Orientation dynamics of a neutrally-buoyant spheroid of arbitrary aspect-ratio in simple shear flow of weakly viscoelastic fluid

Deepak Madival & Ganesh Subramanian
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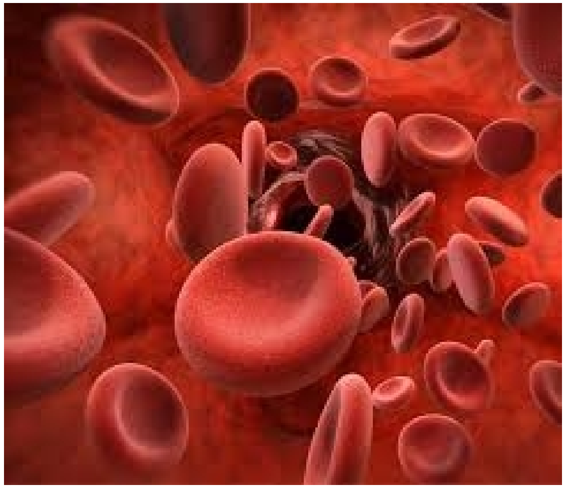
- Non-spherical particles in a fluid are found in nature and technology.

Red Blood Cells

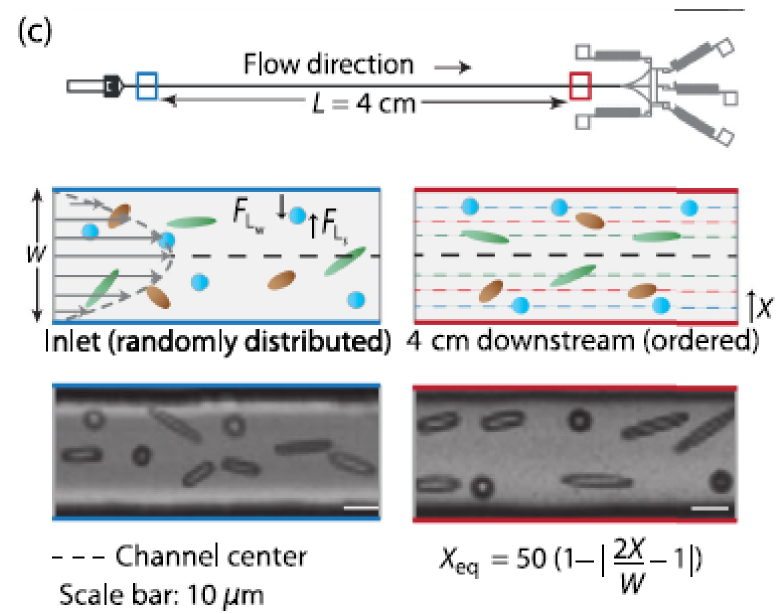


Fibres in pulp used for paper manufactuing

Ice crystals in clouds

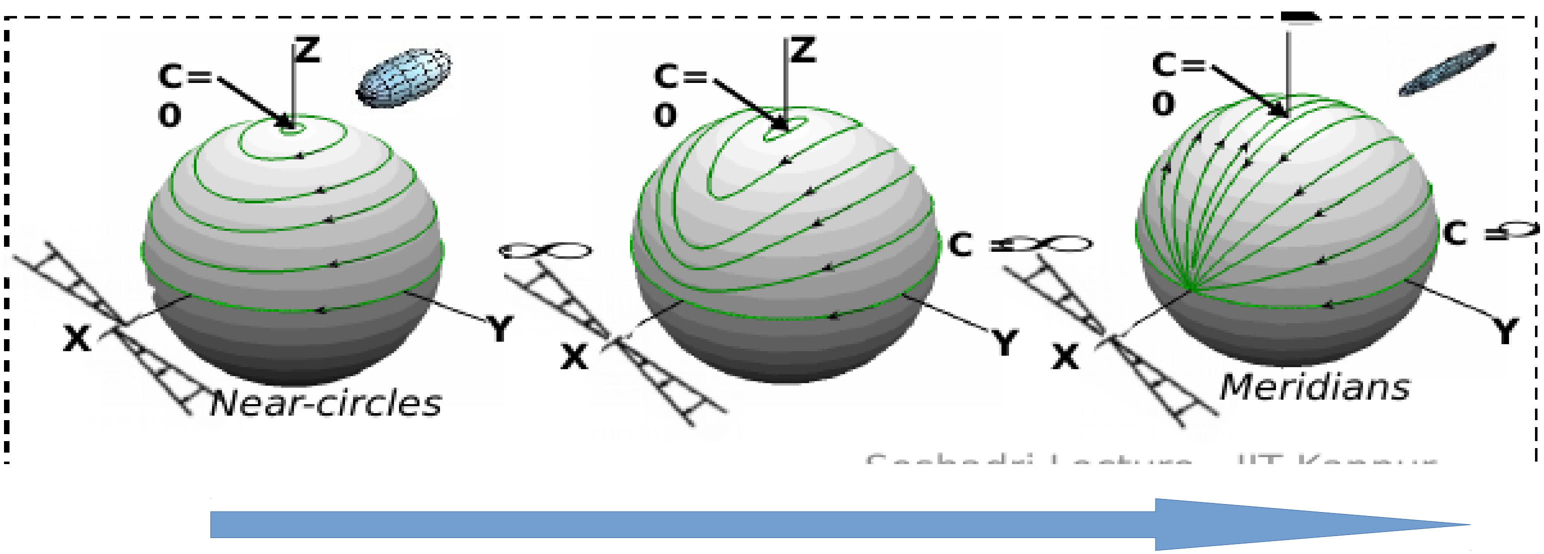
Applications

- The rheological properties of a suspension of non-spherical particles depends on the long-time orientation distribution of those particles.
- In microfluidic applications, shape-based separation of particles in the flow depends on the orientation distribution of particles.



Jeffery orbits

- In Stokes flow, spheroid rotates along one of a one-parameter family of Jeffery orbits, determined by its initial orientation.
- Drift across Jeffery orbits is caused by finite inertia or fluid elasticity.



