Initial conditions for inflation

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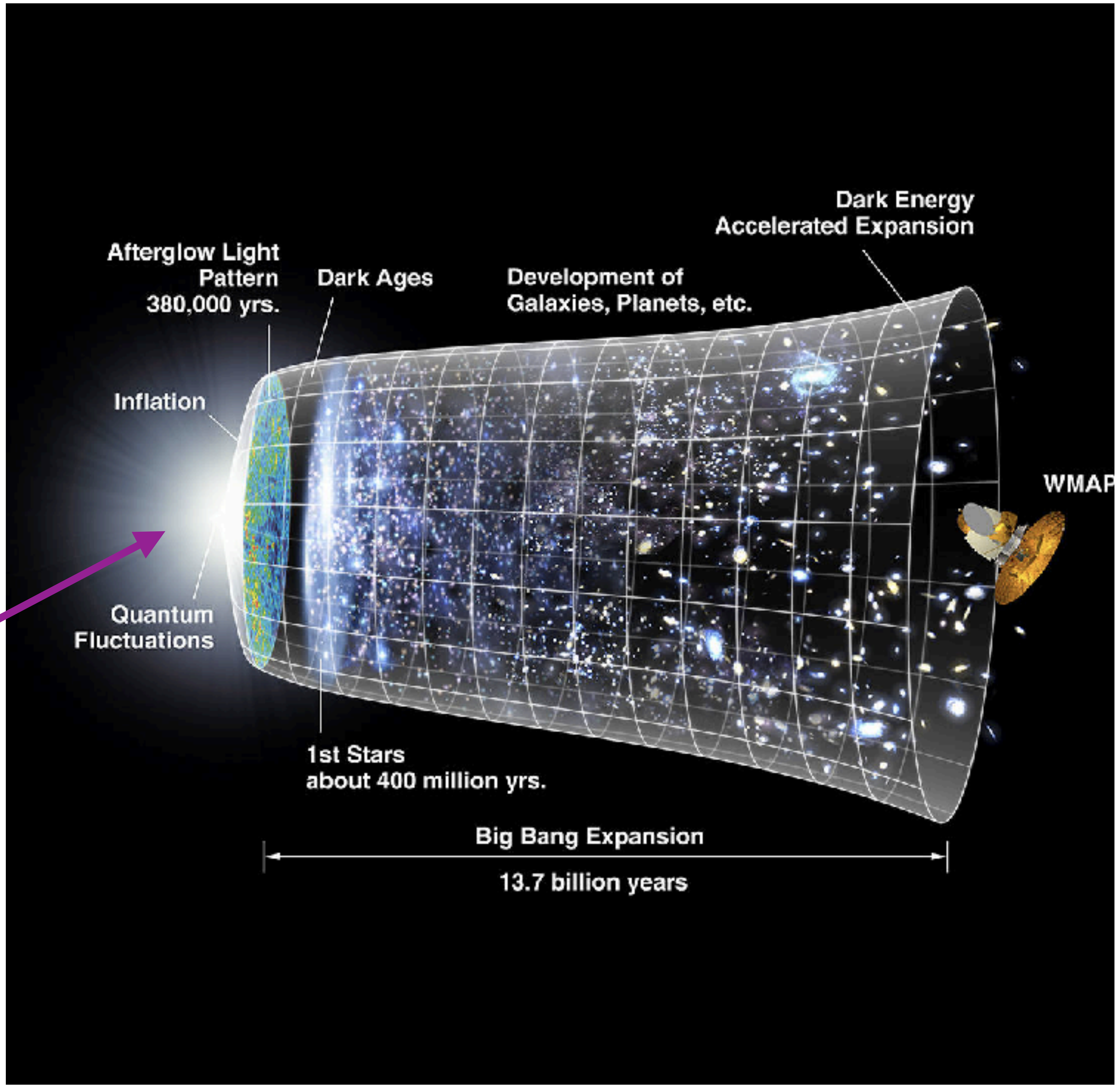
Inflation

- Solves the horizon and flatness problems
- Predicts a spectrum of scale invariant fluctuations



Inflation - the horizon problem

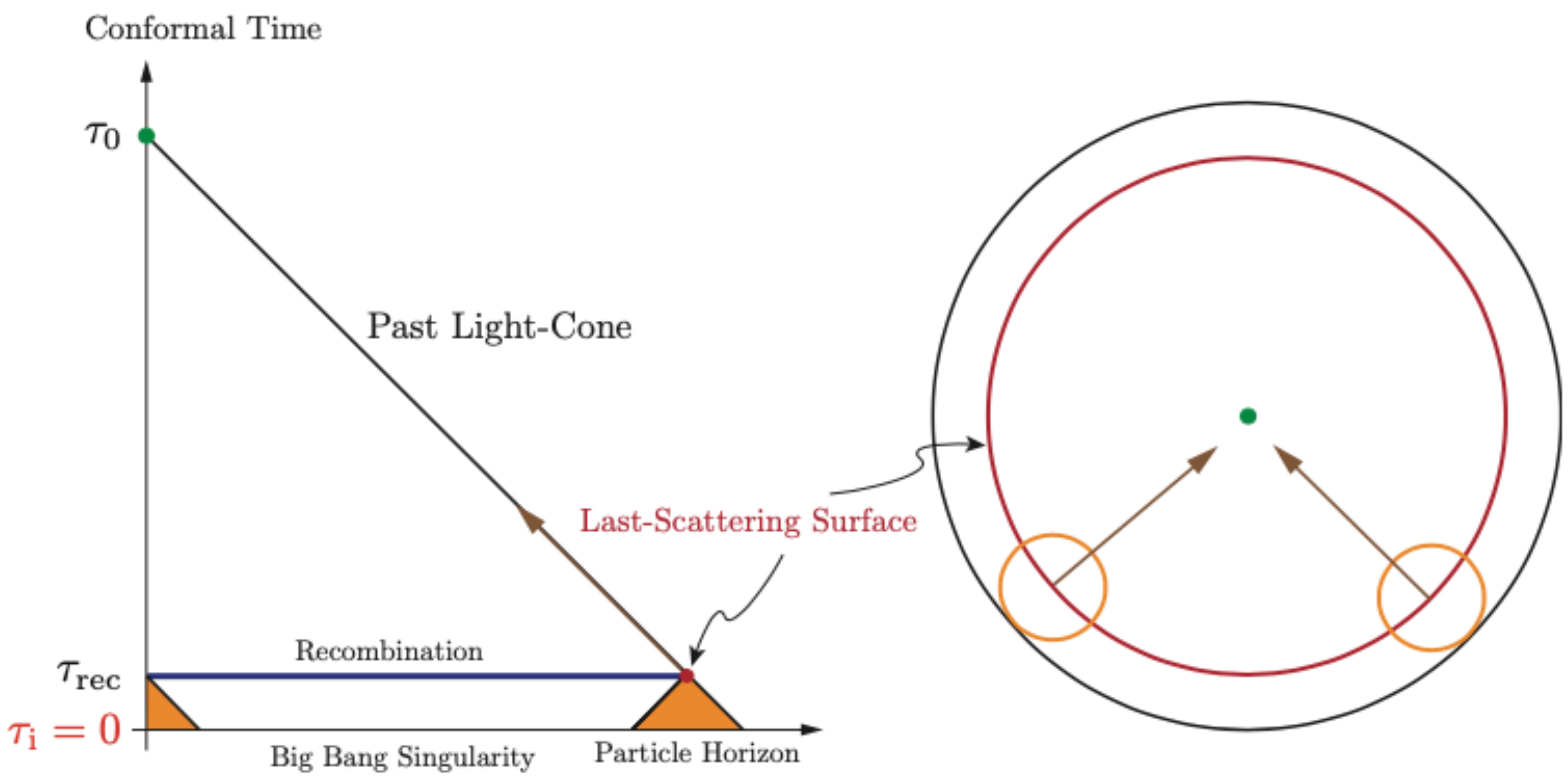


Figure 1.1: *Conformal diagram for the standard FRW cosmology.*

Image credit: D Baumann lecture notes: The Physics of Inflation

Inflation - the horizon problem

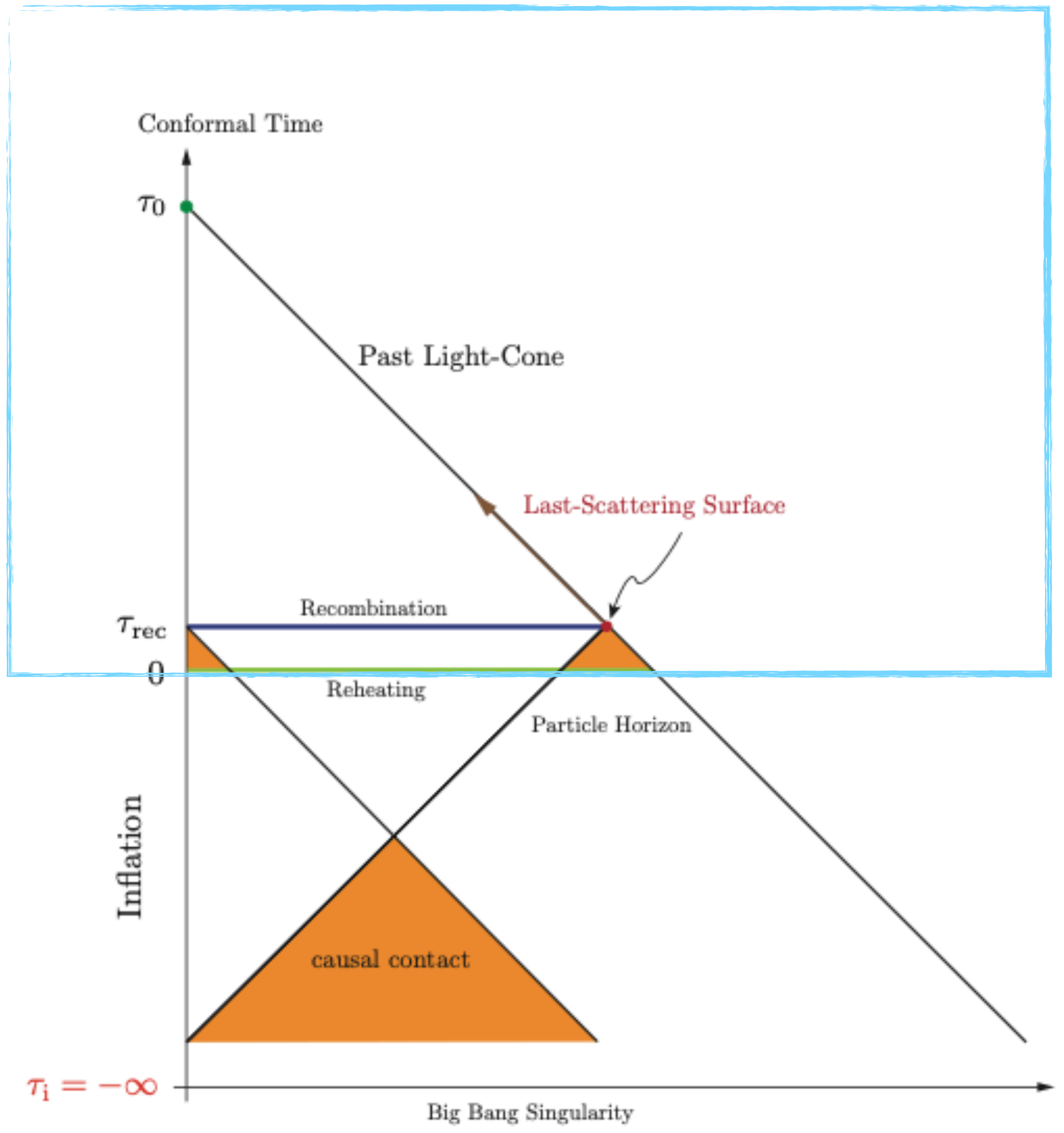


Figure 1.2: *Conformal diagram for inflationary cosmology.*

- We need the comoving Hubble radius to shrink over time:

$$\frac{d}{dt}(aH)^{-1} = -\frac{\ddot{a}}{\dot{a}^2} < 0$$

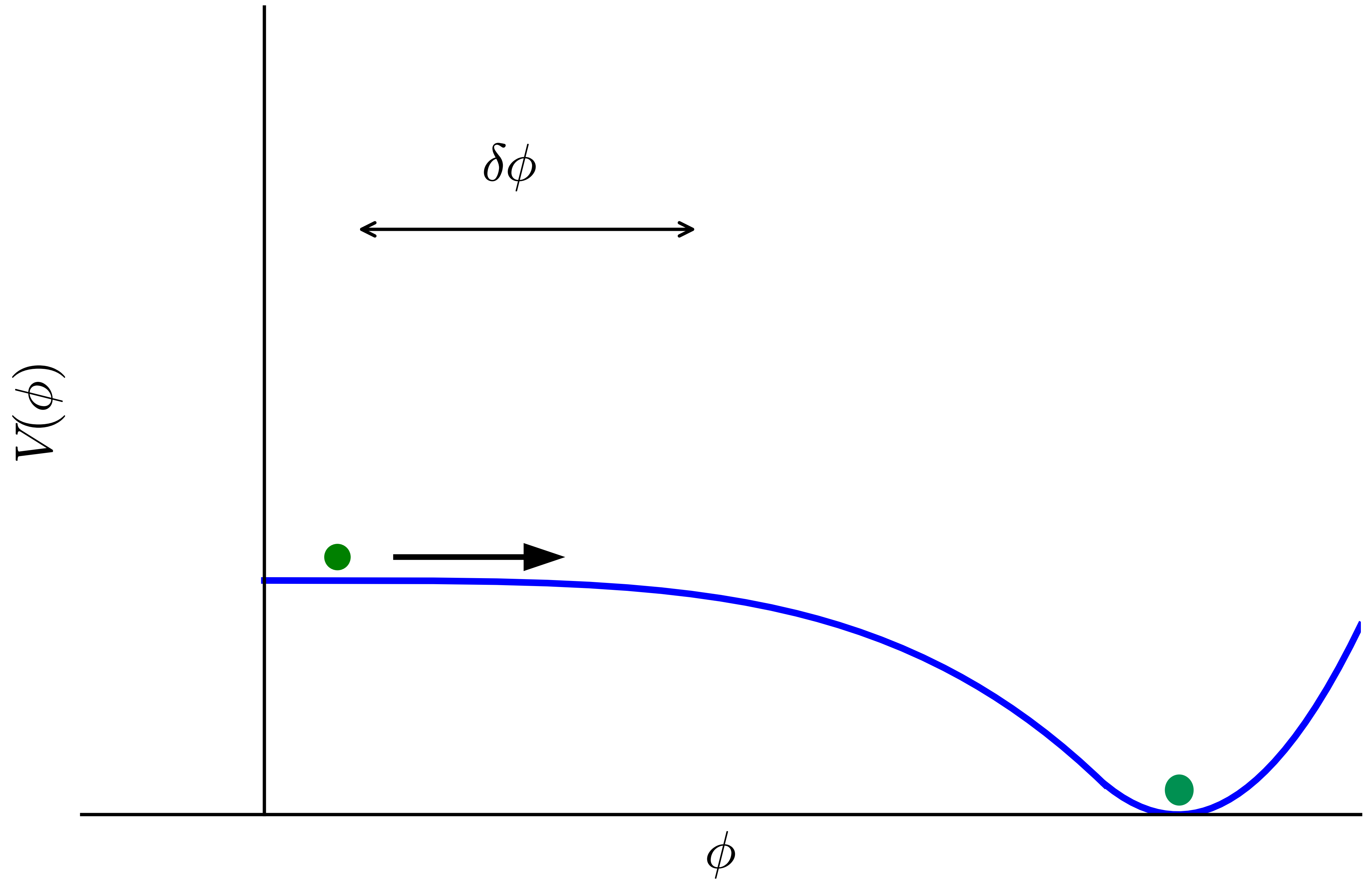
$\Rightarrow$ We need a fluid with an equation of state which breaks the Strong Energy Condition, $\frac{p}{\rho} > -\frac{1}{3}$

