Navigating the Hilbert Space with Models

24th December 2019

ICTS INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

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P.W. Anderson, 13 Dec 2018, Princeton

About my Institute

http://www.imsc.res.in

Institute of Mathematical Sciences

Research in

Theoretical Physics
Pure Mathematics
Computer Science

~ 60 faculy
100 Ph.D. students
20 PDF's
10 visitors

Junior Research Fellow

Entrance through **JEST** Exam

Autonomous Institute

similar to
IIT's
IISc, Bangalore
TIFR, Bombay

Aided by DAE

Post Doctoral Fellowship
Visiting Research Scholar
Summer Program for BSc, BE, MSc students
Faculty Associateship
Adjunct Faculty
Visiting Professor…


India भारत
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National Capital
State or Union Territory Capital
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0 200 400 Miles
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Bhutan
Itanagar
Kathmandu
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Thimphu
Assam
Nagaland
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Jaipur
Lucknow
Rajasthan
Uttar Pradesh
Patna
Shillong
Meghalaya
Manipur
Imphal
Bihar
Bangladesh
Agartala
Dhaka
Tripura
Aizawl
Mizoram
Jharkhand
West Bengal
Ranchi
Kolkata (Calcutta)
Gandhinagar
Gujarat
Bhopal
Madhya Pradesh
Myanmar
Chhattis-garh
Orissa
Daman
Diu
Daman
Silvassa
Maharashtra
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Bhubaneswar
Dadra & Nagar Haveli
Mumbai (Bombay)
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Andhra Pradesh
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Pondicherry
Pondicherry
Tamil Nadu
Kavaratti
Pondicherry
Andaman and Nicobar Islands
Andaman Sea
Lakshadweep
Kerala
Thiruvananthapuram
Nicobar Islands
Sri Lanka
30
15
90



IMS


A busy street in Chennai

2 miles from Matscience as the Crow flies

Madras

Indian Institute of Technology

Invitation to visit Matscience & IIT Madras

Suprisingly rich
ground state manifold
of
spin clusters

Quantum spin quadrumer
Subhankar Khatua, R. Shankar and R. Ganesh
PHYSICAL REVIEW B **97**, 054403 (2018)

Effective theories for quantum spin clusters:
Geometric phases and selection by singularity
Subhankar Khatua, Diptiman Sen and R. Ganesh
PHYSICAL REVIEW B **100**, 134411 (2019)

Order by singularity in Kitaev clusters
Sarvesh Srinivasan, Subhankar Khatua, G. Baskaran and R. Ganesh
arXiv:1912.04341

Hilbert space is incomprehensibly big

People could easily get lost

Experiments, Physical Intuitions and Models
guide us in our Voyage and Explorations

Spin Liquid Models

Surprises in Quantum Spin Clusters

Resonating Valence Bond (RVB) States

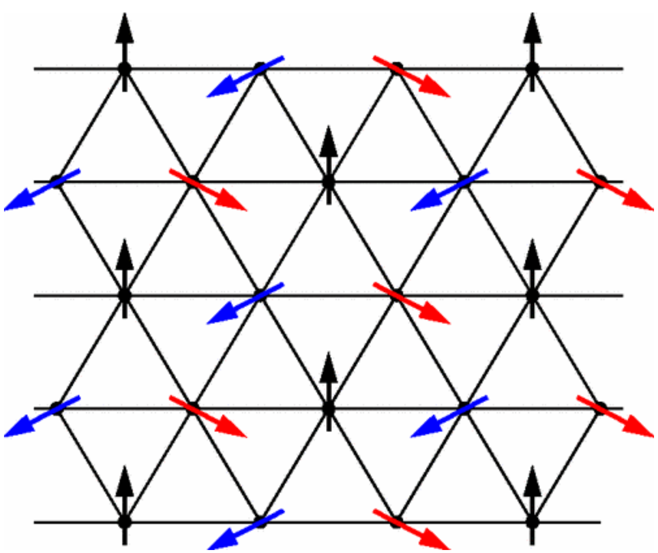
Pauling 1931
Anderson 1973
Fazekas 1974

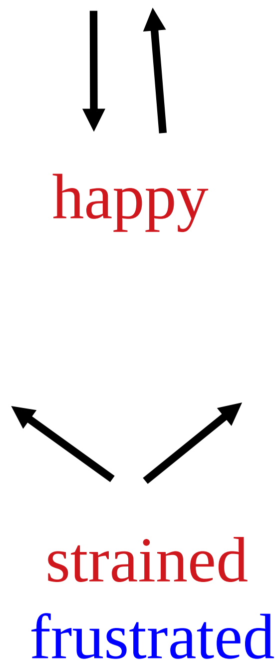
Quantum Heisenberg Antiferromagnets (spin half)