Increasing aspect ratio of the oblate spheroid

Characterizing fluid elasticity

- Two parameters characterize fluid elasticity (at low *De*):

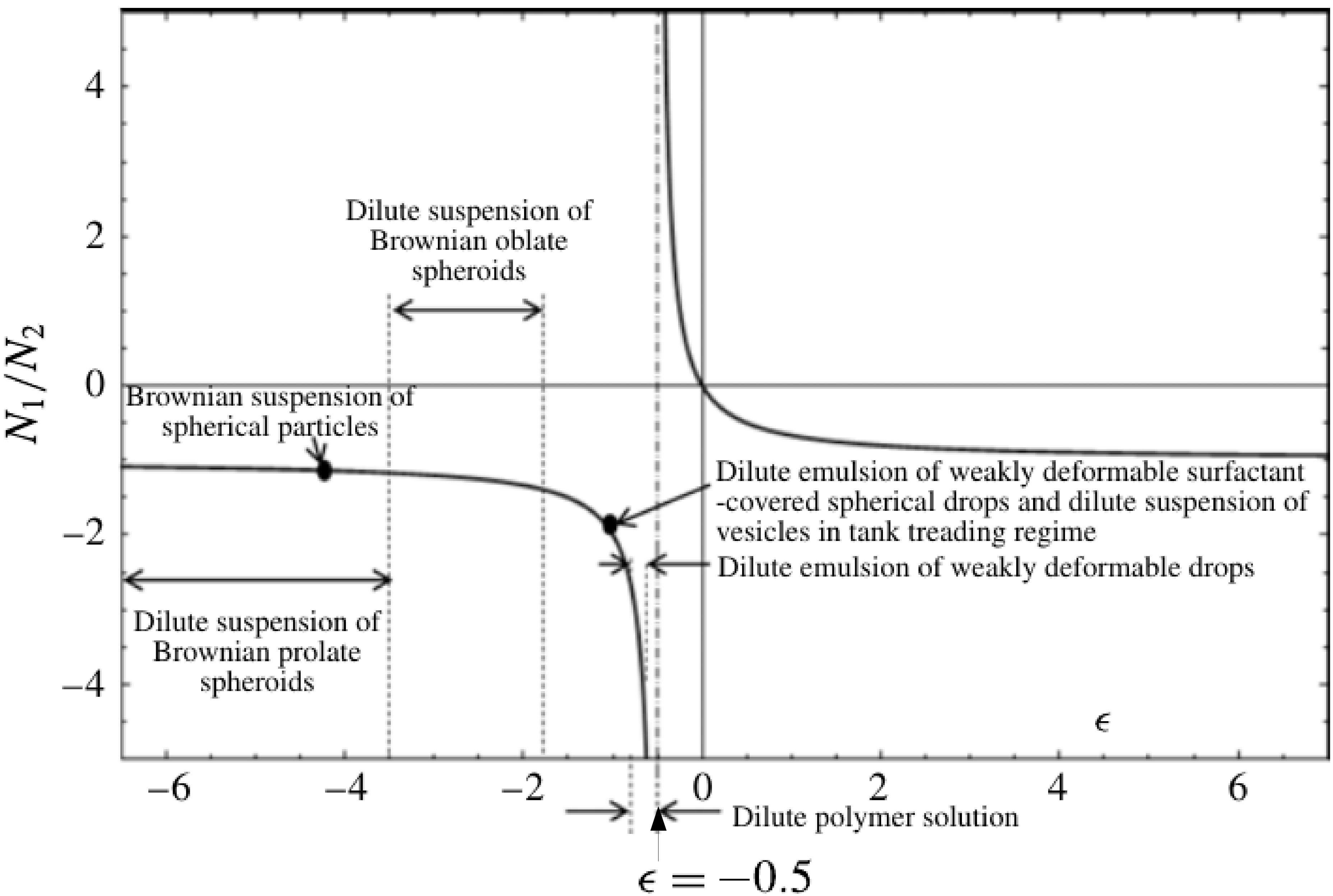
  1. Deborah number, *De*

  2. Ratio of normal stress-differences, N_2/N_1

- Normal stress-difference ratio is related to the second order fluid parameter, ε: $$\frac{N_2}{N_1} = -\left(1 + \frac{1}{2\epsilon}\right)$$

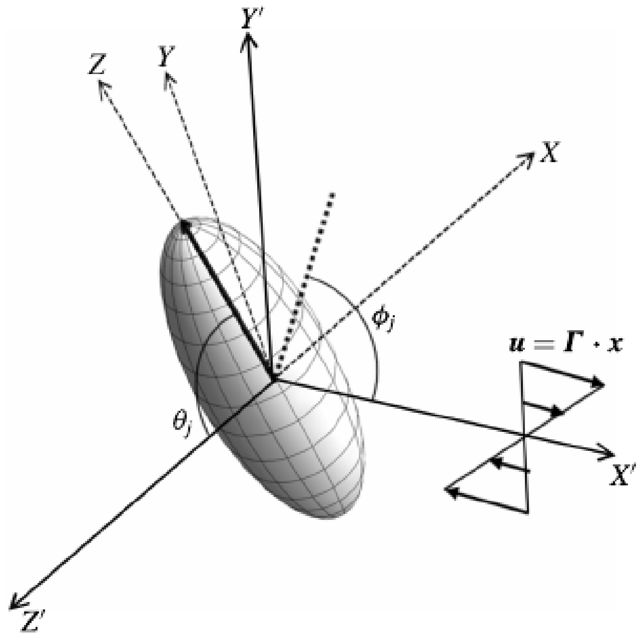
- Usually for polymer solutions, $-0.7 \leq \epsilon \leq -0.5$

$$\frac{N_2}{N_1} = -\left(1 + \frac{1}{2\epsilon}\right)$$



Our problem

- A neutrally buoyant spheroid in a viscoelastic simple shear flow at small Deborah number.
- Deborah number (*De*) is the ratio of fluid relaxation time to a flow time scale (inverse shear rate for simple shear flow).
- Find the long time orientation dynamics of the spheroid.
- Even though *De* is small, the orientation changes will be large.
- Aspect ratio is arbitrary and $-3 \leq \varepsilon \leq +3$ in our study



Methodology-Reciprocal theorem

- Torque, or angular velocity, is found using the Lorentz reciprocal theorem.
- Reciprocal theorem uses **known** Stokes velocity fields from two problems:

1. The *actual problem* of spheroid in shear flow
2. A *test problem* with the same flow domain (spheroid rotating about its transverse axis)

Methodology-Reciprocal theorem

- Angular velocity is given by an integral:

$$\text{Total angular velocity} = \underbrace{\mathbf{L}^{(2),\text{inv}}\Big[\cdot \boldsymbol{\Gamma} : \int_{S_p} dS \; \mathbf{x}(\boldsymbol{\Sigma}\cdot\mathbf{n}) \ldots}_{\text{Jeffery angular velocity}}$$

$$+De \int_{V(t)} dV \; \underbrace{\underbrace{\sigma^{(1)}_{NN}}_{\text{actual problem}} \quad : \quad \underbrace{\nabla \mathbf{U}^{(2)}\Big]}_{\text{test problem}}}_{\text{determined by Stokes velocity fields; completely known!}}$$

