Viscous Deformable Drop in Planar Linear Flow: Evidence of non-trivial Streamline Topology

Sabarish V N
EMU, ***JNCASR***

Ganesh Subramanian
EMU, ***JNCASR***

Motivation:

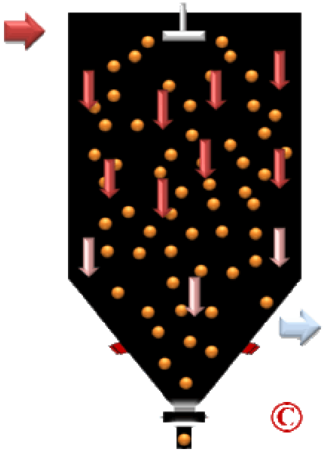
- The study of dynamics of drop immersed in an ambient fluid has a wide range of applications for the engineering community. Although the primary focus has been on predicting the rheology of suspensions of drops, another important aspect relevant to practical applications is the study of convective transport of heat or mass from the drop to the ambient fluid.

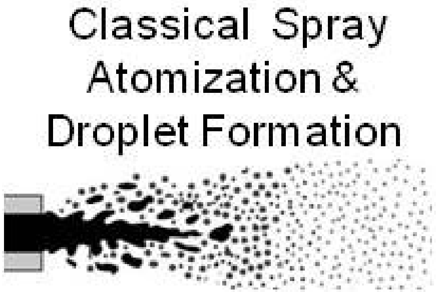
Applications

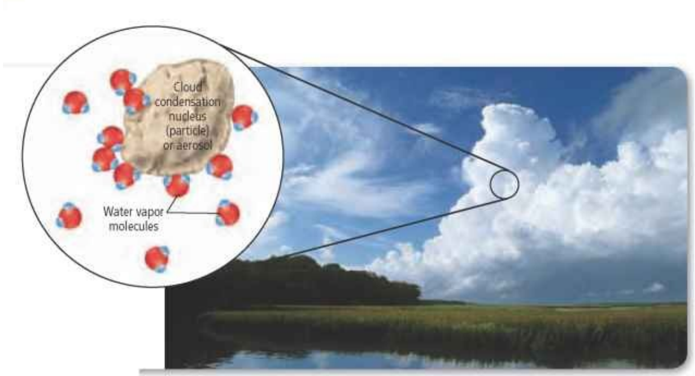
Chemical Engineering

Mechanical Engineering

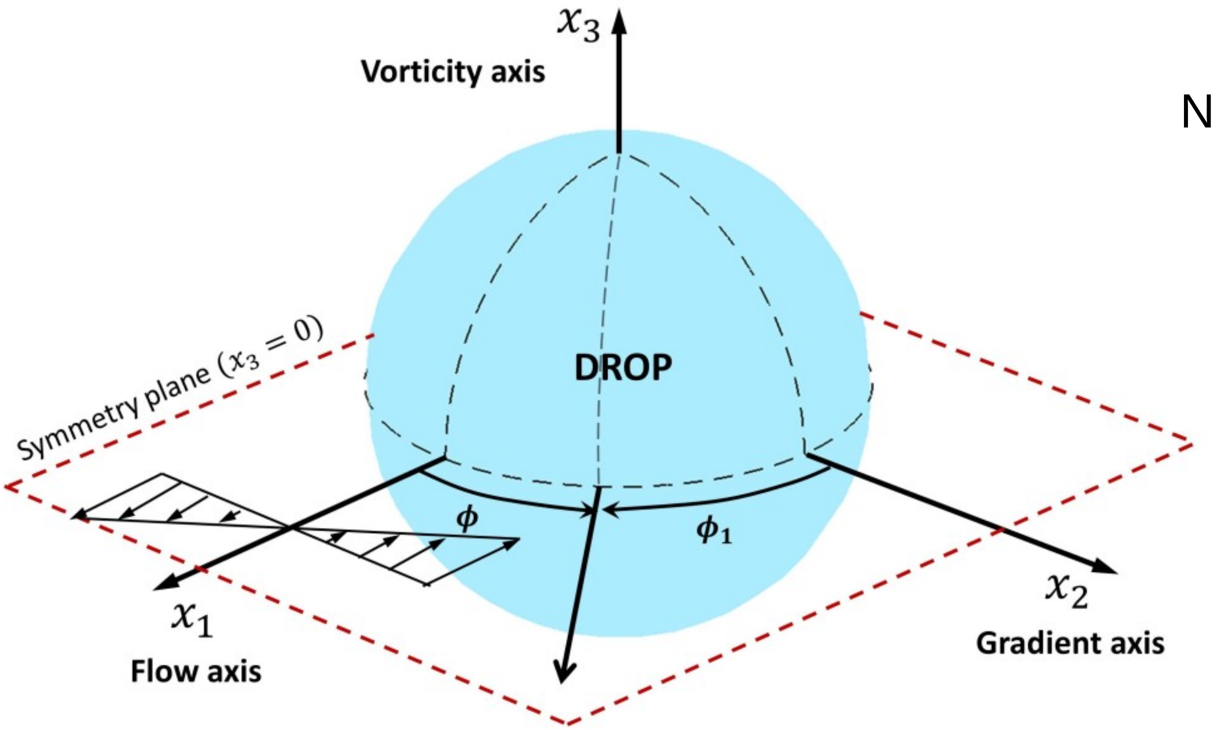
Atmospheric Sciences







Introduction:



Neutrally buoyant drop in a linear ambient flow.

Planar linear flow parameter
(-1 ≤ α ≤ 1)

Dimensionless parameters of interest : ***Ca, Re, λ, α***

Viscosity ratio

Stokes' Flow

$\boldsymbol{u} = f(\lambda, \alpha)$

$$-\boldsymbol{\nabla} p + \mu \nabla^2 \boldsymbol{u} = 0,$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0,$$

$$\left.\begin{aligned} -\boldsymbol{\nabla}\hat{p} + \lambda \nabla^2 \hat{\boldsymbol{u}} &= 0 \\ \boldsymbol{\nabla} \cdot \hat{u} &= 0 \end{aligned}\right\} \quad \text{interior problem,} \quad \left.\begin{aligned} -\boldsymbol{\nabla} p + \nabla^2 \boldsymbol{u} &= 0 \\ \boldsymbol{\nabla} \cdot \boldsymbol{u} &= 0 \end{aligned}\right\} \quad \text{exterior}$$

The Boundary Conditions are,

$$\boldsymbol{u}' \to 0 \quad \text{as} \quad r \to \infty \quad \text{(disturbance velocity decays off at infinity)}$$

$$\hat{\boldsymbol{u}} = \boldsymbol{u}' + \boldsymbol{\Gamma}.\boldsymbol{x} \quad \text{at} \quad r = 1 \quad \text{(continuity of velocity at interface)}$$

$$\boldsymbol{u} \cdot \hat{n} = \hat{\boldsymbol{u}} \cdot \hat{\boldsymbol{n}} = 0 \quad r = 1 \quad \text{(stationary spherical interface),}$$

$$(\sigma \cdot \hat{\boldsymbol{n}}) \cdot (\mathbf{I} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}) = (\hat{\sigma} \cdot \hat{\boldsymbol{n}}) \cdot (\mathbf{I} - \hat{\boldsymbol{n}}\hat{\boldsymbol{n}}) \quad \text{at} \quad r = 1 \quad \text{(continuity of tangential stress at the interface)}$$

Stokes' Exterior field:

$$\mathbf{u} = \mathbf{E}\cdot\mathbf{x} + \frac{1}{2}(\boldsymbol{\omega}\wedge\mathbf{x}) + \mathbf{x}(\mathbf{x}\cdot\mathbf{E}\cdot\mathbf{x})\left(\frac{c_1}{2r^5} + \frac{c_3}{r^7}\right) - \frac{2c_3}{5r^5}(\mathbf{E}\cdot\mathbf{x}) + c_4\left(\frac{\boldsymbol{\omega}\wedge\mathbf{x}}{r^3}\right).$$

Stokes' Interior field:

$$\hat{\mathbf{u}} = d_3(\mathbf{E}\cdot\mathbf{x}) + d_4(\boldsymbol{\omega}\wedge\mathbf{x}) - \frac{2}{21}d_2\mathbf{x}(\mathbf{x}\cdot\mathbf{E}\cdot\mathbf{x}) + \frac{5}{21}d_2 r^2\mathbf{E}\cdot\mathbf{x}.$$

Where,

$$c_4 = 0,$$

$$d_4 = \frac{1}{2},$$

$$c_1 = -\frac{5(\lambda-1)}{2\lambda+3}(2) - \frac{8}{2\lambda+3}b_1,$$

$$c_3 = 5\left(\frac{\lambda-1}{2\lambda+3}\right) + \frac{20(3\lambda+2)}{(2\lambda+3)(19\lambda+16)}b_1,$$

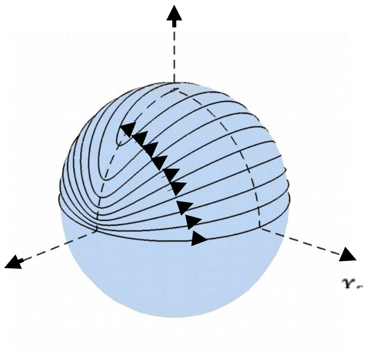
$$d_2 = \frac{84}{19\lambda+16}b_1,$$

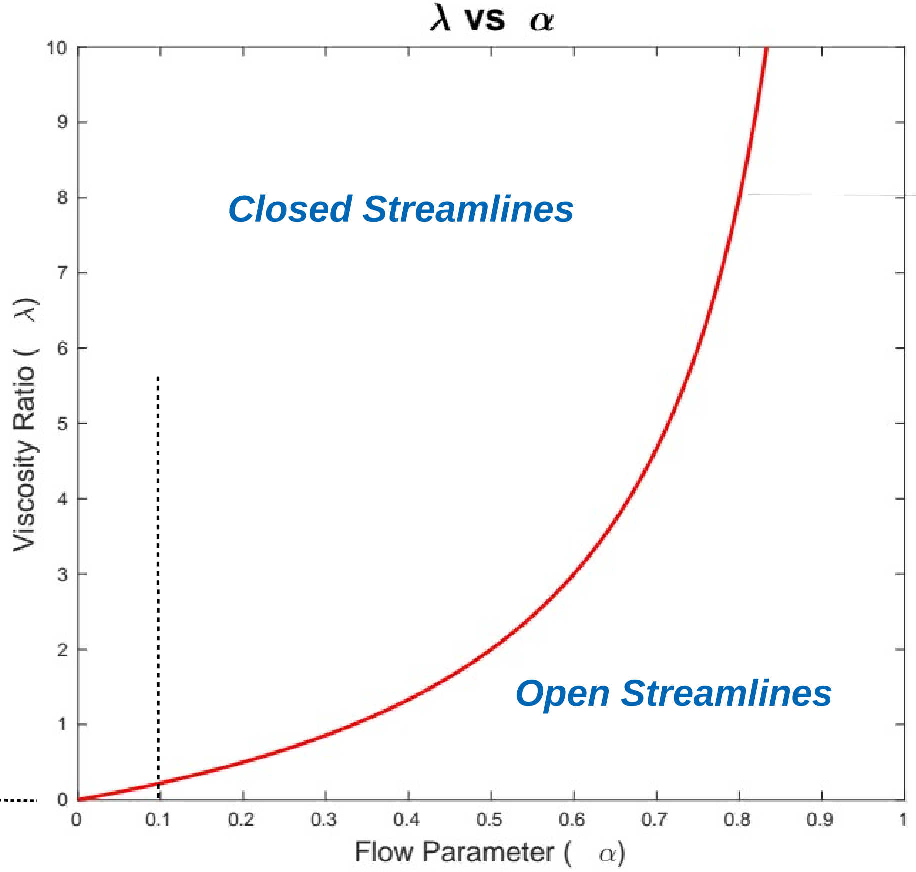
$$d_3 = \frac{5}{2\lambda+3} - \frac{4(16\lambda+19)}{(2\lambda+3)(19\lambda+16)}b_1.$$

$$b_1 = \frac{19\lambda+16}{8(\lambda+1)}$$

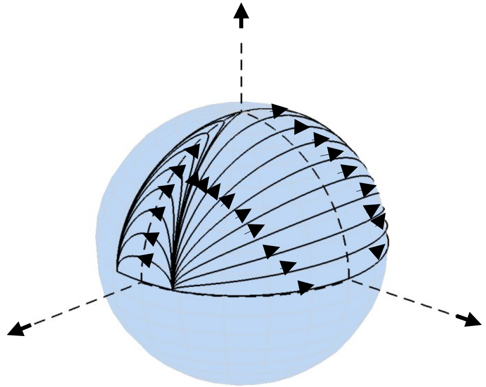
- *Advanced Transport Phenomena, Gary Leal,* ***Cambridge Univ. Press****, 2007.*

