



# Flat bands in twisted bilayer transition metal dichalcogenides.

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Bangalore

January 2, 2020

Novel Phases of Quantum Matter  
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- Recent interest in 'Magic angles' in twisted bilayer graphene [1].

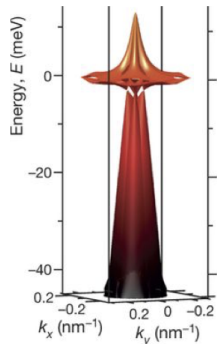
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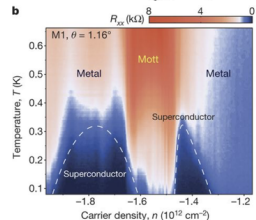
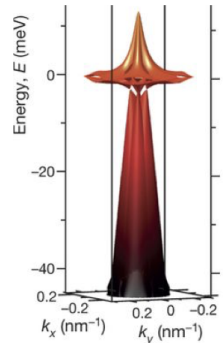


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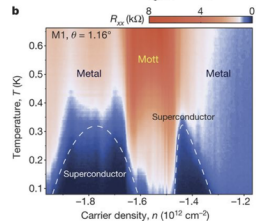
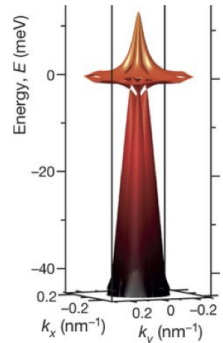




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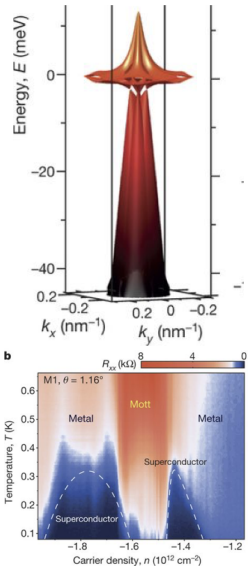
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- What is the origin of these flat bands in twisted bilayer TMDs?

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# Moiré patterns in twisted bilayer MoS<sub>2</sub> (tBLM)



- Introducing a small-angle twist between the two layers of bilayer MoS<sub>2</sub> leads to the formation of a Moiré pattern.
- We study these Moire patterns for twist angles close to 0° and 60°.
- A geometrically commensurate superlattice is generated for each angle using the Twister code [1].

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1. <http://www.physics.iisc.ernet.in/~mjain/software.html>



- DFT calculations for the larger angles<sup>[1]</sup> are performed using Quantum Espresso.
- Smaller angles contain too many atoms.
- Use *ab initio* calculations to parametrize Kolmogorov-Crespi Force-fields<sup>[2]</sup> for relaxation.
- Use SIESTA code for the electronic structure.

Angle	Atoms
5.1, 54.9	762
3.5, 56.5	1626
2.6, 57.4	2814
2, 58	5514
1.5, 58.5	8322

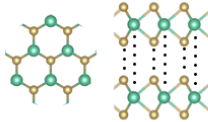
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  2. Mit H. Naik, Indrajit Maity, Prabal K. Maiti and Manish Jain, J. Phys. Chem. C **123** 9770-9778 (2019).

# Stackings in BLM

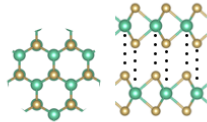


Five unique high-symmetry stackings.

**AA**



**AB**

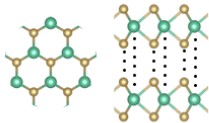


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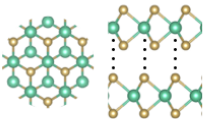


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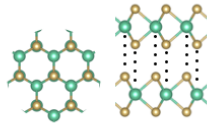
**AA**



**BMo/S**



**AB**



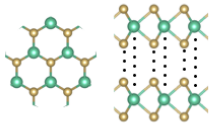


# Stackings in BLM

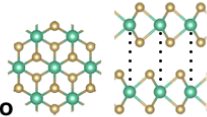


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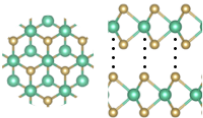
**AA**



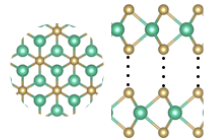
**BMo/Mo**



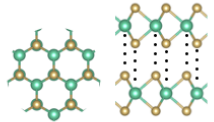
**BMo/S**



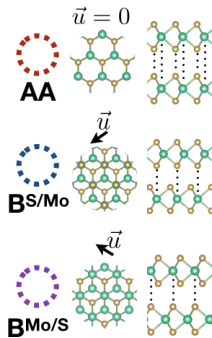
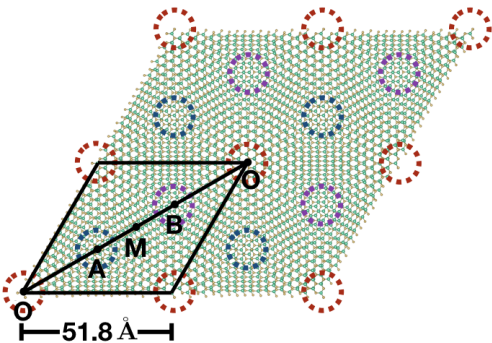
**BS/S**