Prefactor of *De* implies that only Stokes velocity fields are needed inside the integral

Viscoelastic stress tensor

- Stress tensor for viscoelastic fluid to first order in *De*:

$$\boldsymbol{\sigma} = \underbrace{-p\mathbf{I} + 2\mu\mathbf{e}}_{\text{Newtonian stress}} + \underbrace{De\ \boldsymbol{\sigma}_{NN}}_{\text{viscoelastic correction}}$$

$$\boldsymbol{\sigma}_{NN} = \boldsymbol{\sigma}_{NNC} + \boldsymbol{\sigma}_{NNQ}$$

corotational

quadratic

$$\boldsymbol{\sigma}_{NNQ} = 4(1+\epsilon)\mathbf{e}\cdot\mathbf{e}$$

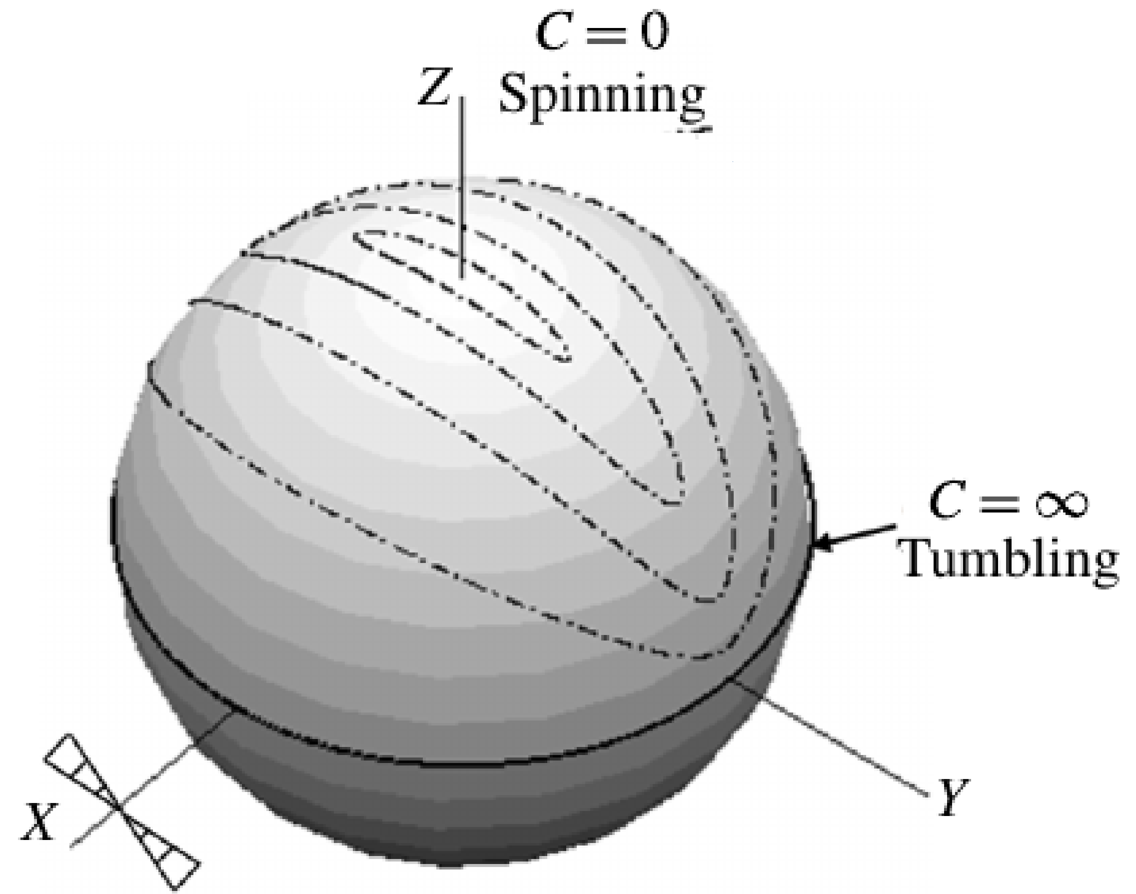
$$\boldsymbol{\sigma}_{NNC} = 2\epsilon\left(\frac{D\mathbf{e}}{Dt} + (\mathbf{w}\cdot\mathbf{e})^{\text{antisym}}\right)$$

