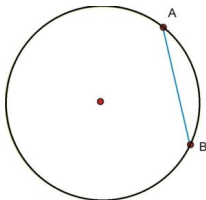# Random Transits Across Bounded Domains

Prabodh Shukla, NEHU Shillong (retired).

based on
Surprising variants of Cauchy's formula for mean chord length,
PS and Diana Thongjaomayum
Phys Rev E 100, 050103(R) 2019.

February 19, 2020

# For simplicity, take bounded domain to be a circle

An object enters at A and exits at B. A could be anywhere on boundary and direction of entry is isotropic. The object could be,

- a photon moving in a straight line
- a meandering ant
- an agitated molecule
- a random walker

Surprisingly, $<AB> = <L> = \ell_1 + \ell_2 + \ell_3 + \ldots \approx$ volume/area. The distribution of ℓ_i is not so clear ! We want to understand these issues.

Cauchy's formula for mean chord length
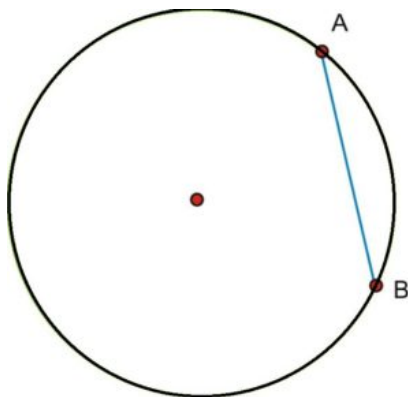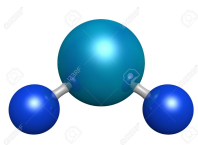

$< AB >=< L >= \eta_d V/S$

$\eta_2 = \pi, \eta_3 = 4$

Ant

Ants do not follow a straight path and do not tell us exactly what equations they follow so analytic calculation is not possible, However, experimentally $< AB >=< L >\approx$ Area/Length of Boundary.

Molecule

Boltzmann transport equation and mean field approximation gives
$< AB >=< L >\approx$ Volume/Area. $P(\ell_i) = \frac{1}{\lambda}e^{-\frac{\ell_i}{\lambda}}$
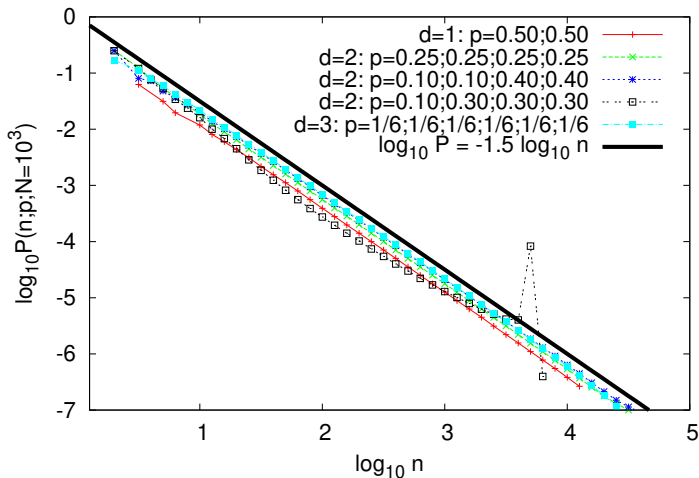
Random walker on a small lattice: a test run
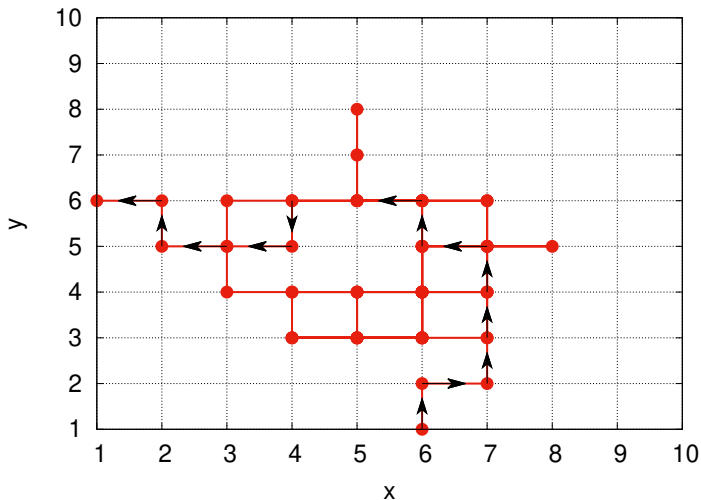
computer generated walk on 10×10 lattice, taking 68 steps from boundary-to-boundary and visiting 27 lattice points on the way,