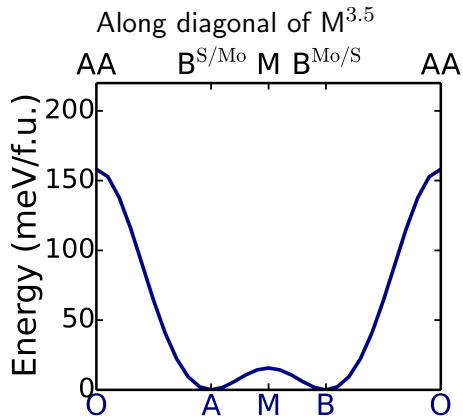
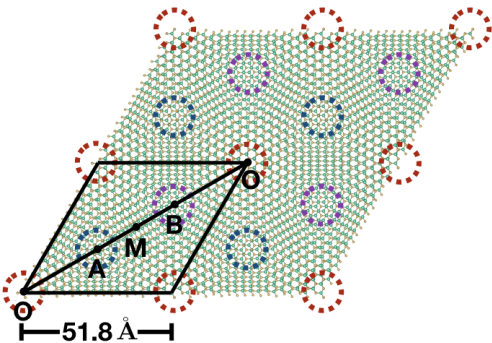
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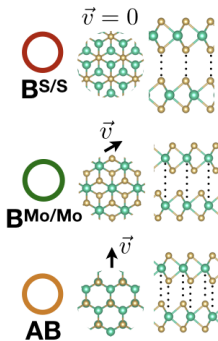
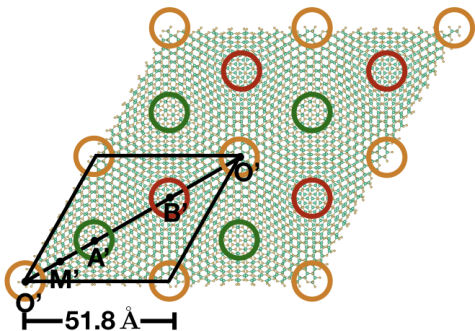
# 3.5° twist angle, $M^{3.5}$



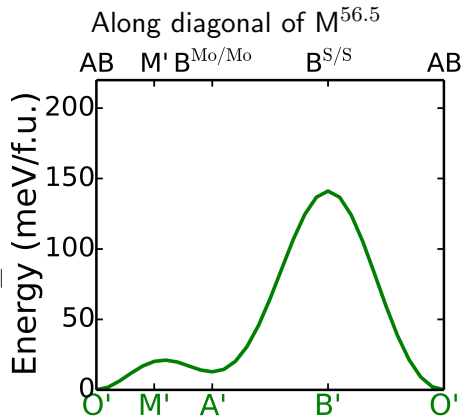
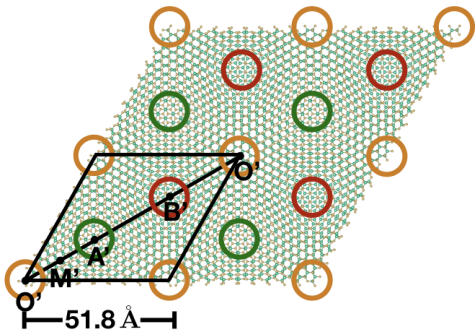
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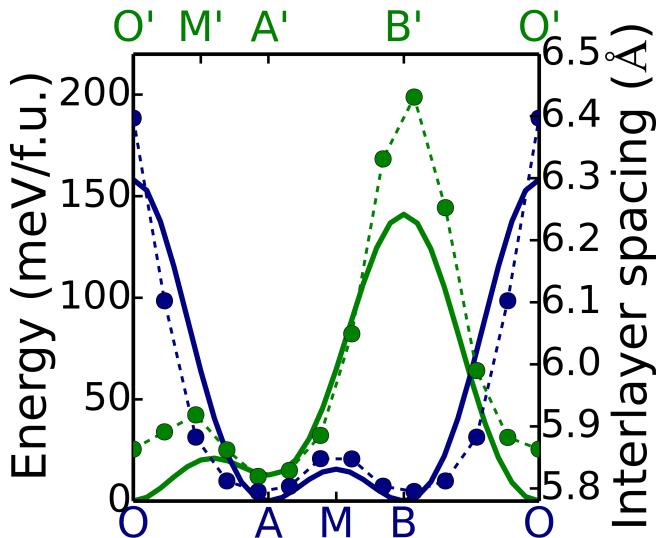
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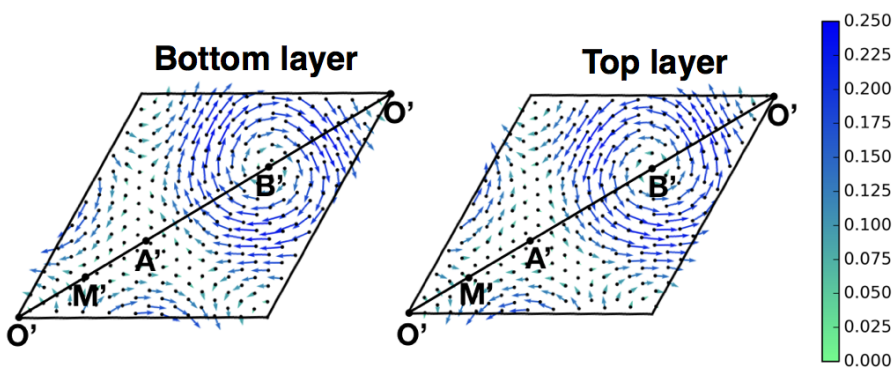


# 56.5° twist angle, $M^{56.5}$



# Interlayer spacings (ILS) in the Moiré







- Order-parameter defined as the shortest displacement vector that takes any given stacking to the highest energy stacking in the corresponding Moiré pattern.

