INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Lagrangian statistics in rotating turbulent flows

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In collaboration with
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Turbulent particle transport

Time irreversibility

$$\partial_t \mathbf{u} + ((\mathbf{u} \cdot \nabla)\mathbf{u}) = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{f} \qquad \ldots(1),$$

$$\nabla \cdot \mathbf{u} = 0 \qquad \ldots(2),$$

$$t \to t', \quad \mathbf{u} \to -\mathbf{u}$$

Time reversal symmetry is broken

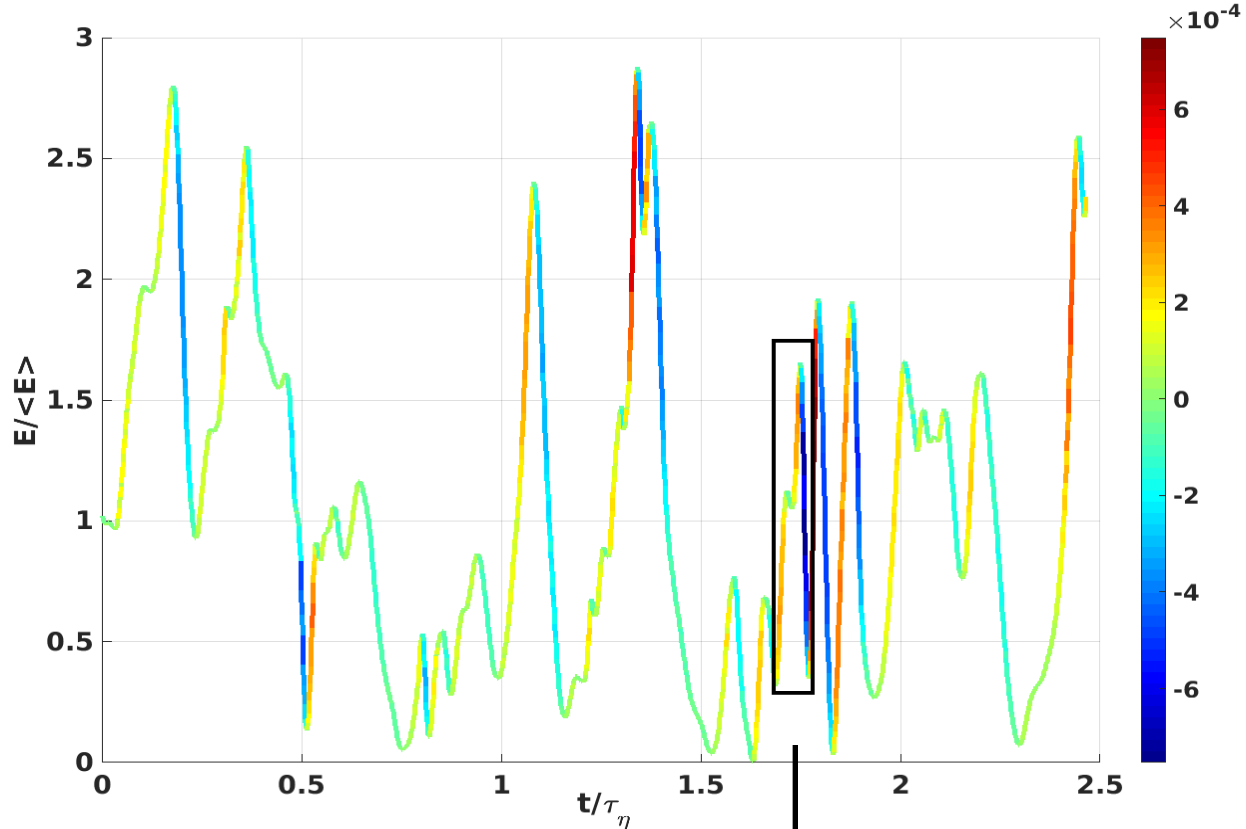
Eulerian Framework

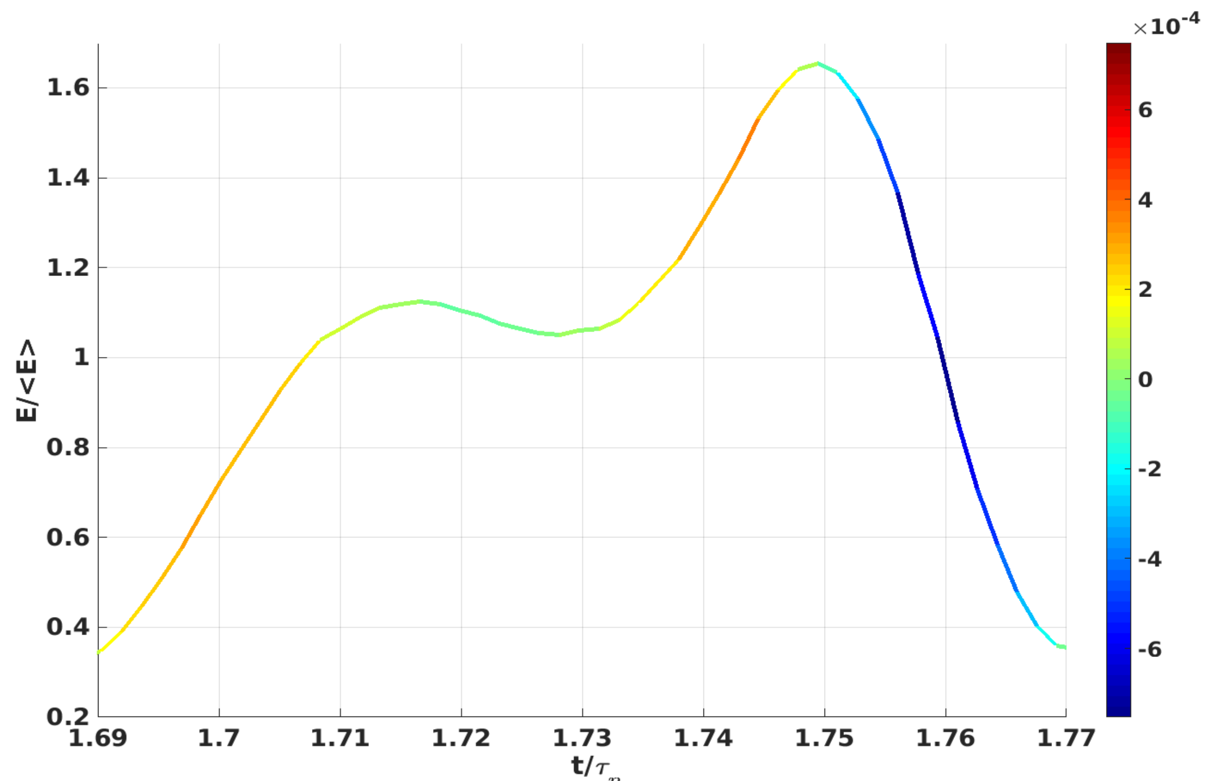
1. M. K. Verma, "Asymmetric energy transfers in driven nonequilibrium systems and arrow of time", Eur. Phys. J. B **92**, 190 (2019).

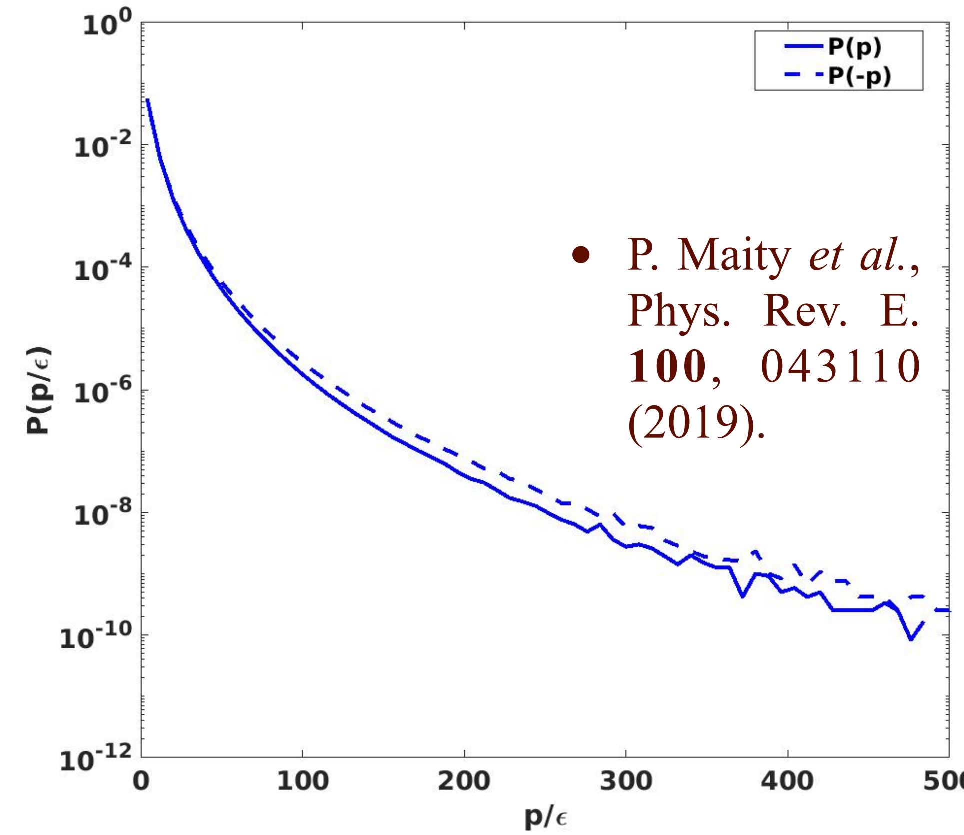
Lagrangian Framework

1. H. Xu *et al.*, Proc. Natl. Acad. Sci. (USA) **111**, 7558 (2014).
2. J. Jucha, H. Xu, A. Pumir, Phys. Rev. Lett. **113**, 054501 (2014).
3. M. Cencini, L. Biferale, G. Boffetta, M. De Pietro, Phys. Rev. Fluids 2, 104604 (2017).
4. A. Bhatnagar, A. Gupta, D. Mitra, and R. Pandit, Phys. Rev. E. **97**, 033102 (2018).

