(Linear) Resistivity in Metals

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In most clean metals, electrical resistivity increases linearly with temperature, except at rather low temperatures

The resistivity is due to electrons scattering off phonons which have a quantum energy scale ($\hbar\omega_D$) and a corresponding temperature scale θ_D. At temperatures higher than θ_D , the resistivity is proportional to $< x_i^2 > \sim k_B T$ (classical equipartition law, so 'universal'). The slope is not universal; it depends on the strength of electron phonon coupling.

But, there is a lot of recent churning in this field. Why?

1. Large size of the resistivity over a wide temperature range with no signs of saturation at a (T=0) quantum scale
2. Association of this phenomenon with electronic systems which are strongly correlated/have no low excitation energy quasiparticles

I will show some data and mention some theoretical attempts Will also present a point of view, not quite a theory yet.

Quantum scale of resistivity (Mott, Ioffe and Regel; 1950s and 1940s)

Crudely, resistivity $\rho = (mv_F / ne^2 l)$ where l is the mean free path.

(Drude, Boltzmann transport theory)

- Hard to think of electron as a quasiparticle when its de Broglie wavelength λ is smaller than the mean free path l. Since these are electrons with Fermi energy, $\lambda_F \sim l$ is the characteristic quantum scale. The resistivity corresponding to this is the Mott maximum metallic resistivity (the more common phrase is Mott minimum metallic conductivity) .
- Ioffe and Regel argued that since the electron has to collide with something for resistive scattering, and that something is at least an atomic spacing 'a' away, the minimum possible l ~ 'a'.
- In three dimensions, for most metallic densities, the Mott and Ioffe-Regel limits of resistivity are close to each other. The number is called ρ_{MIR} . The absolute value is about 0.2 m-ohm cm for typical metallic electron densities. These are guesses for a crossover scale.
- In two dimensions, the scale is a resistance. The quantum of resistance is (h/e^2).

As observed by von Klitzing in quantum Hall effect, it is

$$R_K = h/e^2 = 25812.80745\ldots\ \Omega.$$

(This is a topological invariant there: known to about 1 part in 10^9).

What is this quantum scale?

- Some people assume the relation between n and k_F to be that for an electron with spin ($n=(k_F^3/3\pi^2)$ in 3d; $n=(k_F^2/2\pi)$ in 2d). Or, to be that without spin ($n=(k_F^3/6\pi^2)$ in 3d ; $n=(k_F^2/4\pi)$ in 2d).
- Some assume that the Mott criterion is $k_F l = 1$; others that $k_F l = 2\pi$.
- The assumptions about the Mott criterion reflect our belief that roughly there, something changes, and the fact that this is a crossover scale.
- I will assume spinful electrons , namely , that $n=(k_F^3/3\pi^2)$ in 3d $n=(k_F^2/2\pi)$ in 2d
- I will assume that the Mott criterion is $k_F l = 2\pi$ (i.e. that $l=\lambda_F$)
- In this case, $\rho_M = R_K (3/4k_F)$ for 3d and $R_M = (R_K/2\pi)$ for 2d.

(For $k_F l = 1$, however, the values are $\rho_M = R_K (3\pi/2k_F)$ for 3d and $R_M = (R_K)$ for 2d)

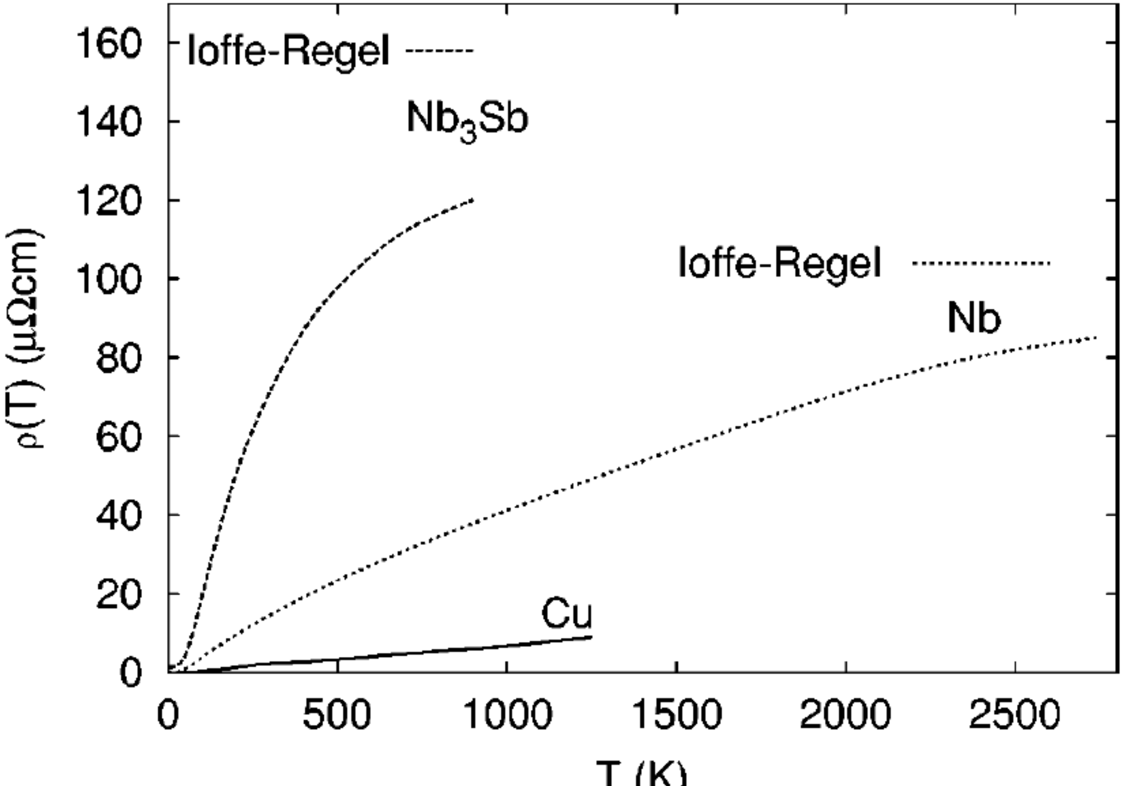
The Ioffe Regel values are: $\rho_{IR} = R_K (3\pi/2k_F)(1/k_F a)$ for 3d; $R_{IR} = (R_K)(1/k_F a)$ for 2d

Numbers (for the case $l=\lambda_F$)

We take $k_F = 1.5\times10^8$ cm^{-1} ; $a=3\times10^{-8}$ cm.

$\rho_M = 0.13$ mΩcm; $\rho_{IR} = 0.12$ mΩcm. $R_{M\Box} = 4.11$kΩ; $R_{IR} = 5.73$kΩ.

- Experimentally, something does happen at low temperatures around this resistivity (resistance) : crudely, this value separates metal from insulator. (To have this kind of resistivity at low T, the metal has to be very disordered).
- For pure metals, resistivity increases linearly with temperature, but is generally small. In some, in which it is large but linear, there are clear signs of its bending over as if to saturate at ρ_{MIR} (the resistivity saturation phenomenon, much discussed in the seventies and eighties) The figure below shows it.(Ioffe Regel numbers are for Nb_3Sb and Nb).

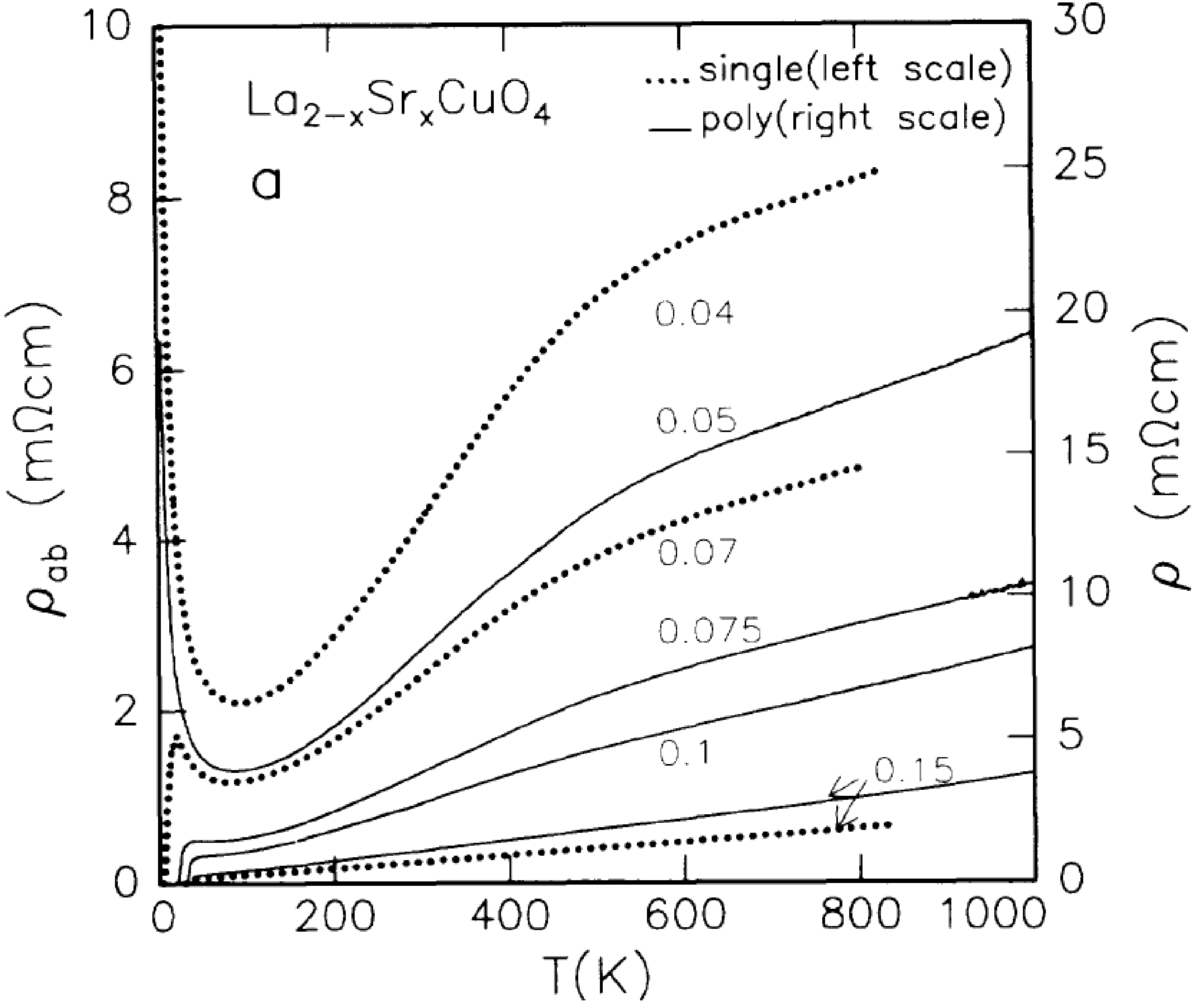


- The saturation value seems to be of order of σ_{MIR}.
- Still not understood. A phenomenological model of parallel resistors (Wiesmann et. al.)

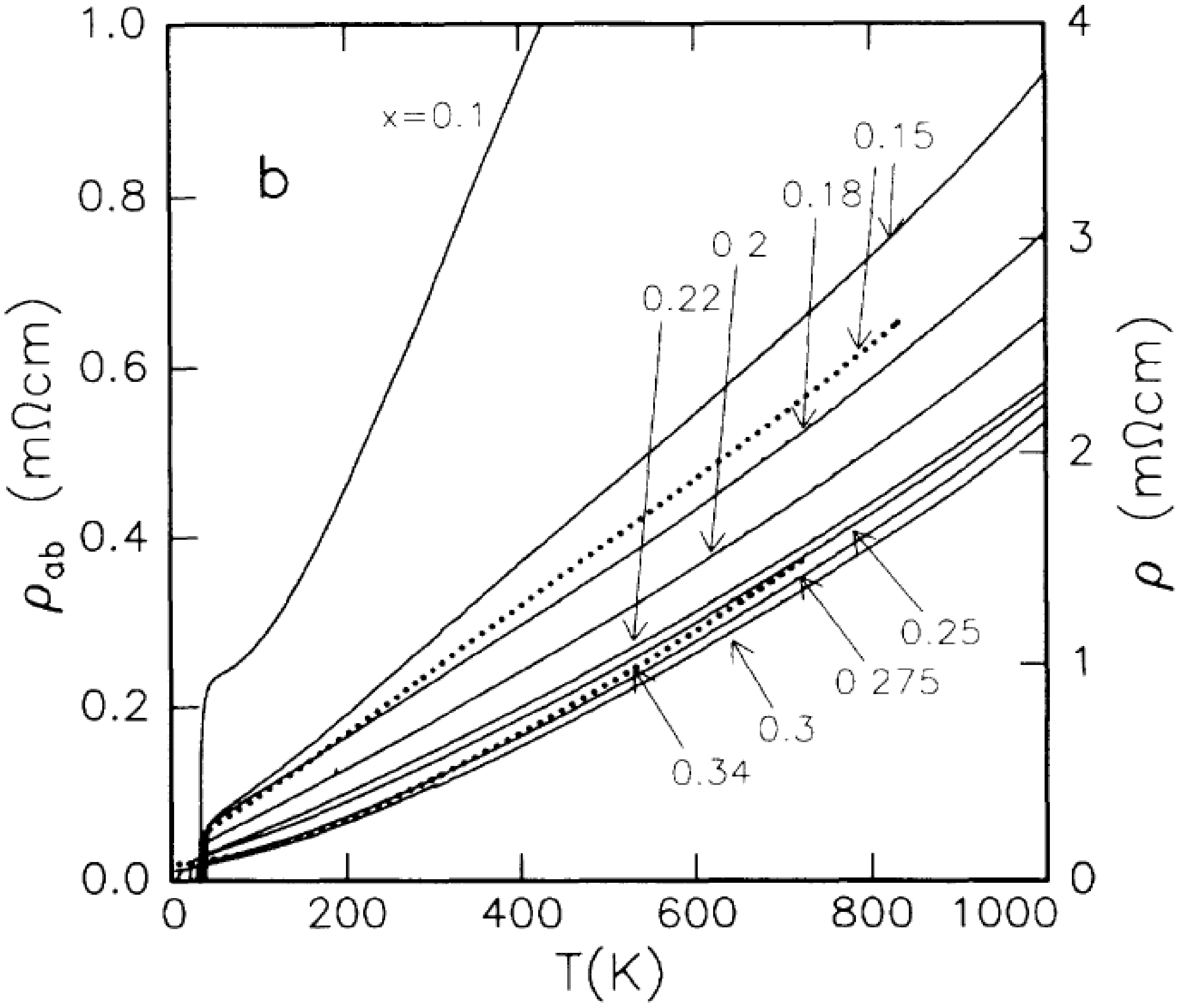
$$\sigma = \sigma_{Drude} + \sigma_{MIR}$$

seems to work well. What is the origin of this model? Why does it involve σ_{MIR} ?