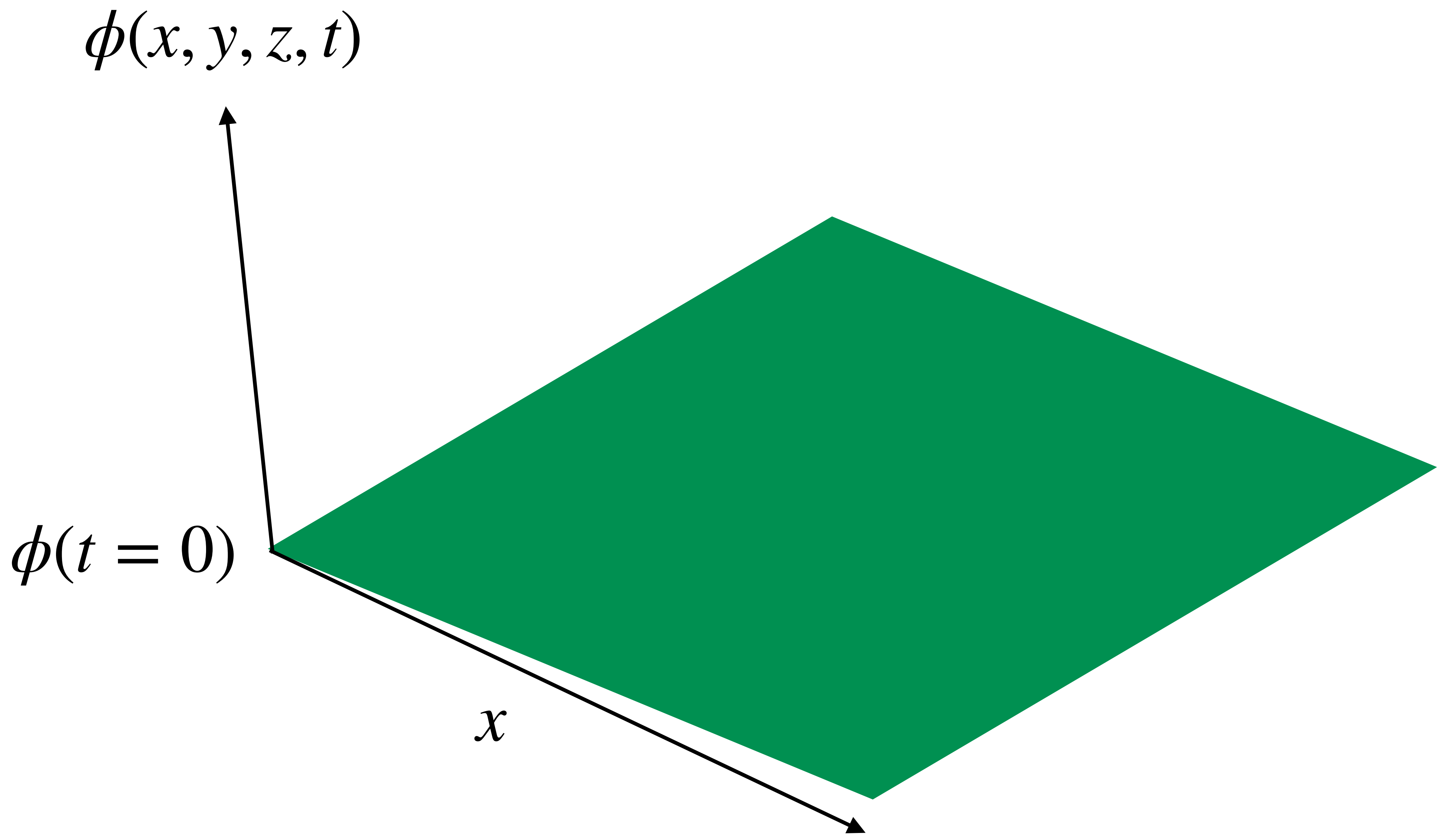
Image credit: D Baumann lecture notes: The Physics of Inflation

The single field slow roll story

Once upon a time, a long long time ago, there was a scalar field on a hill, which started to slowly roll down...

$$\frac{p}{\rho} \approx -1 \quad \begin{cases} p \sim \cancel{\dfrac{\dot{\phi}^2}{2}} + \cancel{\dfrac{\nabla_a\phi\nabla^a\phi}{2}} - V(\phi) \\ \rho \sim \cancel{\dfrac{\dot{\phi}^2}{2}} - \cancel{\dfrac{\nabla_a\phi\nabla^a\phi}{6}} + V(\phi) \end{cases}$$



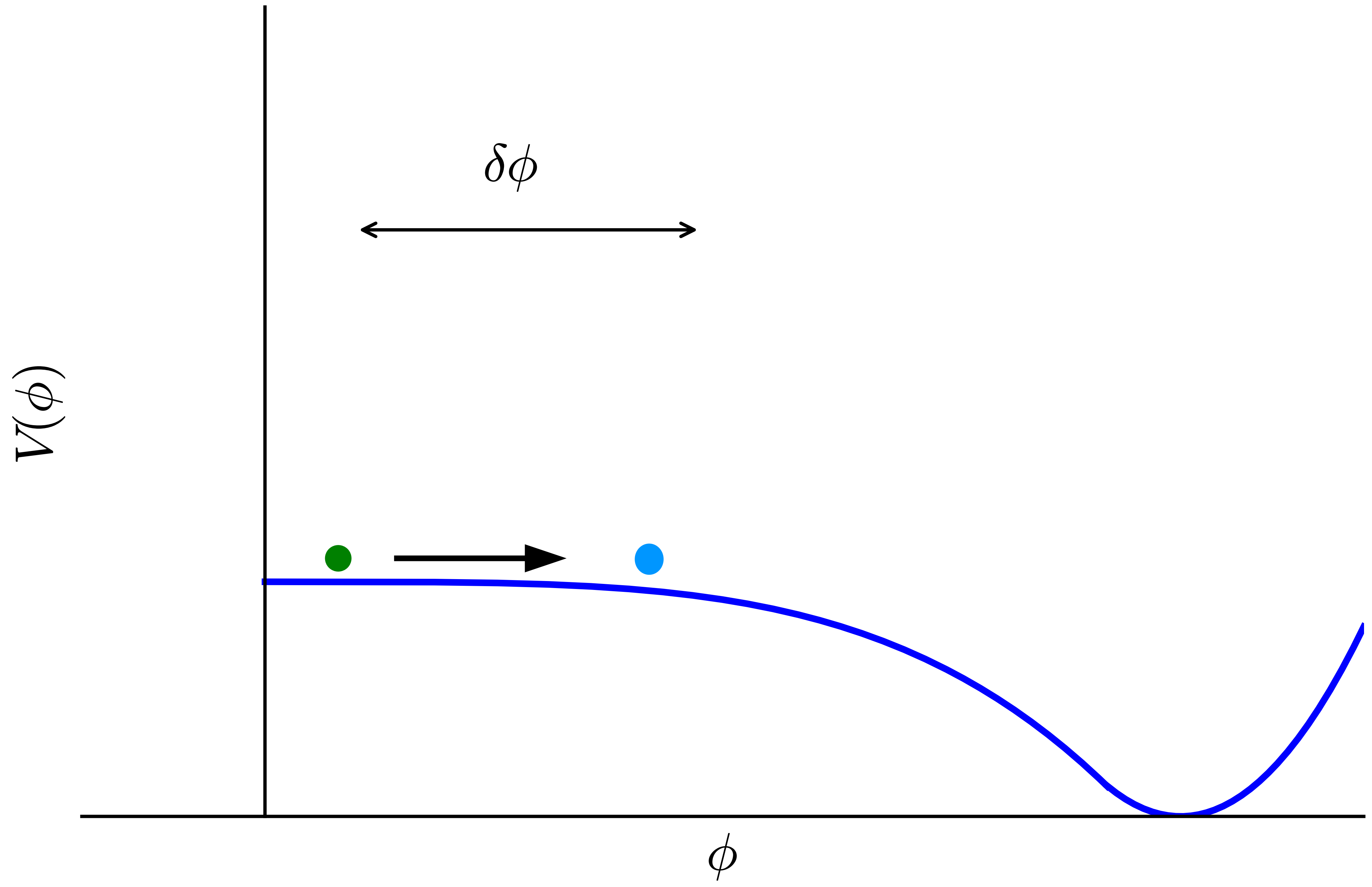


The single field slow roll story

Once upon a time, a long long time ago, there was a fairly complicated set of initial conditions…

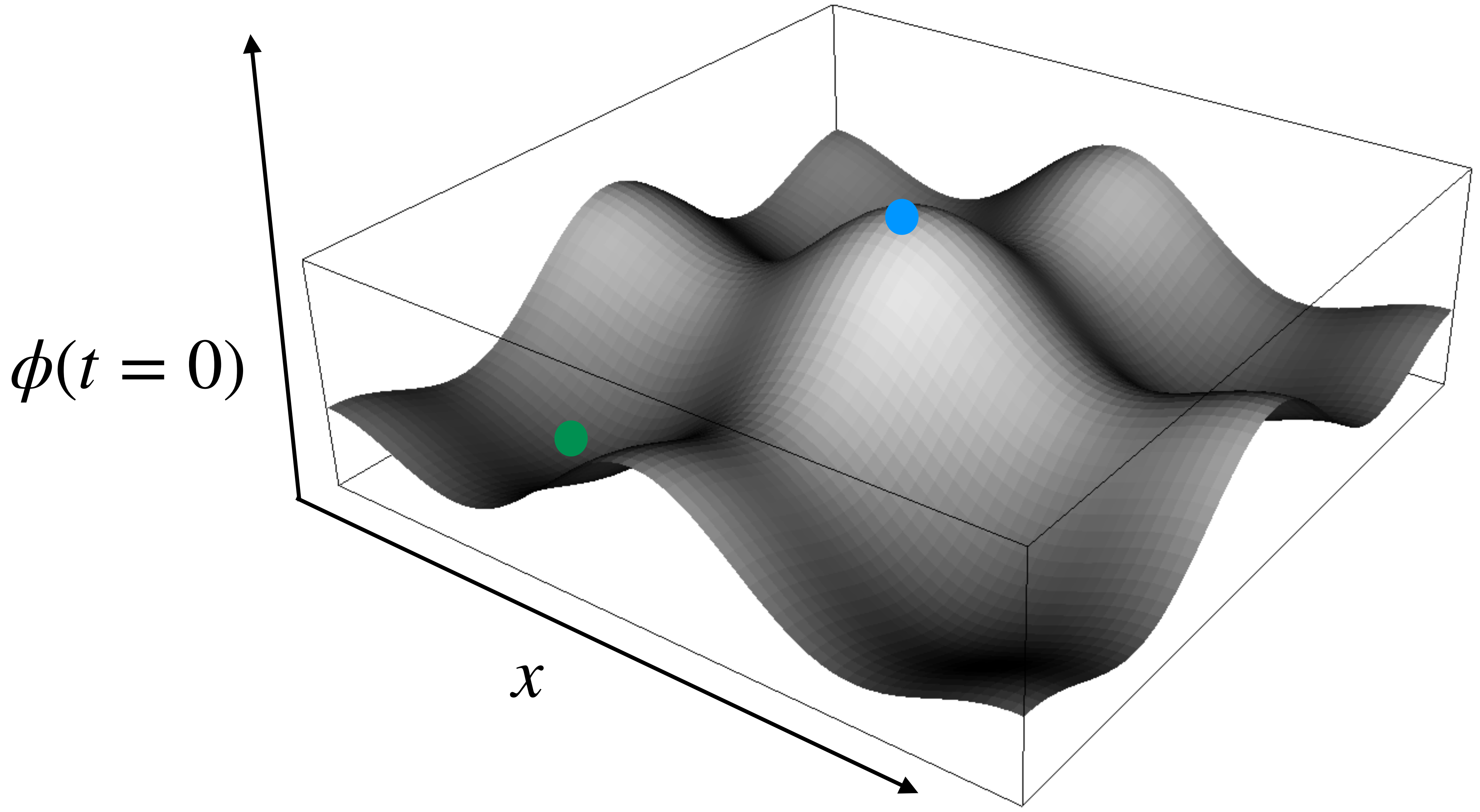
$$p \sim \frac{\dot{\phi}^2}{2} + \frac{\nabla_a \phi \nabla^a \phi}{2} - V(\phi)$$

$$\rho \sim \frac{\dot{\phi}^2}{2} - \frac{\nabla_a \phi \nabla^a \phi}{6} + V(\phi)$$