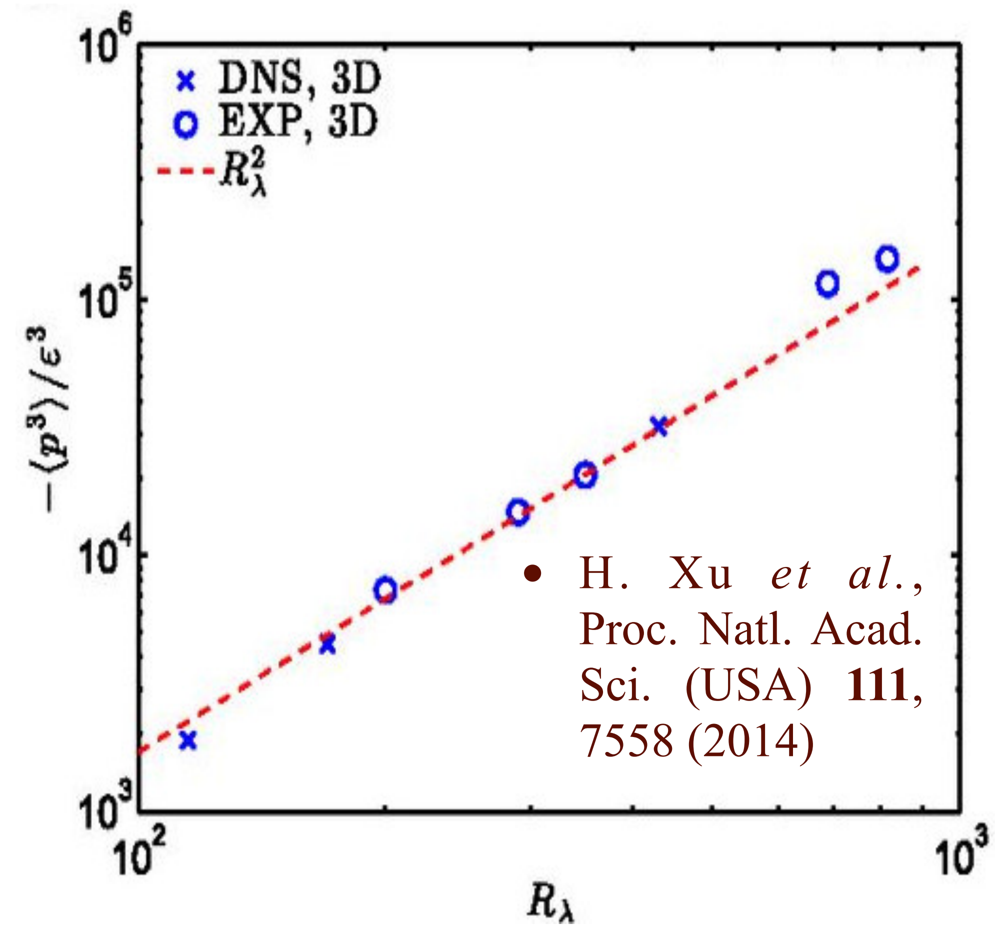
Particle trajectories of tracers in turbulent flow







- P. Maity *et al.*, Phys. Rev. E. **100**, 043110 (2019).



- H. Xu *et al.*, Proc. Natl. Acad. Sci. (USA) **111**, 7558 (2014).

Rotating turbulence

Eulerian fluid fields

$$\partial_t \mathbf{u} + ((\mathbf{u} \cdot \nabla)\mathbf{u}) + 2(\boldsymbol{\Omega} \times \mathbf{u}) = -\nabla P + \nu \nabla^2 \mathbf{u} + \mathbf{f} + \alpha \triangle^{-1} \mathbf{u} \qquad \ldots(3),$$

$$\nabla \cdot \mathbf{u} = 0 \qquad \ldots(4),$$

$$P = p - \frac{1}{2} |\boldsymbol{\Omega} \times \mathbf{r}|^2 \qquad \ldots(5),$$

Coriolis force

Large scale friction

Centrifugal force

Where **u**(x,y,z,t) = (u_1, u_2, u_3) is the flow velocity, ν is the kinematic viscosity, and P is the pressure field. The external forcing term **f** is active for k ≤ 3.

Lagrangian Particle

$$Re = \frac{uL}{\nu} \qquad \frac{d\mathbf{r_p}}{dt} = \mathbf{v_p} \qquad \ldots(6)$$

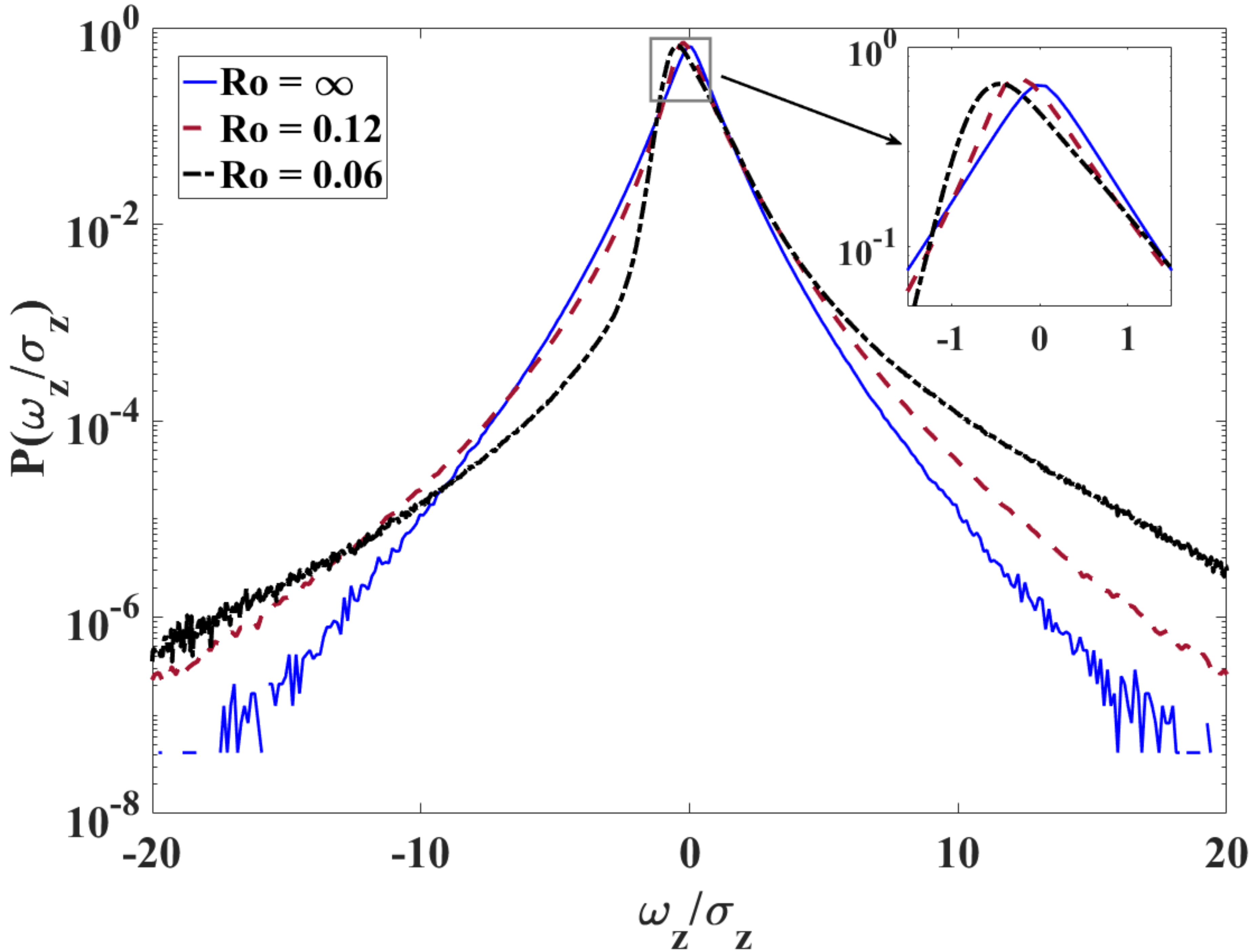
$$St = \frac{\tau_p}{\tau_\eta} \qquad \frac{d\mathbf{v_p}}{dt} = \frac{\mathbf{v_p} - \mathbf{u_p}}{\tau_p} - 2\boldsymbol{\Omega} \times \mathbf{v_p} - (\boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \mathbf{r_p}) \qquad \ldots(7)$$

$$Ro = \frac{u}{L\Omega} \qquad \tau_p \rightarrow 0, \quad \mathbf{v_p} = \mathbf{u_p} \qquad \ldots(8)$$

$\mathbf{u_p}$ is the flow velocity at the particle position , $\mathbf{V_p}$ and $\mathbf{r_p}$ are the particle velocity and position vector respectively.

1. Luca Biferale *et. al.*, "Coherent Structures and Extreme Events in Rotating Multiphase Turbulent Flows", Phys. Rev. E **6**, 041036 (2016).
2. L. Del Castello and H. J. H. Clercx, Phys. Rev. Lett. 107, 214502 (2011).
3. L. Del Castello and H. J. H. Clercx, Phys. Rev. E 83, 056316 (2011)