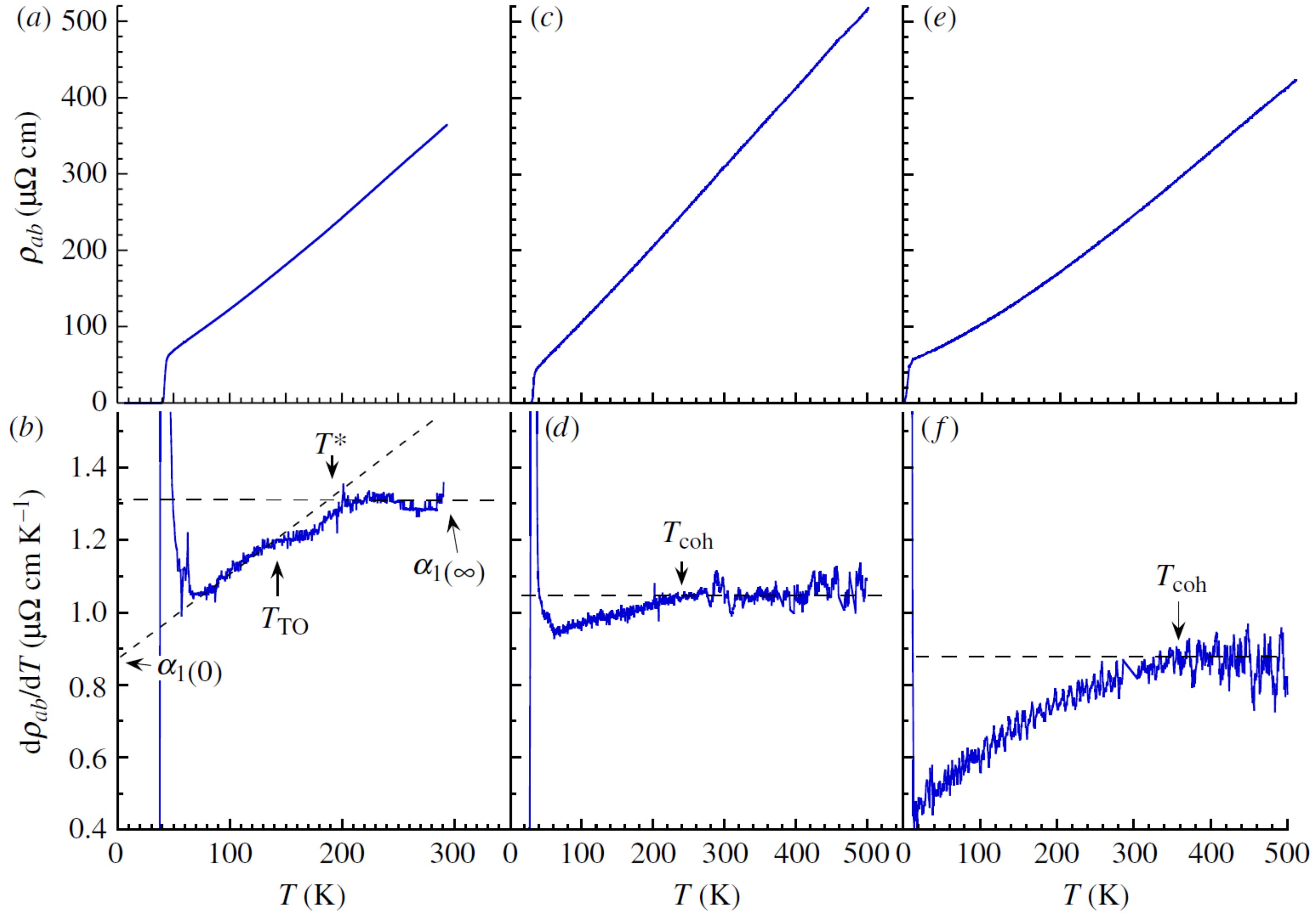
- One microscopic theoretical attempt I remember (Chakraborty and Allen? 1980?)is that interband matrix elements are the cause of σ_{MIR} , and the Kubo formula for conductivity is the cause of the parallel resistor like form for σ.
- In the late eighties and early nineties, the novel phenomenon of large linear, nonsaturating resistivity was discovered in the cuprates which had been observed to harbour high temperature superconductivity. The resistivity seems to keep increasing without showing signs of saturation, well beyond ρ_{MIR} .
- (Crudely, the bare TB t~0.4eV ~ 4,600K is their characteristic electronic energy scale. The TB bandwidth is 8t~37,000K)



Takagi, Batlogg et. al., PRL 1992, reported ρ_{ab} of high quality single crystal films of $La_{2-x} Sr_x CuO_4$ with x ranging from 0.1 (underdoped) to 0.34 (overdoped) and temperature from T_c to 1000K. They saw a linear(ish), nonsaturating increase all through. For larger x, they fitted ρ(T) to a form T^{α} , and found the best fit for α =1.5. They discussed the possible violation of the MIR limit. For d=6x10^{-8} cm and $\rho_{ab} \sim \rho_{IR}$ ~0.5 mΩcm, resistance per layer is 6-7 kΩ. We see that for x<0.1, resistance per layer highr, often much higher, than R_K .



b
x=0.1
0.15
0.18
0.2
0.22
0.25
0 275
0.3
0.34
ρ_{ab} (mΩcm)
ρ (mΩcm)
T(K)
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
0
200
400
600
800
1000



A more recent, detailed look at the same region (Hussey,Cooper, Proust, Takagi et.al. , Science 2009, Phil Trans Roy Soc 2011) suggested the following picture.

Assume $\rho(T)= \rho_o + \alpha_1 T+ \alpha_2 T^2$

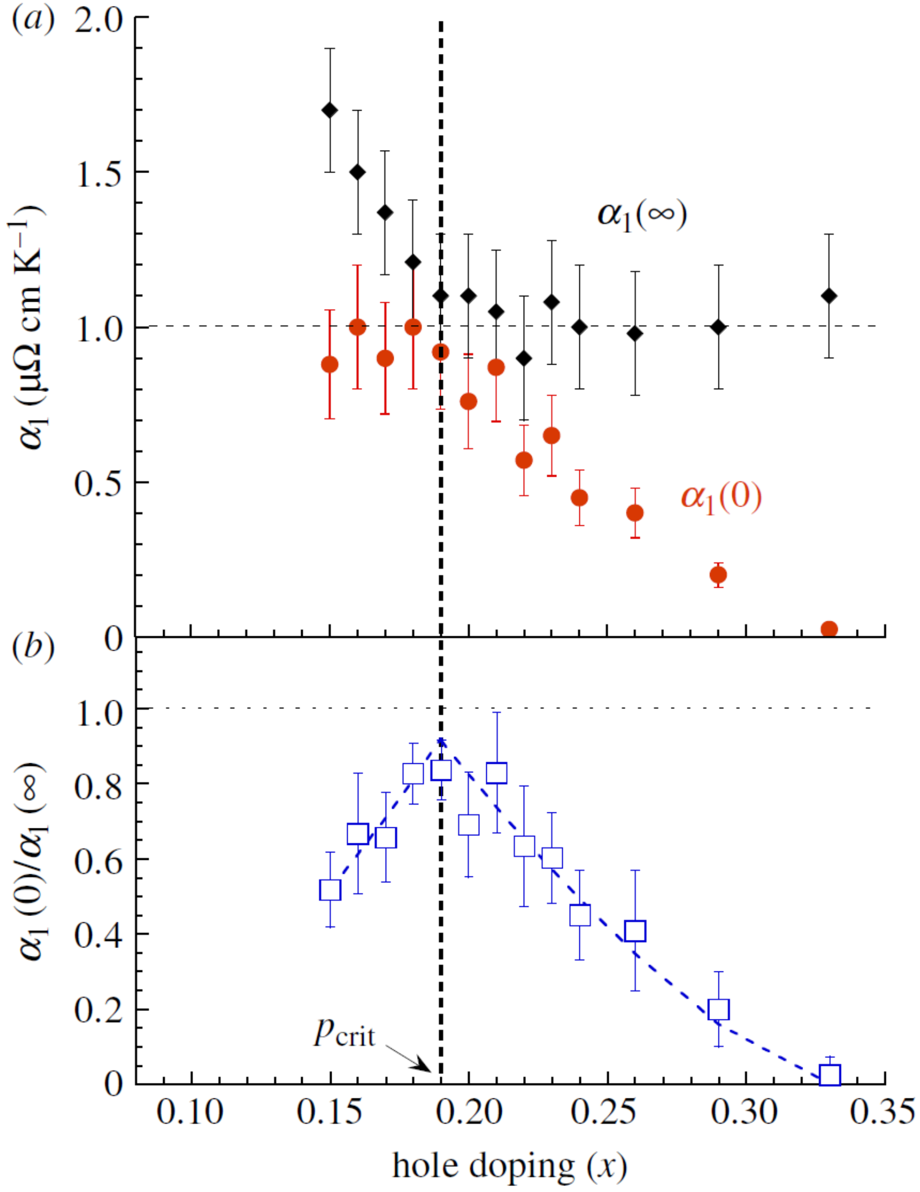
α_1 and α_2 depend on temperature!

Data shown for x=0.17,0.21 and 0. 26.

$d\rho_{ab}/dT$ reveals the α's better.

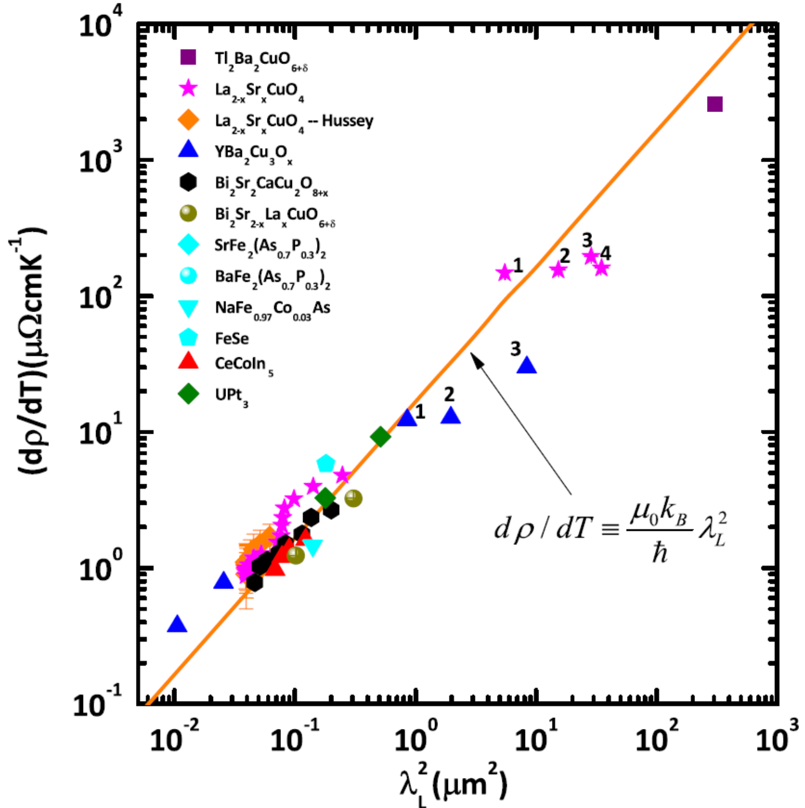
Above a certain temperature, there is only the linear term. ($\alpha_2 \rightarrow 0$ above T^* or T_{coh}) . Since the T^2 term is a signature of a coherent Fermi fluid, this implies that at high temperatures, one always has only an incoherent Fermi fluid, characterized by a $\alpha_1(\infty)$.

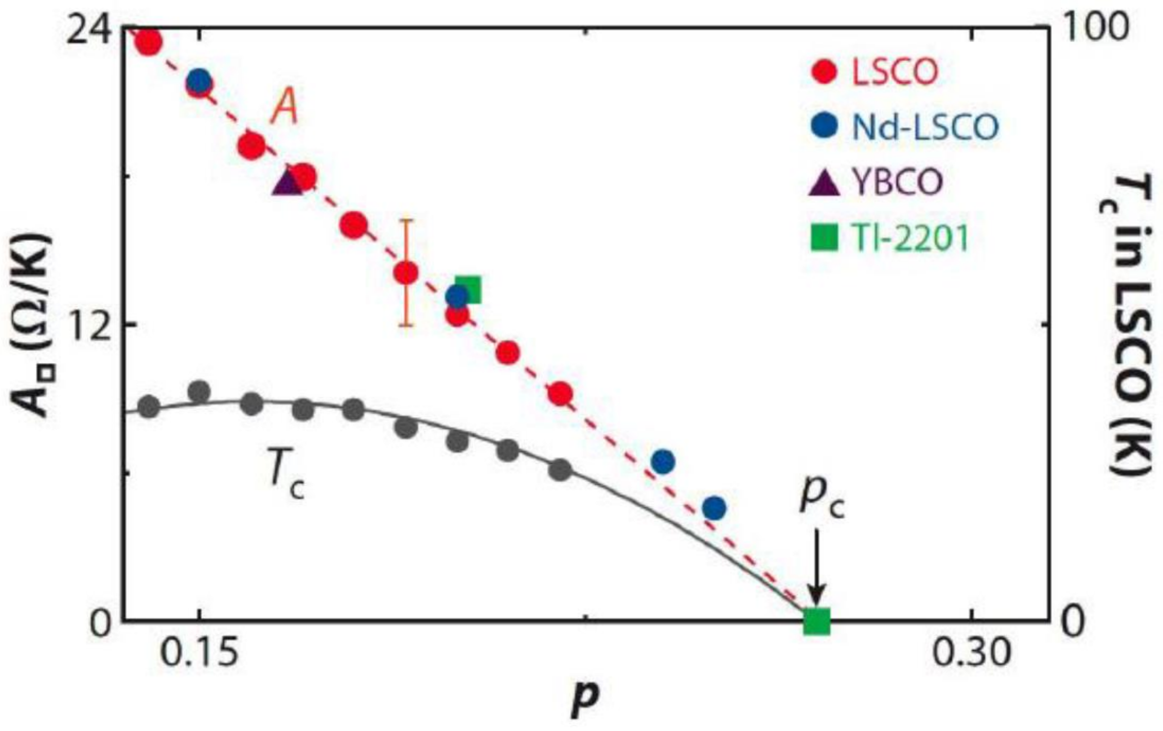
The p dependence of $\alpha_1(0)$ and of $\alpha_1(\infty)$ are shown in the next slide.