Vorticity tensor

Strain tensor

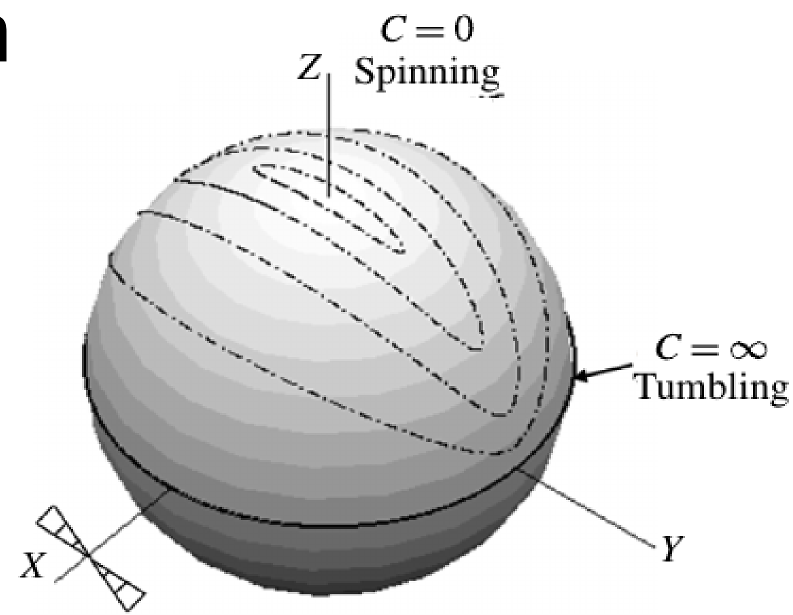
(C,τ) coordinates

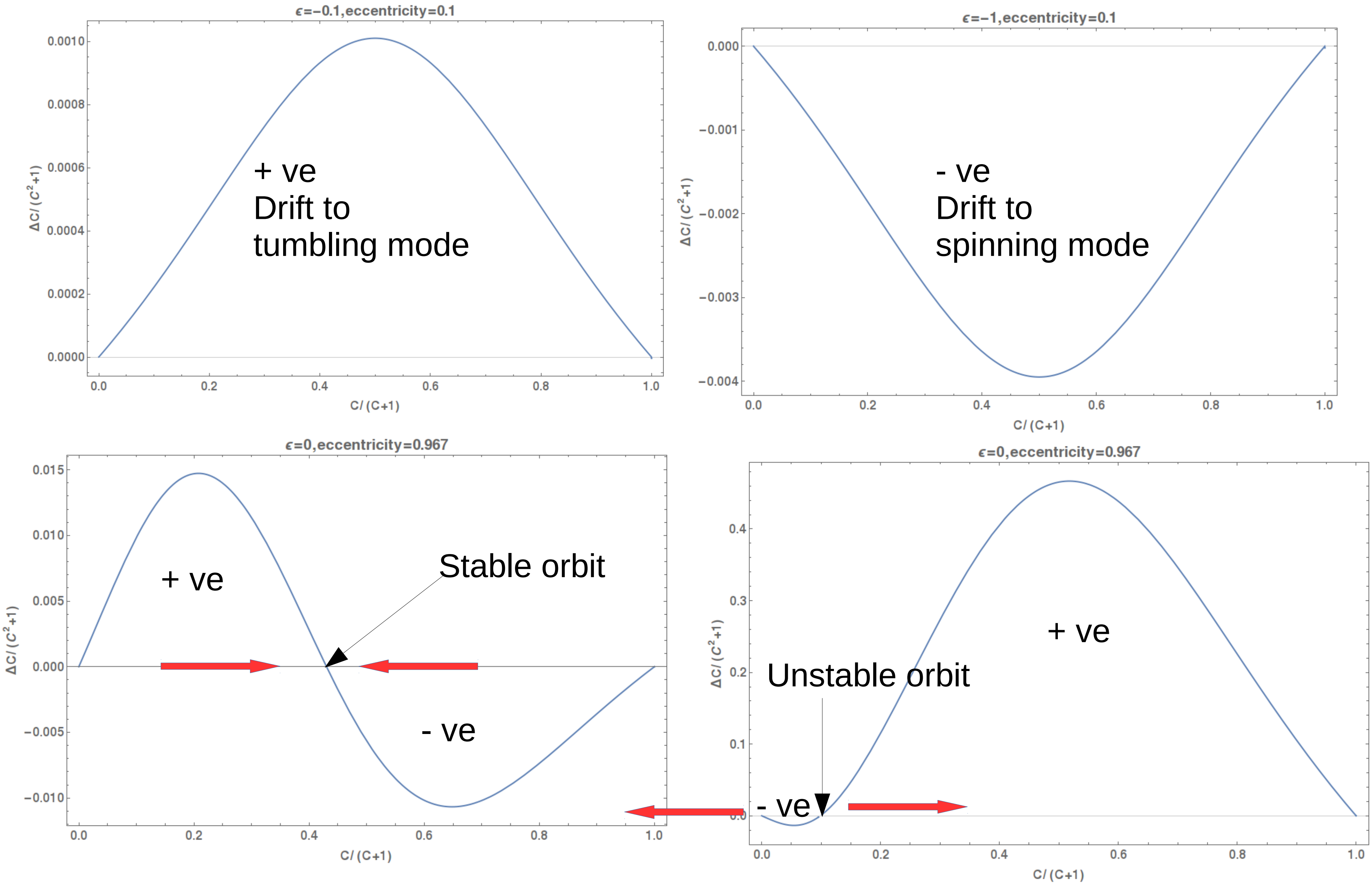
- C is the Jeffery orbit constant.
- τ is the phase along Jeffery orbit.
- In Stokes flow a spheroid rotates along a fixed Jeffery orbit.



Measuring drift across Jeffery orbits

- We compute: ΔC = Change in C over one period of revolution in the Jeffery orbit.
- ΔC = 0 in Stokes flow.
- ΔC > 0 implies drift towards tumbling mode.
- ΔC < 0 implies drift towards spinning mode.



