Symmetry-broken topological phases

Tanmoy Das

Department of Physics

Indian Institute of Science, Bangalore, India

Arpit Raj
Nepal Banerjee
Diptiman Sen

A. Raj, N. Banerjee, TD (under review)
Bansil, Lin, TD, RMP **88**, 021004 (2016).

Outlook

- How robust are the topological phases with broken symmetry?
- Robustness of quantum spin-Hall effect with **broken** time-reversal symmetry.
- New quantization phenomena with electrical **potential**.
- Helical anomaly, helical magnetic effect, magneto-electric coupling in 2+1D.

Classifications of phases of matter

Broken symmetry

Quantum phases

$\underline{T}$: Magnetism

$\underline{P}$: Ferroelectricity

$\underline{PT}$: Multiferroics

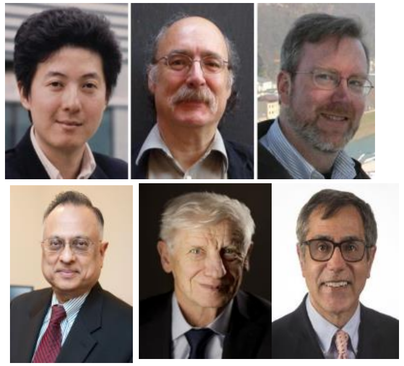
$\underline{U(1)}$: Superconductivity

<u>Translational:</u> Density waves

Symmetry invariant

Topological phases

Symmetry			Dimension			
T	C	S	1	2	3	4
0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	0	0	0	0	0	$\mathbb{Z}$
1	1	1	$\mathbb{Z}$	0	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
1	-1	1	0	0	$\mathbb{Z}$	0



How robust is the topological table to symmetry breaking? Surprisingly robust up to a critical coupling.

Topology indicator in symmetry broken topological phases? Quantum anomalies.

<u>This work</u>: Time-reversal broken Z_2 topology and Helical anomaly.

A somewhat general model

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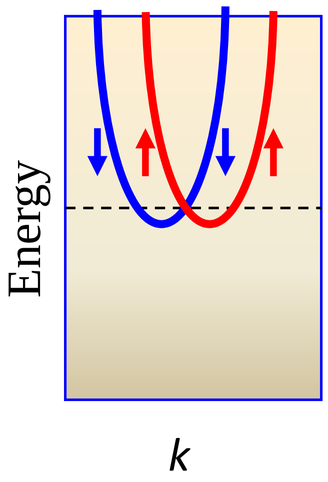
$$\psi = (\psi_{A\uparrow} \quad \psi_{A\downarrow})^T$$

$$H = \begin{matrix} \xi_A & \alpha_k \\ \alpha_k^* & \xi_A \end{matrix}$$

$$\xi_A = k^2/m$$

$$\alpha_k = \alpha_R(k_y - ik_x)$$



A somewhat general model

$$\psi = (\psi_{A\uparrow} \quad \psi_{A\downarrow})^T \; \psi_{B\uparrow} \quad \psi_{B\downarrow})^T$$

$$H = \begin{pmatrix} \xi_A & \alpha_k & & \\ \alpha_k^* & \xi_A & & \\ & & \xi_B & -\alpha_k \\ & & -\alpha_k^* & \xi_B \end{pmatrix}$$

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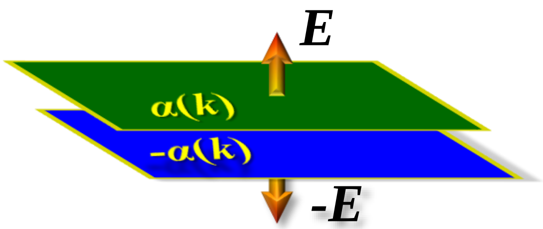
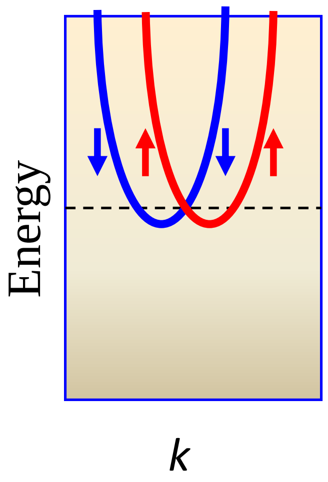


(Rashba bilayer)



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$$\psi = (\psi_{A\uparrow} \quad \psi_{A\downarrow} \quad \psi_{B\uparrow} \quad \psi_{B\downarrow})^T$$

$$H = \begin{pmatrix} \xi_A & \alpha_k & \xi_{AB} & 0 \\ \alpha_k^* & \xi_A & 0 & \xi_{AB} \\ & c.c. & \xi_B & -\alpha_k \\ & & -\alpha_k^* & \xi_B \end{pmatrix}$$

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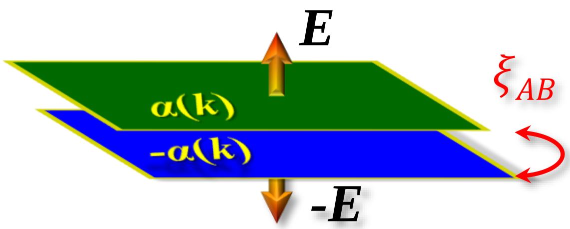
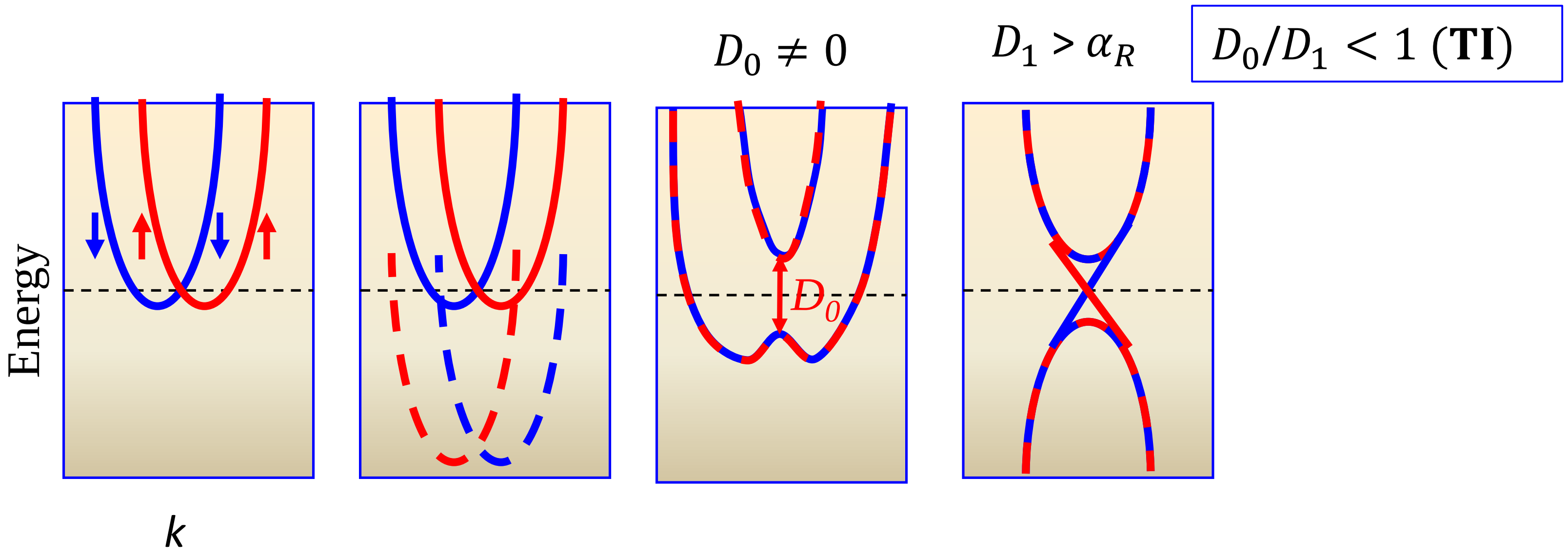


This principle of **helicity inversion** induced topological phase can be achieved intrinsically, artificially, or via interaction in condensed matter and optical lattices.

