014), 1307.5238.
[103] A. Ashtekar, T. Pawlowski, P. Singh, and K. Vandersloot, Phys. Rev. **D75**, 024035 (2007), gr-qc/0612104.
[104] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. **D74**, 084003 (2006), gr-qc/0607039.
[105] A. Ashtekar, T. Pawlowski, and P. Singh, Phys. Rev. Lett. **96**, 141301 (2006), gr-qc/0602086.
[106] E. Wilson-Ewing, JCAP **1303**, 026 (2013), 1211.6269.
[107] Y. Shtanov and V. Sahni, Phys. Lett. **B557**, 1 (2003), gr-qc/0208047.
[108] M. G. Brown, K. Freese, and W. H. Kinney, JCAP **0803**, 002 (2008), astro-ph/0405353.
[109] T. Battefeld and G. Geshnizjani, Phys. Rev. **D73**, 064013 (2006), hep-th/0503160.
[110] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999), hep-ph/9905221.
[111] M. Bojowald and G. M. Paily, Phys. Rev. **D86**, 104018 (2012), 1112.1899.
[112] P. Singh, Phys.Rev. **D85**, 104011 (2012), 1112.6391.
[113] T. Thiemann, Lect. Notes Phys. **721**, 185 (2007), hep-th/0608210.
[114] P. Holland, Phys. Rept. **224**, 95 (1993).
[115] A. S. Sanz and S. Miret-Artés, Lect. Notes Phys. **850**, 187 (2012).
[116] N. Pinto-Neto and J. Fabris, Class. Quant. Grav. **30**, 143001 (2013), 1306.0820.
[117] J. Acacio de Barros, N. Pinto-Neto, and M. Sagioro-Leal, Phys. Lett. **A241**, 229 (1998), gr-qc/9710084.
[118] R. Casadio, Int.J.Mod.Phys. **D9**, 511 (2000), gr-qc/9810073.
[119] P. Peter, E. J. Pinho, and N. Pinto-Neto, Phys. Rev. **D73**, 104017 (2006), gr-qc/0605060.
[120] E. J. Pinho and N. Pinto-Neto, Phys. Rev. **D76**, 023506 (2007), hep-th/0610192.
[121] R. Y. Donagi, J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, JHEP **0111**, 041 (2001), hep-th/0105199.
[122] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok (2001), hep-th/0105212.
[123] R. Kallosh, L. Kofman, and A. D. Linde, Phys. Rev. **D64**, 123523 (2001), hep-th/0104073.
[124] R. Kallosh, L. Kofman, A. D. Linde, and A. A. Tseytlin, Phys. Rev. **D64**, 123524 (2001), hep-th/0106241.
[125] A. Lukas, B. A. Ovrut, K. Stelle, and D. Waldram, Nucl.Phys. **B552**, 246 (1999), hep-th/9806051.
[126] J. Martin, P. Peter, N. Pinto Neto, and D. J. Schwarz, Phys. Rev. **D65**, 123513 (2002), hep-th/0112128.
[127] J. Martin, P. Peter, N. Pinto-Neto, and D. J. Schwarz, Phys. Rev. **D67**, 028301 (2003), hep-th/0204222.
[128] H. Kragh (2013), 1308.0932.
[129] P. J. Steinhardt and N. Turok, Phys. Rev. **D65**, 126003 (2002), hep-th/0111098.
[130] P. J. Steinhardt and N. Turok (2001), hep-th/0111030.
[131] J. Khoury, B. A. Ovrut, N. Seiberg, P. J. Steinhardt, and N. Turok, Phys. Rev. **D65**, 086007 (2002), hep-th/0108187.
[132] G. N. Felder, A. V. Frolov, L. Kofman, and A. D. Linde, Phys.Rev. **D66**, 023507 (2002), hep-th/0202017.
[133] A. Mithani and A. Vilenkin (2012), 1204.4658.
[134] R. Brandenberger and F. Finelli, JHEP **0111**, 056 (2001), hep-th/0109004.
[135] D. H. Lyth, Phys.Lett. **B524**, 1 (2002), hep-ph/0106153.
[136] R. Durrer and F. Vernizzi, Phys. Rev. **D66**, 083503 (2002), hep-ph/0203275.
[137] P. Creminelli, A. Nicolis, and M. Zaldarriaga, Phys. Rev. **D71**, 063505 (2005), hep-th/0411270.
[138] C.-Y. Tseng, Phys. Rev. **D87**, 023518 (2013), 1210.0581.
[139] L. Battarra and J.-L. Lehners, Phys.Rev. **D89**, 063516 (2014), 1309.2281.
[140] A. Notari and A. Riotto, Nucl. Phys. **B644**, 371 (2002), hep-th/0205019.
[141] J.-L. Lehners, P. McFadden, N. Turok, and P. J. Steinhardt, Phys. Rev. **D76**, 103501 (2007), hep-th/0702153.
[142] E. I. Buchbinder, J. Khoury, and B. A. Ovrut, Phys. Rev. **D76**, 123503 (2007), hep-th/0702154.
[143] P. Creminelli and L. Senatore, JCAP **0711**, 010 (2007), hep-th/0702165.
[144] E. I. Buchbinder, J. Khoury, and B. A. Ovrut, JHEP **0711**, 076 (2007), 0706.3903.
[145] R. Kallosh, J. U. Kang, A. Linde, and V. Mukhanov, JCAP **4**, 018 (2008), 0712.2040.
[146] R. P. Woodard, Lect.Notes Phys. **720**, 403 (2007),
[182] P. C. Stavrinos and S. I. Vacaru, Class. Quant. Grav. **30**, 055012 (2013), 1206.3998.
[183] R. I. Ivanov and E. M. Prodanov, Phys. Rev. **D86**, 083536 (2012), 1210.0186.
[184] C. Gao, Y. Lu, and Y.-G. Shen, Gen.Rel.Grav. **46**, 1791 (2014), 1402.3863.
[185] J. D. Barrow, D. Kimberly, and J. Magueijo, Class.Quant.Grav. **21**, 4289 (2004), astro-ph/0406369.
[186] T. Clifton and J. D. Barrow, Phys.Rev. **D75**, 043515 (2007), gr-qc/0701070.
[187] J. D. Barrow and D. Sloan, Phys.Rev. **D88**, 023518 (2013), 1304.6699.
[188] R. H. Brandenberger and C. Vafa, Nucl. Phys. **B316**, 391 (1989).
[189] A. A. Tseytlin and C. Vafa, Nucl. Phys. **B372**, 443 (1992), hep-th/9109048.
[190] S. Alexander, R. H. Brandenberger, and D. Easson, Phys. Rev. **D62**, 103509 (2000), hep-th/0005212.
[191] J. Kripfganz and H. Perlt, Class. Quant. Grav. **5**, 453 (1988).
[192] R. H. Brandenberger (2005), hep-th/0509099.
[193] R. Hagedorn, Nuovo Cim. Suppl. **3**, 147 (1965).
[194] S. C. Frautschi, Phys. Rev. **D3**, 2821 (1971).
[195] M. J. Bowick and L. Wijewardhana, Phys. Rev. Lett. **54**, 2485 (1985).
[196] B. Sundborg, Nucl. Phys. **B254**, 583 (1985).
[197] E. Alvarez, Nucl. Phys. **B269**, 596 (1986).
[198] J. Polchinski, Commun. Math. Phys. **104**, 37 (1986).
[199] P. Salomonson and B.-S. Skagerstam, Nucl. Phys. **B268**, 349 (1986).
[200] D. Mitchell and N. Turok, Nucl. Phys. **B294**, 1138 (1987).
[201] K. O'Brien and C. Tan, Phys. Rev. **D36**, 1184 (1987).
[202] R. H. Brandenberger, C. Kounnas, H. Partouche, S. P. Patil, and N. Toumbas (2013), 1312.2524.
[203] R. H. Brandenberger (2008), 0808.0746.
[204] R. H. Brandenberger, A. Nayeri, S. P. Patil, and C. Vafa, Int. J. Mod. Phys. **A22**, 3621 (2007), hep-th/0608121.
[205] T. Battefeld and S. Watson, JCAP **0406**, 001 (2004), hep-th/0403075.
[206] N. Kaloper and S. Watson, Phys. Rev. **D77**, 066002 (2008), 0712.1820.
[207] I. Florakis, C. Kounnas, H. Partouche, and N. Toumbas, Nucl. Phys. **B844**, 89 (2011), 1008.5129.
[208] C. Kounnas, H. Partouche, and N. Toumbas, Class. Quant. Grav. **29**, 095014 (2012), 1111.5816.
[209] C. Kounnas and N. Toumbas, PoS **Corfu2012**, 083 (2013), 1305.2809.
[210] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998), hep-th/9711200.
[211] N. Turok, B. Craps, and T. Hertog (2007), 0711.1824.
[212] T. Hertog and G. T. Horowitz, JHEP **0407**, 073 (2004), hep-th/0406134.
[213] J. M. Maldacena, pp. 155–203 (2003), hep-th/0309246.
[214] A. D. Linde, Pisma Zh. Eksp. Teor. Fiz. **30**, 479 (1979).
[215] J. J. M. Carrasco, W. Chemissany, and R. Kallosh, JHEP **1401**, 130 (2014), 1311.3671.
[216] R. Kallosh and A. Linde, JCAP **1401**, 020 (2014), 1311.3326.
[217] L. Cornalba and M. S. Costa, Phys.Rev. **D66**, 066001 (2002), hep-th/0203031.
[218] H. Liu, G. W. Moore, and N. Seiberg, JHEP **0206**, 045 (2002), hep-th/0204168.
astro-ph/0601672.
[147] J. Khoury, P. J. Steinhardt, and N. Turok, Phys. Rev. Lett. **92**, 031302 (2004), hep-th/0307132.
[148] J. Khoury, B. A. Ovrut, P. J. Steinhardt, and N. Turok, Phys. Rev. **D66**, 046005 (2002), hep-th/0109050.
[149] J.-L. Lehners and P. J. Steinhardt, Phys. Rev. **D79**, 063503 (2009), 0812.3388.
[150] J.-L. Lehners, Phys. Rev. **D84**, 103518 (2011), 1107.4551.
[151] J.-L. Lehners, Phys. Rept. **465**, 223 (2008), 0806.1245.
[152] J.-L. Lehners and P. J. Steinhardt, Phys. Rev. **D87**, 123533 (2013), 1304.3122.
[153] J.-L. Lehners, P. J. Steinhardt, and N. Turok, Int. J. Mod. Phys. **D18**, 2231 (2009), 0910.0834.
[154] J.-L. Lehners, Class. Quant. Grav. **28**, 204004 (2011), 1106.0172.
[155] M. C. Johnson and J.-L. Lehners, Phys. Rev. **D85**, 103509 (2012), 1112.3360.
[156] K. Enqvist and M. S. Sloth, Nucl.Phys. **B626**, 395 (2002), hep-ph/0109214.
[157] D. H. Lyth and D. Wands, Phys. Lett. **B524**, 5 (2002), hep-ph/0110002.
[158] K. Koyama, S. Mizuno, and D. Wands, Class. Quant. Grav. **24**, 3919 (2007), 0704.1152.
[159] K. Koyama and D. Wands, JCAP **0704**, 008 (2007), hep-th/0703040.
[160] K. Koyama, S. Mizuno, F. Vernizzi, and D. Wands, JCAP **0711**, 024 (2007), 0708.4321.
[161] J.-L. Lehners, P. McFadden, and N. Turok, Phys. Rev. **D75**, 103510 (2007), hep-th/0611259.
[162] J.-L. Lehners, P. McFadden, and N. Turok, Phys. Rev. **D76**, 023501 (2007), hep-th/0612026.
[163] T. Battefeld, Phys. Rev. **D77**, 063503 (2008), 0710.2540.
[164] P. Ade et al. (Planck Collaboration), Astron.Astrophys. **571**, A22 (2014), 1303.5082.
[165] P. Ade et al. (Planck Collaboration) (2013), 1303.5083.
[166] J.-L. Lehners and P. J. Steinhardt, Phys. Rev. **D77**, 063533 (2008), 0712.3779.
[167] J.-L. Lehners and P. J. Steinhardt, Phys. Rev. **D78**, 023506 (2008), 0804.1293.
[168] J.-L. Lehners, Adv. Astron. **2010**, 903907 (2010), 1001.3125.
[169] T. Qiu, X. Gao, and E. N. Saridakis, Phys. Rev. **D88**, 043525 (2013), 1303.2372.
[170] M. Li, Phys. Lett. **B724**, 192 (2013), 1306.0191.
[171] A. Fertig, J.-L. Lehners, and E. Mallwitz, Phys.Rev. **D89**, 103537 (2014), 1310.8133.
[172] A. Ijjas, J.-L. Lehners, and P. J. Steinhardt (2014), 1404.1265.
[173] B. Xue and E. Belbruno (2014), 1403.2122.
[174] E. Saridakis, Nucl. Phys. **B808**, 224 (2009), 0710.5269.
[175] T. Biswas (2008), 0801.1315.
[176] M. Novello, A. N. Araujo, and J. Salim, Int. J. Mod. Phys. **A24**, 5639 (2009), 0802.1875.
[177] L. Medeiros, Int.J.Mod.Phys. **D23**, 1250073 (2012), 1209.1124.
[178] C.-Y. Sun, Phys. Rev. **D78**, 064060 (2008), 0806.1080.
[179] E. J. Son and W. Kim, Mod.Phys. Lett. **A26**, 719 (2011), 1007.5371.
[180] C. Li, L. Wang, and Y.-K. E. Cheung, Phys.Dark Univ. **3**, 18 (2014), 1101.0202.
[181] V. Sahni and A. Toporensky, Phys. Rev. **D85**, 123542 (2012), 1203.0395.
[219] G. T. Horowitz and J. Polchinski, Phys. Rev. **D66**, 103512 (2002), hep-th/0206228.
[220] L. Cornalba and M. S. Costa, Fortsch.Phys. **52**, 145 (2004), hep-th/0310099.
[221] M. Berkooz and D. Reichmann, Nucl. Phys. Proc. Suppl. **171**, 69 (2007), 0705.2146.
[222] M. Oltean and R. Brandenberger, Phys.Rev. **D90**, 083505 (2014), 1406.7318.
[223] E. I. Buchbinder, J. Khoury, and B. A. Ovrut, Phys. Rev. Lett. **100**, 171302 (2008), 0710.5172.
[224] R. H. Brandenberger (2012), 1206.4196.
[225] Y.-F. Cai, D. A. Easson, and R. Brandenberger, JCAP **1208**, 020 (2012), 1206.2382.
[226] Y.-F. Cai, R. Brandenberger, and P. Peter, Class. Quant. Grav. **30**, 075019 (2013), 1301.4703.
[227] Y.-F. Cai, E. McDonough, F. Duplessis, and R. H. Brandenberger, JCAP **1310**, 024 (2013), 1305.5259.
[228] Y.-F. Cai, W. Xue, R. Brandenberger, and X. Zhang, JCAP **0905**, 011 (2009), 0903.0631.
[229] D. A. Easson, I. Sawicki, and A. Vikman, JCAP **1111**, 021 (2011), 1109.1047.
[230] A. Nicolis, R. Rattazzi, and E. Trincherini, Phys. Rev. **D79**, 064036 (2009), 0811.2197.
[231] C. de Rham and G. Gabadadze, Phys. Rev. **D82**, 044020 (2010), 1007.0443.
[232] C. de Rham, G. Gabadadze, and A. J. Tolley, Phys. Rev. Lett. **106**, 231101 (2011), 1011.1232.
[233] C. de Rham and A. J. Tolley, JCAP **1005**, 015 (2010), 1003.5917.
[234] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, JCAP **1010**, 026 (2010), 1008.0048.
[235] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, Phys. Rev. Lett. **105**, 231302 (2010), 1008.0603.
[236] C. Deffayet, S. Deser, and G. Esposito-Farese, Phys. Rev. **D82**, 061501 (2010), 1007.5278.
[237] P. Creminelli, A. Nicolis, and E. Trincherini, JCAP **1011**, 021 (2010), 1007.0027.
[238] P. Creminelli, K. Hinterbichler, J. Khoury, A. Nicolis, and E. Trincherini, JHEP **1302**, 006 (2013), 1209.3768.
[239] G. W. Horndeski, Int.J.Theor.Phys. **10**, 363 (1974).
[240] C. Deffayet and D. A. Steer, Class. Quant. Grav. **30**, 214006 (2013), 1307.2450.
[241] M. Osipov and V. Rubakov, JCAP **1311**, 031 (2013), 1303.1221.
[242] O. Pujolas, I. Sawicki, and A. Vikman, JHEP **1111**, 156 (2011), 1103.5360.
[243] T. Qiu (2014), 1404.3060.
[244] J. D. Barrow, Class. Quant. Grav. **21**, L79 (2004), gr-qc/0403084.
[245] T. J. Battefeld and R. Brandenberger, Phys. Rev. **D70**, 121302 (2004), hep-th/0406180.
[246] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. **A173**, 211 (1939).
[247] H. van Dam and M. Veltman, Nucl. Phys. **B22**, 397 (1970).
[248] V. Zakharov, JETP Lett. **12**, 312 (1970).
[249] D. Boulware and S. Deser, Phys. Rev. **D6**, 3368 (1972).
[250] A. De Felice, A. E. Gümrükçüoğlu, and S. Mukohyama, Phys. Rev. Lett. **109**, 171101 (2012), 1206.2080.
[251] A. De Felice, A. E. Gümrükçüoğlu, C. Lin, and S. Mukohyama, JCAP **1305**, 035 (2013), 1303.4154.
[252] G. D'Amico, C. de Rham, S. Dubovsky, G. Gabadadze, D. Pirtskhalava, et al., Phys. Rev. **D84**, 124046 (2011), 1108.5231.
[253] A. E. Gumrukcuoglu, C. Lin, and S. Mukohyama, Phys. Lett. **B717**, 295 (2012), 1206.2723.
[254] Q.-G. Huang, Y.-S. Piao, and S.-Y. Zhou, Phys. Rev. **D86**, 124014 (2012), 1206.5678.
[255] E. N. Saridakis, Class. Quant. Grav. **30**, 075003 (2013), 1207.1800.
[256] K. Hinterbichler, Rev. Mod. Phys. **84**, 671 (2012), 1105.3735.
[257] C. de Rham, Living Rev.Rel. **17**, 7 (2014), 1401.4173.
[258] T. Biswas, T. Koivisto, and A. Mazumdar, JCAP **1011**, 008 (2010), 1005.0590.
[259] Y.-F. Cai, C. Gao, and E. N. Saridakis, JCAP **1210**, 048 (2012), 1207.3786.
[260] T. Biswas, E. Gerwick, T. Koivisto, and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012), 1110.5249.
[261] T. Biswas, A. S. Koshelev, A. Mazumdar, and S. Y. Vernov, JCAP **1208**, 024 (2012), 1206.6374.
[262] A. S. Koshelev, Class. Quant. Grav. **30**, 155001 (2013), 1302.2140.
[263] J.-L. Lehners, Phys.Rev. **D86**, 043518 (2012), 1206.1081.
[264] J. Garriga, A. Vilenkin, and J. Zhang, JCAP **1311**, 055 (2013), 1309.2847.
[265] B. Gupt and P. Singh, Phys.Rev. **D89**, 063520 (2014), 1309.2732.
[266] Y.-S. Piao, Phys. Rev. **D70**, 101302 (2004), hep-th/0407258.
[267] Y.-S. Piao, Phys. Lett. **B677**, 1 (2009), 0901.2644.
[268] Z.-G. Liu and Y.-S. Piao (2014), 1404.5748.
[269] A. Vilenkin and J. Zhang (2014), 1403.1599.
[270] P. Horava, JHEP **0903**, 020 (2009), 0812.4287.
[271] P. Horava, Phys. Rev. **D79**, 084008 (2009), 0901.3775.
[272] P. Horava, Phys. Rev. Lett. **102**, 161301 (2009), 0902.3657.
[273] G. Calcagni, JHEP **0909**, 112 (2009), 0904.0829.
[274] E. Kiritsis and G. Kofinas, Nucl. Phys. **B821**, 467 (2009), 0904.1334.
[275] R. Brandenberger, Phys. Rev. **D80**, 043516 (2009), 0904.2835.
[276] R. Maier, Class. Quant. Grav. **30**, 115011 (2013), 1302.0139.
[277] M. Henneaux, A. Kleinschmidt, and G. Lucena Gómez, Phys.Rev. **D81**, 064002 (2010), 0912.0399.
[278] I. Kimpton and A. Padilla, JHEP **1304**, 133 (2013), 1301.6950.
[279] T. Lee and G. Wick, Nucl. Phys. **B9**, 209 (1969).
[280] T. Lee and G. Wick, Phys. Rev. **D2**, 1033 (1970).
[281] B. Grinstein, D. O'Connell, and M. B. Wise, Phys. Rev. **D77**, 025012 (2008), 0704.1845.
[282] R. Caldwell, Phys. Lett. **B545**, 23 (2002), astro-ph/9908168.
[283] J. M. Cline, S. Jeon, and G. D. Moore, Phys. Rev. **D70**, 043543 (2004), hep-ph/0311312.
[284] Y.-F. Cai, T.-t. Qiu, R. Brandenberger, and X.-m. Zhang, Phys. Rev. **D80**, 023511 (2009), 0810.4677.
[285] L. E. Allen and D. Wands, Phys. Rev. **D70**, 063515 (2004), astro-ph/0404441.
[286] K. Bhattacharya, Y.-F. Cai, and S. Das, Phys. Rev. **D87**, 083511 (2013), 1301.0661.
[287] B. Feng, X.-L. Wang, and X.-M. Zhang, Phys. Lett. **B607**, 35 (2005), astro-ph/0404224.
[288] B. Feng, M. Li, Y.-S. Piao, and X. Zhang, Phys. Lett. **B634**, 101 (2006), astro-ph/0407432.
[289] Z.-K. Guo, Y.-S. Piao, X.-M. Zhang, and Y.-Z. Zhang, (2008), 0809.2022.
[324] J. Martin and P. Peter, Phys. Rev. Lett. **92**, 061301 (2004), astro-ph/0312488.
[325] S. Weinberg (2008).
[326] D. Battefeld and T. Battefeld, Phys. Rev. **D80**, 063518 (2009), 0907.2443.
[327] J. J. Heckman, A. Tavanfar, and C. Vafa, JHEP **1004**, 054 (2010), 0812.3155.
[328] B. S. Acharya, P. Kumar, K. Bobkov, G. Kane, J. Shao, et al., JHEP **0806**, 064 (2008), 0804.0863.