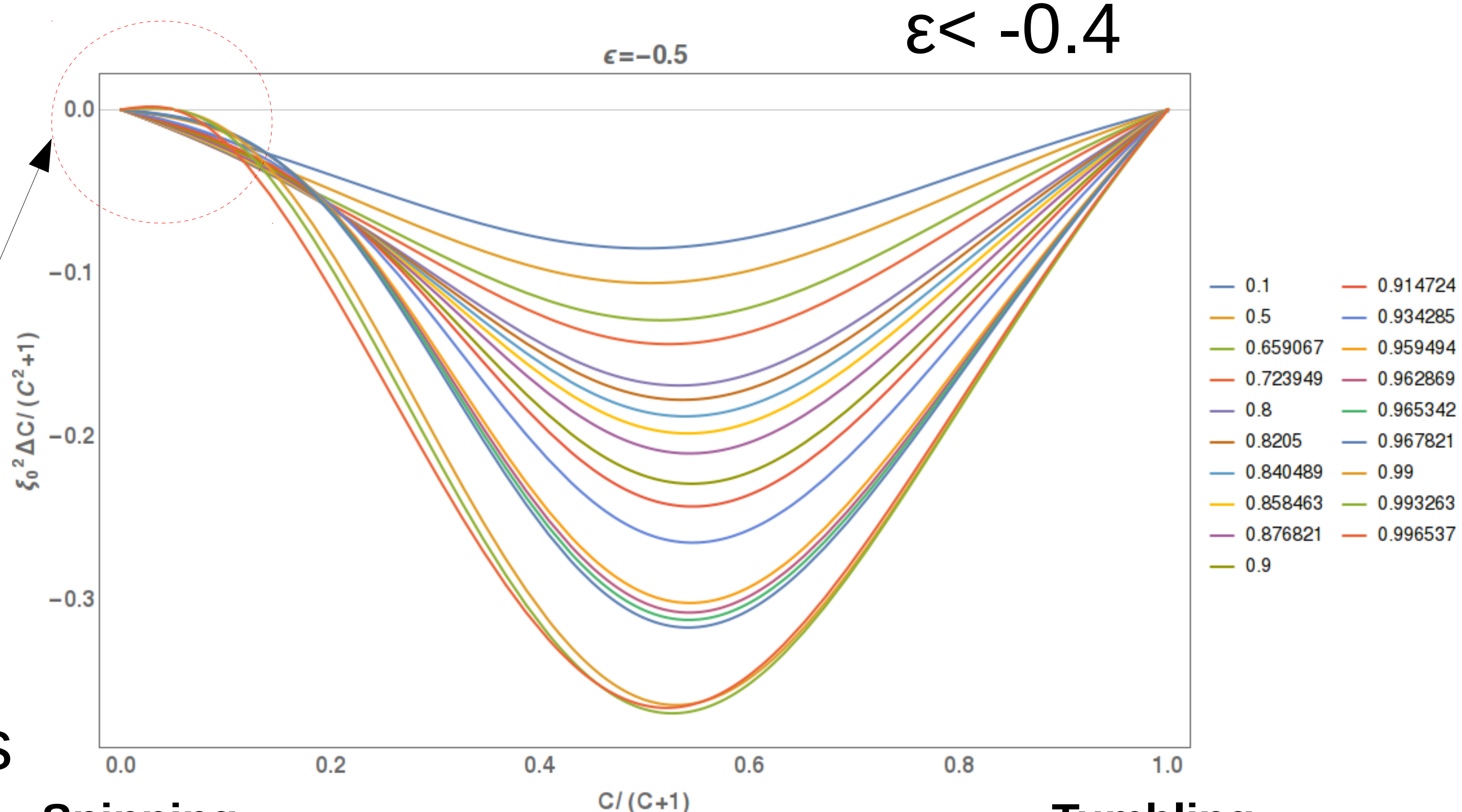


Zero crossing with POSITIVE slope implies UNSTABLE orbit

Zero crossing with NEGATIVE slope implies STABLE orbit

$\Delta C/(C^2+1)$ vs $C/(C+1)$

