Sedimenting Anisotropic Particles in Turbulence

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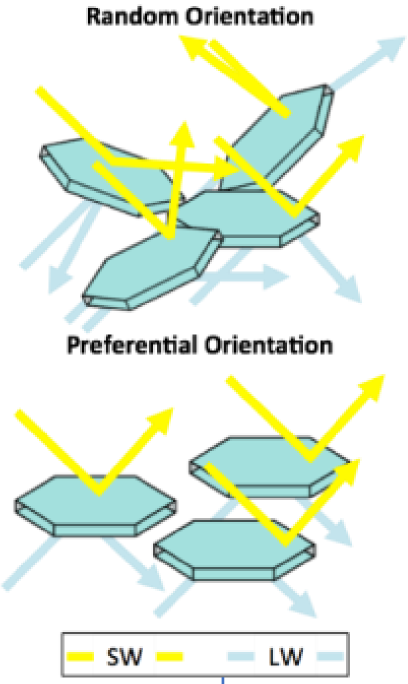
Motivation

CIRRUS CLOUDS

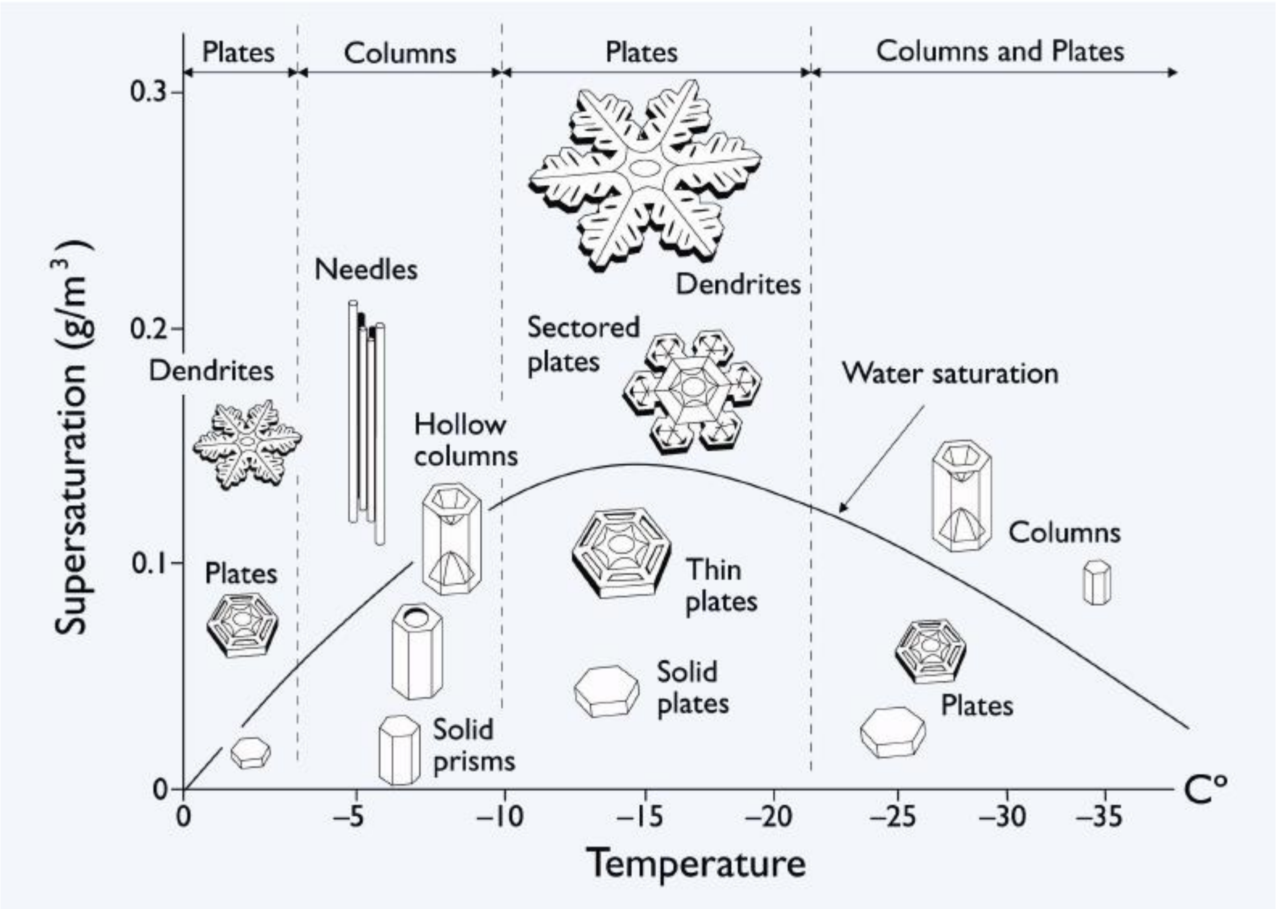


Ice-crystals settling under gravity

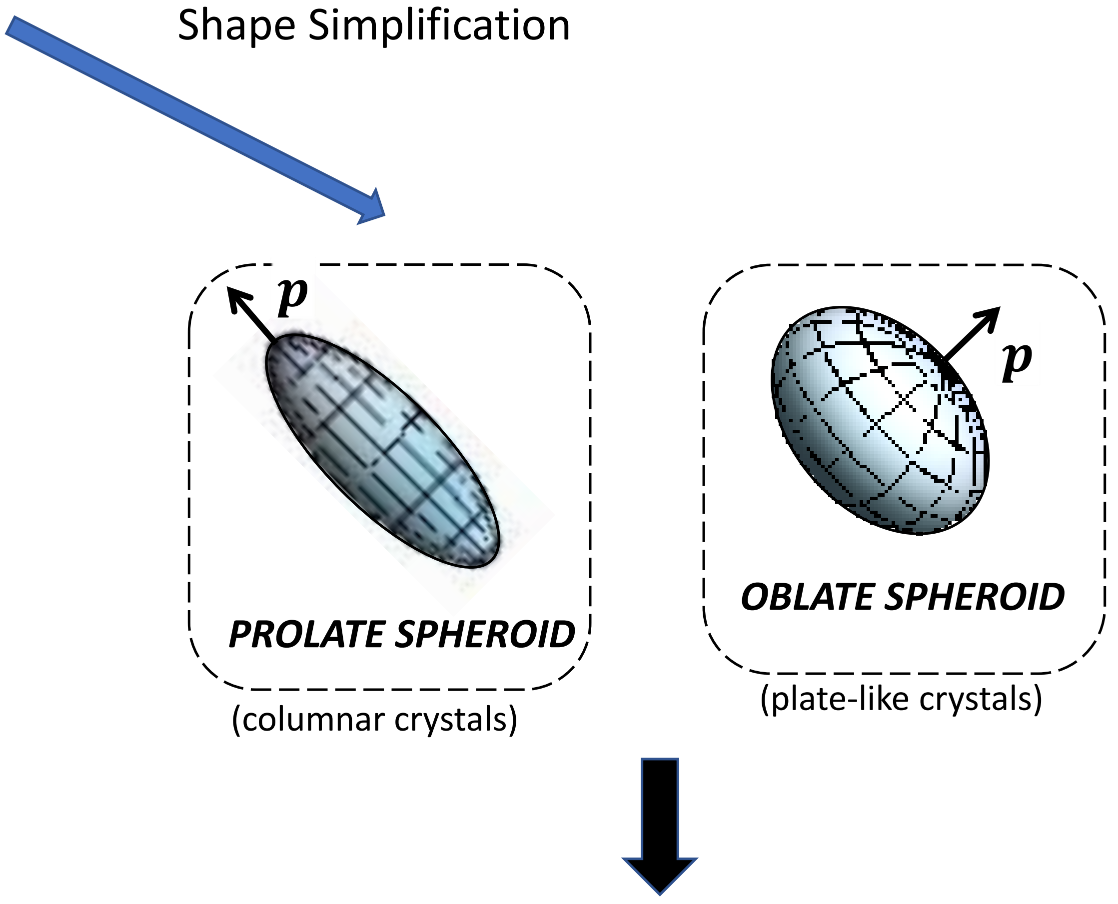
Particle orientation matters!

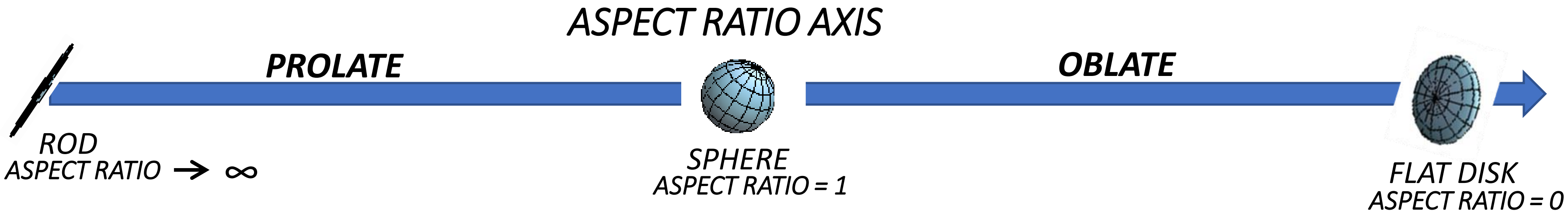
Light scattering affects Earth-atmosphere radiation budget

Ice-crystals come in a variety of shapes and sizes



http://www.cas.manchester.ac.uk/resactivities/cloudphysics/topics/lightscattering/