[329] E. Huguet and P. Peter, Astropart.Phys. **12**, 277 (2000), hep-ph/9901370.
[330] Y.-F. Cai, R. Brandenberger, and X. Zhang, Phys. Lett. **B703**, 25 (2011), 1105.4286.
[331] J. Polchinski (1998).
[332] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, JHEP **0605**, 043 (2006), hep-th/0512177.
[333] V. Braun, Y.-H. He, and B. A. Ovrut, JHEP **1309**, 008 (2013), 1301.6767.
[334] R. Maier, S. Pereira, N. Pinto-Neto, and B. B. Siffert, Phys. Rev. **D85**, 023508 (2012), 1111.0946.
[335] J. K. Erickson, D. H. Wesley, P. J. Steinhardt, and N. Turok, Phys. Rev. **D69**, 063514 (2004), hep-th/0312009.
[336] D. Garfinkle, W. C. Lim, F. Pretorius, and P. J. Steinhardt, Phys. Rev. **D78**, 083537 (2008), 0808.0542.
[337] J.-L. Lehners and P. J. Steinhardt, Phys. Rev. Lett. **106**, 081301 (2011), 1008.4567.
[338] P. Ade et al. (Planck Collaboration) (2013), 1303.5076.
[339] A. D. Linde, Newton Centenary Volume (Cambridge U. Press) (1986).
[340] J. Garriga, A. D. Linde, and A. Vilenkin, Phys.Rev. **D69**, 063521 (2004), hep-th/0310034.
[341] L. Abbott, Sci.Am. **258**, 106 (1988).
[342] P. J. Steinhardt and N. Turok, Science **312**, 1180 (2006), astro-ph/0605173.
[343] M. G. Romania, N. Tsamis, and R. Woodard, Lect.Notes Phys. **863**, 375 (2013), 1204.6558.
[344] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).
[345] R. H. Brandenberger and J. Martin, Class. Quant. Grav. **30**, 113001 (2013), 1211.6753.
[346] S. D. Vitenti and N. Pinto-Neto, Phys.Rev. **D85**, 023524 (2012), 1111.0888.
[347] N. Pinto-Neto and S. Vitenti, Phys.Rev. **D89**, 028301 (2014), 1312.7790.
[348] I. P. Heard and D. Wands, Class.Quant.Grav. **19**, 5435 (2002), gr-qc/0206085.
[349] B. Xue, PhD dissertation, Princeton University (2014).
[350] V. Belinsky, I. Khalatnikov, and E. Lifshitz, Adv. Phys. **19**, 525 (1970).
[351] E. Lifshitz and I. Khalatnikov, Adv. Phys. **12**, 185 (1963).
[352] C. W. Misner, Phys. Rev. Lett. **22**, 1071 (1969).
[353] J. D. Barrow and K. Yamamoto, Phys. Rev. **D82**, 063516 (2010), 1004.4767.
[354] S. Dubovsky, T. Gregoire, A. Nicolis, and R. Rattazzi, JHEP **0603**, 025 (2006), hep-th/0512260.
[355] V. Rubakov (2014), 1401.4024.
[356] N. Arkani-Hamed, H.-C. Cheng, M. A. Luty, and S. Mukohyama, JHEP **0405**, 074 (2004), hep-th/0312099.
[357] P. Creminelli, M. A. Luty, A. Nicolis, and L. Senatore, JHEP **0612**, 080 (2006), hep-th/0606090.
[358] K. Hinterbichler, A. Joyce, J. Khoury, and G. E. Miller,
Phys. Lett. **B608**, 177 (2005), astro-ph/0410654.
[290] Y.-F. Cai, T. Qiu, R. Brandenberger, Y.-S. Piao, and X. Zhang, JCAP **0803**, 013 (2008), 0711.2187.
[291] A. De Felice and S. Tsujikawa, Living Rev.Rel. **13**, 3 (2010), 1002.4928.
[292] K. Bamba, A. N. Makarenko, A. N. Myagky, S. Nojiri, and S. D. Odintsov (2013), 1309.3748.
[293] A. V. Astashenok, Astrophys.Space Sci. **351**, 377 (2014), 1308.0581.
[294] I. Brevik, V. Obukhov, and A. Timoshkin (2014), 1404.1887.
[295] M. Bouhmadi-Lopez, J. Morais, and A. B. Henriques, Phys. Rev. **D87**, 103528 (2013), 1210.1761.
[296] K. Bamba, A. N. Makarenko, A. N. Myagky, and S. D. Odintsov (2014), 1403.3242.
[297] A. S. Koshelev, Class. Quant. Grav. **30**, 155001 (2013), 1302.2140.
[298] K. Hayashi and T. Shirafuji, Phys. Rev. **D19**, 3524 (1979).
[299] E. V. Linder, Phys. Rev. **D81**, 127301 (2010), 1005.3039.
[300] Y.-F. Cai, S.-H. Chen, J. B. Dent, S. Dutta, and E. N. Saridakis, Class. Quant. Grav. **28**, 215011 (2011), 1104.4349.
[301] H. Maeda, Phys. Rev. **D85**, 124012 (2012), 1112.5178.
[302] S. Mukherji and M. Peloso, Phys.Lett. **B547**, 297 (2002), hep-th/0205180.
[303] N. Turok, M. Perry, and P. J. Steinhardt, Phys.Rev. **D70**, 106004 (2004), hep-th/0408083.
[304] G. Geshnizjani, T. J. Battefeld, and G. Geshnizjani, Phys. Rev. **D73**, 048501 (2006), hep-th/0506139.
[305] V. Bozza and G. Veneziano, JCAP **0509**, 007 (2005), gr-qc/0506040.
[306] R. Maier, I. D. Soares, and E. Tonini, Phys. Rev. **D79**, 023522 (2009), 1212.3766.
[307] R. Maier, N. Pinto-Neto, and I. D. Soares, Phys. Rev. **D87**, 043528 (2013), 1301.5250.
[308] R. Maier, F. Pace, and I. D. a. Soares, Phys. Rev. D **88**, 106003 (2013), 1310.5038.
[309] Y.-F. Cai and E. N. Saridakis, JCAP **0910**, 020 (2009), 0906.1789.
[310] A. H. Chamseddine and V. Mukhanov, JHEP **1311**, 135 (2013), 1308.5410.
[311] A. H. Chamseddine, V. Mukhanov, and A. Vikman (2014), 1403.3961.
[312] M. Novello and J. Salim, Phys. Rev. **D20**, 377 (1979).
[313] W. Heisenberg and H. Euler, Z.Phys. **98**, 714 (1936), physics/0605038.
[314] M. Artymowski and Z. Lalak, Phys. Lett. **B707**, 203 (2012), 1109.5901.
[315] T. Kibble, J.Math.Phys. **2**, 212 (1961).
[316] D. W. Sciama, Rev. Mod. Phys. **36**, 463 (1964).
[317] S. Brechet, M. Hobson, and A. Lasenby, Class.Quant.Grav. **24**, 6329 (2007), 0706.2367.
[318] S. Brechet, M. Hobson, and A. Lasenby (2008), 0807.2523.
[319] N. J. Popławski, Phys. Rev. **D85**, 107502 (2012), 1111.4595.
[320] S. Alexander, C. Bambi, A. Marciano, and L. Modesto (2014), 1402.5880.
[321] A. Borde, A. H. Guth, and A. Vilenkin, Phys.Rev.Lett. **90**, 151301 (2003), gr-qc/0110012.
[322] P. Peter (2009), 0912.1293.
[323] P. Peter and N. Pinto-Neto, Phys. Rev. **D78**, 063506 Phys. Rev. Lett. **110**, 241303 (2013), 1212.3607.
[359] B. Elder, A. Joyce, and J. Khoury, Phys. Rev. **D89**, 044027 (2014), 1311.5889.
[360] A. Nicolis, R. Rattazzi, and E. Trincherini, JHEP **1005**, 095 (2010), 0912.4258.
[361] P. Creminelli, M. Serone, and E. Trincherini, JHEP **1310**, 040 (2013), 1306.2946.
[362] C. de Rham, M. Fasiello, and A. J. Tolley (2013), 1308.2702.
[363] V. Rubakov, Phys. Rev. **D88**, 044015 (2013), 1305.2614.
[364] L. R. Abramo and N. Pinto-Neto, Phys. Rev. **D73**, 063522 (2006), astro-ph/0511562.
[365] L. R. Abramo, I. Yasuda, and P. Peter, Phys. Rev. **D81**, 023511 (2010), 0910.3422.
[366] L. P. Chimento and M. I. Forte, Phys.Rev. **D73**, 063502 (2006), astro-ph/0510726.
[367] J. De-Santiago, J. L. Cervantes-Cota, and D. Wands, Phys.Rev. **D87**, 023502 (2013), 1204.3631.
[368] B. Xue, D. Garfinkle, F. Pretorius, and P. J. Steinhardt, Phys. Rev. **D88**, 083509 (2013), 1308.3044.
[369] R. V. Buniy, S. D. Hsu, and B. M. Murray, Phys. Rev. **D74**, 063518 (2006), hep-th/0606091.
[370] M. Koehn, J.-L. Lehners, and B. Ovrut, Phys. Rev. **D87**, 065022 (2013), 1212.2185.
[371] B. Xue and P. J. Steinhardt, Phys. Rev. **D84**, 083520 (2011), 1106.1416.
[372] D. A. Easson, I. Sawicki, and A. Vikman, JCAP **1307**, 014 (2013), 1304.3903.
[373] D. Wands, Phys. Rev. **D60**, 023507 (1999), gr-qc/9809062.
[374] E. Kiritsis and C. Kounnas, Phys. Lett. **B331**, 51 (1994), hep-th/9404092.
[375] J. J. Atick and E. Witten, Nucl. Phys. **B310**, 291 (1988).
[376] I. Antoniadis and C. Kounnas, Phys. Lett. **B261**, 369 (1991).
[377] J. Barbon and E. Rabinovici, JHEP **0203**, 057 (2002), hep-th/0112173.
[378] J. Barbon and E. Rabinovici (2004), hep-th/0407236.
[379] S. Kalyana Rama, Phys. Lett. **B408**, 91 (1997), hep-th/9701154.
[380] B. A. Bassett, M. Borunda, M. Serone, and S. Tsujikawa, Phys. Rev. **D67**, 123506 (2003), hep-th/0301180.
[381] R. Easther, B. R. Greene, M. G. Jackson, and D. N. Kabat, JCAP **0502**, 009 (2005), hep-th/0409121.
[382] B. Greene, D. Kabat, and S. Marnerides, Phys. Rev. **D80**, 063526 (2009), 0809.1704.
[383] L. Boubekeur, P. Creminelli, J. Norena, and F. Vernizzi, JCAP **0808**, 028 (2008), 0806.1016.
[384] W. Israel, Nuovo Cim. **B44S10**, 1 (1966).
[385] X. Gao, M. Lilley, and P. Peter (2014), 1403.7958.
[386] X. Gao, M. Lilley, and P. Peter (2014), 1406.4119.
[387] R. H. Brandenberger, Class. Quant. Grav. **28**, 204005 (2011), 1105.3247.
[388] A. Nayeri, R. H. Brandenberger, and C. Vafa, Phys. Rev. Lett. **97**, 021302 (2006), hep-th/0511140.
[389] R. H. Brandenberger, S. Kanno, J. Soda, D. A. Easson, J. Khoury, et al., JCAP **0611**, 009 (2006), hep-th/0608186.
[390] N. Kaloper, L. Kofman, A. D. Linde, and V. Mukhanov, JCAP **0610**, 006 (2006), hep-th/0608200.
[391] R. H. Brandenberger, A. Nayeri, S. P. Patil, and C. Vafa, Phys. Rev. Lett. **98**, 231302 (2007), hep-
th/0604126.
[392] A. Nayeri (2006), hep-th/0607073.
[393] P. A. R. Ade et al. (BICEP2 Collaboration) (2014), 1403.4302.
[394] F. Finelli and R. Brandenberger, Phys. Rev. **D65**, 103522 (2002), hep-th/0112249.
[395] D. H. Lyth, Phys. Lett. **B526**, 173 (2002), hep-ph/0110007.
[396] K. Enqvist, E. Keski-Vakkuri, and S. Rasanen, Nucl. Phys. **B614**, 388 (2001), hep-th/0106282.
[397] T. Battefeld, S. P. Patil, and R. Brandenberger, Phys. Rev. **D70**, 066006 (2004), hep-th/0401010.
[398] H. Kodama and T. Hamazaki, Prog.Theor.Phys. **96**, 949 (1996), gr-qc/9608022.
[399] M. Gasperini, M. Giovannini, and G. Veneziano, Nucl. Phys. **B694**, 206 (2004), hep-th/0401112.
[400] J. M. Bardeen, Phys. Rev. **D22**, 1882 (1980).
[401] A. Kumar, Phys. Rev. **D86**, 123522 (2012), 1205.6637.
[402] V. F. Mukhanov, H. Feldman, and R. H. Brandenberger, Phys. Rept. **215**, 203 (1992).
[403] A. A. Starobinsky, JETP Lett. **42**, 152 (1985).
[404] M. Sasaki and E. D. Stewart, Prog.Theor.Phys. **95**, 71 (1996), astro-ph/9507001.
[405] M. Sasaki and T. Tanaka, Prog.Theor.Phys. **99**, 763 (1998), gr-qc/9801017.
[406] F. Vernizzi and D. Wands, JCAP **0605**, 019 (2006), astro-ph/0603799.
[407] G. Rigopoulos, E. Shellard, and B. van Tent, Phys. Rev. **D73**, 083522 (2006), astro-ph/0506704.
[408] D. H. Lyth and Y. Rodriguez, Phys. Rev. Lett. **95**, 121302 (2005), astro-ph/0504045.
[409] C. Gordon, D. Wands, B. A. Bassett, and R. Maartens, Phys. Rev. **D63**, 023506 (2001), astro-ph/0009131.
[410] E. F. Bunn, A. R. Liddle, and . White, Martin J., Phys. Rev. **D54**, 5917 (1996), astro-ph/9607038.
[411] G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. **D69**, 023505 (2004), astro-ph/0303591.
[412] A. J. Tolley and D. H. Wesley, JCAP **0705**, 006 (2007), hep-th/0703101.
[413] L. A. Boyle, P. J. Steinhardt, and N. Turok, Phys. Rev. **D69**, 127302 (2004), hep-th/0307170.
[414] M. Gasperini and G. Veneziano, Mod.Phys. Lett. **A8**, 3701 (1993), hep-th/9309023.
[415] M. Gasperini and G. Veneziano, Phys. Rev. **D50**, 2519 (1994), gr-qc/9403031.
[416] J.-c. Hwang and H. Noh, Phys. Rev. **D65**, 124010 (2002), astro-ph/0112079.
[417] C. Cartier, R. Durrer, and E. J. Copeland, Phys. Rev. **D67**, 103517 (2003), hep-th/0301198.
[418] R. H. Brandenberger, Int. J. Mod. Phys. Conf.Ser. **01**, 67 (2011), 0902.4731.
[419] J.-c. Hwang and E. T. Vishniac, Astrophys. J. **382**, 363 (1991).
[420] V. Bozza, M. Giovannini, and G. Veneziano, JCAP **0305**, 001 (2003), hep-th/0302184.
[421] D. Baumann, P. J. Steinhardt, K. Takahashi, and K. Ichiki, Phys.Rev. **D76**, 084019 (2007), hep-th/0703290.
[422] A. Starobinskii, JETP Lett. **30**, 682 (1979).
[423] P. Ade et al. (Planck Collaboration) (2013), 1303.5075.
[424] B. Audren, D. G. Figueroa, and T. Tram (2014), 1405.1390.
[425] R. Adam et al. (Planck Collaboration) (2014), 1409.5738.
A. Mazumdar, Ann.Rev.Nucl.Part.Sci. **60**, 27 (2010), 1001.2600.
[461] J. M. Cline (2006), hep-th/0612129.
[462] L. McAllister and E. Silverstein, Gen. Rel. Grav. **40**, 565 (2008), 0710.2951.
[463] C. Burgess and L. McAllister, Class. Quant. Grav. **28**, 204002 (2011), 1108.2660.
[464] K. Dasgupta, R. Gwyn, E. McDonough, M. Mia, and R. Tatar, JHEP **1407**, 054 (2014), 1402.5112.
[465] E. Silverstein and A. Westphal, Phys. Rev. **D78**, 106003 (2008), 0803.3085.
[466] S. P. Patil and R. Brandenberger, Phys.Rev. **D71**, 103522 (2005), hep-th/0401037.
[467] S. P. Patil and R. H. Brandenberger, JCAP **0601**, 005 (2006), hep-th/0502069.
[468] S. P. Patil (2005), hep-th/0504145.
[469] R. J. Danos, A. R. Frey, and R. H. Brandenberger, Phys.Rev. **D77**, 126009 (2008), 0802.1557.
[470] S. Mishra, W. Xue, R. Brandenberger, and U. Yajnik, JCAP **1209**, 015 (2012), 1103.1389.
[471] S. Watson and R. Brandenberger, JCAP **0311**, 008 (2003), hep-th/0307044.
[472] S. Watson, Phys.Rev. **D70**, 066005 (2004), hep-
[426] W. N. Colley and J. R. Gott (2014), 1409.4491.
[427] T. Battefeld and R. Easther, JCAP **0703**, 020 (2007), astro-ph/0610296.
[428] D. Battefeld and T. Battefeld, JCAP **0911**, 010 (2009), 0908.4269.
[429] D. Babich, P. Creminelli, and M. Zaldarriaga, JCAP **0408**, 009 (2004), astro-ph/0405356.
[430] T. Battefeld and J. Grieb, JCAP **1112**, 003 (2011), 1110.1369.
[431] P. Ade et al. (Planck Collaboration) (2013), 1303.5084.
[432] J. M. Maldacena, JHEP **0305**, 013 (2003), astro-ph/0210603.
[433] P. Creminelli, JCAP **0310**, 003 (2003), astro-ph/0306122.
[434] P. Creminelli and M. Zaldarriaga, JCAP **0410**, 006 (2004), astro-ph/0407059.
[435] K. Enqvist and S. Nurmi, JCAP **0510**, 013 (2005), astro-ph/0508573.
[436] M. Sasaki, J. Valiviita, and D. Wands, Phys. Rev. **D74**, 103003 (2006), astro-ph/0607627.
[437] K. Enqvist and T. Takahashi, JCAP **0809**, 012 (2008), 0807.3069.
[438] M. Kawasaki, T. Kobayashi, and F. Takahashi, Phys. Rev. **D84**, 123506 (2011), 1107.6011.
[439] G. Leung, E. R. Tarrant, C. T. Byrnes, and E. J. Copeland, JCAP **1209**, 008 (2012), 1206.5196.
[440] G. Leung, E. R. Tarrant, C. T. Byrnes, and E. J. Copeland, JCAP **1308**, 006 (2013), 1303.4678.
[441] C. T. Byrnes and G. Tasinato, JCAP **0908**, 016 (2009), 0906.0767.
[442] J. Elliston, D. J. Mulryne, D. Seery, and R. Tavakol, JCAP **1111**, 005 (2011), 1106.2153.
[443] X. Gao, M. Lilley, and P. Peter (2014), 1406.xxxx.
[444] J.-L. Lehners and S. Renaux-Petel, Phys. Rev. **D80**, 063503 (2009), 0906.0530.
[445] G. N. Felder, L. Kofman, and A. D. Linde, Phys. Rev. **D60**, 103505 (1999), hep-ph/9903350.
[446] J.-c. Hwang, Phys. Rev. **D65**, 063514 (2002), astro-ph/0109045.
[447] S. Tsujikawa, Phys. Lett. **B526**, 179 (2002), gr-qc/0110124.
[448] J. Martin and P. Peter, Phys. Rev. **D69**, 107301 (2004), hep-th/0403173.
[449] J. Khoury, P. J. Steinhardt, and N. Turok, Phys. Rev. Lett. **91**, 161301 (2003), astro-ph/0302012.
[450] L. Kofman, A. D. Linde, and A. A. Starobinsky, Phys. Rev. **D56**, 3258 (1997), hep-ph/9704452.
[451] J. Meyers and E. R. M. Tarrant (2013), 1311.3972.
[452] J. Zhang, Y. Cai, and Y.-S. Piao (2013), 1307.6529.
[453] B. Underwood and Y. Zhai (2013), 1312.3006.
[454] J. Braden, L. Kofman, and N. Barnaby, JCAP **1007**, 016 (2010), 1005.2196.
[455] D. Battefeld and S. Kawai, Phys. Rev. **D77**, 123507 (2008), 0803.0321.
[456] H. L. Child, J. Giblin, John T., R. H. Ribeiro, and D. Seery, Phys. Rev. Lett. **111**, 051301 (2013), 1305.0561.
[457] L. Perreault Levasseur, R. Brandenberger, and A.-C. Davis, Phys. Rev. **D84**, 103512 (2011), 1105.5649.
[458] I. Rudenok, Y. Shtanov, and S. Vilchinskii, Phys.Lett. **B733**, 193 (2014), 1401.7298.
[459] A. Mazumdar and J. Rocher, Phys. Rept. **497**, 85 (2011), 1001.0993.
[460] R. Allahverdi, R. Brandenberger, F.-Y. Cyr-Racine, and
[473] R. Brandenberger, Y.-K. E. Cheung, and S. Watson, JHEP **0605**, 025 (2006), hep-th/0501032.
[474] S. Cremonini and S. Watson, Phys.Rev. **D73**, 086007 (2006), hep-th/0601082.
[475] L. Liu and H. Partouche, JHEP **1211**, 079 (2012), 1111.7307.
[476] T. Battefeld and N. Shuhmaher, Phys.Rev. **D74**, 123501 (2006), hep-th/0607061.
[477] F. Ferrer and S. Rasanen, JHEP **0602**, 016 (2006), hep-th/0509225.
[478] D. Wands, K. A. Malik, D. H. Lyth, and A. R. Liddle, Phys. Rev. **D62**, 043527 (2000), astro-ph/0003278.
[479] L. R. Abramo and P. Peter, JCAP **0709**, 001 (2007), 0705.2893.
[480] J. Elliston, S. Orani, and D. J. Mulryne, Phys.Rev. **D89**, 103532 (2014), 1402.4800.
[481] D. H. Lyth, Phys.Rev.Lett. **78**, 1861 (1997), hep-ph/9606387.
[482] Y.-F. Cai, R. Brandenberger, and X. Zhang, JCAP **1103**, 003 (2011), 1101.0822.
[483] Y.-F. Cai, J. Quintin, E. N. Saridakis, and E. Wilson-Ewing, JCAP **1407**, 033 (2014), 1404.4364.