There **is** an incoherent linear T regime

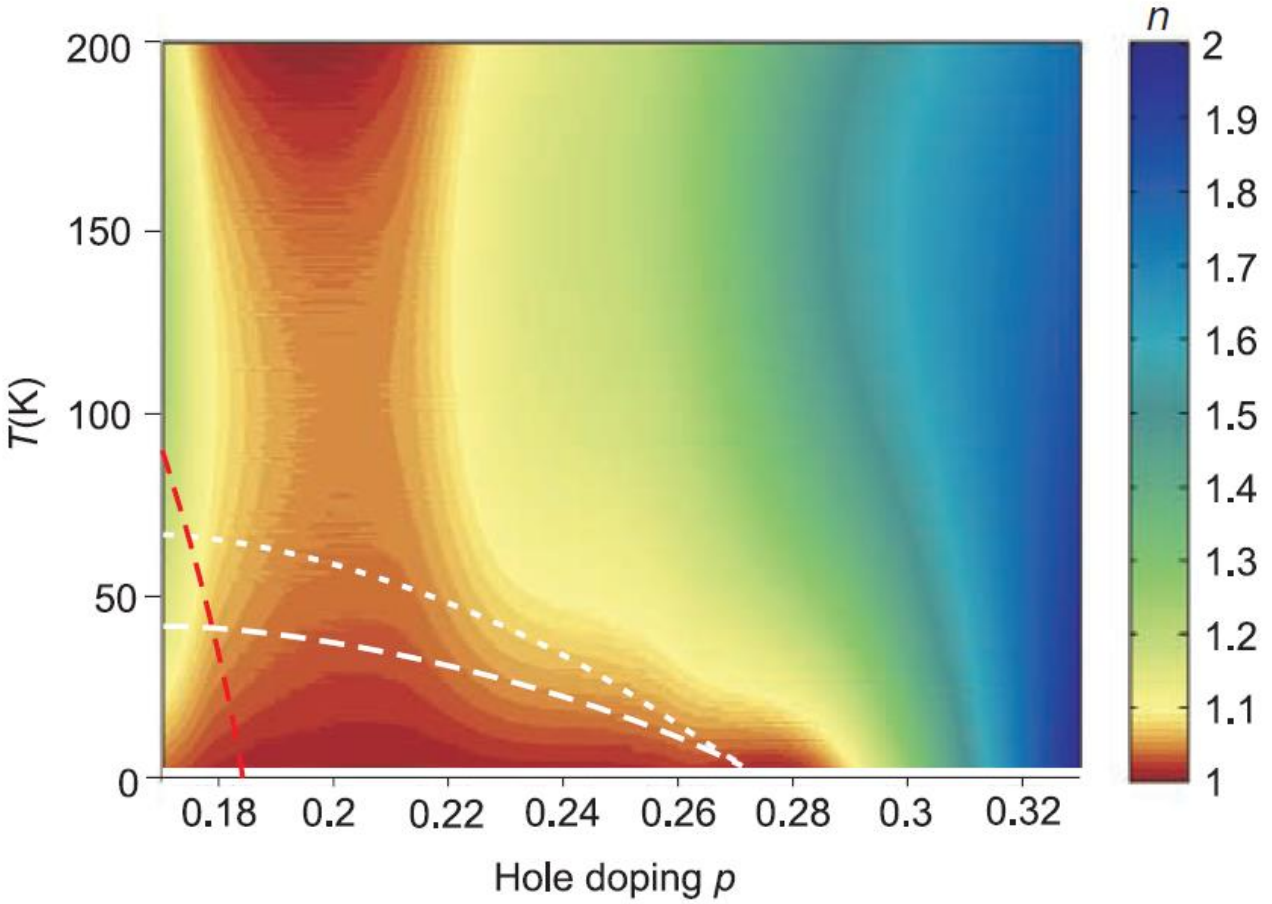
1. Only intrinsic(?), $\alpha_1(\infty)$ for $p>p_{crit}$.
2. Besides α_2 ,tThere is a term α_1 (0) for $p<p_{crit}$ (Origin :pairing fluctuations?).There are many examples of experimental correlations bet. T_c and α_1 (0) (including in Bechgaard salts). Two are shown now. One recent example is Hu et.al. Scient. Rep. 2017, below. Another is on the next slide.



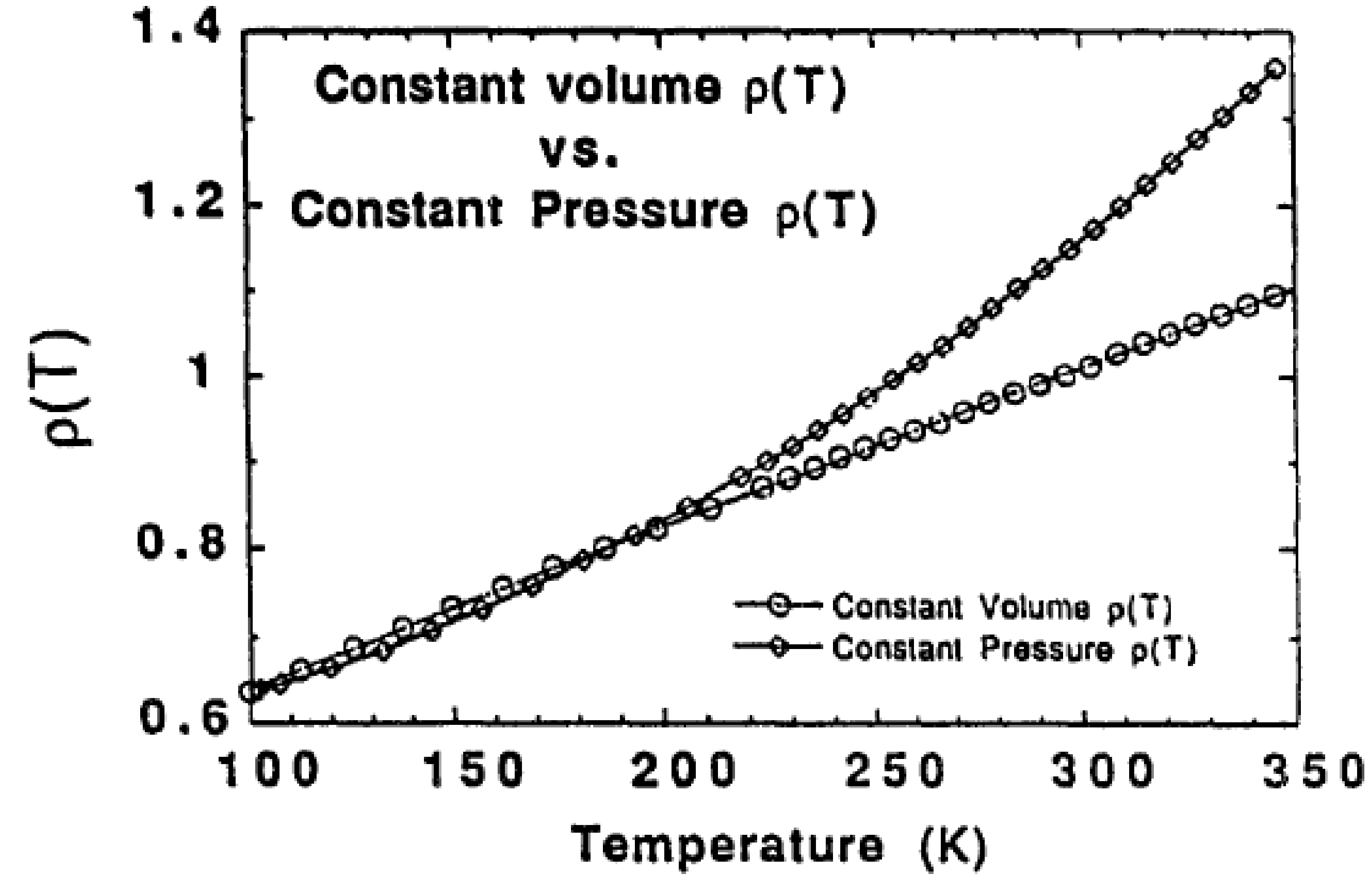


The low temperature coefficient A is common to many superconducting cuprates, as shown in the left.

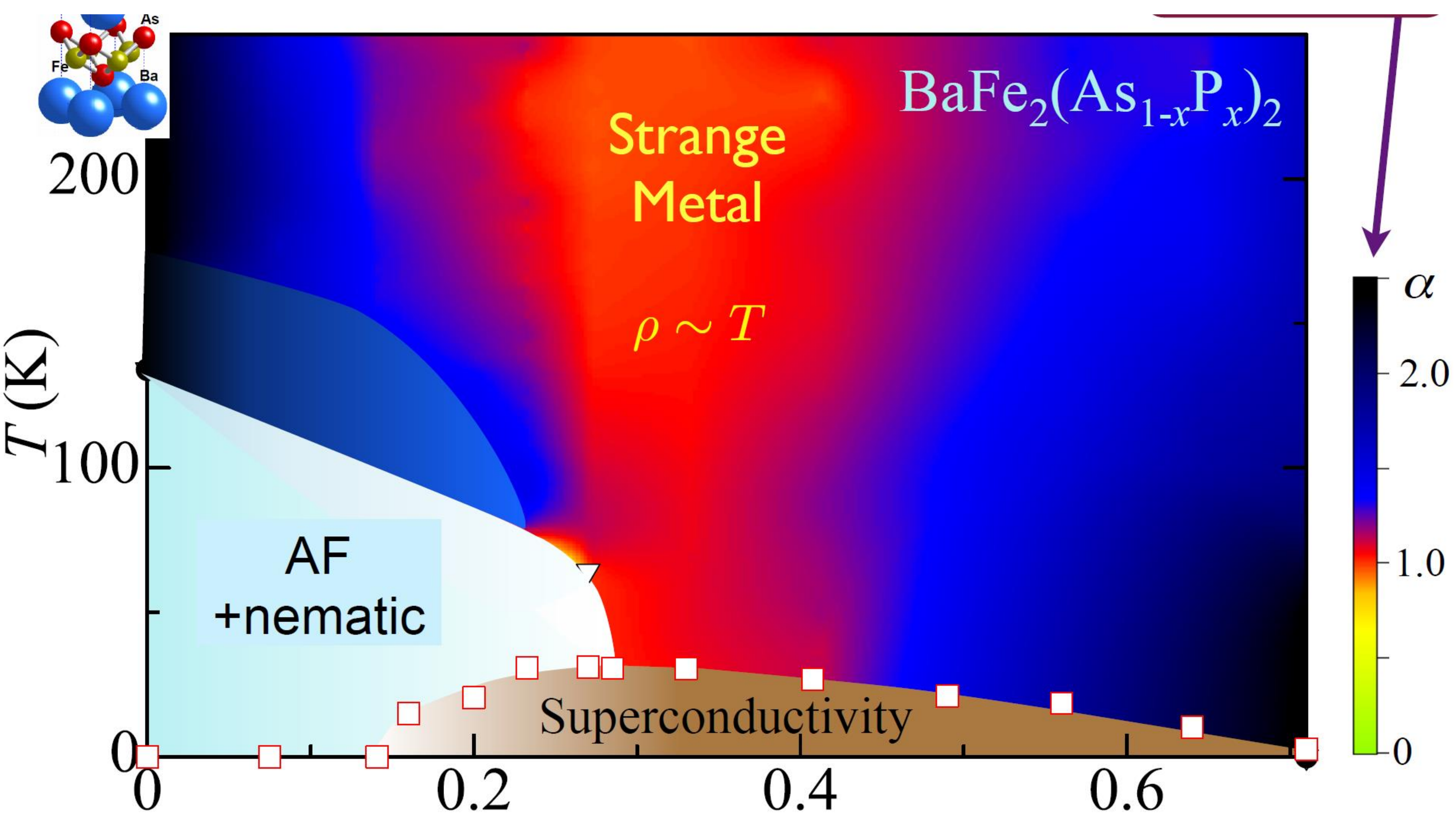
- In all cuprates, there is clear observation of a robust linear nonsaturating resistivity regime at high T above the optimum doping (maximum T_c). Though $T>T_c^{opt}$, it is still <<t~3000-4000K. $\rho > \rho_{MIR}$ already in this regime.The linear regime $\alpha_1(0)T$ at low accessed by the magnetic field destruction of superconductivity, over a wide range of doping does not sit well with the idea of a standard QCP.
- One example of both is the plot of ln {ρ(T)-ρ(0)} vs. lnT for different hole dopings, for the LSCO samples above (Cooper et. al. 2009). The value n=1 is seen at high temperatures near p_{opt} and at low T over a wide range of p.
- From the plot of $\alpha_1(0)$, we see that it is nonzero for 0.17<p < 0.29, including p beyond superconducting dome; the putative QCP is at p_c = 0.19.

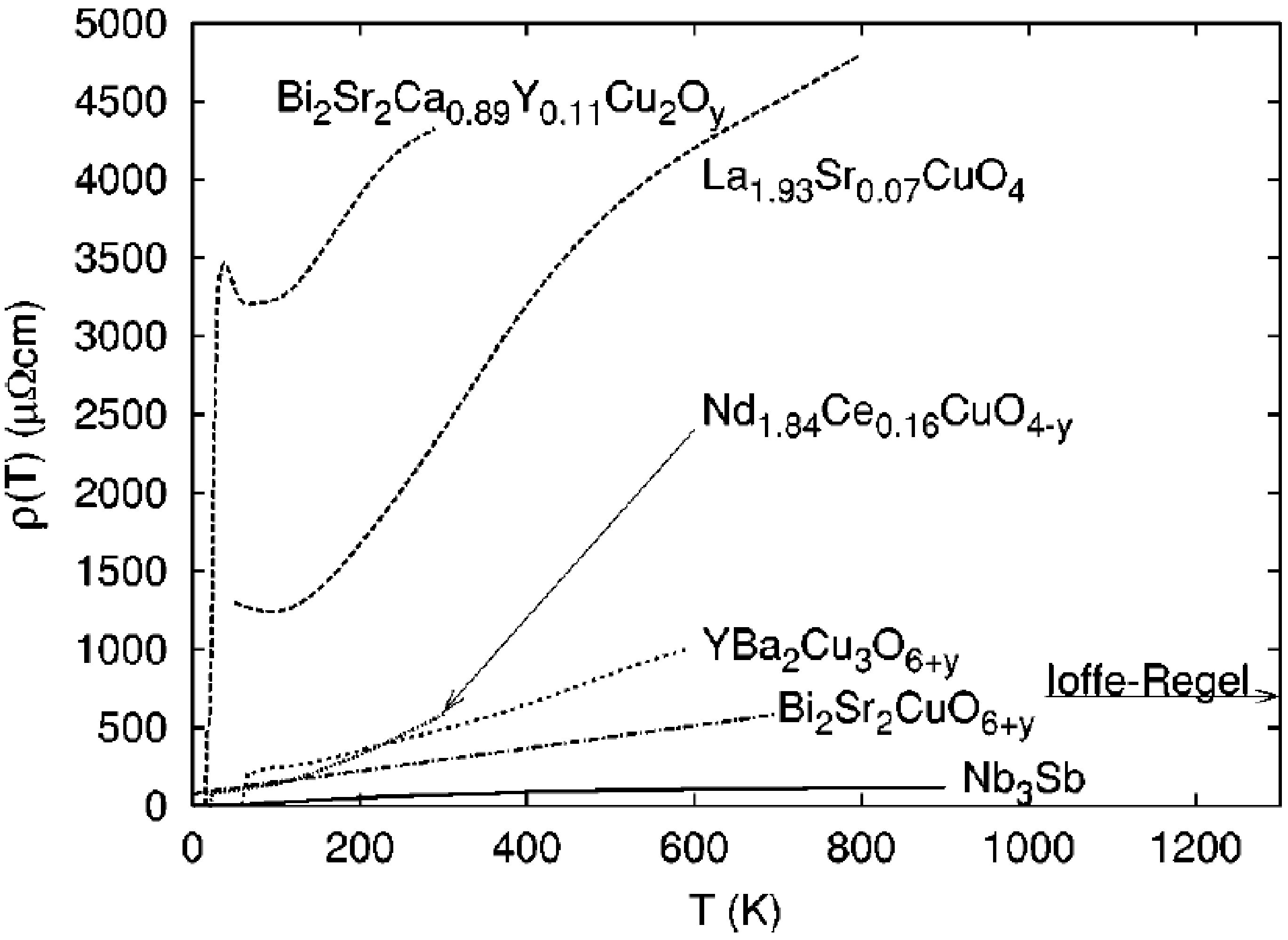


- High temperature nonsaturating linear resistivity is endemic to strongly correlated systems. It is observed not only in cuprates (strange metal behaviour?) and other HTSCs but in many other strongly correlated systems (bad metals?). (Several slides below).
- Perhaps this is different in origin from the low temperature linear resistivity, which is 'exposed' in superconducting compounds by killing superconductivity by a high magnetic field.
- I will not talk further about the latter, QCP, etc. but try to focus on the former, as perhaps a generic feature of transport in strongly correlated systems **without** external disorder. I will start by giving a few examples and mention a few theoretical attempts.
- The resistivity at high temperatures is often not quite exactly linear. (Could be due to difference between constant pressure/constant volume; escape of oxygen on heating so electron/hole density changes; melting; structural changes……. See the data of Zettl for $Rb_3 C_{60}$)