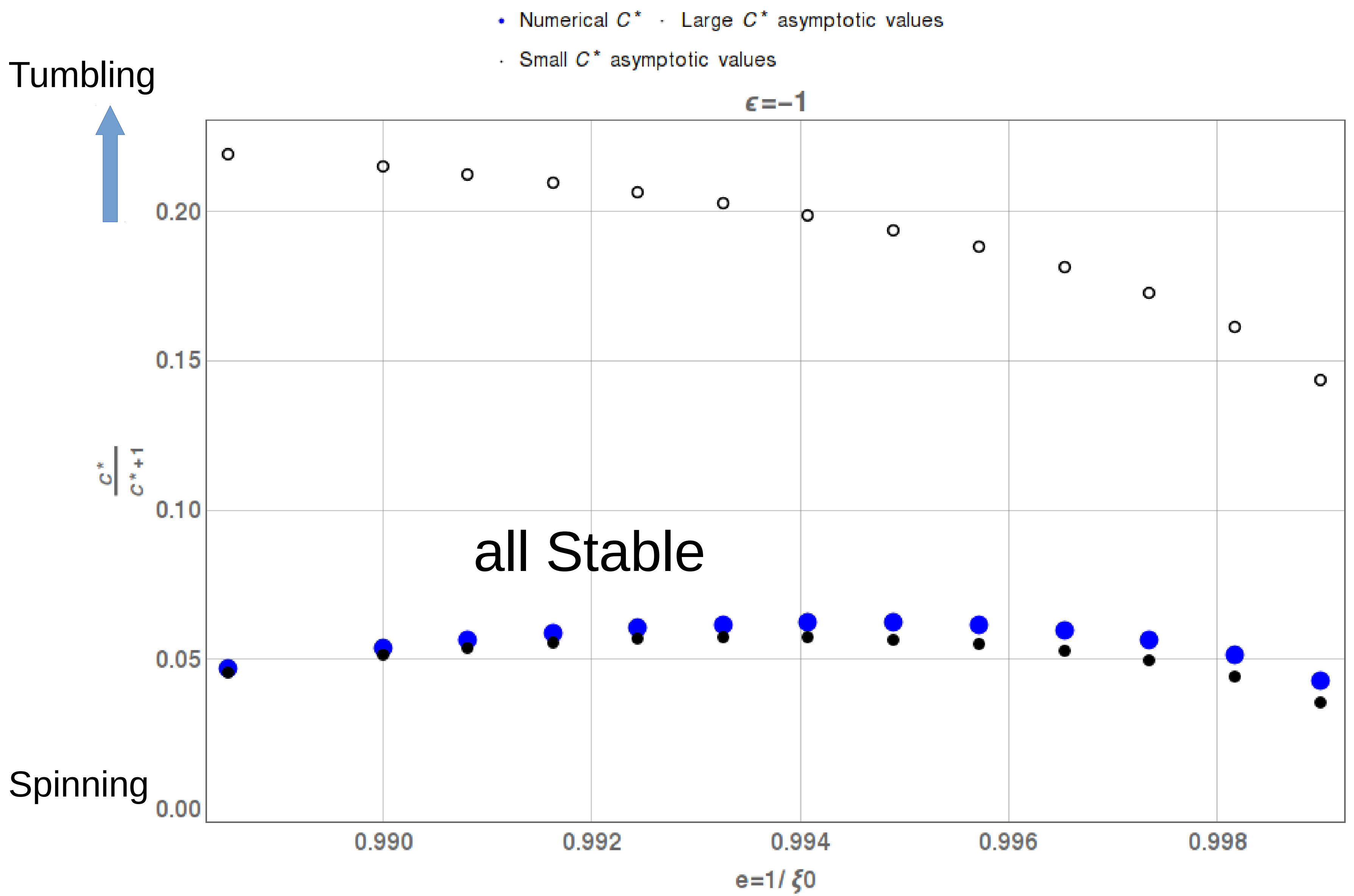
Typical curves for ε< -0.4



Zero crossings

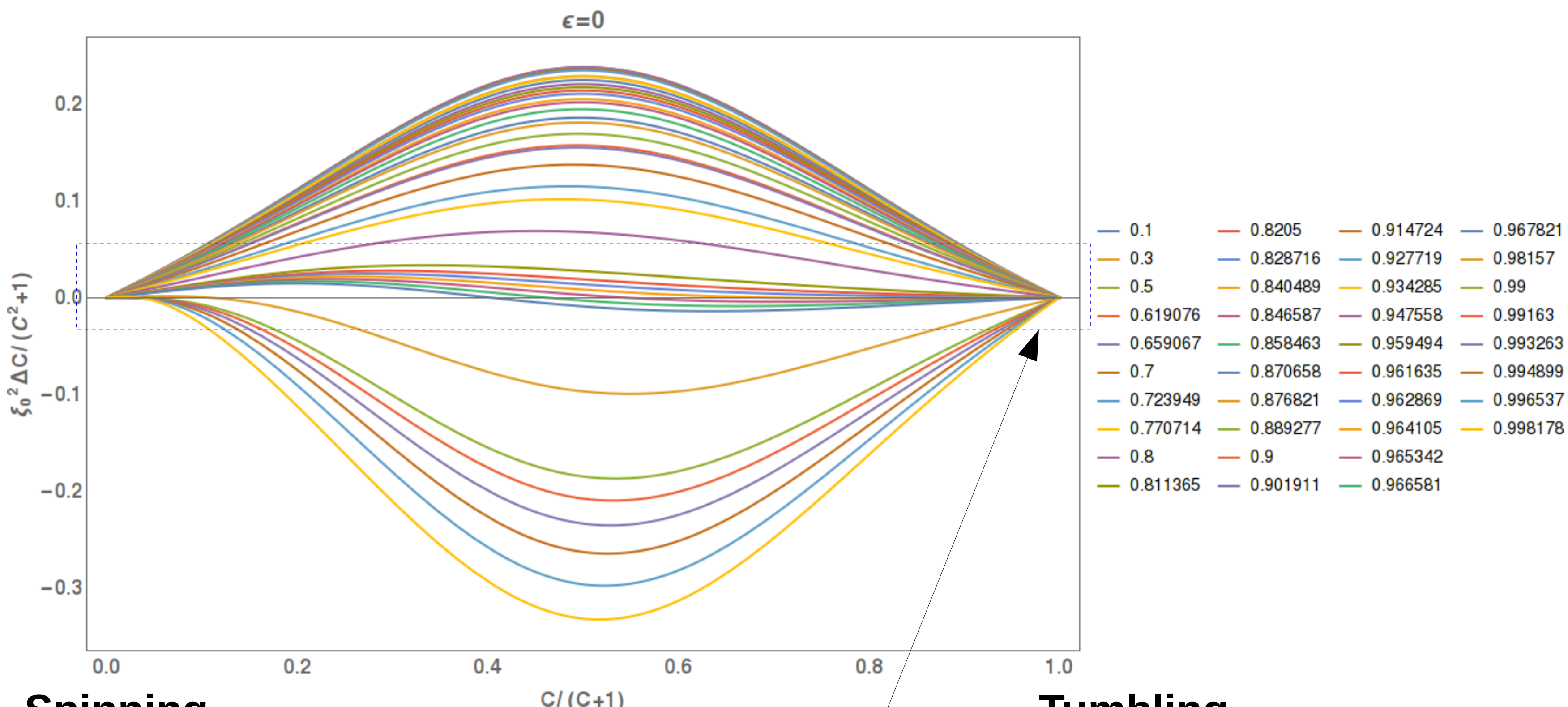
Spinning **Tumbling**

Zero crossing values C*/(C*+1)