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Standard Failures and bouncing solutions

Singularity Merely a non issue in the bounce case!

Horizon $d_{\mathrm{H}} \equiv a(t) \int_{t_{\mathrm{i}}}^{t} \frac{\mathrm{d}\tau}{a(\tau)}$ can be made divergent easily if $t_{\mathrm{i}} \to -\infty$

Flatness $\frac{\mathrm{d}}{\mathrm{d}t} |\Omega - 1| = -2 \frac{\ddot{a}}{\dot{a}^3}$ $\quad \ddot{a} < 0 \;\&\; \dot{a} < 0$

accelerated expansion (inflation) or decelerated contraction (bounce)

Homogeneity Large & flat Universe + low initial density + diffusion

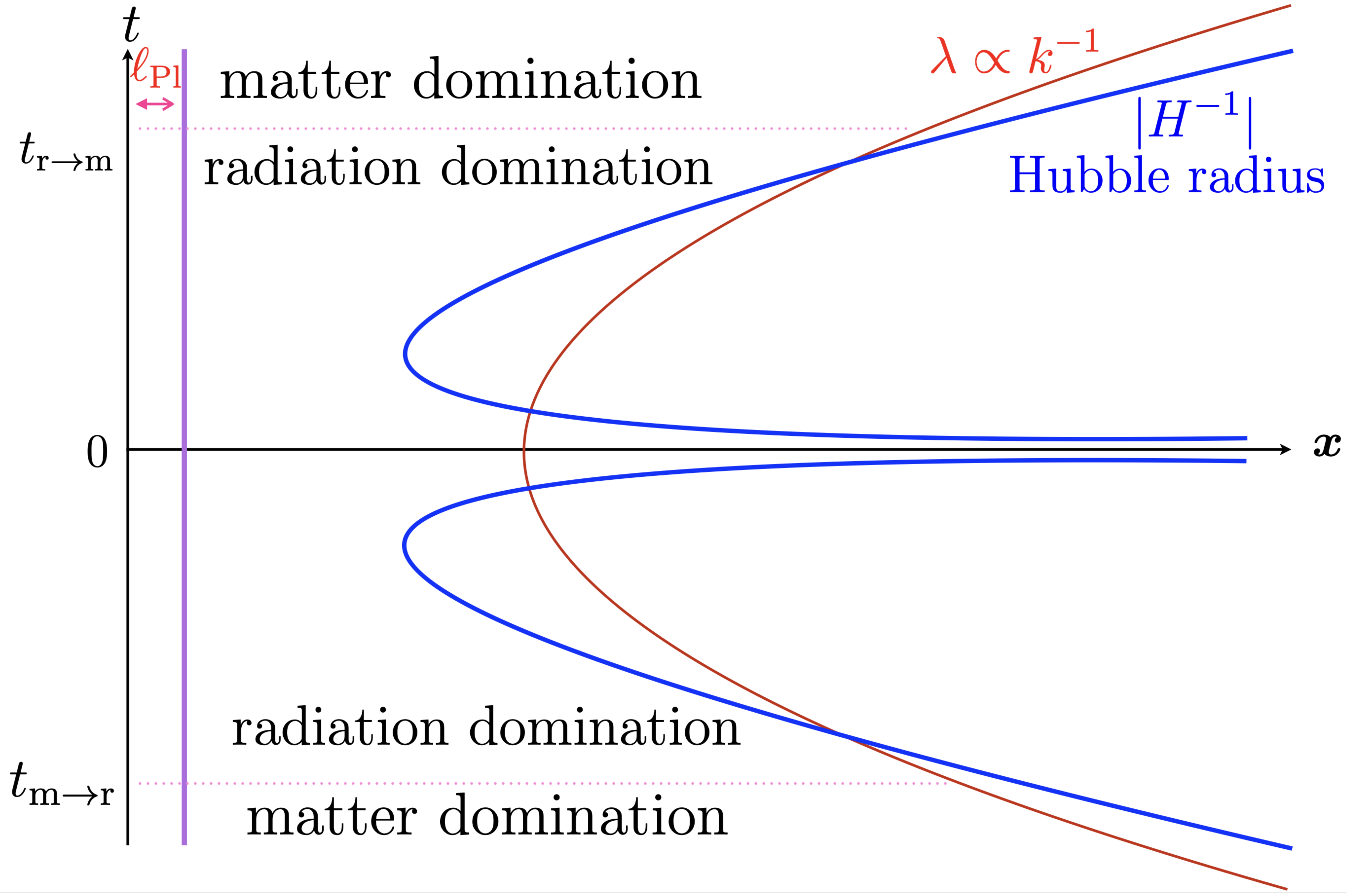
$$\frac{t_{\mathrm{dissipation}}}{t_{\mathrm{Hubble}}} \propto \frac{\lambda}{R_{\mathrm{H}}^{1/3}} \left(1 + \frac{\lambda}{A R_{\mathrm{H}}^2}\right)$$

enough time to dissipate any wavelength

$\Longrightarrow$ quantum vacuum fluctuations...

Isotropy Potentially problematic: model dependent

Others dark matter/energy, baryogenesis, ...



t
$\ell_{\rm Pl}$
matter domination
$\lambda \propto k^{-1}$
$|H^{-1}|$
Hubble radius
$t_{\rm r\to m}$
radiation domination
0
$\boldsymbol{x}$
radiation domination
$t_{\rm m\to r}$
matter domination

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$$d_{\mathrm{H}}^{\mathrm{cont}} = \frac{3(1+w)}{1+3w} t_{\mathrm{end}} \left[1 - \left(\frac{t_{\mathrm{ini}}}{t_{\mathrm{end}}} \right)^{(1+3w)/[3(1+w)]} \right]$$

$$t_{\mathrm{ini}} \to -\infty$$

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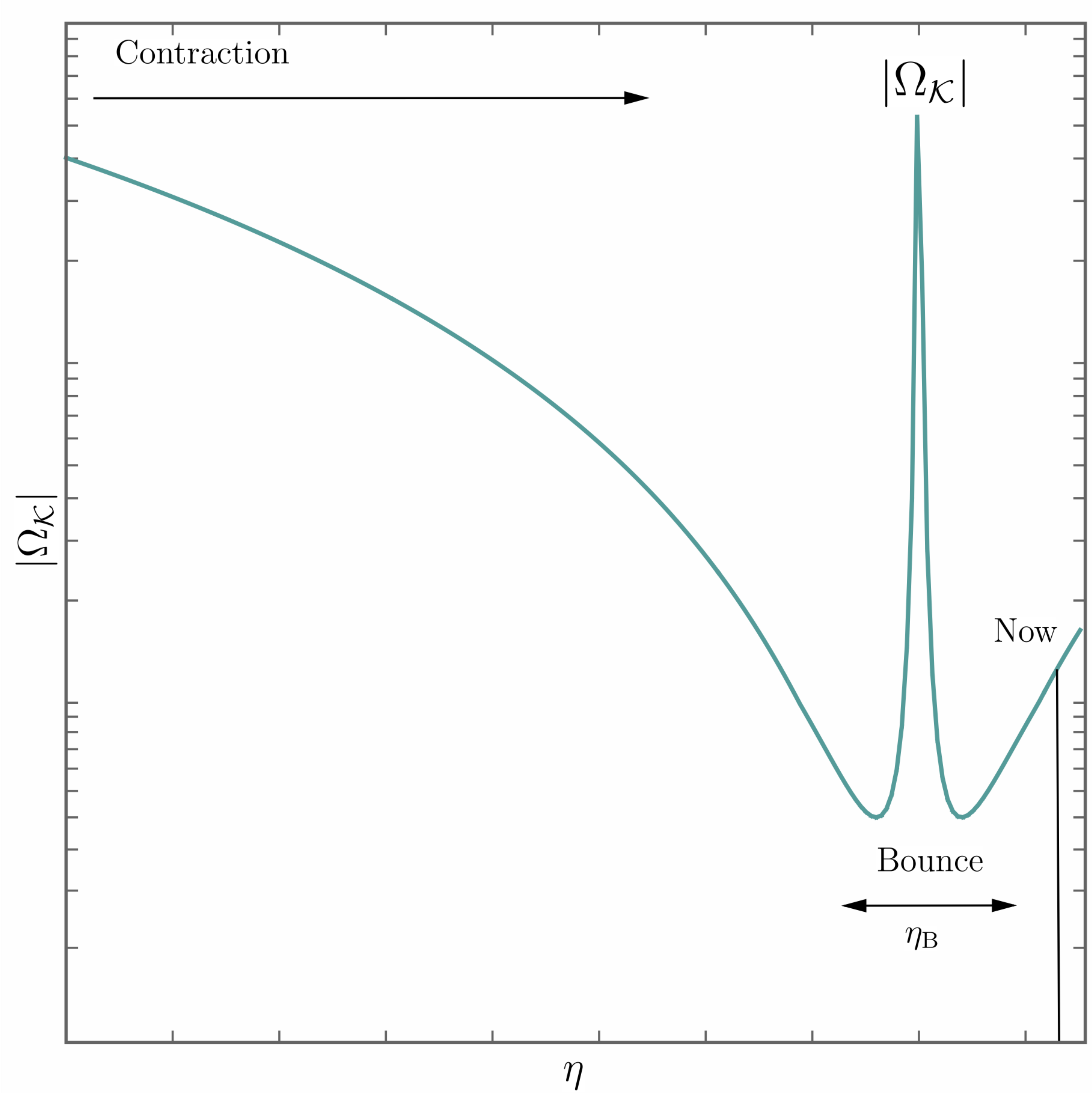
$$H^2 = \frac{1}{3}\left[-\frac{3\mathcal{K}}{a^2} + \frac{\rho_{\mathrm{m}0}}{a^3} + \frac{\rho_{\mathrm{r}0}}{a^4} + \frac{\rho_{\theta 0}}{a^6} + \ldots + \frac{\rho_{\phi 0}}{a^{3(1+w_\phi)}}\right]$$

Critical density

$$\rho_{\mathrm{c}} \equiv \frac{3H^2}{8\pi G_{\mathrm{N}}} \implies \Omega \equiv \frac{\rho}{\rho_{\mathrm{c}}}$$

Density parameter



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$\Longrightarrow$ quantum vacuum fluctuations...

Shear Potentially problematic: model dependent

Others dark matter/energy, baryogenesis, ...

The problem with contraction: BKL/shear instability

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\sum_i \mathrm{e}^{2\theta_i(t)}\sigma^i\sigma^i$$

Ricci flat:

$$\sigma^i = \mathrm{d}x^i$$

$$\sum_i \theta_i = 0$$

Average scale factor

$$\frac{\dot{a}}{a}$$ Mean Hubble parameter

$$H_i \equiv \frac{1}{a\mathrm{e}^{\theta_i}}\frac{\mathrm{d}}{\mathrm{d}t}\left(a\mathrm{e}^{\theta_i}\right) = H + \dot{\theta}_i$$

Friedman equations

$$H^2 = \frac{\rho_\mathrm{T}}{3M_\mathrm{Pl}^2} + \frac{1}{6}\sum_i \dot{\theta}_i^2$$

$$\dot{H} = -\frac{\rho_\mathrm{T}+p_\mathrm{T}}{2M_\mathrm{Pl}^2} - \frac{1}{2}\sum_i \dot{\theta}_i^2$$

$$\ddot{\theta}_i + 3H\dot{\theta}_i = 0$$

$$\rho_\mathrm{shear} \propto a^{-6}$$

Ekpyrotic solution:

$$w_{\rm ekp} \gg 1 \quad \Longrightarrow \rho_{\rm ekp} \propto a^{-3\left(1+w_{\rm ekp}\right)} \gg a^{-6} \text{ when } \quad a \to 0$$

ultra stiff eq. of state

Problem: regular bounce ➡ $\exists$ phase with $w_{\rm bounce} < -1$

So finally...

$$\rho_{\rm Shear} \equiv \frac{M_{\rm Pl}^2}{2} \sum_i \dot{\theta}_i^2 \propto a^{-6} \gg \rho_{\rm Fluid}$$

➡ Singularity!