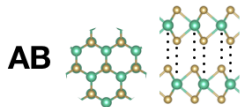
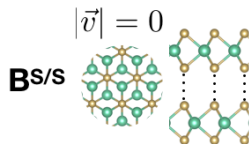
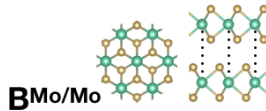
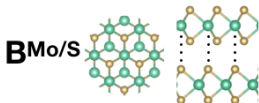
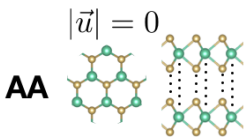


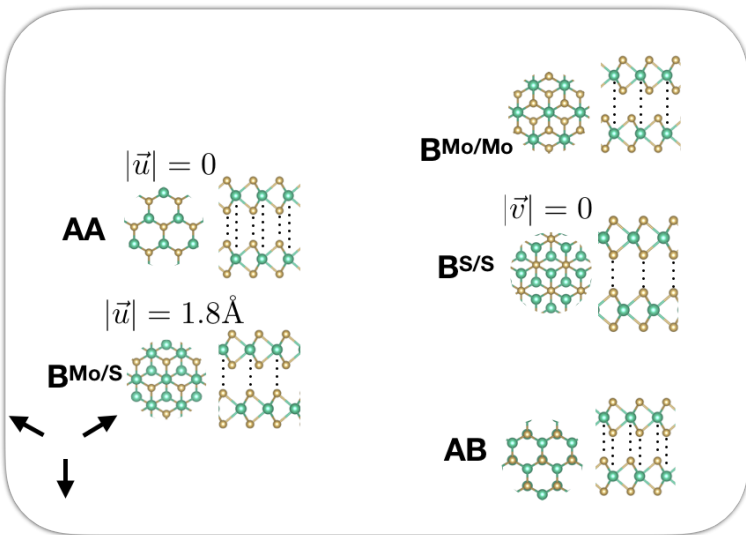


- Order-parameter defined as the shortest displacement vector that takes any given stacking to the highest energy stacking in the corresponding Moiré pattern.
- A simple translation cannot transform AB to AA stacking. We hence need two order parameters.

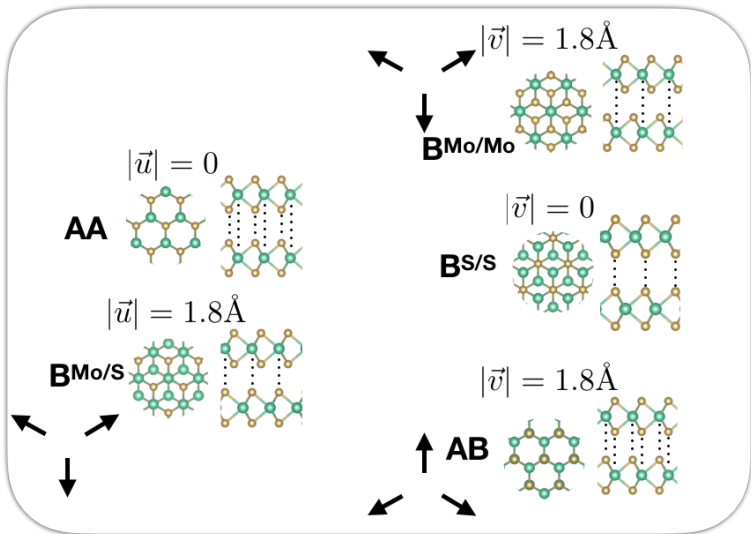


- Order-parameter defined as the shortest displacement vector that takes any given stacking to the highest energy stacking in the corresponding Moiré pattern.
- A simple translation cannot transform AB to AA stacking. We hence need two order parameters.
- $\vec{u}$  for angles close to  $0^\circ$ , and  $\vec{v}$  for angles close to  $60^\circ$





# Order-parameter

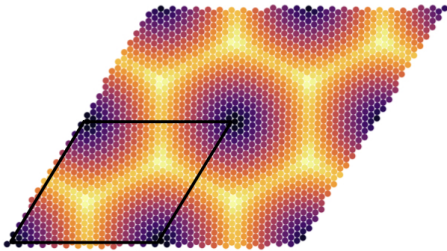


# Order-parameter distribution

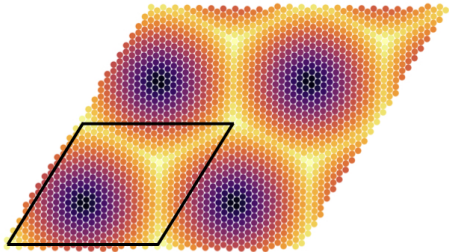


Unrelaxed:

$2.65^\circ$



$57.35^\circ$

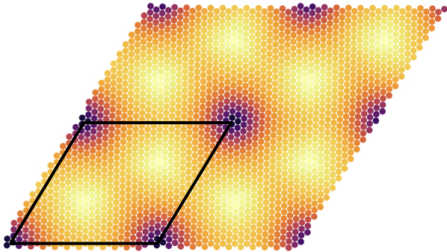


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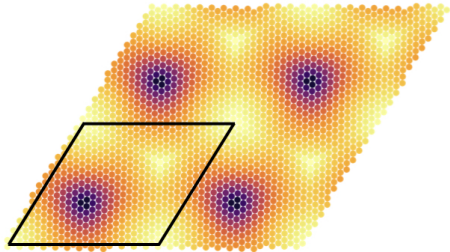


Upon relaxation, trade-off between induced strain and energy minimization by displacing to a stable stacking.