TD, Balatsky, Nat. Commun. **4**, 1972 (2013).



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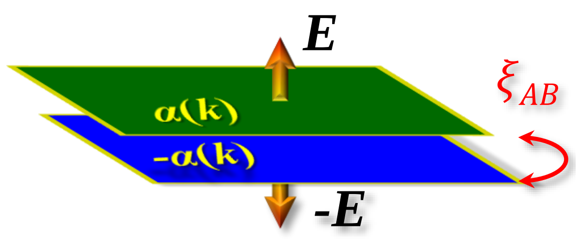

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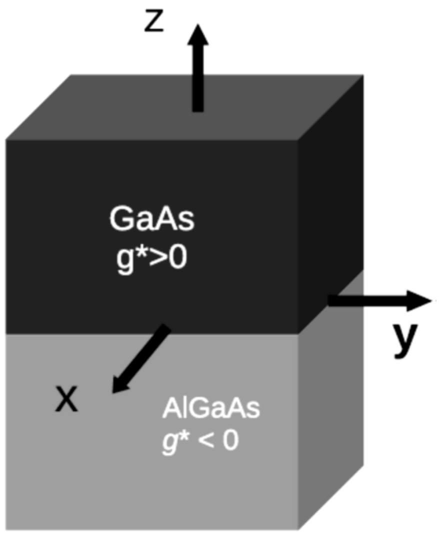
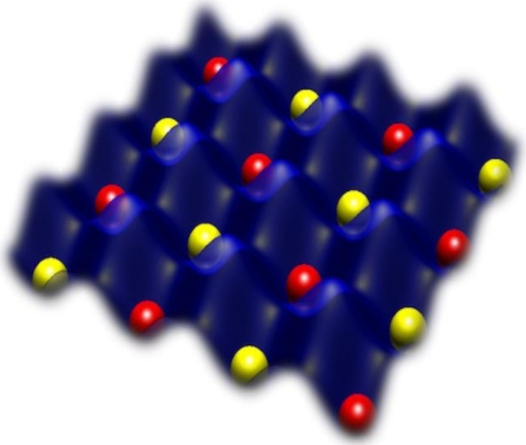
GaAs/AlGaAs



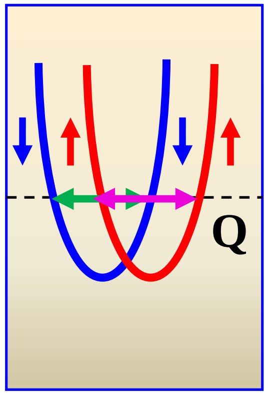
Nepal Banerjee, TD (2019)

Optical lattices



S. Ray, K. Roy, TD (2016)

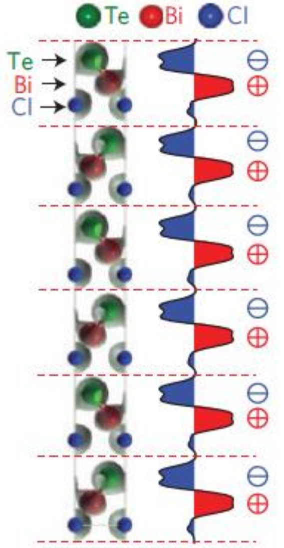
Interaction



$$\alpha_{k+Q} = -\alpha_k$$

Gaurav Gupta, TD (2016).

Bulk materials



Y. L. Chen et al. Nat. Phys. **9**, 714 (2013)

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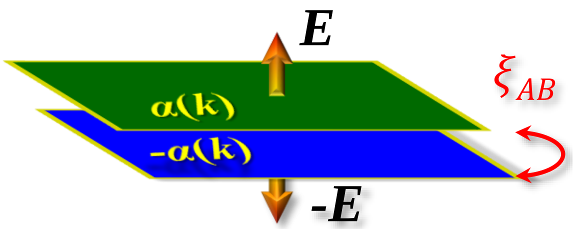

Quantum Spin-Hall state : TR invariant

$$H = \xi_A I_{4\times 4} + \begin{pmatrix} h_k^+ & \mathbf{0} \\ \mathbf{0} & h_k^- \end{pmatrix}, \quad h_k^\pm = \xi_{AB}\sigma_z \pm \alpha_k' \sigma_x + \alpha_k'' \sigma_y$$

Chern numbers $C_\pm = \pm 1$

Condition: Dirac mass (ξ_{ABk}) must change sign

$$\xi_{ABk} = 0 \quad at \quad \frac{D_0}{D_1} = k_0^2 > 0$$

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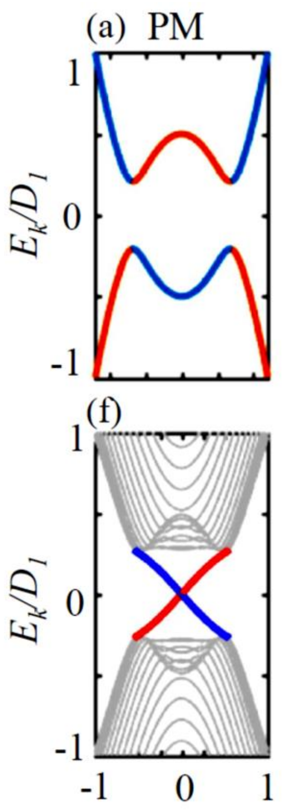

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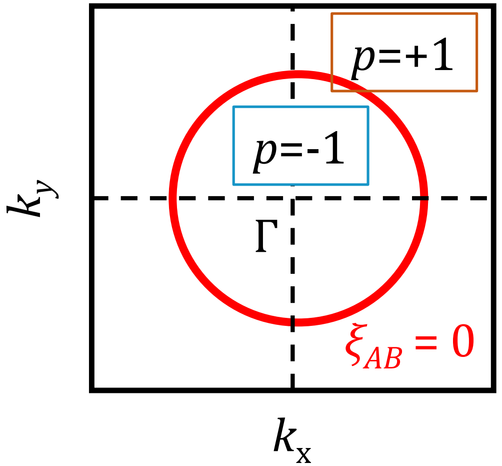


Symmetry breaking topological phases

$$H_{\text{int}} = U \sum_{i\in(A,B)} n_{i\uparrow} n_{i\downarrow} + V \sum_{j\neq i\in(A,B)} n_i n_j$$

Spin: $\mathbf{S}_i = \psi_i^\dagger \boldsymbol{\sigma} \psi_i$

Chiral: $\mathbf{T}_\sigma = \psi_\sigma^\dagger \boldsymbol{\tau} \psi_\sigma$

***T*-breaking** order parameters:

FM/AF: $\mathcal{M}^\pm = \frac{1}{2}(\langle S_A^z \rangle \pm \langle S_B^z \rangle)$

Chiral magnet/chiral sublattice: $\mathcal{N}^{x/y} = \frac{1}{2}\left(\left\langle T_\uparrow^{x/y} \right\rangle \mp \left\langle T_\downarrow^{x/y} \right\rangle\right)$

Exchange energy:

$E_m = U\mathcal{M}^\pm$

$E_m = V\mathcal{N}^{x/y}$

Symmetry breaking topological phases

T^2	C^2	S^2	P	$(PT)^2$	$(CP)^2$	$(CPT)^2$	AZ (2D)	Our result (2D)	Sym broken order
-1	1	1					$\mathbb{Z}_2$ (QSH)		
0	1	0	1	0	-1 ↓	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (QSH)	FM
0	1	0	1	0	0	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}$ (QAH)	FM

Symmetry invariant

Topological phases

Symmetry T	C	S	Dimension 1	2	3	4
0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	0	0	0	0	0	$\mathbb{Z}$
1	1	1	$\mathbb{Z}$	0	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
1	-1	1	0	0	$\mathbb{Z}$	0

T = Time-reversal
C = Charge conjugation
S = TC =Chiral/sublattice
P = Parity

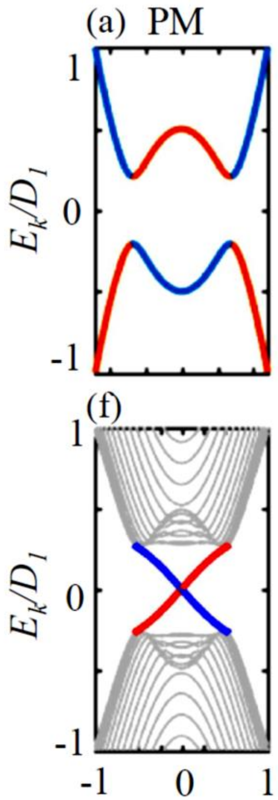
Altland, Zirnbauer (AZ), PRB **55**, 1142 (1997).
Chiu, Teo, Schnyder, Ryu RMP **88**, 035005 (2016).