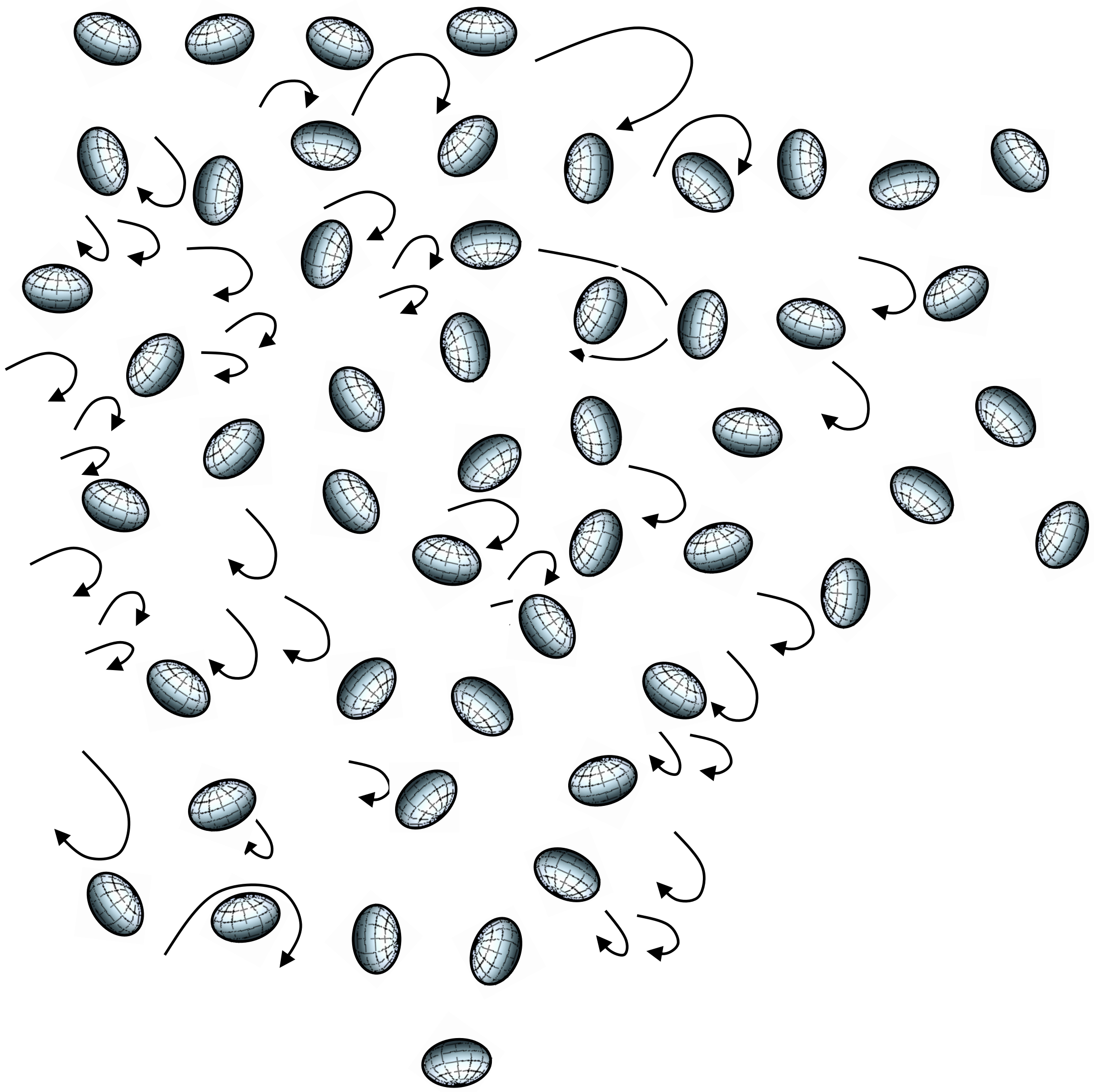




CIRRUS CLOUDS

Problem set-up

A dilute suspension of spheroids in a turbulent flow

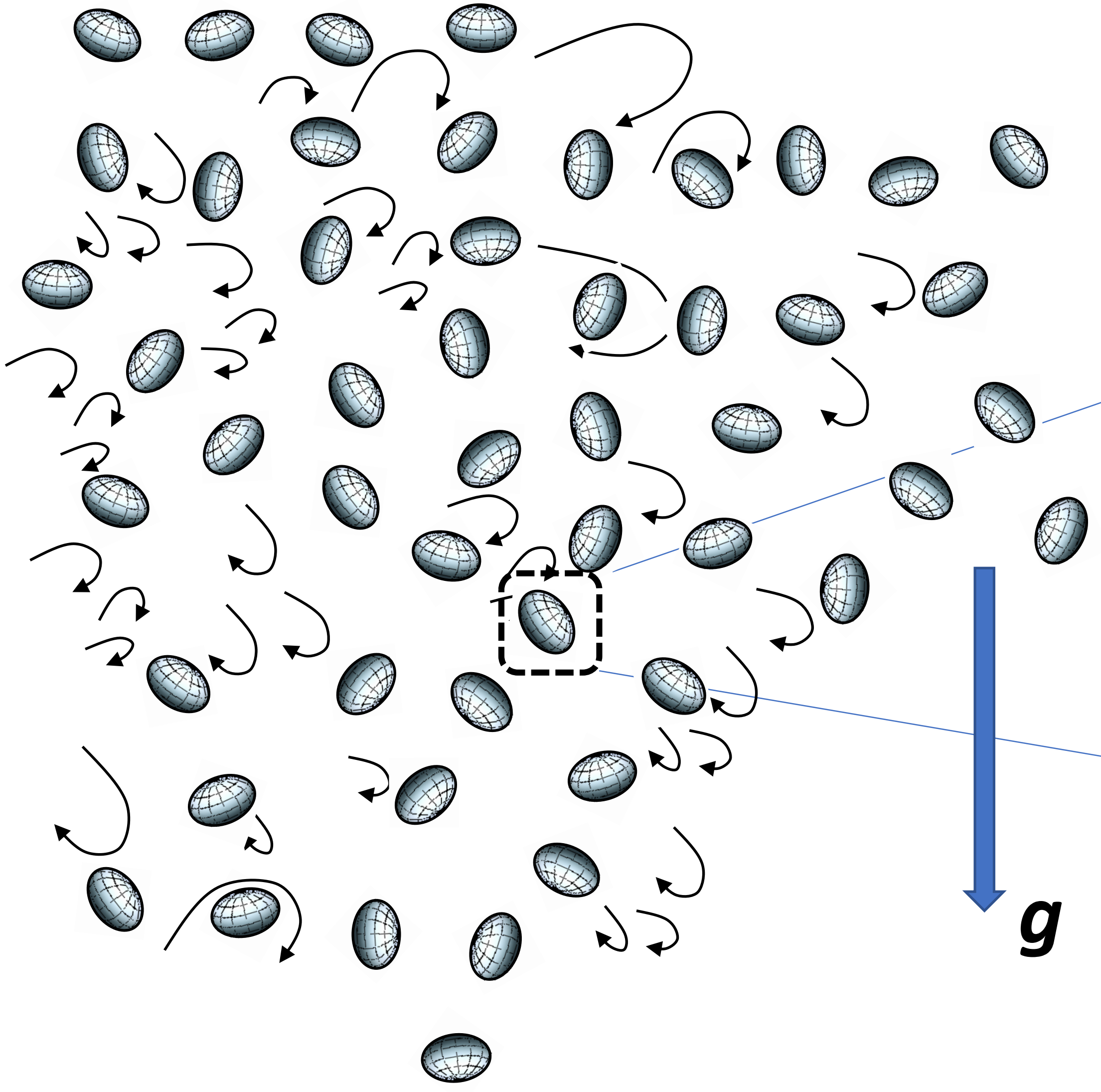


CIRRUS CLOUDS

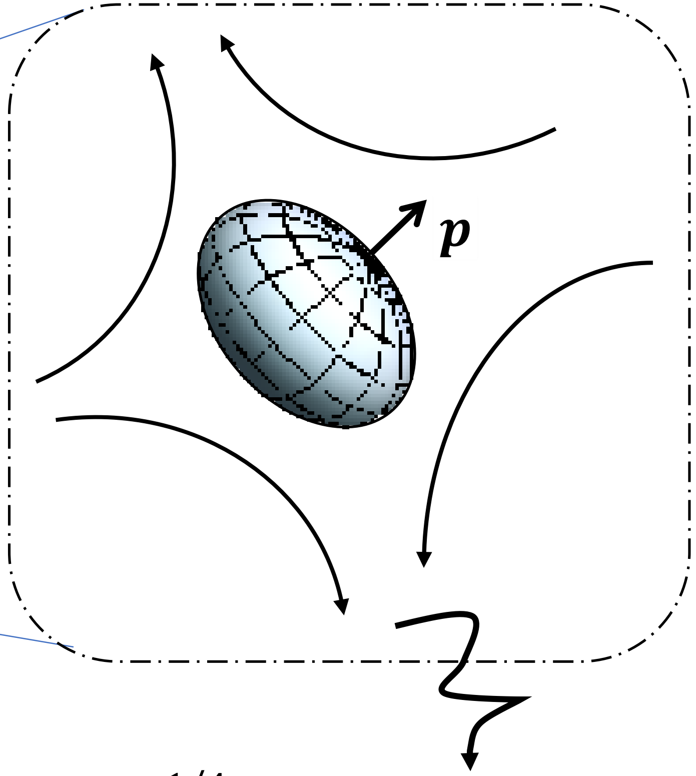
Problem set-up

A dilute suspension of spheroids in a turbulent flow

Ice crystal size:	10’s to 1000’s of microns
Atmospheric : turbulence	O(1 mm) Kolmogorov length



Spheroids are smaller than the Kolmogorov scale



Kolmogorov scale $\sim \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$

Turbulence appears as a fluctuating linear flow

Governing Equations:

Sub-Kolmogorov particles $\Rightarrow Re_{shear} \ll 1$

Force balance: $$\frac{d\boldsymbol{U}_p}{dt} = \boldsymbol{g} + \frac{1}{\tau_p X_A}\boldsymbol{M}_t^{-1}.(\boldsymbol{U}_p - \boldsymbol{u})$$

Torque balance: $$\frac{d\boldsymbol{\omega}_p}{dt} + \boldsymbol{I}_p^{-1}.\left[\boldsymbol{\omega}_p \wedge \left(\boldsymbol{I}_p.\boldsymbol{\omega}_p\right)\right] = K_{sed}\boldsymbol{I}_p^{-1}.[(\boldsymbol{M}_t.\widehat{\boldsymbol{g}}).\boldsymbol{p}(\boldsymbol{M}_t.\widehat{\boldsymbol{g}}) \wedge \boldsymbol{p}]$$
$$+8\pi\mu L^3 \boldsymbol{I}_p^{-1}.[\boldsymbol{M}_r^{-1}.\left(\frac{1}{2}\boldsymbol{\Omega} - \boldsymbol{\omega}_p\right)$$
$$-Y_H(\boldsymbol{E}.\boldsymbol{p}) \wedge \boldsymbol{p}]$$

Angular acceleration

*Gravity-induced torque for $Re_s \ll 1$

Turbulent shear-induced torque(Jeffery's torque)

Earlier works have neglected this crucial contribution

Gustavsson et al, *PRL* **119**, 254501 (2017); Siewert et al, *Atmos. Res.* **142**, 45-56 (2014); Siewert et al, *JFM* **758**, 686-701 (2014); Jucha et al, Phys. Rev. Fluids **3**,014604 (2018).