$2.65^\circ$



$57.35^\circ$

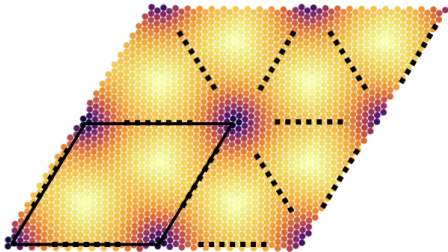


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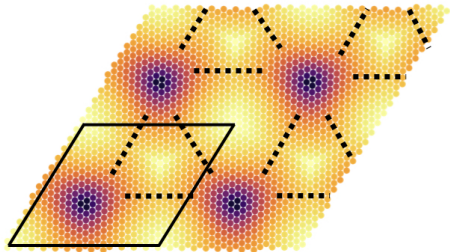


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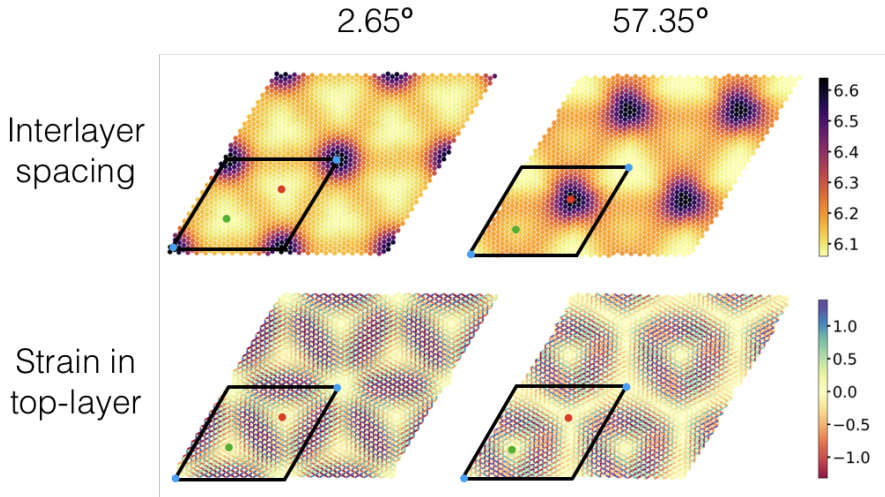


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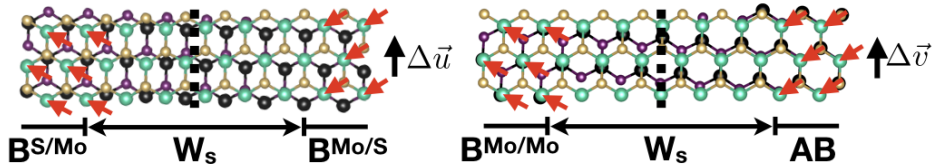