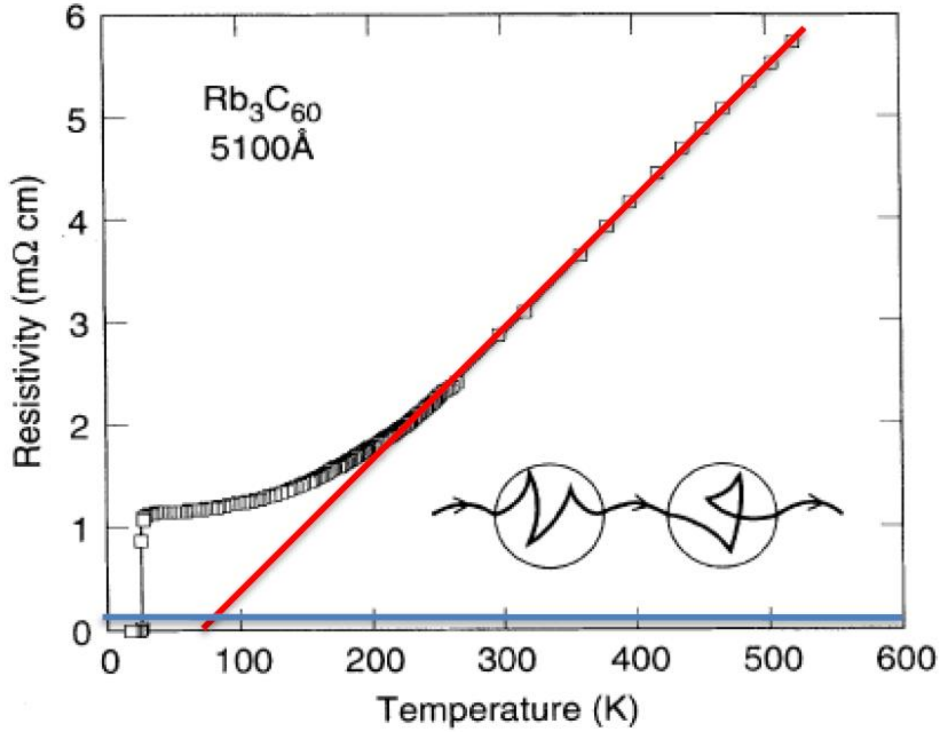


ρ(T) vs.T under constant pressure (measured) and constant volume (generally calculated) for Rb_3C_{60} . (Zettl, 1994, Physica C) . The constant volume graph is really linear!

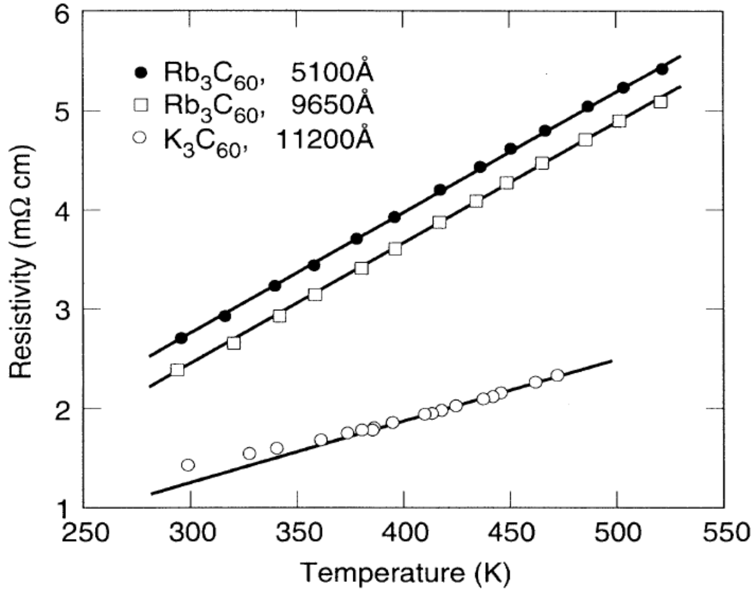

As
Fe
Ba
BaFe$_2$(As$_{1-x}$P$_x$)$_2$
Strange Metal
$\rho \sim T$
AF +nematic
Superconductivity
T (K)
200
100
0
0
0.2
0.4
0.6
α
2.0
1.0
0

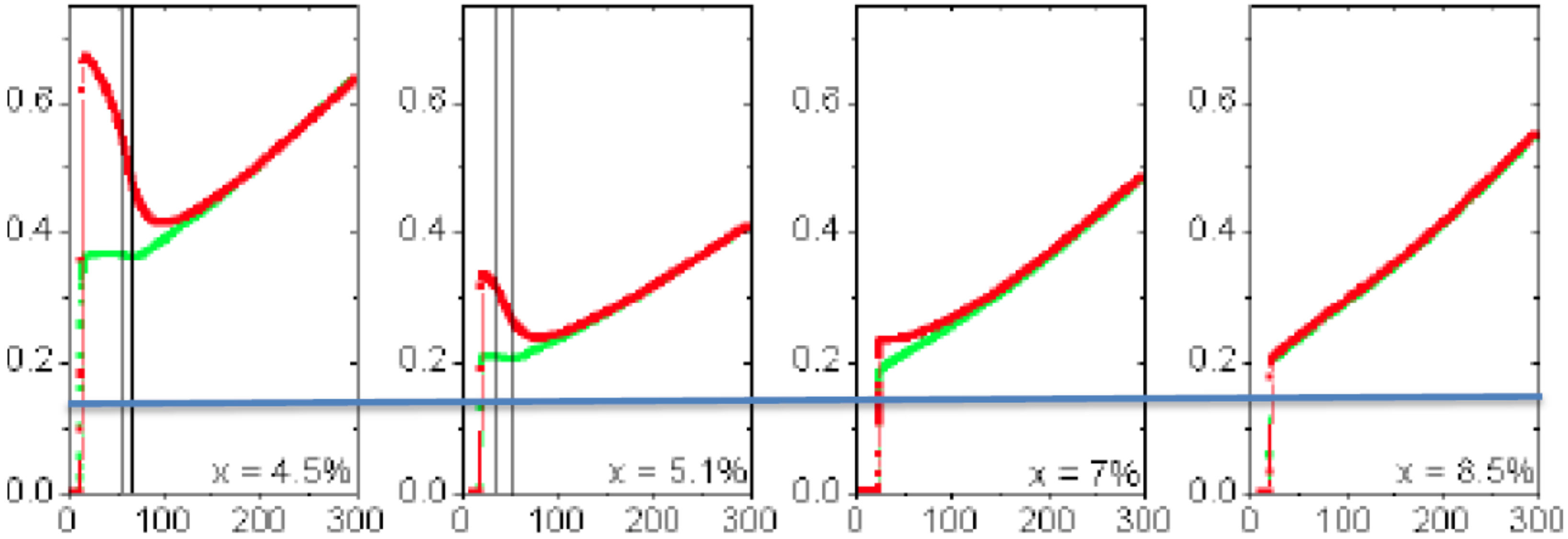


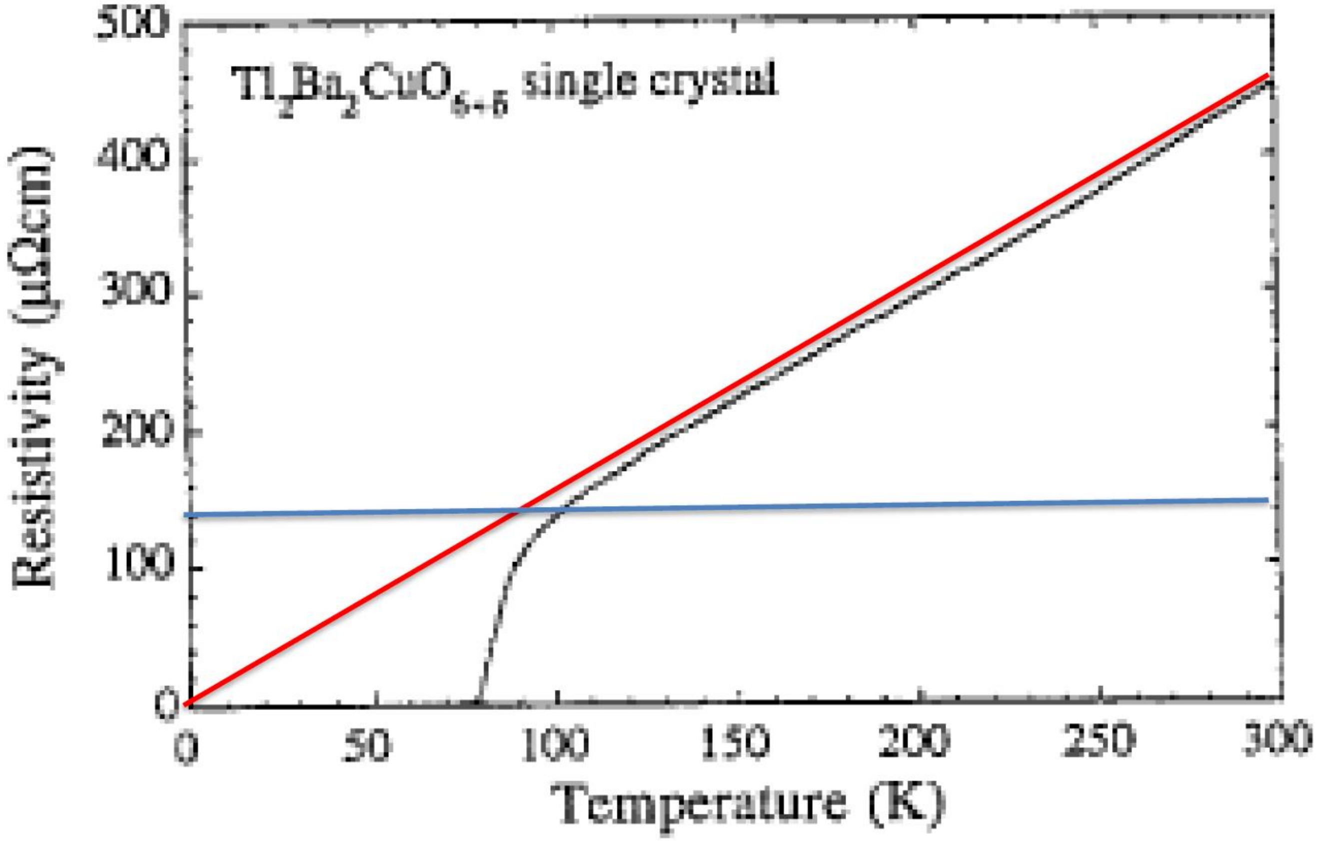
From saturation of electrical resistivity by Gunnarson, Calandra and Han, RMP 2003 Ioffe Regel limit shown for LSCO with Sr 0.07.



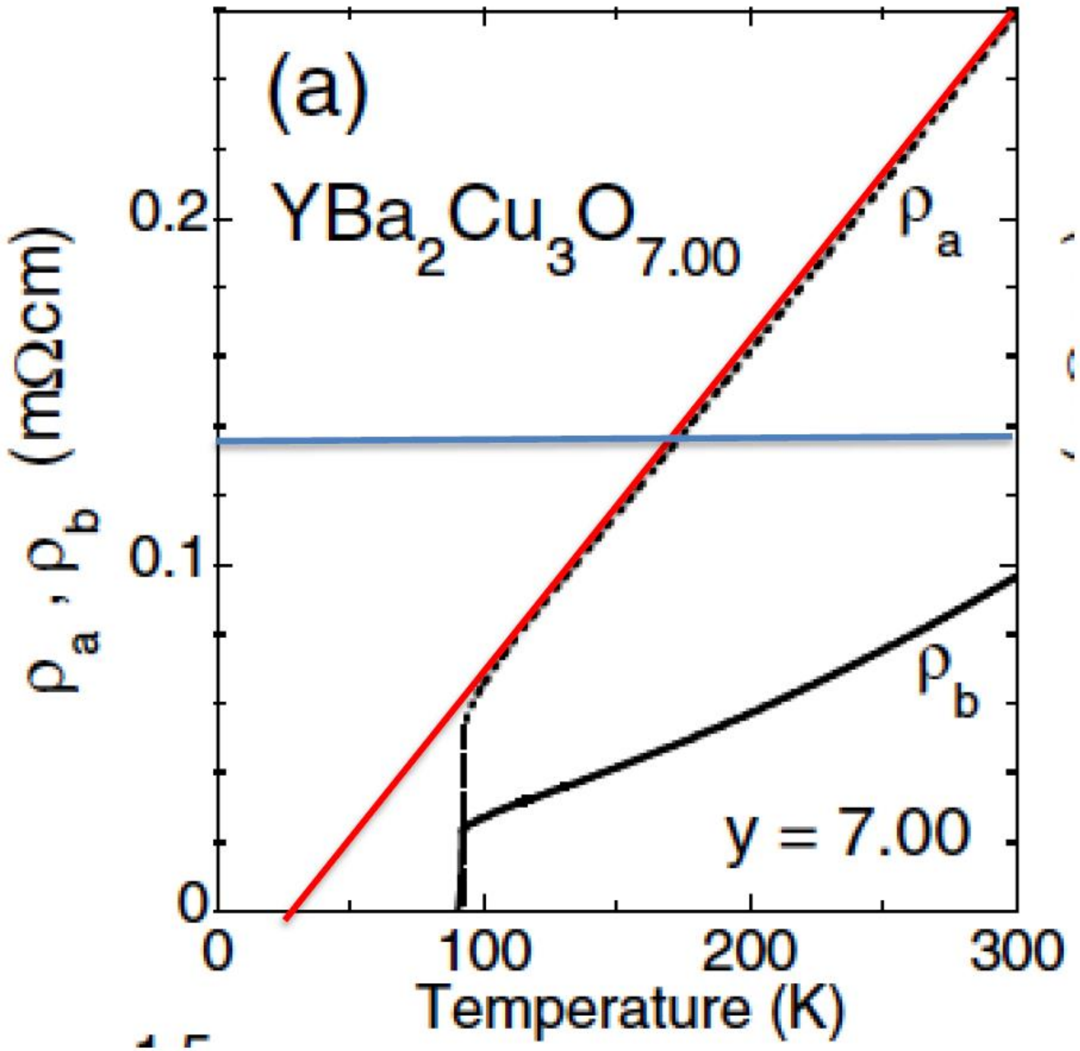
Two superconductors
Rb_3C_{60} and iron oxypnictides. Horizontal lines are the Ioffe Regel limit values. The Rb_3C_{60} data are from Hebard, Palstra et al Phys Rev B , 1995.