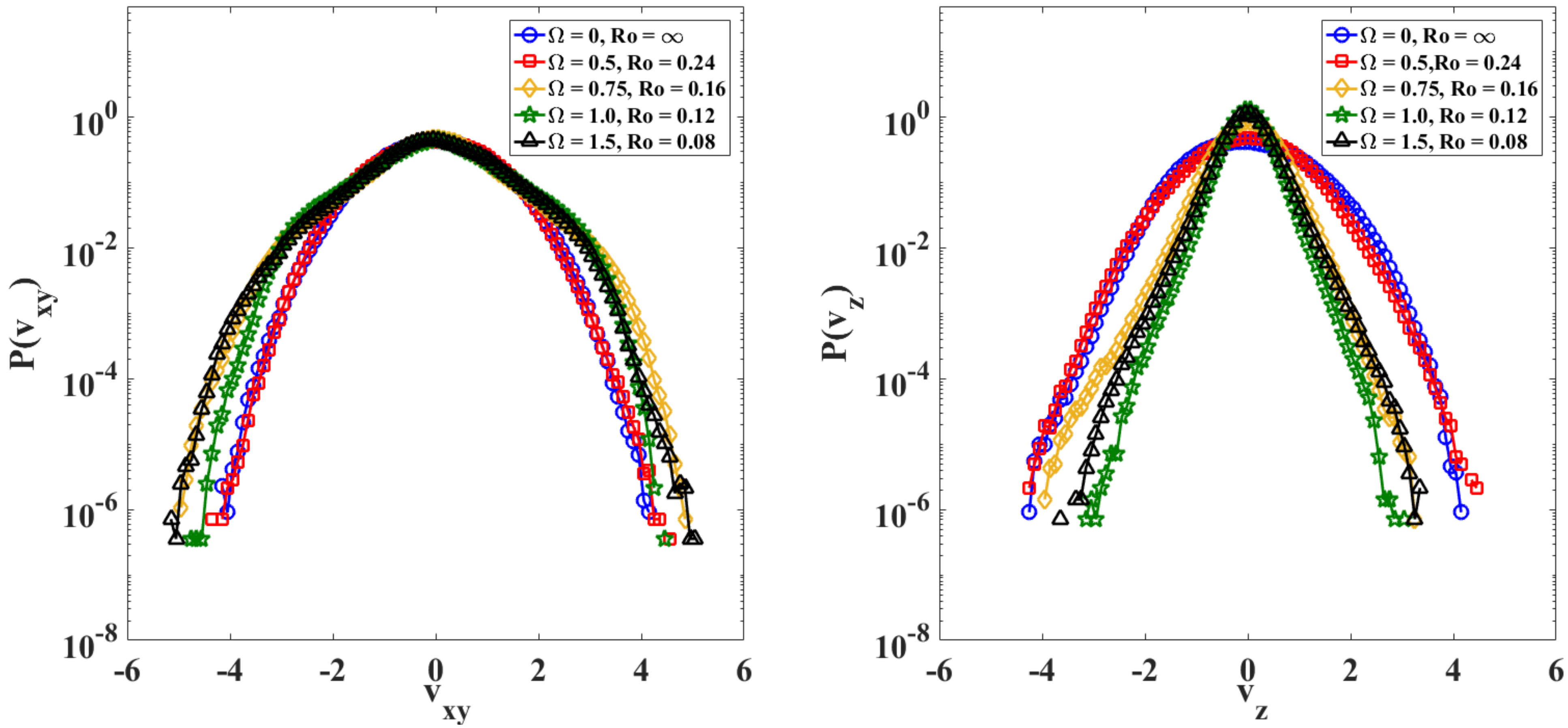
Rotational effects



- **P. Maity**, R. Govindarajan, and S. S. Ray, "*Statistics of Lagrangian trajectories in a rotating turbulent flow*", Phys. Rev. E. **100**, 043110 (2019).
- C. Morize, F. Moisy, and M. Rabaud, Phys. Fluids **17**, 095105 (2005).

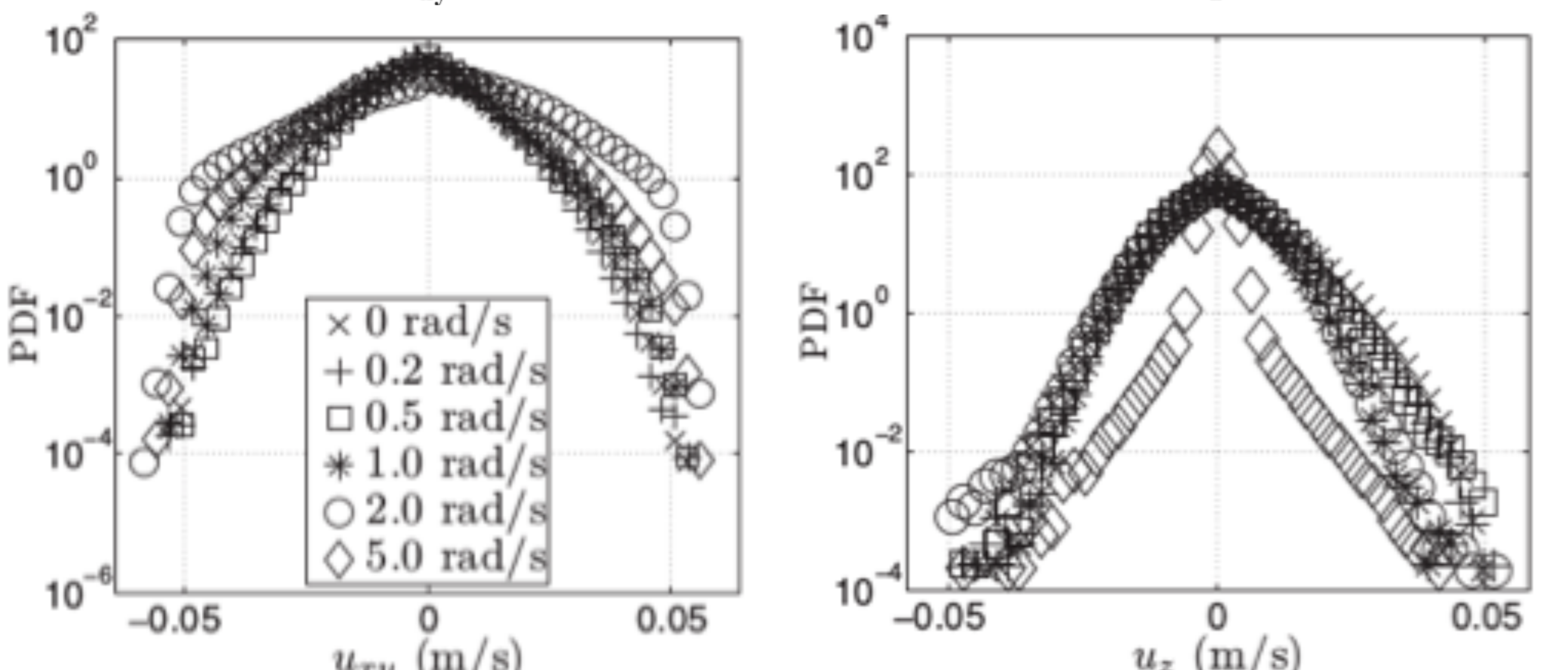
Anisotropy due to rotation

Simulations Results
Maity et. al 2020.



Experimental results

- L. D. Castello & H. J. H. Clercx, PRE **83**, 056316 (2011)



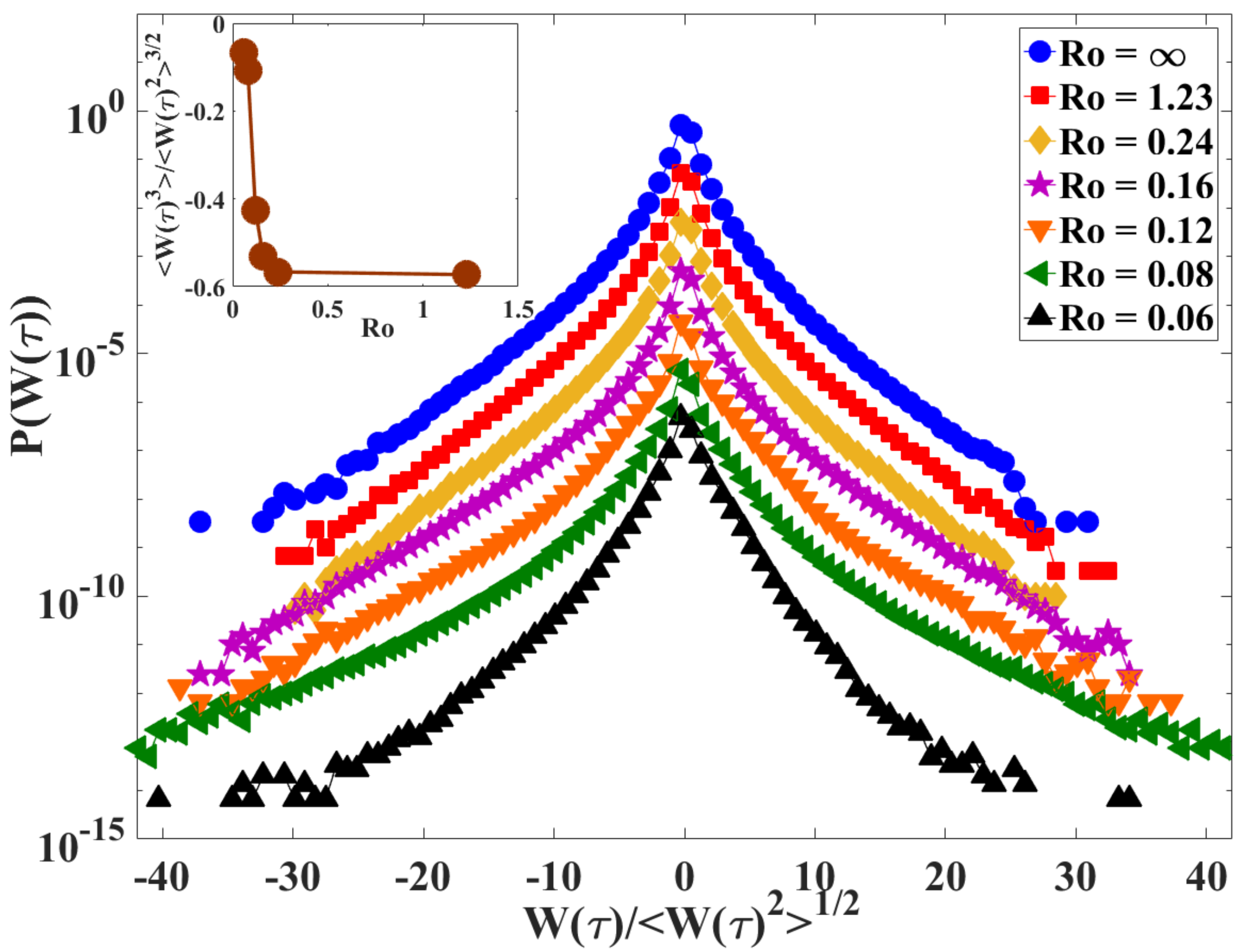
Simulation details

N Coallocation points	ν Kinematic viscosity	δt Time step	N_p Number of tracers	Re_λ Taylor Reynold's number	$k_{max}\eta$	τ_η Kolmogorov time	α Coefficient of large scale friction	ε Mean energy dissipation
512	10^{-3}	4×10^{-4}	10^6	90	2.56	3.33×10^{-3}	5×10^{-3}	0.89