FM order: *CP*-protected $\mathbb{Z}_2$ invariant

$$H_{int} = |E_m|\tau_z \otimes \sigma_z$$

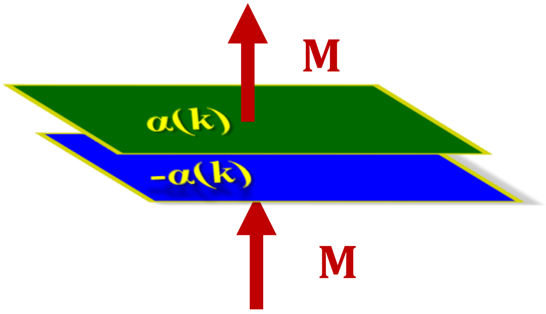
Dirac mass become different: $\xi_{AB}^{\pm} = \xi_{AB} \pm |E_m|$

$$C_{\pm} = \pm 1 \quad \text{if} \quad \frac{(D_0 \pm E_m)}{D_1} > 0 \qquad E_m < D_0$$



$C_+ = 1$

$C_- = -1$



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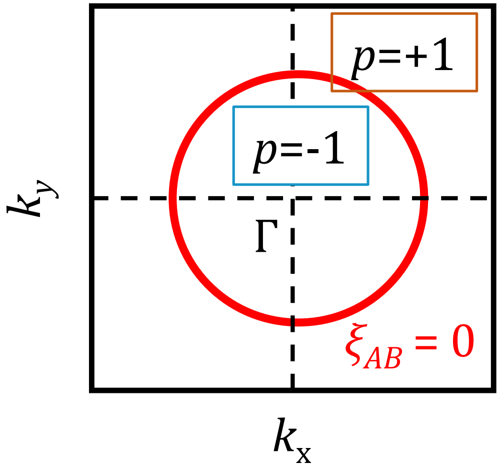

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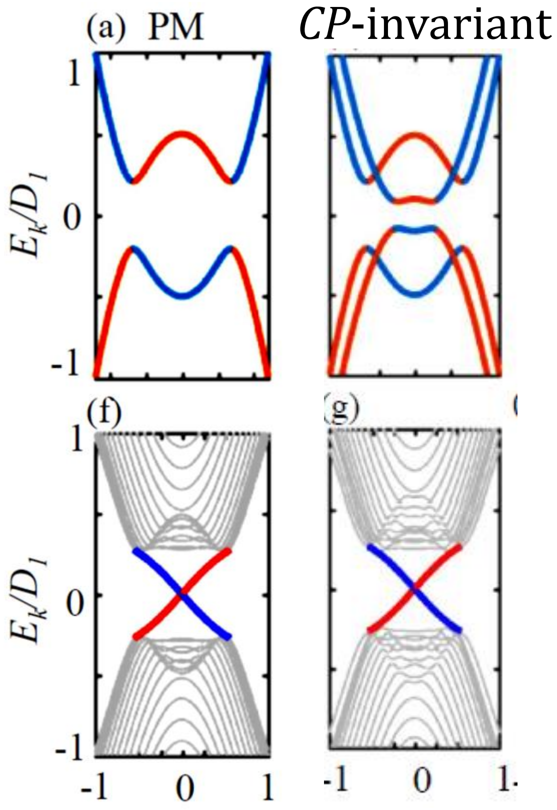
$C_{\pm} = \pm 1$ if $\frac{(D_0 \pm E_m)}{D_1} > 0$ $\quad E_m < D_0$ QSH

$(CP)^2 = -1$ Antiunitary. Zero energy states are protected



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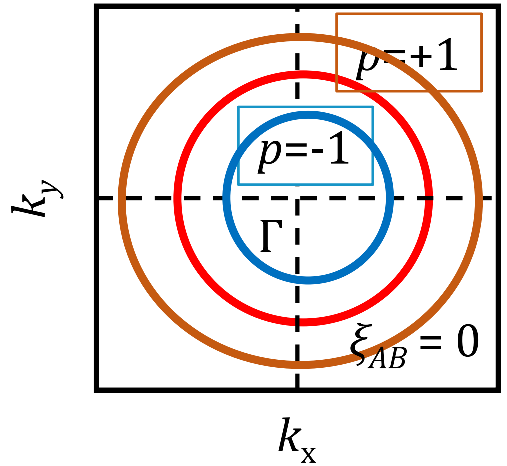

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$C_{+} = 1$, $C_{-} = 0$ $\qquad E_m > D_0$ QAH

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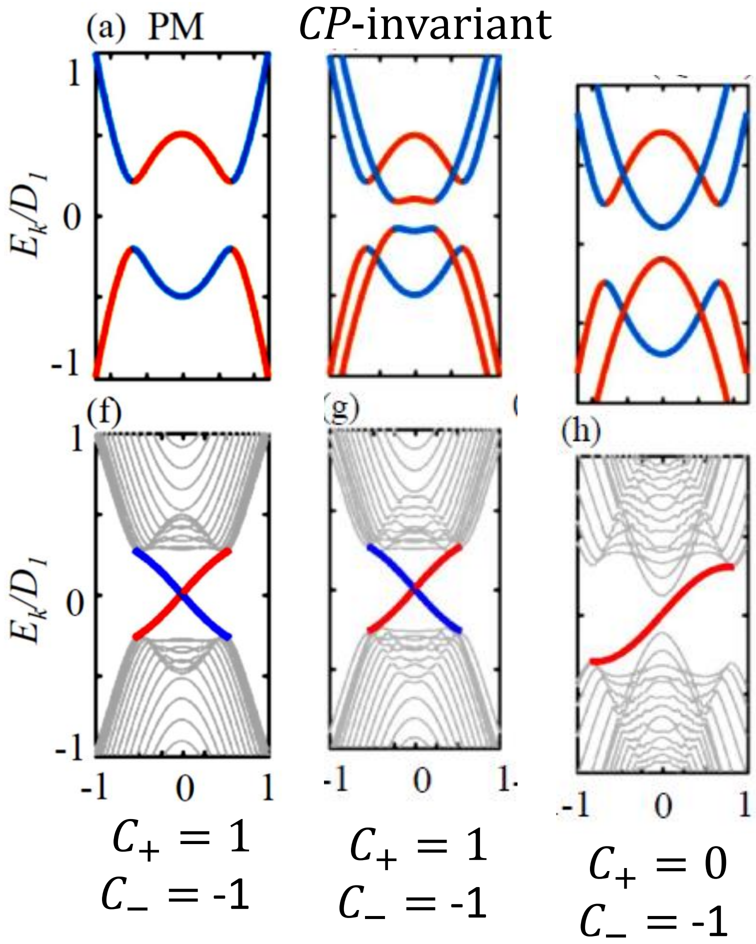