*Dabade, V., Marath, N., & Subramanian, G. (2015). Effects of inertia and viscoelasticity on sedimenting anisotropic particles. *Journal of Fluid Mechanics, 778*, 133-188. doi:10.1017/jfm.2015.360

Direct Numerical Simulations:

Kolmogorov Froude number : $Fr_\eta = \frac{\tau_p g}{u_\eta}$

Kolmogorov Stokes number : $St_\eta = \frac{\tau_p}{\tau_\eta}$

Settling Reynolds number : $Re_s = \frac{\tau_p g L}{\nu}$

Particle aspect ratios : κ

Taylor Reynolds number : R_λ

Parameter ranges

$\kappa = 0.01$–0.5 (OBLATES) $\kappa = 2$–100 (PROLATES)

$R_\lambda = 47,\ 96,\ 150,\ 200$

$St_\eta \equiv (0.0037, 0.4)$

$Fr_\eta \equiv (0.5, 25)$

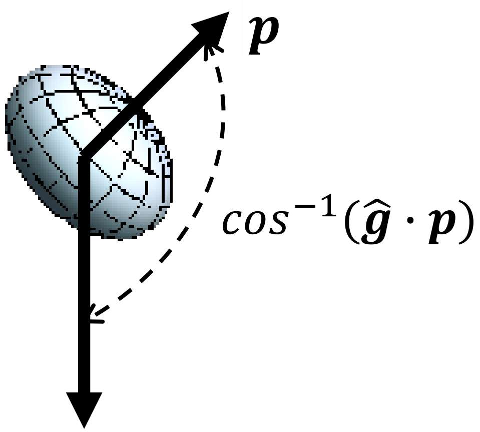
- Parameters relevant to atmospheric scenario
- Estimates indicate a dominant gravity-induced torque

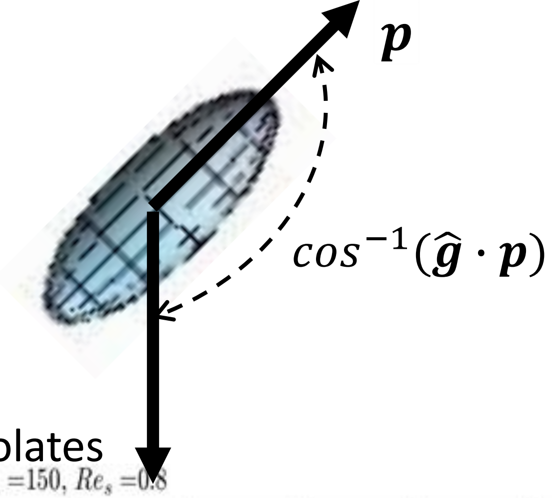
10^5 particles initialized in the computational box for each run