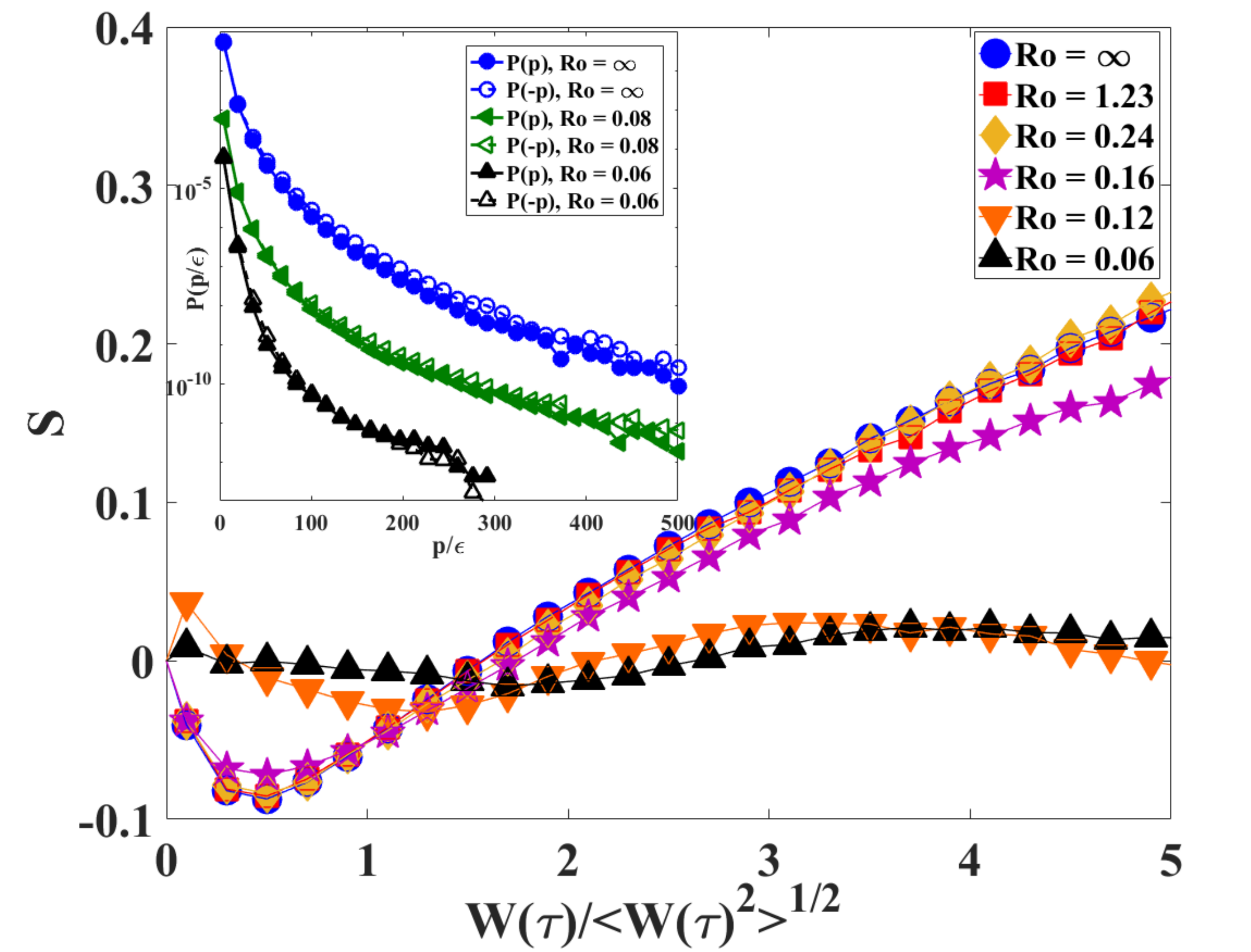
$\mathbf{\Omega}=(0,0,\Omega)$

Ω	Rossby number $Ro=U/L\Omega$
0	∞
0.1	1.43
0.5	0.24
0.75	0.16
1.0	0.12
1.5	0.08
2.0	0.06

Effect of rotation on energy increment statistics



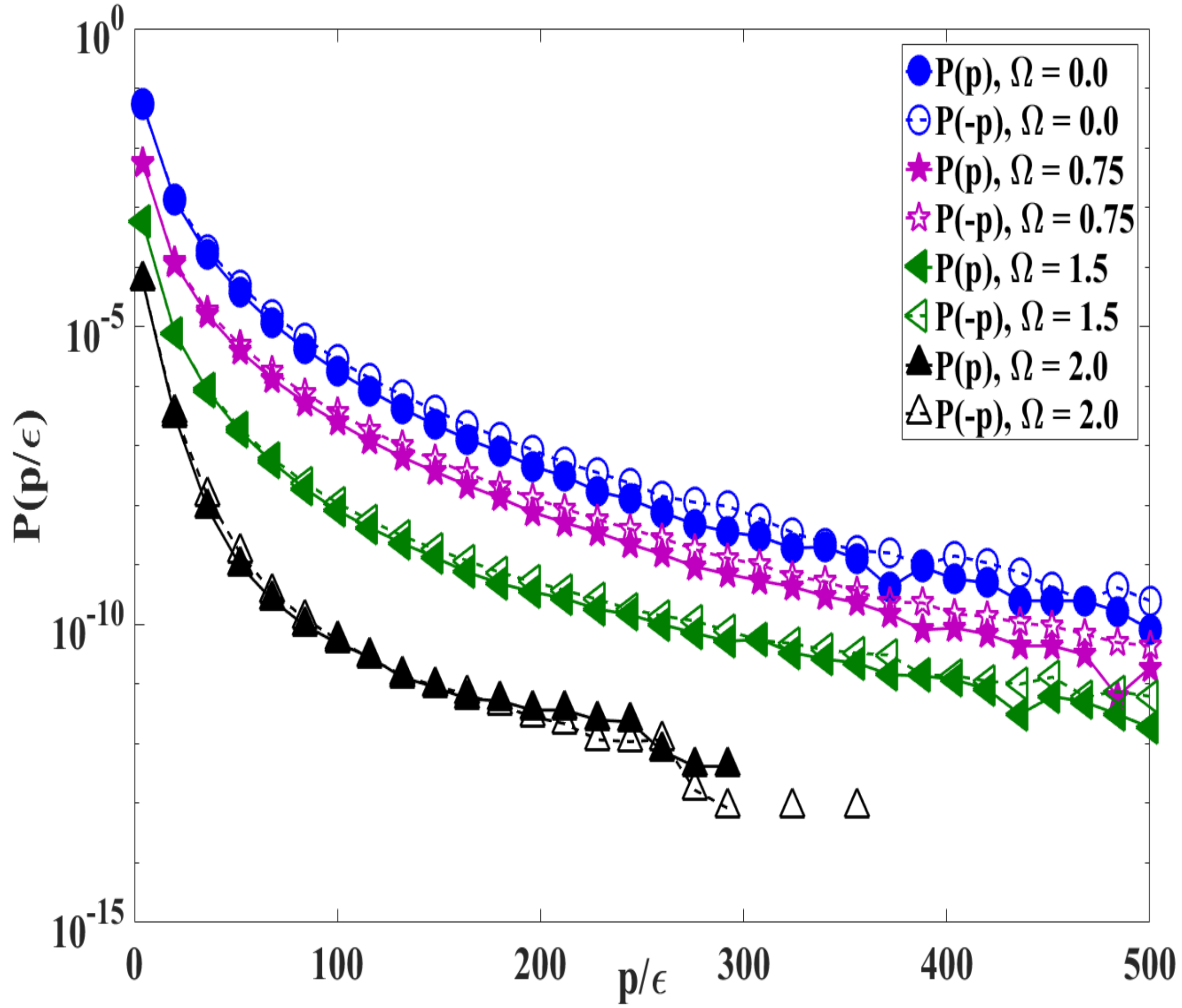
PDFs of energy increment **$W(\tau) = E(t+\tau) - E(\tau)$**, for time increments $\tau/\tau_\eta = 2$, τ_η being the Kolmogorov time scale.
The PDFs have been shifted by a factor of 10 for clarity.

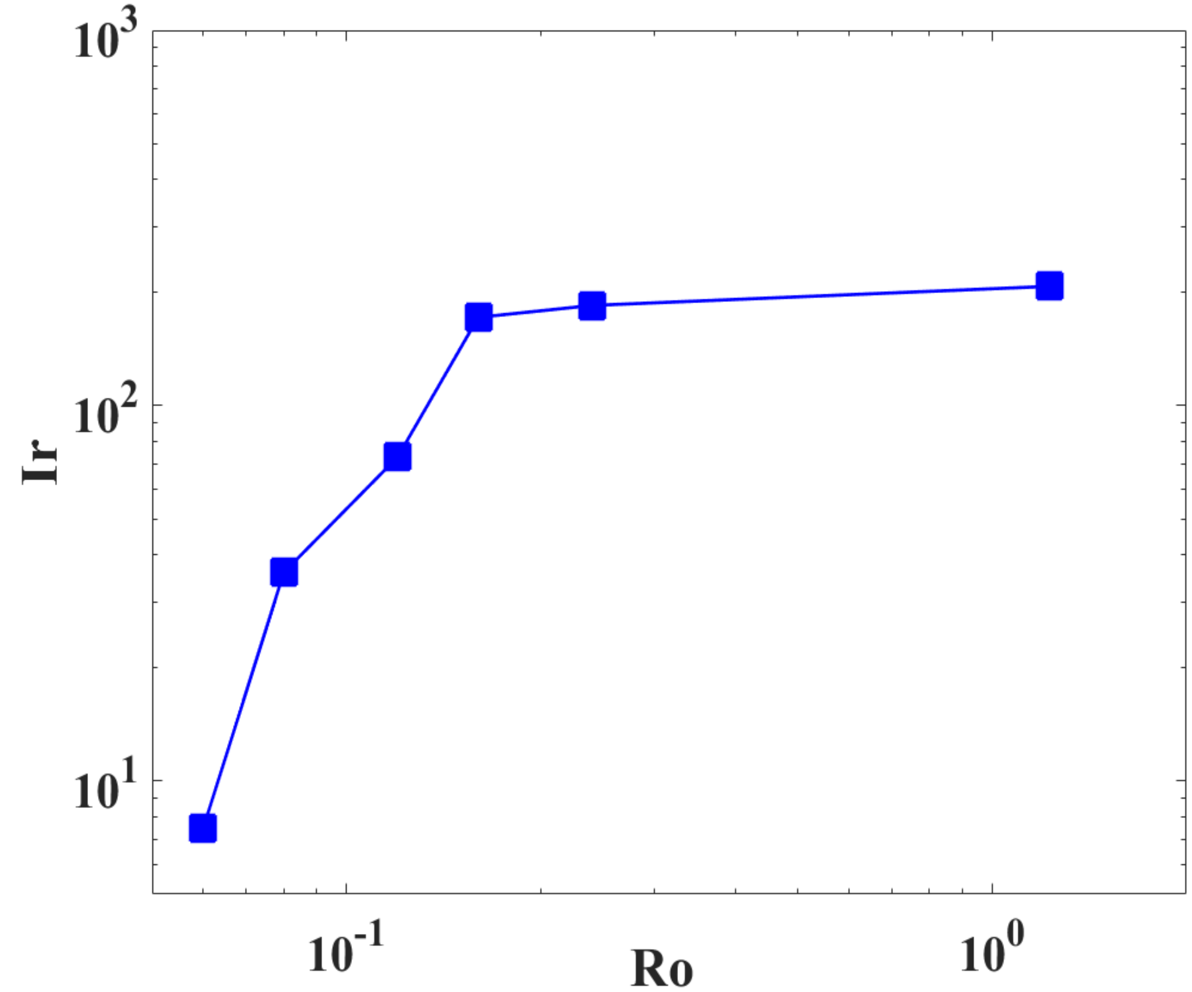


Variation of symmetry function **$S = \ln(P(-W)/P(W))$** as a function of $W(\tau)$, $\tau/\tau_\eta = 2$ for various values of rotation rates given by *Ro*.