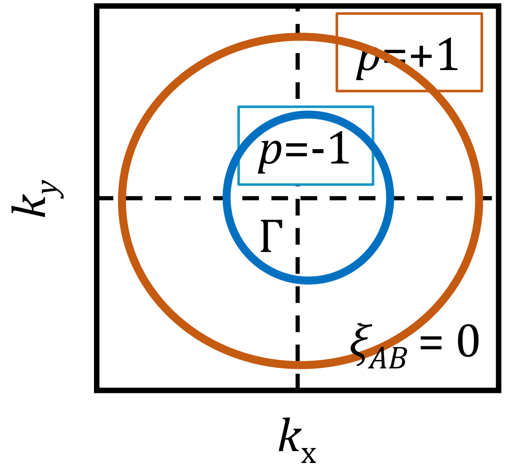

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0	0	1	1	0	0	-1	0	$\mathbb{Z}_2$ (QSH)	Chiral magnet (CM)
0	0	0	0	0	0	-1	$\mathbb{Z}$ (IQH)	$\mathbb{Z}_2$ (QSH)	CM + P broken

Symmetry invariant

Topological phases

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Chiral Magnet : *CPT-invariant* TI

$$H_{int} = |E_m|\tau_z \otimes \sigma_0$$

Different Newtonian mass: $h_k^{\pm} = h_k^{\pm} \pm |E_m|$

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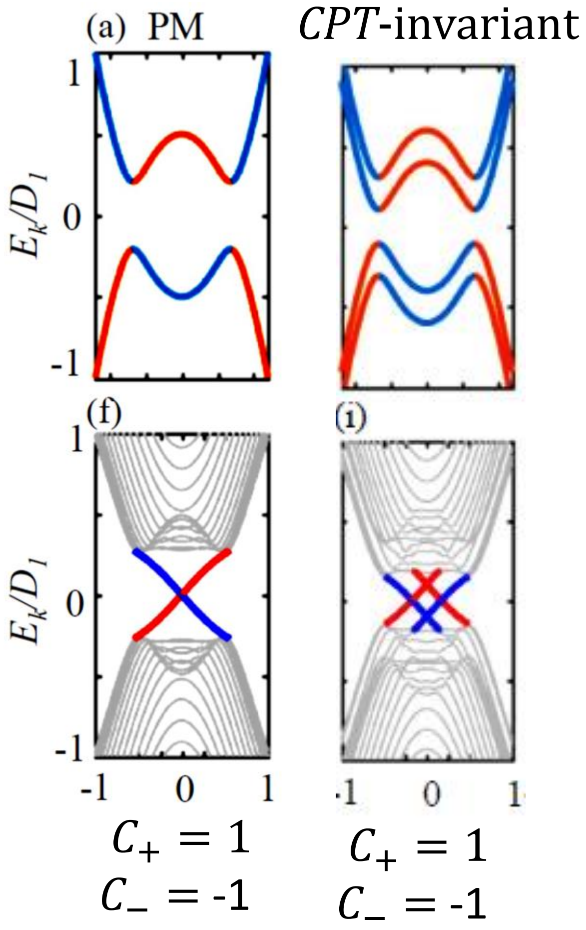
Newtonian mass does **not** affect the Chern number

QSH

$(CPT)^2 = -1$ But a linear operator



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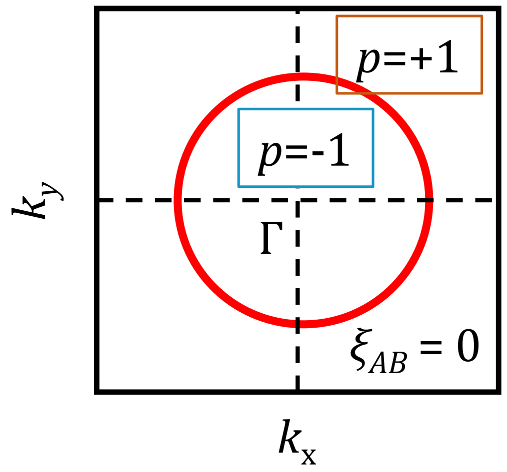

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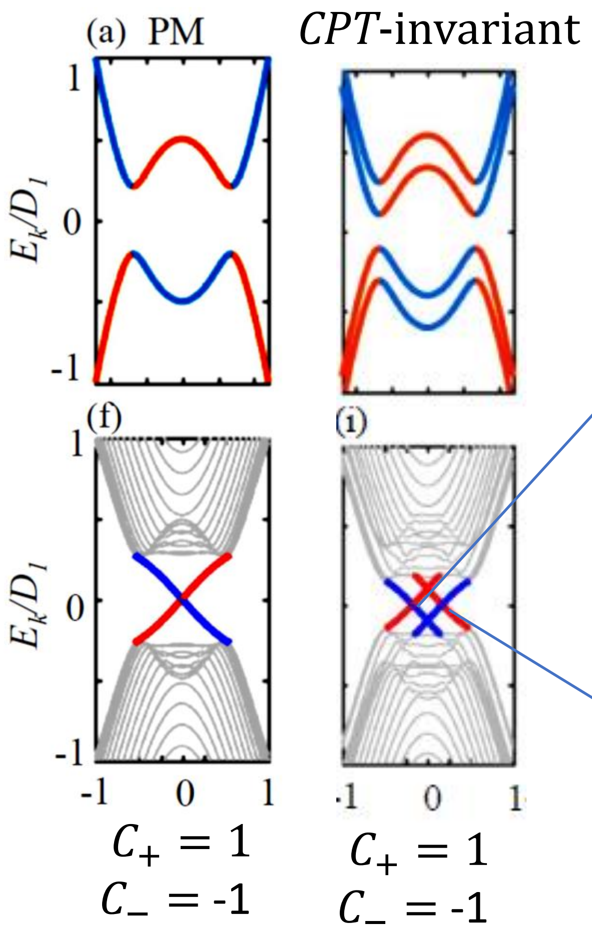

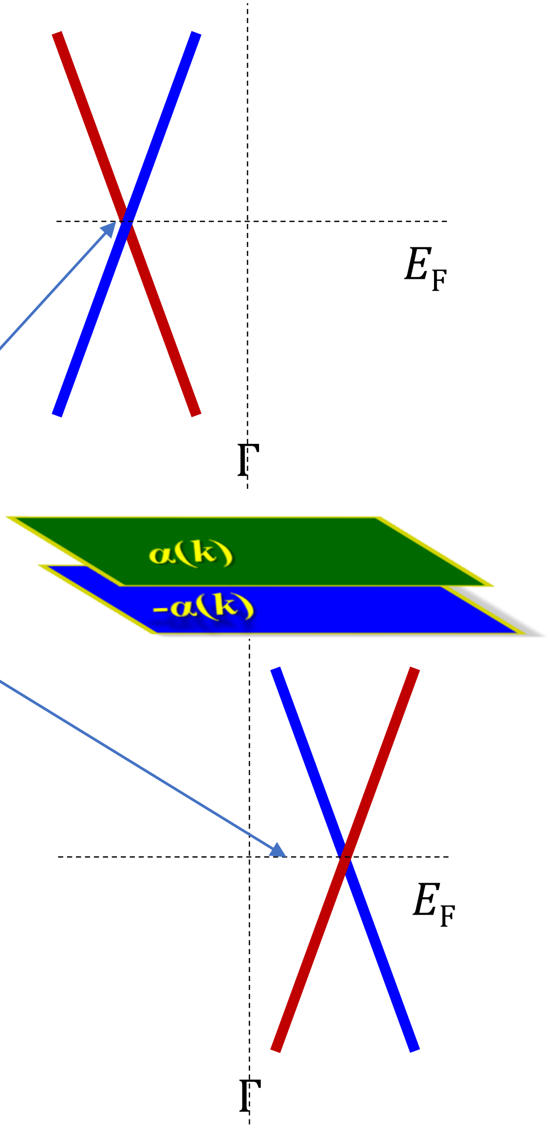

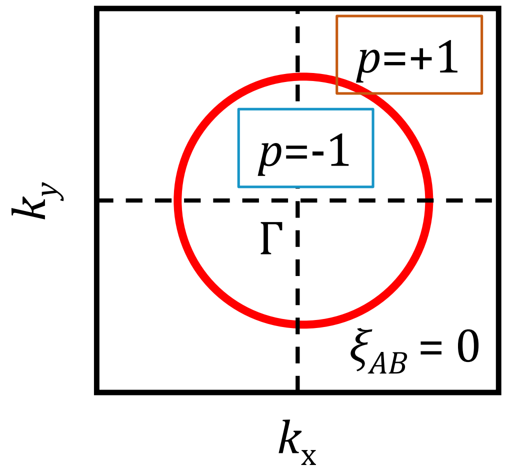

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Helical anomaly

With in-plane electric field: $\partial_\mu J_\sigma^\mu = \sigma \frac{e^2}{h} E$

$J_\sigma^\mu = (\rho_\sigma, \mathbf{J}_\sigma)$: **Chiral** charge, current per spin $\sigma = \pm$.