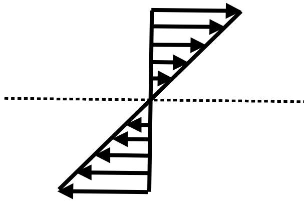
Streamline Topologies in Planar shearing flows

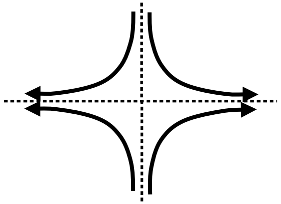


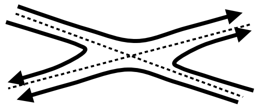


$$\lambda_c(\alpha) = 2\alpha/(1-\alpha).$$





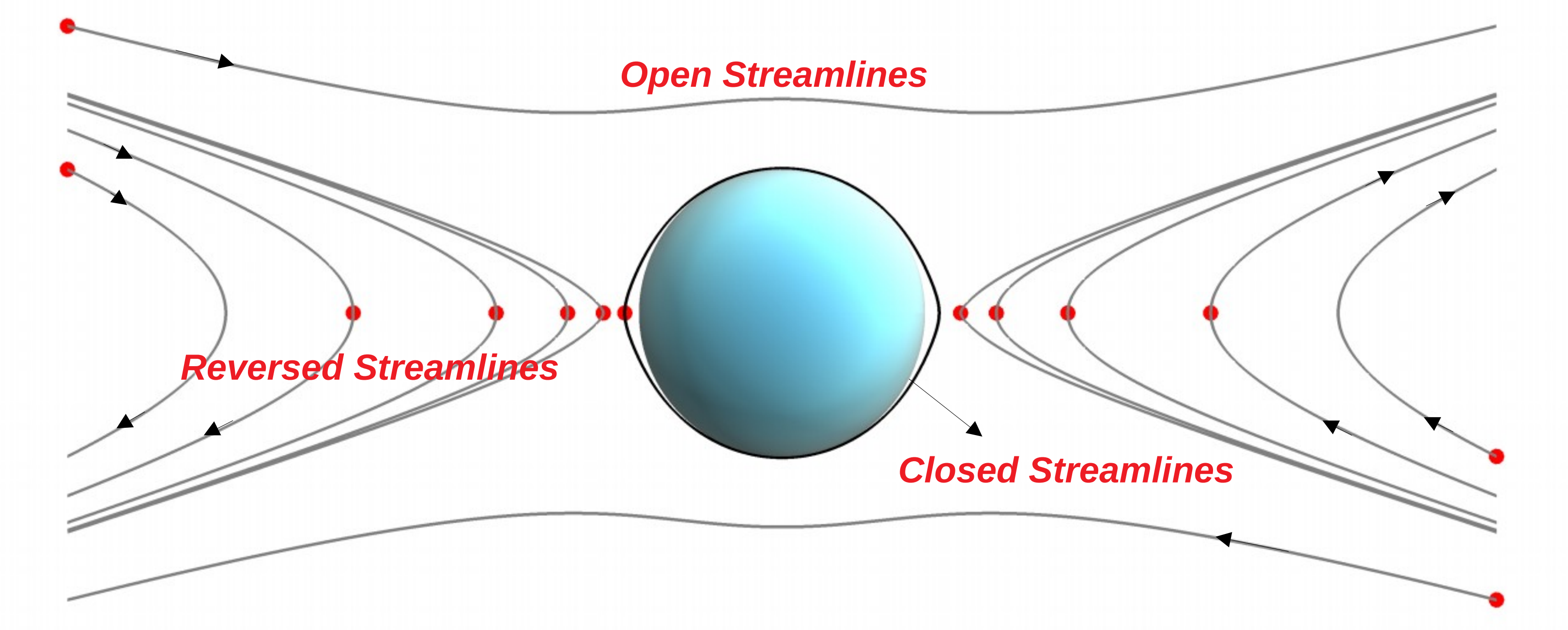


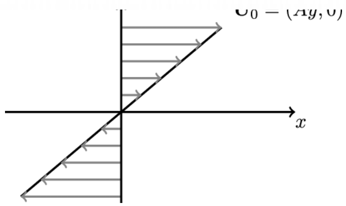


- *External and internal streamlines and deformation of drops in linear two-dimensional flows.*
  *Powell R L, **J. Colloid. Interface Sci. 95**, 148-162, 1983.*

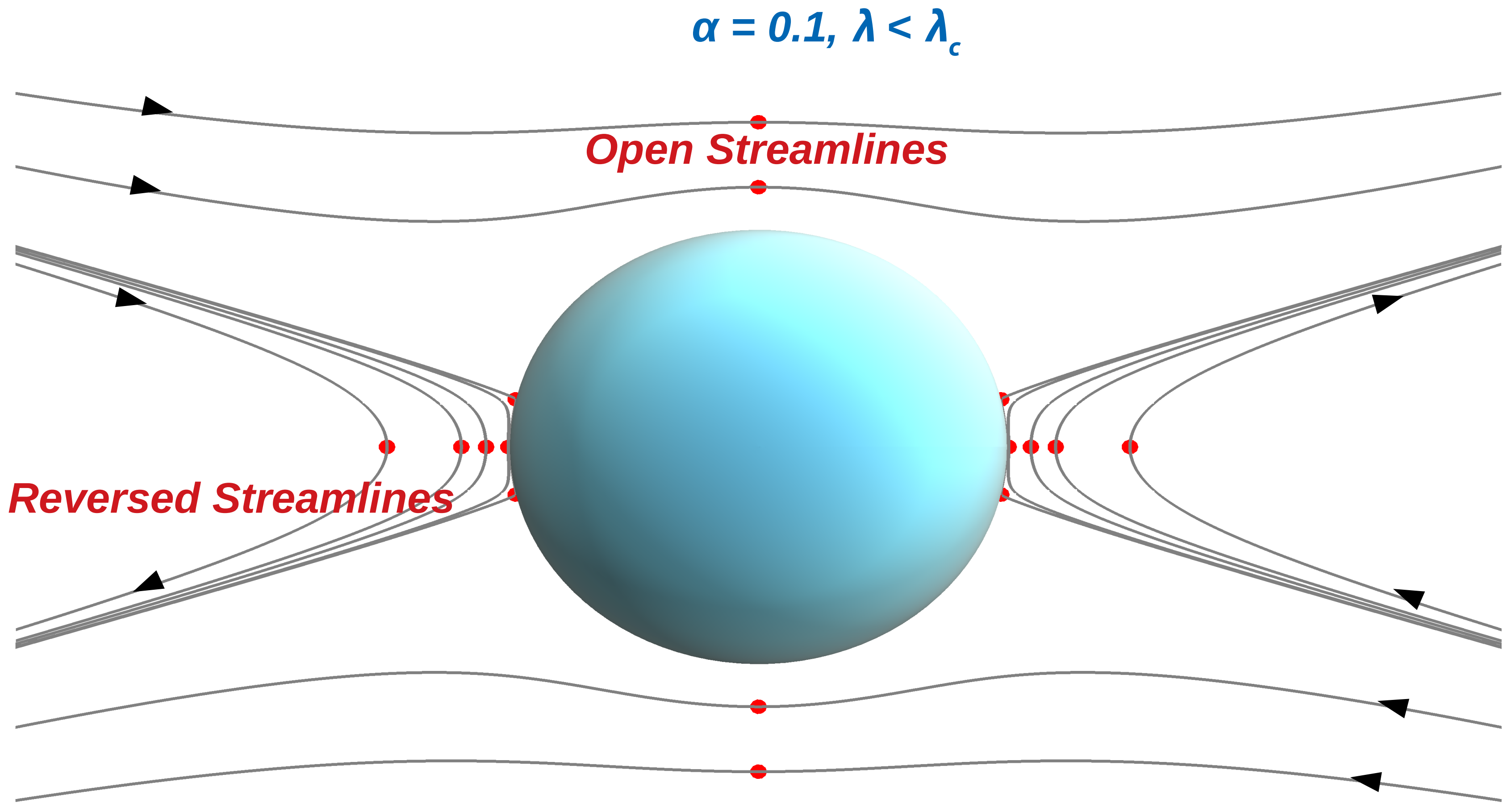
In Plane Streamline Topology – Stoke's Flow

$\alpha = 0.1, \lambda > \lambda_c$

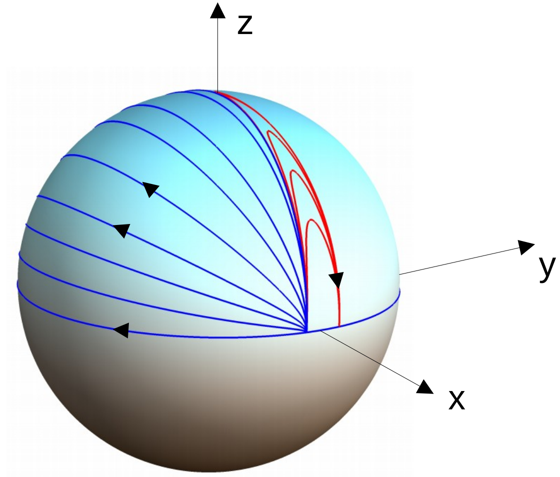




In Plane Streamline Topology – Stoke's Flow



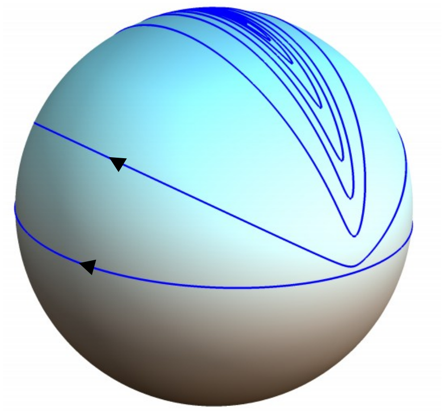
Surface Streamline Topology



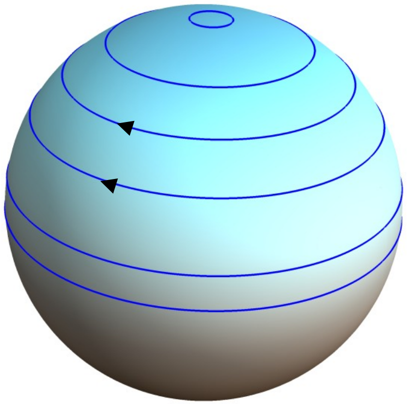
Open streamline case

$\lambda < \lambda_c$

Increasing λ



$\lambda > \lambda_c$



$\lambda \to \infty$ ***Solid particle***

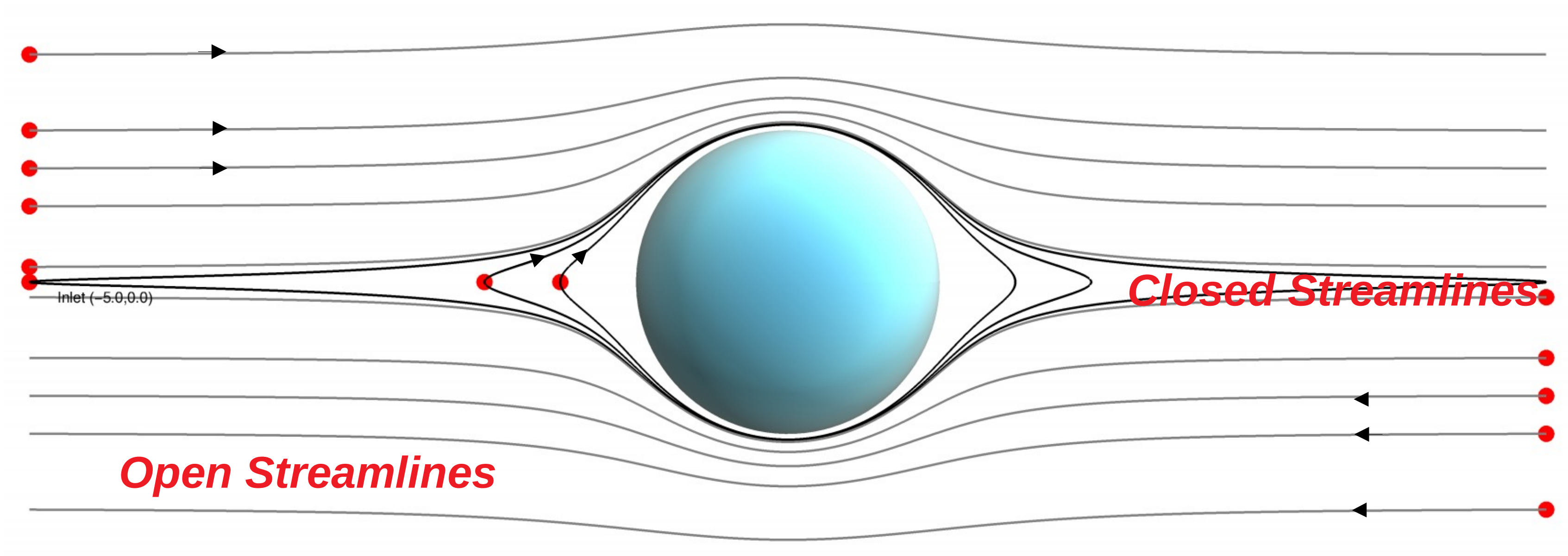
Surface Streamlines are ***JEFFERY ORBITS***