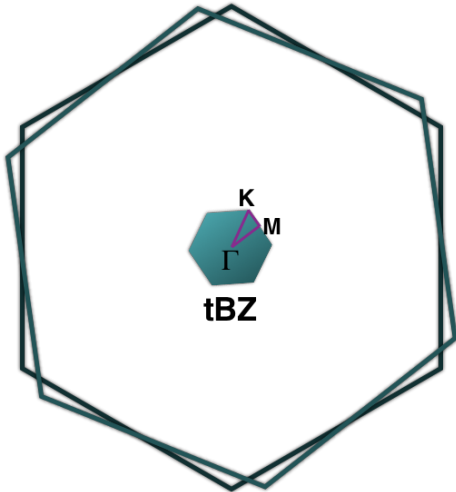


# Interlayer spacing and strain distribution

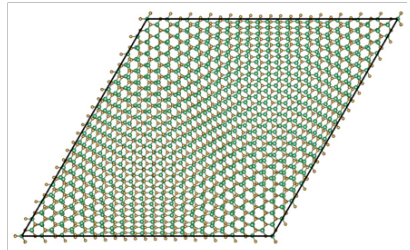


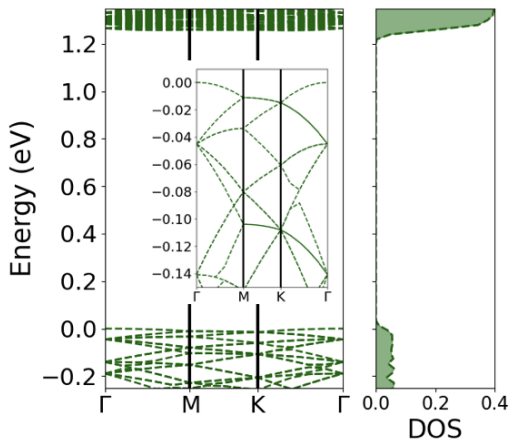
# Shear-strain soliton



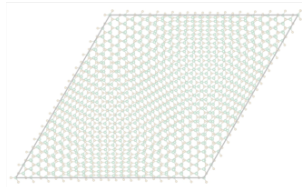


**Twist angle:  $2.6^\circ$**

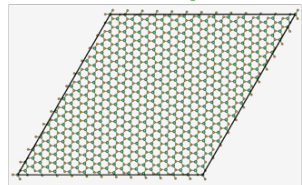


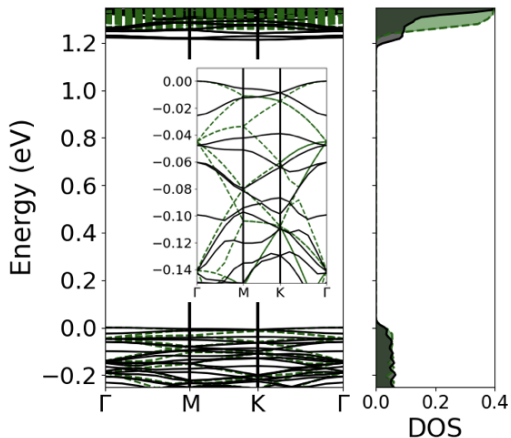


2.65° moiré superlattice

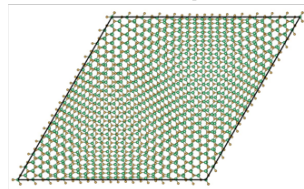


Pristine AB superlattice

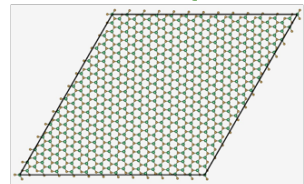


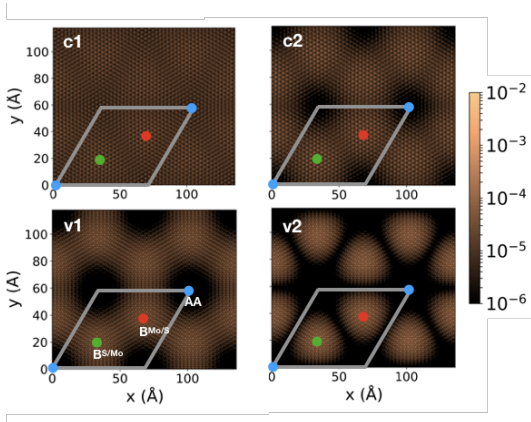
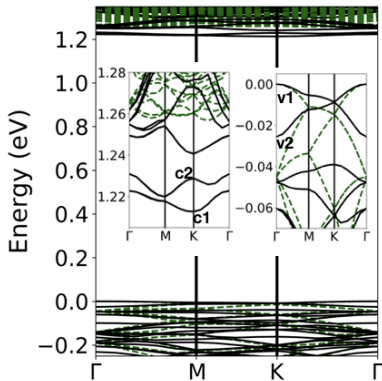


## 2.65° moiré superlattice



## Pristine AB superlattice

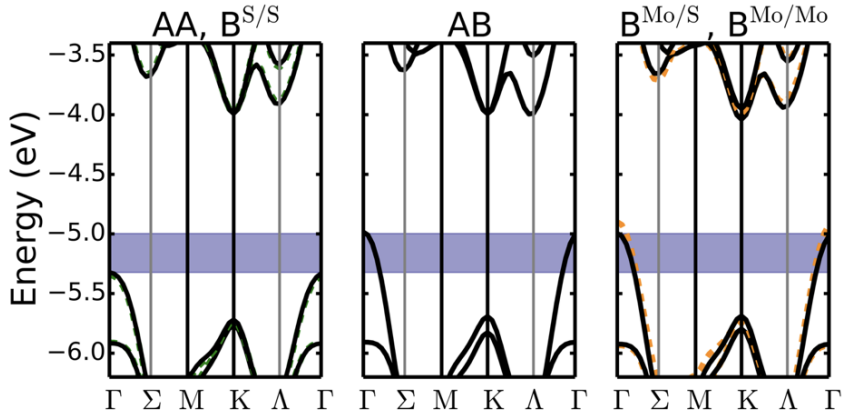




# Electronic structure of individual stackings



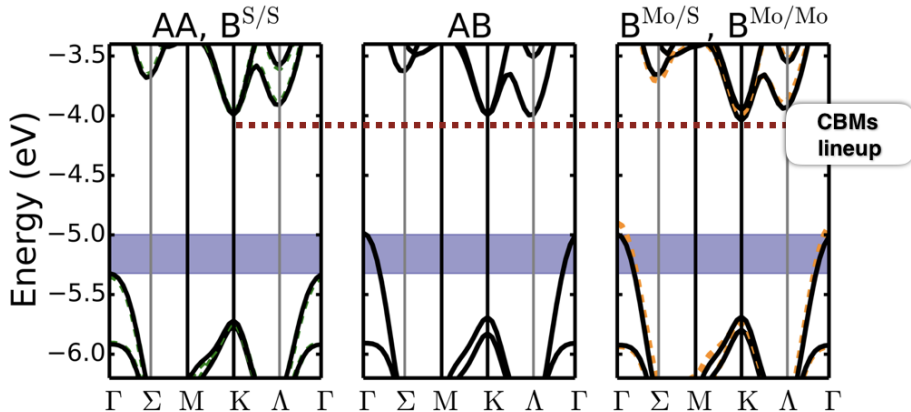
At their equilibrium interlayer spacing.



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At their equilibrium interlayer spacing.

