Parton paradigm for the quantum Hall effect

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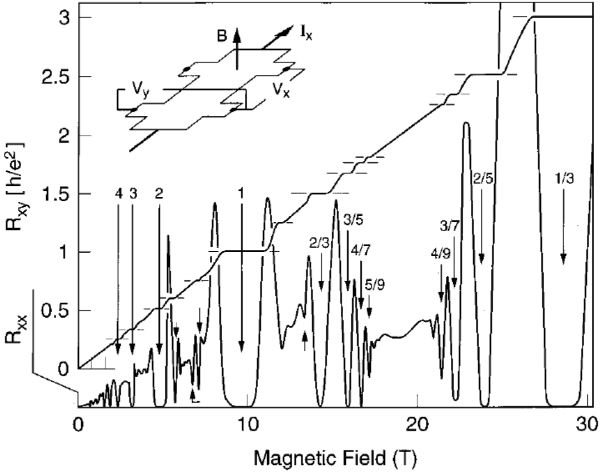
Plan of the talk

- Lowest Landau level FQHE: composite fermion theory
- Second Landau level FQHE: parton framework
- Conclusion and outlook

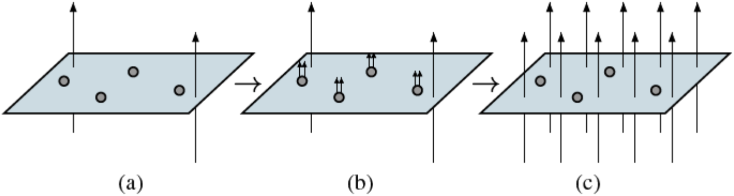
FQHE in the LLL predominantly occurs at $\nu = n/(2pn \pm 1)$



J. P. Eisenstein and H. L. Stormer, Science **248**, 4962, 1510-1516 (1990)

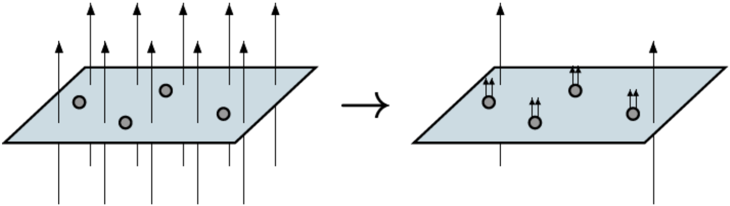
FQHE as IQHE of composite fermions

A composite fermion (CF) is a bound state of an electron and even number of vortices/flux quanta.



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

Composite fermions experience a reduced magnetic field

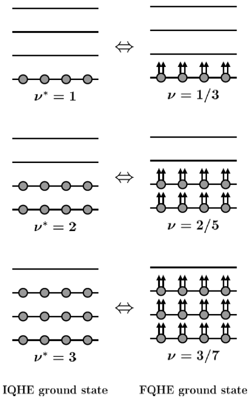


$$B^* = B - 2p\rho\phi_0, \quad \phi_0 - h/e$$

$$\nu = \frac{\rho\phi_0}{B}, \quad \nu^* = \frac{\rho\phi_0}{|B^*|}, \quad \nu = \frac{\nu^*}{2p\nu^* \pm 1}$$

J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE ground states are analogous to IQHE ones



J. K. Jain, Composite Fermions, Cambridge University Press (2007)

FQHE wave functions are analogous to IQHE ones

- Jain wave functions at $\nu = n/(2pn \pm 1)$:

$$\Psi^{\rm CF}_{\nu=\frac{n}{2pn\pm1}} = \mathcal{P}_{\rm LLL}\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p}\right).$$

(dropped Gaussian factor for ease of notation)

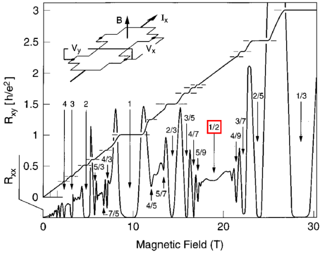
Φ_n wave function of n filled LLs.

$\mathcal{P}_{\rm LLL}$ implements lowest Landau level projection.