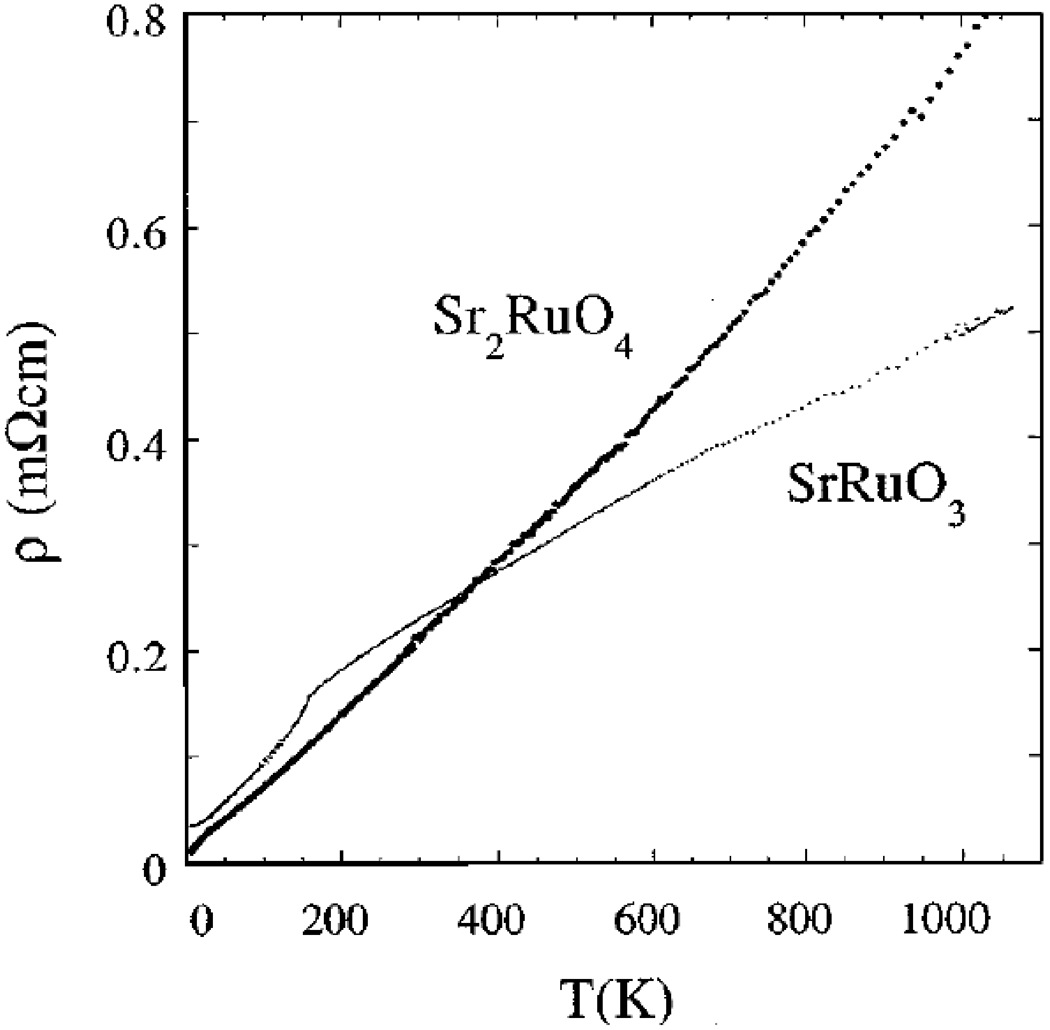






The temperature dependence of resistivity of Tl 2201 crystals for different hole concentrations can be fitted with exactly the same expression as for LSCO.

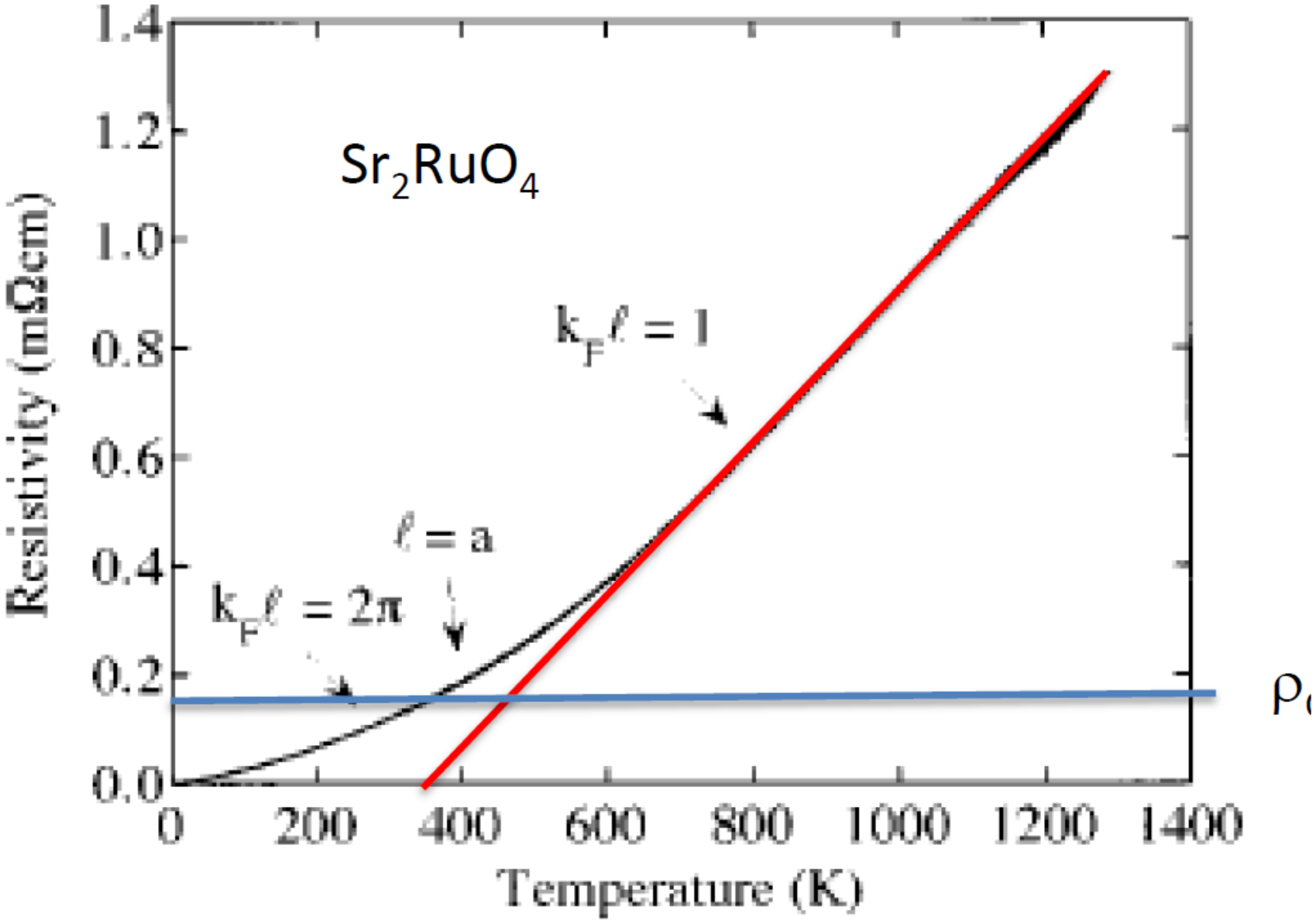




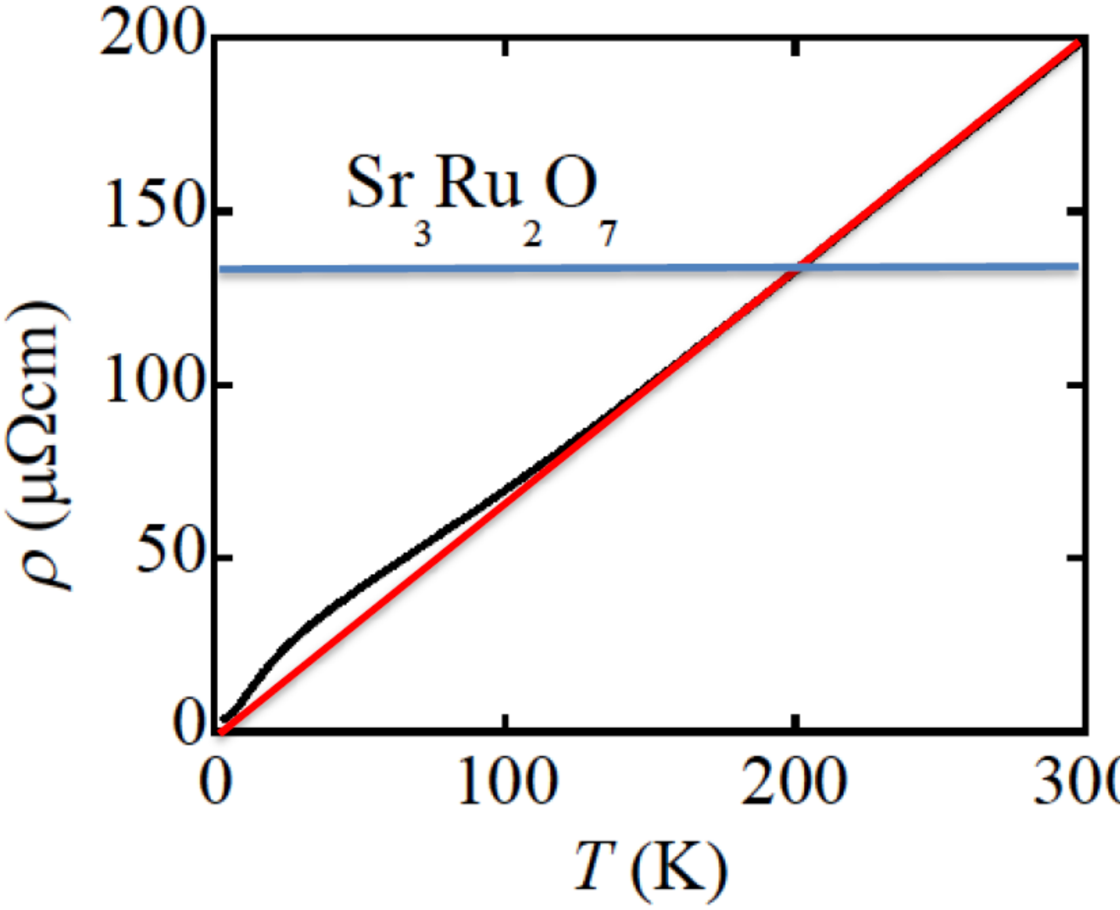
Strontium Ruthenate : A mystery compound. (See Armitage in Nature Dec 2019). Structurally similar to cuprates. T_c ~1K. First argued to be p wave superconductor. (Rice and Sigrist; Baskaran) Some experimental supporting evidence found. Accumulating evidence to the contrary . Not clear what it is. A kind of topological superconductor.?

Believed to be a Fermi liquid from properties at low T below about 30K. Fermi surface (three Fermi surfaces?) Nearly 2d. But all that maybe at low T. At highT ,seems to be a standard incoherent Fermi fluid with linear resistivity

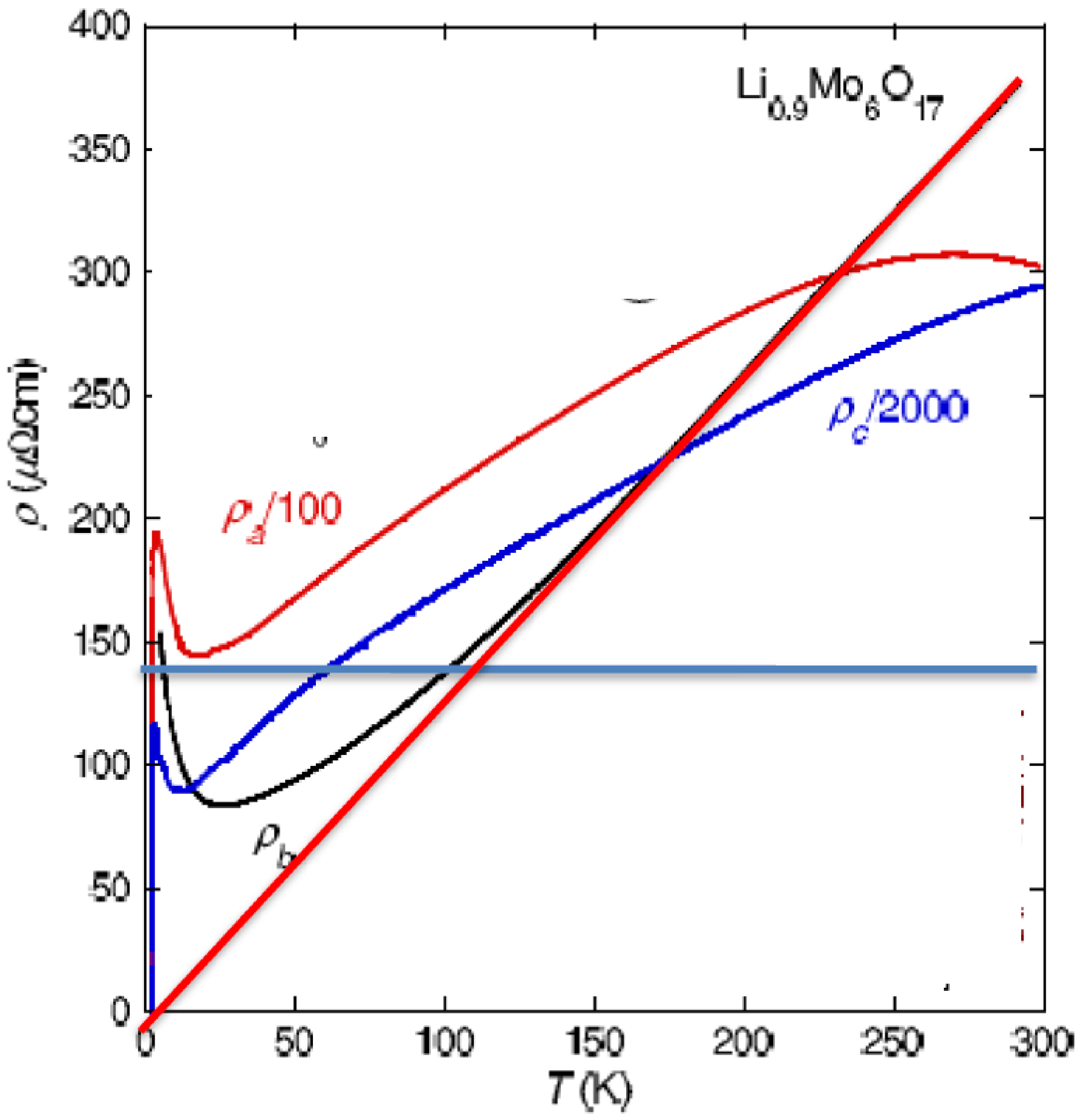
The slide shows resistivity data till about 1000K (Pavuna, Forro, Berger , EPL, 1998).