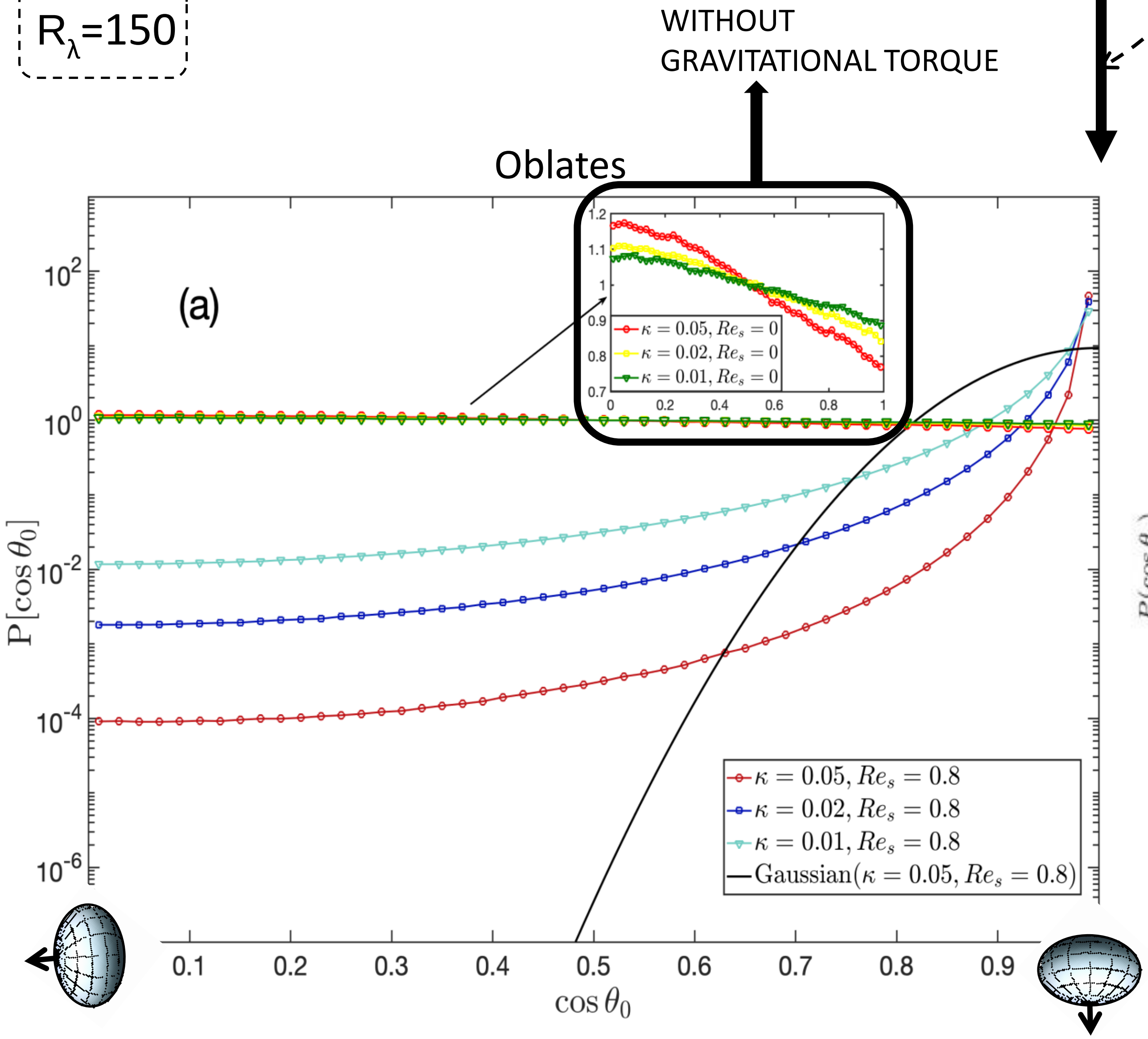
DIRECT NUMERICAL SIMULATIONS

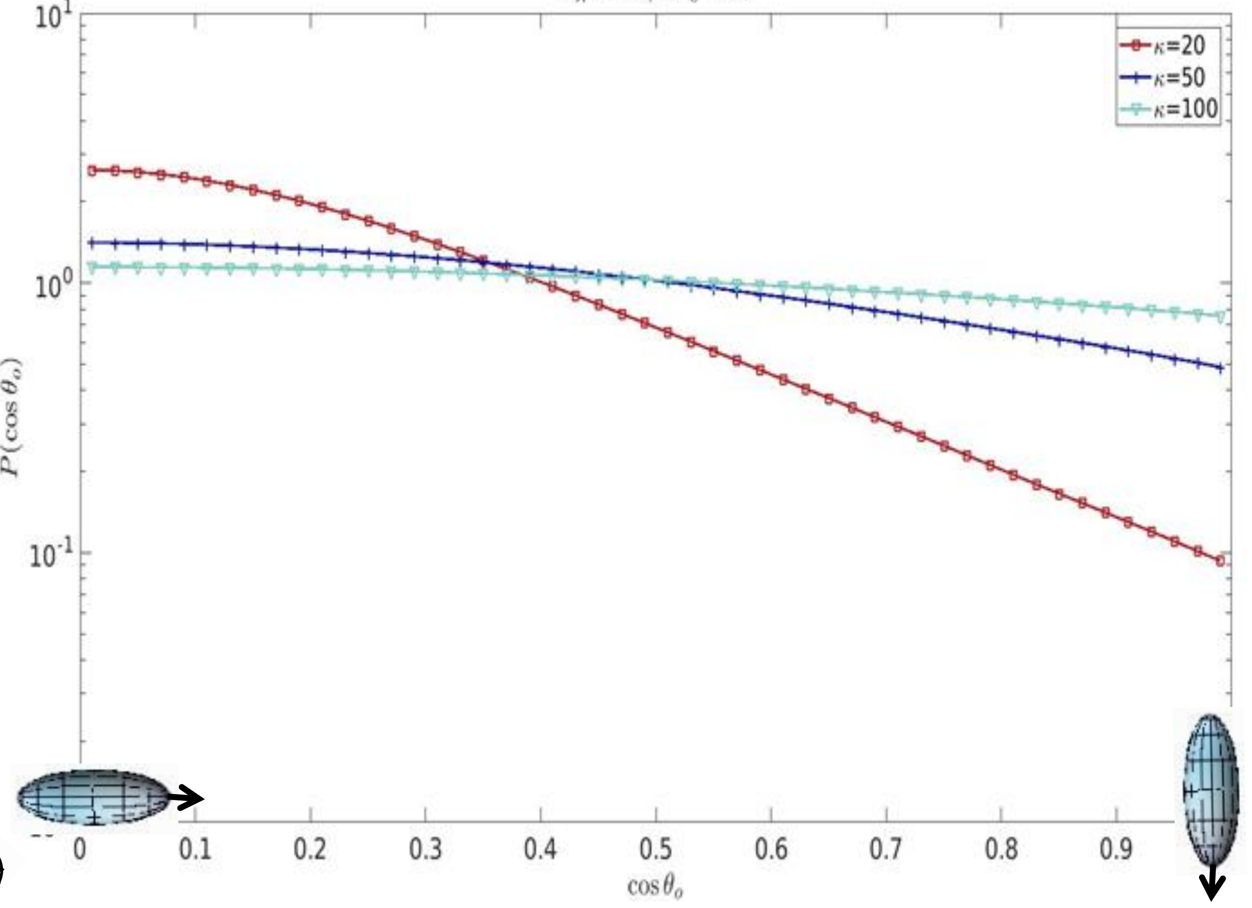
Results

R_λ=150





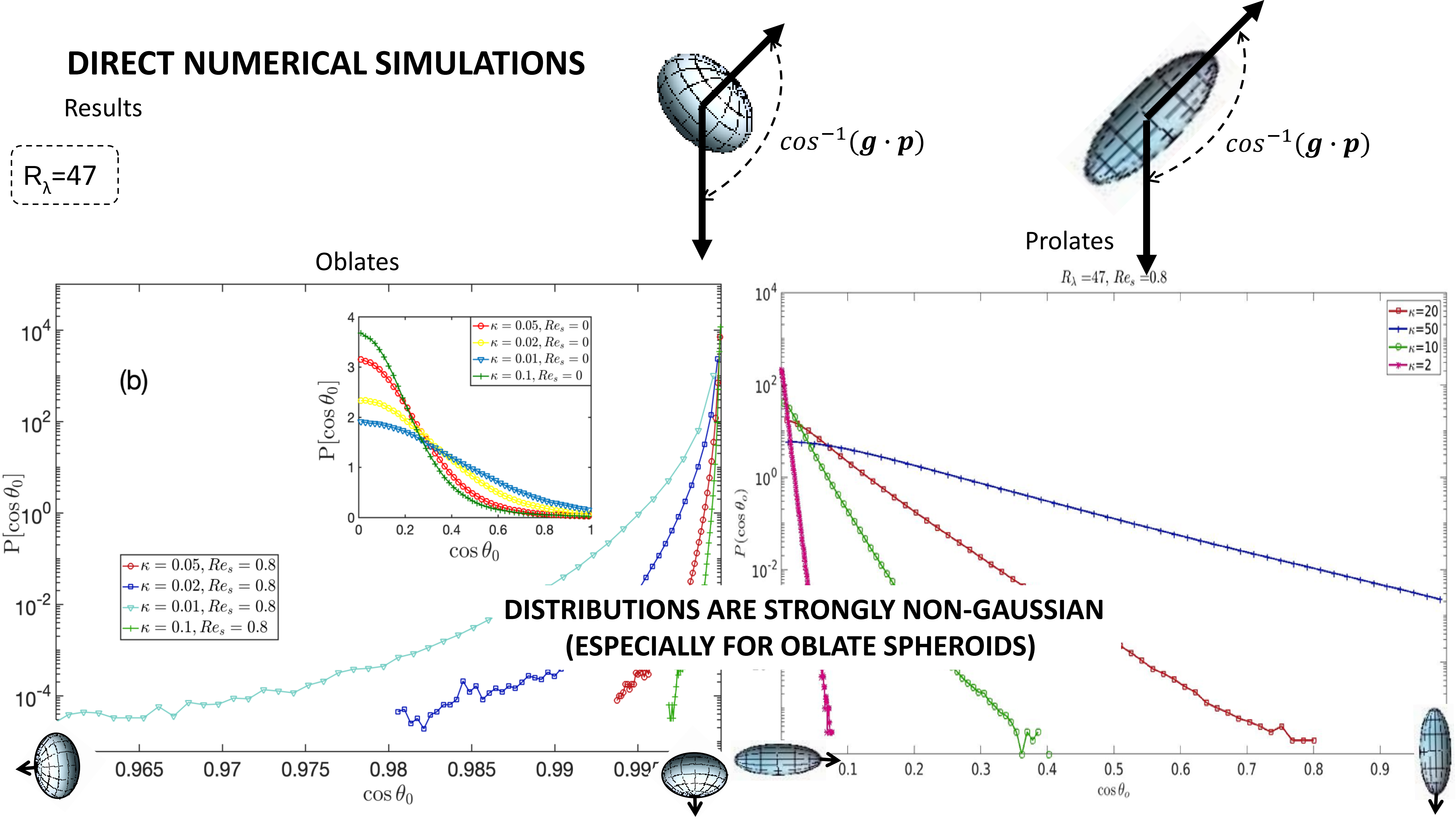




DIRECT NUMERICAL SIMULATIONS

Results

R_λ=47



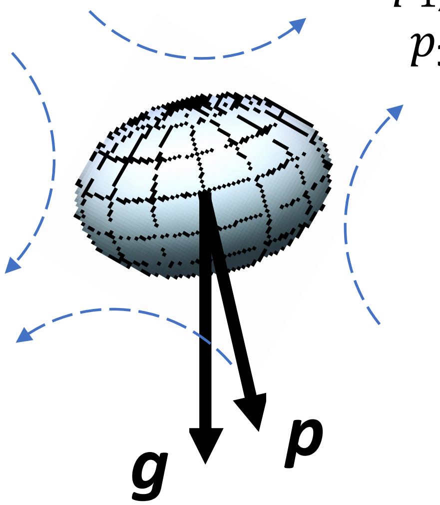
DISTRIBUTIONS ARE STRONGLY NON-GAUSSIAN (ESPECIALLY FOR OBLATE SPHEROIDS)

RAPID SETTLING THEORY

$$Fr_\eta \gg 1, St_\eta \ll 1$$

$$Fr_\eta = \frac{U_{sed}}{u_\eta} \qquad St_\eta = \frac{\tau_p}{\tau_\eta}$$

$$p_{1,2} \ll 1$$
$$p_3 \approx 1$$



- Dominant gravity-induced torque leads to broadside-on orientation

- Quasi-static balance between gravity-induced and (weak) turbulent torques in the plane normal to gravity.

The Kolmogorov-scale velocity gradients are distinctly *non-Gaussian* (DISSIPATION-RANGE INTERMITTENCY)

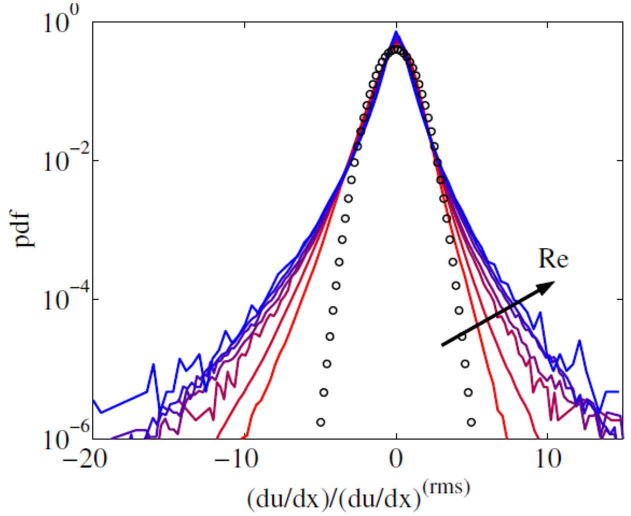


Figure 1. The p.d.f.s of the longitudinal velocity gradient for several Reynolds numbers, increasing in the direction of the arrow. Normalized with the standard deviation. $Re_L = 260 - 3.5 \times 10^6$. Symbols are Gaussian. Data from *Jiménez et al.* [1993]; *Belin et al.* [1997]; *Antonia and Pearson* [1999].

Using RST-

Final relation between components of orientation and (turbulent) velocity gradient

$$\begin{cases} p_1 = \dfrac{1}{K_{sed}}\left(S_{31} + \dfrac{Y^H}{Y^C} E_{31}\right) \\ p_2 = \dfrac{1}{K_{sed}}\left(S_{32} + \dfrac{Y^H}{Y^C} E_{32}\right) \end{cases}$$

RAPID SETTLING THEORY

- Second Moment of the Orientation Distribution

$$< p_1^2 + p_2^2 > = \frac{1}{(K^{sed})^2} \sum_{i=1}^{2} \left(< W_{3i}\, W_{3i} > + \left(\frac{Y^H}{Y^C}\right)^2 < E_{3i}\, E_{3i} > \right)$$

Second Moment of the Turbulent Velocity Gradient

$$< \frac{\partial u_i}{\partial x_j} \frac{\partial u_k}{\partial x_l} > = \frac{\epsilon}{15\nu} \left(\delta_{ik}\delta_{jl} - \frac{1}{4}\delta_{ij}\delta_{kl} - \frac{1}{4}\delta_{il}\delta_{jk} \right)$$

$$< E_{3i} E_{3i} > = \frac{\gamma_\eta^2}{20} \qquad < W_{3i} W_{3i} > = \frac{\gamma_\eta^2}{12} \qquad i = 1,2$$

$$< p_1^2 + p_2^2 > = \frac{32\pi^2 Y_A^2 Y_C^2}{f_I^2(\kappa) X_A^2} \left[\frac{1}{3} + \left(\frac{Y^H}{Y^C}\right)^2 \frac{1}{5} \right] \frac{1}{Fr_\eta^4}$$

$$\frac{F(\kappa)}{Fr_\eta^4}$$

RAPID SETTLING THEORY

- Second Moment of the Orientation Distribution

$$< p_1^2 + p_2^2 > = \frac{1}{(K^{sed})^2} \sum_{i=1}^{2} \left(< W_{3i}\, W_{3i} > + \left(\frac{Y^H}{Y^C}\right)^2 < E_{3i}\, E_{3i} > \right)$$

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$$\frac{F(\kappa)}{Fr_\eta^4}$$

- Fourth Moment

Dissipation-range intermittency leads to an additional R_λ dependence

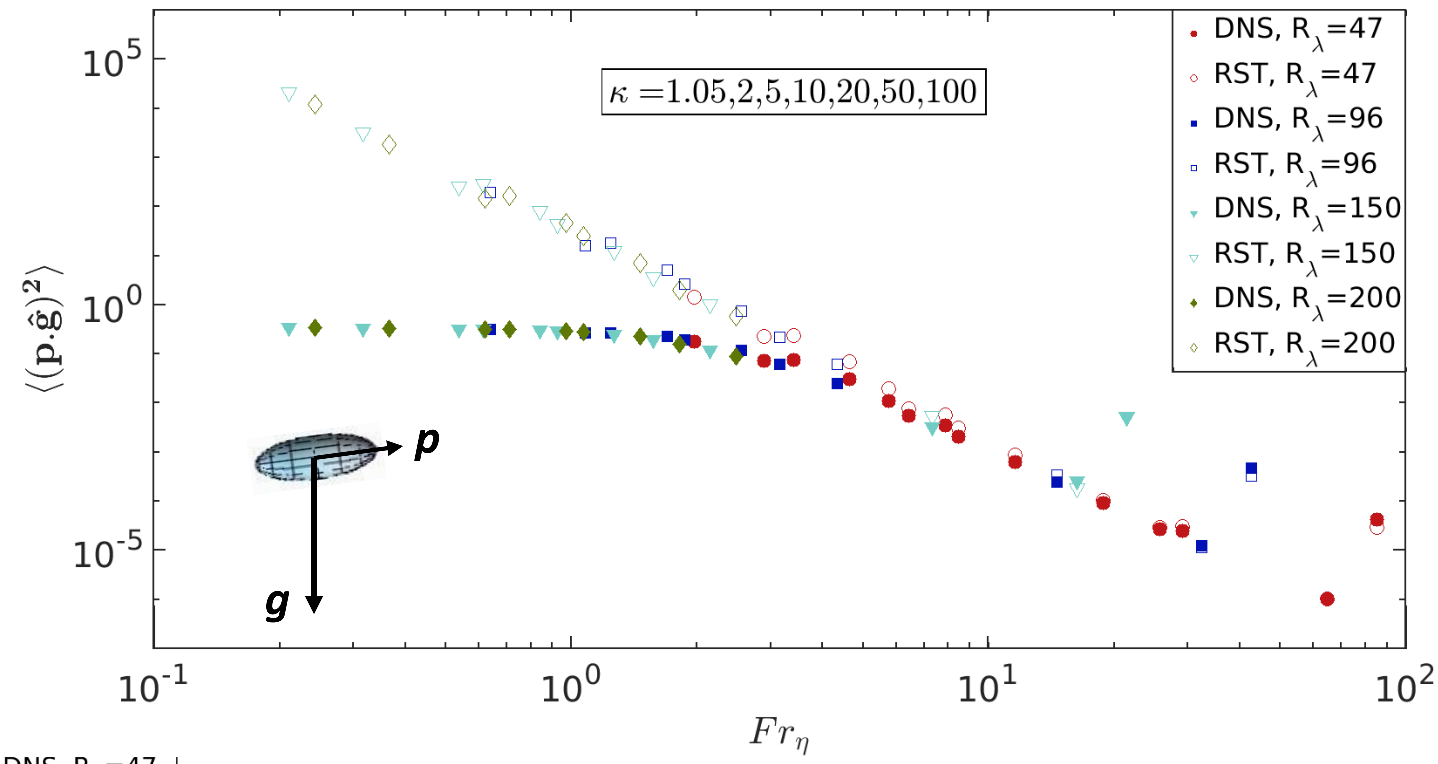
$$< (p_1^2 + p_2^2)^2 > \propto G(\kappa, R_\lambda) \frac{1}{Fr_\eta^8}$$