Random walker on larger $N \times N$ lattices: $< n >$

Plot legend:
- $<n>=N$
- d=1: p=0.50;0.50
- d=1: p=0.75;0.25
- d=2: p=0.25;0.25;0.25;0.25
- d=2: p=0.10;0.10;0.40;0.40
- d=2: p=0.10;0.30;0.30;0.30
- d=3: p=1/6;1/6;1/6;1/6;1/6;1/6
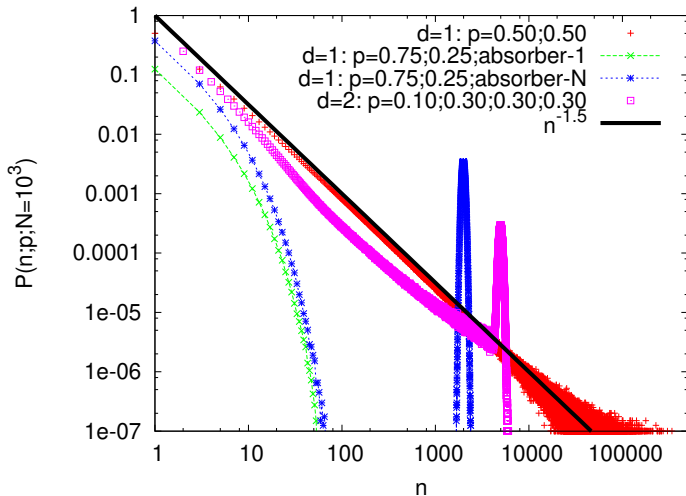
Axes: N (horizontal), <n> (vertical)
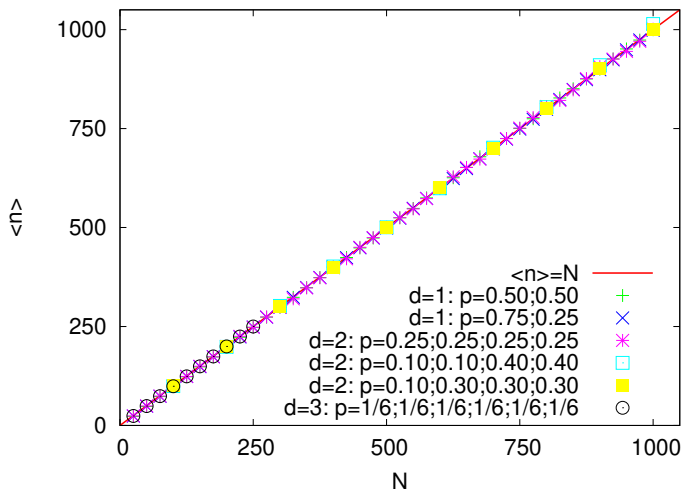
Random walker on large lattice: $P(n)$

Power laws seem to be the norm not exponentially damped distributions!

Random walker on large lattice
understanding departures from power law

Conclusion

The remarkable universality of Cauchy's formula and power laws seems to emerge from the uniformity and isotropy of the incident flux on the bounding surface and also of the scattering mechanism inside the bounded domain. Other factors do not seem to affect it.

Poetry! Faiz ka sher!

sar-ba-sar thee raah manzil apnee,
ham jahan pahunche kaamyaab aaye,