Total chiral charge: $\rho = \frac{\rho_\uparrow + \rho_\downarrow}{2}$

Total chiral current: $\mathbf{J} = \frac{\mathbf{J}_\uparrow + \mathbf{J}_\downarrow}{2}$

$\partial_\mu J^\mu = 0$

No chiral anomaly

Helical charge: $\rho_s = \frac{\rho_\uparrow - \rho_\downarrow}{2}$

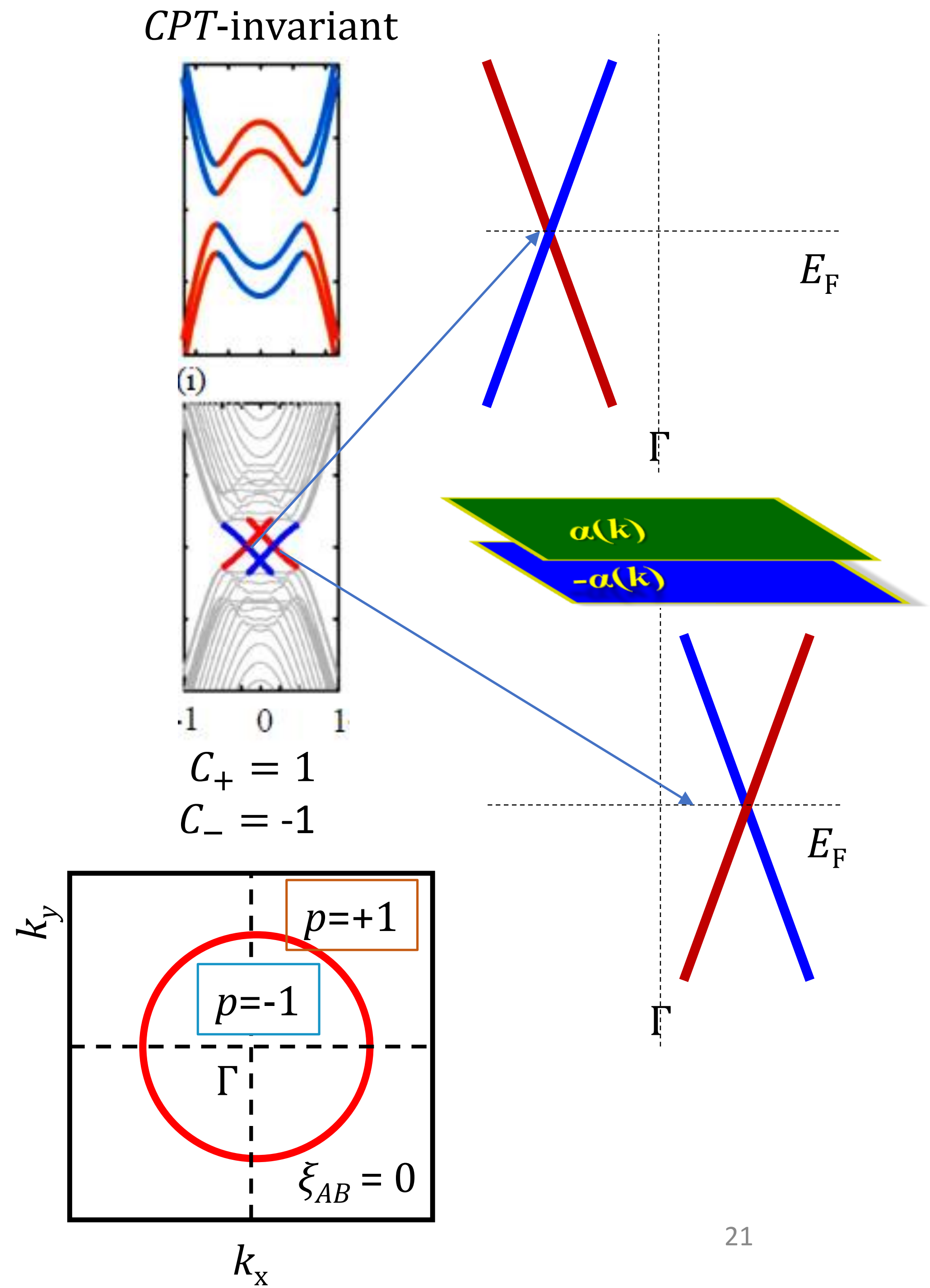
Total chiral current: $\mathbf{J}_s = \frac{\mathbf{J}_\uparrow - \mathbf{J}_\downarrow}{2}$

$$\partial_\mu J_s^\mu = \frac{e^2}{h} E$$

Helical anomaly

Steady state: $J_s = \frac{e^2}{h} V$

A new quantized anomaly indicator



Chiral Magnet : *CPT-invariant* TI

$$H_{int} = |E_m| \tau_z \otimes \sigma_0$$

Different Newtonian mass: $h_k^{\pm} = h_k^{\pm} \pm |E_m|$

Helical anomaly

With in-plane electric field: $\partial_\mu J_\sigma^\mu = \sigma \frac{e^2}{h} E$

$J_\sigma^\mu = (\rho_{\sigma}, \mathbf{J}_\sigma)$: **Chiral** charge, current per spin $\sigma = \pm$.

Total chiral charge: $\rho = \frac{\rho_\uparrow + \rho_\downarrow}{2}$

Total chiral current: $\mathbf{J} = \frac{\mathbf{J}_\uparrow + \mathbf{J}_\downarrow}{2}$

$$\partial_\mu J^\mu = 0$$

No chiral anomaly

Helical charge: $\rho_s = \frac{\rho_\uparrow - \rho_\downarrow}{2}$

Total chiral current: $\mathbf{J}_s = \frac{\mathbf{J}_\uparrow - \mathbf{J}_\downarrow}{2}$

$$\partial_\mu J_s^\mu = \frac{e^2}{h} E$$

Helical anomaly

Steady state: $J_s = \frac{e^2}{h} V$

A new quantized anomaly indicator

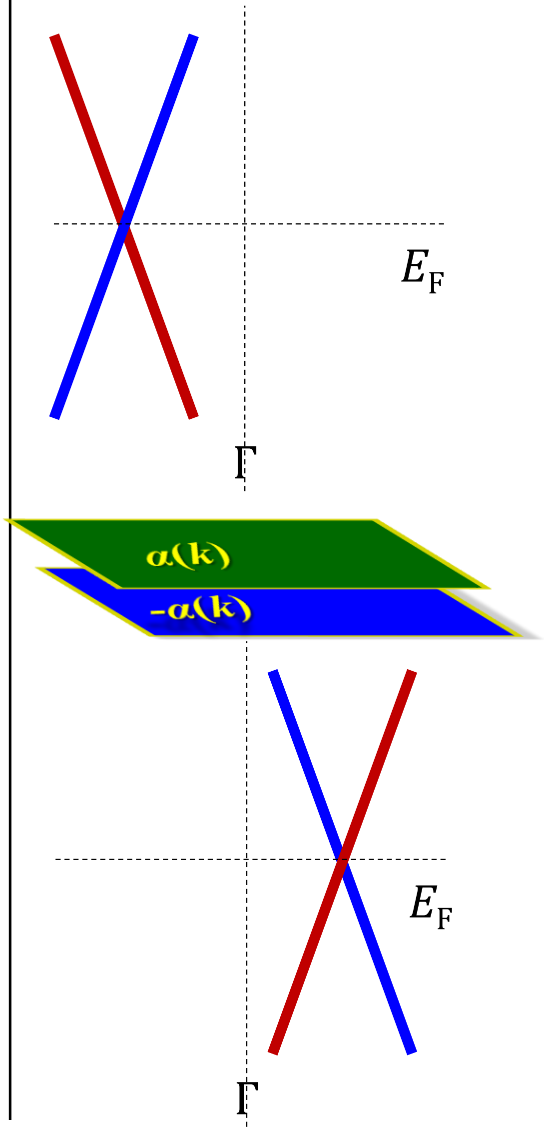
With in-plane magnetic field:

Zeeman energy density :

$$E_z = \mu_B \, \rho_s \, B$$

$$\mathbf{J_s} = \mu_B \frac{e^2}{h} \mathbf{B}$$

Helical magnetic effect



Symmetry breaking topological phases

T^2	C^2	S^2	P	$(PT)^2$	$(CP)^2$	$(CPT)^2$	AZ (2D)	Our result (2D)	Sym broken order
-1	1	1					$\mathbb{Z}_2$ (QSH)		
0	1	0	1	0	-1	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (QSH)	FM
0	1	0	1	0	0	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}$ (QAH)	FM
0	0	1	1	0	0	-1	0	$\mathbb{Z}_2$ (QSH)	Chiral magnet (CM)
0	0	0	0	0	0	-1	$\mathbb{Z}$ (IQH)	$\mathbb{Z}_2$ (QSH)	CM + P broken
0	1	0	0	-1	0	-1	$\mathbb{Z}$	$\mathbb{Z}_2$ (Anomalous spin Hall)	AF

Symmetry invariant

Topological phases

Symmetry T	C	S	Dimension 1	2	3	4
0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	0	0	0	0	0	$\mathbb{Z}$
1	1	1	$\mathbb{Z}$	0	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
1	-1	1	0	0	$\mathbb{Z}$	0

T = Time-reversal
C = Charge conjugation
S = TC =Chiral/sublattice
P = Parity

Altland, Zirnbauer (AZ), PRB **55**, 1142 (1997).
Chiu, Teo, Schnyder, Ryu RMP **88**, 035005 (2016)

AF order: *PT*-invariant TI

$$H_{int} = |E_m|\tau_x \otimes \sigma_x$$

Off diagonal terms.   Chern number is not defined.

Bulk bands are adiabatically connected to the QSH state.

So what is the topological anomaly?

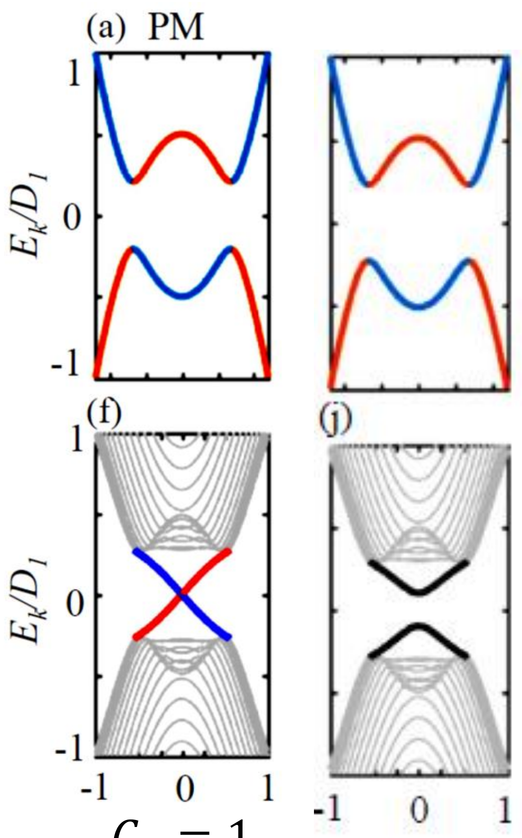
Quantum Spin-Hall state : TR invariant

$$H = \xi_A I_{4\times4} + \begin{pmatrix} h_k^+ & \mathbf{0} \\ \mathbf{0} & h_k^- \end{pmatrix}, \quad h_k^\pm = \xi_{AB}\sigma_z \pm \alpha_k'\sigma_x + \alpha_k''\sigma_y$$

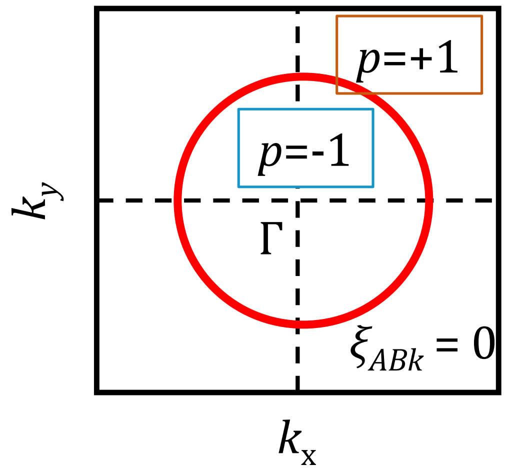
Chern numbers $C_\pm = \pm 1$

Condition: Dirac mass (ξ_{ABk}) much change sign

$$\xi_{ABk} = 0 \quad at \quad \frac{D_0}{D_1} = k_0^2 > 0$$



$C_+ = 1$
$C_- = -1$



AF order: *PT*-invariant TI

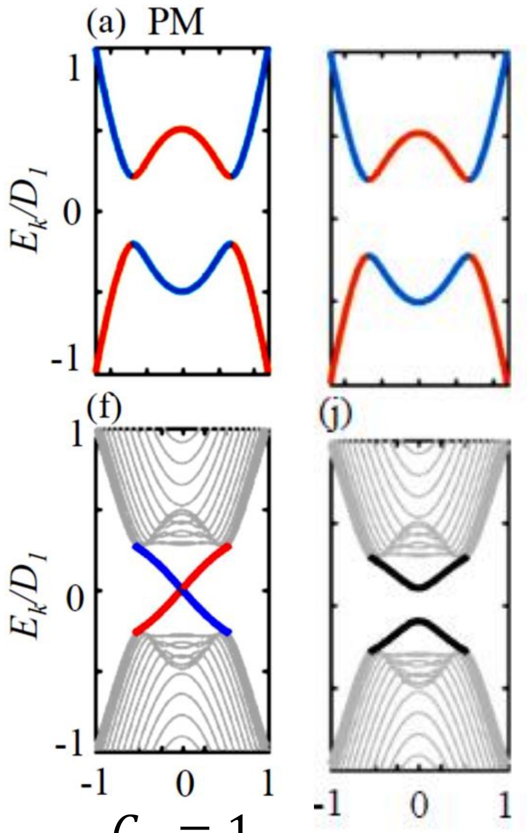
$$H_{int} = |E_m|\tau_x \otimes \sigma_x$$

Off diagonal terms. Chern number is not defined.

Bulk bands are adiabatically connected to the QSH state.