- no adjustable parameters in these wave functions
- wave functions can be evaluated for large system sizes

J. K. Jain, Phys. Rev. Lett. **63**, 199 (1989)

Mystery of the $\nu = 1/2$ state

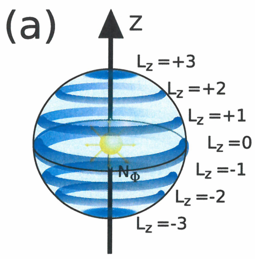


- composite fermions absorb all of the magnetic flux: $B^* = 0$

  Halperin, Lee and Read, Phys. Rev. B **47**, 7312 (1993)

- In zero effective magnetic field CFs form a Fermi sea

Spherical geometry



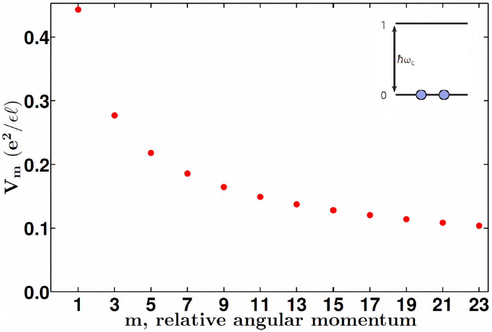
$l = |Q|, |Q|+1, |Q|+2, \cdots \quad l_n = |Q| + n \quad m = -l, -l+1, \cdots, l-1, l$

L and its z-component L_z are good quantum numbers

$$N_\phi = 2Q = \nu^{-1} N - \mathcal{S}, \qquad \mathcal{S} \rightarrow \text{shift, characterizes the state}$$

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Haldane pseudopotentials parametrize the interaction



V_m: energy of two electrons in a state of relative angular momentum m

fully spin-polarized electrons → only odd pseudopotentials relevant

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Overlaps of CF states with LLL Coulomb ground states

overlaps obtained from direct projected states

ν	N	Hilbert space dimension	$\lvert\langle\Psi^{0\mathrm{LL}}\vert\Psi^{\mathrm{CF}}\rangle\rvert$
1/3	15	2×10^9	0.9876 (Laughlin)
1/5	10	4×10^7	0.9228 (Laughlin)
2/5	12	3×10^5	0.9971
3/7	12	6×10^4	0.9988
2/9	8	1×10^7	0.9744

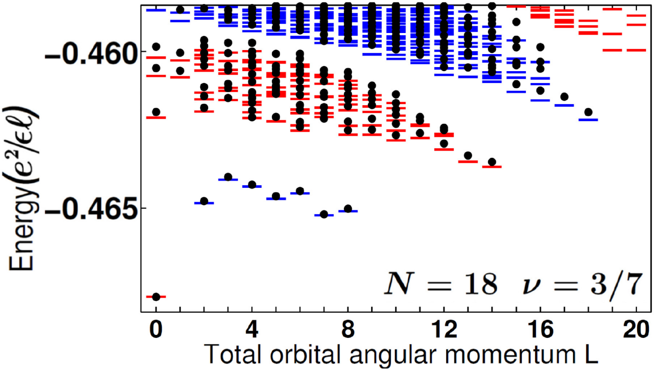
$\vert\Psi^{0\mathrm{LL}}\rangle$ is obtained by brute-force exact diagonalization

B. Kusmierz and A. Wójs, Phys. Rev. B **97**, 245125 (2018)

Ajit C. Balram (unpublished)

CF theory is extremely accurate in the lowest Landau level

energies obtained using Jain-Kamilla projected states



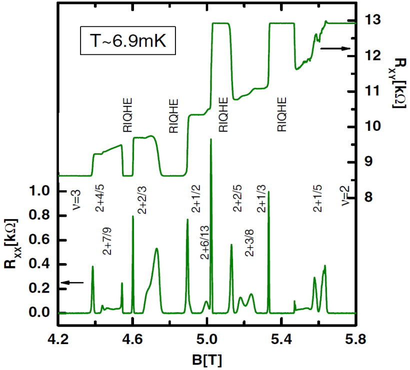
dashes are obtained by brute-force exact diagonalization

$\sim 10^6$ states at each total orbital angular momentum L

Ajit C. Balram, A. Wójs and J. K. Jain, Phys. Rev. B **88**, 205312 (2013)