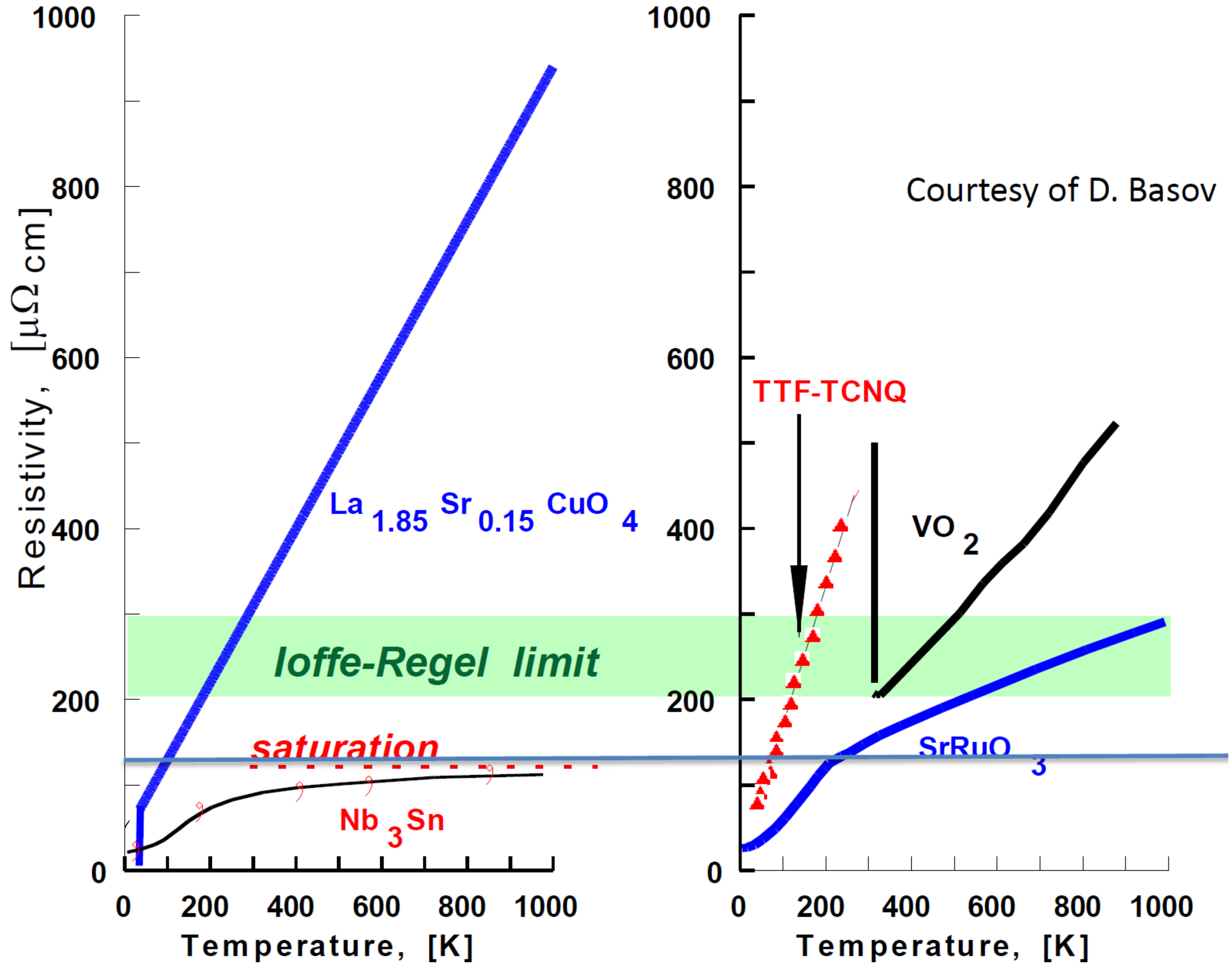
Sr2RuO4
Resistivity (mΩcm)
Temperature (K)
kFℓ = 1
ℓ = a
kFℓ = 2π
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
200
400
600
800
1000
1200
1400



Sr3Ru2O7
ρ (μΩcm)
T (K)
0
50
100
150
200
0
100
200

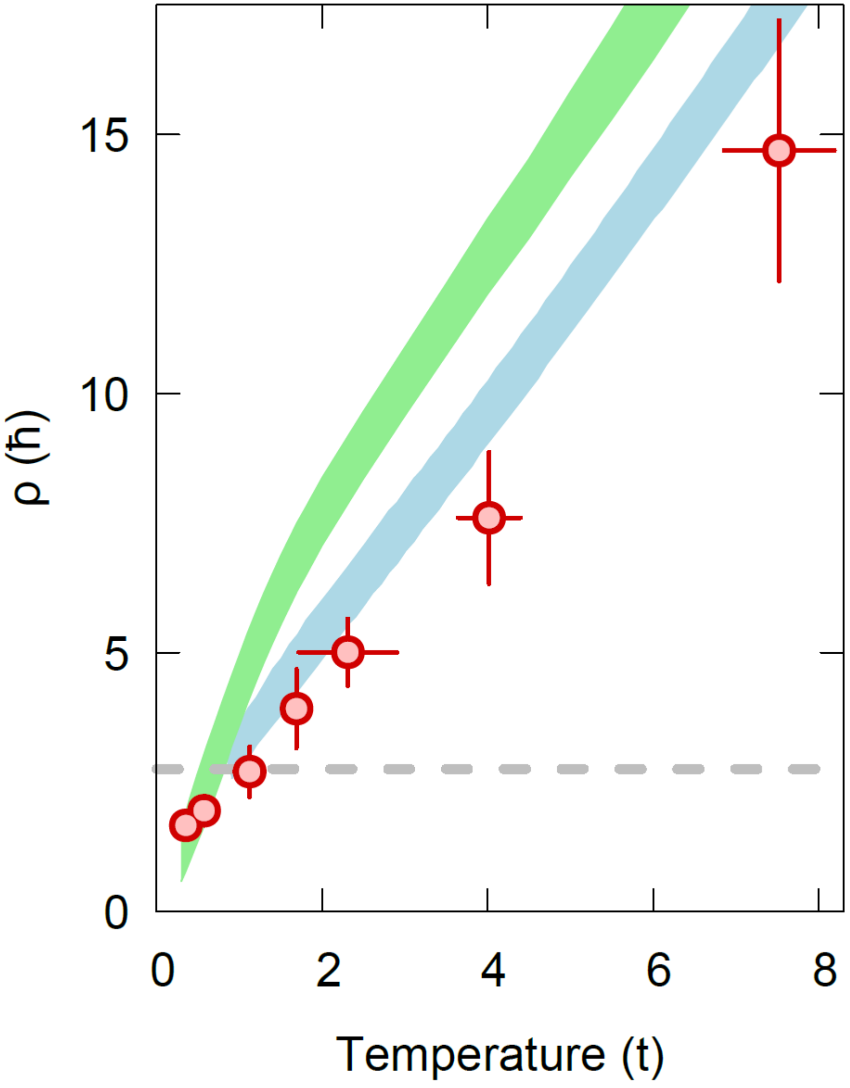


Purple bronze. Another mysterious compound. Monoclinic.Very anisotropic. Resistivity linear in T from 30 to 300K, though θ_D ~ 400K. Metal insulator transition at ~20K. Superconductor below 2K(?).



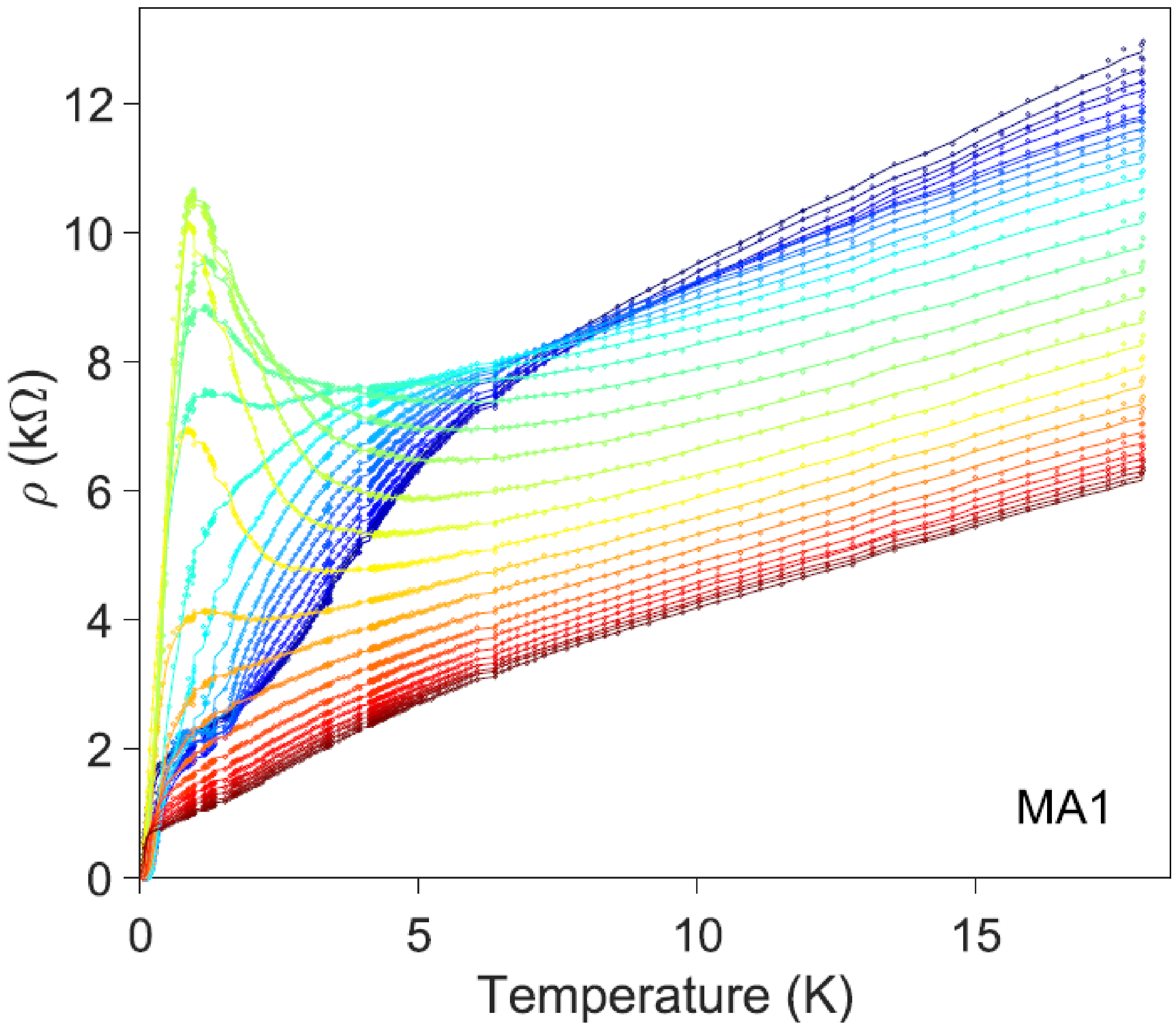
From a talk by Kivelson, ITP, 2011.

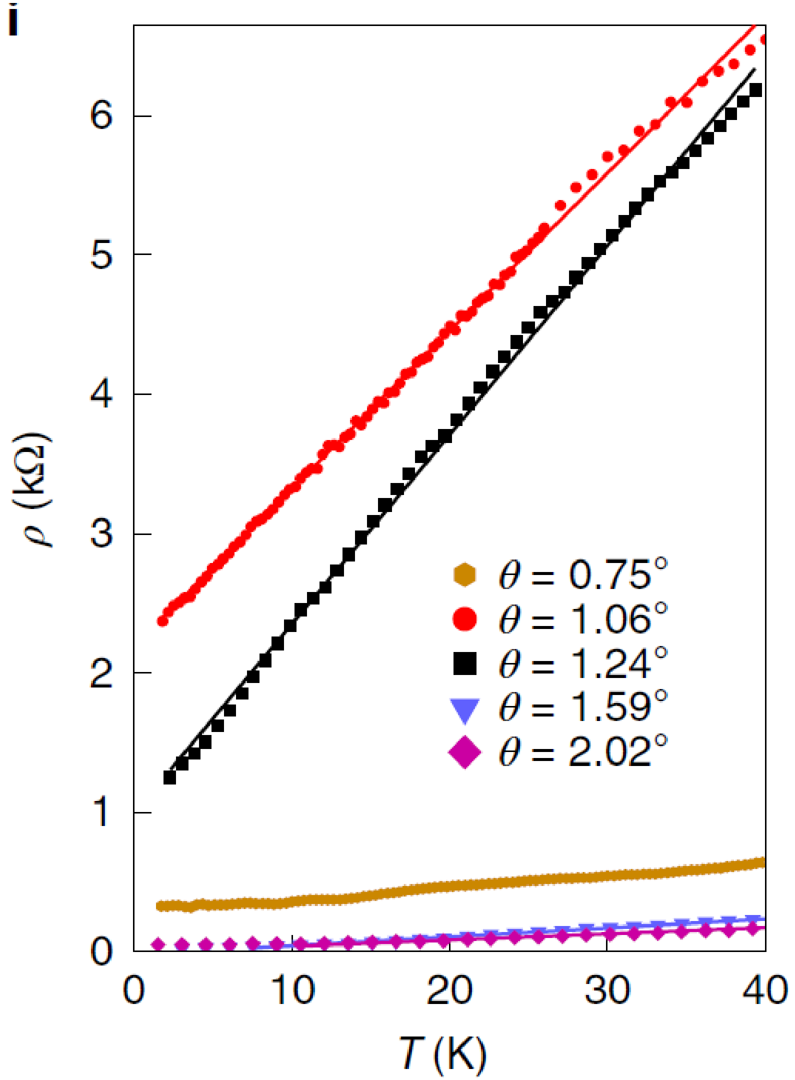
- More recently, this behaviour has been observed in Fermi systems over an extremely wide temperature range, in cold atom systems and in twisted bilayer graphene.