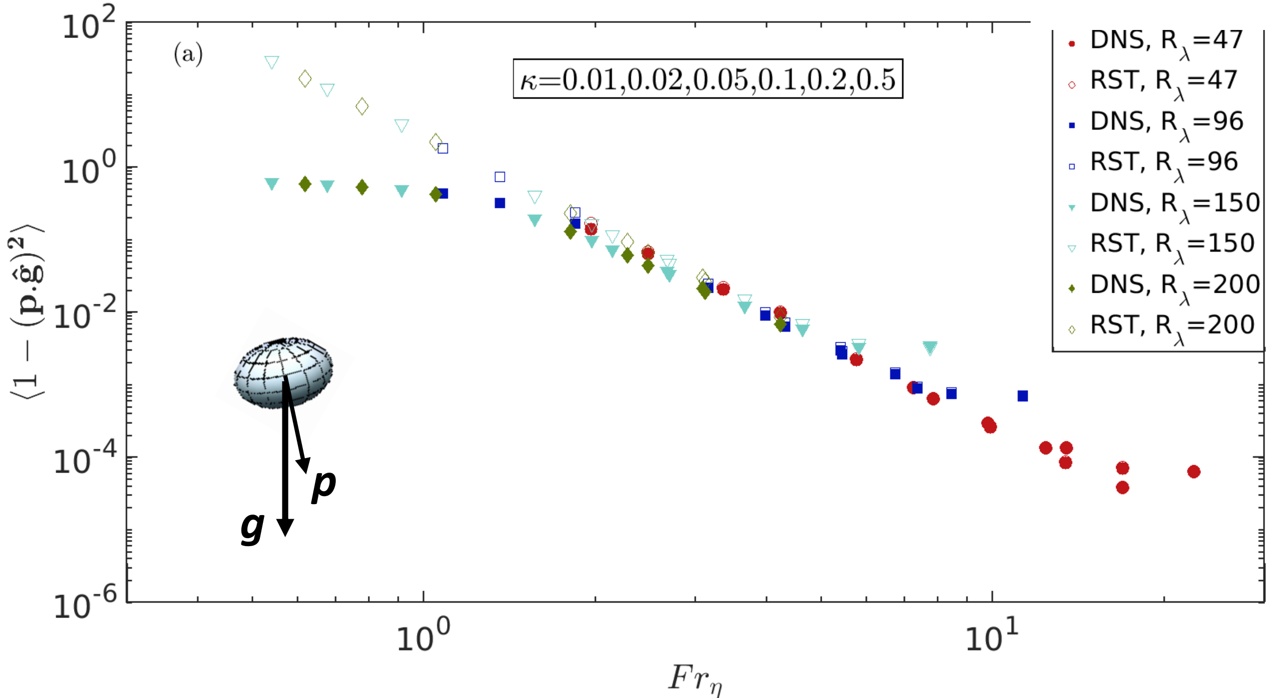
RAPID SETTLING THEORY

- Second moment of the orientation distribution

$$p \cdot \hat{g} = cos\theta \qquad (\theta \approx 0)$$
$$< 1 - (p \cdot \hat{g})^2 > \approx \; < \theta^2 >$$

Oblates





Prolates

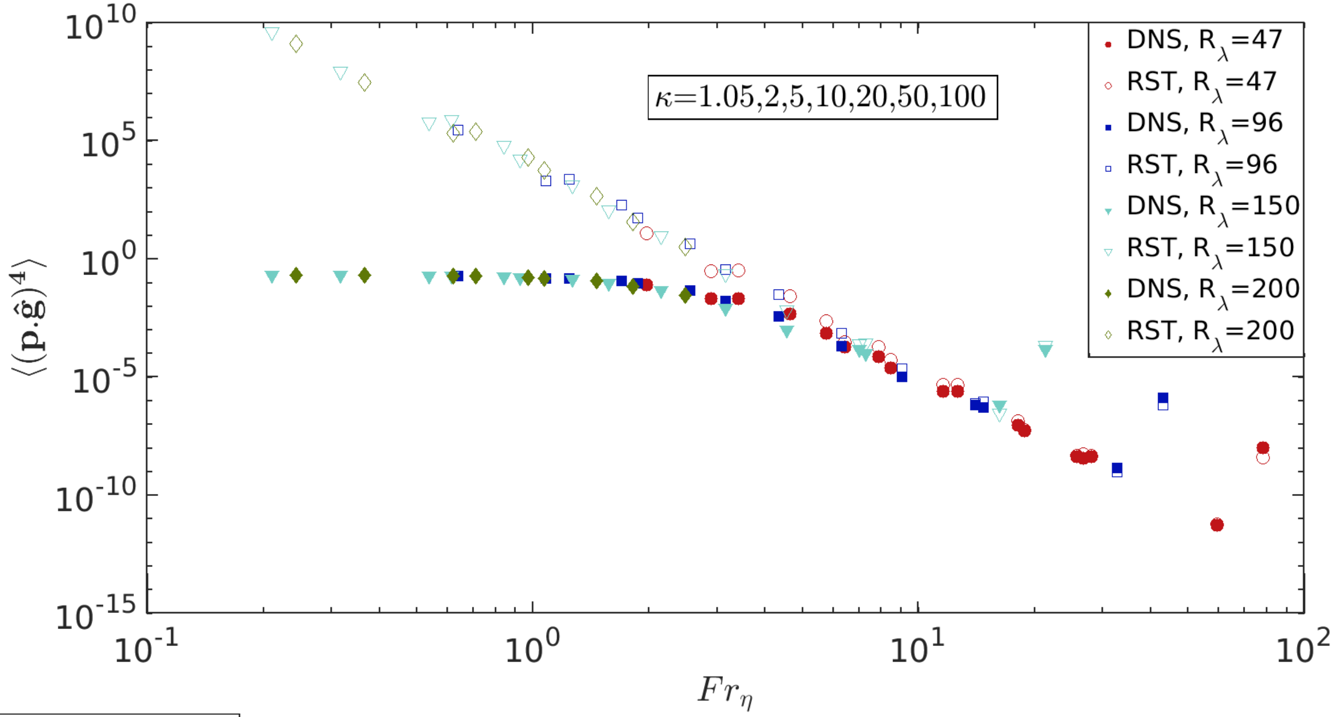
$$p \cdot \hat{g} = cos\theta \qquad (\theta \approx \frac{\pi}{2})$$
$$< (p \cdot \hat{g})^2 > \approx \; < (\frac{\pi}{2} - \theta)^2 >$$

