Linear Instability Of Transient Flows: Numerical Approach And Experimental Validation

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Outline

1. Motivation
2. Problem Formulation
3. Impulsively Blocked Channel Flow
   - Quasi-steady Normal Mode Analysis
   - Farrell's Approach
   - Optimal Perturbation: Transient Base Flow
4. A Special Case: Trapezoidal Flow rate variation
5. Summary

Motivation

To suggest a suitable approach to study instability of decaying flows in a duct

- Type of flow
  - Transient flows
  - No further supply of energy
  - Bounded flows
- Flow characteristics
  - Reverse flow region
  - Adverse pressure gradient
  - High shear stress
- Development of flow
  - Redistribution of energy viscous diffusion
  - Decay of energy viscous dissipation

Motivation

Applications of decaying flows in a duct

- Sudden blockage in internal flow systems
  - Application specific requirement or sudden blockage
  - Hydraulic devices and other physiological flows
  - Substantial change in velocity, pressure and shear stress
- Examples
  - Valve operation in hydraulic systems, chemical and natural gas pipelines
  - Blood flow in arteries and flow in respiratory system
  - Dynamic stall behavior: dynamics of the reverse flow regions

Linear Stability Approaches

To Compare

1. Quasi-steady normal mode analysis
2. Farrell's approach applied in quasi-steady sense
3. Optimal growth analysis considers base flow decay

Based on

- Perturbation energy growth
- Radial Distribution of each mode
- Conformity with experiment

Experimental Investigation of a Transient Flow

Set-up

- a steady fully developed duct flow imposed with sudden blockage
- an unsteady equivalent of the steady problem
  - a trapezoidal flow rate is maintained
  - the deceleration phase of the flow and the gradual flow development due to viscous dissipation: emulate the phenomena of impulsively blocked flow

Objective

- PIV measurement provides the velocity field data
- approximated with the analytical velocity profiles
- Observation of the vorticity field

Experimental Investigation of a Transient Flow

Analysis

- Spatial dynamic mode decomposition
    - applied to the vorticity field at a time instance
- temporal dynamic mode decomposition
    - velocity fields of the whole spatial domain as a single data
    - its dynamic evolution: the temporal sequence of the data fields starting from the
    - piston stoppage time to a time of interest

Suddenly Blocked Channel Flow

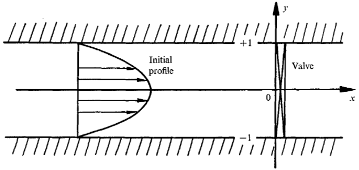


FIGURE: Sketch of Channel flow with initial profile before application of the valve.

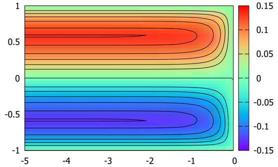


FIGURE: Streamlines near the end wall after the passage of the pressure wave.

- $\frac{\partial \boldsymbol{\omega}}{\partial t} = M\left[\nabla\times\mathbf{u}\times\boldsymbol{\omega} + \frac{1}{Re}\nabla^2\boldsymbol{\omega}\right]$, $\quad M = U/c.$
- For $M \ll 1$ and $M/Re \ll 1$: $\partial\boldsymbol{\omega}/\partial t = 0$.
- Following the passage of pressure wave (small timescale of h/c), the vorticity is essentially frozen.
- Velocity distribution just immediately after the passage of the pressure wave satisfies

$$\nabla\times\mathbf{u} = \boldsymbol{\omega}(0^-) \quad \text{with } u(x=0)=0 \text{ and } v(y=\pm 1)=0\ . \tag{1}$$

Base Flow

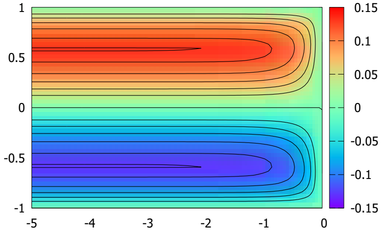


FIGURE: Streamlines near the end wall after the passage of the pressure wave.