Onward to the second Landau level

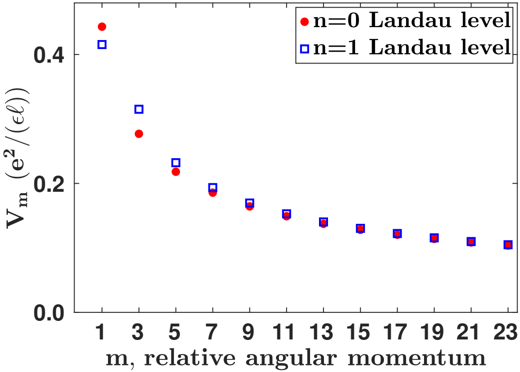
FQH states in the second Landau level



- appearance of *even* denominator fractions
- 6/13 appears "out of order"

Kumar *et al.* Phys. Rev. Lett. **105**, 246808 (2010)

Landau levels differ in their Haldane pseudopotentials



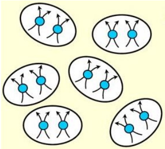
V_m energy of two electrons in a state of relative angular momentum m

stronger repulsion at shortest approach in the LLL

F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983)

Candidate states for $\nu = 5/2$: Pfaffian

$$\Psi^{\rm MR}_{\nu=1/2} = {\rm Pf}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)^2$$



p-wave paired state of composite fermions

G. Moore and N. Read, Nucl. Phys. B **360**, 362 (1991)

N	Hilbert space dimension	$\lvert\langle\Psi^{\rm 1LL}\lvert\Psi^{\rm MR}_{\nu=1/2}\rangle\rvert$
20	4×10^8	0.6736

B. Kusmierz and A. Wójs, Phys. Rev. B **97**, 245125 (2018)

Candidate states for $\nu = 5/2$: anti-Pfaffian

- anti-Pfaffian is the particle-hole conjugate of Pfaffian

$$\Psi^{\mathrm{aPf}}_{\nu=1/2} = \mathcal{P}_{ph}\left(\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)^2\right)$$

Levin *et al.*, Phys. Rev. Lett. **99**, 236806 (2007), Lee *et al.*, Phys. Rev. Lett. **99**, 236807 (2007)

- construction extremely difficult to implement numerically
- recent numerics suggest anti-Pfaffian is favored in the presence of LL mixing

E. H. Rezayi, Phys. Rev. Lett. **119**, 026801 (2017)

Candidate states for $\nu = 5/2$: PH-Pfaffian

$$\Psi^{\mathrm{PH-Pf,TJ}}_{\nu=1/2} = \mathcal{P}_{\mathrm{LLL}}\left(\left\{\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\prod_{i<j}(z_i - z_j)\right\}^{*}\prod_{i<j}(z_i - z_j)^3\right)$$

Th. Jolicoeur, Phys. Rev. Lett. **99**, 036805 (2007)

$$\Psi^{\mathrm{PH-Pf,ZF}}_{\nu=1/2} = \mathcal{P}_{\mathrm{LLL}}\left(\left\{\mathrm{Pf}\left[\frac{1}{z_i - z_j}\right]\right\}^{*}\prod_{i<j}(z_i - z_j)^2\right)$$

P.T. Zucker and D.E. Feldman, Phys. Rev. Lett. **117**, 096802 (2016)

- state is particle-hole symmetric to a good extent

  Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

  R. V. Mishmash *et. al.*, Phys. Rev. B **98**, 081107(R) (2018)

- consistent with recent thermal Hall measurements

  M. Banerjee *et. al.*, Nature **559**, 205-210 (2018)

Candidate states in the second Landau level

- 1/3 and 2/3: dressed Laughlin or $k = 4$ Read-Rezayi (k-cluster states)

  N. Read and E. H. Rezayi, Phys. Rev. B **59**, 8084 (1999)

- 2/5: particle-hole conjugate of $k = 3$ anti-Read-Rezayi or Bonderson-Slingerland state (Pfaffian times composite boson)

  Parsa Bonderson and J. K. Slingerland, Phys. Rev. B **78**, 125323 (2008)

- 3/8: Bonderson-Slingerland state
- 6/13: Levin-Halperin state

  Levin and Halperin, Phys. Rev. B **79**, 205301 (2009)

Can we find a unified description of the second LL FQHE?