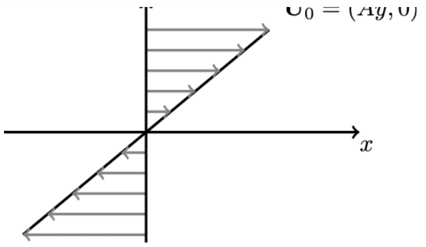
$$\dot{p} = \Omega \cdot p + \frac{\gamma^2 - 1}{\gamma^2 + 1}\left[E \cdot p - p\left(\mathrm{E:pp}\right)\right] \qquad \boldsymbol{\gamma = f(\lambda, \alpha)}$$

- *External and internal streamlines and deformation of drops in linear two-dimensional flows. Jeffery G B,* ***Proc. R. Soc. London A. 102****, 161-179, 1922.*

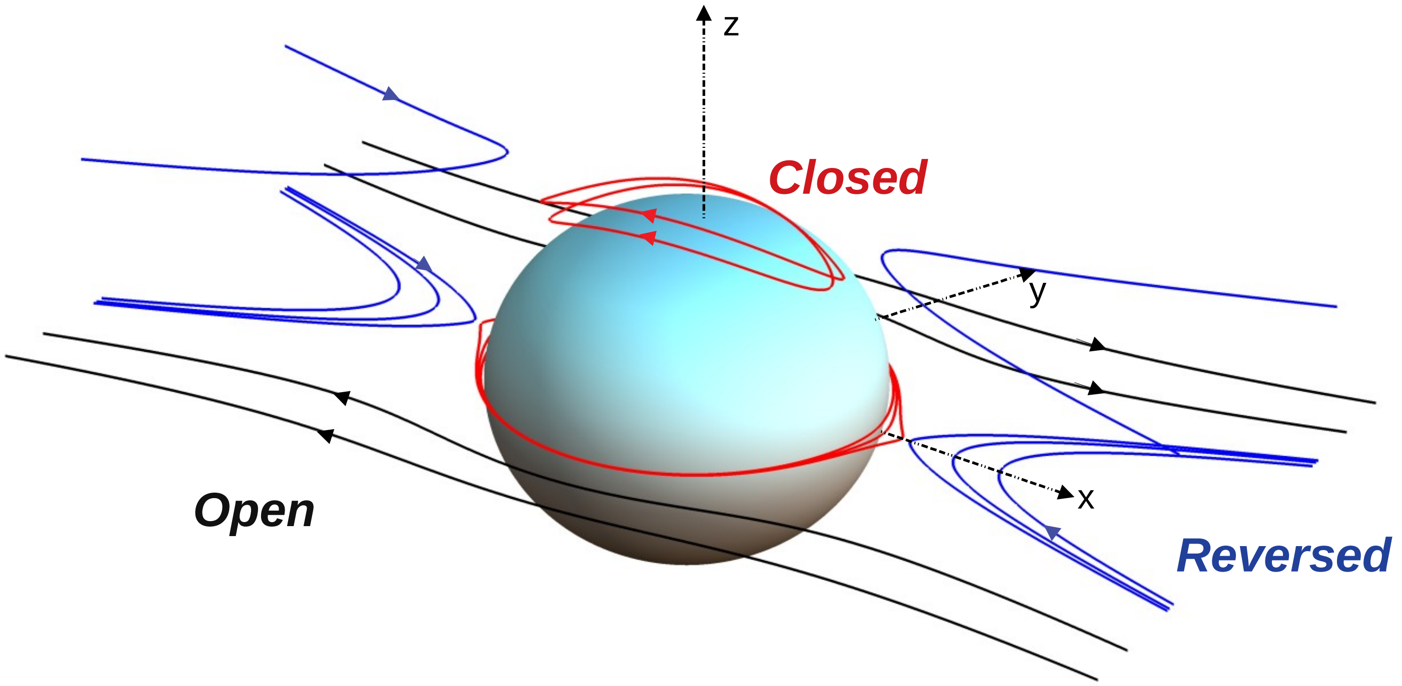
In Plane Streamline Topology – Stoke’s Flow

α = 0



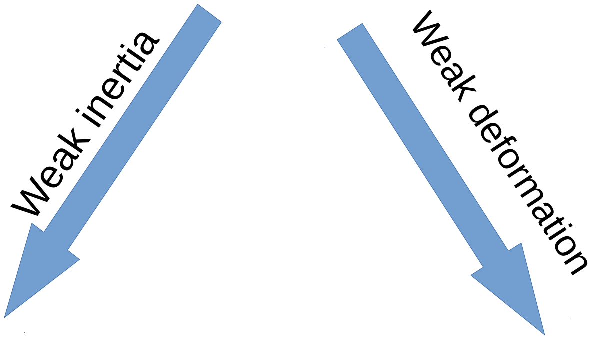


3D STREAMLINE TOPOLOGY FOR Re = 0 and Ca = 0



Method of Analysis:

Stokes' Velocity field



$$\boldsymbol{u} = \boldsymbol{u}^{(0)} + Re\,\boldsymbol{u}^{(1)} + O(Re^{3/2})$$

$$\mathbf{u} = \mathbf{u}^{(0)} + Ca\,\mathbf{u}^{(1)} + Ca^2\,\mathbf{u}^{(2)} + \cdots$$

$\mathbf{u} = \mathrm{f}(Ca,\lambda,\alpha)$

$\mathbf{u} = \mathrm{f}(Re,\lambda,\alpha)$

(In the Closed streamline Regime)

- *Heat or mass transport from drops in shearing flows. Part 1. The open streamline regime Krishnamurthy D and GS, **JFM 850**, 439-483, 2018.*
- *Heat of mass transport from drops in shearing flows. Part 2. Inertial effects on transport Krishnamurthy D and GS, **JFM 850**, 439-483, 2018.*

Reynolds field:

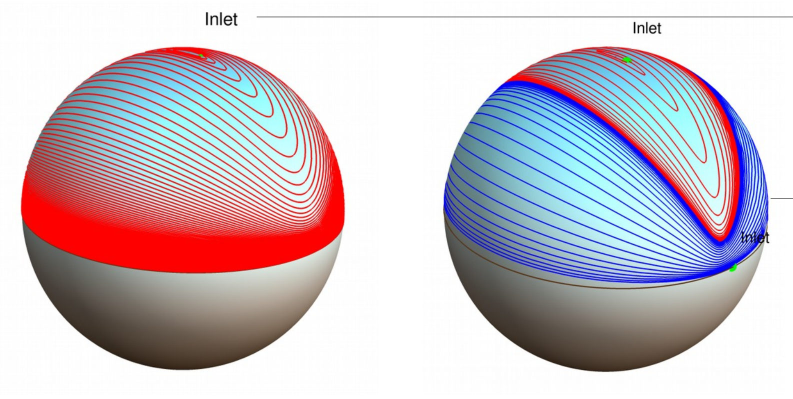
$$\mathbf{u}_i = u_i^{(0)} + \mathrm{Re}\, u_i^{(1)}$$

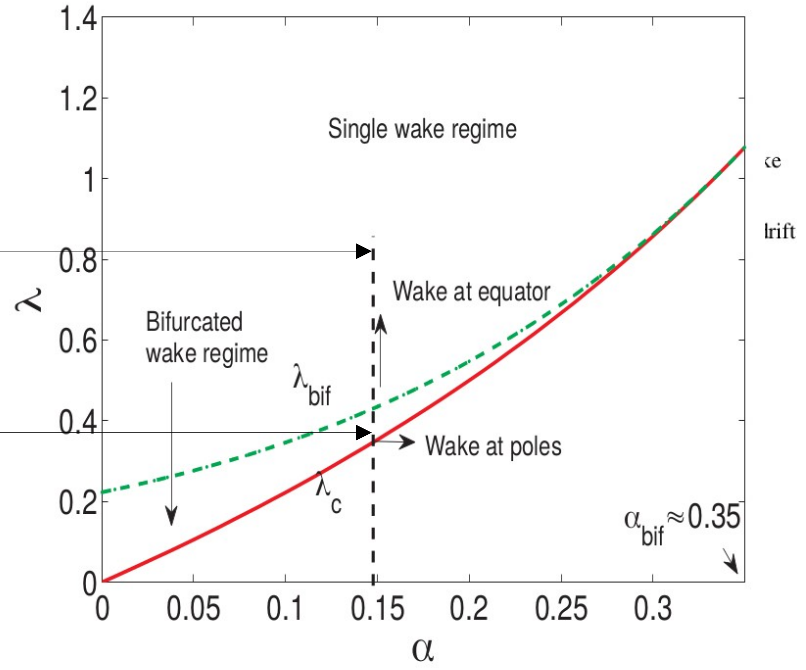
$$\mathbf{u}_i^{(1)} = f(\lambda,\alpha,\Gamma_{ij}),\ Ca = 0$$

Closed Streamlines

$\lambda > \lambda_{bif}$ → *Single Wake regime*

$\lambda < \lambda_{bif}$ → *Bifurcated Wake regime*





Capillary Velocity field:

$$\vec{u} = v1\,(\mathbf{E} : \hat{n}\hat{n})^2\hat{n} \;+\; v2\,(\mathbf{E}.\mathbf{E} : \hat{n}\hat{n})\hat{n} \;+\; v3\,(\mathbf{E} : \mathbf{E})\hat{n} \;+$$
$$v4\,(\mathbf{E} : \hat{n}\hat{n})\mathbf{E}.\hat{n} \;+\; v5\,\mathbf{E}.\mathbf{E}.\hat{n} \;+\; v6\,(\mathbf{A}_2 : \hat{n}\hat{n})\hat{n} \;+\; v7\,\mathbf{A}_2.\hat{n}$$

Where, $\mathbf{A}_2 = 2(\mathbf{E}.\Omega - \Omega.\mathbf{E}) \;+\; 4(\mathbf{E}.\mathbf{E})$ *(Valid only when **Ca << O(1 / λ)** (Weak Flow))*

The integral equation to be solved in BEM is given by,

$$u_j(\mathbf{x}_0) = \frac{2}{1+\lambda}U_j(\mathbf{x}_0) - \frac{1}{4\pi\mu_1(1+\lambda)}\int_D G_{ji}(\mathbf{x}_0,\mathbf{x})\,\Delta f_i(\mathbf{x})\,dS(\mathbf{x})$$