So what is the topological anomaly?

$$H = \begin{pmatrix} \xi_A + E_m & \alpha_k & \xi_{AB} & 0 \\ \alpha_k^* & \xi_A - E_m & 0 & \xi_{AB} \\ & & \xi_B - E_m & -\alpha_k \\ & c.c. & -\alpha_k^* & \xi_B + E_m \end{pmatrix}$$



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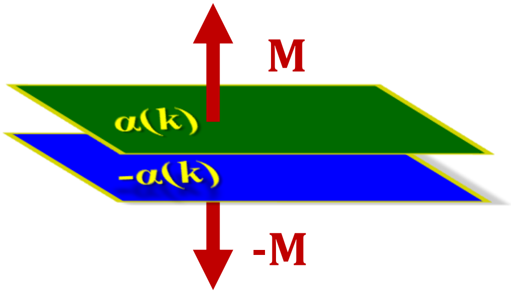
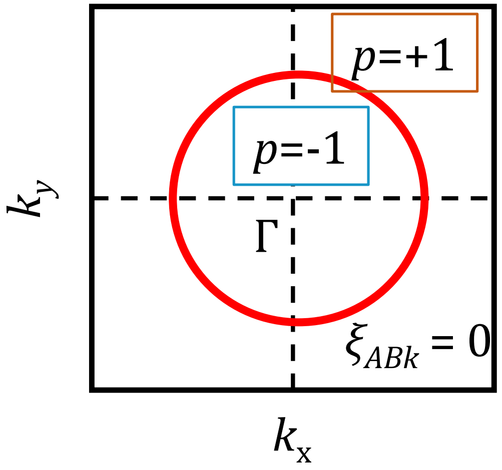




AF order: *PT*-invariant TI

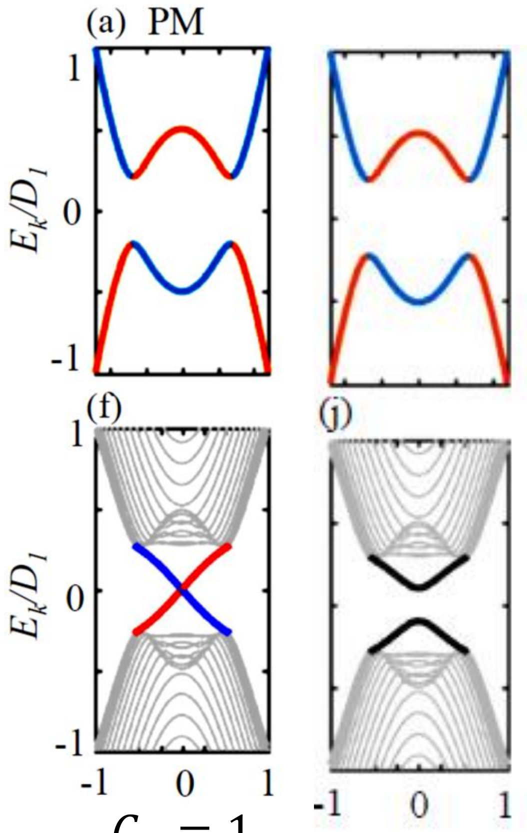
$$H_{int} = |E_m|\tau_x \otimes \sigma_x$$

Off diagonal terms. Chern number is not defined.

Bulk bands are adiabatically connected to the QSH state.

So what is the topological anomaly?

$$H = \begin{pmatrix} \xi_A + E_m & \alpha_k & \xi_{AB} + iE_m & 0 \\ \alpha_k^* & \xi_A - E_m & 0 & \xi_{AB} - iE_m \\ & & \xi_B - E_m & -\alpha_k \\ & c.c. & -\alpha_k^* & \xi_B + E_m \end{pmatrix}$$



$C_+ = 1$
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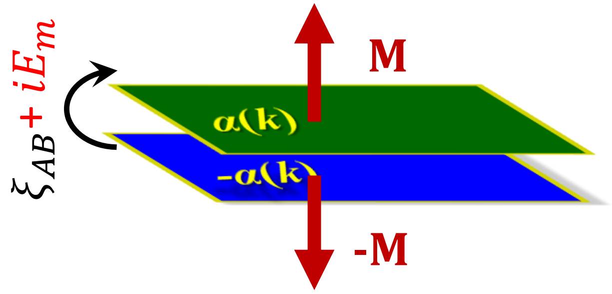


Jackiw-Rossi model: $H = \boldsymbol{\alpha}_{\mathbf{k}}.\boldsymbol{\gamma} + \tilde{\xi}_{AB}.\boldsymbol{\Gamma}$ Nucl. Phys. B 190, 681 (1981)

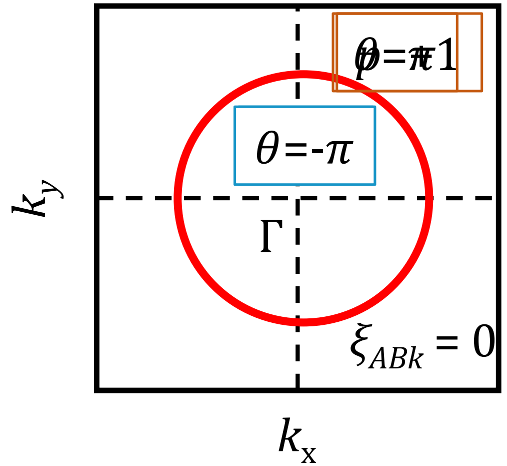
Complex hopping (vortex): $\tilde{\xi}_{AB} = \xi_{AB} + iE_m$

$$\theta_k = \tan^{-1}\frac{E_m}{\xi_{AB}} \qquad \nabla\theta_k = \pi\delta(k - k_0)\hat{k}$$

(Across the band inversion: $\xi_{ABk} = 0$)

$$\sigma_{xy}^{\pm} = \pm\frac{e^2}{\pi h}\frac{1}{(2\pi)^2}\int d\mathbf{k}.\nabla\theta_k = \pm\frac{e^2 k_0}{2\pi h}$$

Anomalous spin edge current:
$$\partial_\mu J_\pm^\mu = (\partial_\mu \sigma_{xy}^{\pm})E$$



Symmetry breaking topological phases

T^2	C^2	S^2	P	$(PT)^2$	$(CP)^2$	$(CPT)^2$	AZ (2D)	Our result (2D)	Sym broken order
-1	1	1					$\mathbb{Z}_2$ (QSH)		
0	1	0	1	0	-1	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (QSH)	FM
0	1	0	1	0	0	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}$ (QAH)	FM
0	0	1	1	0	0	-1	0	$\mathbb{Z}_2$ (QSH)	Chiral magnet (CM)
0	0	0	0	0	0	-1	$\mathbb{Z}$ (IQH)	$\mathbb{Z}_2$ (QSH)	CM + P broken
0	1	0	0	-1	0	-1	$\mathbb{Z}$	$\mathbb{Z}_2$ (Anomalous spin Hall)	AF
0	1	0	1	0	-1	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (ASH)	Chiral sublattice (CS)

Symmetry invariant

Topological phases

Symmetry T	C	S	Dimension 1	2	3	4
0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	0	0	0	0	0	$\mathbb{Z}$
1	1	1	$\mathbb{Z}$	0	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
1	-1	1	0	0	$\mathbb{Z}$	0

T = Time-reversal
C = Charge conjugation
S = TC =Chiral/sublattice
P = Parity

Altland, Zirnbauer (AZ), PRB **55**, 1142 (1997).
Chiu, Teo, Schnyder, Ryu RMP **88**, 035005 (2016)

CS order (non-magnetic): PT-invariant TI

⇕

AF order: PT-invariant TI

$$H = \begin{pmatrix} \xi_A + E_m & \alpha_k & \xi_{AB} + iE_m & 0 \\ \alpha_k^* & \xi_A - E_m & 0 & \xi_{AB} - iE_m \\ & & \xi_B - E_m & -\alpha_k \\ c.c. & & -\alpha_k^* & \xi_B + E_m \end{pmatrix}$$