Cosmic no-hair for ekpyrosis

- Bianchi I-VIII curvature always negative
- Impose $\rho \geq 0$
- Ultra stiff e.o.s., i.e. $\gamma = 1 + w > 2$

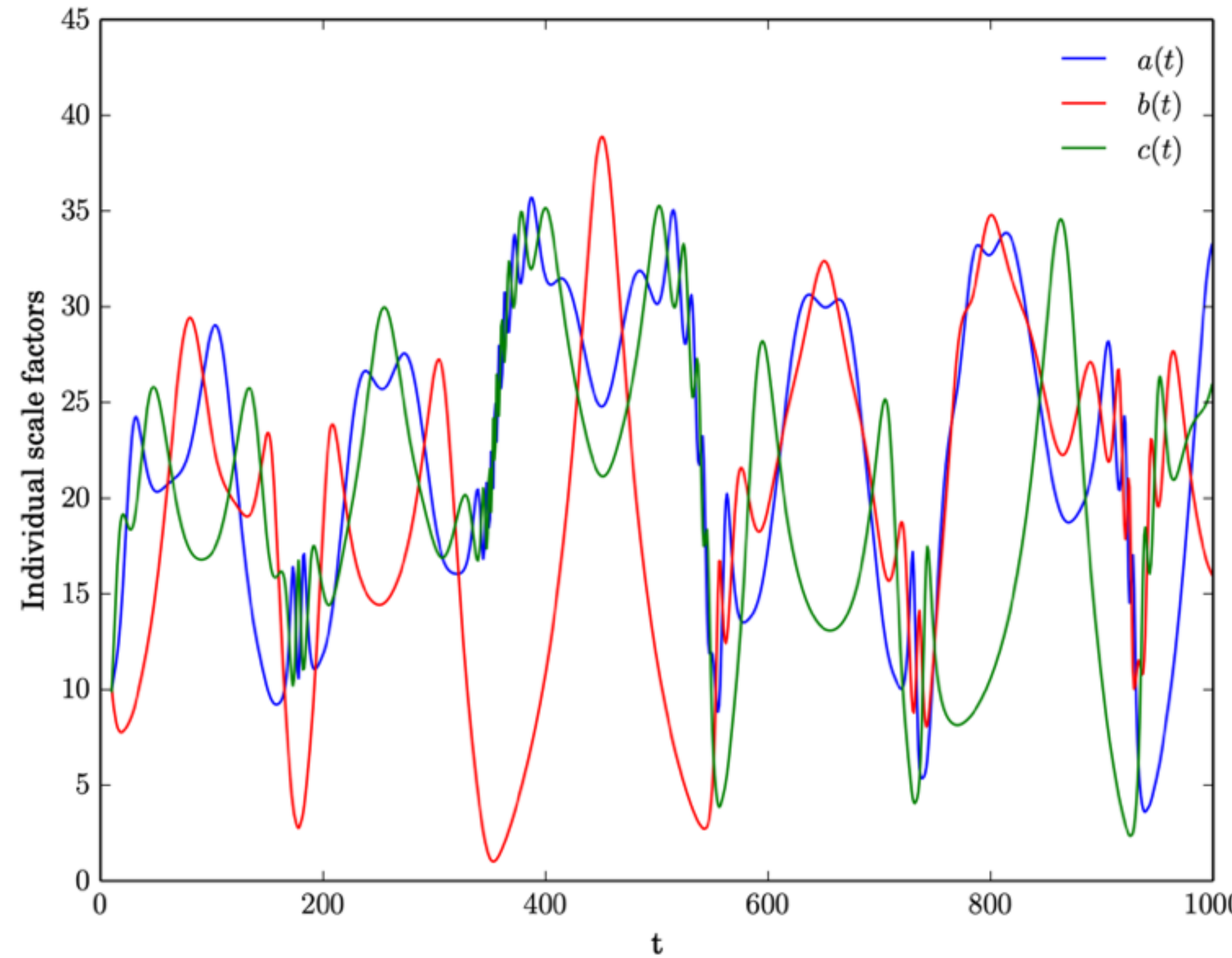
All initially contracting, spatially homogeneous, orthogonal Bianchi Type I-VIII cosmologies and all Bianchi type IX universes sourced by an ultra-stiff fluid with an equation of state such that $(\gamma - 2)$ is positive definite collapse into an isotropic singularity, where the sink is a spatially flat and isotropic FRW universe.

J. Lidsey, CQG**23** (2006) 3517

Anisotropic pressures and ekpyrosis

- Add stress-energy tensor anisotropies (& flat anisotropic Universe):
- $\dot{\sigma}_{ab} + 3H\sigma_{ab} = \pi_{ab}$
- Energy density anisotropies $\sigma^2 \sim \sigma_{ab}\sigma^{ab}$ can grow faster than V^{-2}
- Ekpyrotic fluid must grow faster… fine-tuned w.r.t. the anisotropic stress
- Expansion + 3-curvature anisotropy (Mixmaster) doesn't re-expand

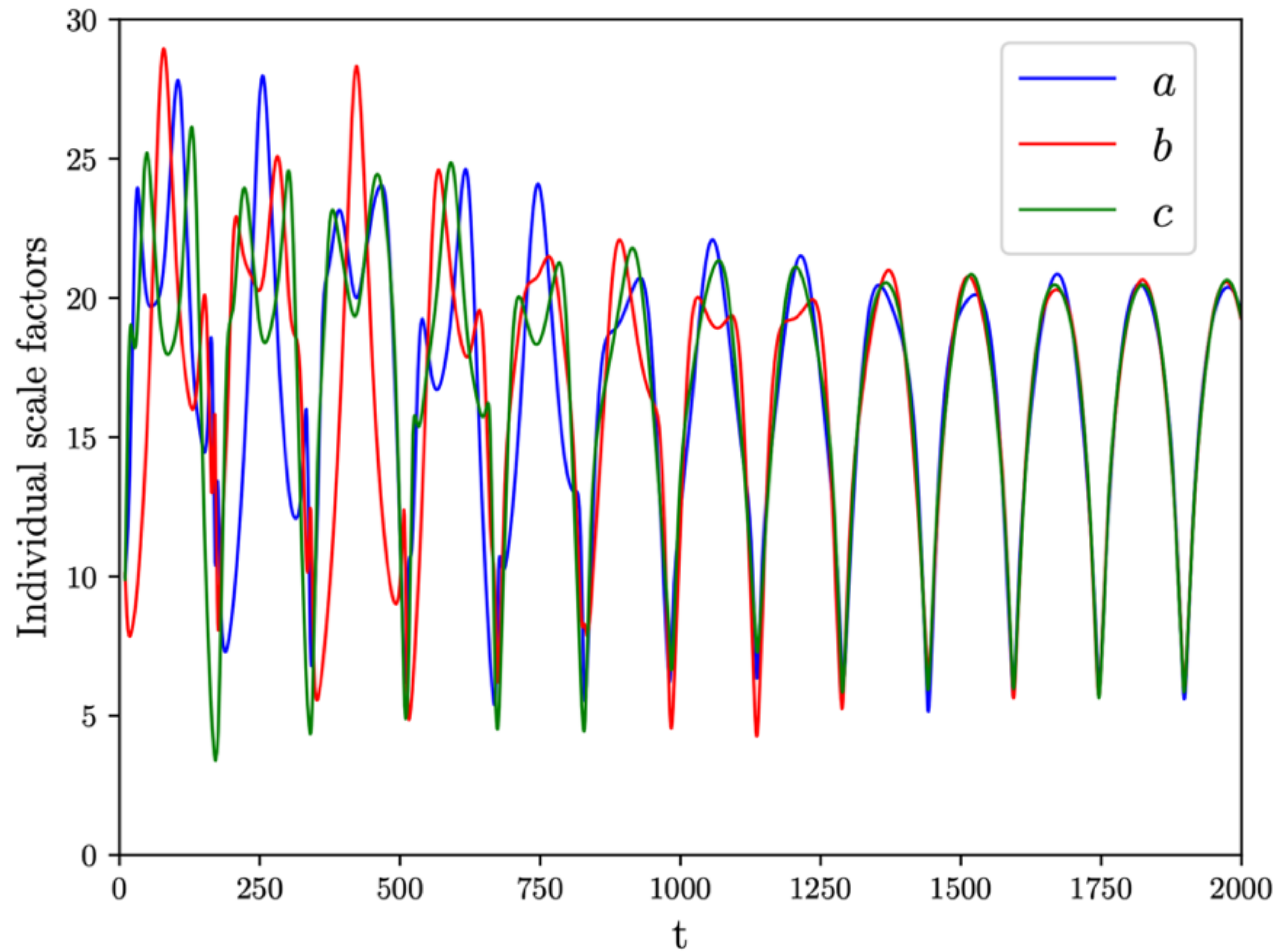
Mixmaster



- Curvature + expansion anisotropies
- Chaotic oscillations - infinite in number on approach to singularity
- Chaotic behaviour = attractor
- Anisotropy growth → collapse

Isotropisation by shear viscosity

$$\dot{\sigma}_{ab} + 3H\sigma_{ab} = \pi_{ab} \qquad \& \qquad \pi_{ab} = -\kappa\rho^{1/2}\sigma_{ab}$$



C. Ganguly & M. Bruni, PRL **123** (2019) 201301

Cosmic no-hair reloaded

- Bianchi I-VIII curvature always negative
- impose $\rho \geq 0$
- Matter obeys Strong Energy Condition, i.e. $\rho + 3p > 0$
- Viscosity coefficient κ obeys $\kappa > 3(2 - \gamma)$

> All initially contracting, spatially homogeneous, orthogonal Bianchi Type I-VIII cosmologies and all Bianchi type IX universes sourced by fluid obeying the Strong Energy Condition and with a shear viscous term such that the coefficient of viscosity κ obeys the condition that $\kappa > 3(2 - \gamma)$ collapse into an isotropic singularity, where the sink is a spatially flat and isotropic FRW universe.

C. Ganguly, arXiv:2008.02286

Quantum cosmology

Hamiltonian GR (3+1)

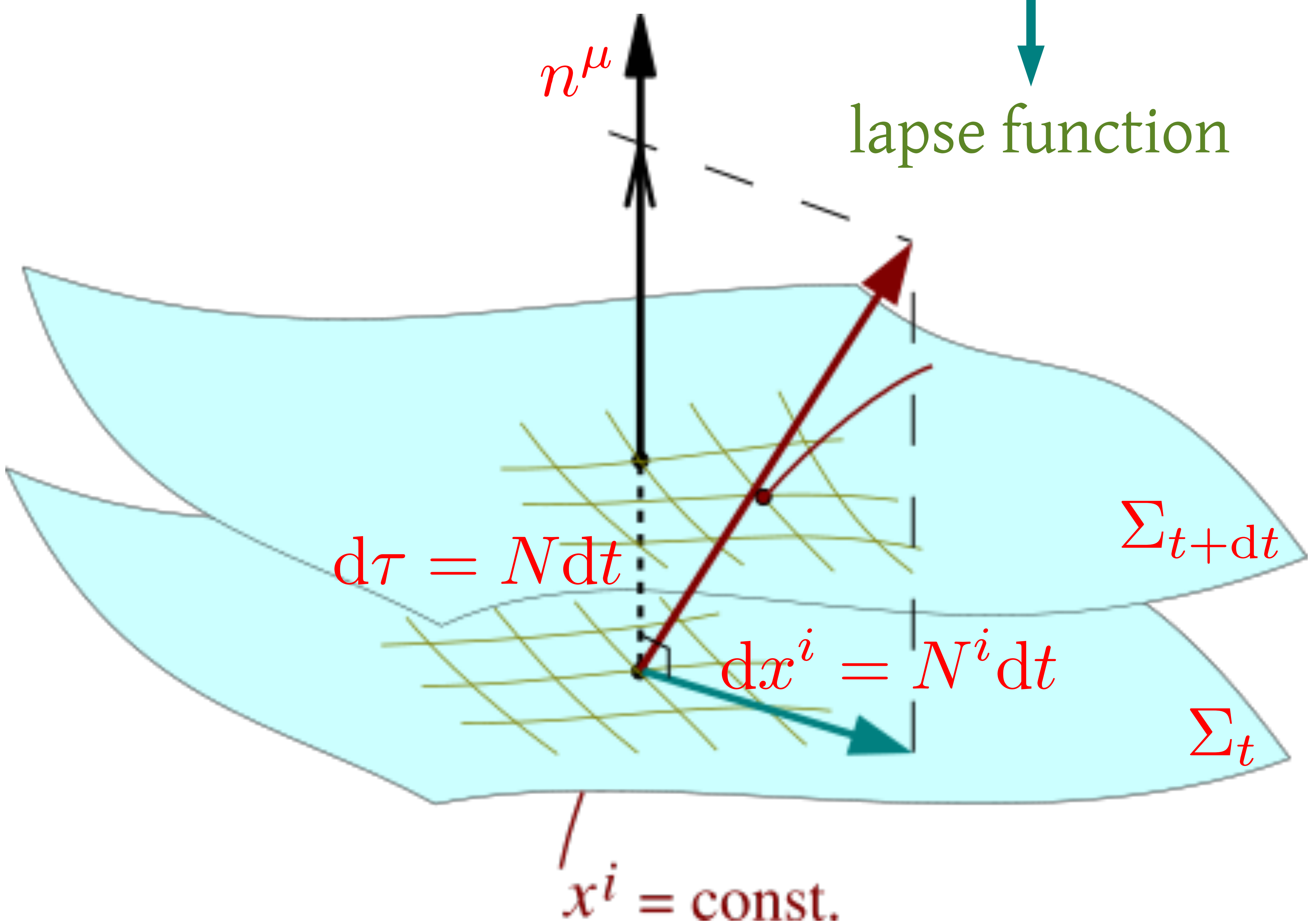
$$\mathrm{d}s^2 = g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -N^2\mathrm{d}t^2 + h_{ij}\left(\mathrm{d}x^i + N^i\mathrm{d}t\right)\left(\mathrm{d}x^j + N^j\mathrm{d}t\right)$$

lapse function

intrinsic metric =
first fundamental form

shift vector

intrinsic curvature tensor
${}^3R^i{}_{jkl}(h)$



extrinsic curvature =
second fundamental form:

$$K_{ij} = -\nabla_j^{(h)} n_i = \frac{1}{2N}\left(\nabla_j^{(h)} N_i + \nabla_i^{(h)} N_j - \frac{\partial h_{ij}}{\partial t}\right)$$

Action (Einstein-Hilbert, compact space):

$$\mathcal{S} = \frac{1}{16\pi G_{\mathrm{N}}}\left[\int_{\mathcal{M}} \sqrt{-g}\ (R - 2\Lambda)\, \mathrm{d}^4x + 2\int_{\partial\mathcal{M}} \sqrt{h} K^i{}_i \mathrm{d}^3x\right] + \mathcal{S}_{\text{matter}}\left[\Phi\left(x\right)\right]$$

$$\longrightarrow \mathcal{S} = \int L \mathrm{d}t = \frac{1}{16\pi G_{\mathrm{N}}}\int \mathrm{d}t \left[\int \mathrm{d}^3x\, N\sqrt{h}\left(K_{ij}K^{ij} - K^2 + {}^3R - 2\Lambda\right) + L_{\text{matter}}\right]$$

Canonical momenta

$$\pi^{ij} \equiv \frac{\delta L}{\delta \dot{h}_{ij}} = -\frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(K^{ij} - h^{ij}K\right)$$

$$\pi^{\Phi} \equiv \frac{\delta L}{\delta \dot{\Phi}} = -\frac{\sqrt{h}}{N}\left(\dot{\Phi} - N\frac{\partial \Phi}{\partial x^i}\right)$$

$$\left.\begin{aligned} \pi^{0} &\equiv \frac{\delta L}{\delta \dot{N}} \approx 0 \\ \pi^{i} &\equiv \frac{\delta L}{\delta \dot{N}^i} \approx 0 \end{aligned}\right\} \text{primary constraints}$$

Hamiltonian $H \equiv \int d^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + \pi^{ij} \dot{h}_{ij} + \pi^\Phi \dot{\Phi}\right) - L$

$$= \int d^3x \left(\pi^0 \dot{N} + \pi^i \dot{N}_i + N\mathcal{H} + N_i \mathcal{H}^i\right)$$

$$\mathcal{H} = \frac{1}{\sqrt{h}} \left(h_{ik} h_{jl} - \frac{1}{2} h_{ij} h_{kl}\right) \pi^{ij} \pi^{kl} - \sqrt{h}\, {}^3\!R$$

$$\mathcal{H}^i = -2\sqrt{h} \nabla_j \left(\frac{\pi^{ij}}{\sqrt{h}}\right)$$

variation wrt lapse: $\mathcal{H} = 0 \rightarrow$ Hamiltonian constraint
variation wrt shift: $\mathcal{H}^i = 0 \rightarrow$ momentum constraint

$\Longrightarrow$ classical description complete

Superspace & canonical quantization

relevant configuration space $\mathrm{Riem}\left(\Sigma\right) \equiv \left\{h_{ij}\left(x^{\mu}\right), \Phi\left(x^{\mu}\right) \middle| x \in \Sigma\right\}$

→ parameters

GR $\Longrightarrow$ invariance/diffeomorphisms $\Longrightarrow$ $\mathrm{Conf} = \dfrac{\mathrm{Riem}\left(\Sigma\right)}{\mathrm{Diff}\left(\Sigma\right)}$: superspace

Wave functional $\Psi\left[h_{ij}\left(x\right), \Phi\left(x\right)\right] = \langle h_{ij}, \Phi | \Psi \rangle$

+ Dirac canonical quantization procedure

$$\pi^{ij} \rightarrow -i\frac{\delta}{\delta h_{ij}} \qquad \pi^{\Phi} \rightarrow -i\frac{\delta}{\delta \Phi} \qquad \pi^{0} \rightarrow -i\frac{\delta}{\delta N} \qquad \pi^{i} \rightarrow -i\frac{\delta}{\delta N_i}$$

primary constraints
$$\begin{cases} \hat{\pi}^0 = -i\dfrac{\delta\Psi}{\delta N} = 0 \\ \hat{\pi}^i = -i\dfrac{\delta\Psi}{\delta N_i} = 0 \end{cases}$$

momentum $\hat{\mathcal{H}}^i\Psi = 0 \quad \Longrightarrow \quad i\nabla_j^{(h)}\left(\dfrac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\mathrm{N}}\hat{T}^{0i}\Psi$

same Ψ for configurations related by a coordinate transformation

Hamiltonian

$$\hat{\mathcal{H}}\Psi = \left[-16\pi G_{\mathrm{N}}\mathcal{G}_{ijkl}\frac{\delta^2}{\delta h_{ij}\delta h_{kl}} + \frac{\sqrt{h}}{16\pi G_{\mathrm{N}}}\left(-{}^3R + 2\Lambda + 16\pi G_{\mathrm{N}}\hat{T}^{00}\right)\right]\Psi = 0$$

Wheeler - De Witt equation

$$\mathcal{G}_{ijkl} = \frac{1}{2\sqrt{h}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right)$$

De Witt metric

primary constraints
$$\begin{cases} \hat{\pi}^0 = -i\dfrac{\delta\Psi}{\delta N} = 0 \\ \hat{\pi}^i = -i\dfrac{\delta\Psi}{\delta N_i} = 0 \end{cases}$$

momentum $\hat{\mathcal{H}}^i\Psi = 0 \implies i\nabla_j^{(h)}\left(\dfrac{\delta\Psi}{\delta h_{ij}}\right) = 8\pi G_{\mathrm{N}}\hat{T}^{0i}\Psi$

same Ψ for configurations related by a coordinate transformation

Hamiltonian

$$\hat{\mathcal{H}}\Psi = 0$$

TIMELESS Schrödinger equation

mini-superspace

restrict attention from an infinite dimensional configuration space to a 2 dimensional space = mini-superspace

$$h_{ij}\mathrm{d}x^i\mathrm{d}x^j \quad \mapsto \quad a^2(t)\left[\frac{\mathrm{d}r^2}{1-\mathcal{K}r^2}+r^2\left(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2\right)\right]$$

WDW equation becomes Schrödinger like for $\Psi\left[a(t),\phi(t)\right]$

Conceptual & technical issues

infinite # d.o.f. to a few: mathematical consistency?