Waseem Bakr et.al. (Brown et.al. Science 2019) Ultra cold Fermi-Hubbard system ^6Li with (U/t) = 7.5. Atom density n= 0.82. Energies in unit of t.Bandwidth W=8t. Brinkman Rice temperature T_{BR} = (1-n)W (W=8t). Obtain diffusion coefficient. Find conductivity from Einstein relation. Measure compressibility. Resistivity is plotted in units of R_K as a function of T in units of t. We see that resistivity is linear upto very high temperatures of order W=8t. In the cuprate language, this would be ~20,000K. It far exceeds the MIR limit (dashed line).

Clean system.Blue curve :DMFT. Green curve: a variant of the Lanczos method.

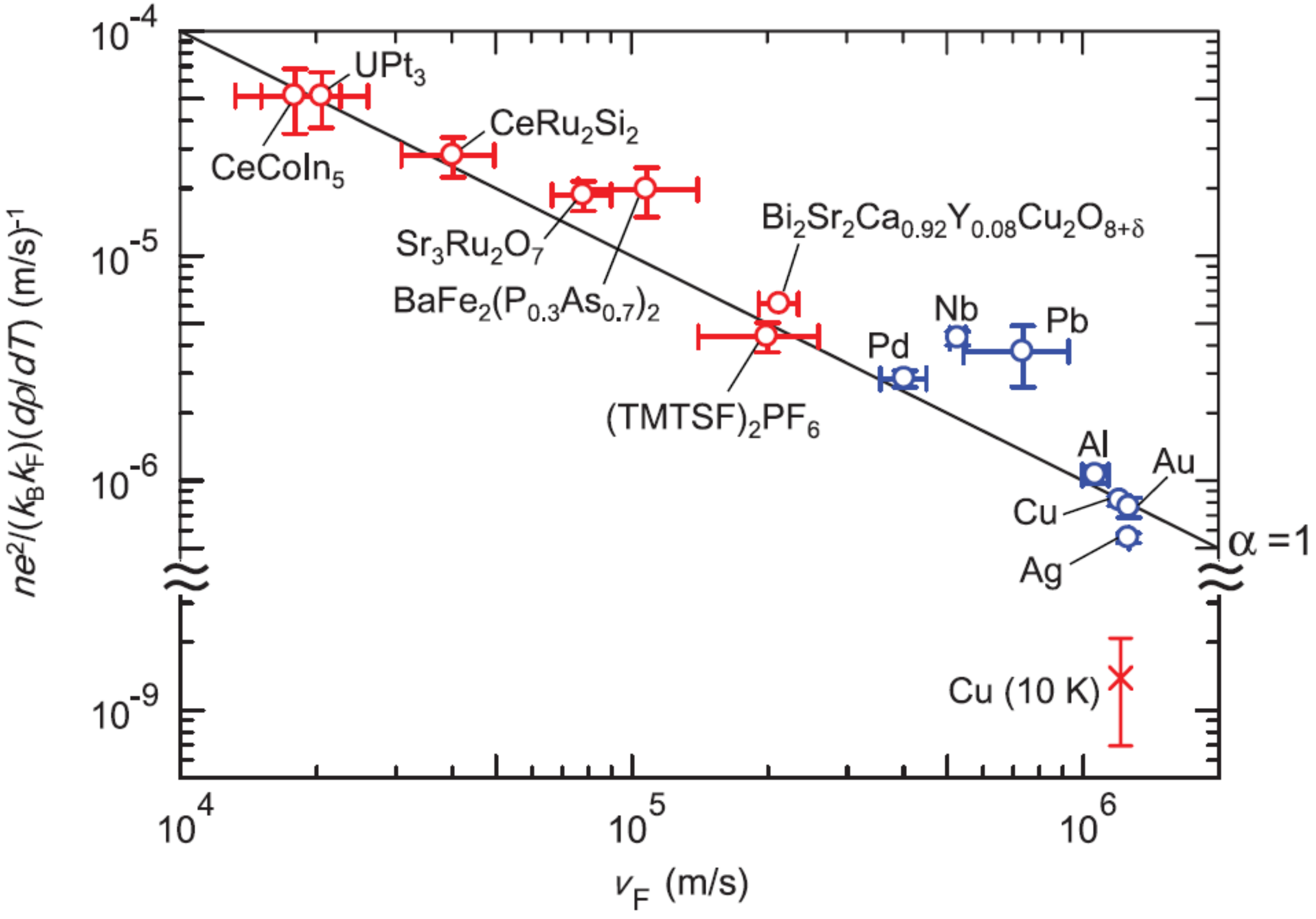




Twisted bilayer graphene: θ=1.1° and hole densities ~10^{12} cm^{-2} ; Various θ's as shown.
T_F ~ 28K. (Cai, Herrero, Senthil, .. ; Polshyn et. Al. both 2019).

- Broad ‘takeaways’:
- In a large class of metals, resistivity is not only linear with T; it is also large, linear over a very wide temperature range starting with unusually low temperatures, continuing on to the highest temperatures accessible. In model systems, much higher temperatures are accessible. So, linear (in the cuprate language again, from 7K to 20,000K; about four orders of magnitude. Often, it is not perfectly linear, but does not show prominent signs of saturation.
- It is often considerably larger than the low temperature quantum estimates of Mott, and of Ioffe and Regel, which seems to be reached at relatively ‘low’ temperatures (Cuprate : 300-500K)
- It seems to be associated with the electron system not having well defined quasiparticles, generally due to strong electron correlation.
- If it is a generic, inevitable, feature , one feels that there should be a single explanation of universal applicability.
- Though QCP is an attractive possibility, the ubiquity and temperature range seem to be against it.

- Phenomenologically, there is at least one well known attempt to drive home the universality:



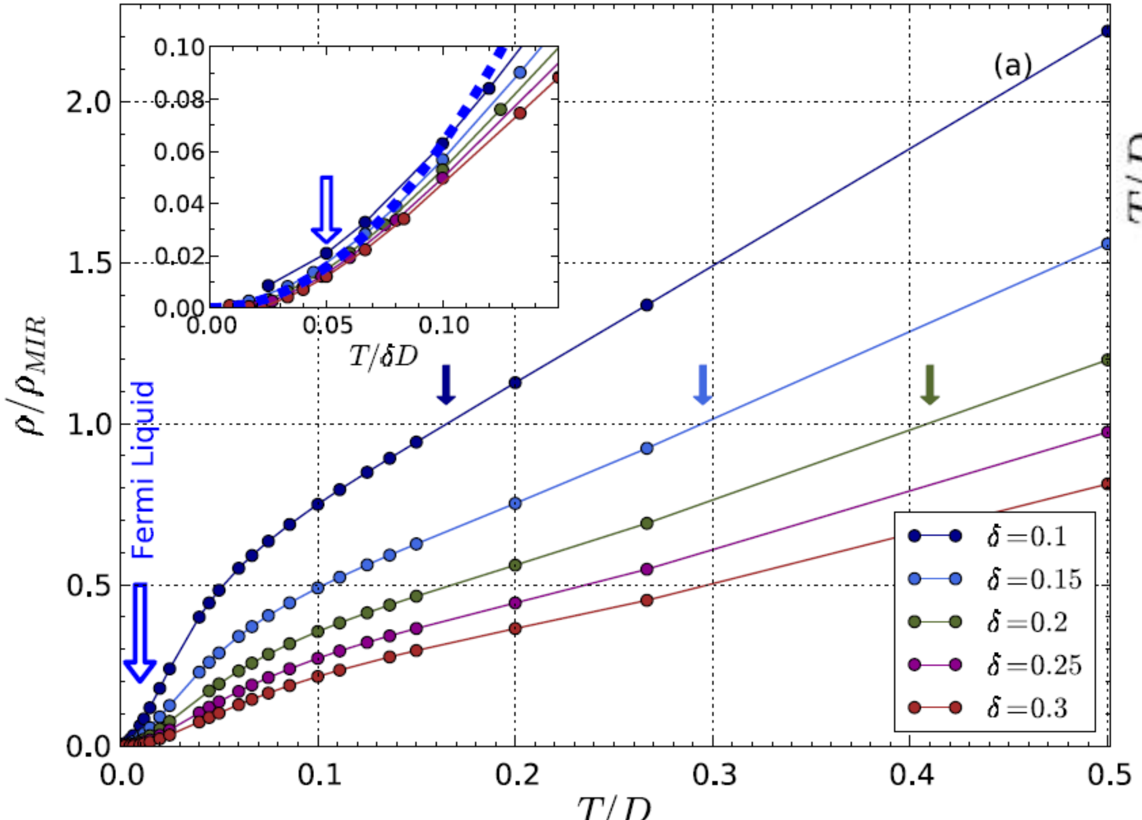
Bruin, Sakai, Perry and Mackenzie, Science, 2013.

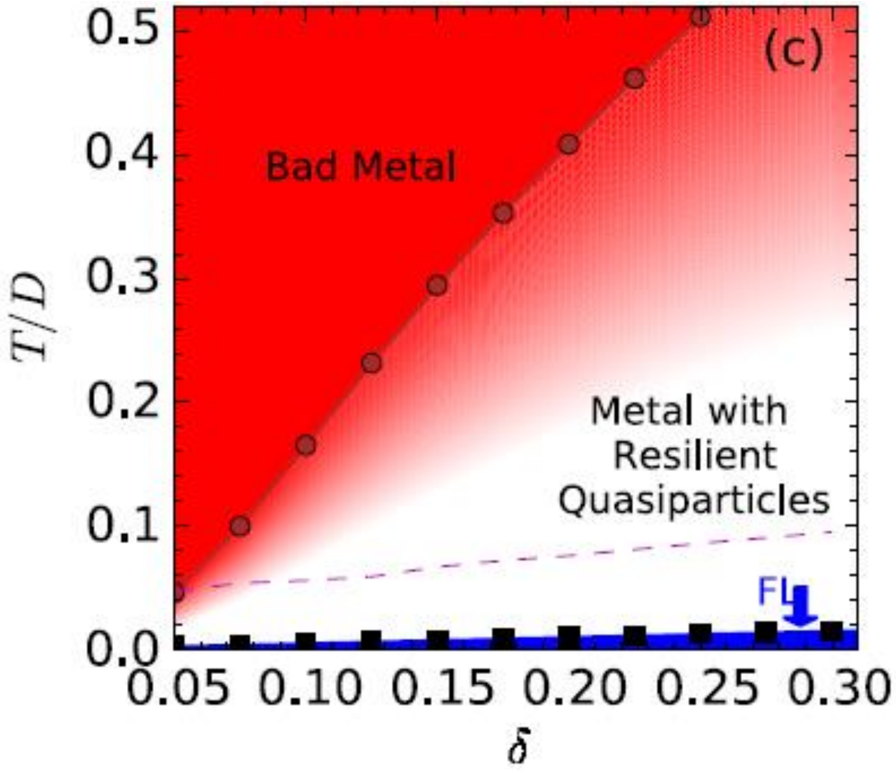
The scattering rate ($1/\tau$) is ($k_B T/\hbar$)(and universal. 'Planckian dissipation limit'). To get from here to resistivity requires a connection (Drude formula is the most popular; $\rho=(m^* /ne^2)(1/\tau)$; or the Boltzmann semiclassical transport equation). This has been obtained using the picture of good quasiparticles colliding randomly with other things. If this is ok, since n is unambiguously known, only need m^*. Via v_F ?

The only calculation is a weak coupling one; perturbation theory in electron phonon coupling (Justified by Migdal's theorem; ok so long as ground state does not change because of strong e-ph coupling, e.g. Jahn-Teller effect) . Here, $(1/\tau) \sim \lambda_{ep}^2 \; (k_B T/\hbar)$. Not universal, nor Planckian dissipation. Fermi golden rule; origin of T is classical equipartition among harmonic fluctuations ; T is above their quantum scale. No quasiparticles in this regime because excitation energy << decay linewidth

- There is a very large number of theoretical attempts: I do not know , and do not understand many of them. Just a few will be mentioned.
- DMFT based calculations
- Limits on charge diffusion (Planckian dissipation limit)
- Some others

DMFT based calculations



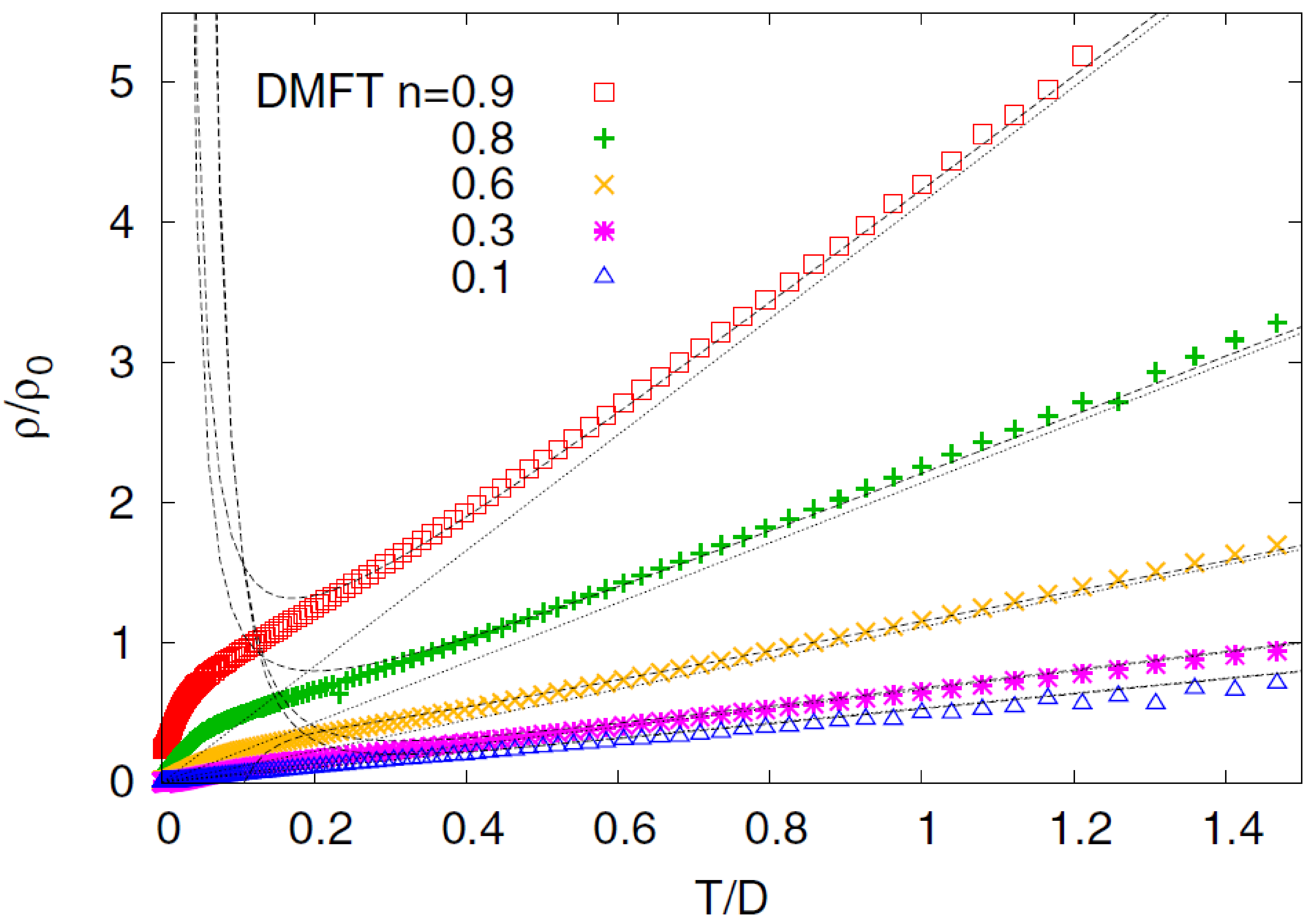


Deng, …. Georges,Kotliar PRL 2013: Single site DMFT; NRG and continuous time QMC (allows one to go to very low T T~0.0025D; e.g. if D~0.04eV, T~10K. d wave sc in cluster DMFT). D=2t/√z =t for a square lattice. Hole doping p=δ.