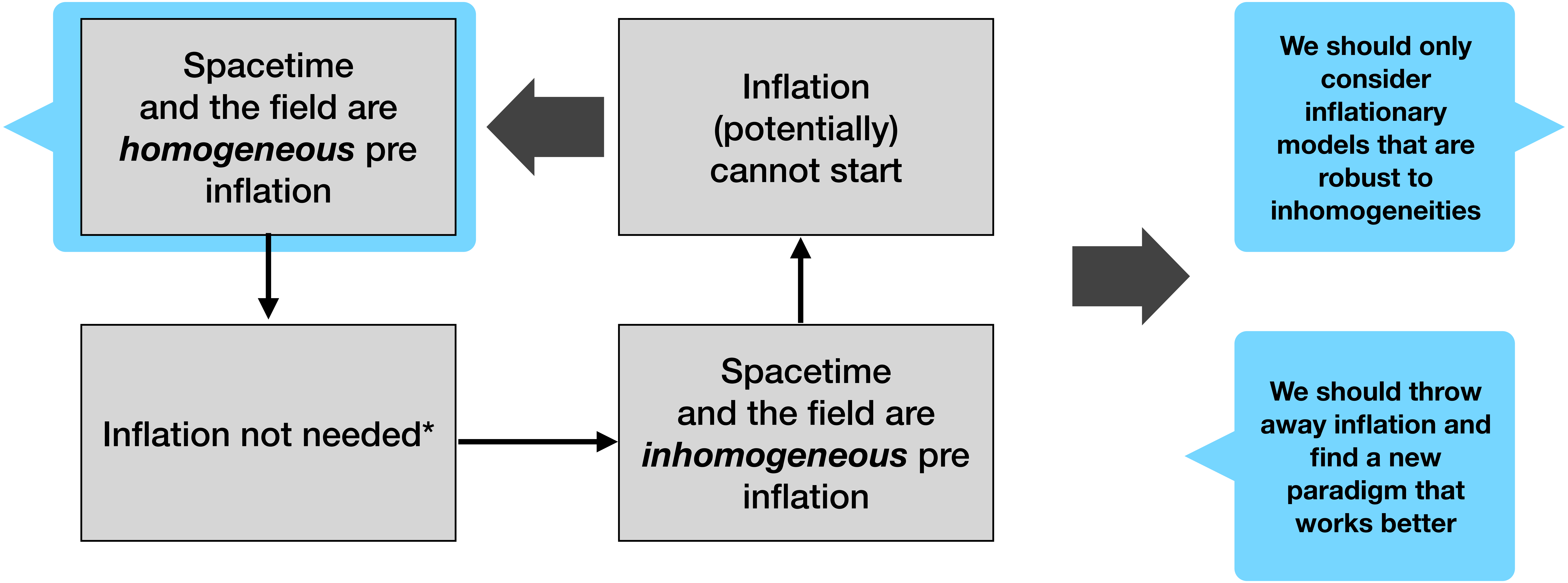


$g_{\mu\nu}(x, y, z, t)$

$\phi(x, y, z, t)$



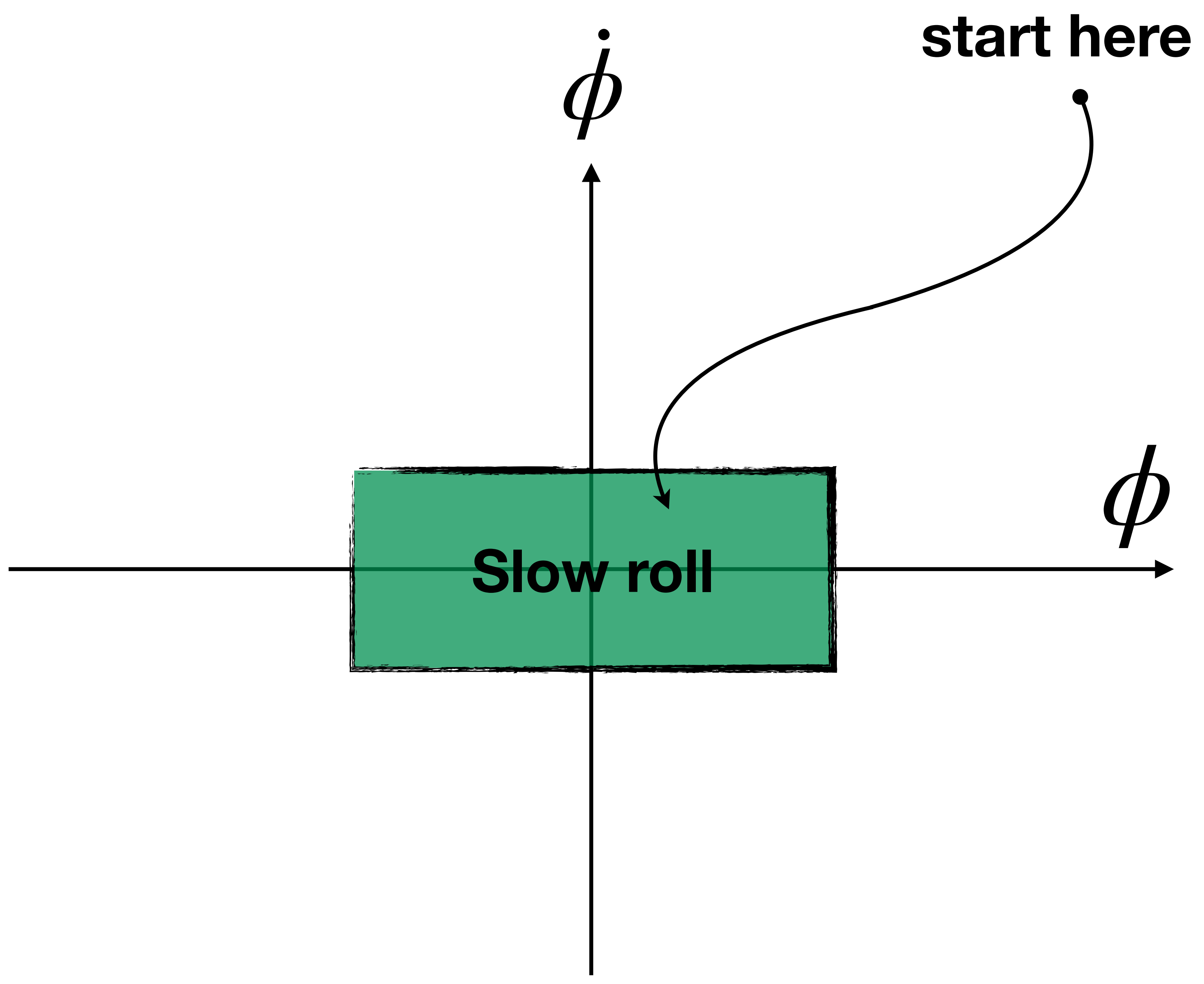
The initial condition problem for inflation



* for horizon problem

Which models of inflation are "robust"?

- Robustness can be understood as the slow roll solution being an attractor in the phase space of initial conditions



Which models of inflation are “robust”?

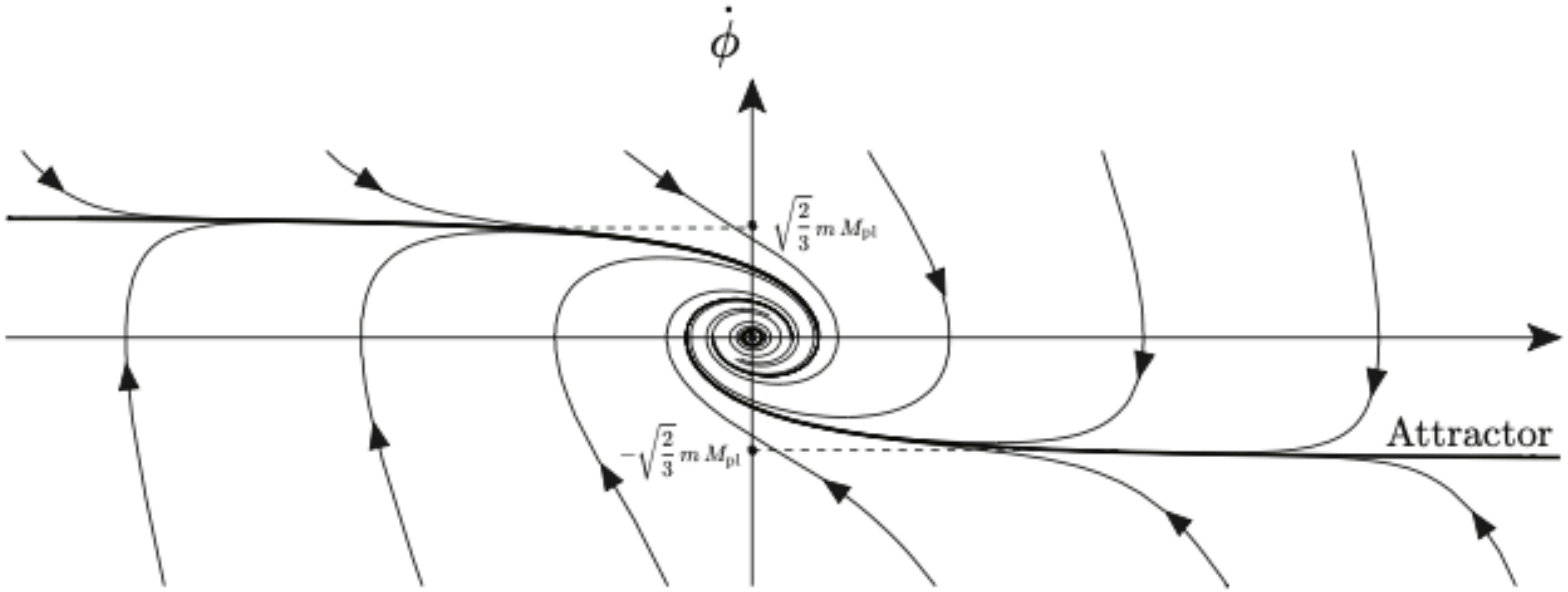


Figure 1.4: *Phase space diagram of $m^2\phi^2$ inflation.*

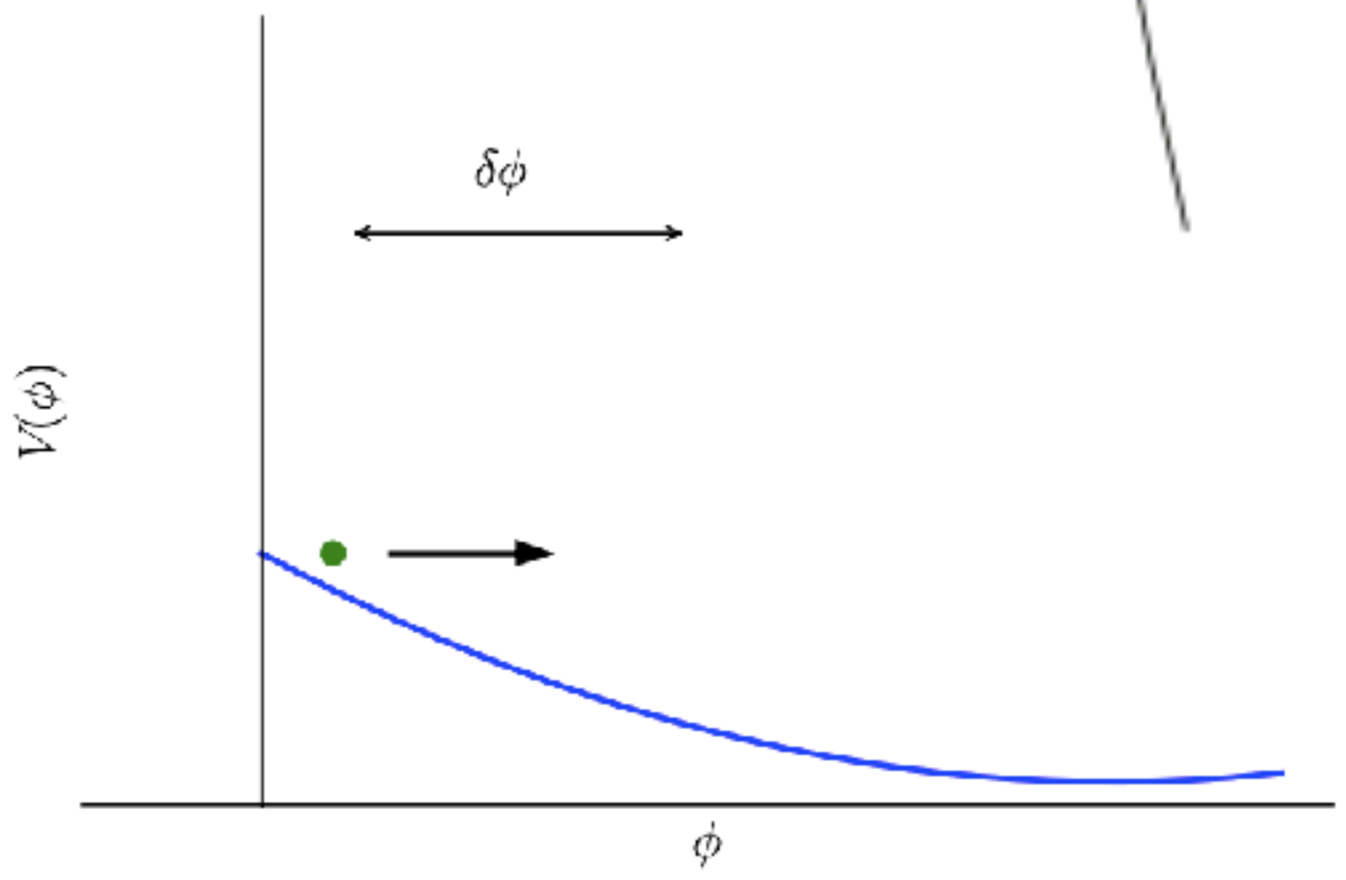


Image credit: D Baumann lecture notes: The Physics of Inflation

Which models of inflation are “robust”?

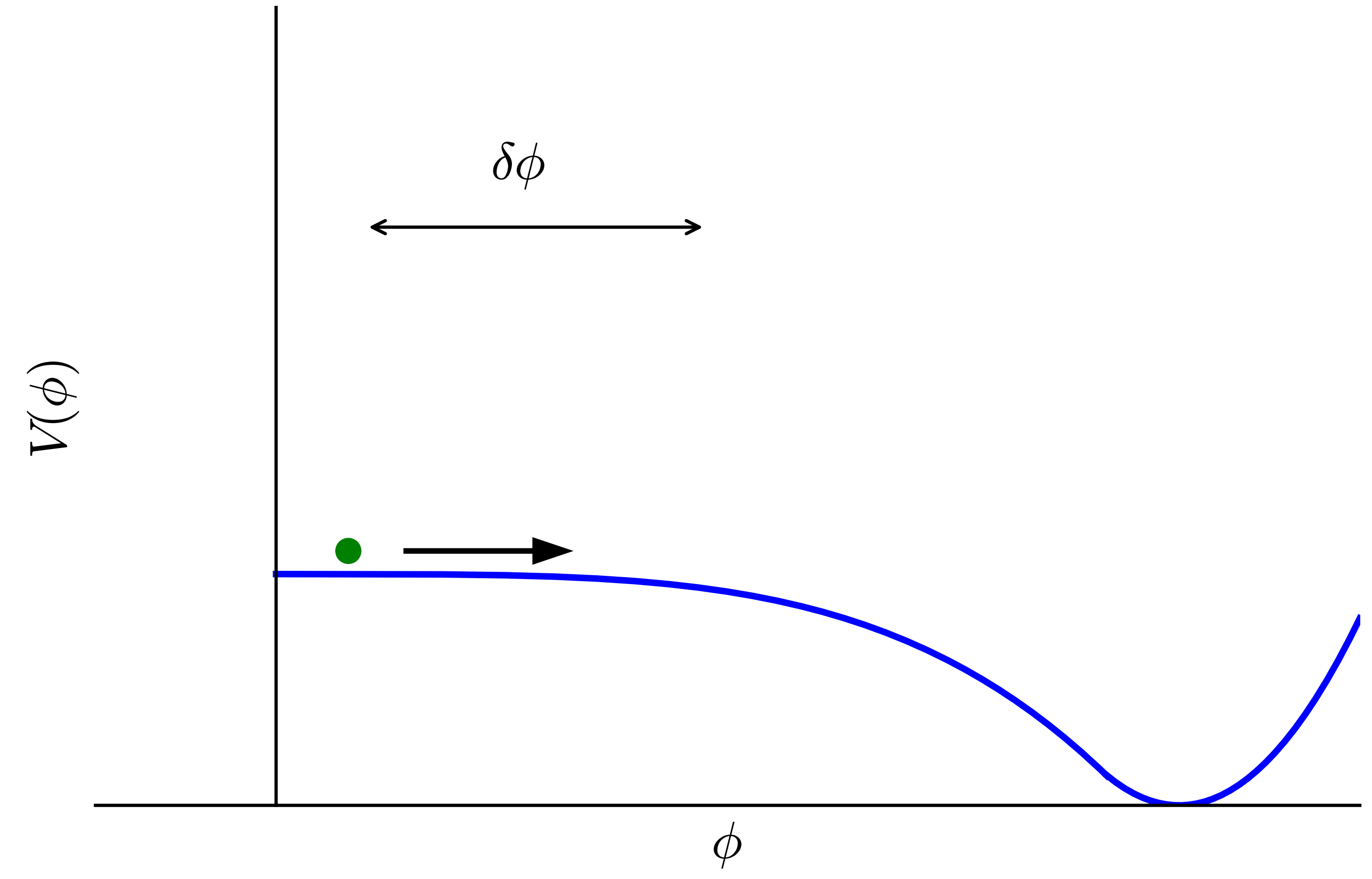
- Rough rule of thumb:

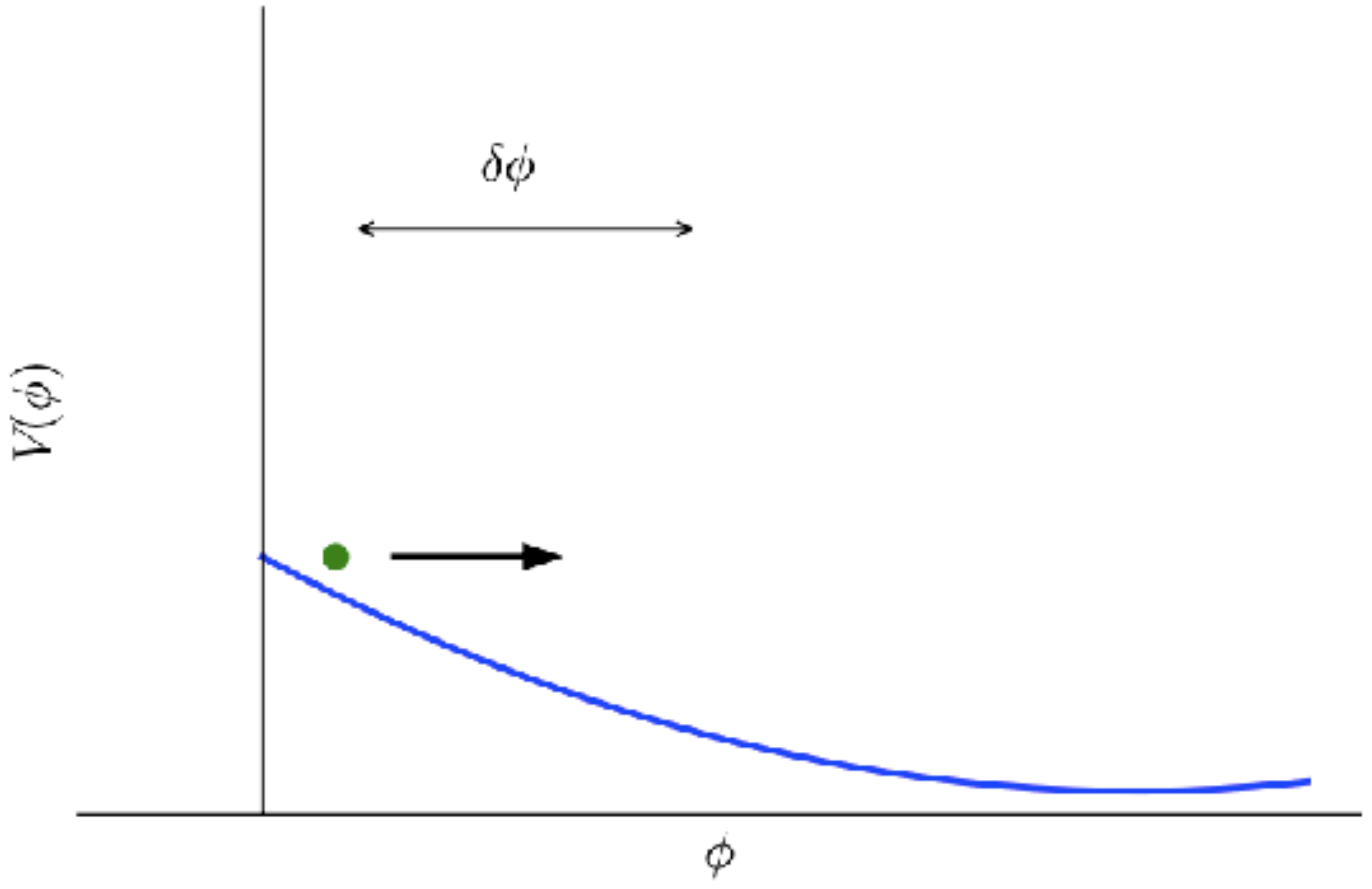
  - Small field models $\delta\phi \ll M_{pl}$ are not robust

  - Large field models $\delta\phi \sim M_{pl}$ are robust

  This is not what we would prefer!

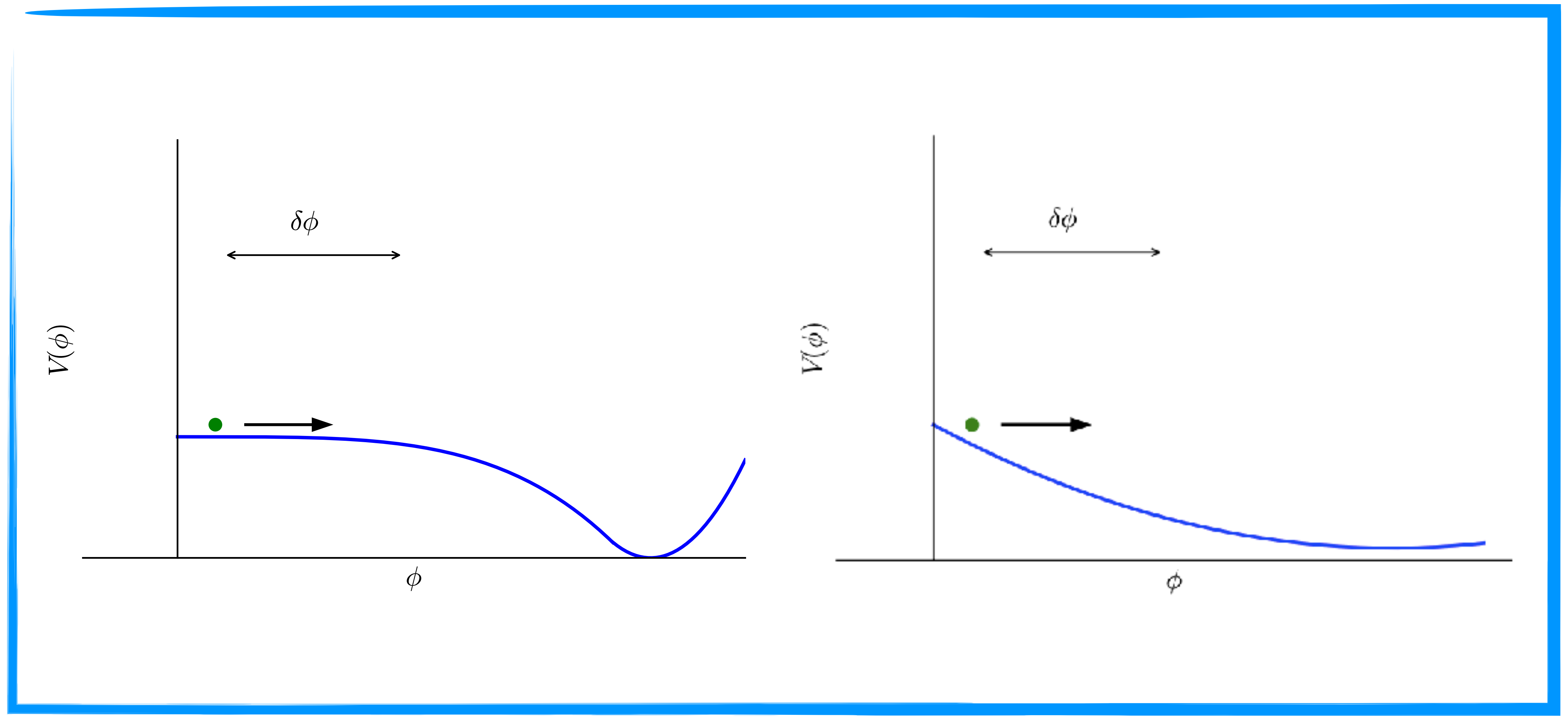
- Suggested reading:
  Initial Conditions for Inflation - A Short Review, Robert Brandenburger
  Int.J.Mod.Phys.D 26 (2016) 01, 1740002 / arXiv:1601.01918



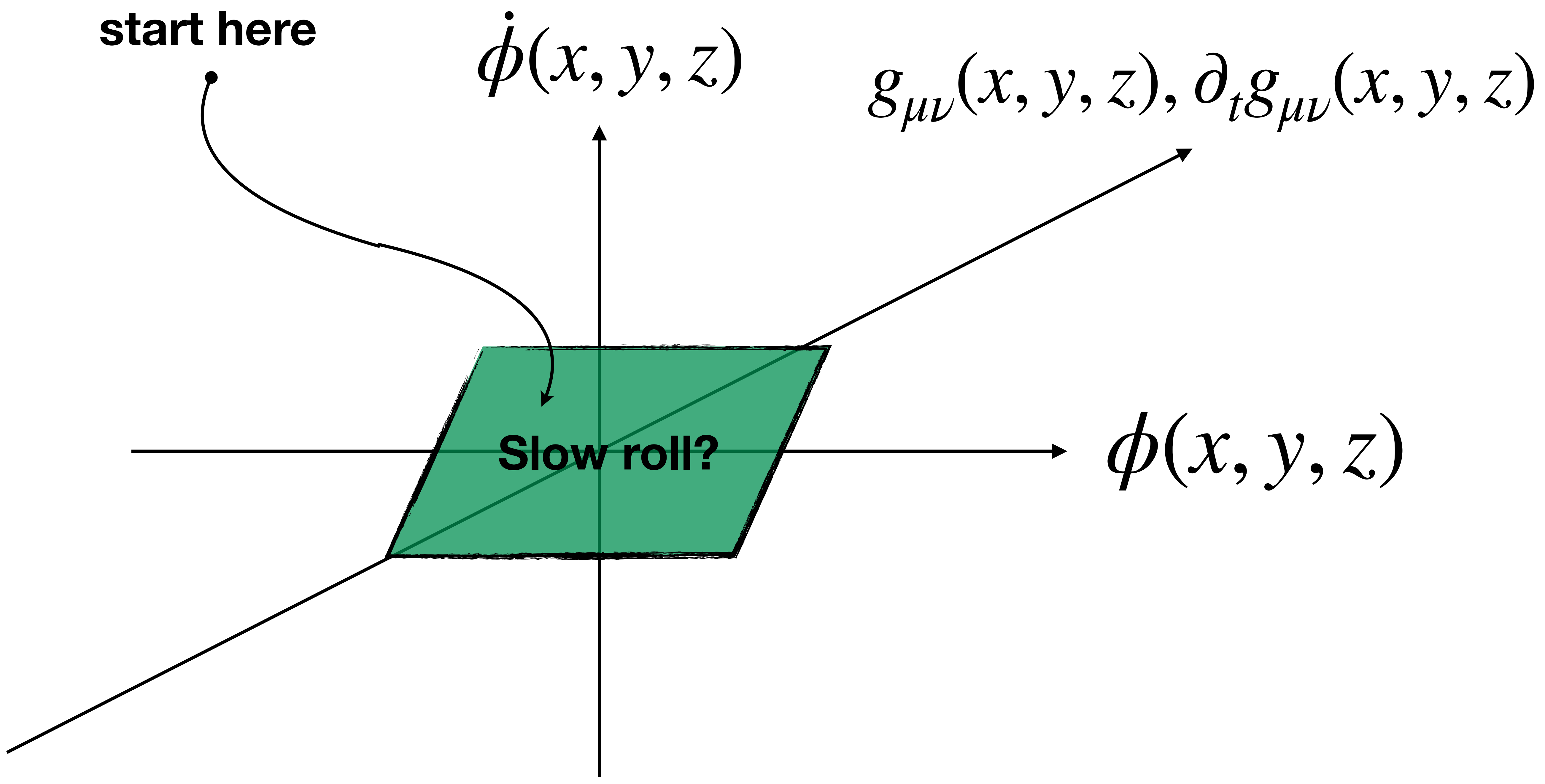


Which models of inflation are “robust”?

- initial condition phase space infinite
- "model space” also infinite
- what is the measure on these spaces?



“Model space”



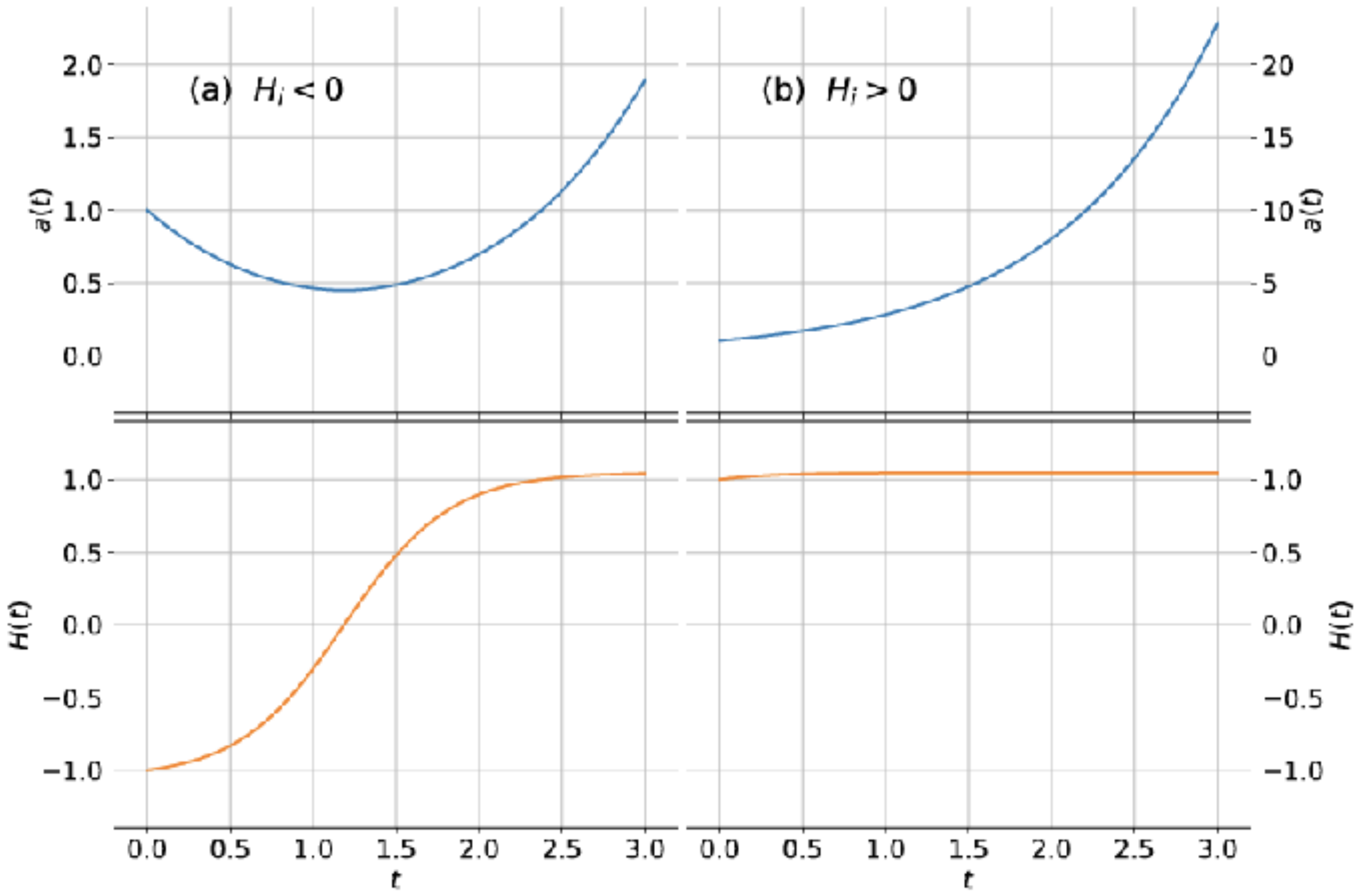
Phase space of initial conditions

A pointer to new physics?

- Consider a rather extreme example:

  Start with a negative H - a collapsing universe - in the absence of matter that violates the Null Energy Condition, the collapse will end at a singularity in a finite time.

  -> All simple single field models fail!