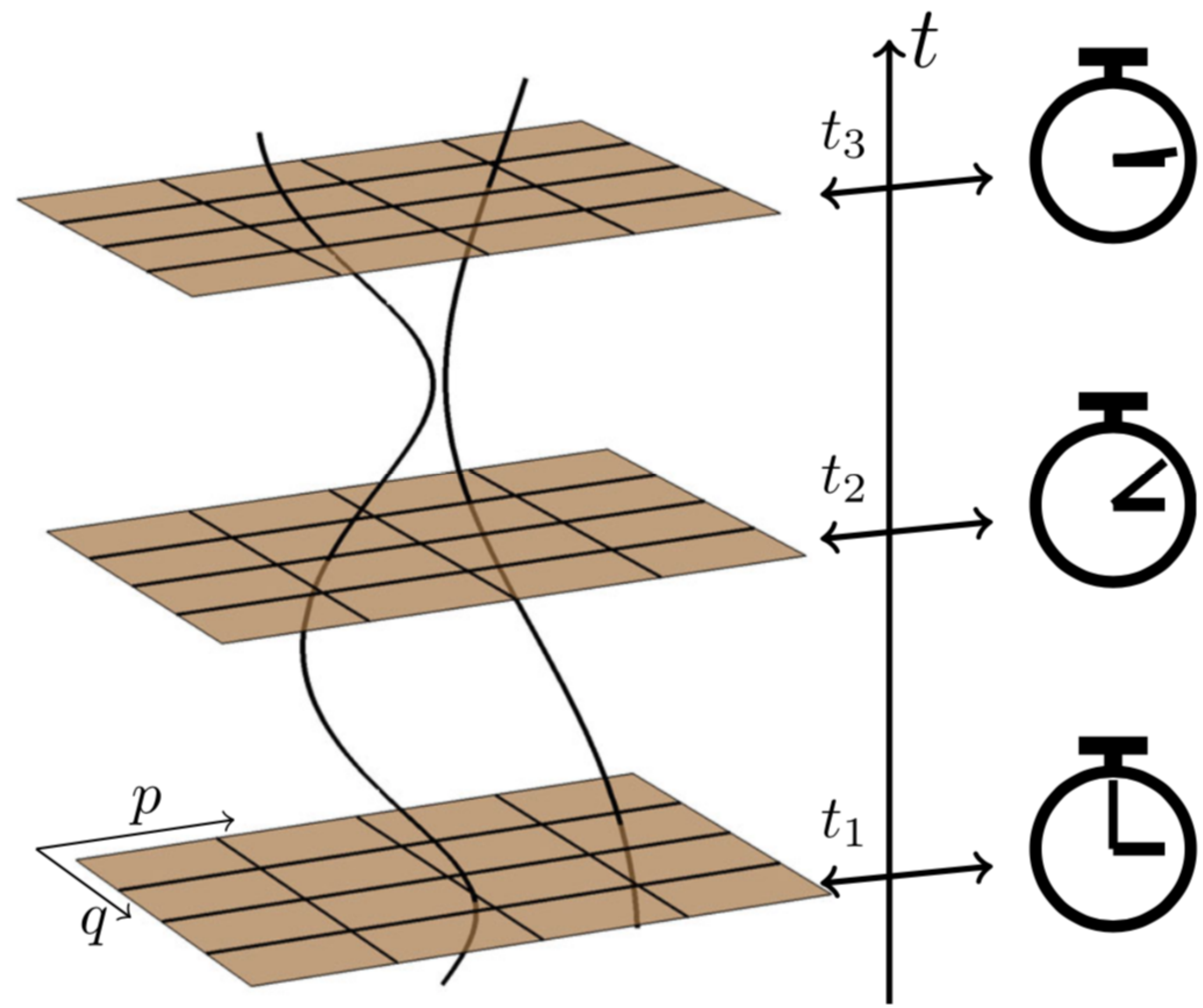
freeze momenta... Heisenberg uncertainties?

$[\text{quantization}, \text{minisuperspace}] \neq 0$

The clock issue in quantum cosmology

→ GR = constrained system: lack of external time

→ arbitrary degree of freedom: internal clock



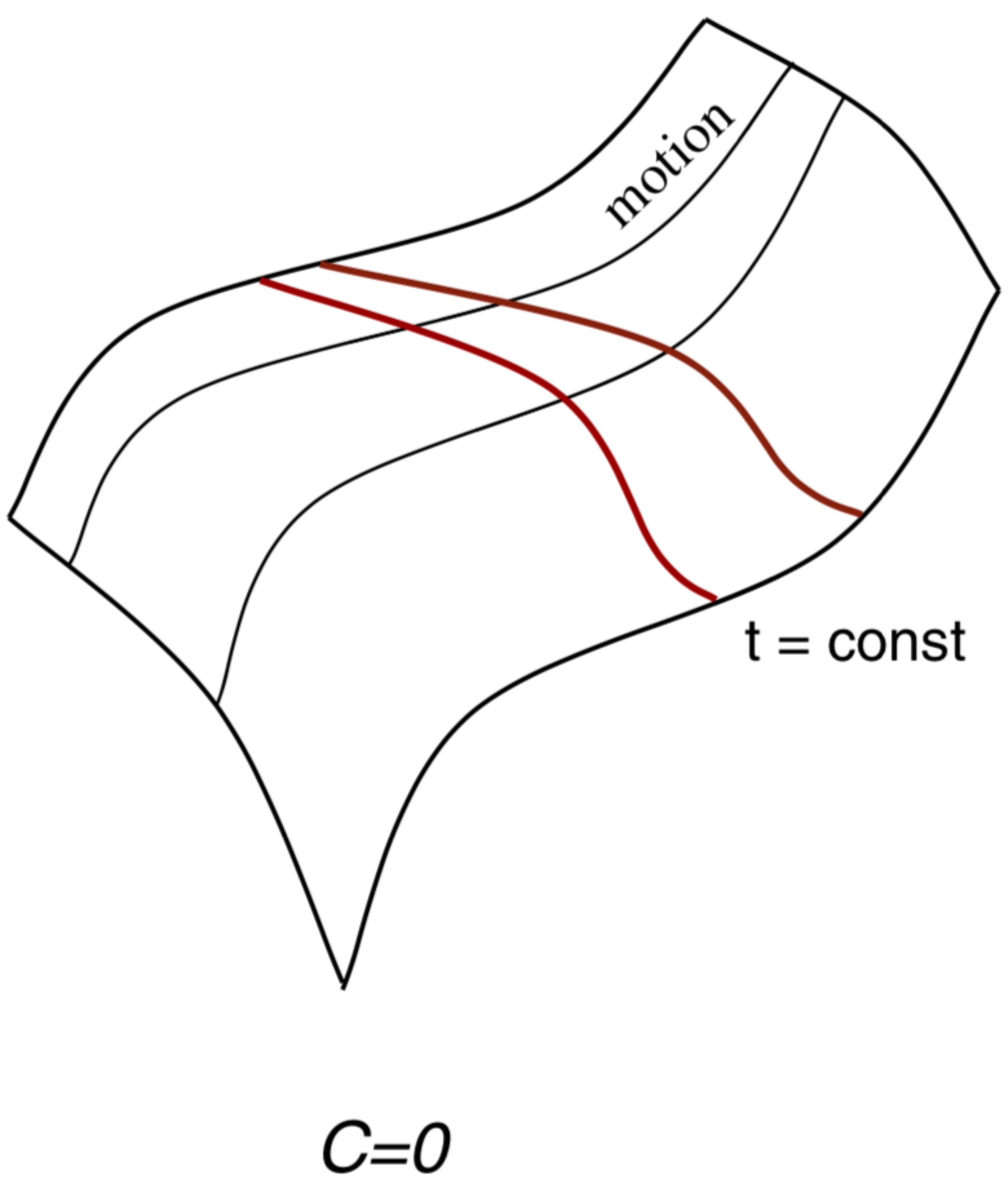
Classical system q_i & p_i

Constraint $C(q_i, p_i) = 0$ & $\frac{\mathrm{d}}{\mathrm{d}\tau}\mathcal{O}(q_i, p_i) = \{\mathcal{O}, C\}_{\mathrm{P.B}}$

evolution parameter (time) ← $\frac{\mathrm{d}}{\mathrm{d}\tau}$; $\mathcal{O}$ → observable

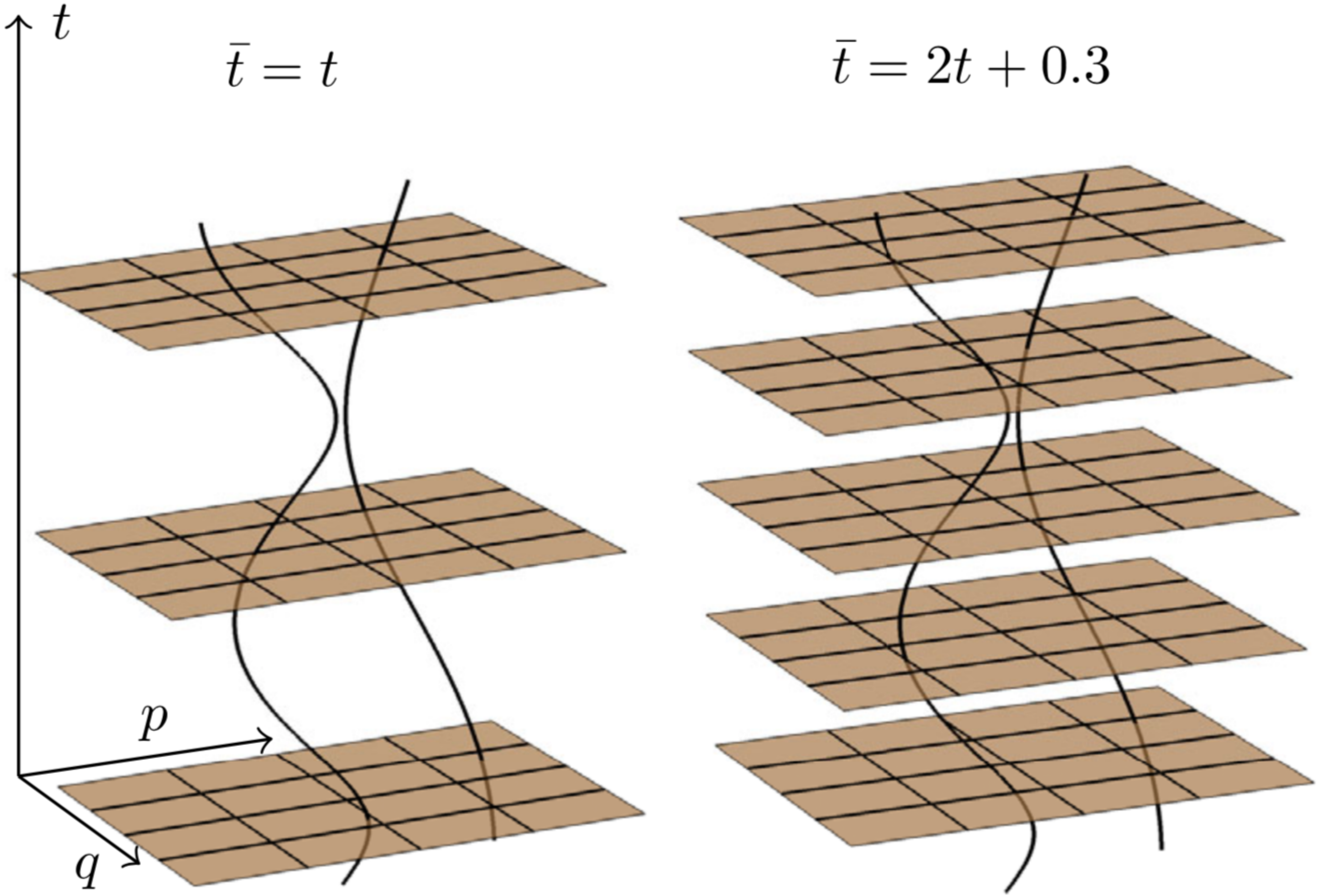
Time parametrization invariance $\tau \to \tau'$ → $N(q_i, p_i, \tau)$

arbitrary non vanishing lapse function



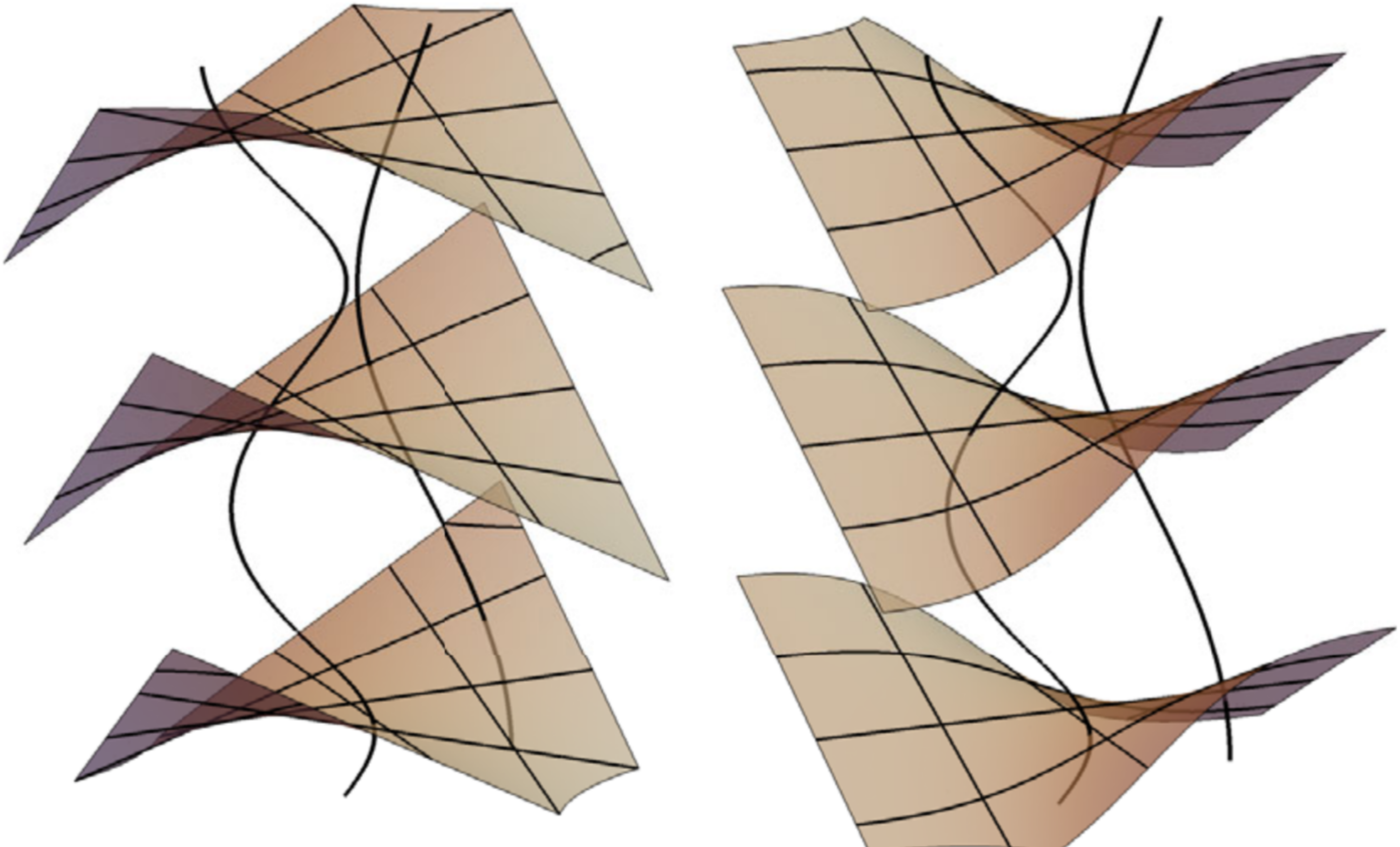
$\mathrm{d}\tau = N\mathrm{d}\tau' \implies \frac{\mathrm{d}}{\mathrm{d}\tau'}\mathcal{O}(q_i, p_i) = \{\mathcal{O}, NC\}_{\mathrm{P.B}}$

hamiltonian $H = NC$



t
$\bar{t} = t$
$\bar{t} = 2t + 0.3$
p
q

$\bar{t} = t + qp$ $\bar{t} = t - \frac{3qp}{3p^2+1}$



Constrained system $C\left(\left\{q^k\right\},\left\{p_k\right\}\right)=H_{\text{tot}}\left(\left\{q^k\right\},\left\{p_k\right\}\right)=0$

Canonical transformation $\left(\left\{q^k\right\},\left\{p_k\right\}\right)\mapsto\left(\left\{Q^a\right\},\left\{P_a\right\}\right)$

→ $\exists Q^\alpha$ such that $\{Q^\alpha, H_{\text{tot}}\}_{\text{P.B.}}=1=\frac{\mathrm{d}Q^\alpha}{\mathrm{d}t}$?

→ defines clock

Quantum system $\hat{H}_{\text{tot}}\Psi\equiv\hat{C}\Psi(Q^a)=0$

$\hat{C}$ linear in $\hat{P}_\alpha=-i\frac{\partial}{\partial Q^\alpha}$

→ will become time

→ $\hat{C}=\hat{P}_\alpha+\hat{H}(P_1,\cdots,P_{\alpha-1},P_{\alpha+1},\cdots,P_n,\{Q^a\})$

$\hat{C}\Psi(Q^a)=0\Longrightarrow$ time-dependent Schrödinger equation

A simple example

$$\mathcal{S} = \frac{1}{2}\int\left(\frac{\dot{x}^2}{z} - zx^2 - \frac{\dot{y}^2}{z} + zy^2\right)\mathrm{d}t$$

First, redefine time: $\mathrm{d}\tau = z\mathrm{d}t$ ⟶ $\mathcal{S} = \frac{1}{2}\int\left[\left(\frac{\mathrm{d}x}{\mathrm{d}\tau}\right)^2 - x^2 - \left(\frac{\mathrm{d}y}{\dot{\tau}}\right)^2 + y^2\right]\mathrm{d}\tau$

Classical EOMs $\begin{cases} \frac{\mathrm{d}^2x}{\mathrm{d}\tau^2} = -x \\ \frac{\mathrm{d}^2y}{\mathrm{d}\tau^2} = -y \end{cases}$ ⟶ 2 independent harmonic oscillators

$$H_{\mathrm{tot}} = H + H_y$$

Canonical transformation $T = \arctan\left(\frac{p_y}{y}\right)$ & $P_T = -\frac{1}{2}\left(p_y^2 + y^2\right) = H_y$

$$\{T, P_T\}_{\text{P.B.}} = 1$$

$$\downarrow$$

$$\{T, H_{\text{tot}}\}_{\text{P.B.}} = 1 \longrightarrow \frac{\mathrm{d}T}{\mathrm{d}\tau} = 1 \longrightarrow \begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}T} = p_x \\ \dfrac{\mathrm{d}p_x}{\mathrm{d}T} = -x \end{cases}$$

$H_{\text{tot}} = P_T + H$ on shell $H_{\text{tot}} \approx 0$

Quantization: x only! $\hat{H}_{\text{tot}}\psi(x, T) = 0 \implies i\frac{\partial\psi}{\partial T} = \hat{H}\psi(x, T)$

& $\int |\psi(x, T)|^2 \mathrm{d}x = 1$

y remains classical (clock)

Bianchi I case $$\mathrm{d}s^2 = -N^2 \mathrm{d}\tau^2 + \sum_{i=1}^{3} a_i^2 \left(\mathrm{d}x^i\right)^2$$

Scale factors
$$\begin{cases} a_1 &= \mathrm{e}^{\beta_0+\beta_+ + \sqrt{3}\beta_-} \\ a_2 &= \mathrm{e}^{\beta_0+\beta_+ - \sqrt{3}\beta_-} \\ a_3 &= \mathrm{e}^{\beta_0 - 2\beta_+} \end{cases}$$

Volume $V \equiv a_1 a_2 a_3 = \mathrm{e}^{3\beta_0}$

$$\mathrm{d}\beta_0 = \frac{1}{3}\mathrm{e}^{-3\beta_0}\mathrm{d}V$$

Action $$\mathcal{S} = \int \mathrm{d}\tau \Big(\underbrace{p_0\dot{\beta}_0 + p_+\dot{\beta}_+ + p_-\dot{\beta}_-}_{\frac{\mathrm{d}\theta}{\mathrm{d}\tau}} - NC \Big)$$

$\frac{\mathrm{d}\theta}{\mathrm{d}\tau}$ → canonical one-form

NC → H

constraint $$C = \frac{\mathrm{e}^{-3\beta_0}}{24}\left(-p_0^2 + p_+^2 + p_-^2\right)$$

ensure canonical one-form remains canonical $p_V \equiv \frac{e^{-3\beta_0}}{3} p_0$