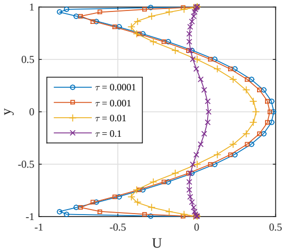
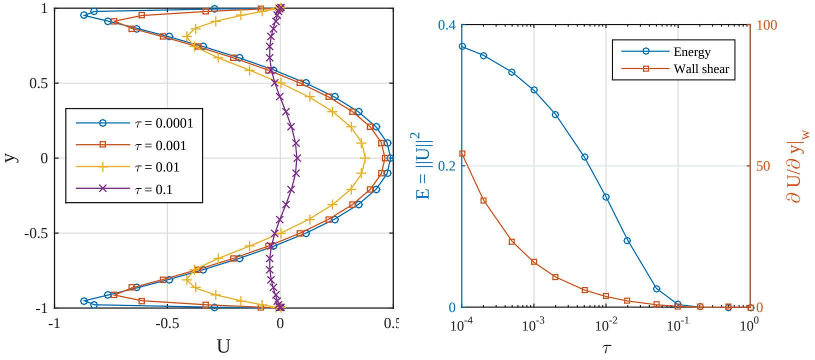


FIGURE: Development of the base velocity profile after the passage of pressure wave.

- The vorticity layer diffuses with a short diffusion time scale of δ^2/nu.
- The velocity profile is given by: $U(y,\tau) = \sum_{n=1}^{\infty} \frac{2}{\nu_n^2}\left[1 - \frac{\cos y\nu_n}{\cos \nu_n}\right] e^{-\nu_n^2 \tau}$, where $\tau = t/Re$ and $\tan \nu_n = \nu_n$.

Impulsively Blocked Channel Flow

Base Flow Characteristics



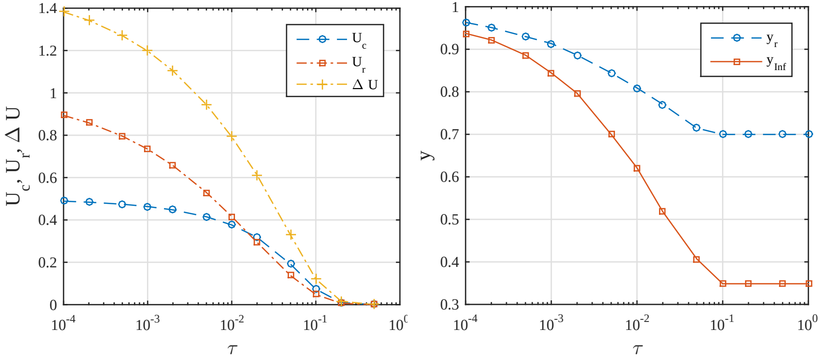
(A) base velocity profile

(B) energy and wall shear

Figure: Variation of the base flow parameters.

Impulsively Blocked Channel Flow

Base Flow Characteristics



(A) center-line velocity, maximum reverse velocity, and velocity difference

(B) location of maximum reverse velocity and inflection point

Figure: Variation of the base flow parameters.

Impulsively Blocked Channel Flow

Base Flow Characteristics

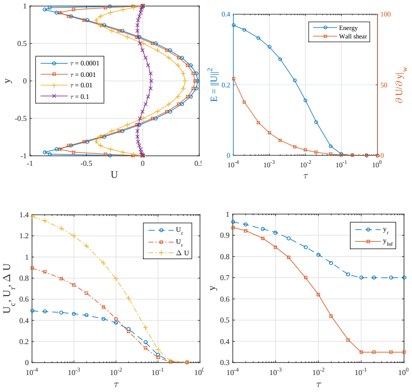
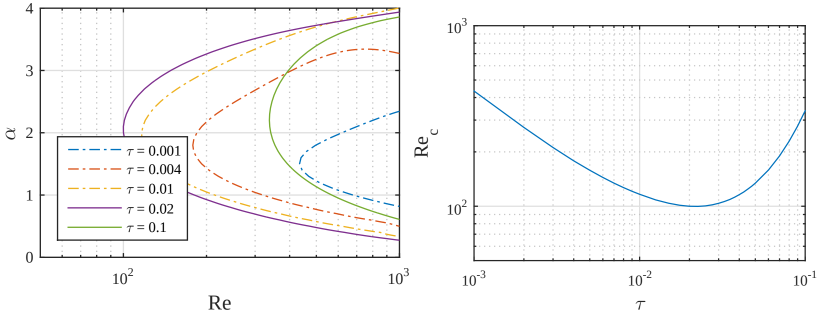


FIGURE: Variation of the base flow parameters.