Single layer Potential

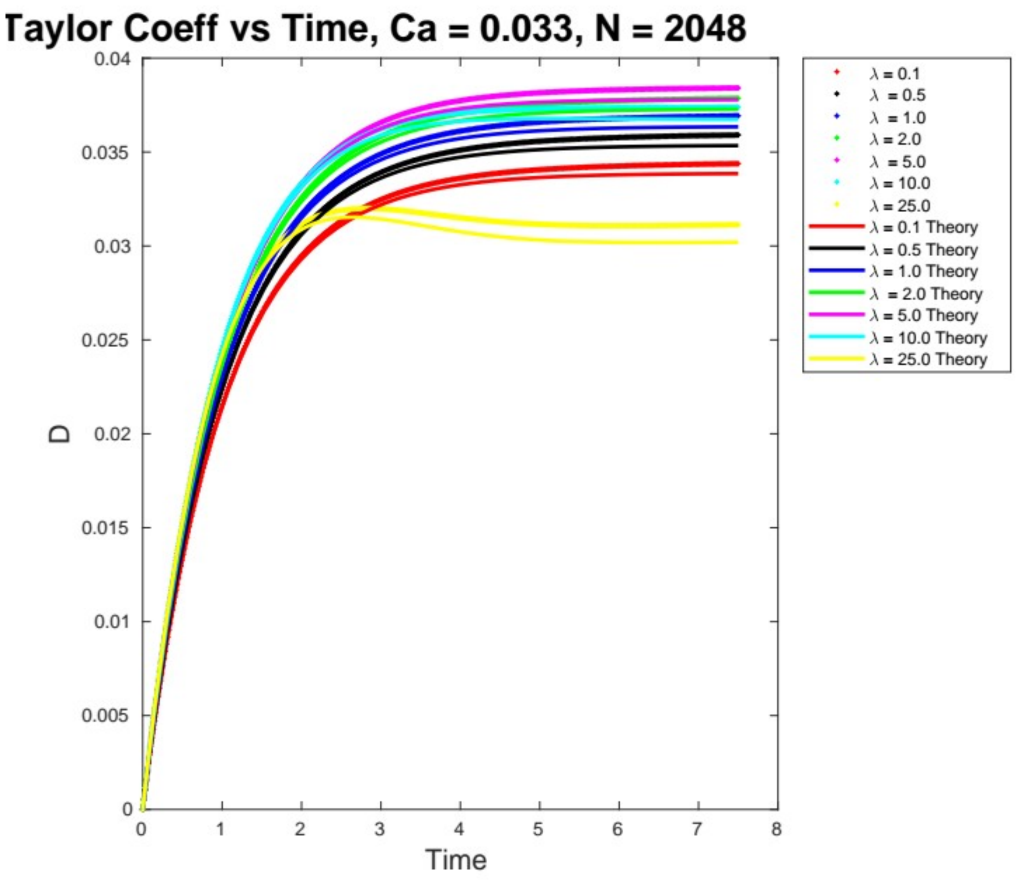
$$+\frac{1-\lambda}{4\pi(1+\lambda)}\int_D^{PV} u_i(\mathbf{x})\,T_{ijk}(\mathbf{x},\mathbf{x}_0)\,n_k(\mathbf{x})\,dS(\mathbf{x}),$$

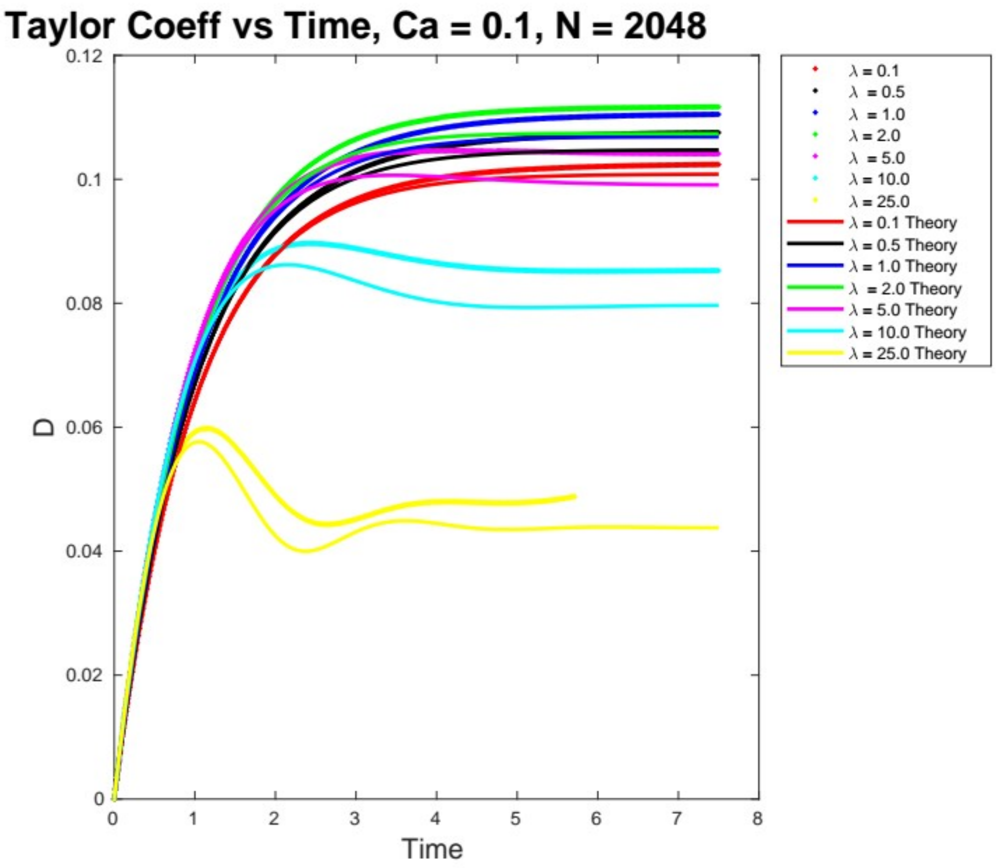
Double layer Potential

- *Second – order theory for the deformation of a Newtonian drop in a stationary flow field*
  *Greco F, **POF 14**, **946-954**, 2002.*
- *Motion and Deformation of Liquid Drops and the Rheology of Dilute Emulsions in Simple Shear Flow*
  *Kennedy. M. R., et.al., **Computers and Fluids. 23**, 251-278, 1994.*