↓

$$\mathrm{d}\theta = p_V \mathrm{d}V + p_+ \mathrm{d}\beta_+ + p_- \mathrm{d}\beta_-$$

constraint $C = \frac{3V}{8}\left(-p_V^2 + \frac{p_+^2 + p_-^2}{9V^2}\right)$

cyclic variable $\dot{p}_\pm = 0$ set $p_+ = k\cos\alpha$ and $p_- = k\sin\alpha$

$$\longrightarrow \mathrm{d}\theta = p_V \mathrm{d}V + p_k \mathrm{d}k + p_\alpha \mathrm{d}\alpha + \underbrace{\mathrm{d}\left(k\cos\alpha\beta_+ + k\sin\alpha\beta_-\right)}$$

$\to$ exact... ignore!

$$\begin{aligned} p_k &\equiv -\left(\cos\alpha\beta_+ + \sin\alpha\beta_-\right), \\ p_\alpha &\equiv \left(k\sin\alpha\beta_+ - k\cos\alpha\beta_-\right) \end{aligned}$$

neither α nor P_α in $H = NC$

the system reduces to
$$\begin{cases} \mathrm{d}\theta &= p_V \mathrm{d}V + p_k \mathrm{d}k \\ C &= \frac{3V}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) \end{cases}$$

Hamilton equations

$$\begin{aligned} \dot{k} &= 0 \\ \dot{p}_k &= -N\frac{k}{12V} \\ \dot{V} &= -N\frac{3Vp_V}{4} \\ \dot{p}_V &= -N\left[\frac{3}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) - \frac{k^2}{12V^2}\right] \end{aligned}$$

+ constraint
$$\frac{3V}{8}\left(-p_V^2 + \frac{k^2}{9V^2}\right) = 0$$

→ closed for V and p_V

Choosing a time

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\left(9\frac{p_k}{k}\right) = -\frac{3}{4}\frac{N}{V}$$ monotonically increasing function

valid time choice $\tau = \frac{9p_k}{k} \implies N = -\frac{4}{3}V$

Solving directly in the action $\mathcal{S} = \int \mathrm{d}\theta = \int \mathrm{d}\tau \left(p_V \dot{V} - \frac{V^2 p_V^2}{2}\right)$

H

classical unconstrained one dimensional system

$$\frac{\mathrm{d}}{\mathrm{d}\tau}(Vp_V) = 0 \implies Vp_V = V_0 p_{V0}$$

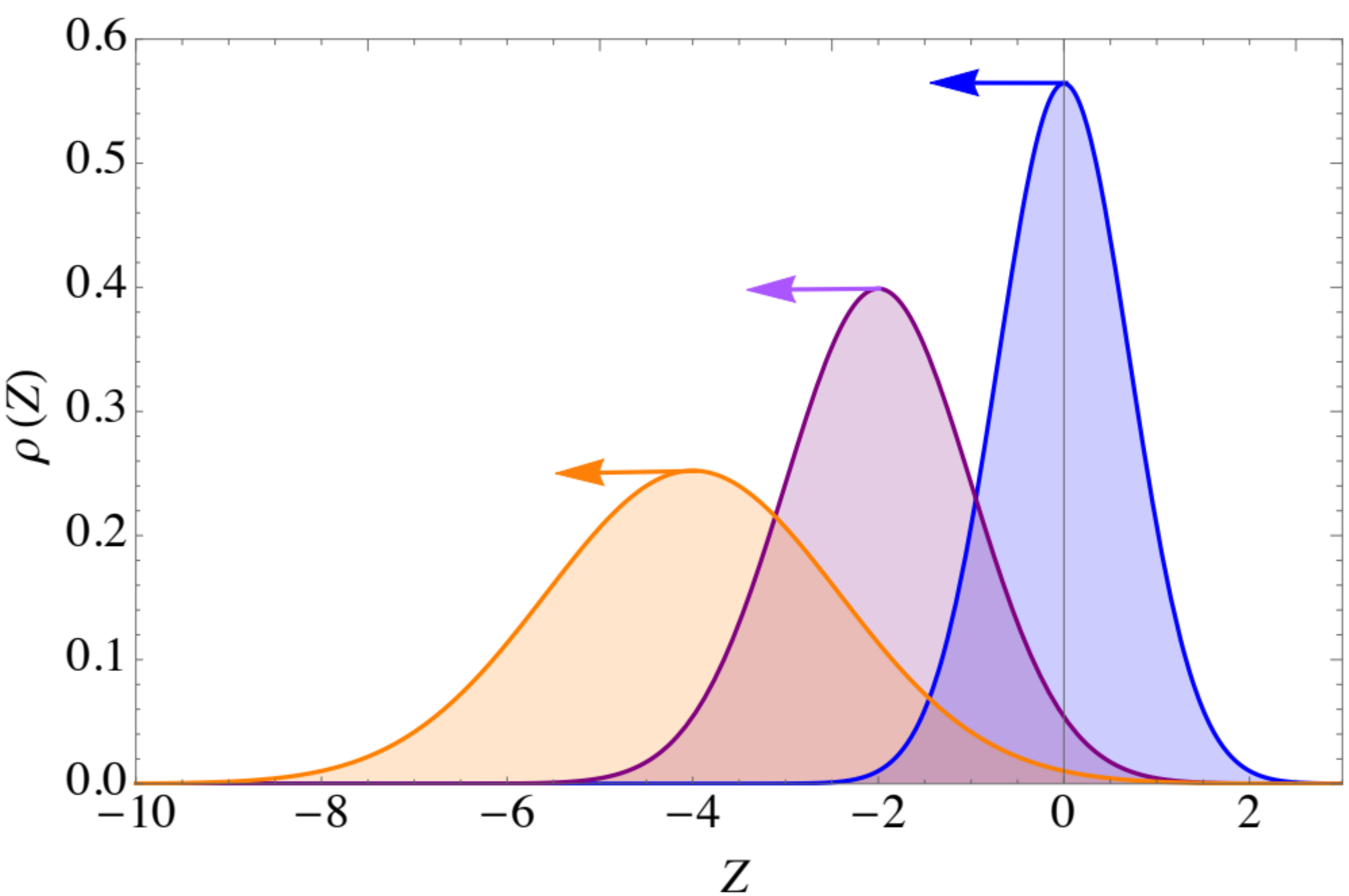
$V = V_0 \mathrm{e}^{(Vp_V)\tau}$ and $p_V = p_{V0}\mathrm{e}^{-(Vp_V)\cdot\tau}$

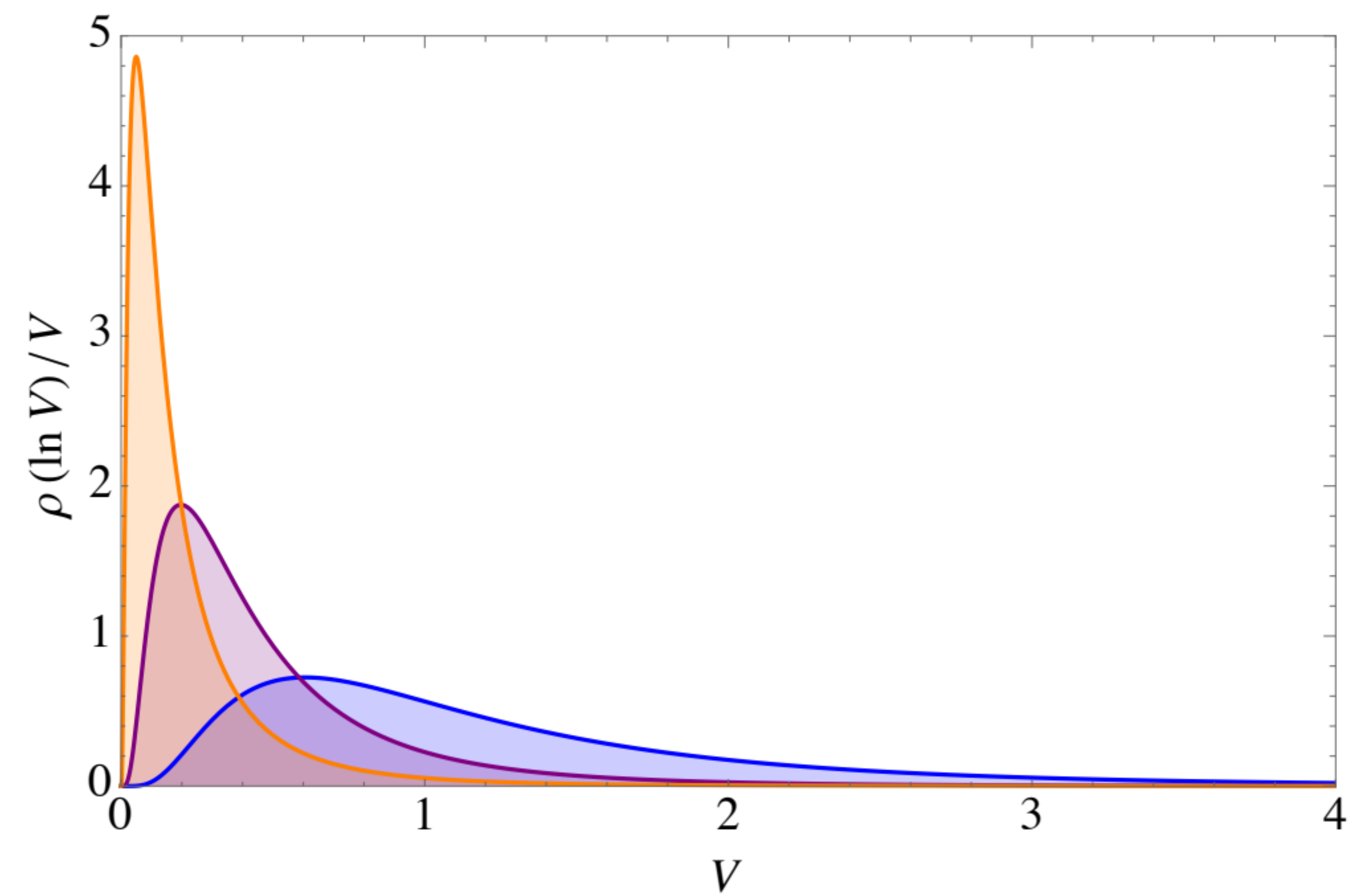
symmetric ordering choice

$$H = V^2 p_V^2 \quad \mapsto \quad \hat{H} = \sqrt{V}\frac{1}{i}\partial_V \sqrt{V} \cdot \sqrt{V}\frac{1}{i}\partial_V \sqrt{V}$$

coordinate transformation $V \mapsto Z = \ln V$

$$\longrightarrow \quad U\hat{H}U^{-1} = -\partial_Z^2, \quad \text{and} \quad Z \in \mathbb{R}$$





slow-gauge time

$$\mathrm{d}\theta = (V p_V)\mathrm{d}V - \left(\frac{V^2 p_V^2}{2}\right)\mathrm{d}\left(\frac{9p_k}{k} + \frac{V - \ln V}{V p_V}\right) + \mathrm{d}\left(\frac{9p_k}{2k}V^2 p_V^2 + \frac{1}{2}V \ln V p_V - \frac{1}{2}V^2 p_V\right)$$

→ Action

$$\mathcal{S} = \int \mathrm{d}\theta = \int \mathrm{d}\eta \left(V p_V \acute{V} - \frac{V^2 p_V^2}{2}\right)$$

new time variable

$$\eta \equiv \frac{9p_k}{k} + \frac{V - \ln V}{V p_V}$$

$$\acute{V} \equiv \mathrm{d}V/\mathrm{d}\eta$$

canonical if

$$\pi_V = p_V V$$

$$H = \frac{1}{2}V^2 p_V^2 = \frac{1}{2}\pi_V^2$$

freely moving particle...

$$(V, \pi_V) \in \mathbb{R}_+ \times \mathbb{R}$$

on the half line

Quantization: a gaussian wave packet

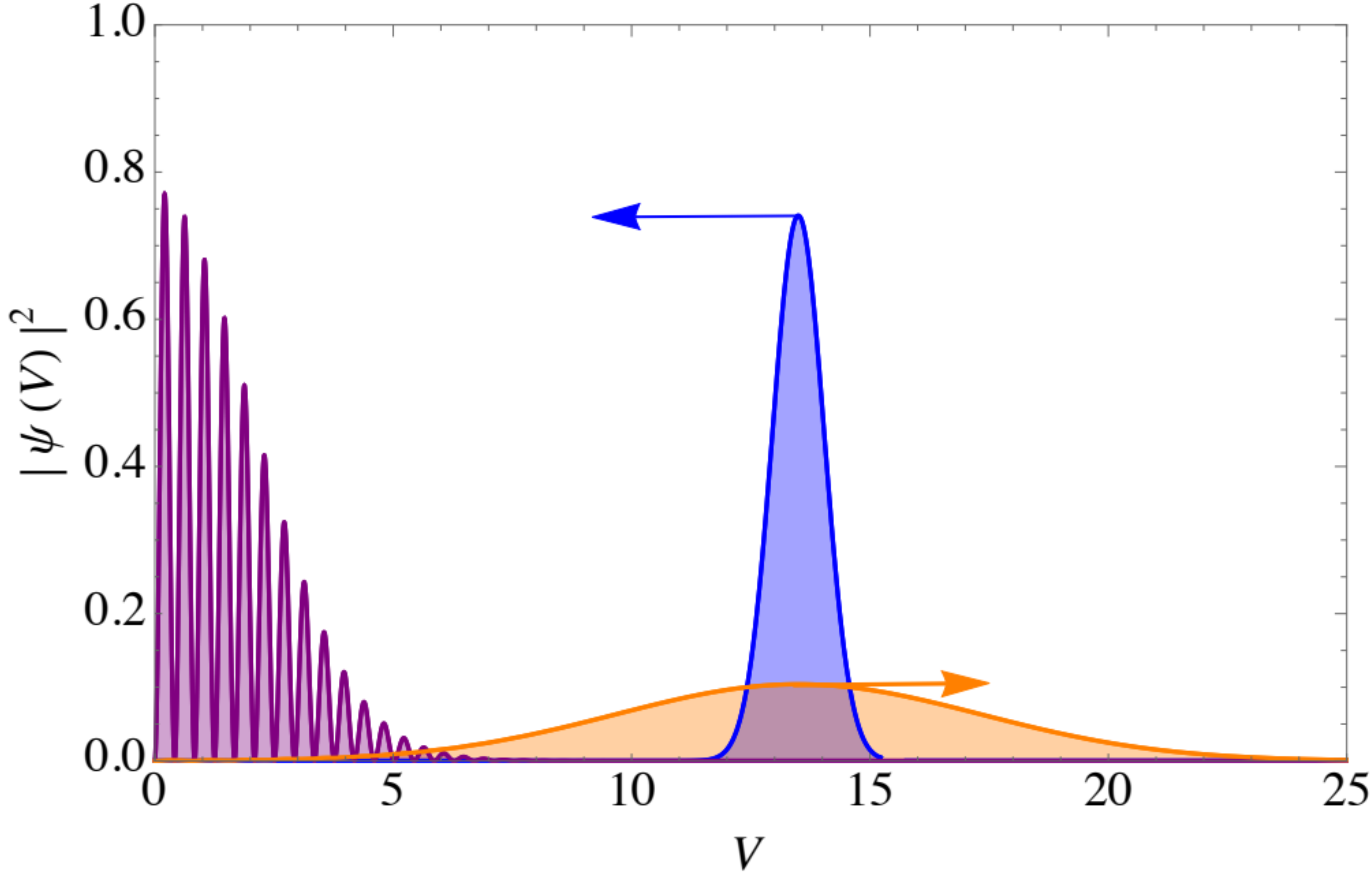
$$u(V,\eta) = \frac{\mathrm{e}^{-k^2/4}}{\sqrt{1+4i\eta}} \exp\left[-\frac{(V-ik/2)^2}{1+4i\eta}\right]$$

implement boundary conditions to ensure self-adjointness

$$\psi(V,\eta) = \frac{u(V+V_0,\eta) - u(-V+V_0,\eta)}{\left[\sqrt{\pi/2}\left(1-\mathrm{e}^{-V_0^2-k^2/2}\right)\right]^{1/2}}$$

→ ***solves the Schrödinger equation***

$$i\frac{\partial}{\partial\eta}\psi = -\triangle_D\,\psi$$

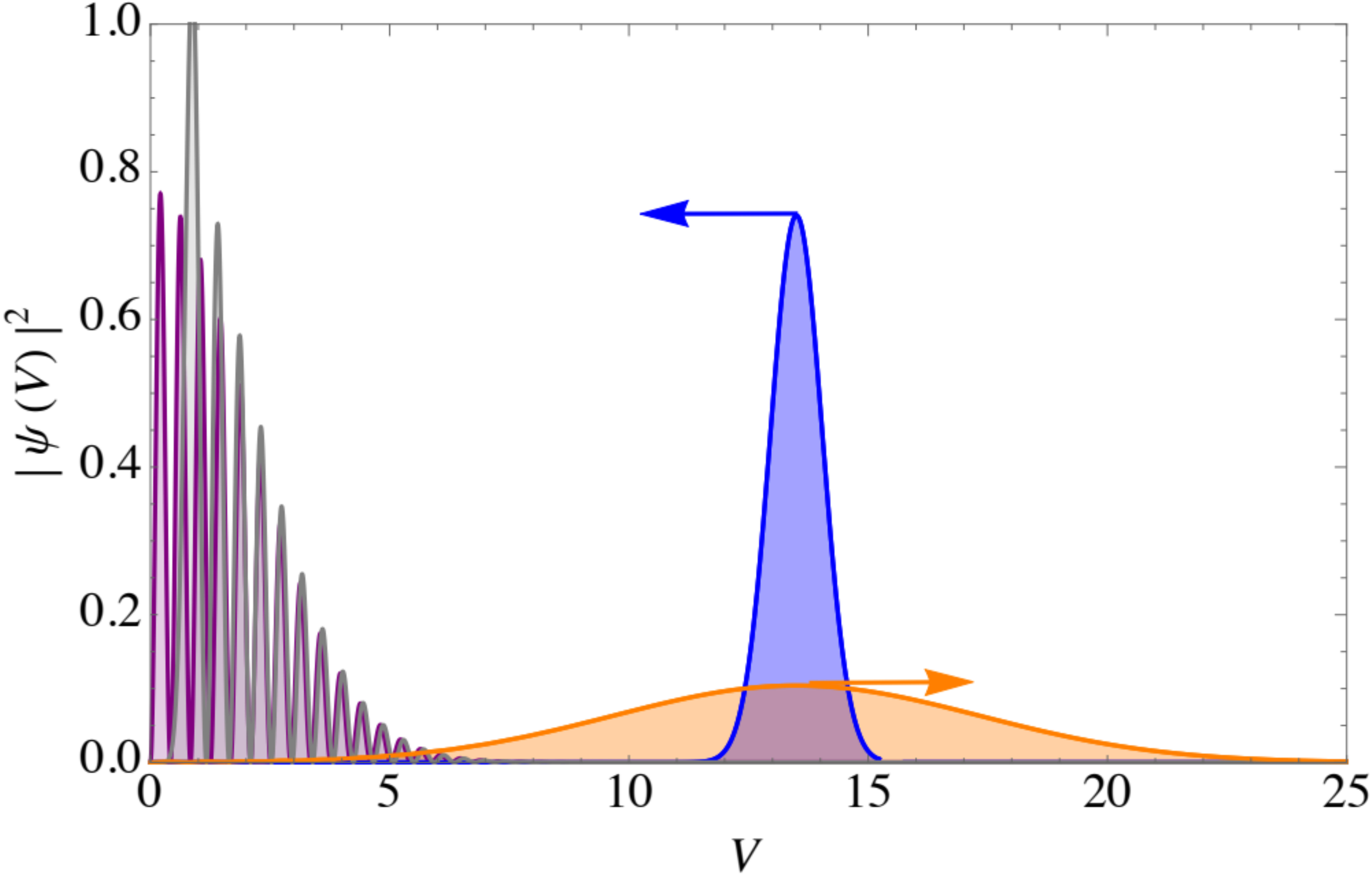

1.0
0.8
0.6
0.4
0.2
0.0
$|\psi(V)|^2$
0
5
10
15
20
25
V

Operator ordering ambiguity $\pi_V^2 \mapsto \hat{V}^s \hat{\pi}_V \hat{V}^{-2s} \hat{\pi}_V \hat{V}^s$