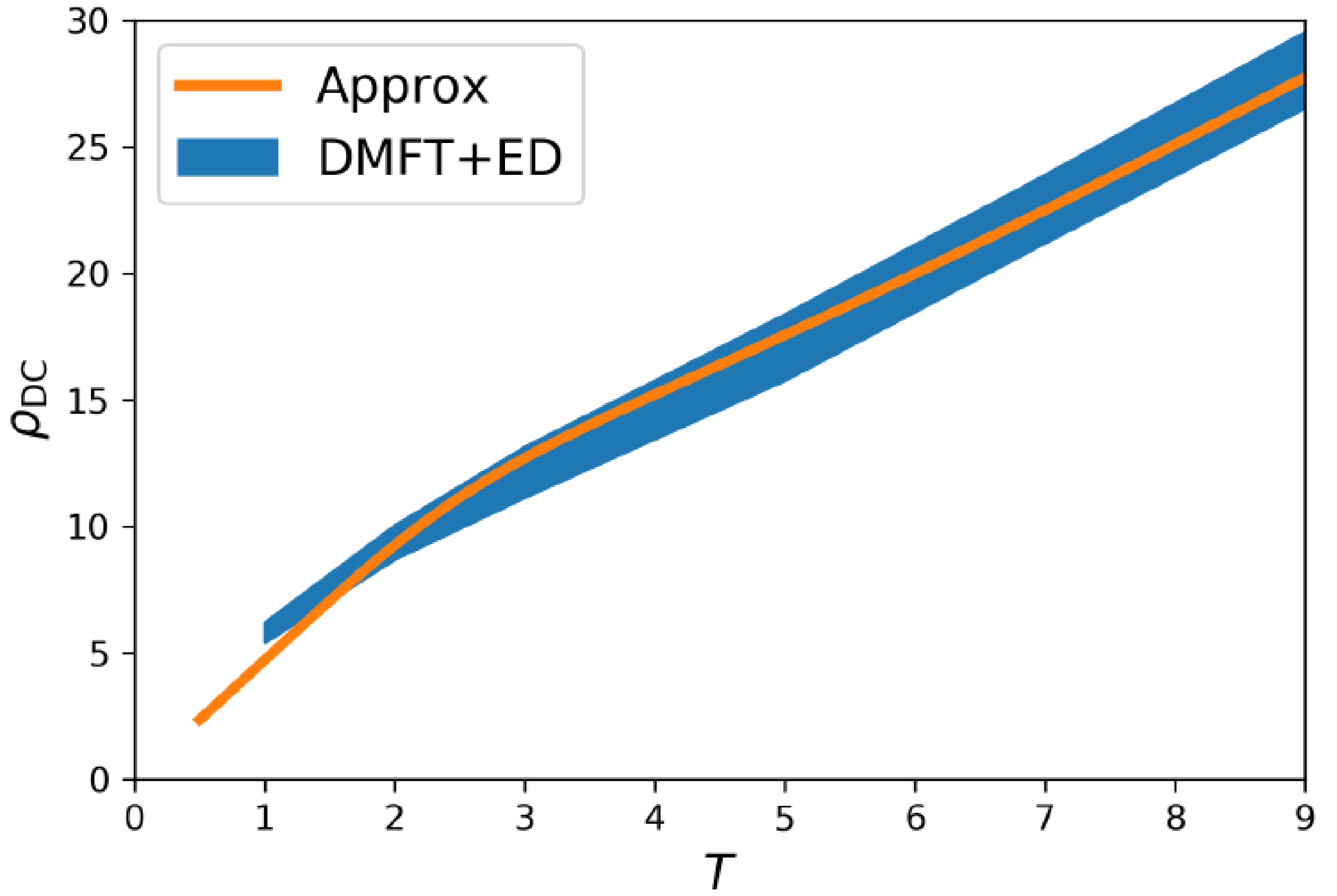
- A large linear T regime; starts at ~0.06 to 0.075 t . And continues to intermediate temperatures .
- There is a clear FL regime, for T< 0.05p t <<p t (which is expected from say the Brinkman-Rice picture for large d)
- There is a large crossover regime .
- There is a large region in T where $\rho > \rho_{MIR}$.
- There is no tendency of resistivity saturation.

There are now at least two calculations of ρ(T) which span a wide temperature range (partly inspired by cold atom and twisted graphene observations)

1. High temperature expansion along with NRG and DMFT (Shastry , Georges…Phys Rev 2016 .)
2. Analytical approximation for DMFT spectral density and exact diagonalization (ED) with DMFT (Cha,Patel,Gull, Kim; arXiv, Oct.2019).



DMFT n=0.9
0.8
0.6
0.3
0.1
ρ/ρ0
T/D
0
1
2
3
4
5
0
0.2
0.4
0.6
0.8
1
1.2
1.4



Patel , Cha, Gull,Kim Arxiv
Oct.16,2019.

- **Limits on charge diffusion (Planckian dissipation limit)**

Thermal equilibration times $\tau_\phi \sim C\,(k_B T/\hbar)$ (many body systems without quasiparticles)

Lyapunaov (divergence) times $\tau_L \sim C'\,(k_B T/\hbar)$

This is the Planckian dissipation limit.

Very exciting because of strong connections to other fundamental questions in Physics

From here , $D \sim v_F^2\, \tau_\phi$; conductivity via the Einstein relation $\sigma = e^2\,(dn/d\mu)\, D$

Many empirical correlations suggesting that linear resistivity corresponds to the PDL.

• **'Some' others**

I mention only two out of the many many: my ignorance is nearly total. I will mainly talk in pictures.

1. Sachdev-Ye-Kitaev (SYK) models.

Start with a SYK quantum dot. Many fermions, each interacts with each other. Strength U.

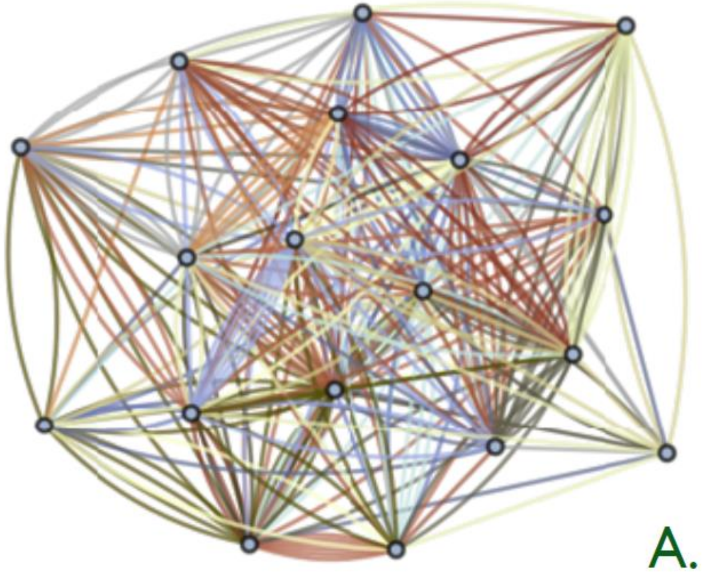
$<U_{ij;kl}>=0$. $<U_{ij;kl}{}^2>= U^2$.

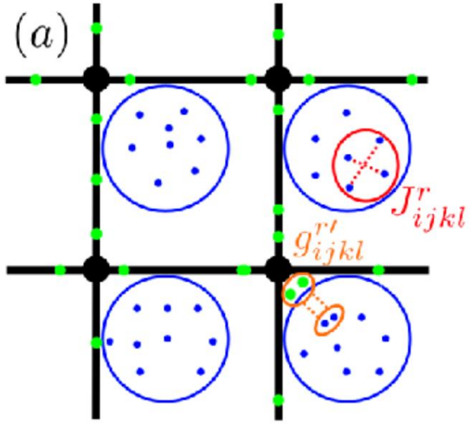
N—inf. Critical strange metal

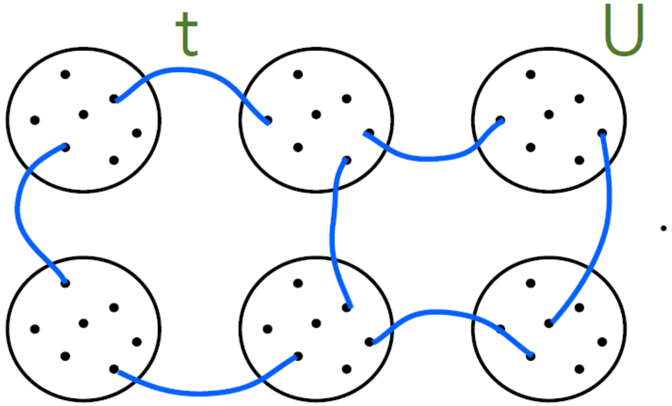
Lattice of such quantum dots (Balents et al 2017)

Incoherent metal with small coherence scale : $(t_o{}^2/U)$.

Linear resistivity metal

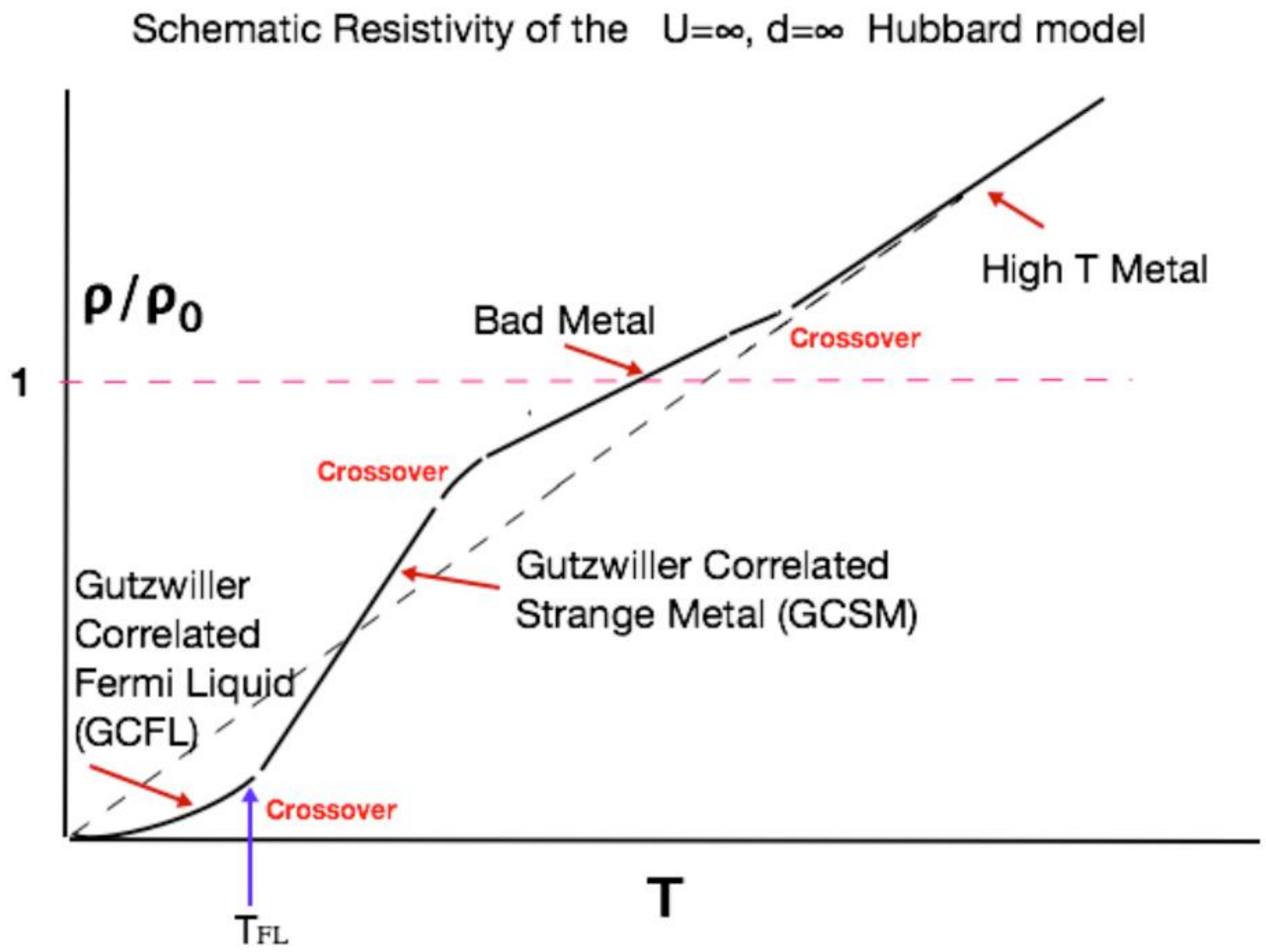






2. ECFL (extremely correlated Fermi liquid) approach (Shastry et al, 2018)

Approximation for the single particle GF, using functional derivatives. Enough to calculate resitivity in d=inf.



- TVR : interested. Only vague thoughts at present.
- The eletron self interaction can be converted exactly to interaction with a time dependent potential. Ignore time dependence; high T limit; no quasiparticles; Hubbard bands, semi circular DOS in large d, etc.. (TVR and DSa, unpublished).
- But there is a time dependence. Leads to FL?
- Mainly interested in why the crossover from FL (?) occurs at such low T.
- Is it an infrared catastrophe effect?
- Quantum dynamics through hopping. So the local charge and the related potential must change quite a bit. There is self consistent average due to 'rapid' fluctuations. But these fluctuations cause low energy particle hole excitations leading to infrared 'catastrophe'. Reduces t to t*. But as electrons hop around, even in a correlated system, the infrared catastrophe must disappear and FL appear.
- In 1970, MH.TVR and GT studied a simple model of a recoiling impurity in an electron gas, and found that the recoil mutes the catastrophe; the crossover scale is $\varepsilon_F(m/M_{imp})$. Is it relevant? Is it somehow the cause of unusually low quantum scale of charge fluctuations? (Like in a single magnetic impurity, the Kondo scale of magnetic fluctuations is exponentially small).
- It seems to be a major unsolved problem in quantum physics.

Thank you