Saving the universe with finite volume effects

J Alexandre and K Clough
Phys. Rev. D 100, 103522

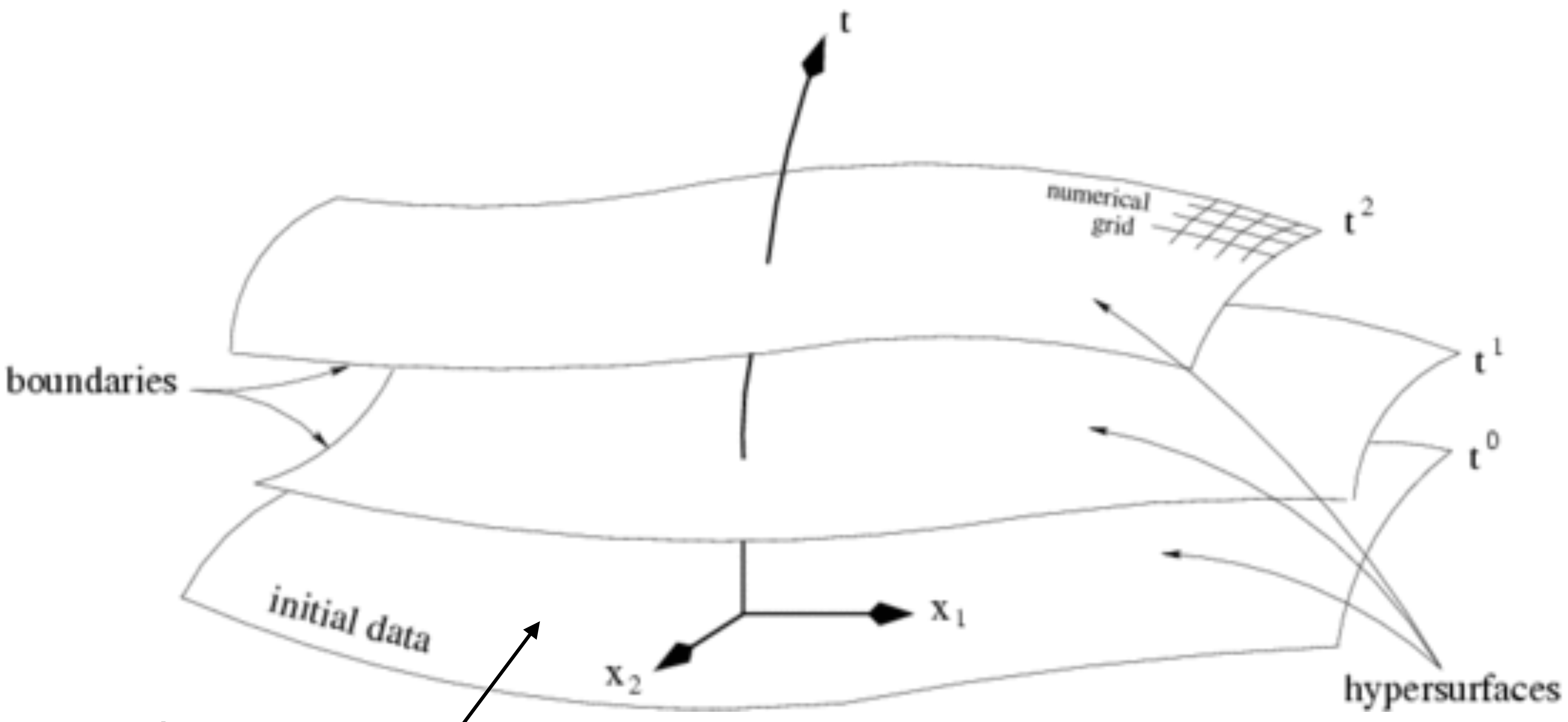
see also :
Quantum Incompleteness of Inflation
A Di Tucci, J Feldbrugge, JL Lehners, N Turok,
Phys.Rev.D 100, 063517

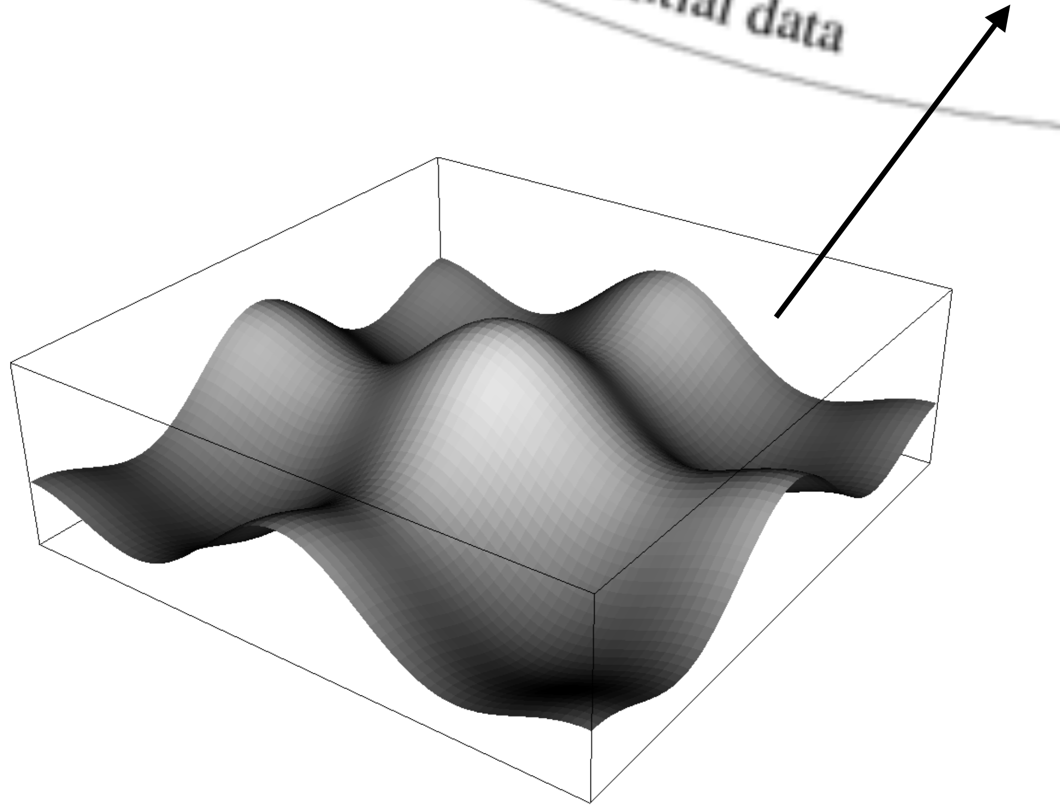
To recap…

- We would like inflationary models to be robust to general initial conditions - both in the metric and the slow-roll field
- That is, starting at a “random” point in this initial phase space, “good” models should proceed to inflate even with “large” deviations from FRW spacetimes
- We assume classical gravity and scalar field dynamics still apply at this point
- This may help us to cut down the space of possible models, or push us to look for new physical mechanisms to explain our homogeneous expanding universe

Studying the initial condition problem with Numerical Relativity

- set up initial conditions with “general” matter and metric fluctuations
- evolve using Einstein Field Equations and see whether inflation proceeds





$$g_{\mu\nu}(x,y,z), \partial_t g_{\mu\nu}(x,y,z)$$

$$\phi(x,y,z) \quad \dot{\phi}(x,y,z)$$

Studying the initial condition problem with Numerical Relativity

Not independent!

- **“Scalar" perturbations:**

$$\phi(x,y,z) = \phi_0 + \Delta\phi\left(\cos\frac{2\pi n x}{L} + \cos\frac{2\pi n y}{L} + \cos\frac{2\pi n z}{L}\right)$$

$\dot{\phi}(x,y,z) = 0 \quad \Longrightarrow \; \rho$ energy density non trivial

leads to “scalar type” metric perturbations

- **“Vector” perturbations:** **Soon!**

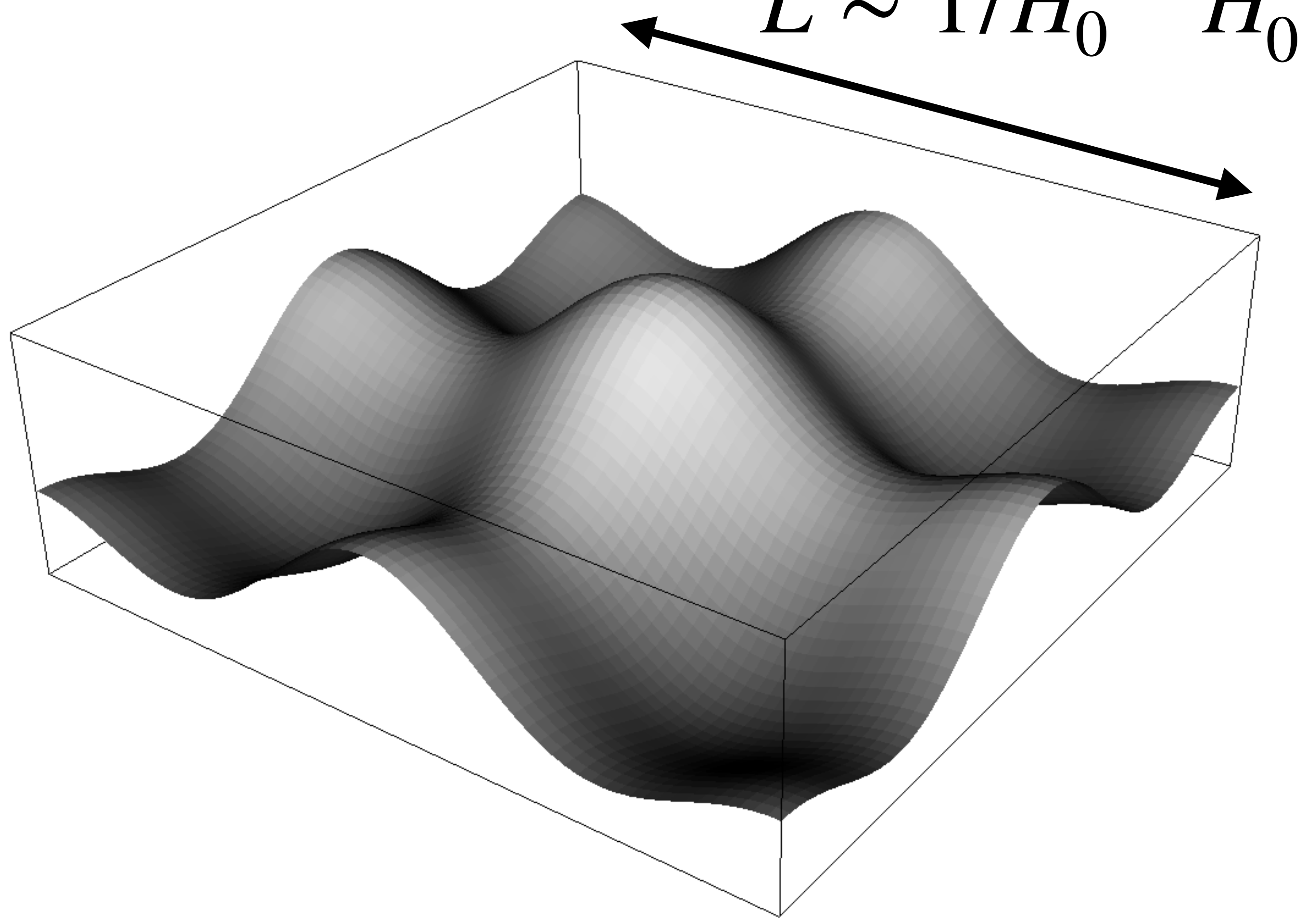
$\phi(x,y,z) \neq 0, \quad \dot{\phi}(x,y,z) \neq 0 \; \Longrightarrow \; S_i$ momentum density non trivial

- **“Tensor” perturbations:**

Vacuum “gravitational wave” perturbations $\tilde{A}_{ij}^{TT} \sim \partial_t g_{ij} \neq 0$

Size = Hubble length in absence of perturbations

$L \sim 1/H_0 \quad H_0^2 = \frac{8\pi G}{3}V(\phi_0)$



$$g_{\mu\nu}(x,y,z), \partial_t g_{\mu\nu}(x,y,z)$$

$$\phi(x,y,z) \quad \dot{\phi}(x,y,z)$$

Studying the initial condition problem with Numerical Relativity

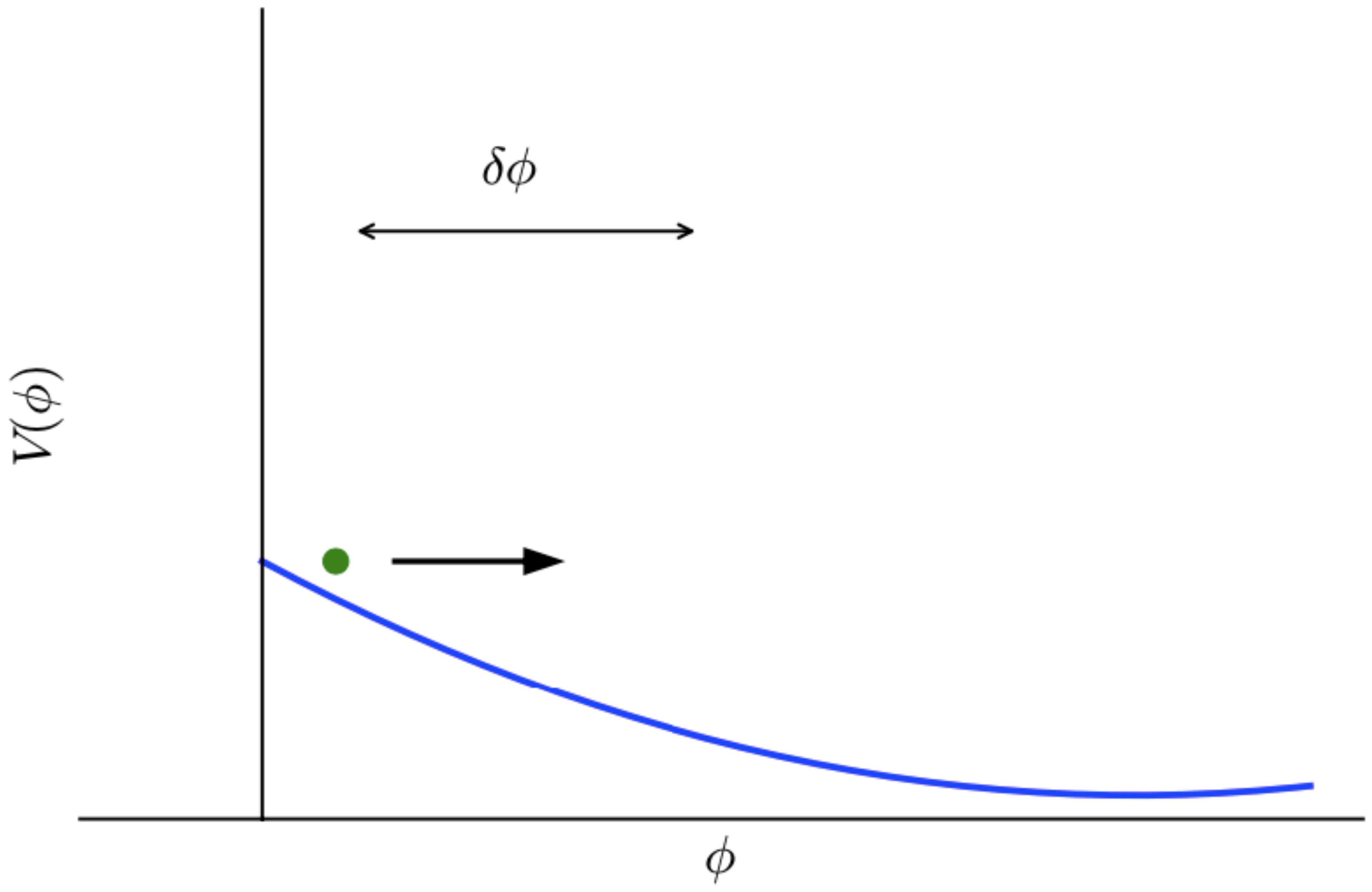
Inflation

- Early works:
  - D. S. Goldwirth and T. Piran
    Inhomogeneity and the onset of inflation
    Phys. Rev. Lett. 64, 2852 (1990)
  - P. Laguna, H. Kurki-Suonio, and R. A. Matzner
    Inhomogeneous Inflation: Numerical Evolution
    Phys.Rev. D48 (1993) 3611-3624
- Recent works:
  - W. E. East, M. Kleban, A. Linde and L. Senatore
    Beginning Inflation in an inhomogeneous universe
    JCAP 1609 (2016) no.09, 010
  - K. Clough, E. A. Lim, B. S. DiNunno, W. Fischler, R. Flauger, S. Paban
    Robustness of Inflation to Inhomogeneous Initial Conditions
    JCAP 1709 (2017) no.09, 025
  - K. Clough, E. A. Lim, R. Flauger
    Robustness of Inflation to Large Tensor Perturbations
    JCAP 05 (2018) 065
  - J. C. Aurrekoetxea, K. Clough, E. A. Lim, R. Flauger
    The Effects of Potential Shape on Inhomogeneous Inflation
    JCAP 05 (2020) 030