Comparison of BEM Transient Results with Theory

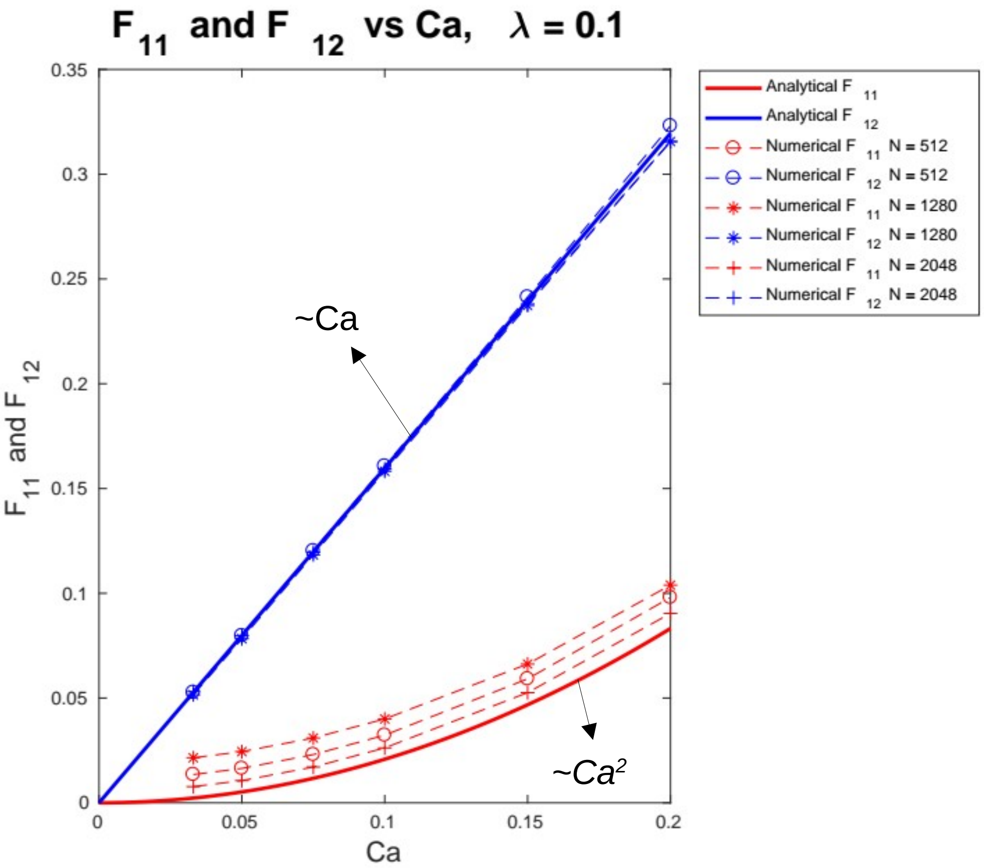
Point-Wise Measures

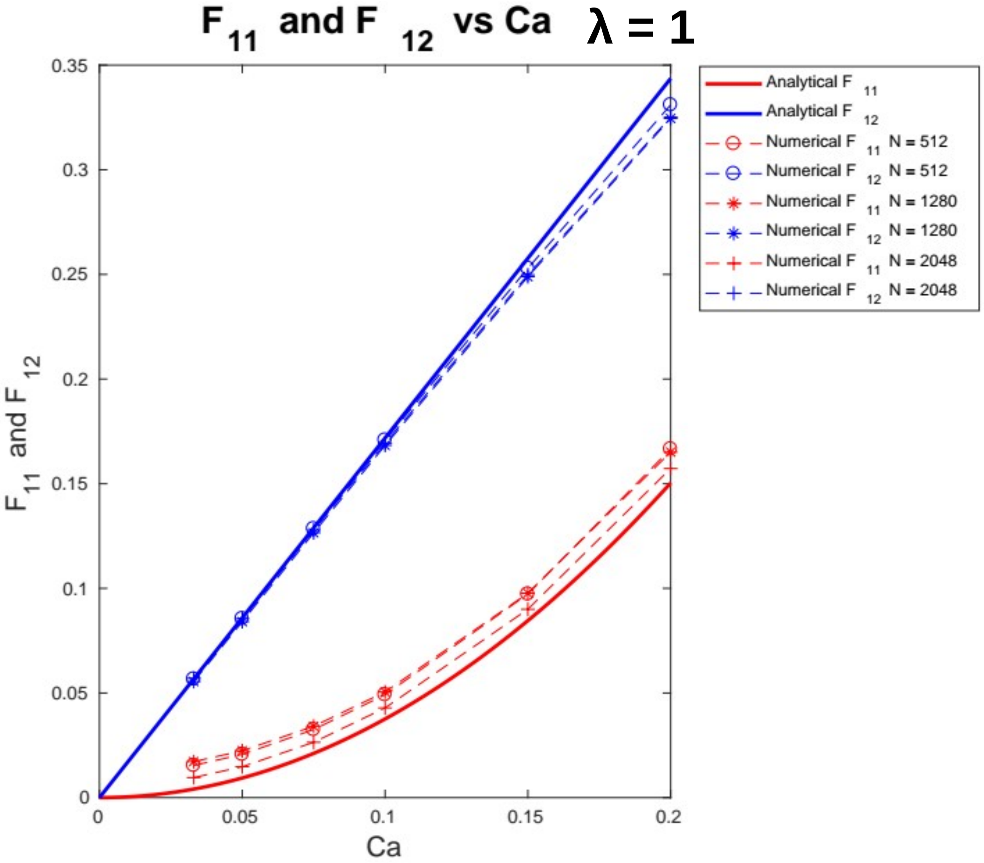




Comparison of BEM Steady State Results with Theory

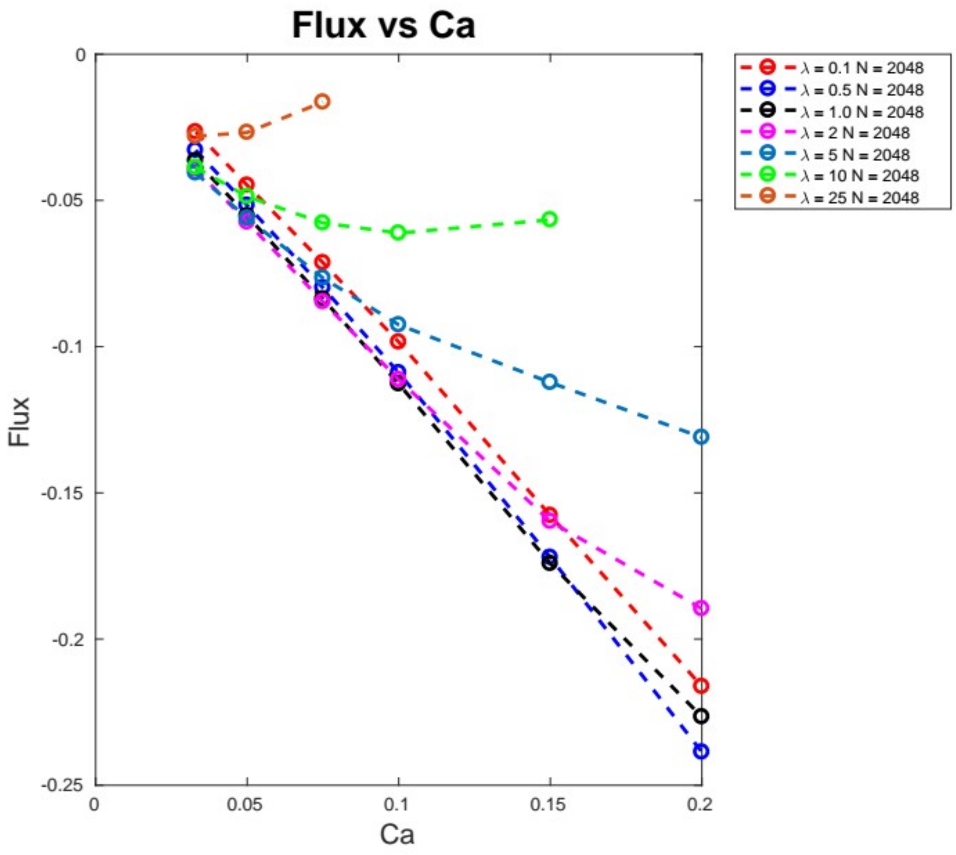
Integral Measures

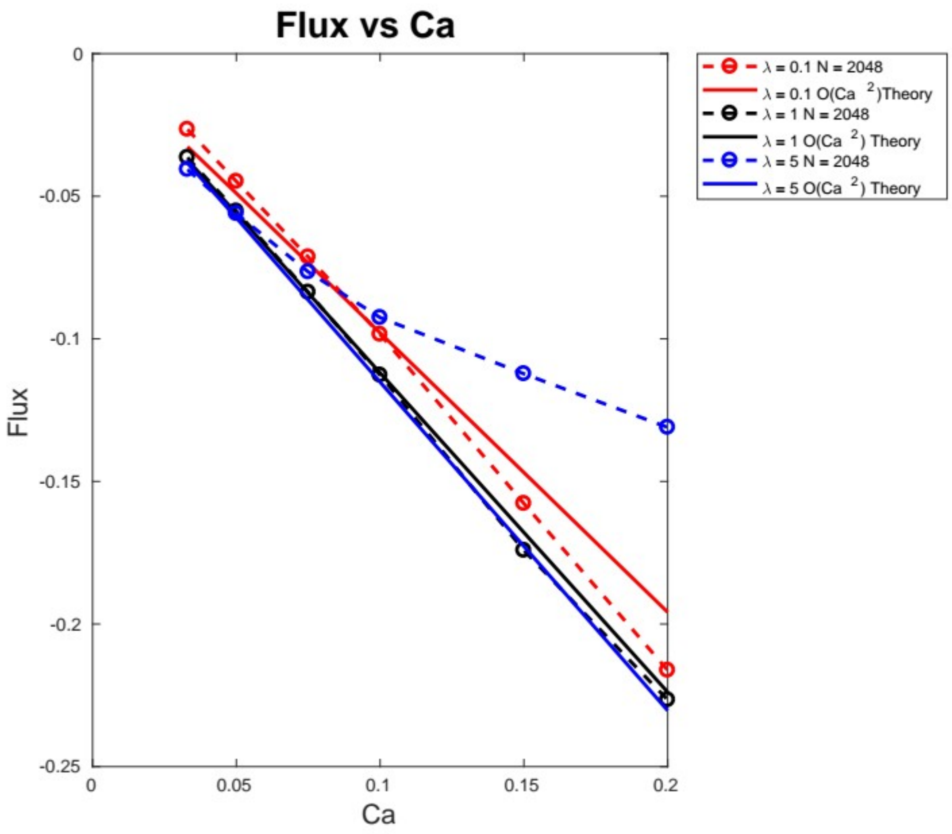




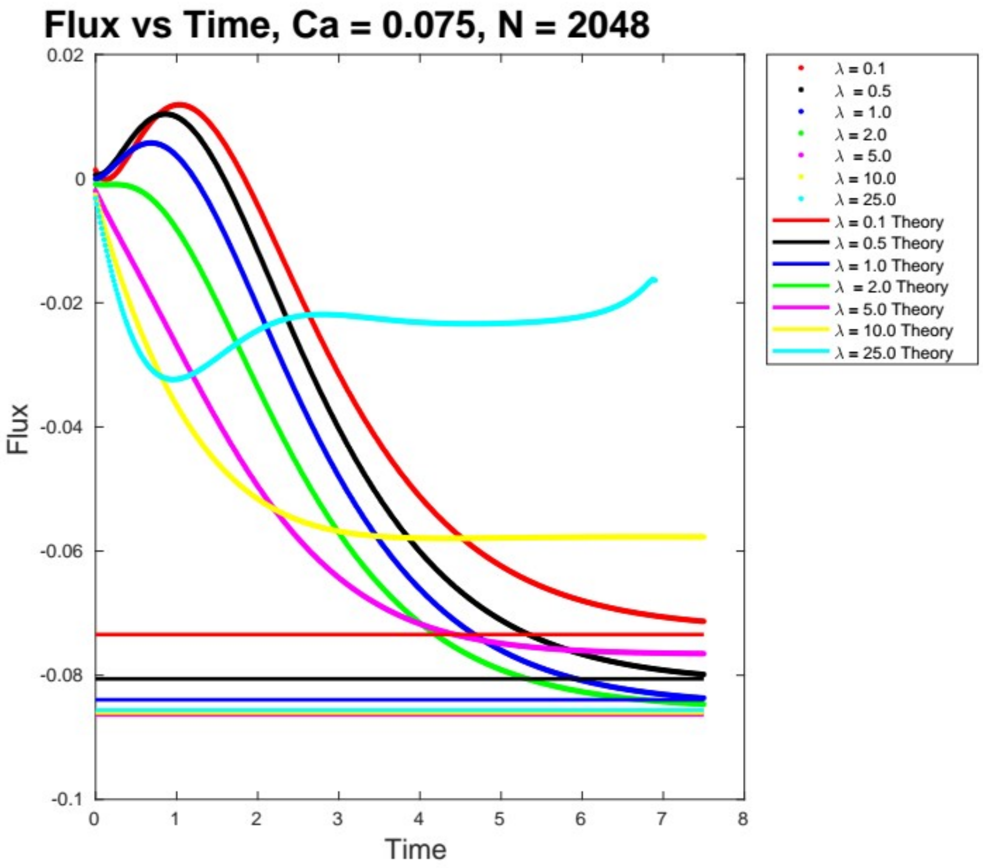
$$F_{ij} = \int (n_i n_j - \delta_{ij})\, r\, d\Phi$$

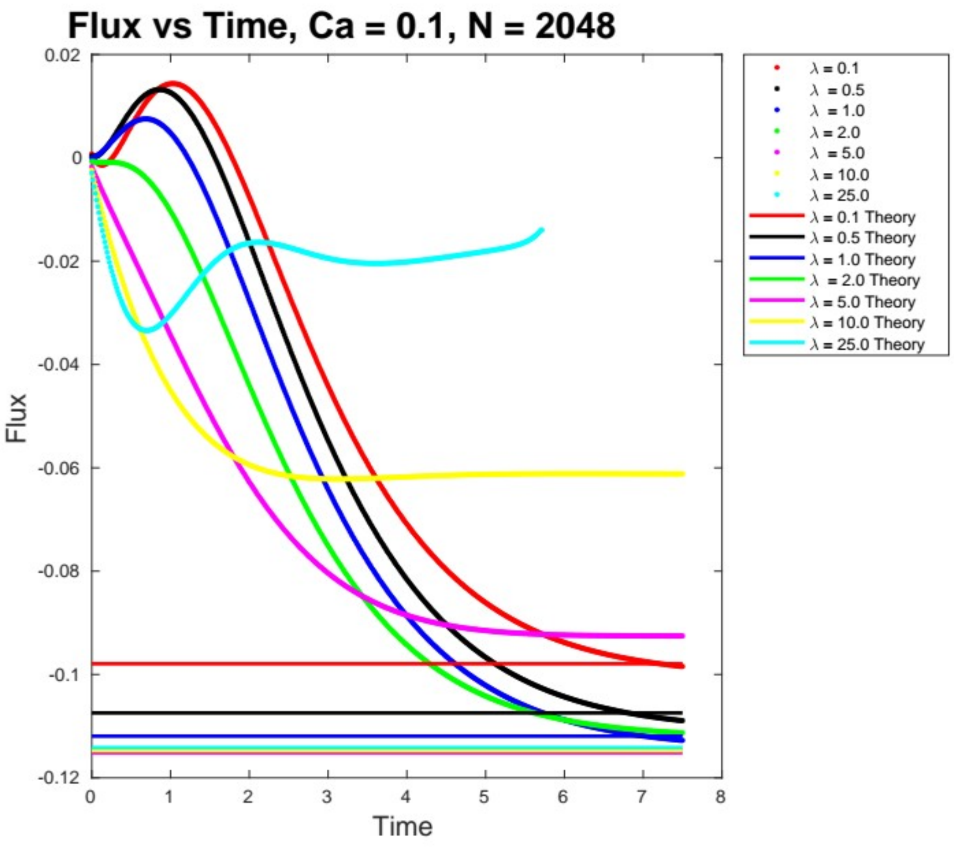
Flux vs *Ca* (Steady State)





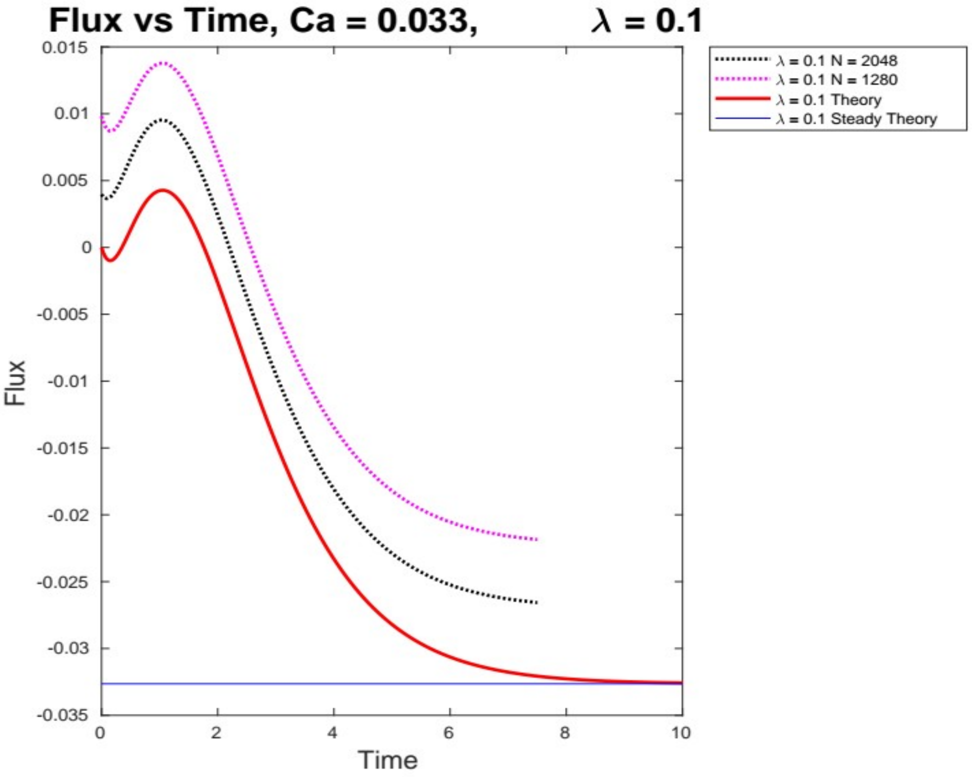
Comparison of BEM Transient Results with Steady state Theory