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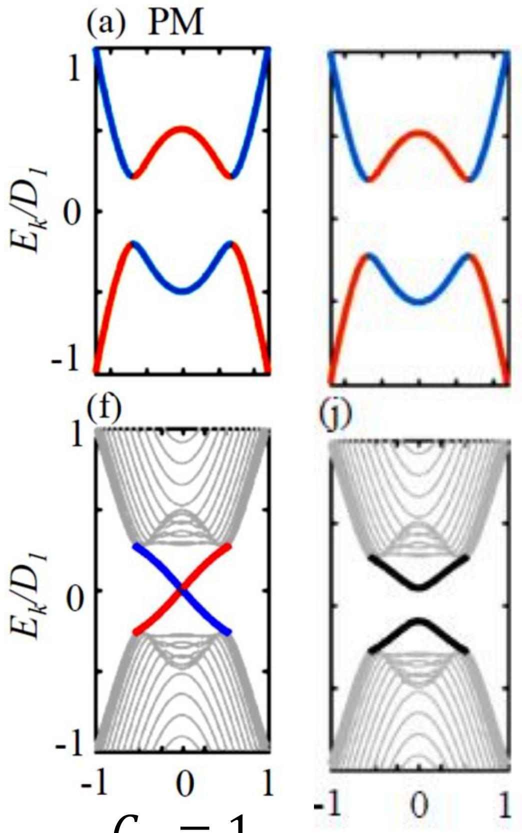
$$\theta_k = \tan^{-1}\frac{E_m}{\xi_{AB}} \qquad \boldsymbol{\nabla}\theta_k = \pi\delta(k - k_0)\hat{k}$$

(Across the band inversion: $\xi_{ABk} = 0$)

$$\sigma_{xy}^{\pm} = \pm\frac{e^2}{\pi h}\frac{1}{(2\pi)^2}\int d\mathbf{k}.\boldsymbol{\nabla}\theta_k = \pm\frac{e^2 k_0}{2\pi h}$$

Anomalous spin edge current:

$$\partial_\mu J_{\pm}^{\mu} = (\partial_\mu \sigma_{xy}^{\pm})E$$



$C_+ = 1$

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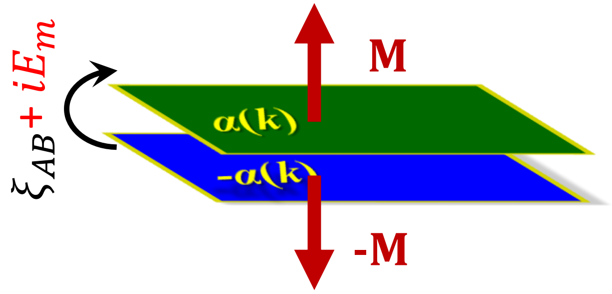

CS order (non-magnetic): *PT*-invariant TI

⇕

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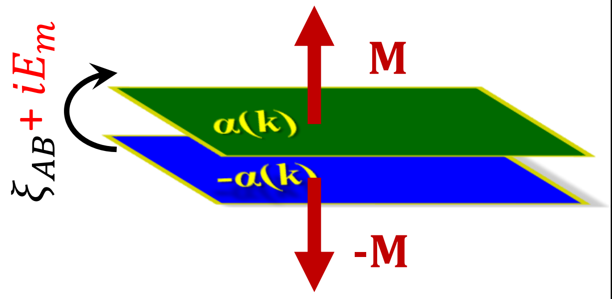
$$H = \begin{pmatrix} \xi_A + E_m & \alpha_k & \xi_{AB} & 0 \\ \alpha_k^* & \xi_A - E_m & 0 & \xi_{AB} \\ & & \xi_B - E_m & -\alpha_k \\ \multicolumn{2}{c}{c.c.} & -\alpha_k^* & \xi_B + E_m \end{pmatrix}$$



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Complex hopping (vortex): $\tilde{\xi}_{AB} = \xi_{AB} + iE_m$

Magneto-electric effect

$$\theta_k = \tan^{-1}\frac{E_m}{\xi_{AB}} \qquad \boldsymbol{\nabla}\theta_k = \pi\delta(k - k_0)\hat{k}$$

(Across the band inversion: $\xi_{ABk} = 0$)

$$J_x^{\pm} = \sigma_{xy}^{\pm} E_y$$

$$\sigma_{xy}^{\pm} = \pm\frac{e^2}{\pi h}\frac{1}{(2\pi)^2}\int d\mathbf{k}.\,\boldsymbol{\nabla}\theta_k = \pm\frac{e^2 k_0}{2\pi h}$$

Surface bound current: $\mathbf{J} = \boldsymbol{\nabla} \times \mathbf{M}$

Anomalous spin edge current:
$$\partial_\mu J_{\pm}^{\mu} = (\partial_\mu \sigma_{xy}^{\pm})E$$

$$M_y^{\pm} = -\sigma_{xy}^{\pm} V$$

Symmetry breaking topological phases

T^2	C^2	S^2	P	$(PT)^2$	$(CP)^2$	$(CPT)^2$	AZ (2D)	Our result (2D)	Sym broken order
-1	1	1					$\mathbb{Z}_2$ (QSH)		
0	1	0	1	0	-1	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (QSH)	FM
0	1	0	1	0	0	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}$ (QAH)	FM
0	0	1	1	0	0	-1	0	$\mathbb{Z}_2$ (QSH)	Chiral magnet (CM)
0	0	0	0	0	0	-1	$\mathbb{Z}$ (IQH)	$\mathbb{Z}_2$ (QSH)	CM + P broken
0	1	0	0	-1	0	-1	$\mathbb{Z}$	$\mathbb{Z}_2$ (Anomalous spin Hall)	AF
0	1	0	1	0	-1	0	$\mathbb{Z}$ (QAH)	$\mathbb{Z}_2$ (ASH)	Chiral sublattice (CS)
0	0	0	0	-1	-1	0	$\mathbb{Z}$ (IQH)	$\mathbb{Z}_2$ (ASH)	CS+ P broken
-1	1	1	1	-1	1	-1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	s, d SC (spinful)
-1	1	1	0	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	p SC (spinful)
-1	-1	1	1	-1	-1	1	0	0	s, d SC (spinless)
-1	-1	1	0	0	0	0	0	0	p SC (spinless)
0	-1	0	1	0	-1	0	$\mathbb{Z}$	$\mathbb{Z}_2$	FM, DM + SC
0	-1	0	0	0	-1	1	$\mathbb{Z}$	$\mathbb{Z}/\mathbb{Z}_2$	AF, SO + SC

Symmetry invariant

Topological phases

Symmetry			Dimension			
T	C	S	1	2	3	4
0	0	0	0	$\mathbb{Z}$	0	$\mathbb{Z}$
0	0	1	$\mathbb{Z}$	0	$\mathbb{Z}$	0
1	0	0	0	0	0	$\mathbb{Z}$
1	1	1	$\mathbb{Z}$	0	0	0
0	1	0	$\mathbb{Z}_2$	$\mathbb{Z}$	0	0
-1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	0
-1	0	0	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
-1	-1	1	$\mathbb{Z}$	0	$\mathbb{Z}_2$	$\mathbb{Z}_2$
0	-1	0	0	$\mathbb{Z}$	0	$\mathbb{Z}_2$
1	-1	1	0	0	$\mathbb{Z}$	0

T = Time-reversal
C = Charge conjugation
S = TC =Chiral/sublattice
P = Parity

Altland, Zirnbauer (AZ), PRB **55**, 1142 (1997).
Chiu, Teo, Schnyder, Ryu RMP **88**, 035005 (2016)

Conclusions

- With symmetry breakings, is the *ten-fold* topological classification table modified?
- Z_2 topology can be robust to time-reversal symmetry breaking.
- Novel helical anomaly in 2+1D systems. (No chiral anomaly due to Z_2 invariance).
- Novel magneto-electric effect in 2+1 D.
- Novel quantization with respect to electric potential.

Spin current: $J_s = \frac{e^2}{h} V$

Helical magnetic effect: $\mathbf{J} = \mu_B \frac{e^2}{h} \mathbf{B}$

$M_y^{\pm} = -\sigma_{xy}^{\pm} V$