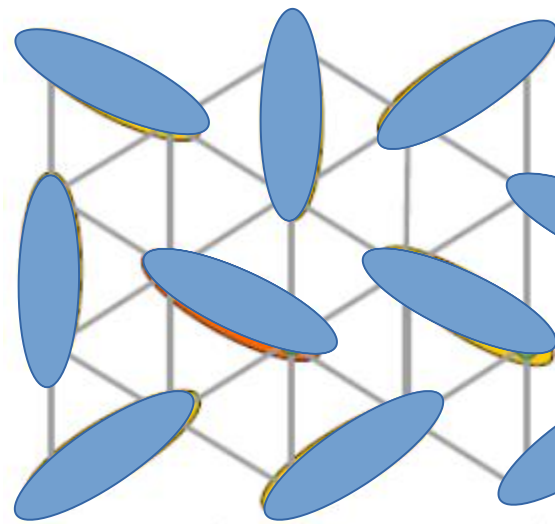
Quantum fluctuations, Encouraged by lattice frustration and lower dimensionality

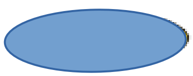
may destroy long range AFM order (spin crystal)

Geometrical Frustration









$$= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$$

RVB State is a Quantum Spin Liquid

Short range or power law spin correlations

$$\langle S_{iz} S_{jz} \rangle \approx e^{\frac{-|i-j|}{\xi}} \quad or \quad \frac{1}{|i-j|^{\alpha}}$$

Robust Spin liquids could harbor fragile long range order

AFM, Valence Bond Order, …

They are highly entangled and are not direct product states

Zero point fluctuations are a measure of entanglement

Ted Hsu (1988)
Tao Li , Ashwin Viswanath, Moessner, Alet ...
Vikram Tripati

In 1987 We were looking for

Spin Liquids, pseudo Fermions & Pseudo Fermi Sea
suggested by Anderson (1987) for Mott insulating La2CuO4

Enlarged Hilbert Space (an aerial view/introspection)
Helped us see what we are looking for

GB, Zou, Anderson 1987

As a Bonus we found
Gauge Structures, Gauge Fields
and other isights into Quantum Spin Liquids

GB, Anderson 1988, …

Square Lattice Heisenberg Antiferromagnet in an Enlarged Hilbert Space

GB, Zou, Anderson 1987

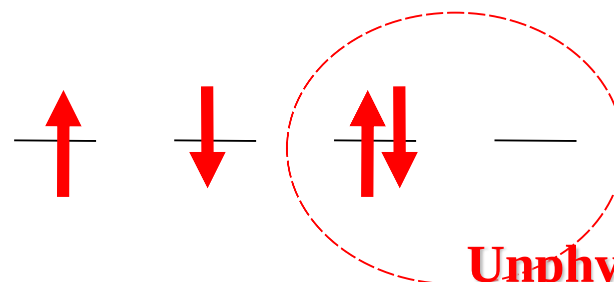
$$H_s = J \sum_{\langle ij \rangle} (S_i \cdot S_j - \frac{1}{4}) = -J \sum_{\langle ij \rangle} b_{ij}^{\dagger} b_{ij}$$

$$\vec{S}_i \equiv C_{i\alpha}^{+} \vec{\sigma}_{\alpha\beta} C_{i\beta}$$

$$n_{i\uparrow} + n_{i\downarrow} = 1$$

Two complex fermion Hilbert space

$$2^2 = 4$$



Unphysical Hilbert space

Extended - S $\longrightarrow$ $\Delta_x = \Delta_y$

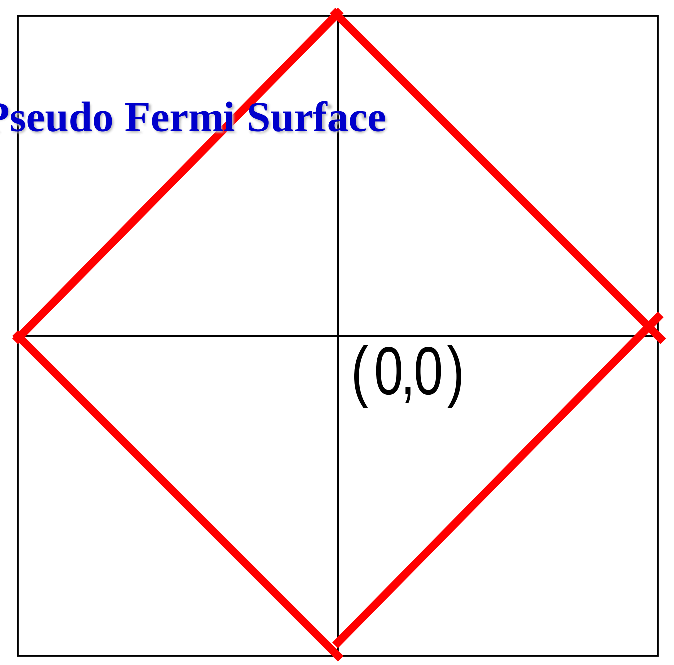
$$H_s = J\sum_{\langle ij\rangle}(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger} b_{ij}$$