- **P. Maity**, R. Govindarajan, and S. S. Ray, "*Statistics of Lagrangian trajectories in a rotating turbulent flow*", Phys. Rev. E. **100**, 043110 (2019).
- S. Ciliberto, S. Joubaud, and A. Petrosyan, J. Stat. Mech. P12003 (2010).

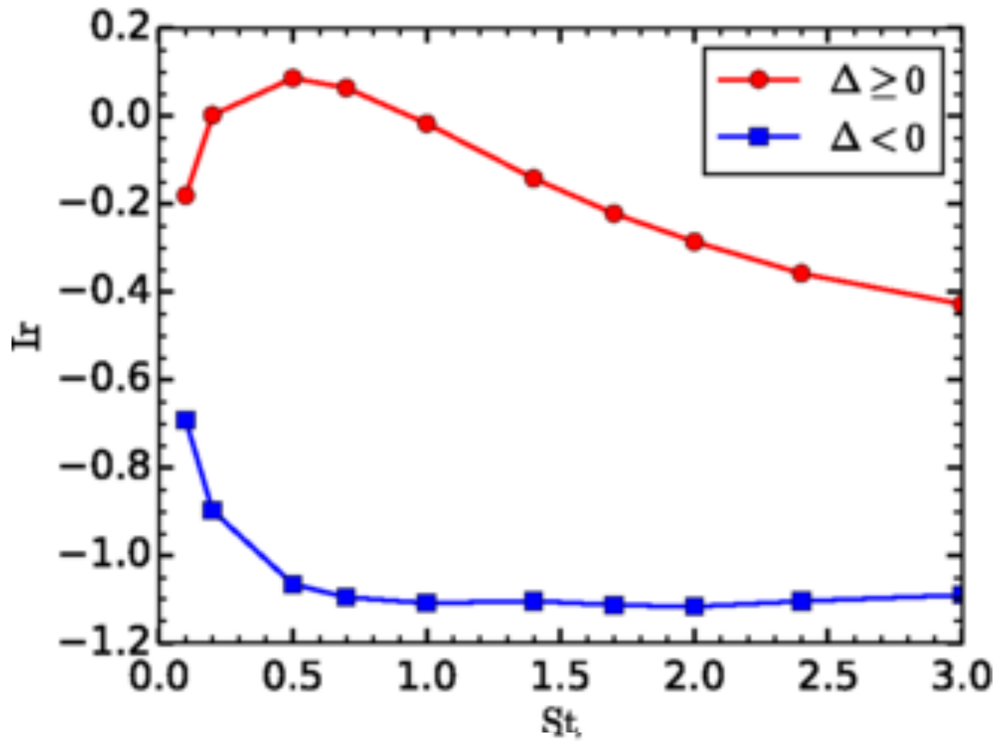
Irreversibility and rotation

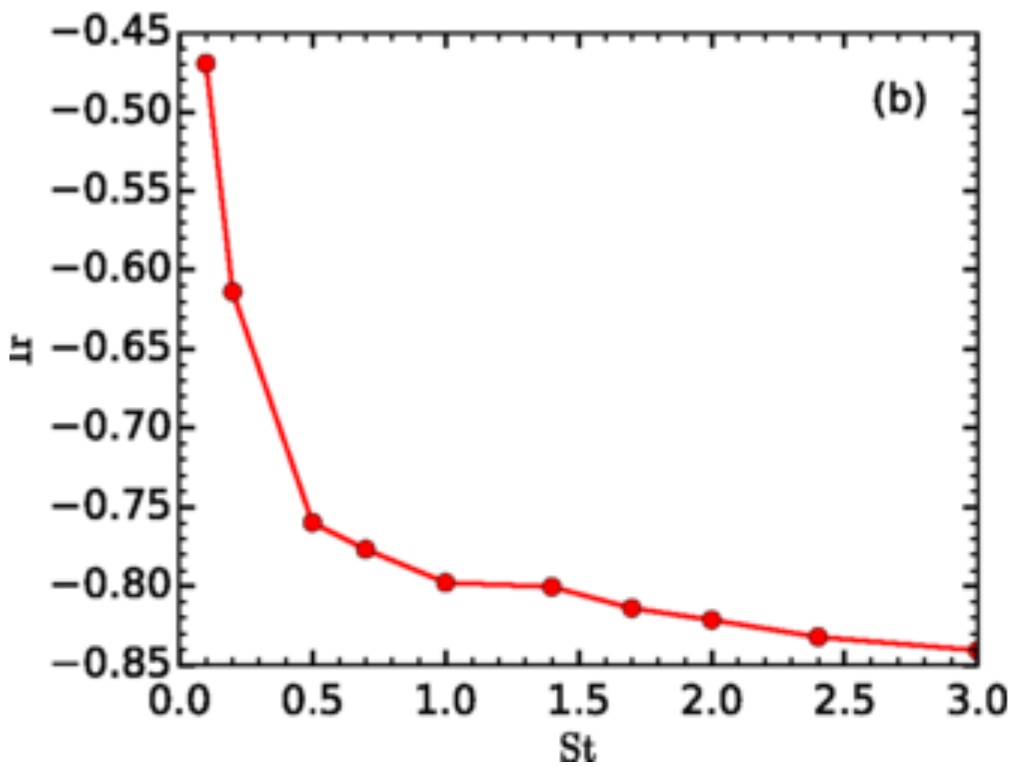




P. Maity, R. Govindarajan, and S. S. Ray, "*Statistics of Lagrangian trajectories in a rotating turbulent flow*", Phys. Rev. E. **100**, 043110 (2019).

Lagrangian Irreversibility (finite St effects)





Irreversibility as a function of Stokes number from Bhatnagar *et al*.

- A. Bhatnagar, A. Gupta, D. Mitra, and R. Pandit, Phys. Rev. E **97**, 033102 (2018).

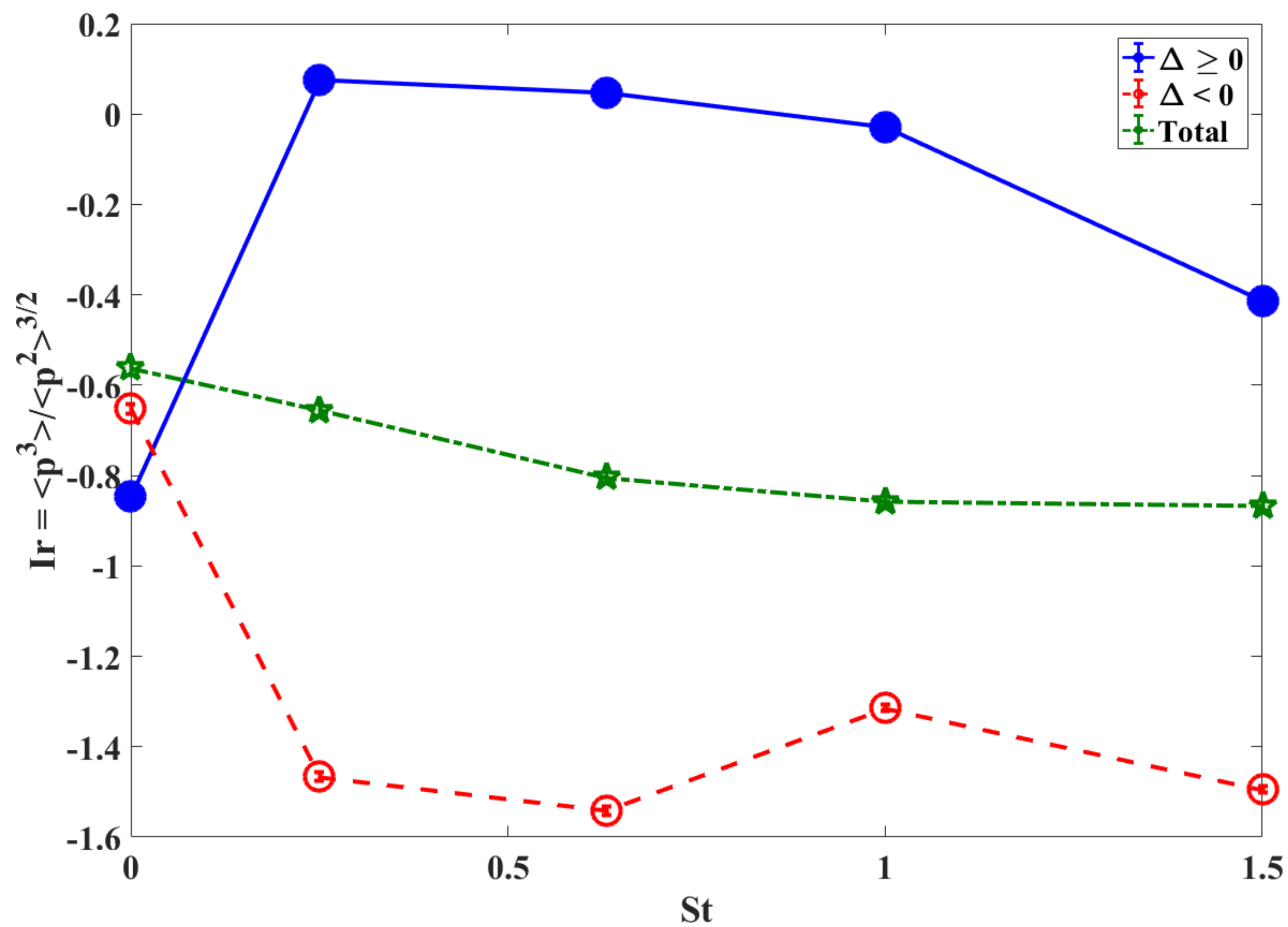
$$\Delta = \frac{27}{4}R^2 + Q^3$$

$\Delta \geq 0$ vortical, $\Delta < 0$ straining

$$Q = \frac{1}{2}(||\omega||^2 - ||S||^2), R = -\frac{1}{4}\omega_i S_{ij}\omega_j - ||S||^3$$