Ekpyrotic Scenarios

- Early works:
  - Evolution to a smooth universe in an ekpyrotic contracting phase with $w > 1$
    D. Garfinkle, W. C. Lim, F. Pretorius, P. J. Steinhardt
    Phys.Rev.D 78 (2008) 083537
  - Nonperturbative analysis of the evolution of cosmological perturbations through a nonsingular bounce
    B. K. Xue, D. Garfinkle, F. Pretorius, P. J. Steinhardt
    Phys.Rev.D 88 (2013) 083509
- Recent works:
  - W. G. Cook, I. A. Glushchenko, A. Ijjas, F. Pretorius, P. J. Steinhardt
    Supersmoothing through Slow Contraction
    Phys.Lett.B 808 (2020) 135690
  - A. Ijjas, W. G. Cook, F. Pretorius, P. J. Steinhardt, E. Y. Davies
    Robustness of slow contraction to cosmic initial conditions
    JCAP 08 (2020) 030

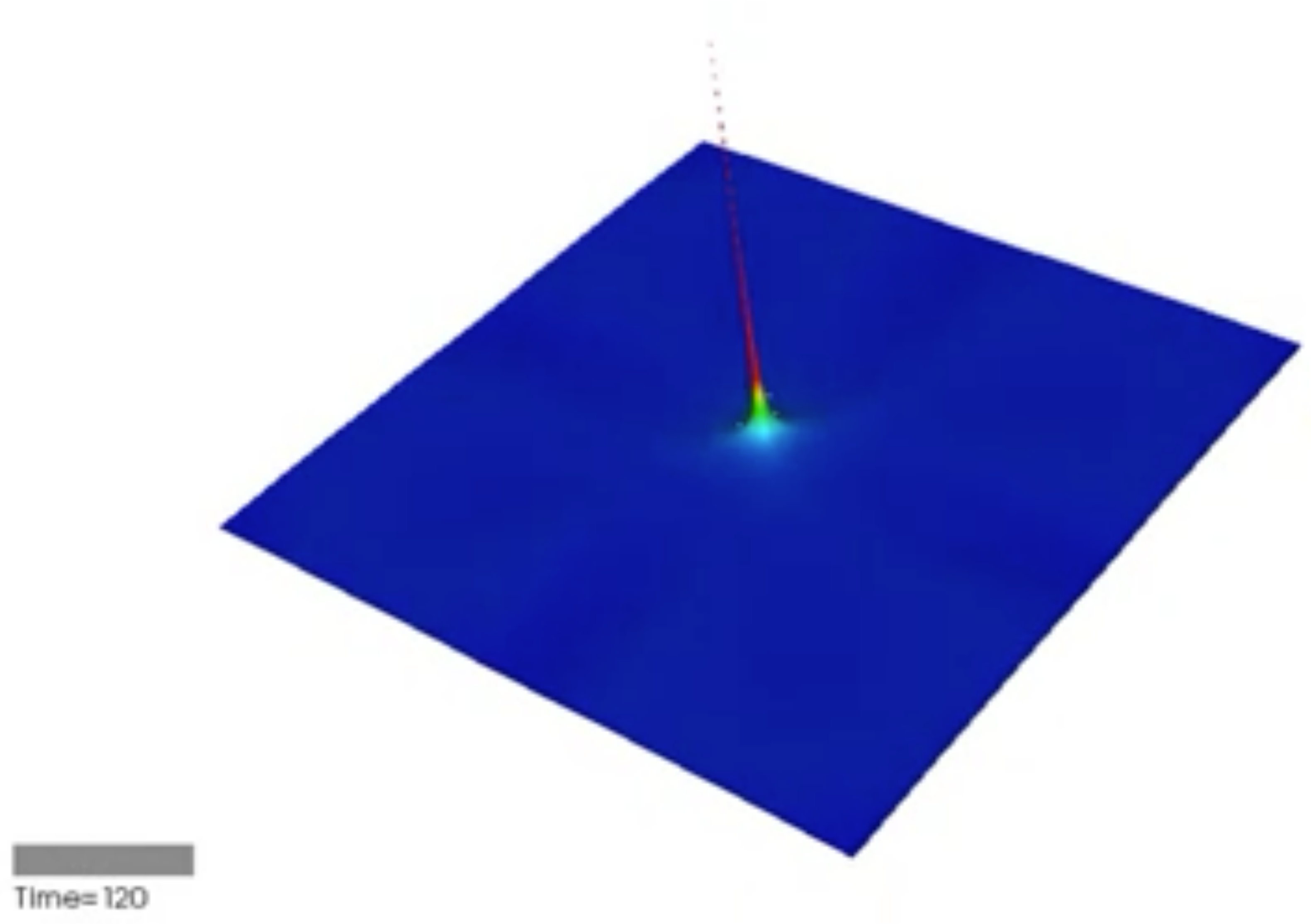
Studying the initial condition problem with Numerical Relativity