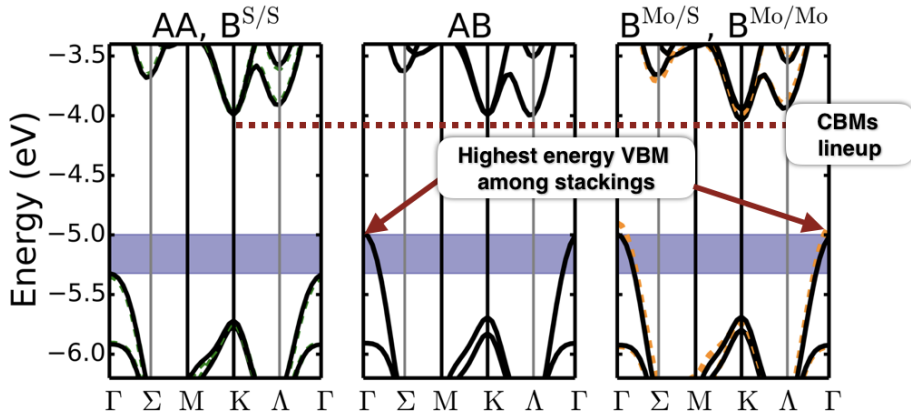




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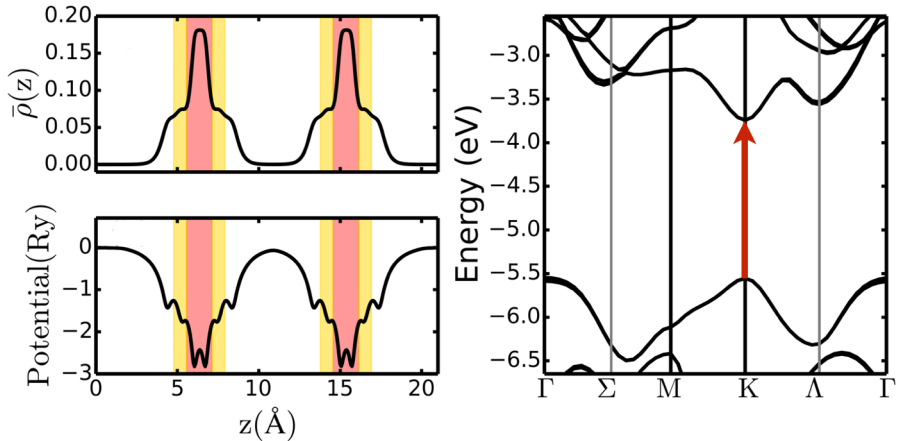


At their equilibrium interlayer spacing.



# Hybridisation between layers

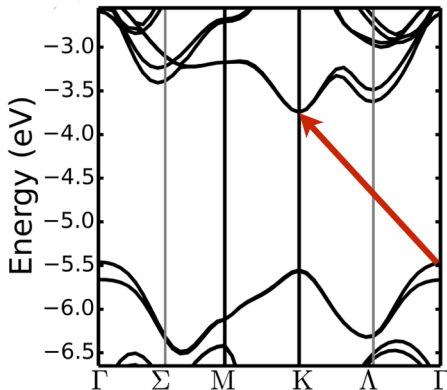
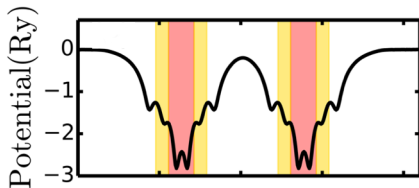
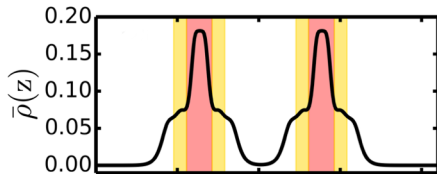
Barrier potential between layers: measure of the hybridisation between the layers.



# Hybridisation between layers



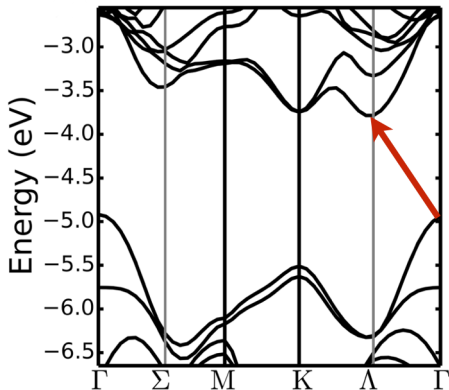
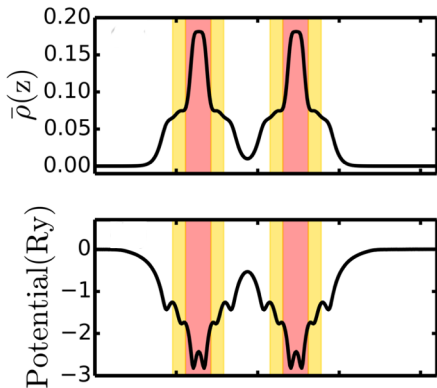
Barrier potential between layers: measure of the hybridisation between the layers.



# Hybridisation between layers



Barrier potential between layers: measure of the hybridisation between the layers.





$$\Delta V(x, y) = V_{\text{MSL}}(x, y) - \bar{V}_{\text{AB}}$$

$$V_{\text{MSL}}(x_0, y_0) = \int_{A_{uc}} \left( \frac{1}{2L_0} \int_{-L_0}^{L_0} V_{\text{MSL}}(x, y, z) dz \right) dx dy$$

is the  $z$  and macroscopic averaged potential in the MSL.