→ $\pi_V^2 \mapsto \hat{\pi}_V^2 + s\hat{V}^{-2}$

↓

self-adjoint hamiltonian on the half-line $s > 3/4$



1.0
0.8
0.6
0.4
0.2
0.0
$|\psi(V)|^2$
0
5
10
15
20
25
V

Closed algebra of operators
$$\begin{cases} [\hat{V}^2, \hat{H}] &= 4i\hat{D}, \\ [\hat{D}, \hat{H}] &= 2i\hat{H}, \\ [\hat{V}^2, \hat{D}] &= 2i\hat{V}^2, \end{cases}$$

$$\hat{D} \equiv \frac{1}{2}\left(\hat{V}\hat{\pi}_V + \hat{\pi}_V\hat{V}\right)$$

→ Heisenberg equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{V}^2 = -i[\hat{V}^2, \hat{H}] = 4\hat{D}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{D} = -i[\hat{D}, \hat{H}] = 2\hat{H}$$

→ Heisenberg equations of motion

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{V}^2 = -i[\hat{V}^2, \hat{H}] = 4\hat{D}$$

$$\frac{\mathrm{d}}{\mathrm{d}\eta}\hat{D} = -i[\hat{D}, \hat{H}] = 2\hat{H}$$

→ solution as time-dependent operators

$$\hat{D}(\eta) = 2\hat{H}\eta + \hat{D}(0) \quad \Longrightarrow \quad \hat{V}^2 = 4\hat{\eta}^2 + 4\hat{D}(0)\eta + \hat{V}^2(0)$$

expectation values follows similar equations…

→ ***semi-classical variables***

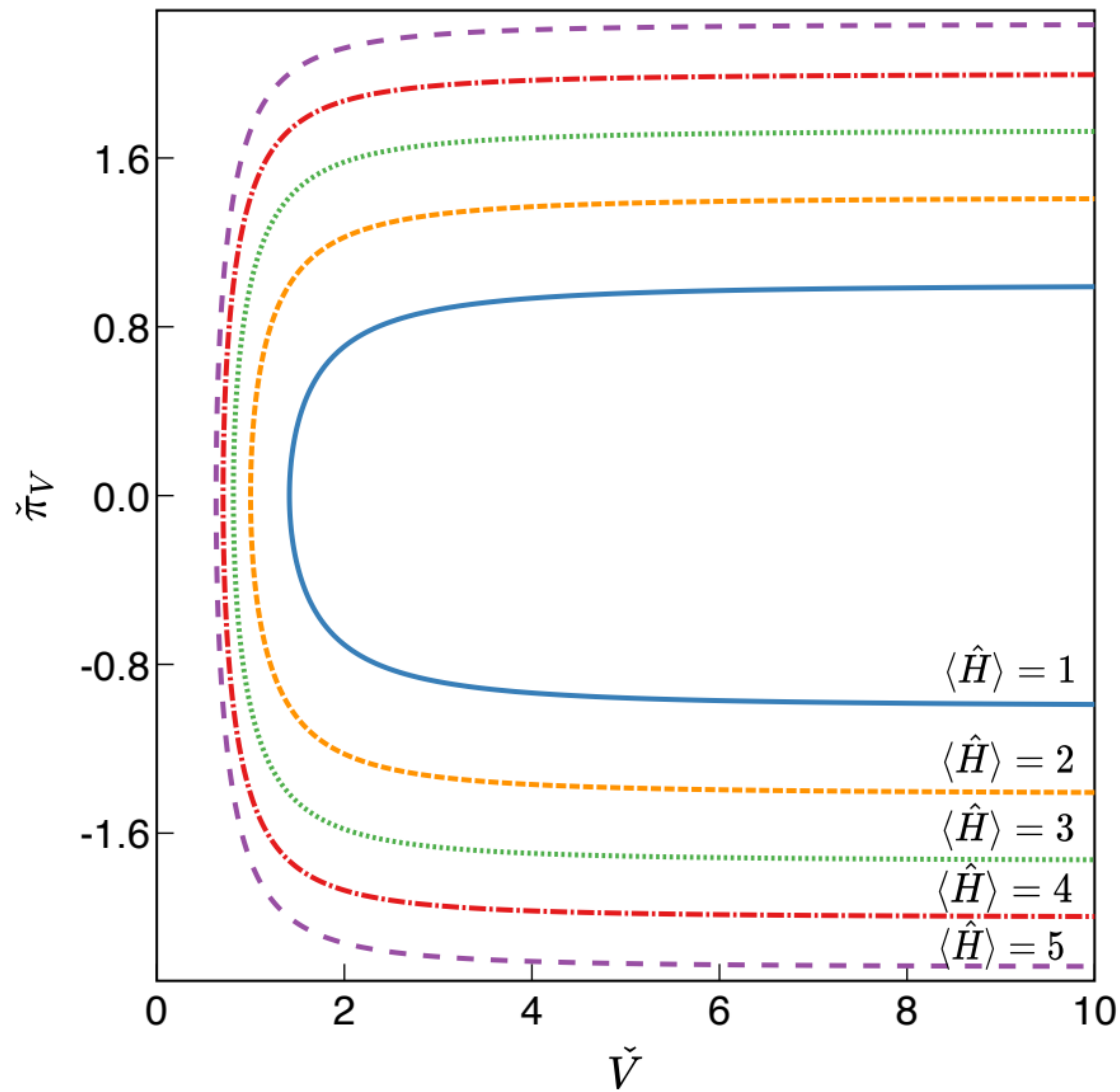
$$\check{V}(t) = \sqrt{\langle \hat{V}^2(t) \rangle}$$

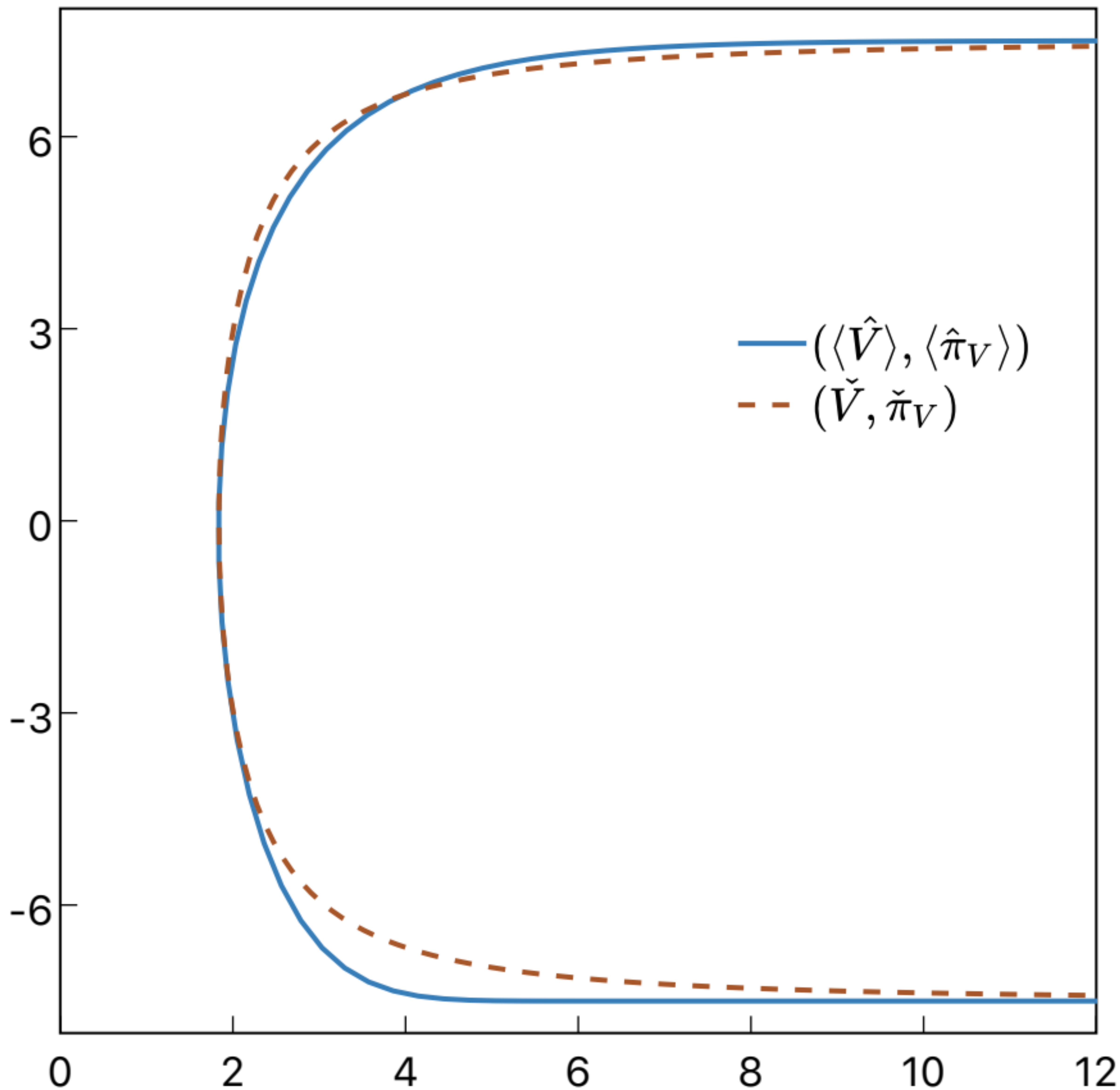
$$\check{\pi}_V(t) = \frac{\langle \hat{D}(t) \rangle}{\check{V}(t)}$$

→ *phase space solution*

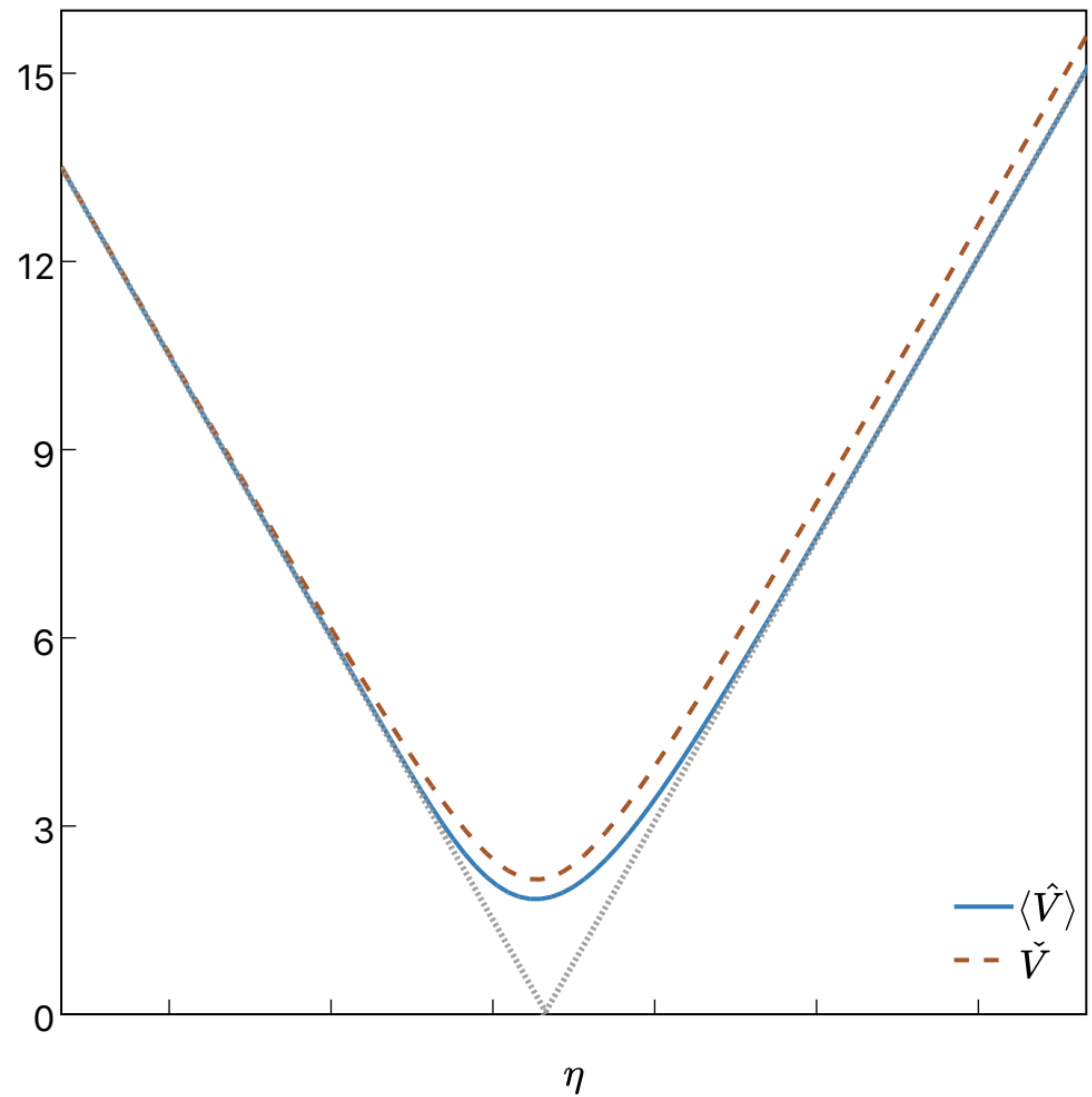
$$\check{V}(t) = \sqrt{4\langle\hat{H}\rangle t^2 + V_0^2},$$
$$\check{\pi}_V(t) = \frac{2\langle\hat{H}\rangle t}{\sqrt{4\langle\hat{H}\rangle t^2 + V_0^2}}.$$

NO SINGULARITY





6
3
0
-3
-6
0
2
4
6
8
10
12
$(\langle\hat{V}\rangle, \langle\hat{\pi}_V\rangle)$
$(\check{V}, \check{\pi}_V)$



15
12
9
6
3
0
$\langle \hat{V} \rangle$
$\check{V}$
η

Changing the time variable $\eta' = \eta'(\eta, V, \pi_V)$

redefining the dynamical variables in the process

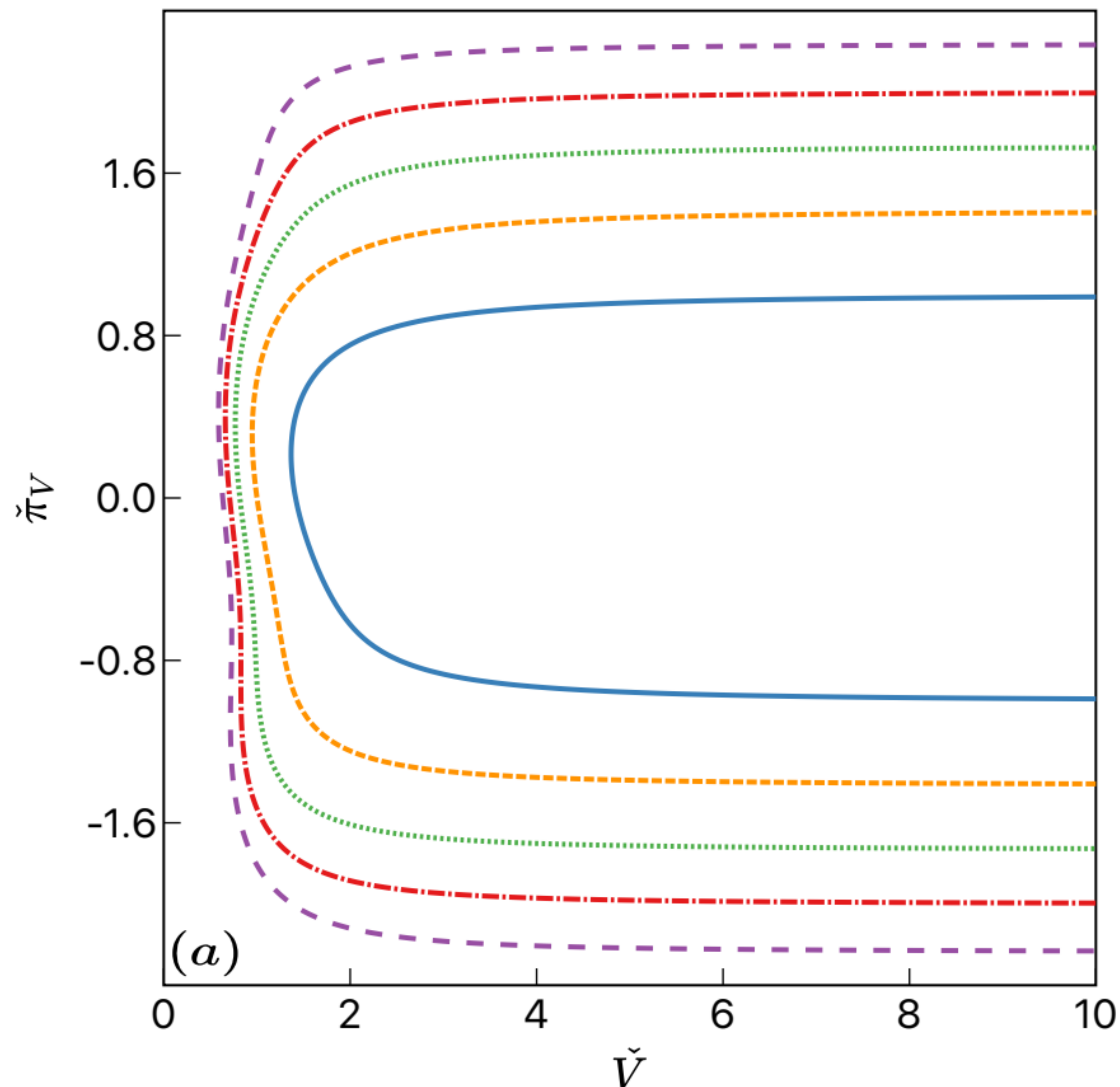
$\pi'_V = \pi_V$ & $V' = V + \pi_V(\eta' - \eta)$ no change of range...

change the canonical one-form

$$\mathrm{d}\theta = \pi_V \mathrm{d}V - \frac{\pi_V^2}{2}\mathrm{d}\eta = \pi'_V \mathrm{d}V' - \frac{\pi_V'^2}{2}\mathrm{d}\eta' + \mathrm{d}\left[(\eta - \eta')\frac{\pi_V'^2}{2}\right]$$