Yes.

In terms of "parton" states.

Parton states: product of integer quantum Hall states

- break each electron into fictitious partons, place partons into IQH states, fuse the partons back to recover the electron

$$\Psi_{\nu}^{\{n_\alpha\}} = \mathcal{P}_{\rm LLL} \prod_{\alpha=1}^{k} \Phi_{n_\alpha}(\{z_i\})$$

- k is odd for fermions

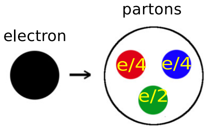
$$\nu^{-1} = \sum_{\alpha=1}^{k} n_\alpha^{-1}, \quad q_\alpha = (-e)\frac{\nu}{n_\alpha}$$

- Laughlin state is a "$111\cdots$" parton state
- Composite fermions states are "$n11\cdots$" parton states

J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

An example of a non-composite fermion state

$$\Psi_{1/2}^{221} = \mathcal{P}_{\rm LLL}\Phi_2\Phi_2\Phi_1$$



- non-Abelian state like Pfaffian and anti-Pfaffian

  X.-G. Wen, Phys. Rev. Lett. **66**, 802 (1991)

- not a good variational state for $\nu = 5/2$
- recent proposals to realize this state in graphene

  Y.-H. Wu, T. Shi and J. K. Jain, Nano Lett. 17 (8), 4643 (2017)

  Y. Kim *et al.*, Nature Physics **15**, 154-158 (2019)

J. K. Jain, Phys. Rev. B **40**, 8079 (1989)

Central result of our work

Two parton sequences (and their particle-hole conjugates) capture almost all the observed FQH states in the second LL

$$\Psi^{\bar{n}\bar{2}111}_{\nu=2n/(5n-2)} = \mathcal{P}_{\rm LLL}[\Phi_n^*][\Phi_2^*]\Phi_1^3$$
$$\Psi^{\bar{n}\bar{2}\bar{2}1111}_{\nu=n/(3n-1)} = \mathcal{P}_{\rm LLL}[\Phi_n^*][\Phi_2^*][\Phi_2^*]\Phi_1^4$$

- These parton states can be evaluated for very large systems
- New candidates for 6/13 and 3/8

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)
Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{n}\bar{2}111$" ansatz

$$\Psi^{\bar{n}\bar{2}111}_{\nu=2n/(5n-2)} = \mathcal{P}_{\rm LLL}[\Phi_n^*][\Phi_2^*]\Phi_1^3 \sim \frac{\Psi^{\rm CF}_{n/(2n-1)}\Psi^{\rm CF}_{2/3}}{\Phi_1}$$

- $n = 1 \implies \nu = 2/3$: standard composite fermion state
- $n = 2 \implies \nu = 1/2$: parton state in the anti-Pfaffian phase
- $n = 3 \implies \nu = 6/13$: a new candidate state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{2}\bar{2}111$" ansatz ∼ anti-Pfaffian

$$\Psi^{\bar{2}\bar{2}111}_{\nu=1/2} = \mathcal{P}_{\rm LLL}[\Phi_2^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm CF}_{2/3}]^2}{\Phi_1}$$

- state occurs at a shift $\mathcal{S} = -1$: same as the anti-Pfaffian shift
- slightly better than anti-Pfaffian for second LL Coulomb

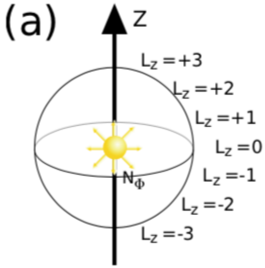
N	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert$	$\lvert\Psi^{\bar{2}\bar{2}111}_{1/2}\lvert\Psi^{\rm aPf}_{1/2}\rangle\rvert$	$\lvert\langle\Psi^{\rm 1LL}_{1/2}\lvert\Psi^{\bar{2}\bar{2}111}_{1/2}\rangle\rvert$
10	0.8194	0.9397	0.8975