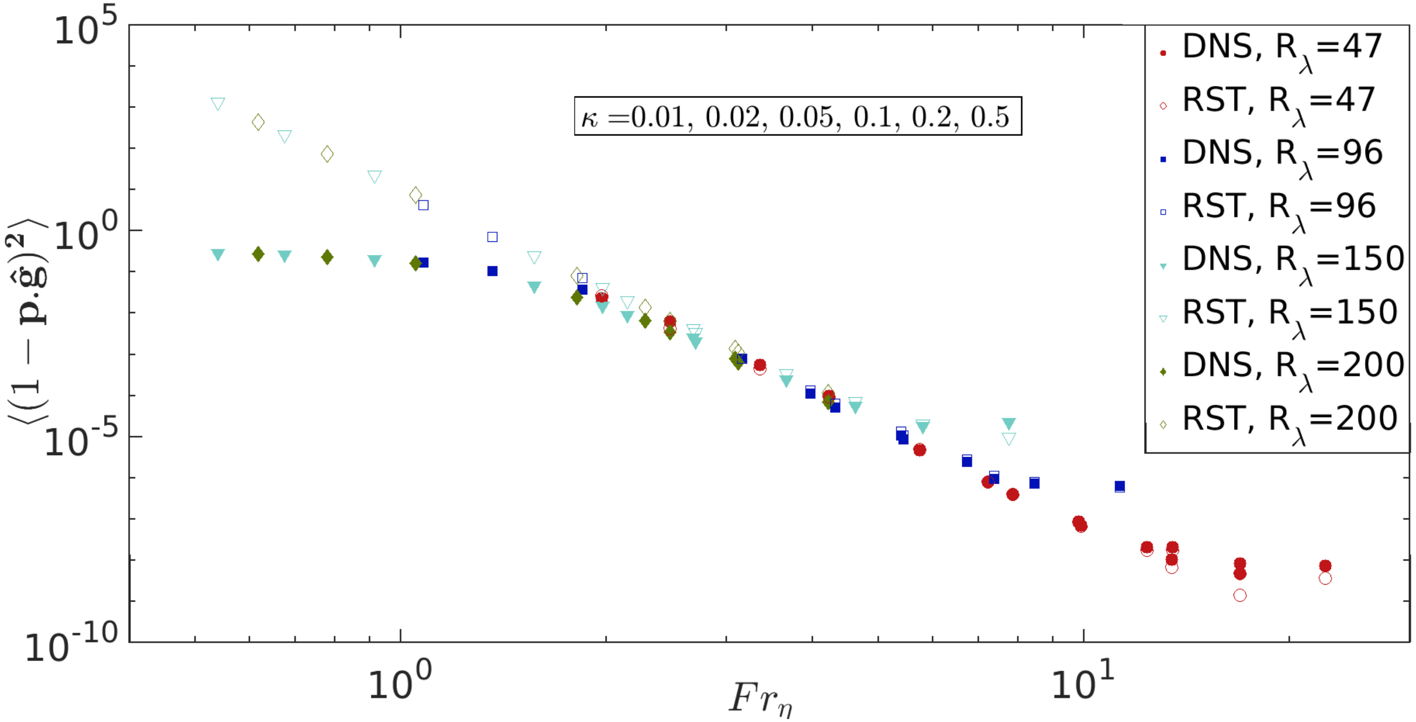
RAPID SETTLING THEORY

- Fourth moment of the orientation distribution

$$< (1 - p.\hat{g})^2 > \approx \; < \theta^4 >$$

Oblates





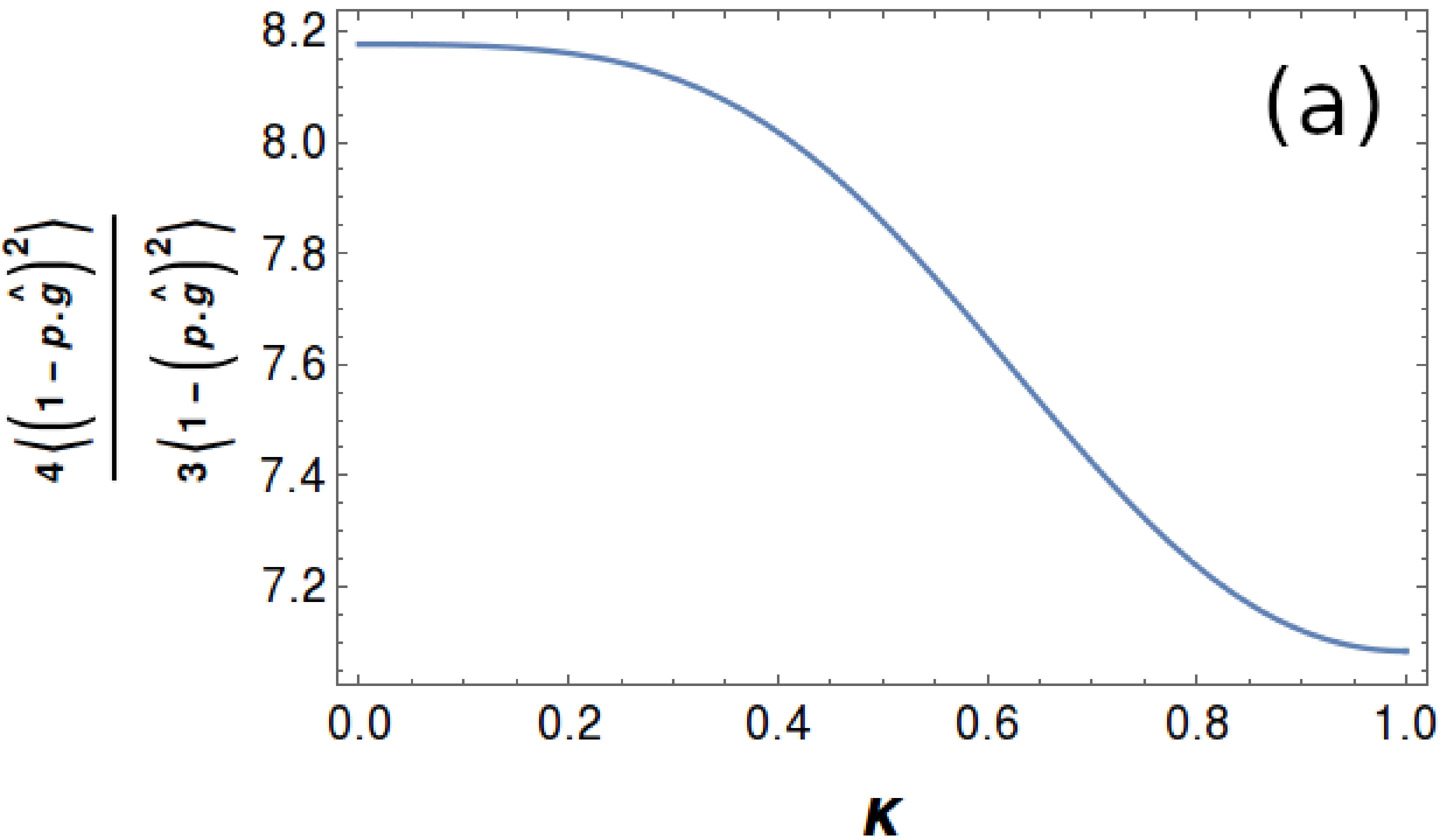
Prolates

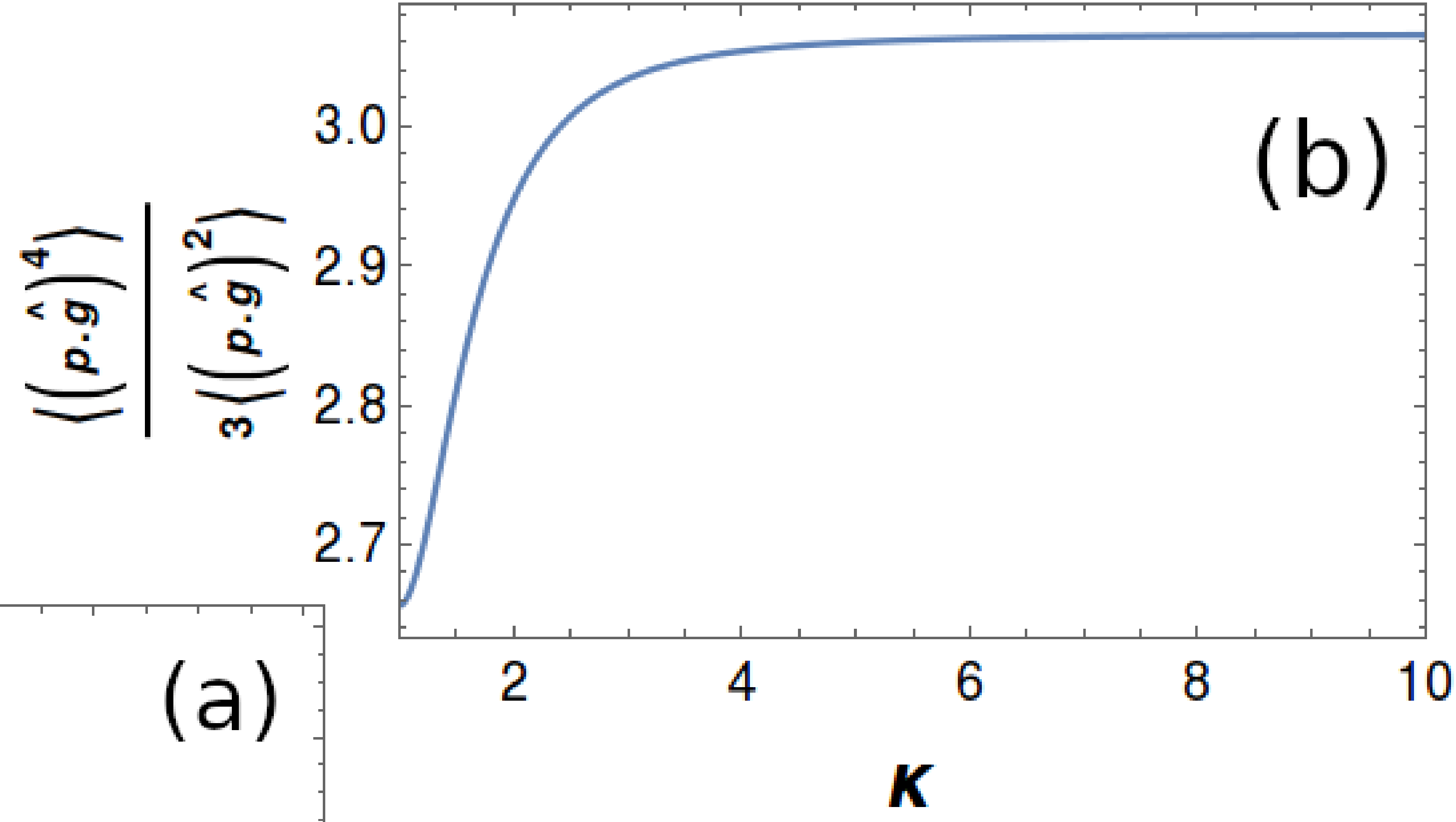
$$< (p \cdot \hat{g})^4 > \approx \; < (\frac{\pi}{2} - \theta)^4 >$$

R_λ=200

Flatness factor is well in excess of 1!