$\Delta C/(C^2+1)$ vs $C/(C+1)$

Typical curves for
$-0.34 < \varepsilon < 0$

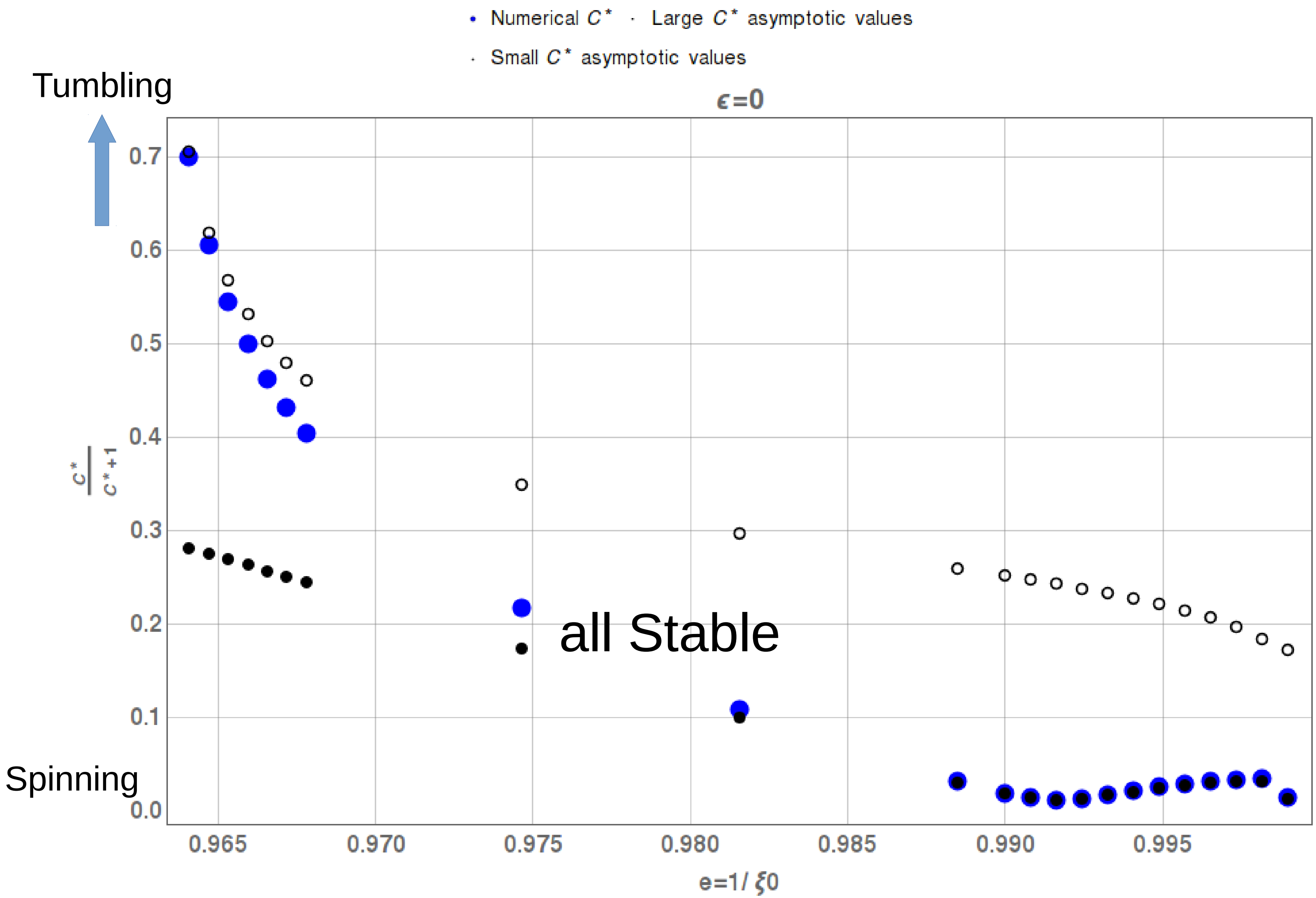


Spinning

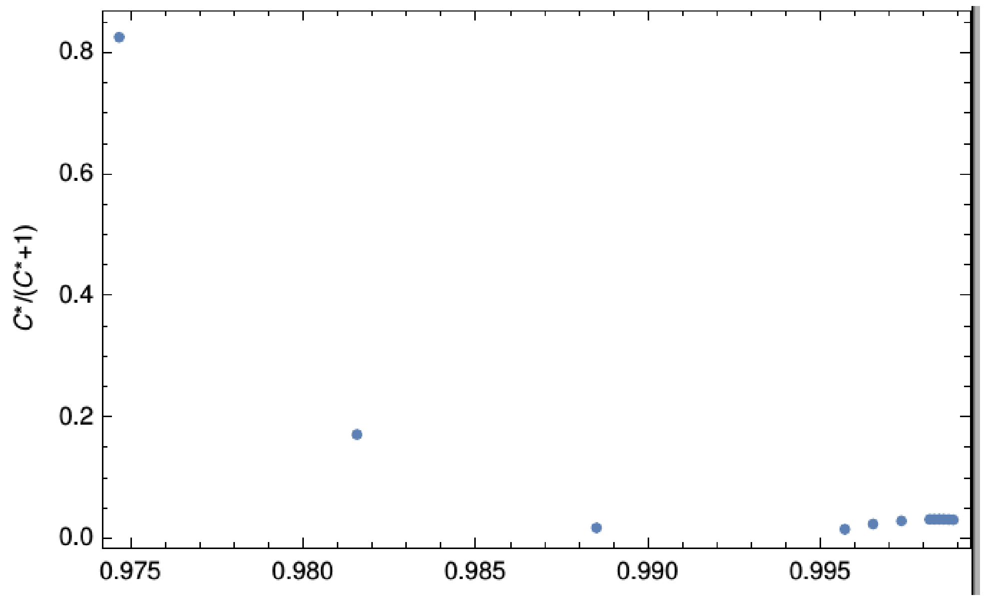
Tumbling

Zero crossings
move from large
C to small C as
eccentricity increases

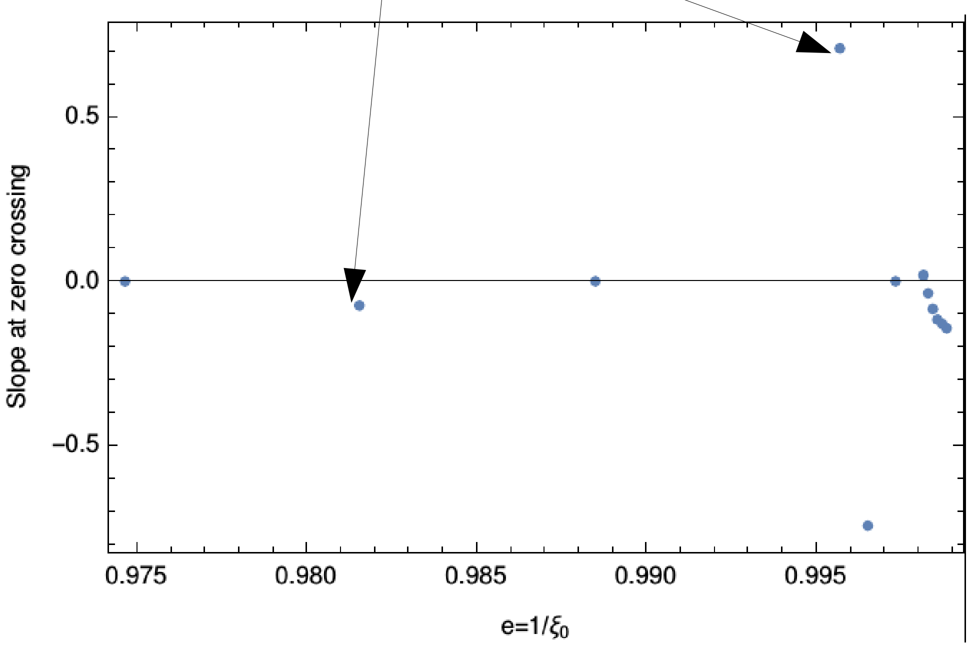
Zero crossing values C*/(C*+1)