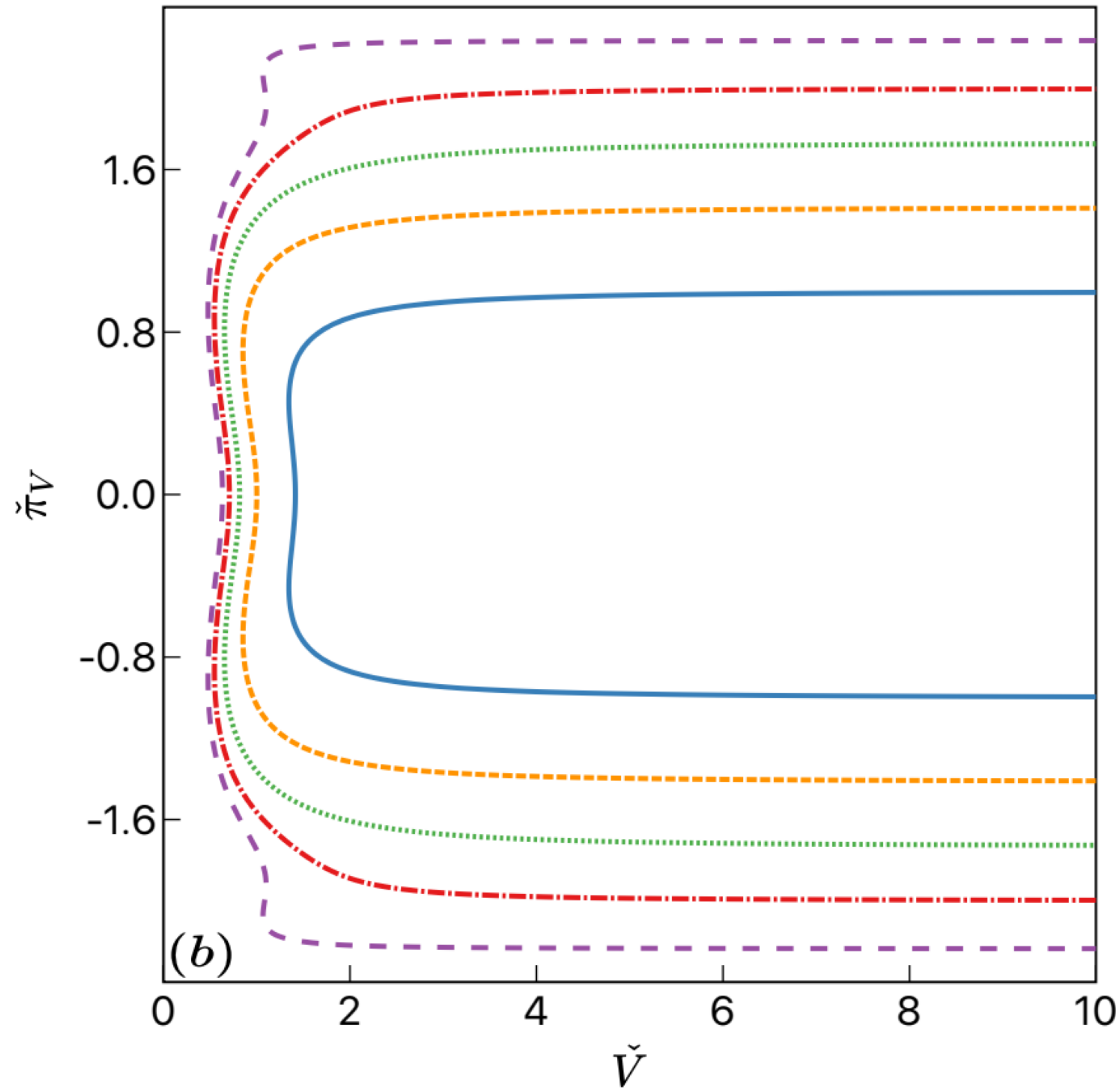
→ ***same system!***

delay function $\Delta(V, \pi_V) = \eta' - \eta$ no dependency on time



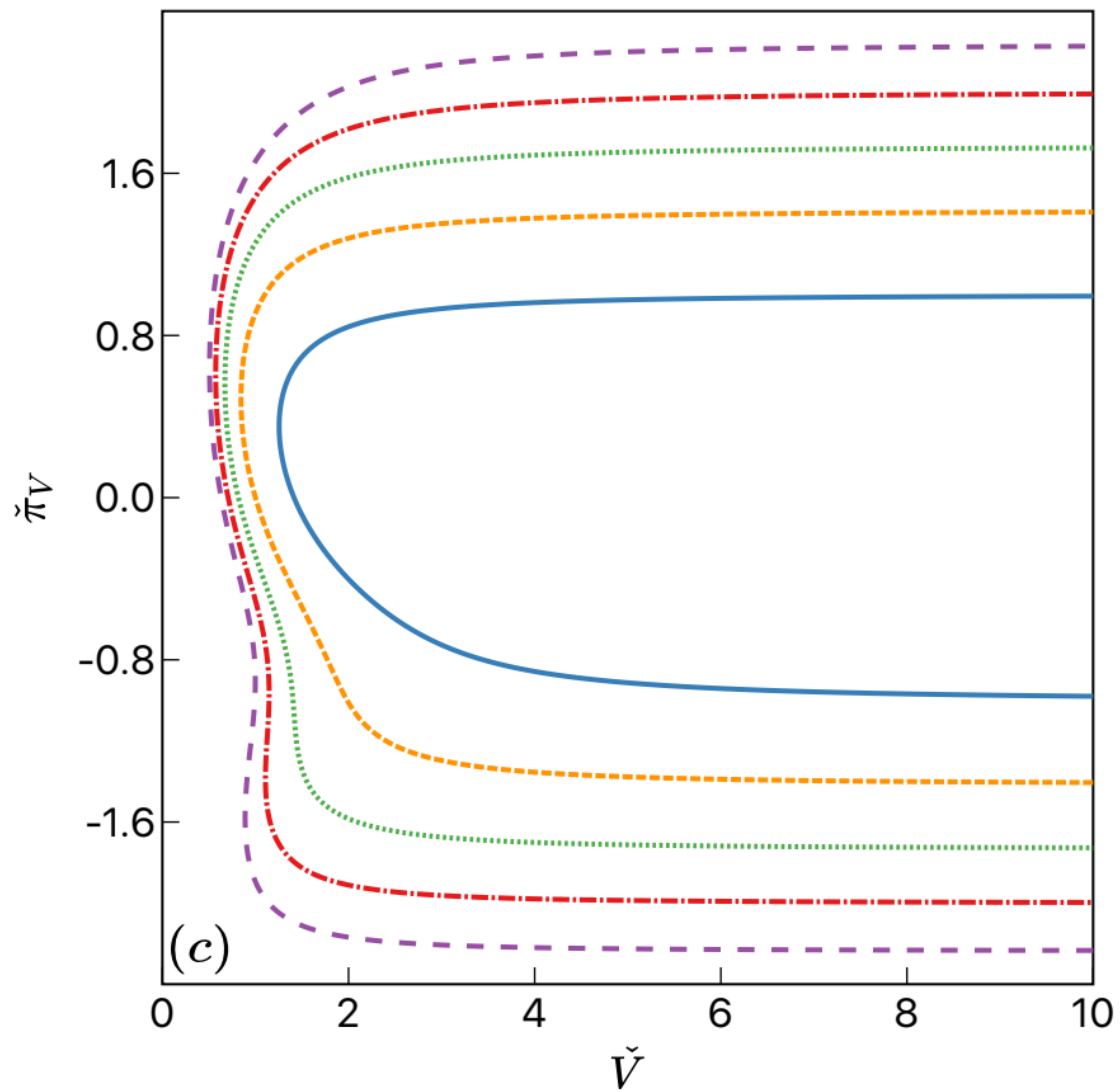
Delay function

$$\Delta = V\mathrm{e}^{-2|\pi_V|/3}\sin(3V\pi_V)/(10\pi_V)$$



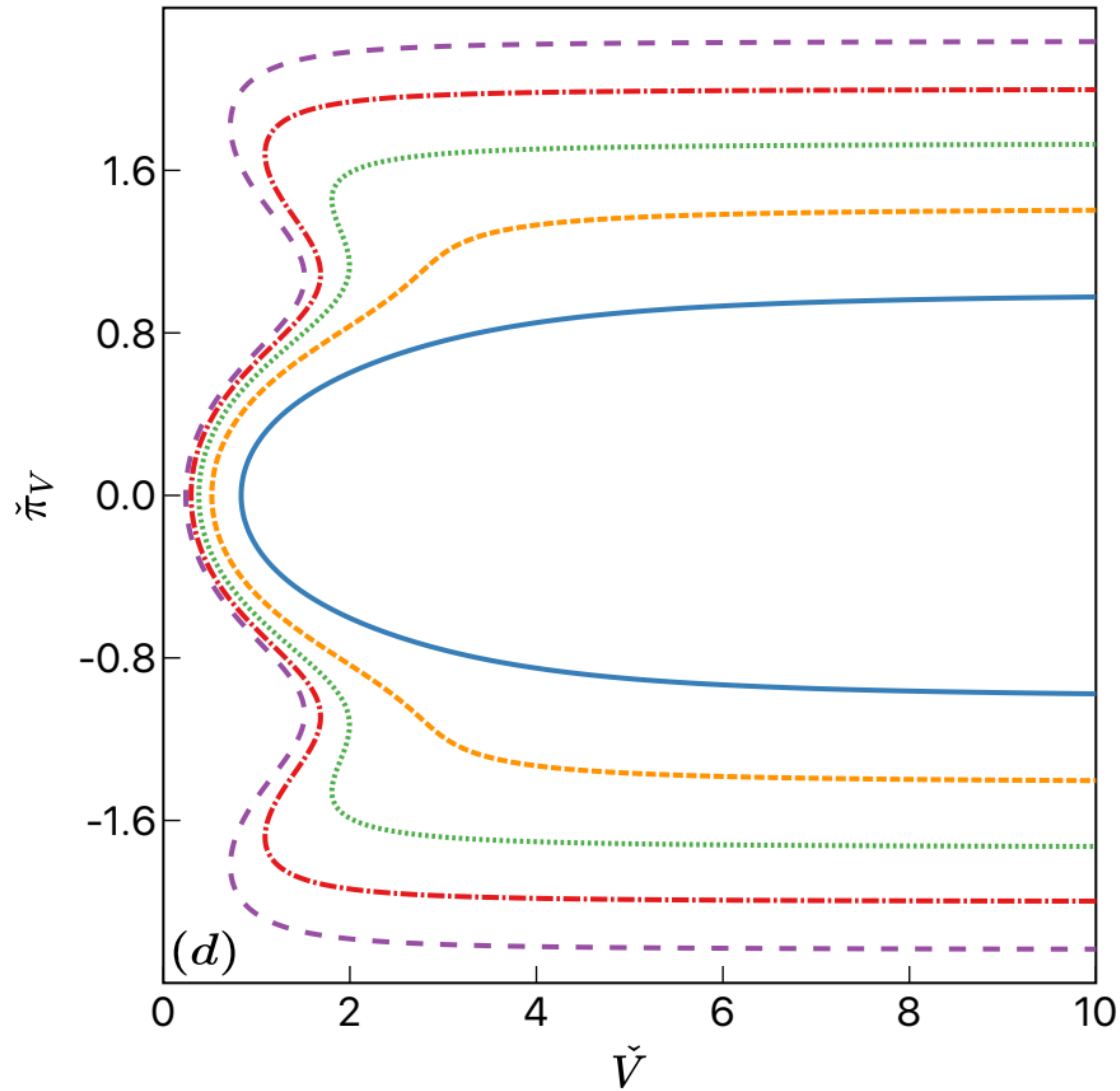
Delay function

$$\Delta = V(\pi_V - 10^{-0.2}\pi_V^3 + \pi_V^5/10)$$



Delay function

$\Delta = 10^{-0.5} V \sin(2\pi_V)/\pi_V$



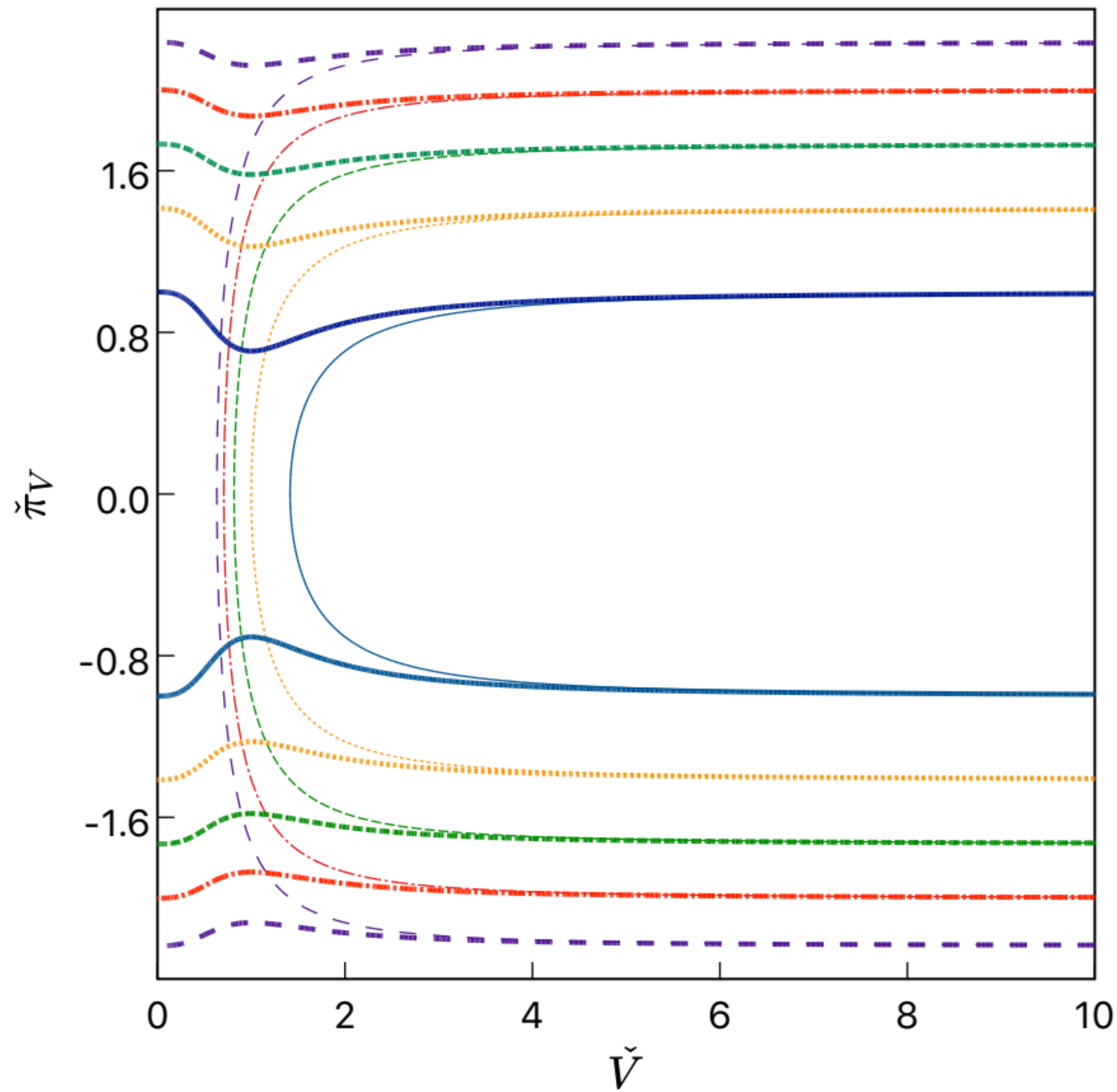
Delay function

$\Delta = 10^{-0.5}(V+1)\cos(3\pi_V)/\pi_V$

Delay function (slow to fast)

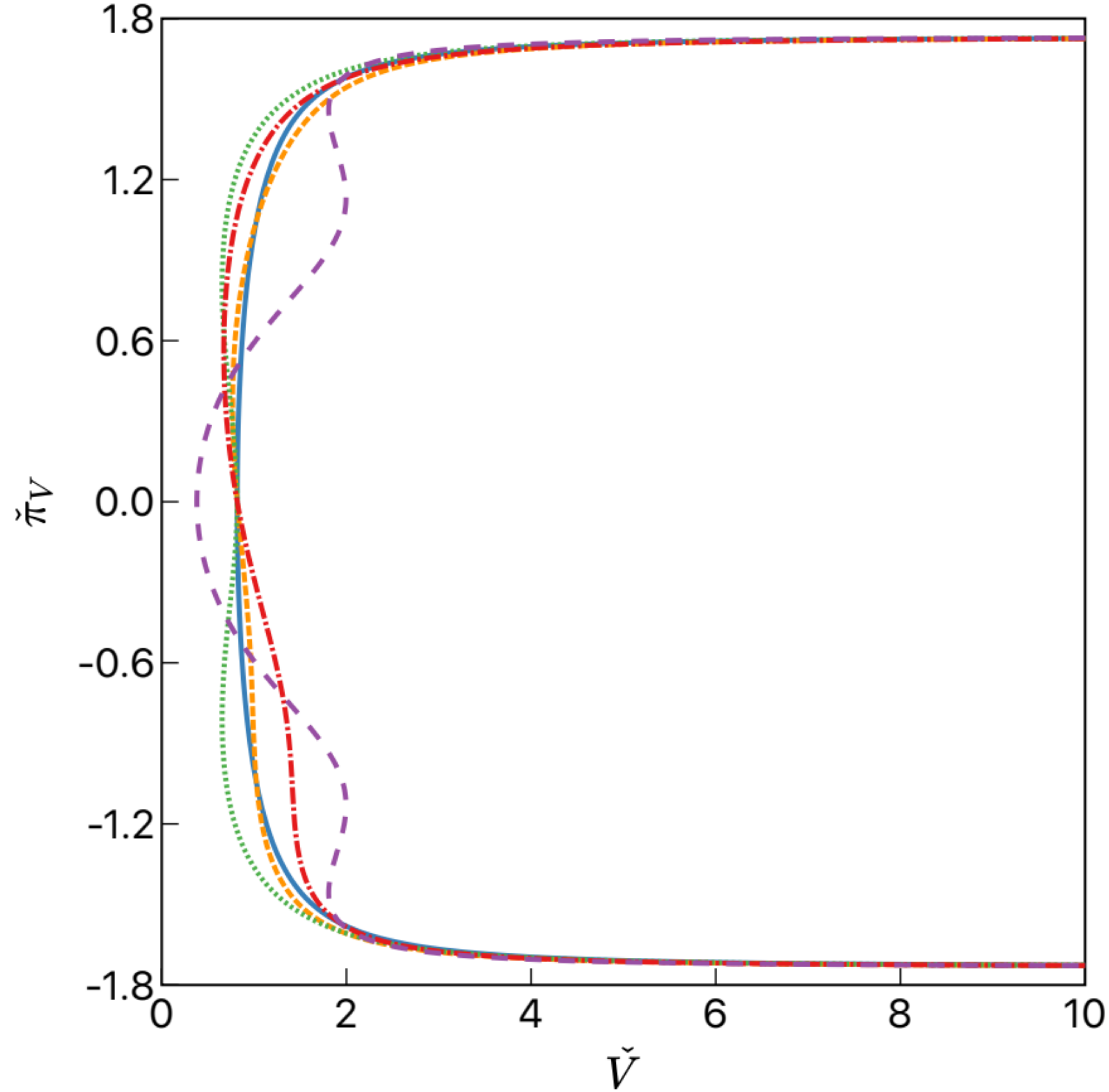
[regular to singular]

$$\Delta_{\mathrm{slow}\to\mathrm{fast}} = \frac{V - \ln V}{V p_V}$$



Comparison between different delay functions

Same asymptotics



P. Małkiewicz, PP and S. Vitenti, *Phys. Rev.* **D101**, 046012 (2020) [arXiv:1911.09892]

Conclusions

→ *Bouncing alternative to inflation still alive*

→ *Shear issue: ekpyrosis or shear viscosity*

→ *Classical / quantum bounce*

→ *Observational consequences & perturbations*

(talks by Shiv Sethi, Yi Fu Cai...)

Thank you for your attention!