Oblates



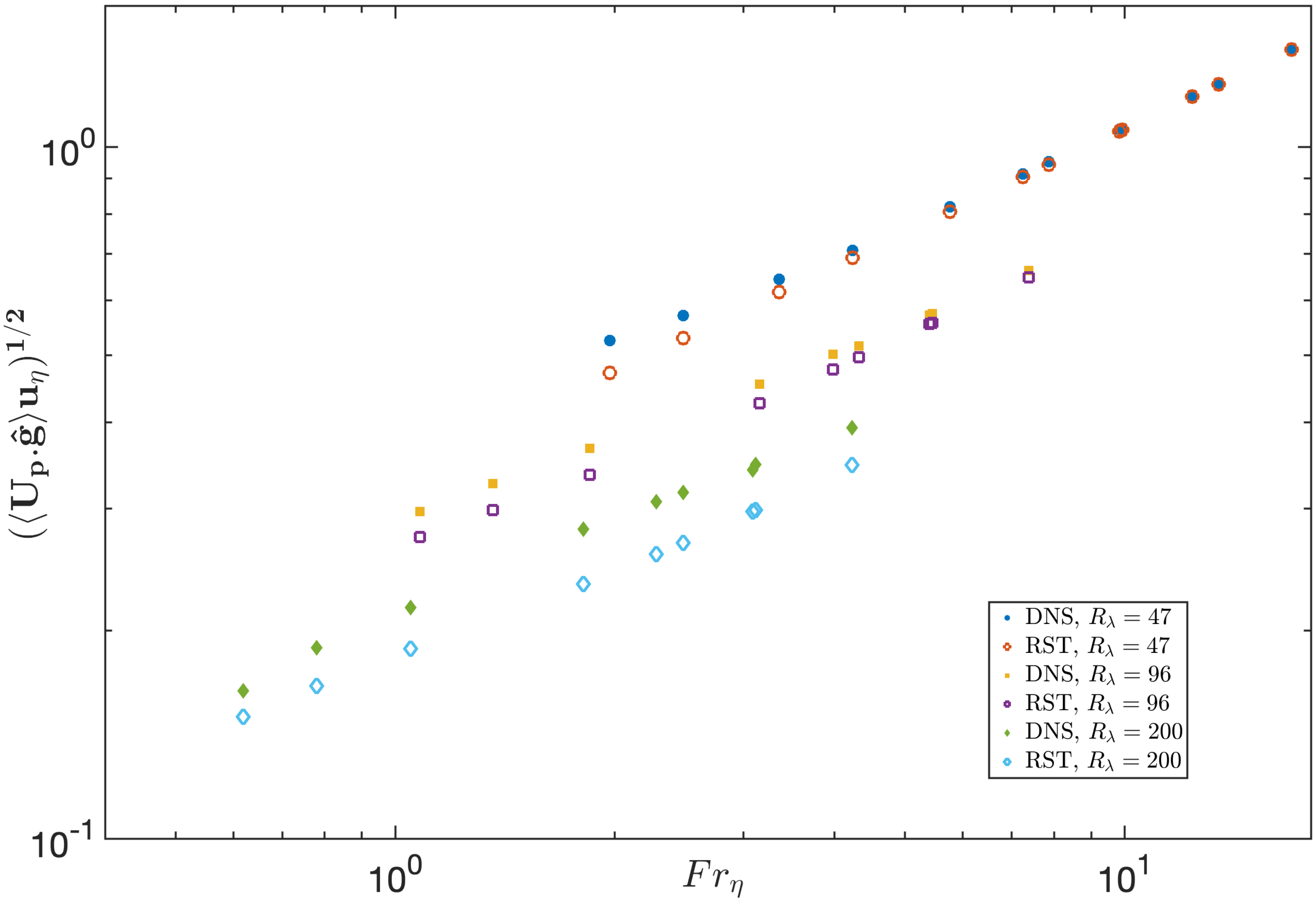


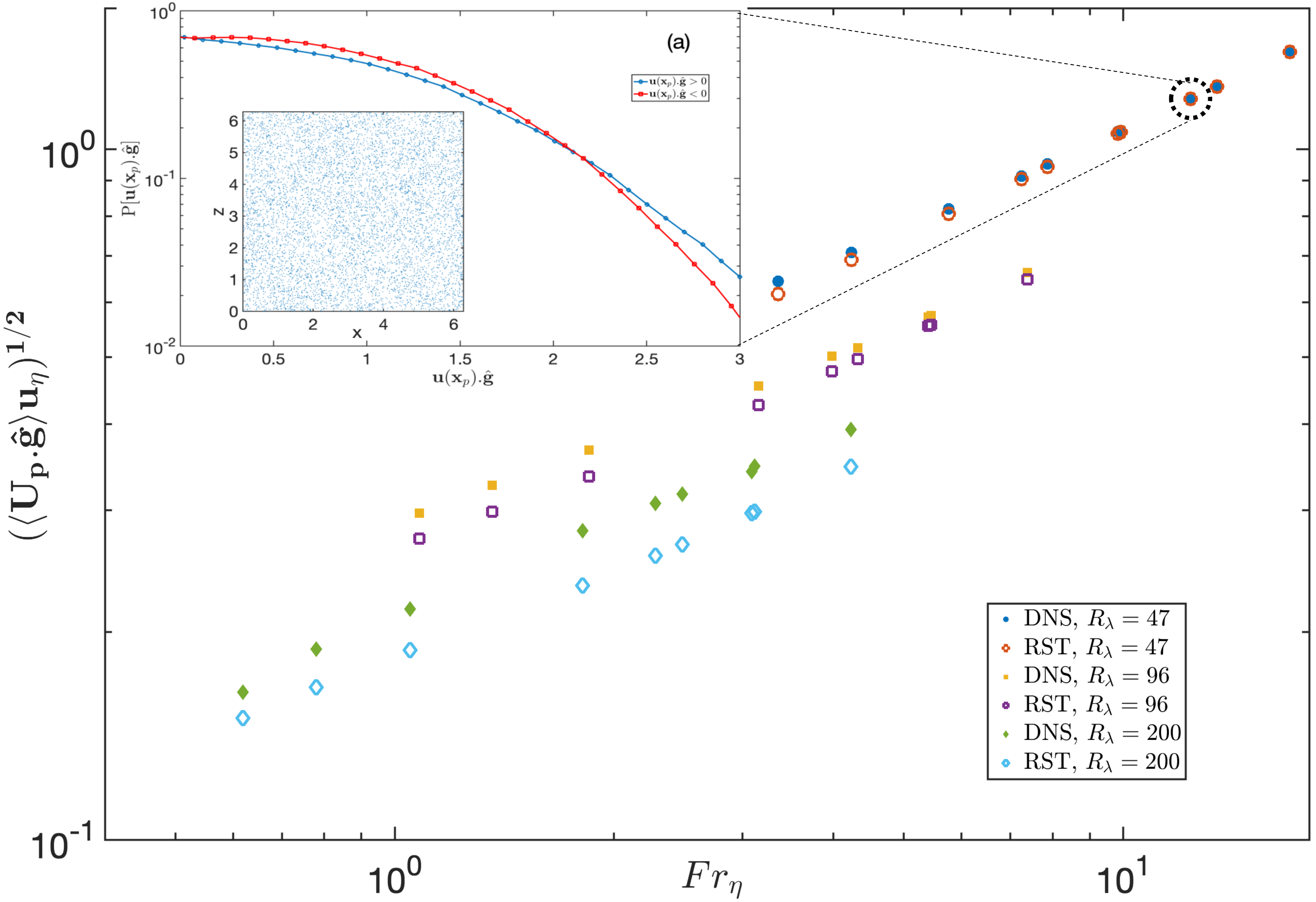
Prolates

DIRECT NUMERICAL SIMULATIONS

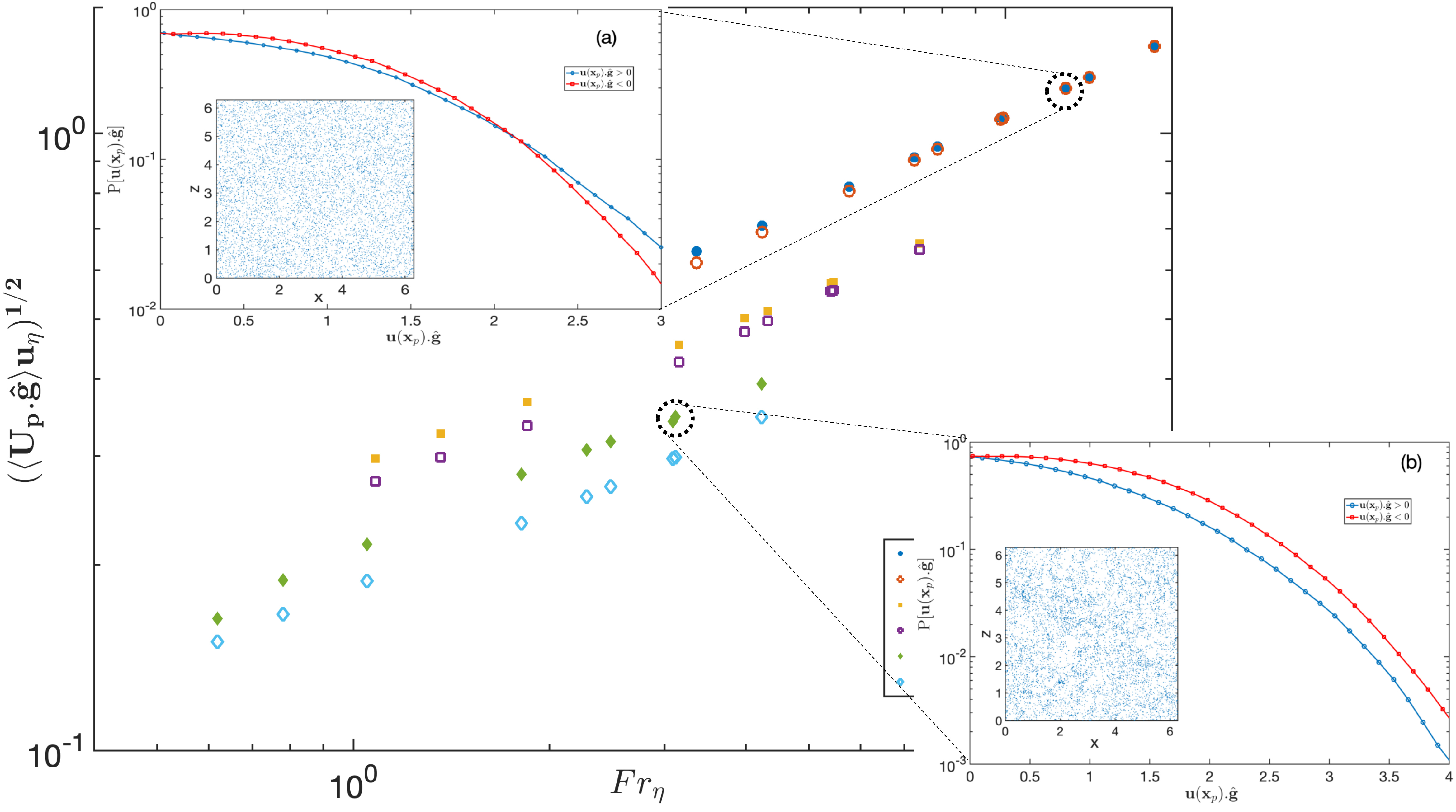
Settling velocity-Enhanced!

But WHY?