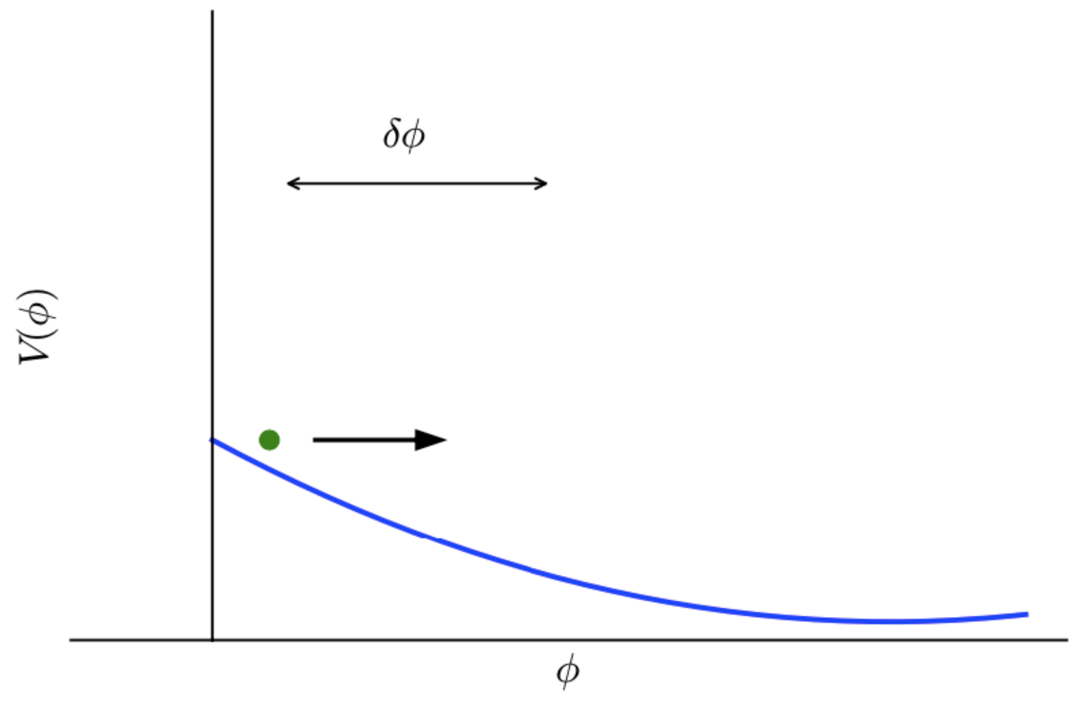


Large field inflation

Studying the initial condition problem with Numerical Relativity

(minus) H -> blue is expanding, red is collapsing

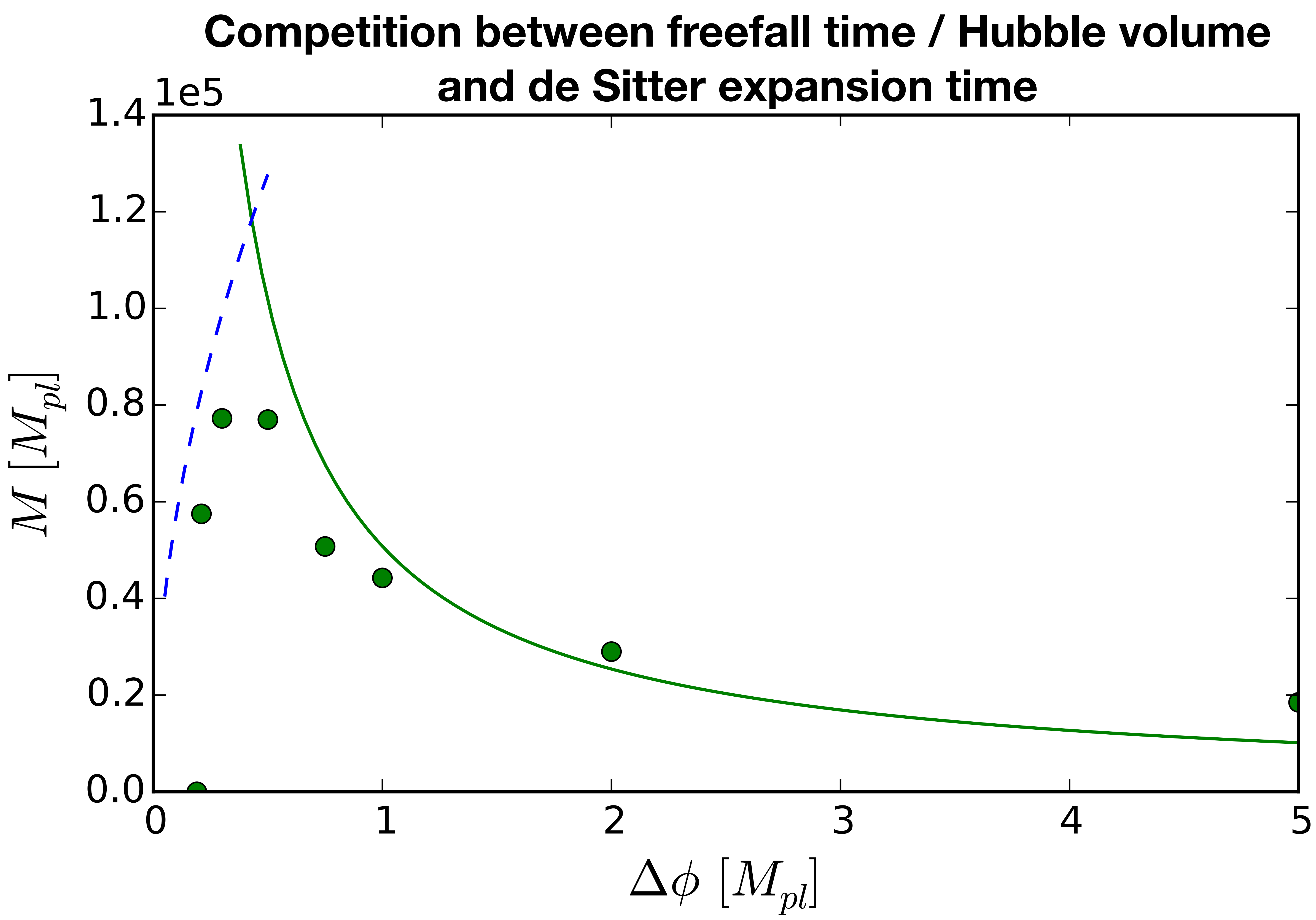




Large field inflation

Studying the initial condition problem with Numerical Relativity

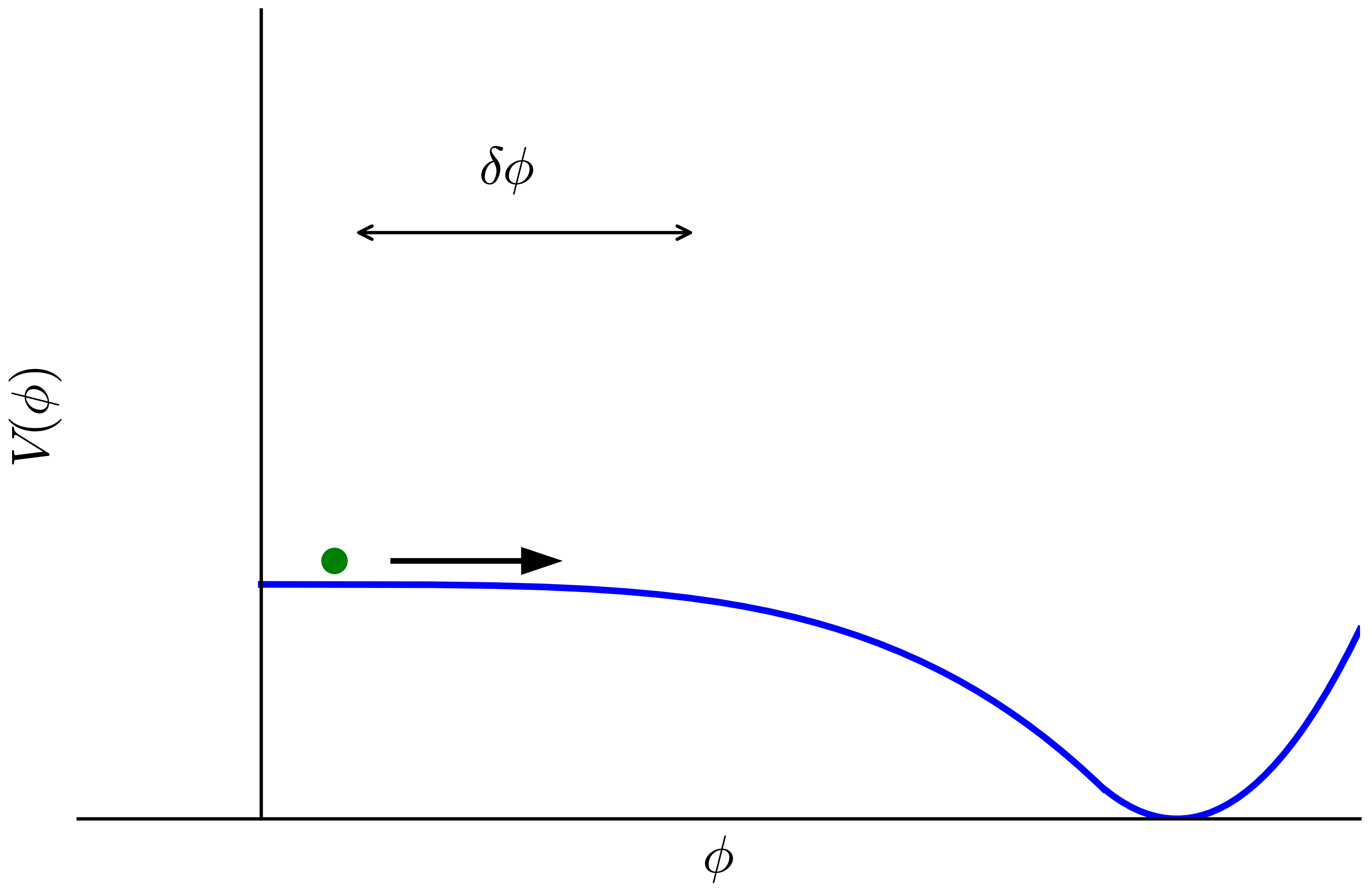
Mass of BH formed



All cases still dominated by inflating regions

Amplitude of scalar perturbations $\frac{\rho_{grad}}{V_0} \sim 10^2$

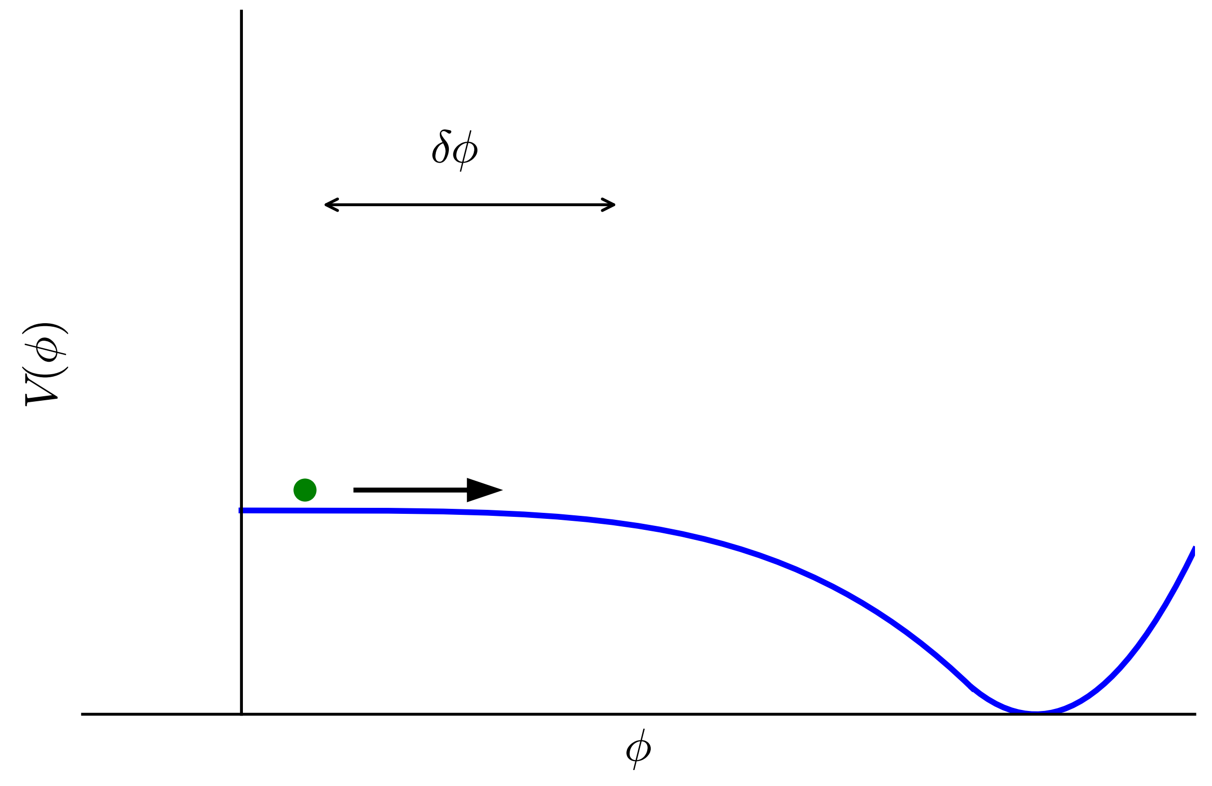
Studying the initial condition problem with Numerical Relativity



Small field inflation

Studying the initial condition problem with Numerical Relativity

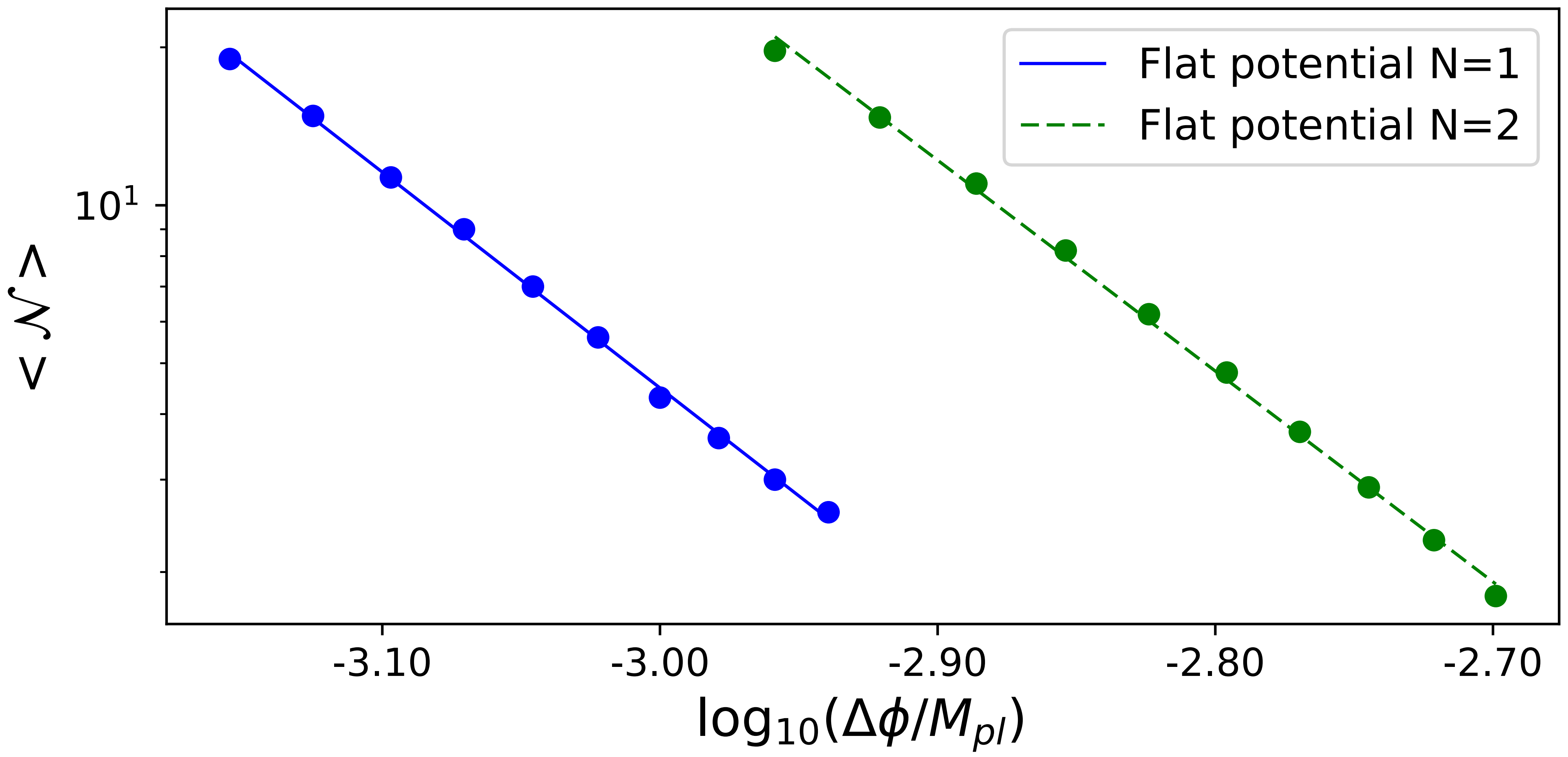
Spatial profile of the field



Small field inflation

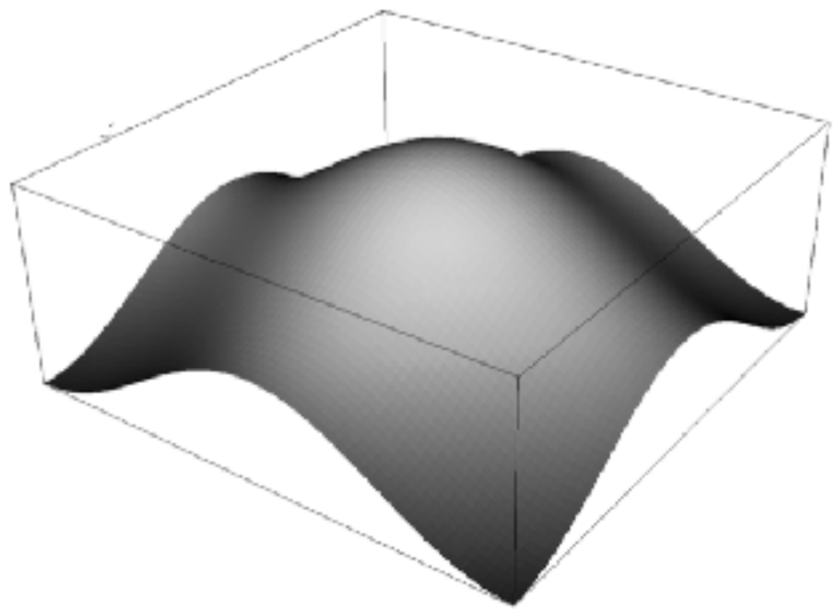
Studying the initial condition problem with Numerical Relativity

Number of e-folds

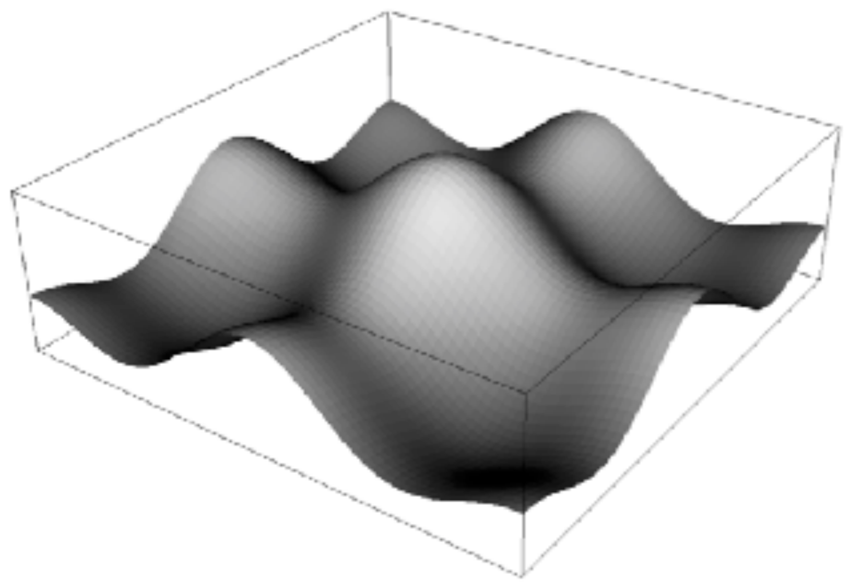


Amplitude of scalar perturbations

$$\frac{\rho_{grad}}{\rho_{V_0}} \sim 10^{-4}$$



N = 1
(n = 1)

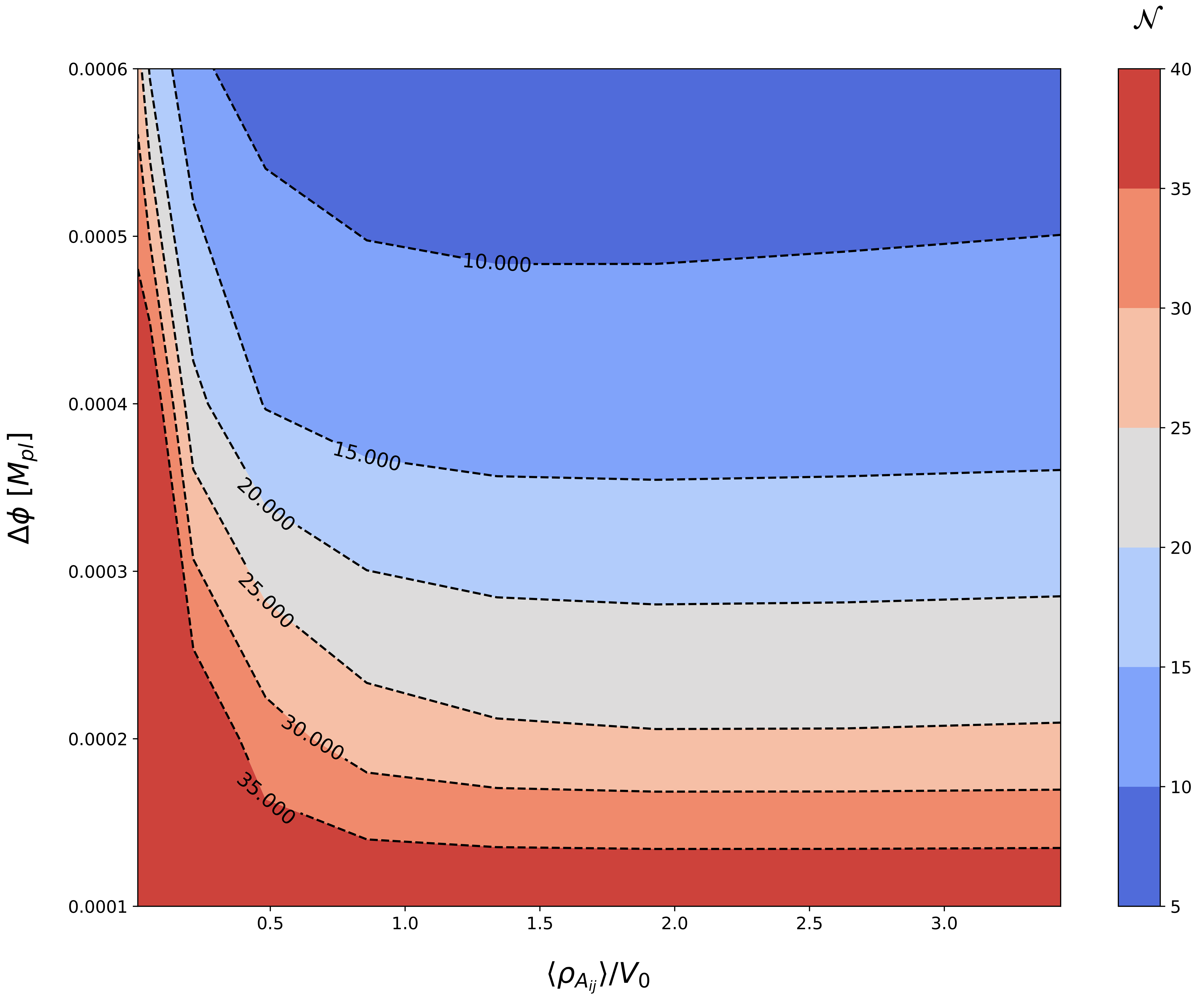


N = 2
1/2 (n = 1 + n = 2)