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Entanglement spectrum

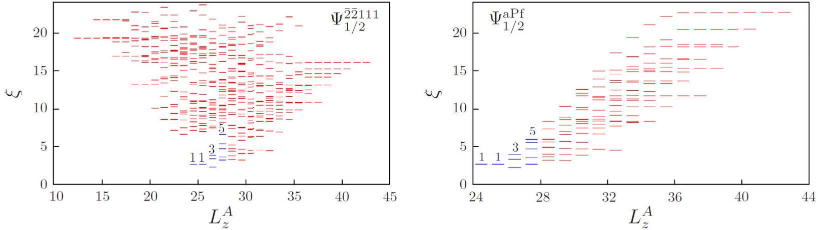
- Logarithm of the eigenvalues of the reduced density matrix
- Counting of low-lying entanglement levels: carries topological fingerprint of the state (Li-Haldane conjecture)
- can be evaluated from just the ground state wave function

related to edge excitations (bulk-edge correspondence)



H. Li and F. D. M. Haldane, Phys. Rev. Lett. **101**, 010504 (2008)

Entanglement spectrum of the "$\bar{2}\bar{2}111$" state



counting: $1, 1, 3, 5, \cdots$

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

The "$[\bar{2}]^k 1^{k+1}$" ansatz $\sim$ anti-Read-Rezayi

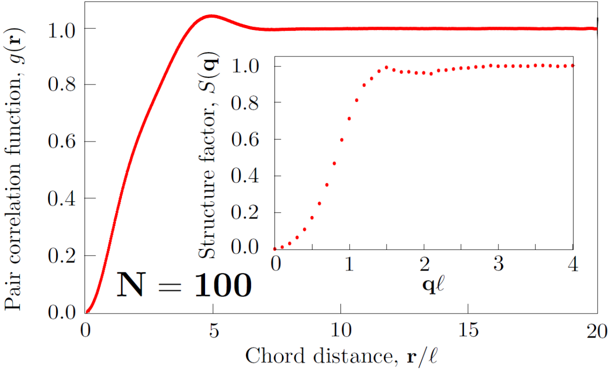
$$\Psi^{[\bar{2}]^k 1^{k+1}}_{\nu=2/(k+2)} = \mathcal{P}_{\rm LLL}[\Phi_2^*]^k \Phi_1^{k+1} \sim \frac{[\Psi^{\rm CF}_{2/3}]^k}{\Phi_1^{k-1}}$$

- $k=1 \implies \nu=2/3$: same as the composite fermion state
- $k=2 \implies \nu=1/2$: parton state in the anti-Pfaffian phase
- $k=3 \implies \nu=2/5$: this state lies in the same phase as the particle-hole conjugate of the Read-Rezayi $k=3$ state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

Quantities from "$\bar{2}\bar{2}111$" ansatz for large system sizes

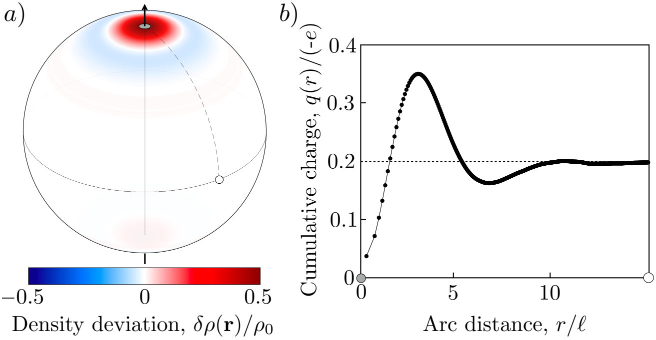


potentially enables numerical studies of braiding

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

Anyon with charge $(-e)/5$

$$\Psi_{2/5}^{2-\text{quasiparticles}} = \mathcal{P}_{\text{LLL}}[\Phi_2^{2-\text{holes}}]^*[\Phi_2^2]^*\Phi_1^4,$$