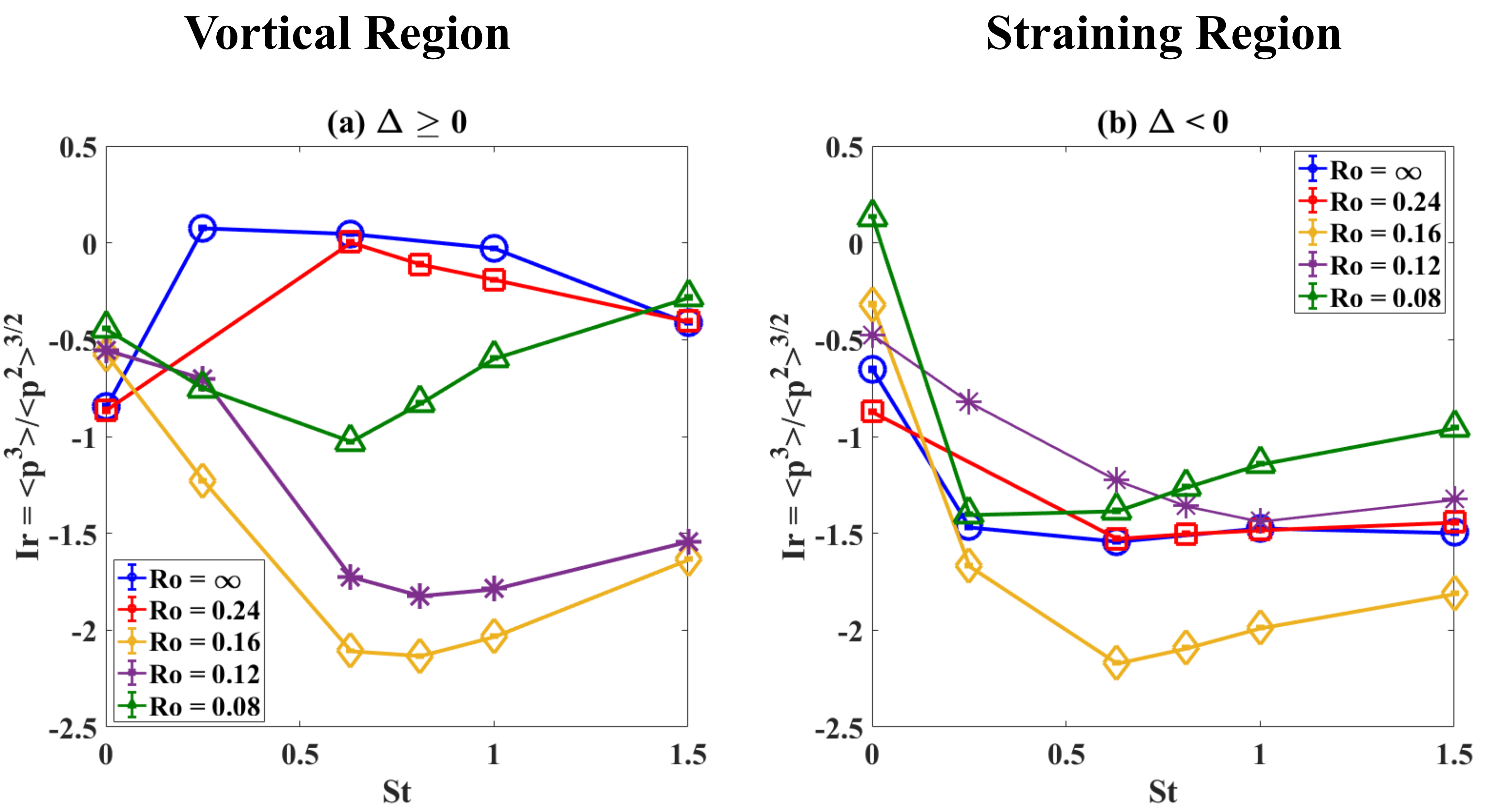
$$||\omega|| = tr[\omega\omega^T], ||\mathbf{S}|| = tr[\mathbf{S}\mathbf{S}^T]$$

$$\mathbf{S} = \frac{1}{2}[\nabla\mathbf{u} + \nabla\mathbf{u}^T], \omega = \frac{1}{2}[\nabla\mathbf{u} - \nabla\mathbf{u}^T]$$



Skewness of Lagrangian power as a function of Stokes number (our simulations).

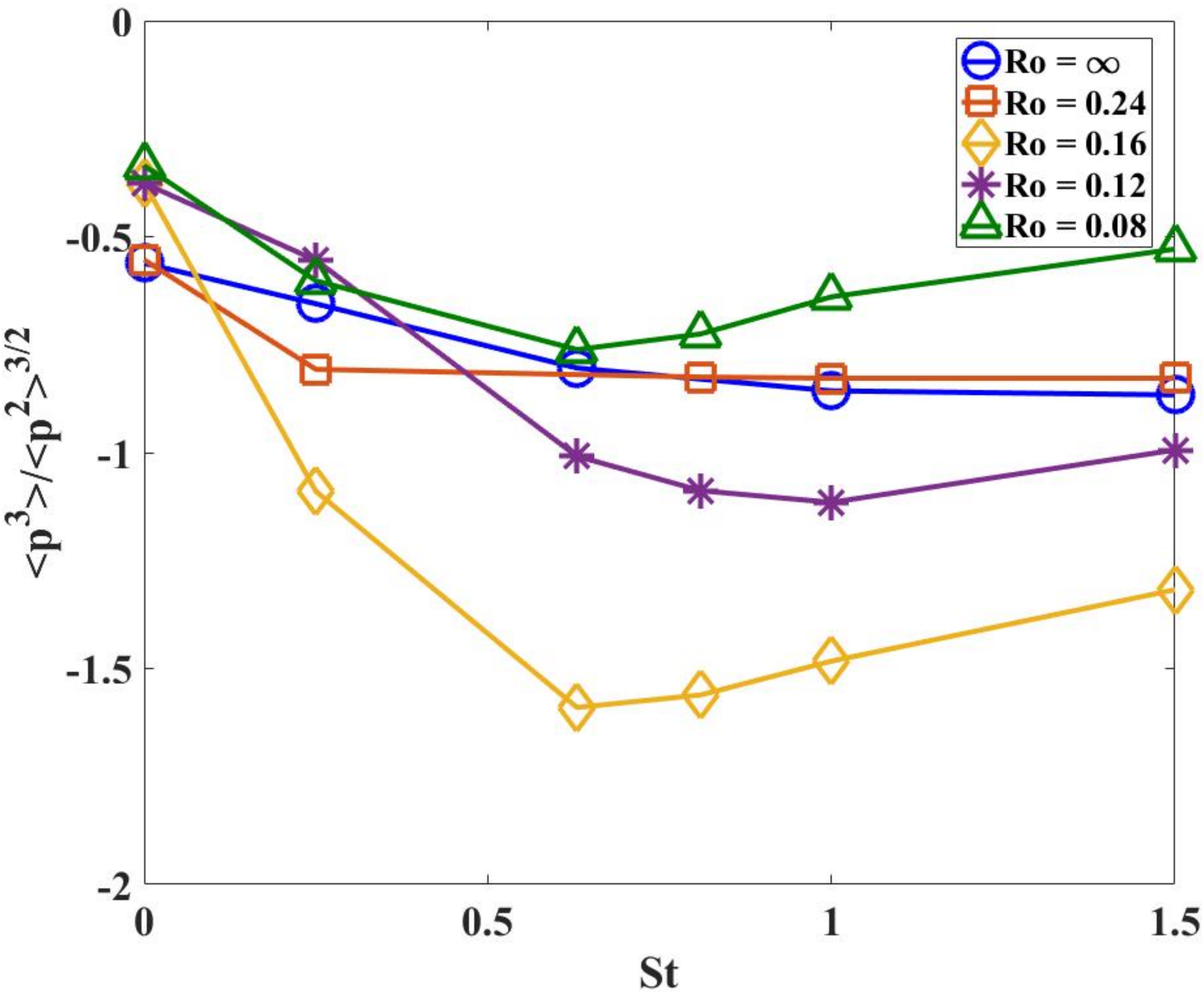
Lagrangian irreversibility with finite St with rotation



Lagrangian irreversibility as a function of Stokes number for various rotation rates (Maity *et al.* 2020)

Lagrangian irreversibility with finite St with rotation

Total



Total Lagrangian irreversibility as a function of Stokes number for various values of rotation rates (Maity *et. al.* 2020)

References:

1. U. Frisch, Turbulence : The legacy of A. N. Kolmogorov (Cambridge University Press, 1995).
2. H. Xu *et al*., Proc. Natl. Acad. Sci. (USA) **111**, 7558 (2018).
3. A. Bhatnagar, A. Gupta, D. Mitra, and R. Pandit, Phys. Rev. E. **97**, 033102 (2018).
4. **P. Maity**, R. Govindarajan, and S. S. Ray, "Statistics of Lagrangian trajectories in a rotating turbulent flow", Phys. Rev. E. **100**, 043110 (2019).
5. Luca Biferale *et. al*., "Coherent Structures and Extreme Events in Rotating Multiphase Turbulent Flows", Phys. Rev. E **6**, 041036 (2016).
6. L. Del Castello and H. J. H. Clercx, Phys. Rev. Lett. 107, 214502 (2011).
7. L. Del Castello and H. J. H. Clercx, Phys. Rev. E 83, 056316 (2011)