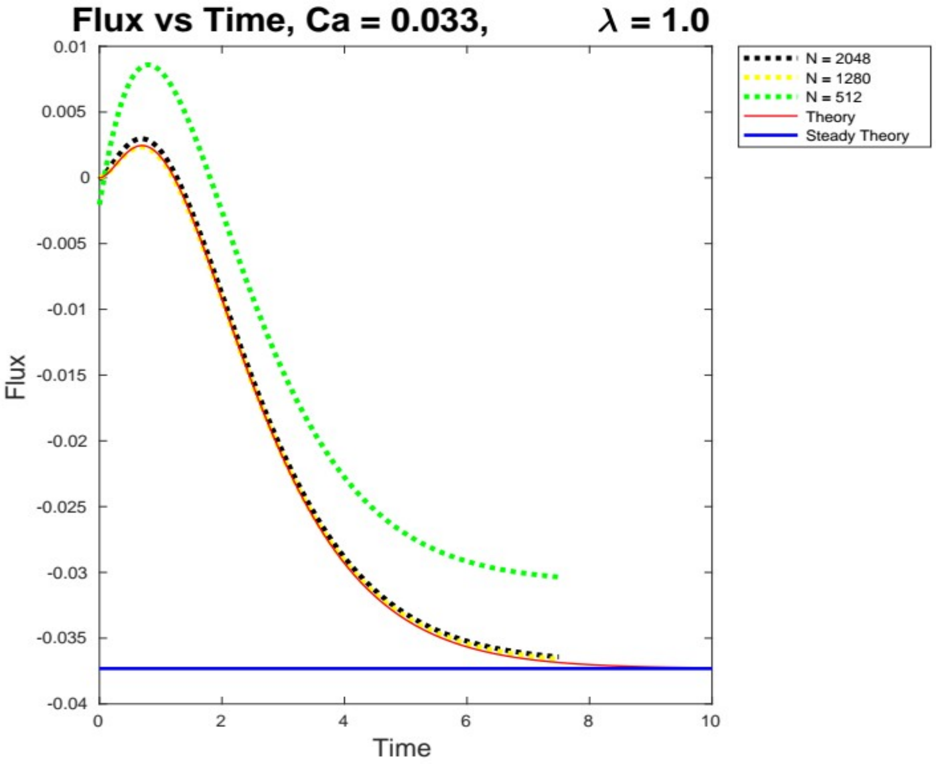




Comparison of BEM Transient Results with Theory

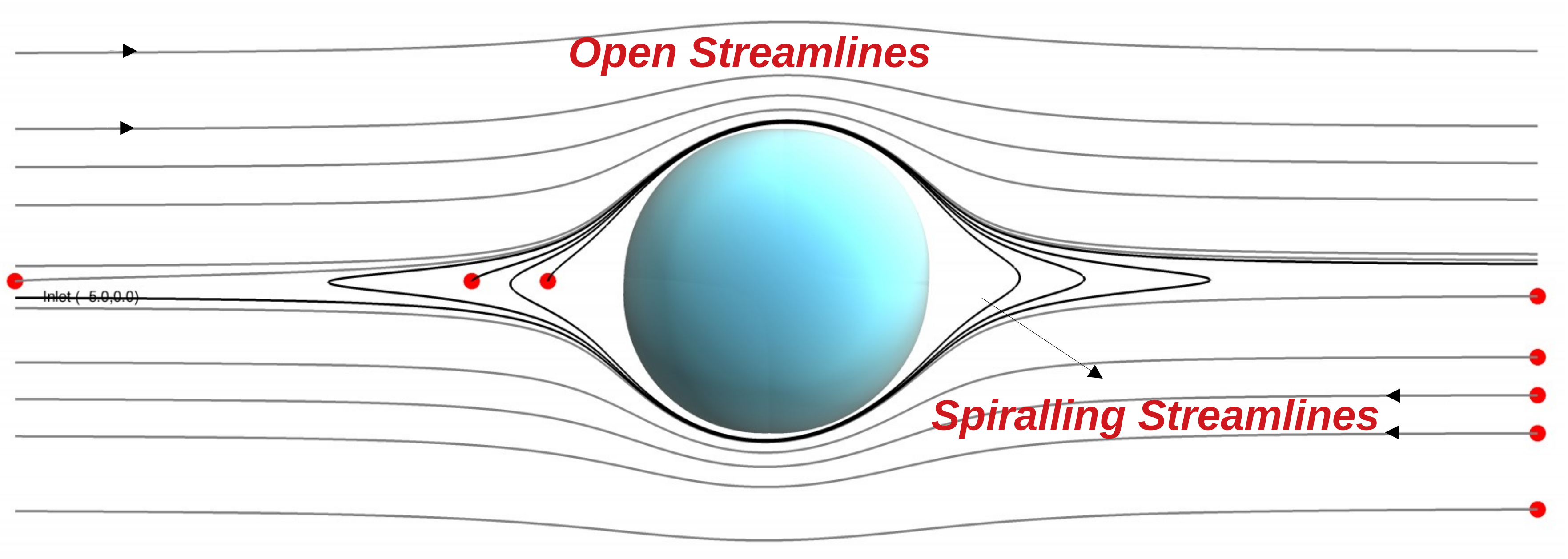
Tranisent Flux in plane





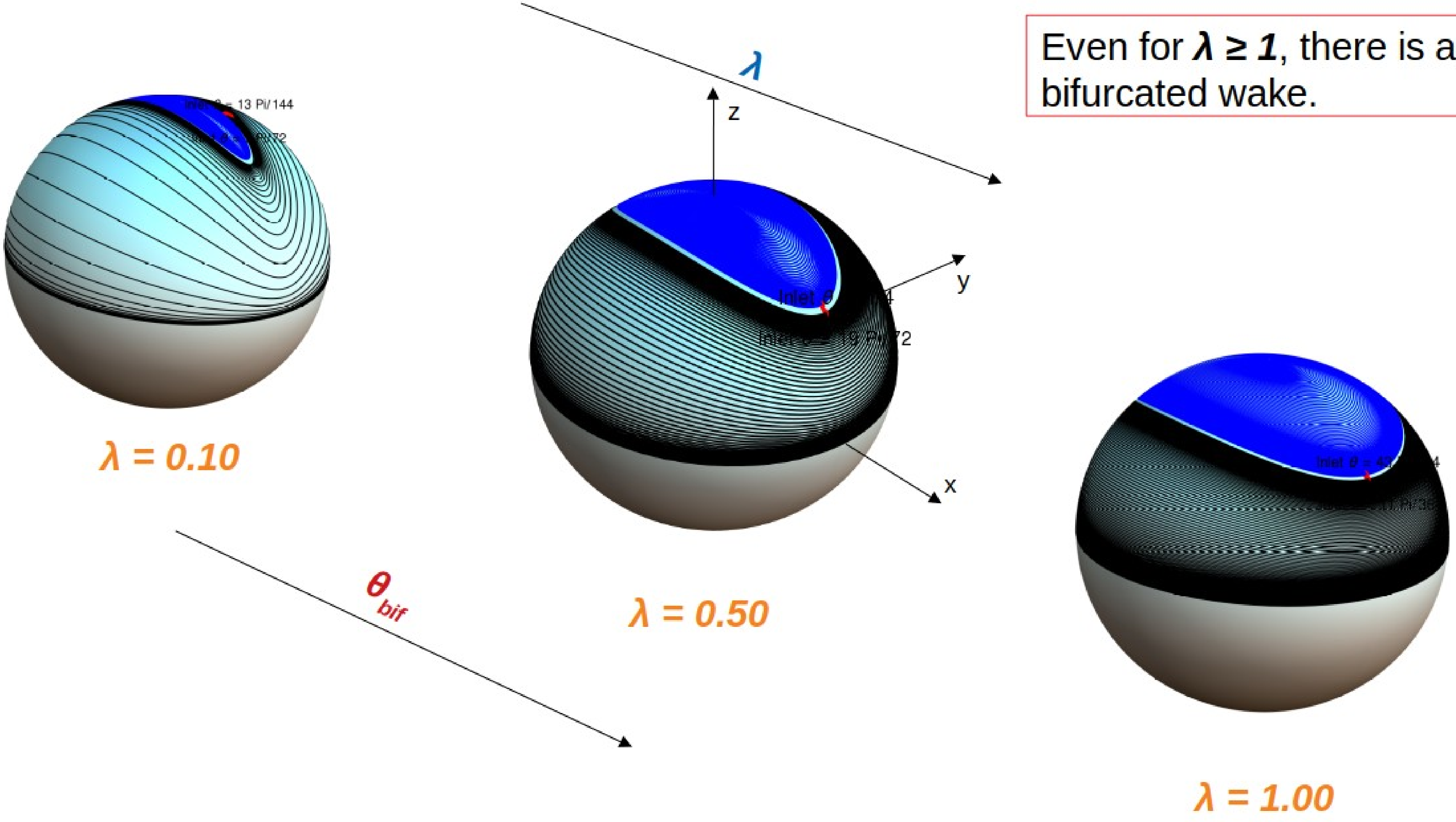
These results prove that deformation alters the streamline topology significantly

Streamlines Topology for α = 0

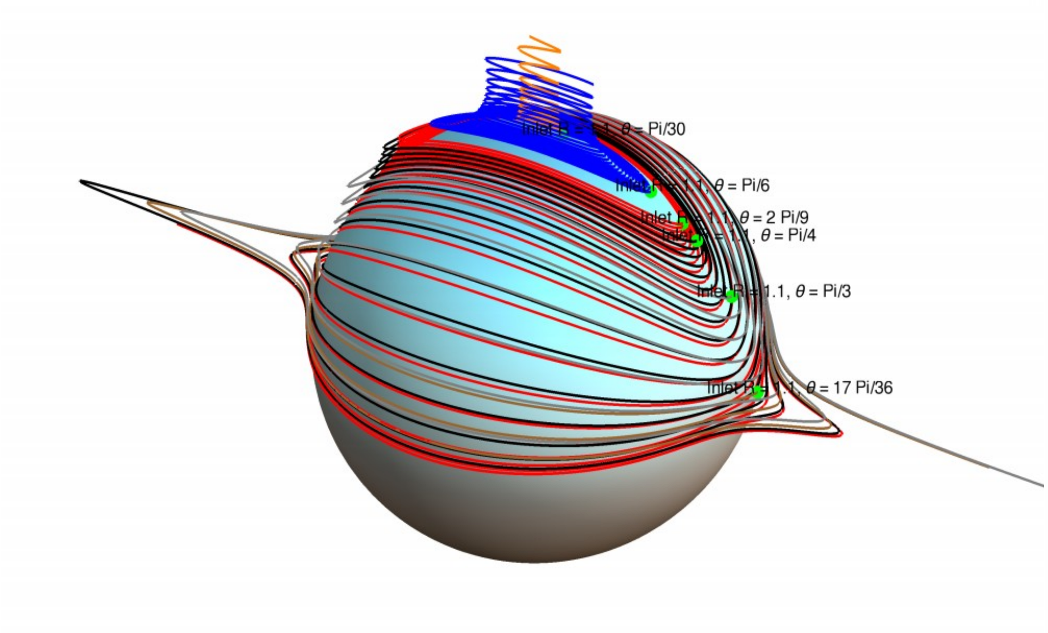


Surface Streamlines for α = 0

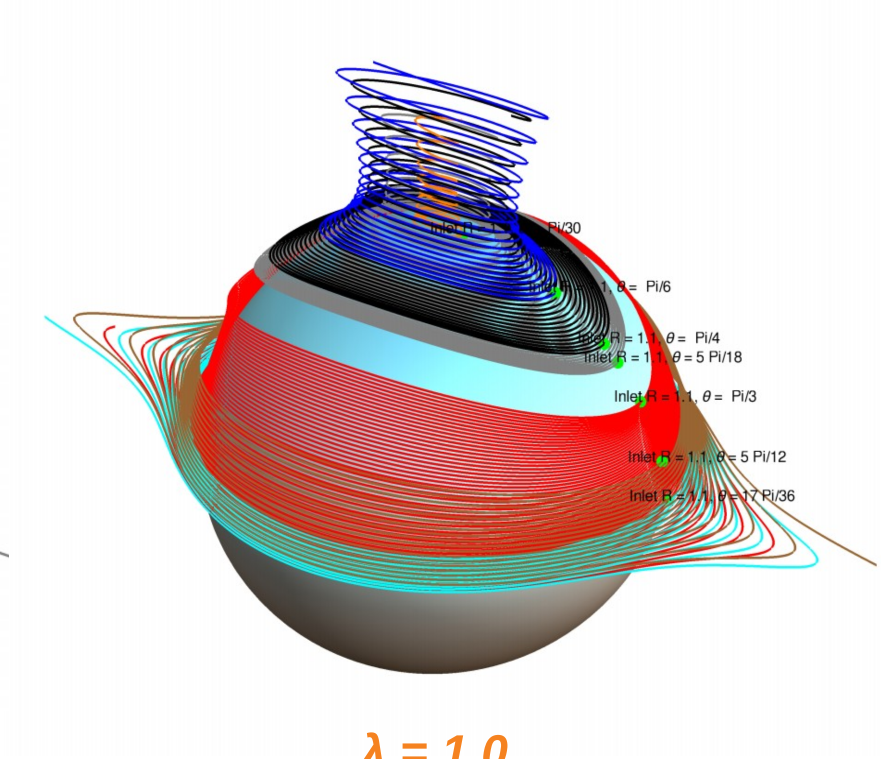
Even for **$\lambda \geq 1$**, there is a bifurcated wake.