- transient flow
- initial decay is fast and mostly exponential
- later on, decays asymptotically to stationary state
- long term stability characteristics may not be important
- intermediate perturbation growth may be exponential or algebraic

Impulsively Blocked Channel Flow

Instantaneous Neutral Curve



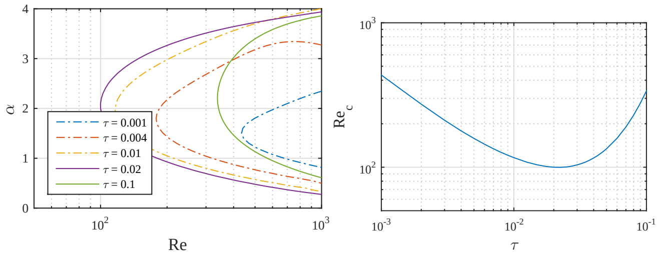
(A) Instantaneous neutral curves corresponding to various time instances.

(B) Variation of the critical Reynolds number with time

Figure: Results of the quasi-steady modal stability analysis.

Impulsively Blocked Channel Flow

Instantaneous Neutral Curve



- Quasi-steady analysis: instantaneous neutral curve
- Instantaneous critical Reynolds number, $Re_c(\tau)$
- Critical time, $\tau^* = dRe_c/d\tau$
- Critical Reynolds number, Re_c^*; necessary condition for instability

Farrell's Approach

Formulation

Key Characteristics

- Transient growth analysis
- Variational formulation
- Quasi-steady base flow

$$\psi(t) = \sum_{j=1}^{N} a_j \tilde{\phi}_j e^{i\alpha(x - c_j t)} = [\mathbf{\Phi}_t \mathbf{a}]\, e^{i\alpha x}$$

A variational problem is formulated with the functional $\|\psi\|^2$ to be maximized, and with the constraint that the initial perturbation has a unit norm, i.e., $\|\psi(0)\|^2 = 1$.

Farrell's Approach

Formulation

Functional of the form: $F = a^* \mathbf{B}_t a + \lambda(a^* \mathbf{B}_0 a - 1)$
where $\mathbf{B}_t = \mathbf{\Phi}_t^* W(\alpha^2 - D^2)\mathbf{\Phi}_t$

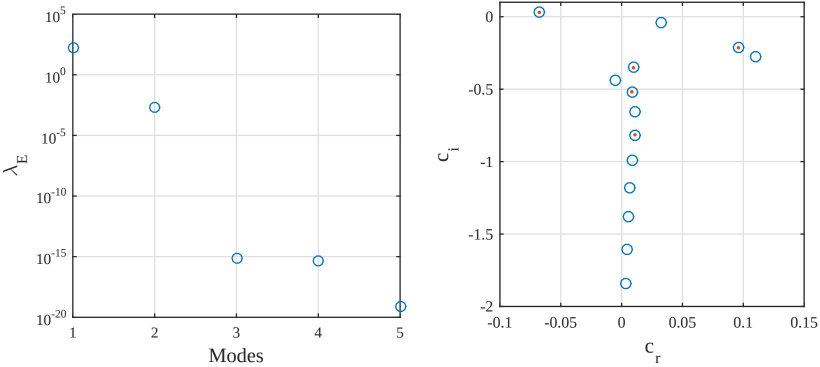
Setting the first variation of the functional F with respect to a to zero leads to an eigenproblem

$$\boxed{\mathbf{B}_t a + \lambda \mathbf{B}_0 a = 0}$$

with eigenvalue $\lambda = \lambda_E$ defining the growth potential and the corresponding eigenvector as the spectral projection of the optimal perturbation (Farrell, 1988). *

*Farrell, B. F. (1988). Optimal excitation of perturbations in viscous shear flow. Physics of Fluids, 31(8):2093.

FARRELL'S APPROACH
EIGEN SPECTRUM



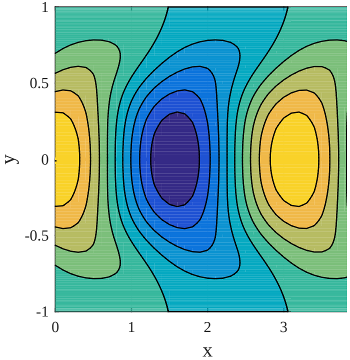
(C) Energy growth (first 5 eigenvalues)

(D) Eigen spectrum

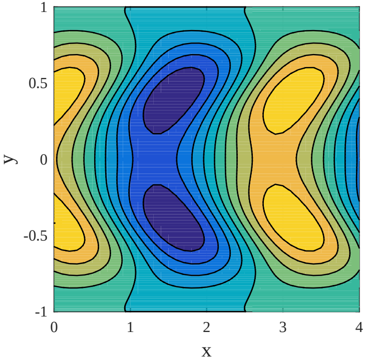
FIGURE: Optimal growth analysis using Farrell's approach; $Re = 150$, $\alpha = 2$ and $\Delta t = 40$ with initial time corresponding to $t = 0.02$.