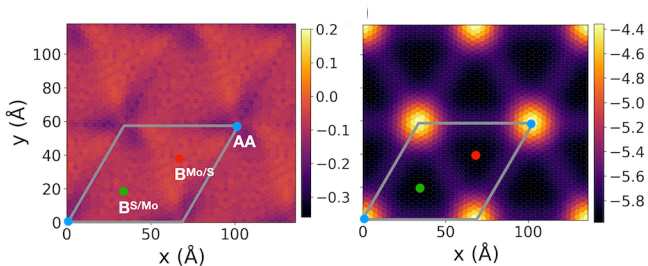
$\bar{V}_{\text{AB}}$  is unit-cell averaged potential of AB stacking.

# Inhomogeneous hybridisation in moiré

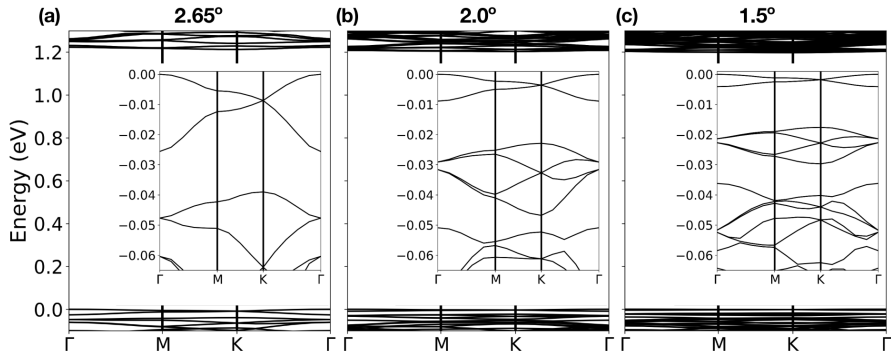


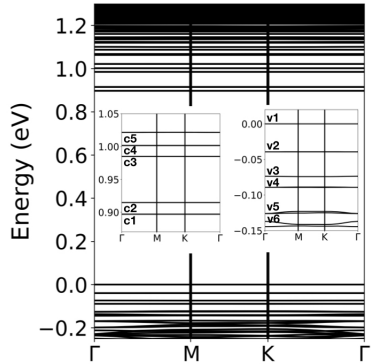
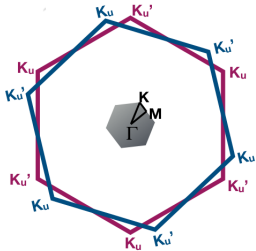
Localization governed by local barrier potential between the layers:

Confining potential      Inhomogeneous hybridization



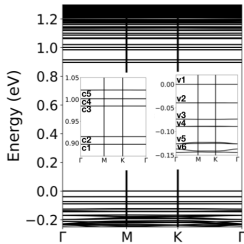
# Evolution of band structure near $0^\circ$



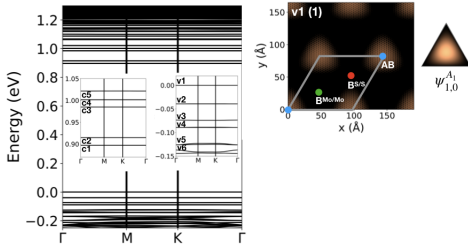




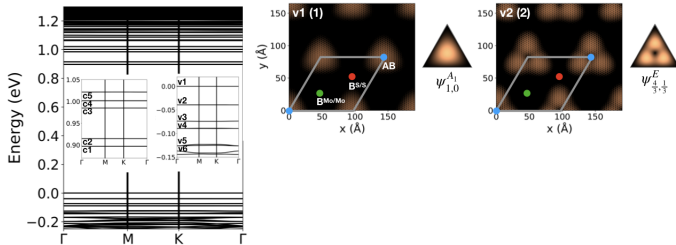
## Equilateral triangle quantum well states.



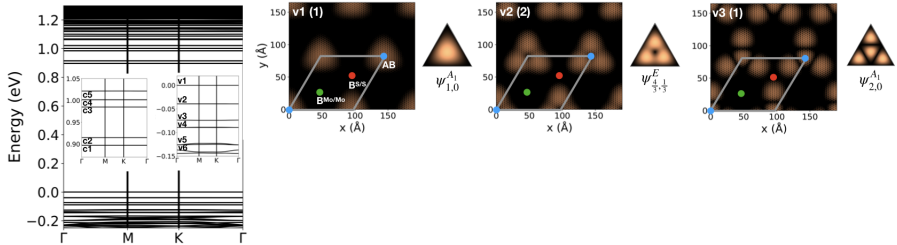
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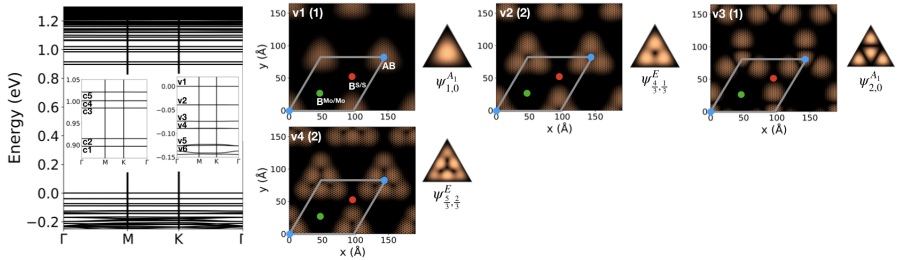
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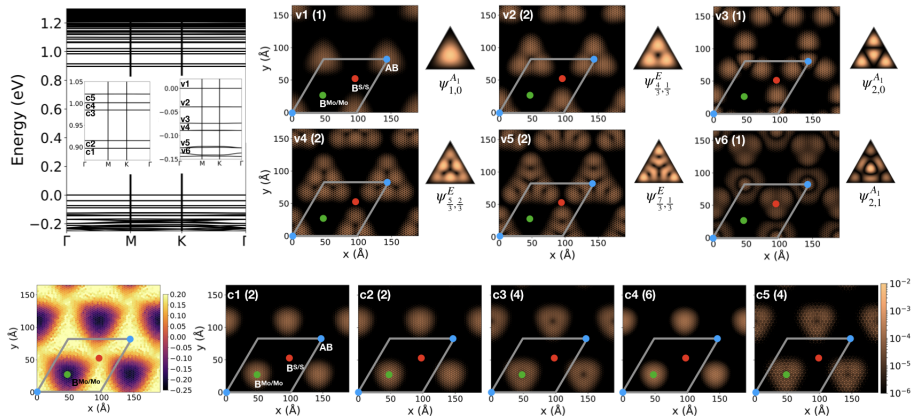