Studying the initial condition problem with Numerical Relativity

Amplitude of scalar perturbations

$$\frac{\rho_{grad}}{V_0} \sim 10^{-4}$$

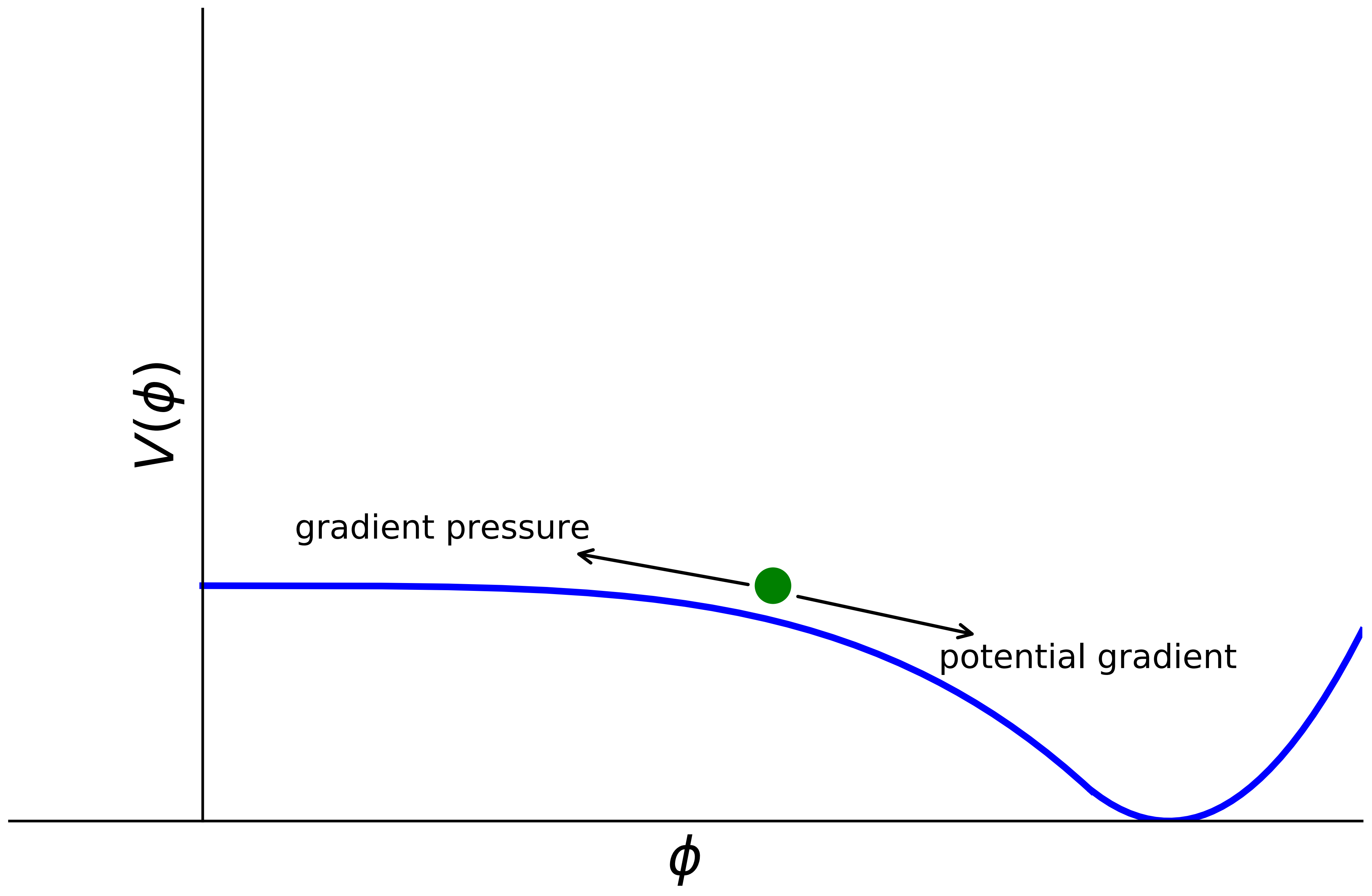


Number of e-folds

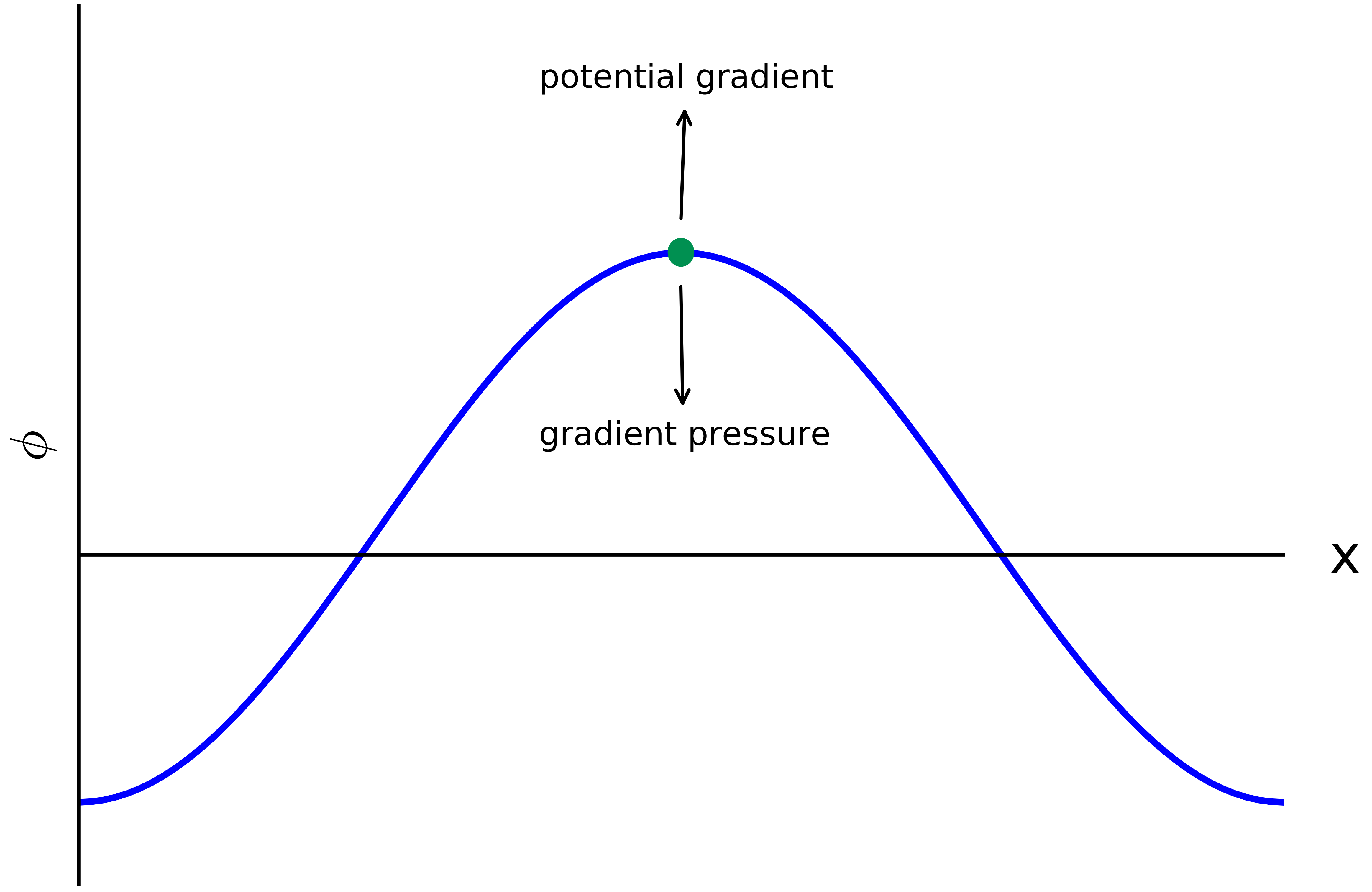
Amplitude of tensor (metric) perturbations

$$\frac{\rho_{tensor}}{V_0} \sim 1$$

Easy to predict when failure occurs

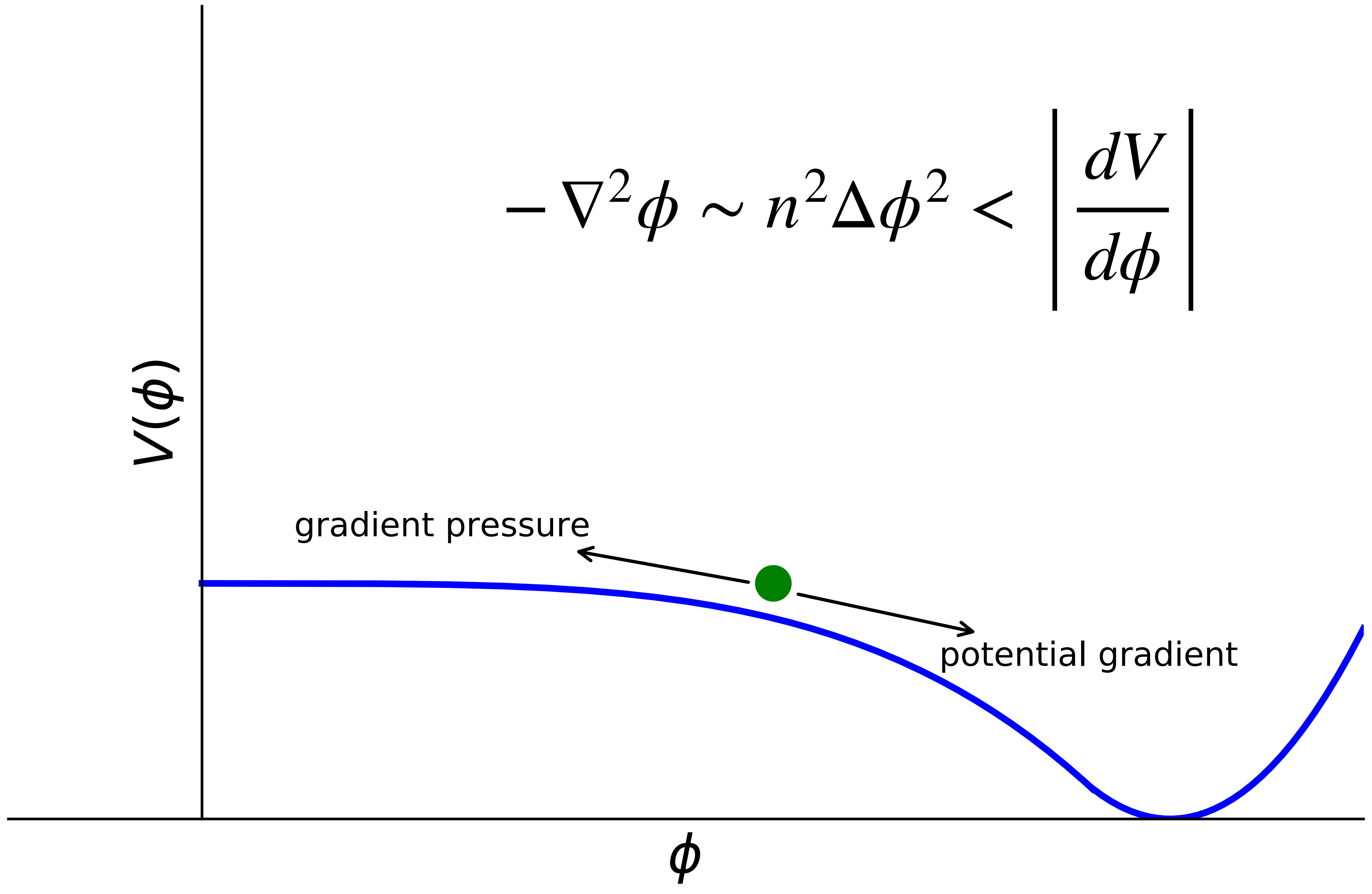


Field space



Physical space

Easy to predict when failure occurs



Field space

Key conclusions

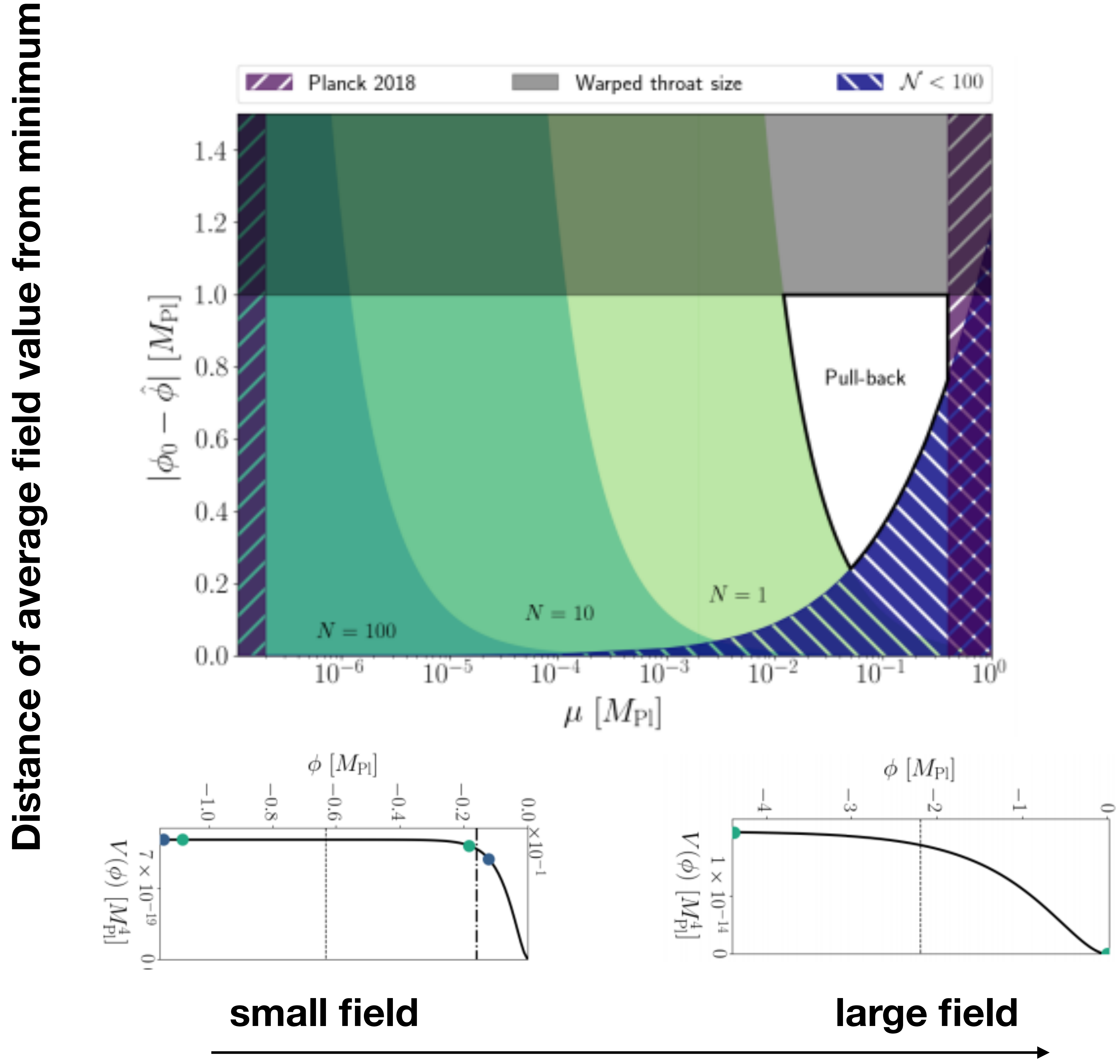
- Small field inflation fails even for very subdominant gradient energies
- Adding additional shorter wavelength modes makes inflation more robust for a given maximum initial value of ϕ (even when increasing energy density in gradients)
- Convexity of the potential is the key driver of recovery to inflation

Model dependence

- Illustrative example: consider D-brane model (concave, but can be large field or small field)

$$V(\phi) = \Lambda^4\left(1 - \frac{\phi^4}{\mu}\right)$$

- For $\mu \ll M_{pl}$ need to put the average value of the field ϕ_0 "further up the hill" to ensure inflation robust to all amplitudes of perturbation



J. C. Aurrekoetxea, K. Clough, E. A. Lim, R. Flauger
The Effects of Potential Shape on Inhomogeneous Inflation
JCAP 05 (2020) 030
https://www.youtube.com/watch?v=yk9sGuG8hdI&feature=youtu.be

Summary

- Inflation solves some problems but introduces new ones regarding how it got started in the first place
- **The solution of these new problems may tell us more about how the Universe began**
- No (minimally coupled, single field) inflationary model is successful for the entire possible phase space of initial conditions, but some are “better” than others.
- Numerical relativity is a powerful tool with which to study the robustness of different models to inhomogeneous initial conditions