Farrell's Approach

Farrell's mode



(A) Unstable Eigenmode



(B) Farrell's mode

Figure: Streamline contours of the most unstable eigen mode and the optimal mode obtained through Farrell's approach.

Quasi-steady vs Transient Base Flow

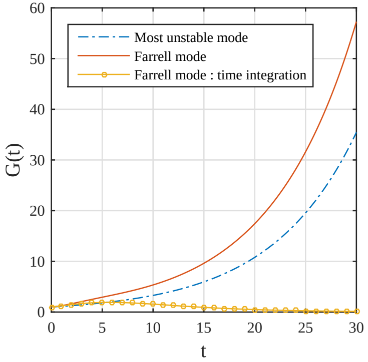


FIGURE: Energy growth of Farrell's mode with time integration considering transient base flow compared with the energy growth with modal assumption.

Optimal Perturbation

Formulation

The basic dynamical equations: $\frac{\partial \phi}{\partial t} = \mathcal{L}_{OS}\phi$

The functional to maximize: $G(T) = \frac{\|\phi(T)\|^2}{\|\phi(0)\|^2}$

With the introduction of adjoint variable $\tilde{\phi}$, an augmented Lagrangian can be defined as:

$$\mathcal{L}(\phi, \tilde{\phi}) = \mathcal{G}(\phi) - \int_0^T (\tilde{\phi}, \dot{\phi} - \mathcal{L}_{OS}\phi)dt$$

To find an unconditional extremum of $\mathcal{L}$ which is given by the zero variation of $\mathcal{L}$ w.r.t. arbitrary variation of both ϕ and $\tilde{\phi}$.

$$\int_0^T (\tilde{\phi}, \dot{\phi} - \mathcal{L}_{OS}\phi)dt = (\tilde{\phi}, \phi)|_0^T - \int_0^T (\dot{\tilde{\phi}} + \mathcal{L}_{OS}^{\dagger}\tilde{\phi}, \phi)dt$$

The corresponding adjoint equation: $-\frac{\partial \tilde{\phi}}{\partial t} = \mathcal{L}_{OS}^{\dagger}\tilde{\phi}$

The terminal conditions: $\tilde{\phi}(T) = \frac{2}{\|\phi_0\|^2}\phi(T)$ and $\phi(0) = \frac{\|\phi_0\|^4}{2\|\phi(T)\|^2}\tilde{\phi}(0)$

Optimal Perturbation
Formulation

Perturbation equation: $\frac{\partial \phi}{\partial t} = \mathcal{L}_{OS}\phi$ with $\phi(\pm 1) = D\phi(\pm 1) = 0$.

Orr-Sommerfeld operator:

$$\mathcal{L}_{OS} \equiv \frac{i\alpha}{(D^2-\alpha^2)}\left[(i\alpha Re)^{-1}(D^2-\alpha^2)^2 - U(D^2-\alpha^2) + D^2U\right]$$

The corresponding adjoint equation with the adjoint variable $\tilde{\phi}$ takes the form

$$-\frac{\partial \tilde{\phi}}{\partial t} = \mathcal{L}^{\dagger}_{OS}\tilde{\phi}$$

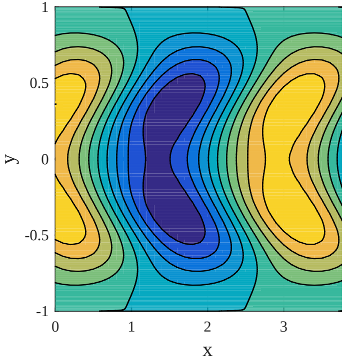
where

$$\mathcal{L}^{\dagger}_{OS} \equiv \frac{i\alpha}{(D^2-\alpha^2)}\left[(i\alpha Re)^{-1}(D^2-\alpha^2)^2 + U(D^2-\alpha^2) + 2(DU)U\right]$$