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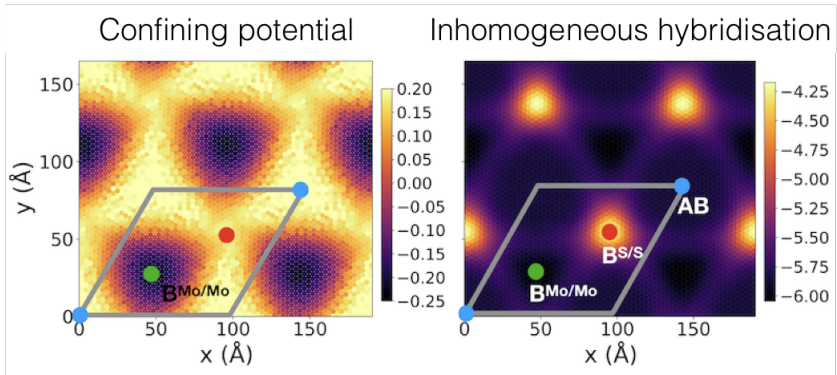


Spatially separated electrons and holes.

# 58° MSL: origin of localisation



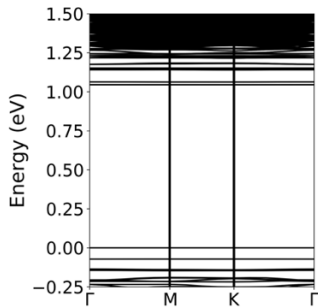
Combined effect of inhomogeneous hybridisation and additional confining potential:



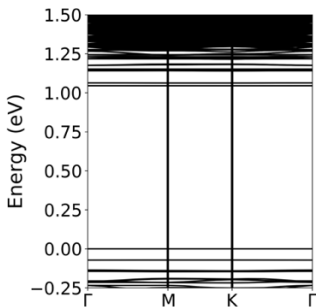




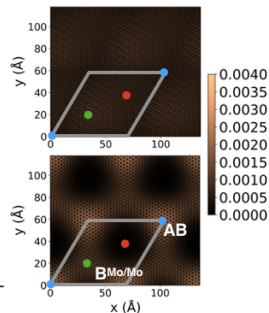
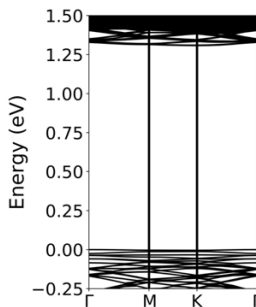
Relaxed structure  
(including strains)



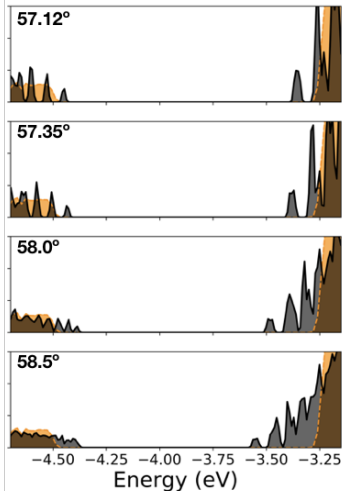
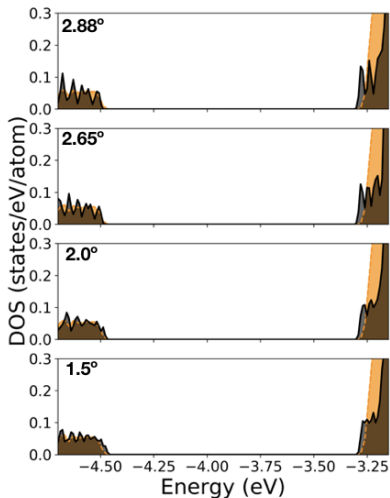
Relaxed structure  
(including strains)



Relaxed structure  
(excluding strains)



# Density of states for twisted Moiré patterns





- We show that there are *no magic angles* in small-angle twisted bilayer  $\text{MoS}_2$  and expect similar phenomenon in other TMD bilayers.



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- Demonstrate the formation of triangular quantum dots with spatially separated electrons and holes for twist angles close to 60°.



- We show that there are *no magic angles* in small-angle twisted bilayer MoS<sub>2</sub> and expect similar phenomenon in other TMD bilayers.
- Demonstrate the formation of triangular quantum dots with spatially separated electrons and holes for twist angles close to 60°.
- Relaxations are crucial to get the correct picture for band localization – unrelaxed structures show spurious flatbands and localization.

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