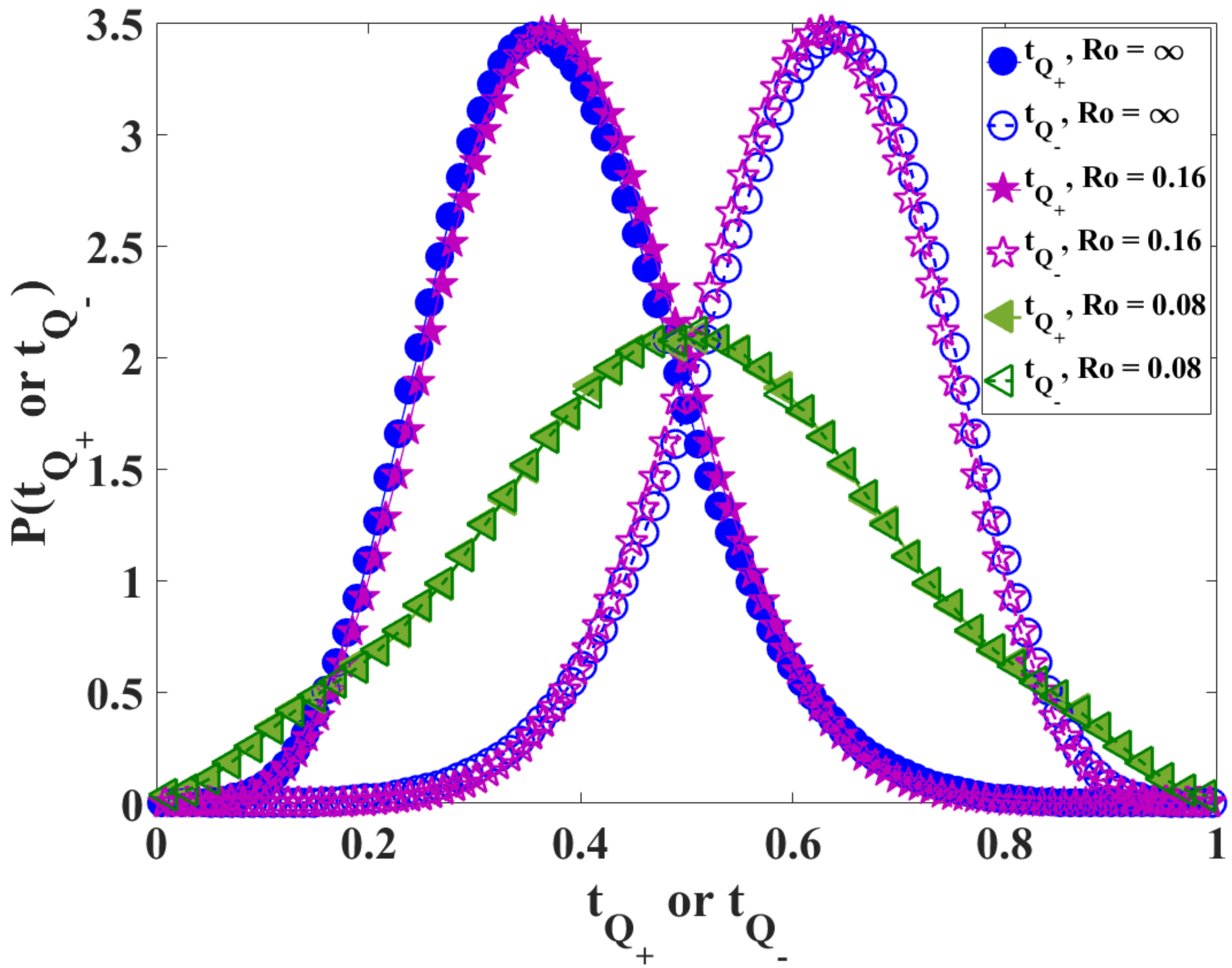
Thanks for you attention

Statistics of Lagrangian Trajectories

$$Q = \frac{1}{2}(||\omega||^2 - ||S||^2), R = -\frac{1}{4}\omega_i S_{ij}\omega_j - ||S||^3$$

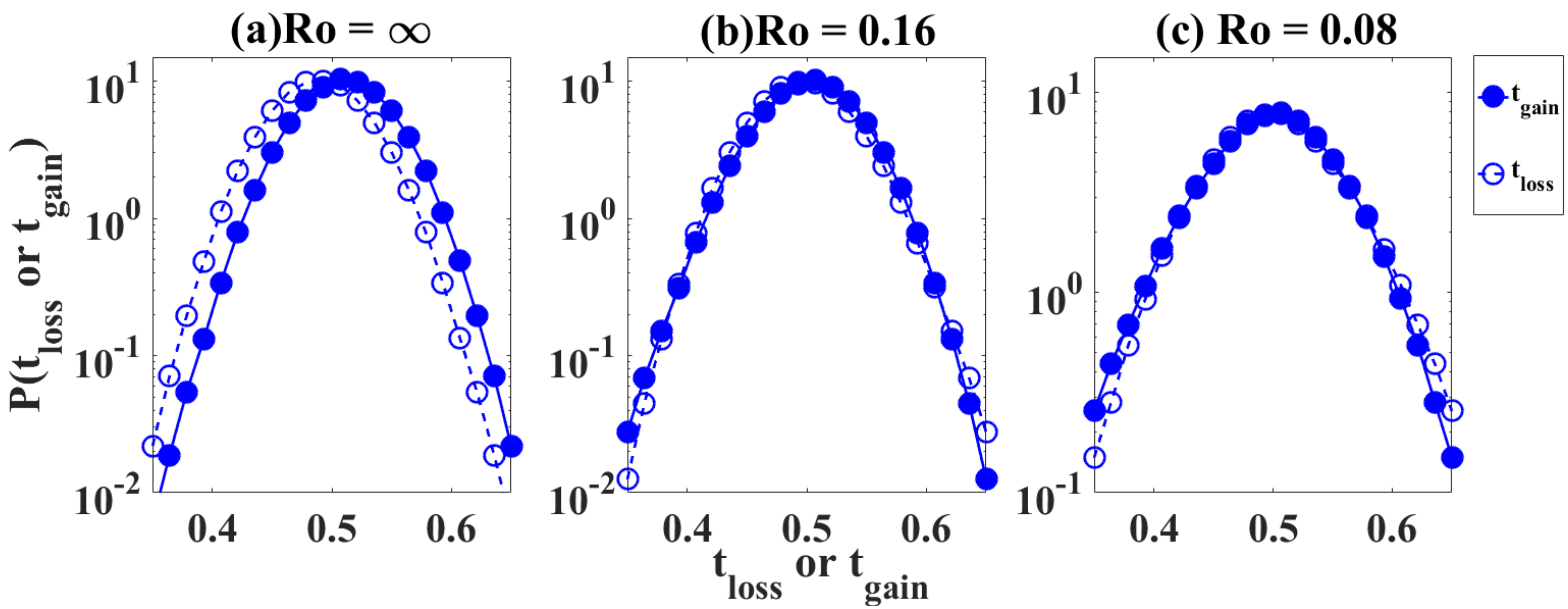
$$||\omega|| = tr[\omega\omega^T], ||\mathbf{S}|| = tr[\mathbf{S}\mathbf{S}^T]$$

$$\mathbf{S} = \frac{1}{2}[\nabla\mathbf{u} + \nabla\mathbf{u}^T], \omega = \frac{1}{2}[\nabla\mathbf{u} - \nabla\mathbf{u}^T]$$



- Akshay Bhatnagar, Anupam Gupta, Dhrubaditya Mitra, Rahul Pandit, and Prasad Perlekar, "How long do particles spend in vortical regions in turbulent flows?", Phys. Rev. E **94**, 053119 (2016).

Statistics of Lagrangian power



P. Maity, R. Govindarajan, and S. S. Ray, "*Statistics of Lagrangian trajectories in a rotating turbulent flow*", Phys. Rev. E. **100**, 043110 (2019).

Flow dynamics

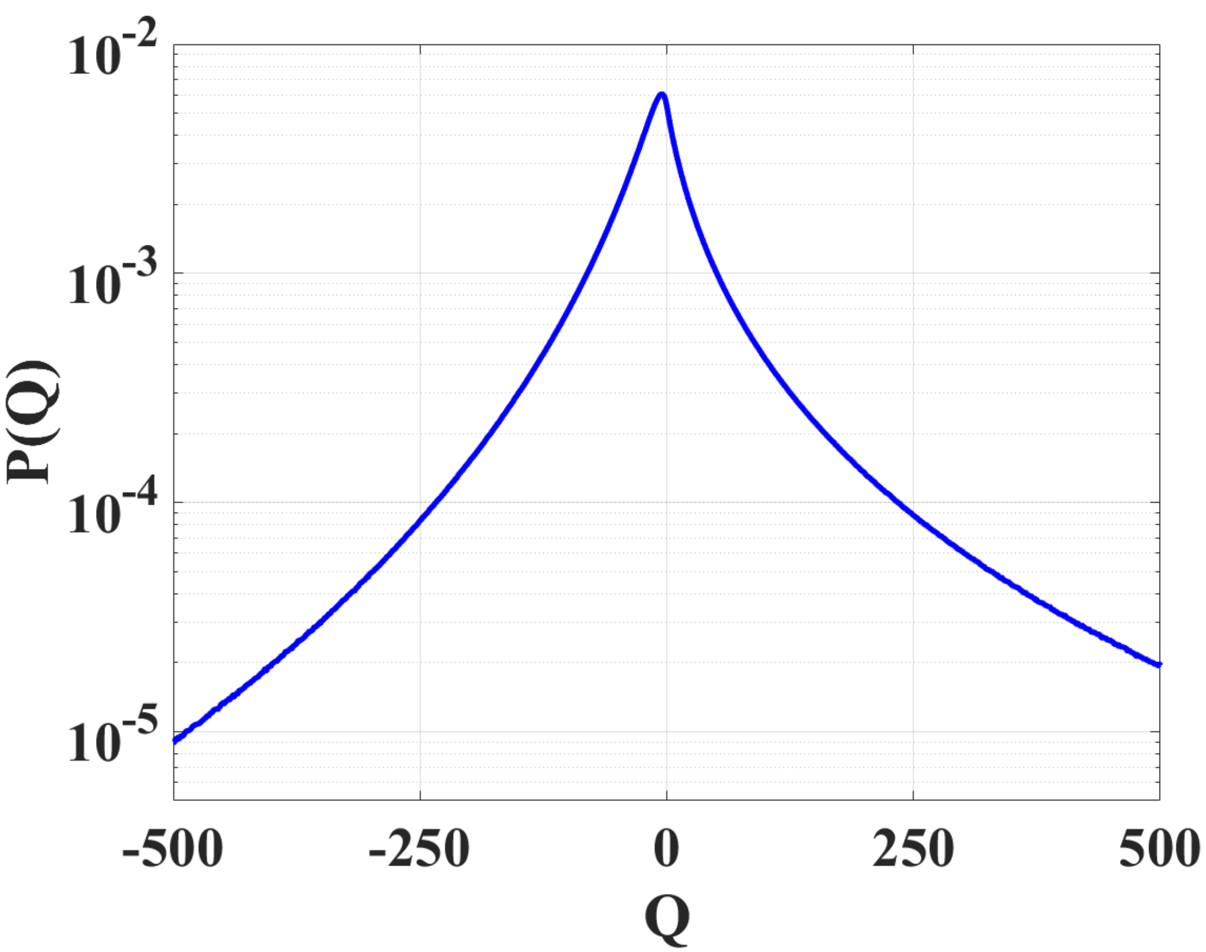
$$Q = \frac{1}{2}(||\omega||^2 - ||S||^2)$$