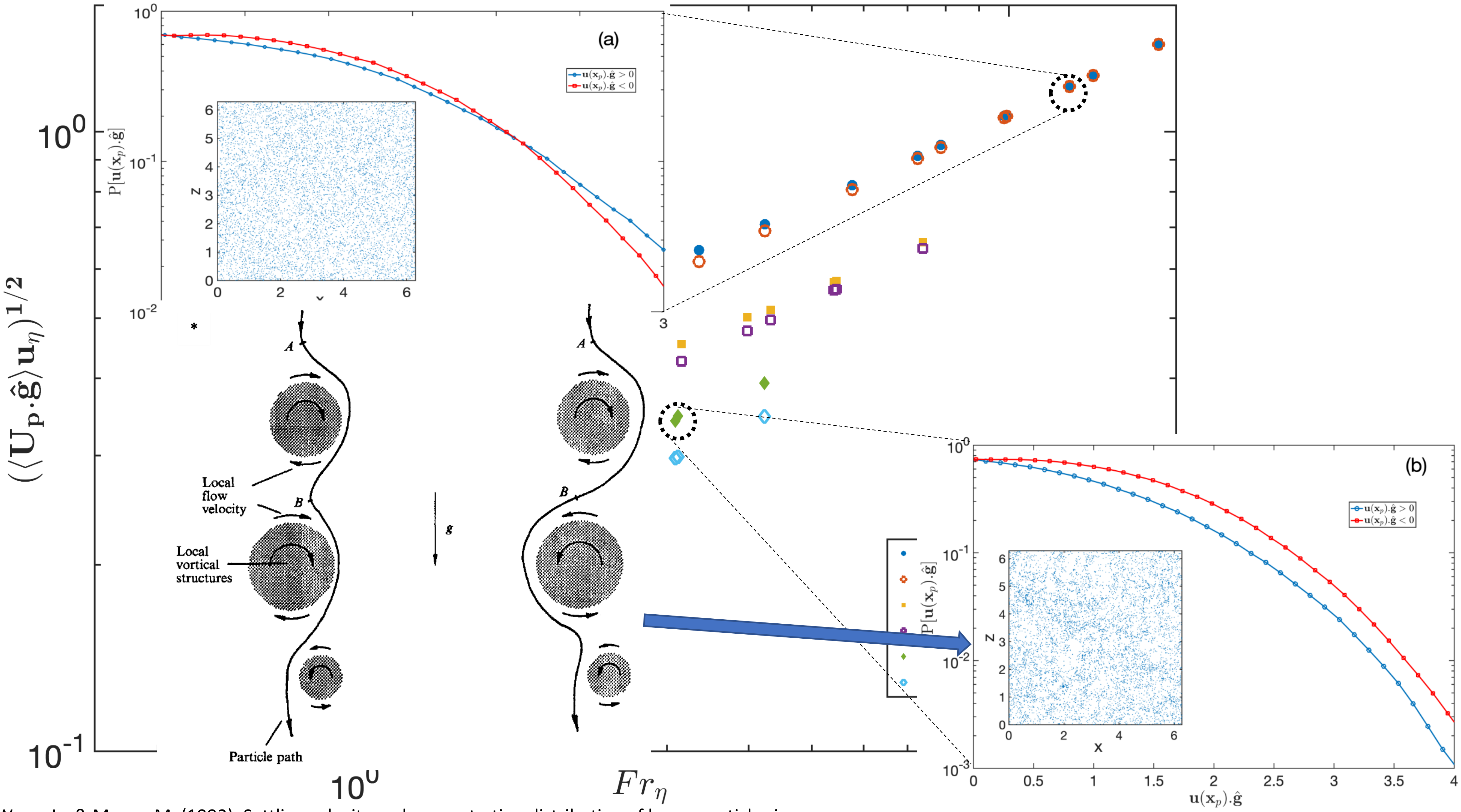




(a)
$\mathbf{u}(\mathbf{x}_p)\cdot\hat{\mathbf{g}} > 0$
$\mathbf{u}(\mathbf{x}_p)\cdot\hat{\mathbf{g}} < 0$
$P[\mathbf{u}(\mathbf{x}_p)\cdot\hat{\mathbf{g}}]$
$\mathbf{u}(\mathbf{x}_p)\cdot\hat{\mathbf{g}}$
x
z
DNS, $R_\lambda = 47$
RST, $R_\lambda = 47$
DNS, $R_\lambda = 96$
RST, $R_\lambda = 96$
DNS, $R_\lambda = 200$
RST, $R_\lambda = 200$
$(\langle \mathbf{U_p}\cdot\hat{\mathbf{g}}\rangle \mathbf{u}_\eta)^{1/2}$
Fr_η

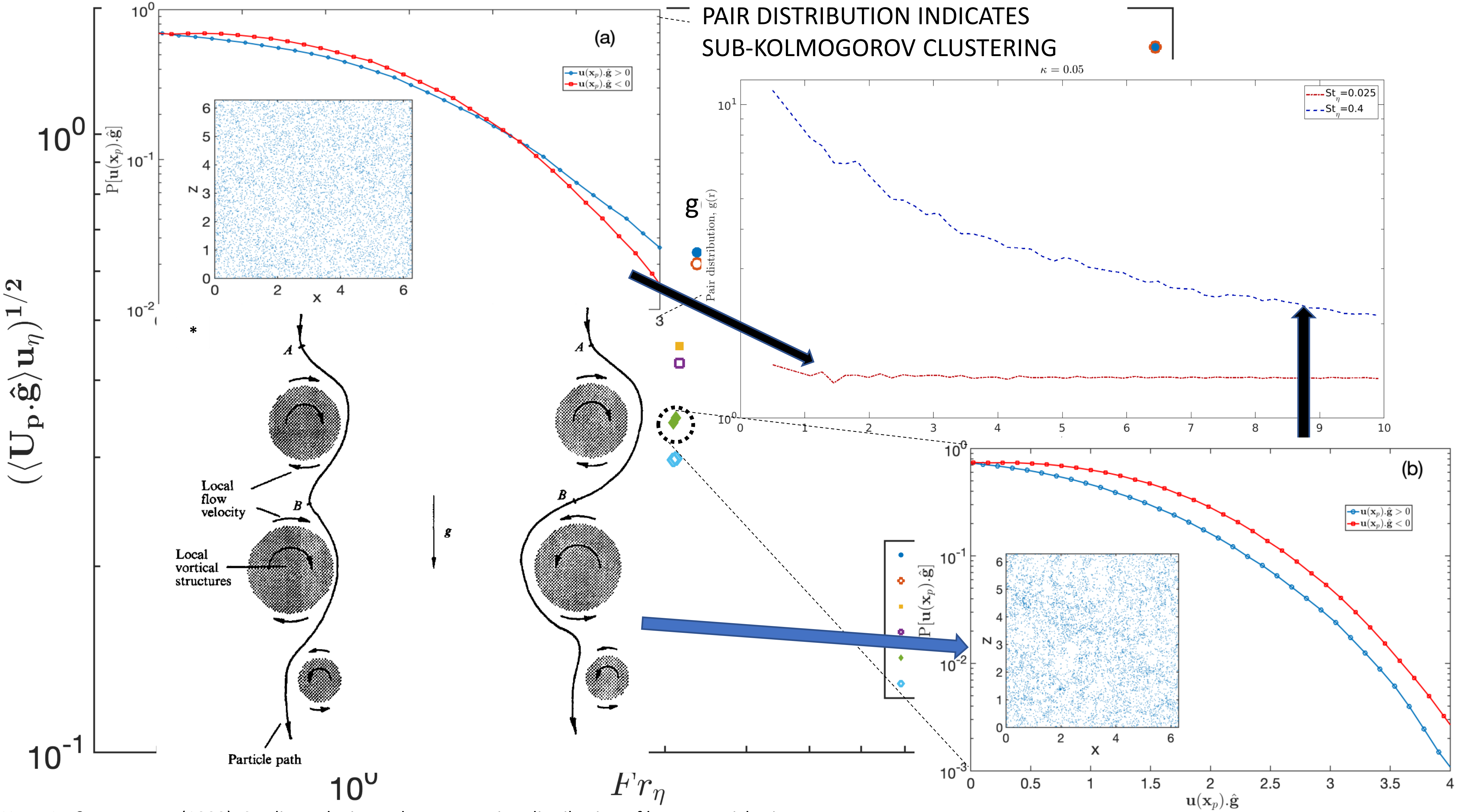


10^0
10^{-1}
$(\langle \mathbf{U_p}\cdot\hat{\mathbf{g}}\rangle \mathbf{u}_\eta)^{1/2}$
Fr_η
(a)
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}} > 0$
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}} < 0$
$\mathrm{P}[\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}}]$
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}}$
x
z
(b)
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}} > 0$
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}} < 0$
$\mathrm{P}[\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}}]$
$\mathbf{u}(\mathbf{x}_p).\hat{\mathbf{g}}$
x
z

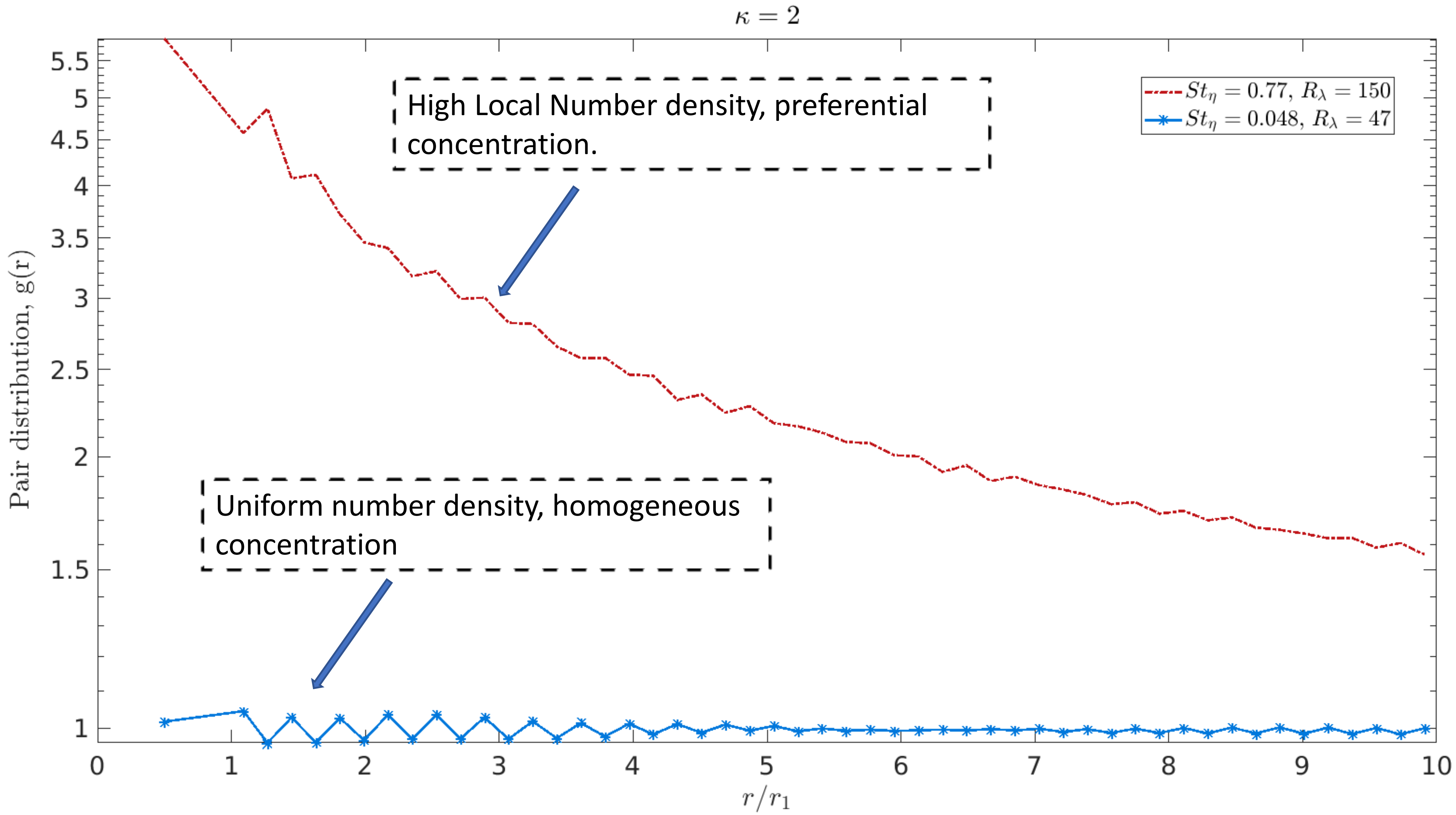


*Wang, L., & Maxey, M. (1993). Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence. *Journal of Fluid Mechanics, 256*, 27-68

PAIR DISTRIBUTION INDICATES SUB-KOLMOGOROV CLUSTERING



*Wang, L., & Maxey, M. (1993). Settling velocity and concentration distribution of heavy particles in homogeneous isotropic turbulence. *Journal of Fluid Mechanics, 256*, 27-68



$\kappa = 2$
High Local Number density, preferential concentration.
Uniform number density, homogeneous concentration
$St_\eta = 0.77,\ R_\lambda = 150$
$St_\eta = 0.048,\ R_\lambda = 47$
Pair distribution, g(r)
r/r_1
5.5
5
4.5
4
3.5
3
2.5
2
1.5
1
0
1
2
3
4
5
6
7
8
9
10

Future Work

- Study of orientation dynamics of ellipsoids settling through a turbulent field.
- How the ellipsoids sample the turbulent field?
- What about the sampling in the Q-R space?

Thank you

Questions…