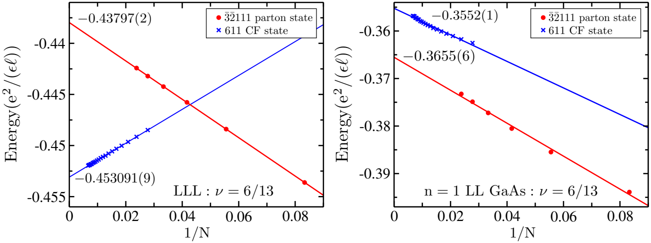
Density profile for $N = 80$ electrons

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

The "$\bar{3}\bar{2}111$" ansatz

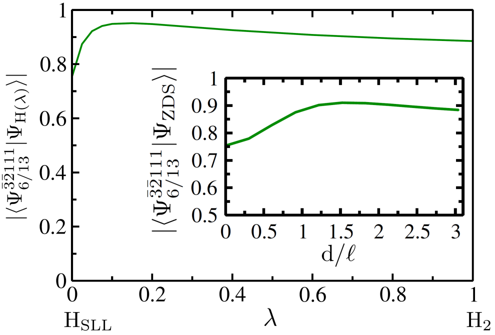
$$\Psi^{\bar{3}\bar{2}111}_{\nu=6/13} = \mathcal{P}_{\rm LLL}[\Phi_3^*][\Phi_2^*]\Phi_1^3 \sim \frac{[\Psi^{\rm CF}_{3/5}][\Psi^{\rm CF}_{2/3}]}{\Phi_1}$$

- occurs at $\mathcal{S}=-2$: topologically different from 6/13 CF state
- energetically better than the 6/13 CF state in the second LL



Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

The "$\bar{3}\bar{2}111$" ansatz gives a good description of $2+6/13$



$H_{\mathrm{ZDS}} \rightarrow$ simulates finite width, $H_2 \rightarrow$ stabilizes Pfaffian

- a good overlap with the second LL Coulomb ground state
- likely in the same universality class as the Levin-Halperin state

Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)

What makes our parton states special?

- Composite fermion ($n11\cdots$ parton) states capture the most prominent LLL plateaus
  $\rightarrow$ placing partons into $\nu = 1$ states, i.e., $\Phi_1 = \prod_{i<j}(z_i - z_j)$ builds good correlations in the many-body state
- Simplest generalization $\rightarrow nm11\cdots$ where $m = 2$ or $m = -2$
- Comes down to energetics: for the second LL interaction our sequence of parton states appear most plausible
- Open problem: for a given interaction which parton state(s) is likely to be stabilized

Outlook

- Composite fermion theory explains almost all the fractional quantum Hall phenomena occurring in the lowest LL.
- Parton sequences:
  - $\bar{n}\bar{2}111$, where $n = 1, 2, 3$ gives $2/3, 1/2, 6/13$
  - $\bar{n}\bar{2}\bar{2}1111$, where $n = 1, 2, 3$ gives $1/2, 2/5, 3/8$

  and their *particle-hole conjugates* contain most of the experimentally observed states in the second Landau level.
- How well does the parton ansatz fare for excitations? Counting works out - overlaps needs to be looked into.

Thank you for your attention!

Most fractional quantum Hall states are products of integer quantum Hall states

References:

- Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)
- Ajit C. Balram *et al.* Phys. Rev. Lett. **121**, 186601 (2018)
- Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

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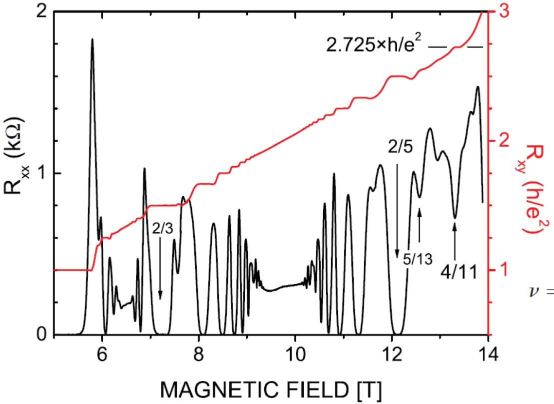
FQHE in the LLL that is not IQHE of CFs

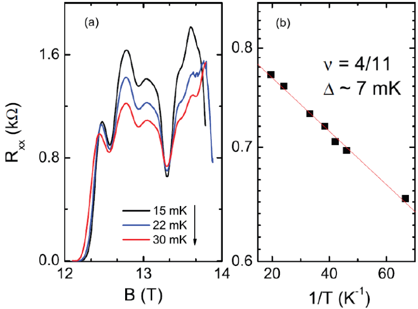


Pan *et al.*, Phys. Rev. B **91**, 041301(R) (2015)