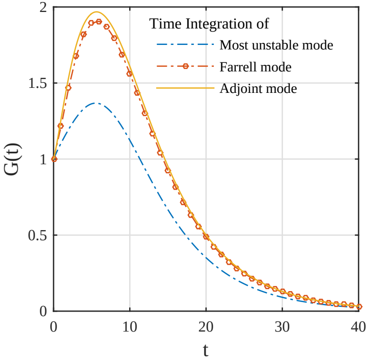
The terminal conditions: $\tilde{\phi}(T) = \frac{2}{||\phi_0||^2}\phi(T)$ and $\phi(0) = \frac{||\phi_0||^4}{2||\phi(T)||^2}\tilde{\phi}(0)$

Optimal Perturbation

Streamline Contour and Energy Growth



(A) Streamline Contour of the optimal mode obtained from adjoint analysis



(B) Energy growth of the optimal mode with time integration

FIGURE: Optimal perturbation obtained by variational approach considering time dependency of base flow.

Optimal Energy growth at vairous *Re*

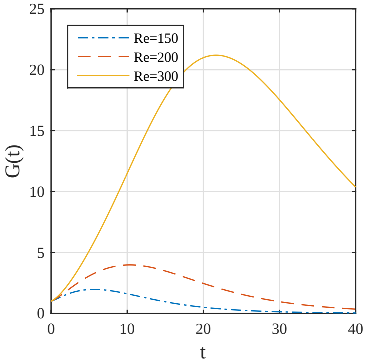


FIGURE: Energy growth of the optimal mode for various Reynolds numbers.

Experimental Setup

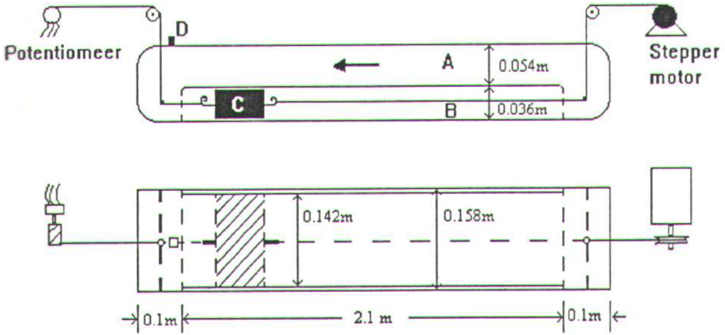


FIGURE: Schematic of the experimental setup.

Dye visualization

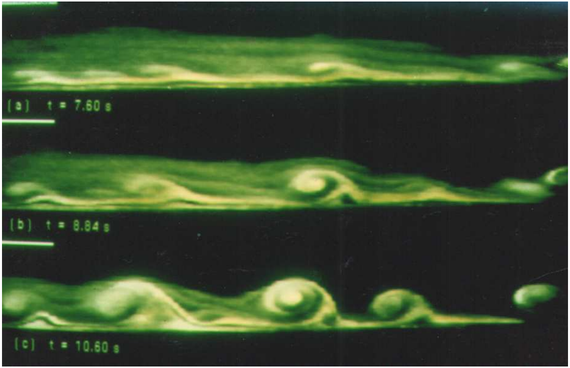
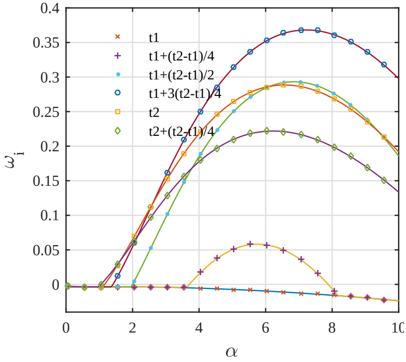
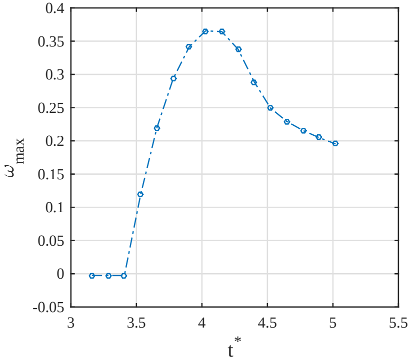


FIGURE: Dye visualization of an unsteady flow (Case 1c of Das (1998)).