Streamlines Topology for $\alpha = 0$



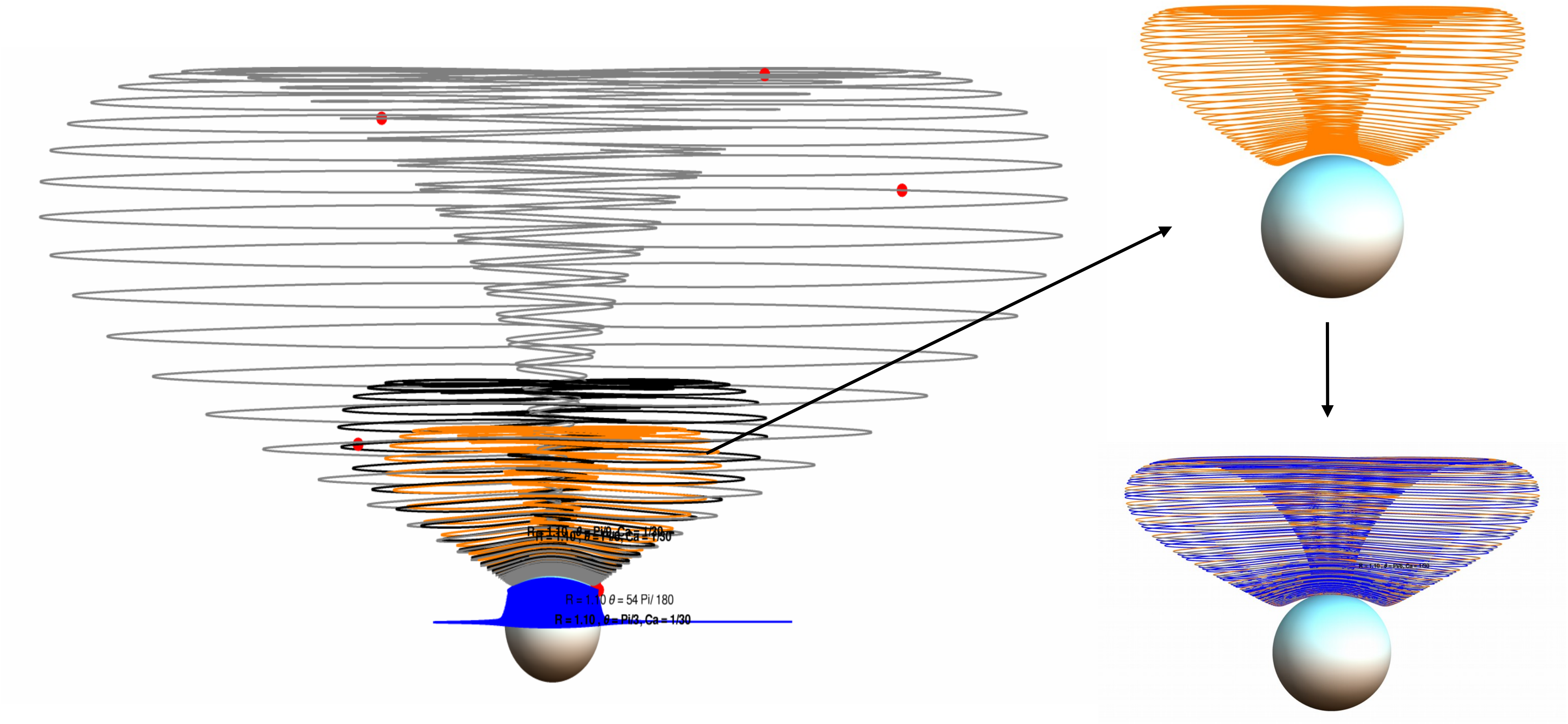
λ = 0.1



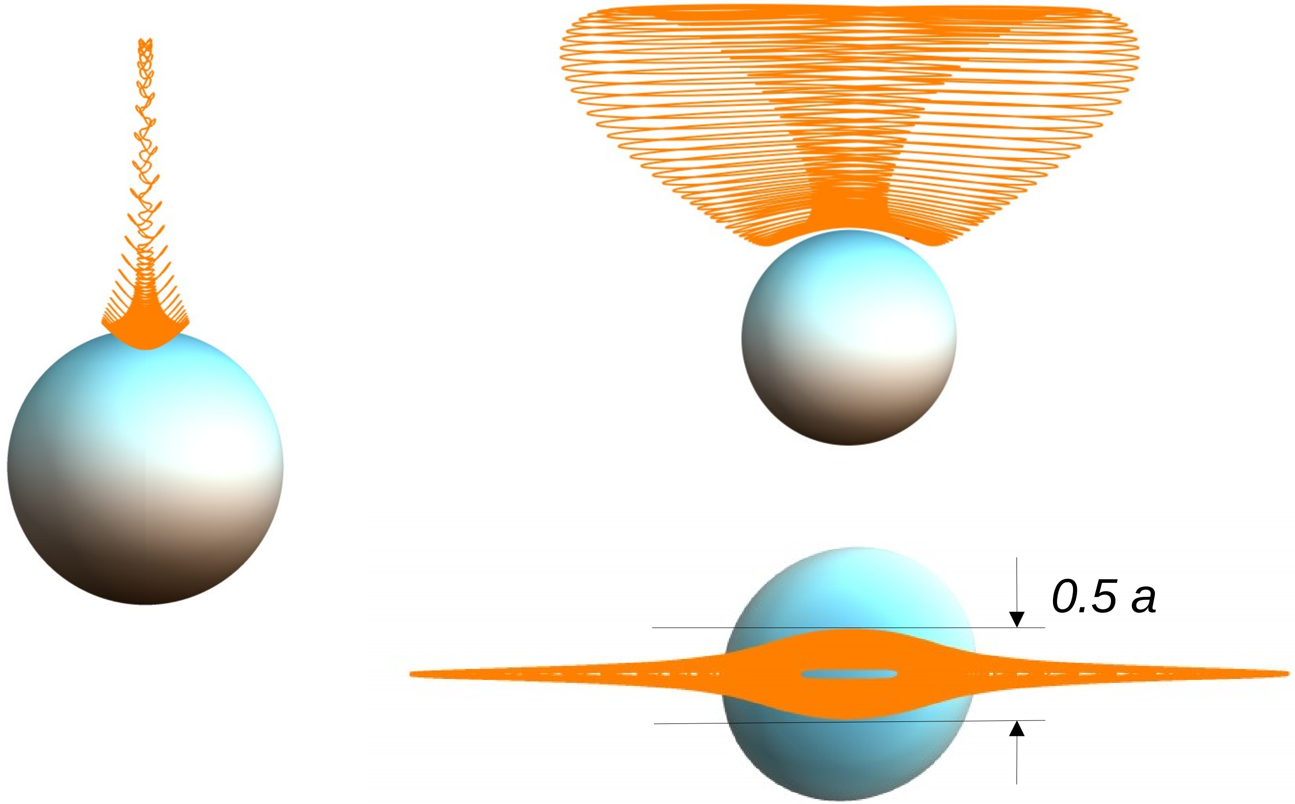
λ = 1.0

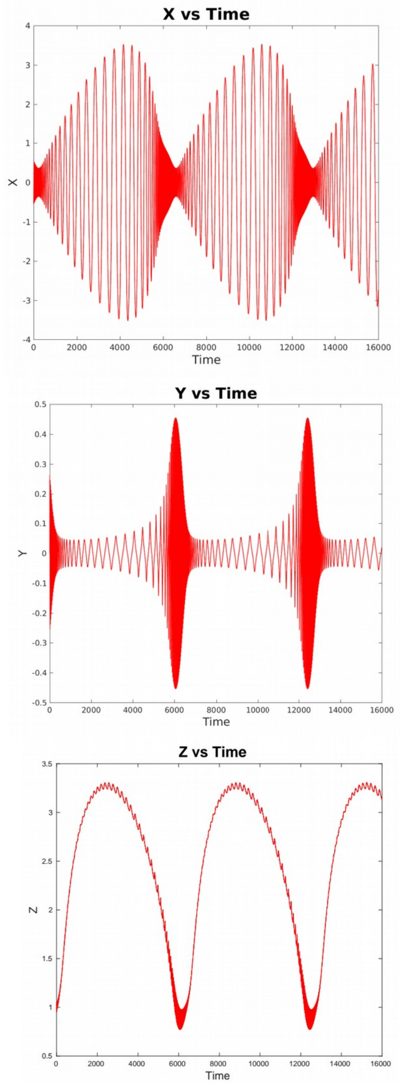
Streamlines Topology for $\alpha = 0$

$\lambda = 1.0$

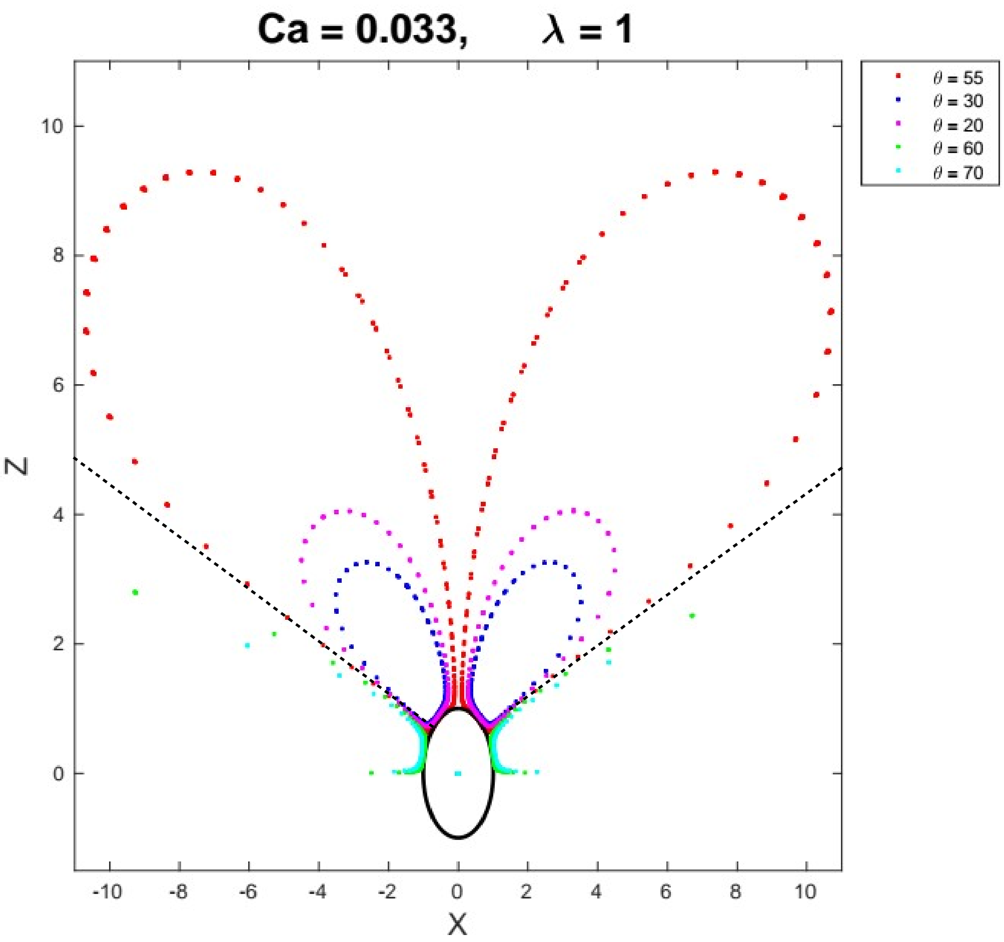


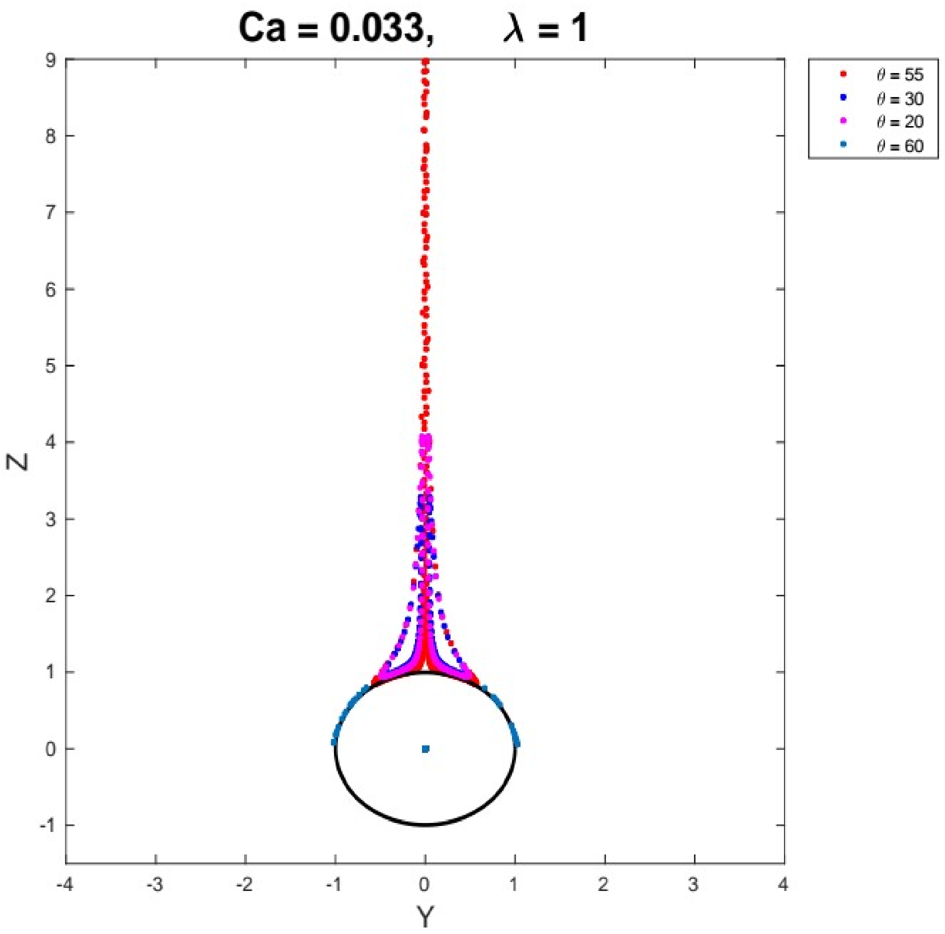
Streamlines Topology for α = 0



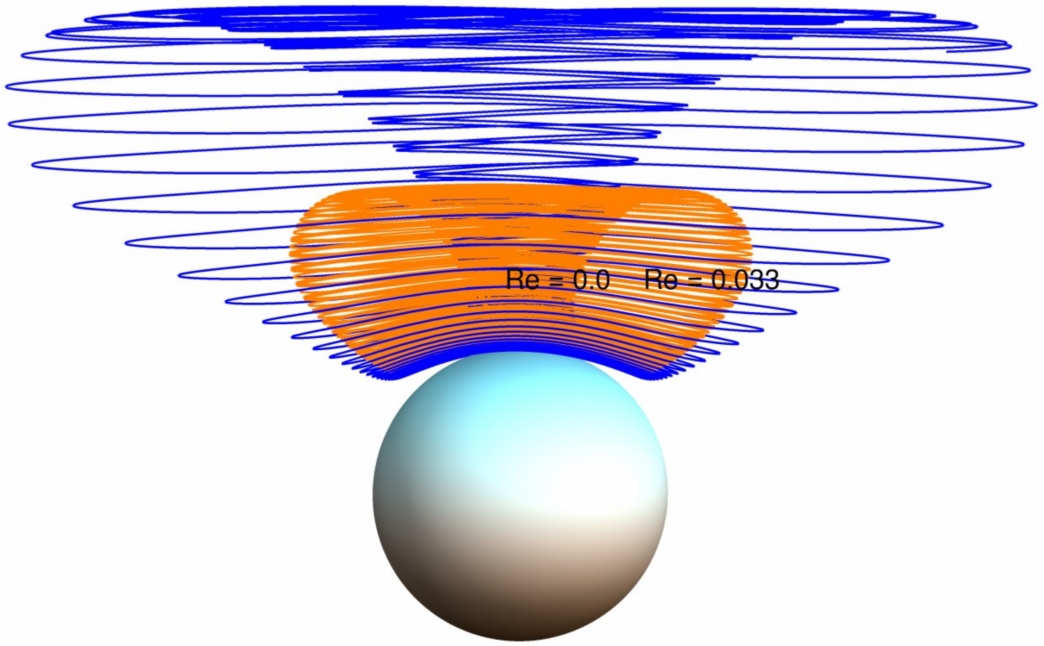


Cut-Section View:

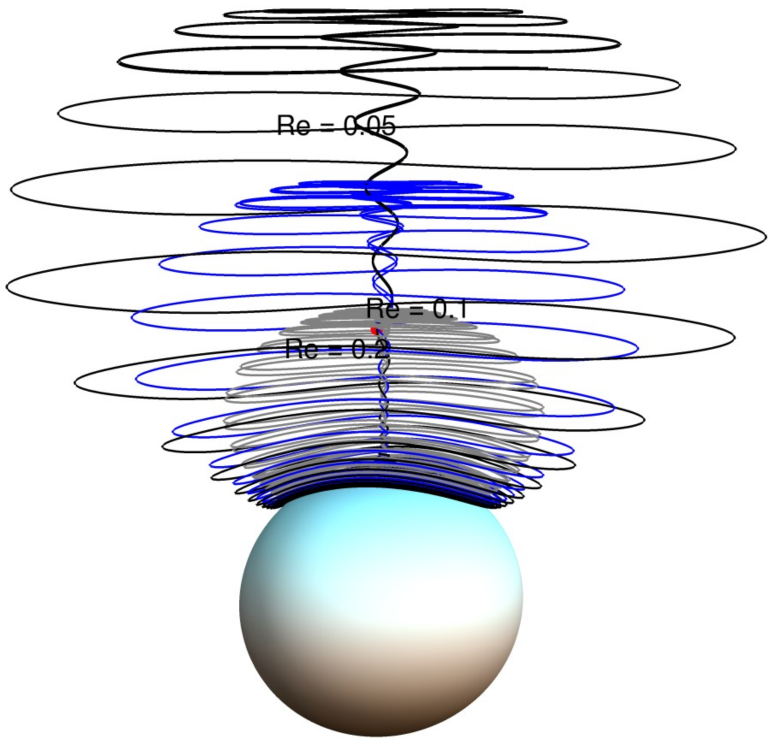




Effect of Inertia on the Streamline Topology:



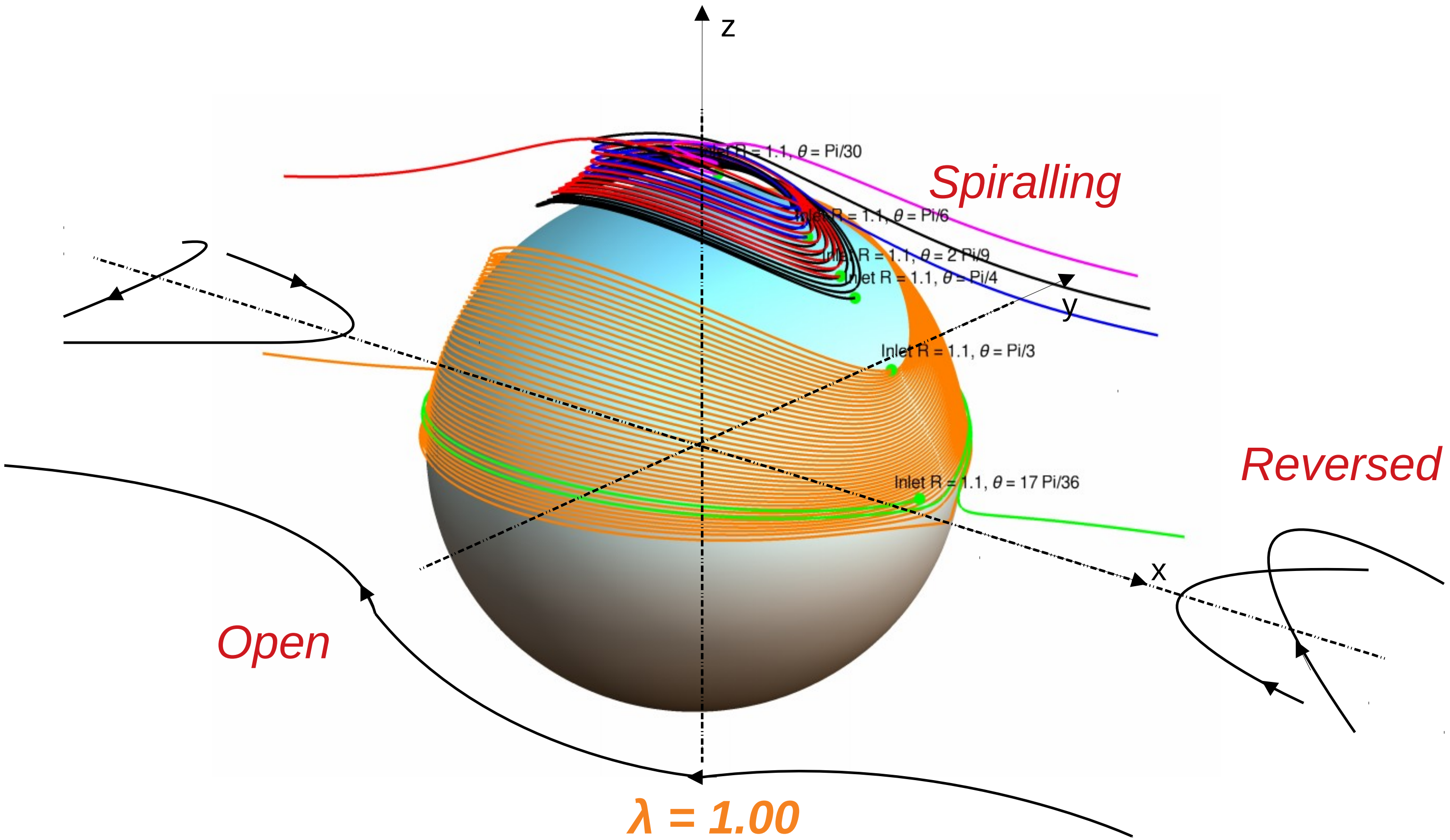
Ca = 0.033



Ca = 0.1

Streamline Topology for a non-zero α

α = 0.1



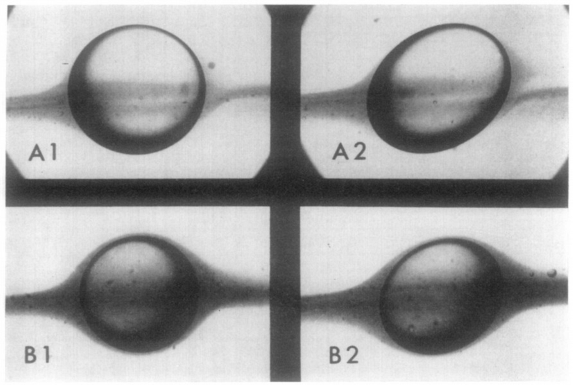
λ = 1.00

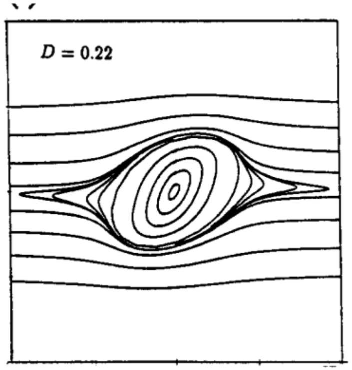
Conclusions:

- We thus find the existense of non-trivial streamline topologies for a drop in a planar linear flow. This has huge implications for the problem of transport from the drop.