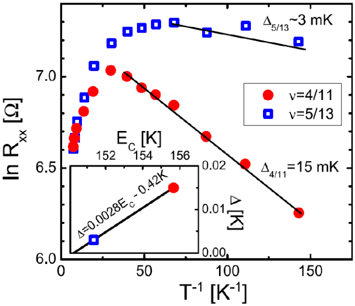
$$\nu = \frac{n}{2pn \pm 1}$$

FQHE of CFs?

Incompressibility at $\nu = 4/11$ and $5/13$



Pan *et al.*, Phys. Rev. B **91**, 041301(R) (2015)



Samkharadze *et al.*, Phys. Rev. B **91**, 081109(R) (2015)

Read-Rezayi states: clustering of electrons

$$\Psi^{\mathrm{RRk}}_{\nu=\frac{k}{k+2}} = \Phi_1 \mathbb{S}\left(\prod_{i_1<j_1}(z_{i_1}-z_{j_1})^2 \prod_{i_2<j_2}(z_{i_2}-z_{j_2})^2 \cdots \prod_{i_k<j_k}(z_{i_k}-z_{j_k})^2\right)$$

- $k=1 \implies \nu=1/3$: same as Laughlin
- $k=2 \implies \nu=1/2$: same as Pfaffian follows from Cauchy identity
- $k=3 \implies \nu=3/5$: particle-hole conjugate candidate for $2/5$ competes with the Bonderson-Slingerland state

  E. H. Rezayi and N. Read, Phys. Rev. B **79**, 075306 (2009)
- $k=4 \implies \nu=2/3$: competitive with Laughlin

  Peterson *et al.*, Phys. Rev. B **92**, 035103 (2015)
- $3/8$ and $6/13$ are not part of this sequence

N. Read and E. H. Rezayi, Phys. Rev. B **59**, 8084 (1999)

Bonderson-Slingerland states

Pfaffian times bosonic Jain

$$\Psi^{\rm BS}_{\nu=\frac{n}{(2p+1)n\pm 1}} = \mathcal{P}_{\rm LLL}{\rm Pf}\left[\frac{1}{z_i - z_j}\right]\left(\Phi_{\pm n}\prod_{i<j}(z_i - z_j)^{2p+1}\right).$$

- $n=1$, $p=0$ and $+ \implies \nu=1/2$: same as the Pfaffian
- $n=2$, $p=0$ and $+ \implies \nu=2/3$: different from Jain $2/3$
  second LL Coulomb ground state not uniform at this shift
- $n=2$, $p=1$ and $- \implies \nu=2/5$: different from Jain $2/5$
  competes with the particle-hole conjugate of the $k=3$ Read-Rezayi

  Bonderson *et al.*, Phys. Rev. B **108**, 036806 (2012)
- $n=3$, $p=1$ and $- \implies \nu=3/8$: feasible in the second LL

  Hutasoit *et al.*, Phys. Rev. B **95**, 125302 (2017)
- $6/13$ is not part of this sequence

Parsa Bonderson and J. K. Slingerland, Phys. Rev. B **78**, 125323 (2008)

analytical argument relating "$\bar{2}\bar{2}111$" state to anti-Pfaffian

- state Φ_2^2 has a central charge of $c = 5/2$

  X.-G. Wen, Phys. Rev. Lett. **66**, 802 (1991)
- write bosonic Φ_2^2 as $\Phi_1 \chi^{\rm CF}_{l=3}$
  analogous to writing bosonic ${\rm Pf}\Phi_1$ as $\Phi_1 \chi^{\rm CF}_{l=1}$
- $[\Phi_2^2]^*$ is essentially $[\Phi_1]^* \chi^{\rm CF}_{l=-3}$
- $\Phi_1^3 [\Phi_2^2]^* \sim |\Phi_1^2| [\Phi_1]^2 \chi^{\rm CF}_{l=-3} \sim \Phi_1^2 \chi^{\rm CF}_{l=-3}$
- This state has central charge $c = 1 - 3/2 = -1/2$ which matches the anti-Pfaffian value
  analogous to ${\rm Pf}\Phi_1^2 \sim \Phi_1^2 \chi^{\rm CF}_{l=1}$ which has $c = 1 + 1/2 = 3/2$
  analogous to $\Phi_2^2 \Phi_1 \sim \Phi_1^2 \chi^{\rm CF}_{l=3}$ which has $c = 1 + 3/2 = 5/2$

  Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **98**, 035127 (2018)

The "$\bar{n}\bar{2}\bar{2}1111$" ansatz

$$\Psi^{\bar{n}\bar{2}\bar{2}1111}_{\nu=n/(3n-1)} = \mathcal{P}_{\rm LLL}[\Phi^{*}_{n}][\Phi^{*}_{2}]^{2}\Phi^{4}_{1} \sim \frac{\Psi^{\rm CF}_{n/(2n-1)}[\Psi^{\rm CF}_{2/3}]^{2}}{\Phi^{2}_{1}}$$

- $n=1 \implies \nu=1/2$: parton state in the anti-Pfaffian phase
- $n=2 \implies \nu=2/5$: same phase as the particle-hole conjugate of the Read-Rezayi $k=3$ state
- $n=3 \implies \nu=3/8$: a new candidate state different from the Bonderson-Slingerland state

Ajit C. Balram, Maissam Barkeshli, and Mark. S. Rudner, Phys. Rev. B **99**, 241108 (2019)

Projecting a state into the lowest Landau level (LLL)

- Direct or exact projection: limited to small sizes
  brute force way to retain the part that resides in the LLL
  bring all the $\bar{z}$'s to left ("normal ordering") and $\bar{z} \rightarrow 2\partial_z$

  S. M. Girvin and T. Jach, Phys. Rev. B **29**, 5617 (1984)

- Jain-Kamilla projection: large systems are accessible

$$\prod_{i<j}(z_i - z_j)^{2p} = \prod_{i \neq j}(z_i - z_j)^p \equiv \prod_j \mathcal{J}_j^p$$

$$\mathcal{J}_j = \prod_k{}' (z_j - z_k) \quad ' \Longrightarrow j \neq k$$

Subsume $\mathcal{J}_j$ into Slater determinant & project each element

J. K. Jain and R. K. Kamilla, Phys. Rev. B **55**, R4895(R) (1997)

221 parton state possibly realized in $n = 3$ LL of graphene

$$\Psi_{1/2}^{221} = \mathcal{P}_{\rm LLL}\Phi_2\Phi_2\Phi_1$$

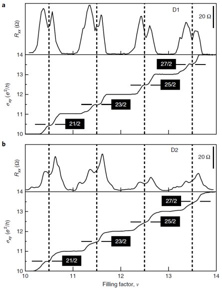


Fig. 2 | Hall plateaus at half filling of the $n = 3$ Landau levels. a, Data recorded on device D1 for $B = 15$ T. **b**, Same as in **a**, but for device D2 and $B = 21.5$ T. All data were acquired at a temperature of ~30 mK.

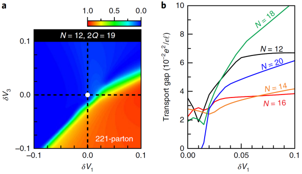


Fig. 3 | Overlaps and transport gaps at half filling of the $n = 3$ Landau level. a, Overlap for the 221-parton state for $N = 12$ electrons seeing a flux of $2Q = 19$ in the spherical geometry. Simulations were performed for a 41×41 grid of parameter pairs. The white dot in the centre marks the exact Coulomb point in the $n = 3$ graphene Landau level, and δV_1 and δV_3 denote changes to the first two relevant Haldane pseudopotentials. **b**, Transport gap for the $n = 3$ Landau level of graphene extracted from exact diagonalization in the vicinity of the Coulomb interaction. The transport gap is shown for 21 values of the interaction defined by $V_m + \delta V_m$, where V_m are the Haldane pseudopotentials for the pure Coulomb interaction, $\delta V_1 = -\delta V_3$, and all other $\delta V_m = 0$.

Y. Kim *et al.*, Nature Physics **15**, 154-158 (2019)