Quasi-steady Results



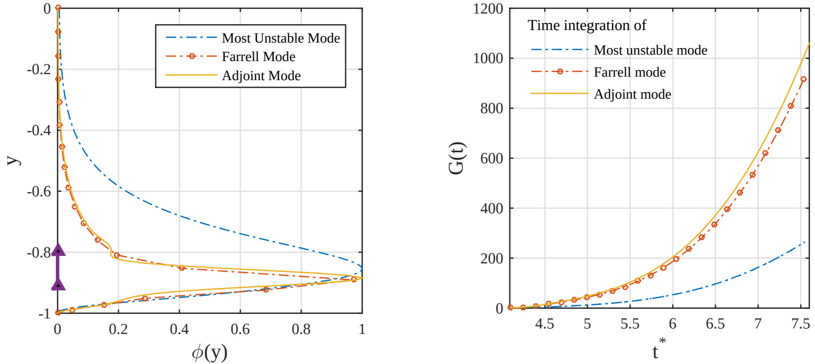
(A) Instantaneous growth rate of the most unstable mode at various time instances.



(B) Variation of the maximum growth rate with time.

Figure: Quasi-steady results of the case with trapezoidal flow rate.

Optimal mode and its growth



(A) Comparison of the modes

(B) Comparison of time integrated growth of perturbation energy

FIGURE: Optimal mode at the critical time and its growth for the trapezoidal case

Summary

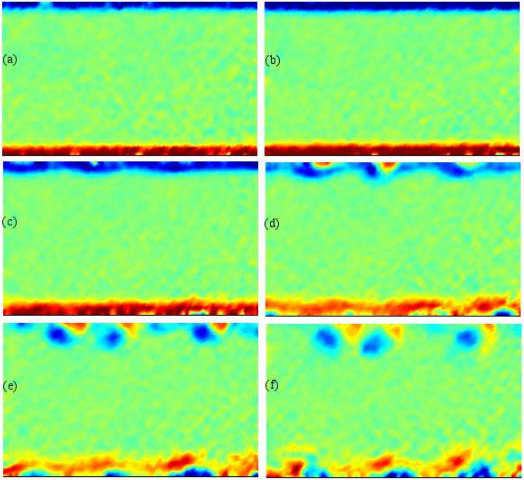
- A comprehensive theoretical study is carried out on the instability of a fully developed channel flow, which is blocked suddenly.
- *Critical Reynolds number*: growth rate near the neutral curve is very low. But, it provides guidelines for choosing the parameters for further analysis.
- *Farrell's analysis*:
  - symmetric modes contribute to the optimal growth
  - most of the energy of the optimal disturbance is concentrated near the critical layer.
- optimal perturbation with an initial-value problem formulation
  - optimal perturbation shows similarity with the Farrell's mode
- A special experimental case of a piston driven channel flow:
  - The wave number of the instability wave observed in the flow visualization, matches well with the theoretical value corresponding to maximum instantaneous growth rate.

Summary

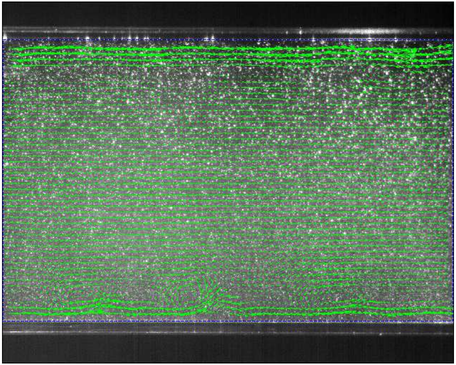
- Thus, it can be concluded that for suddenly blocked channel flow, one can use Farrell's approach with quasi-steady assumption to obtain the optimal initial perturbation while analyzing it as an initial-value problem.
- Farrell's approach has the advantage of its simplicity whereas the variational approach with base flow change is more elaborate and precise.
- However, the existence of instability in the flow cannot be ensured by quasi-steady assumption, and one needs to verify the time integrated perturbation growth to determine the presence of instability.

Future Prospect

Transient Pipe Poiseuille Flow



Thank You



Thank You For Your Attention!!!
Any Questions?

*Avinash Nayak, Debopam Das. (2017) Transient growth of optimal perturbation in a decaying channel flow. *Physics of Fluids* 29, 064104.

*Avinash Nayak, Debopam Das. (2019) A pseudospectral approach applicable for time integration of linearized NS operator that removes pole singularity and physically spurious