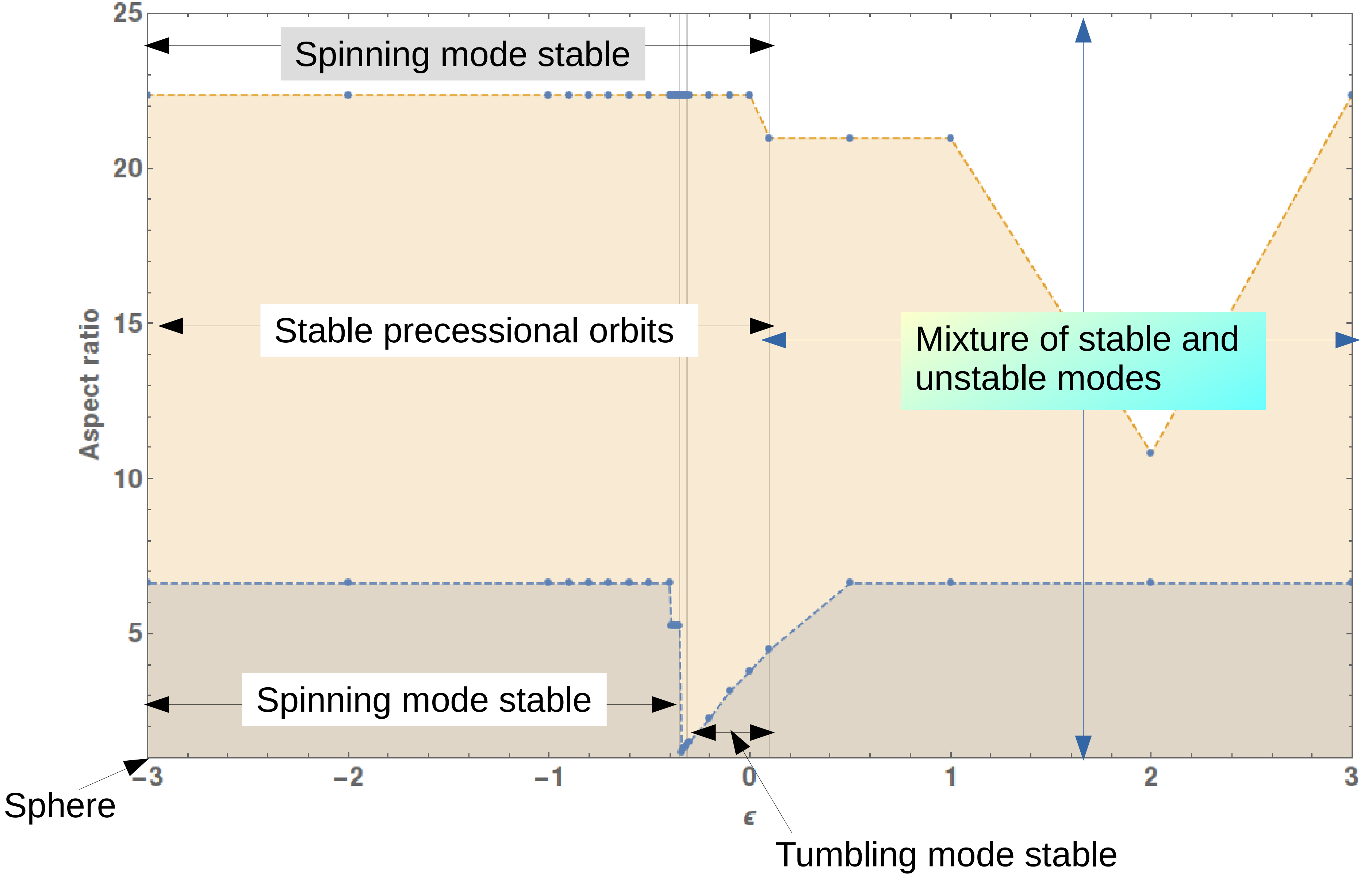
Behavior for ε>0



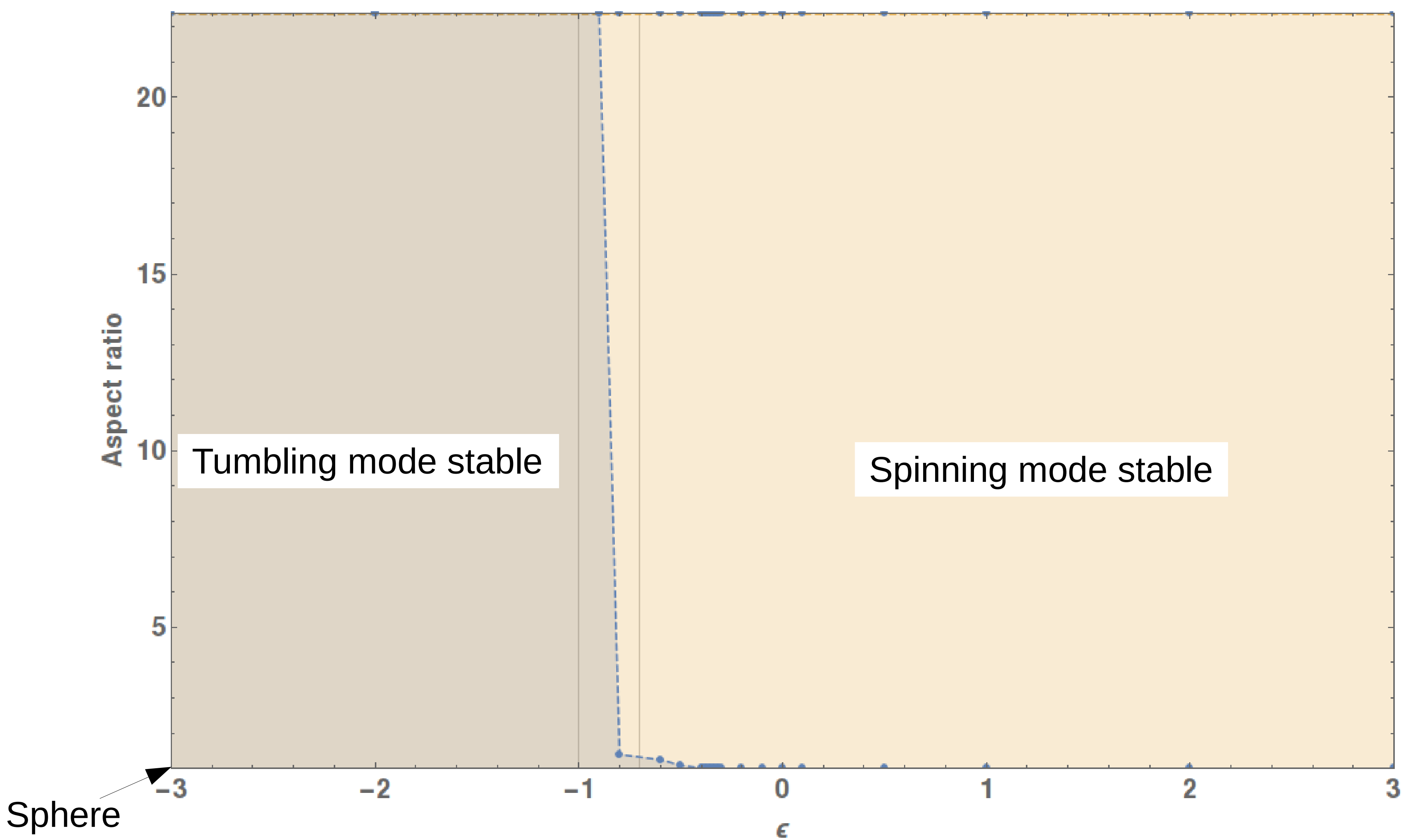
Mixture of stable and unstable orbits



Prolate spheroid – Phase diagram



Oblate spheroid – Phase diagram (ongoing)



Conclusions

- We have a very rich behavior of orientation dynamics depending on the fluid rheology and spheroid geometry, as captured by the phase diagram.
- Earlier experimental studies of prolate spheroidal particles in parallel plate flow cell in a Boger fluid with $\varepsilon \approx -0.5$, $De = 10^{-2}$-1 (Gunes etal 2008;Johnson etal 1990) are in qualitative agreement with our results.

Gunes etal (2008) ***J. Non-Newtonian Fluid Mech.*** **Vol 155, pp. 39-50**
Johnson etal (1990) ***J. Non-Newtonian Fluid Mech.*** **Vol 34, pp. 89-121**

Future work

- Orientation dynamics of oblate spheroid in viscoelastic shear flow.
- Spheroids in non-linear flows, with applications to shape-sorting of particles in microfluidics.
- Combined effect of gravity and shear on the orientation dynamics of spheroids.

Thank You