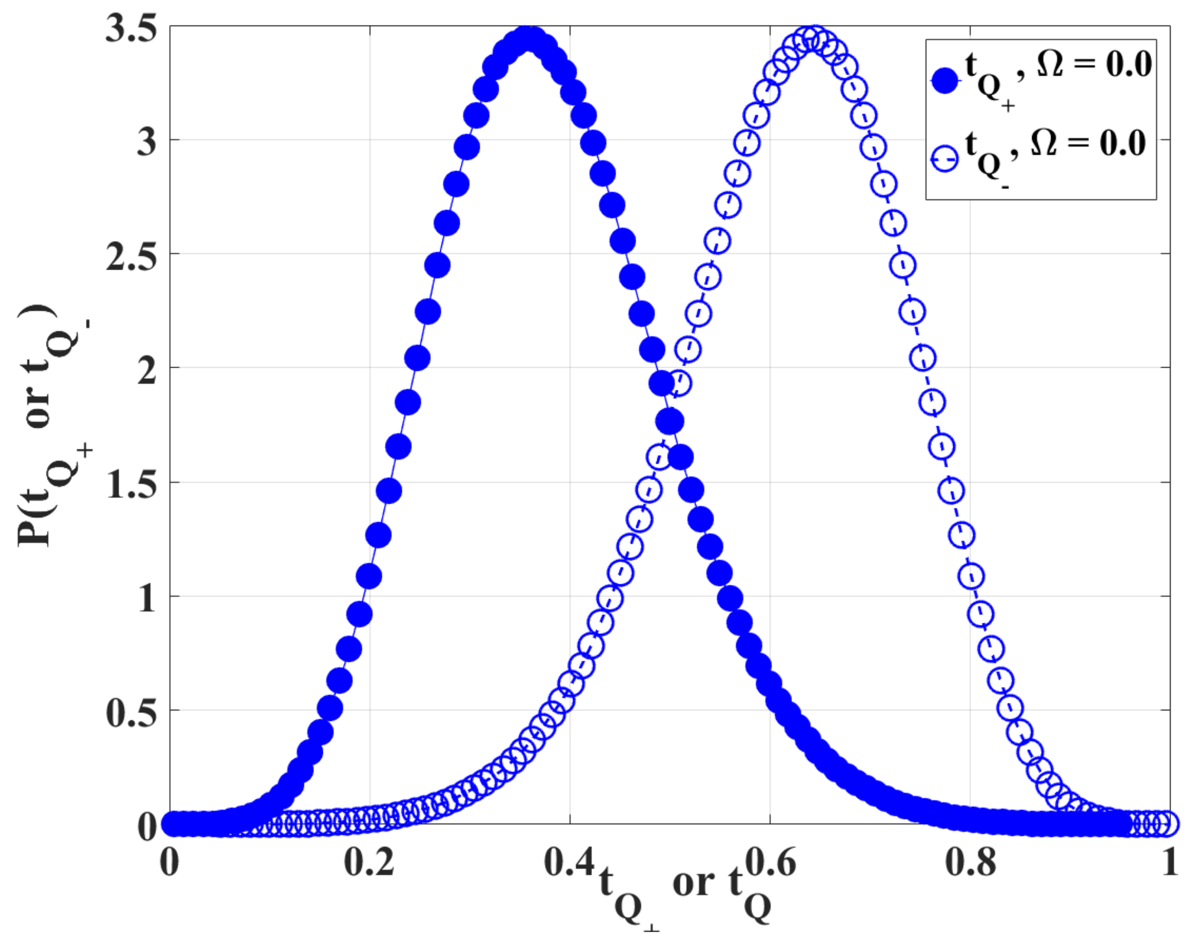
$$||\omega|| = tr[\omega\omega^T], ||\mathbf{S}|| = tr[\mathbf{S}\mathbf{S}^T]$$

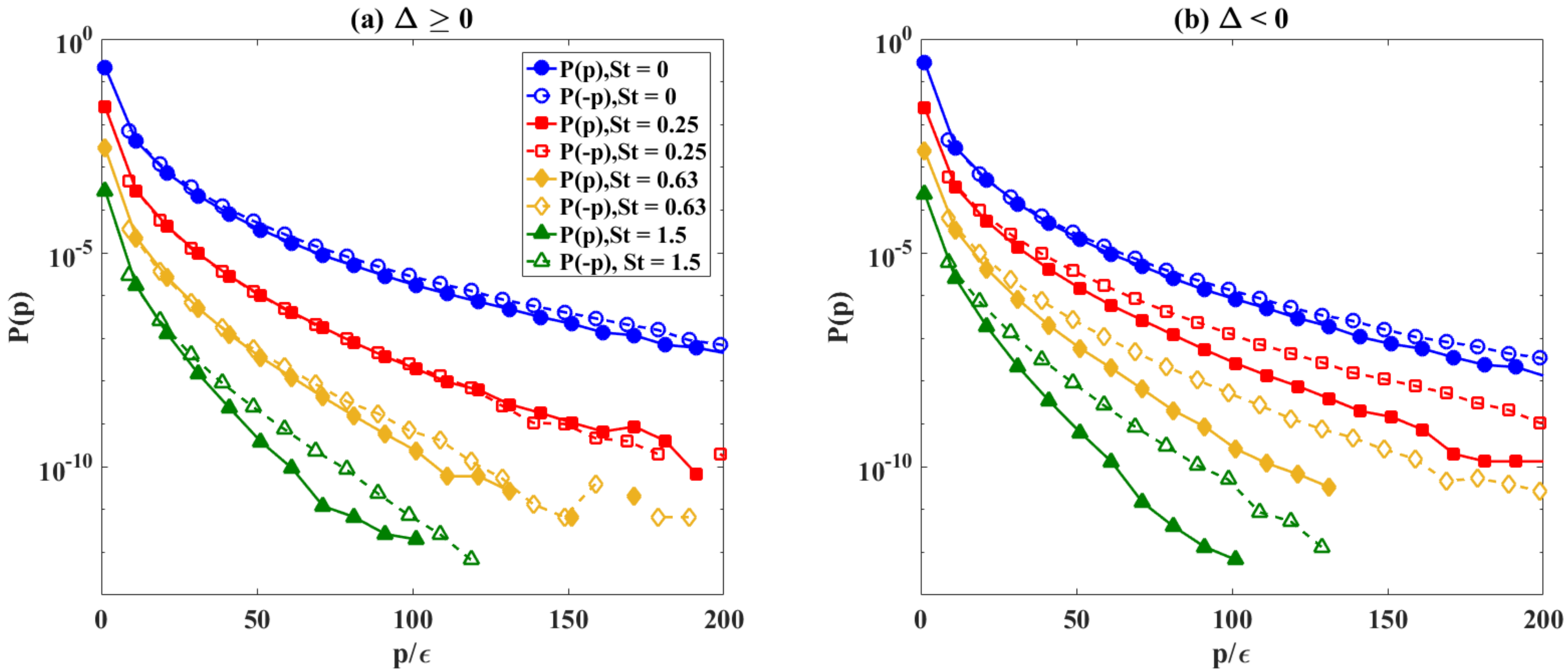
$$\mathbf{S} = \frac{1}{2}[\nabla\mathbf{u} + \nabla\mathbf{u}^T], \omega = \frac{1}{2}[\nabla\mathbf{u} - \nabla\mathbf{u}^T]$$

$Q \geq 0 \rightarrow$ vortical

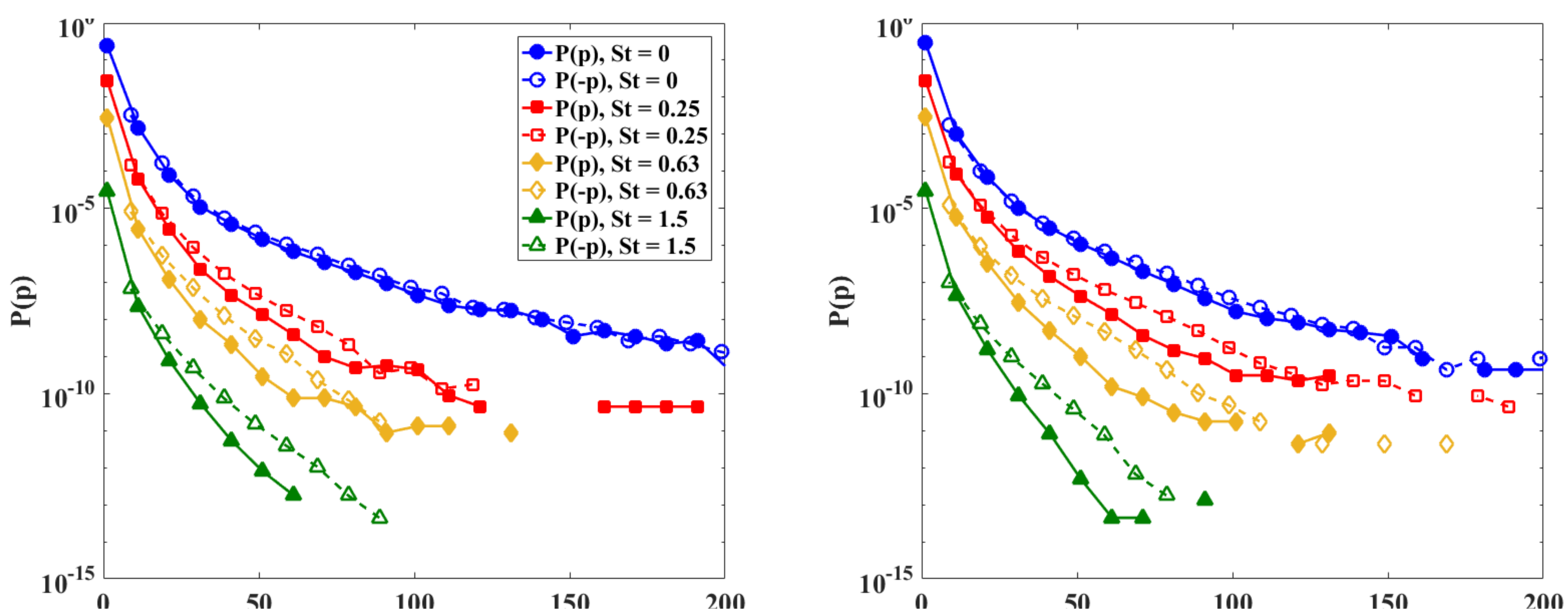
$Q < 0 \rightarrow$ straining







PDFs of instantaneous power p in (a) vortical and (b) straining region for Ro = ∞, Ω = 0.



PDFs of instantaneous power p in (a) vortical and (b) straining region for Ro = 0.12, Ω = 1.0.