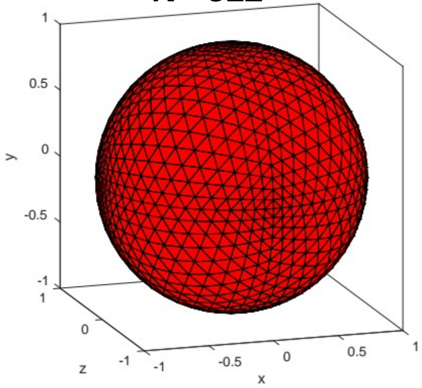
- Moreover the effect of deformation is found to be more dominant than inertia for the same values of *Ca* and *Re*. Thus deformation controls the transport of quantities from the drop.

- The streamline topology for simple shear flow in particular ($\alpha = 0$) is found to be more sophisticated than other linear flows and hence has rich physical implications.

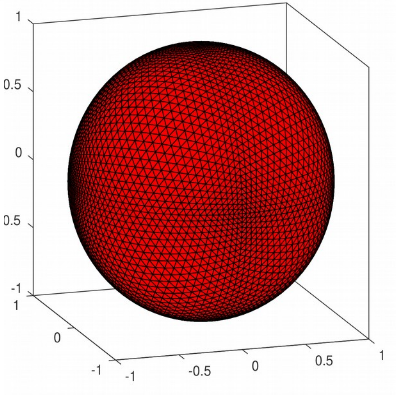




N =512



N =2048



- *Particle Motions in Sheared Suspensions XXVI: Streamlines in and around liquid Drops*
  Torza S, et.al., ***J.Colloid.Interface. Sci. 35****, 529 - 543, 1971.*

- *Motion and Deformation of Liquid Drops and the Rheology of Dilute Emulsions in Simple Shear Flow*
  Kennedy. M. R., et.al., ***Computers and Fluids. 23****, 251-278, 1994.*

Thank You