$$b_{ij}^{+} b_{ij} \to \langle b_{ij}^{+}\rangle b_{ij} + b_{ij}^{+}\langle b_{ij}\rangle$$

$$H_{pair} = -J\sum_{k,k'} \gamma(\mathbf{k}-\mathbf{k}') c_{-k'\downarrow}^{\dagger} c_{k'\uparrow}^{\dagger} c_{k\uparrow} c_{-k\downarrow}$$

$$H_{mF} \sim J\sum_{k\alpha} |\cos k_x + \cos k_y| \; \alpha_{k\sigma}^{\dagger}\alpha_{k\sigma}$$

(π,π)



$$|2D\,RVB\rangle = P_G \prod_k (u_k + v_k c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger})|0\rangle$$

GB, Zou, Anderson 1987

Local U(1) gauge symmetry

$$H_s = J\sum_{\langle ij\rangle}(\mathbf{S}_i\cdot\mathbf{S}_j - \frac{1}{4}) = -J\sum_{\langle ij\rangle} b_{ij}^{\dagger} b_{ij}$$

$$C_{i\alpha}^{+} \to e^{i\theta_i} C_{i\alpha}^{+} \qquad b_{ij}^{+} \to e^{i\theta_i} b_{ij}^{+} e^{i\theta_j} \qquad \Delta_{ij} \to e^{i\theta_i}\Delta_{ij} e^{i\theta_j}$$

GB, Anderson

U(1) RVB magnetic field

$$\Re\ e^{i\oint \mathbf{A}(\mathbf{r}).d\mathbf{l}} \sim \mathbf{S}_i \times (\mathbf{S}_j \times \mathbf{S}_k)$$

Wen Wilczek Zee

Local SU(2) symmetry

$$c_{i\uparrow} \to u_i c_{i\uparrow} + v_i c_{i\downarrow}^{\dagger} \qquad |u_i|^2 + |v_i|^2 = 1$$

Affleck Anderson Zou Hsu

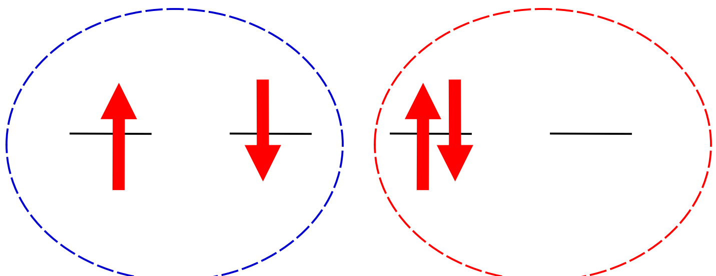
Two complex fermion

Hilbert space $2^2 = 4$

$$2^{2N} = 2^N \times \ldots\ldots\times 2^N$$

$$\leftarrow 2^N\ times \rightarrow$$

Physical spin Pseudo spin



When a physical spin and a Pseudo spin get identified the above 2^N sectors become gauge copies of a Z_2 gauge theory

Sachdev-Ye Model

VOLUME 70, NUMBER 21 PHYSICAL REVIEW LETTERS 24 MAY 1993

Gapless Spin-Fluid Ground State in a Random Quantum Heisenberg Magnet

Subir Sachdev and Jinwu Ye

$$\mathcal{H} = \frac{1}{\sqrt{NM}} \sum_{i>j} J_{ij} \hat{S}_i \cdot \hat{S}_j ,$$

$$P(J_{ij}) \sim \exp[-J_{ij}^2/(2J^2)]$$

group SU(M)

representation labeled by

$n_b = 2S$ for SU(2)

Exact Dynamical Local spin Susceptibility

$$\bar{\chi}(\omega) = X \left[\ln\left(\frac{1}{|\omega|}\right) + i\frac{\pi}{2}\mathrm{sgn}(\omega) \right] + \cdots$$

Create frustration in Spin Space via anisotropic spin- spin coupling

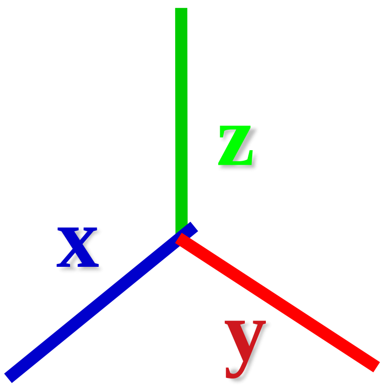
As opposed to

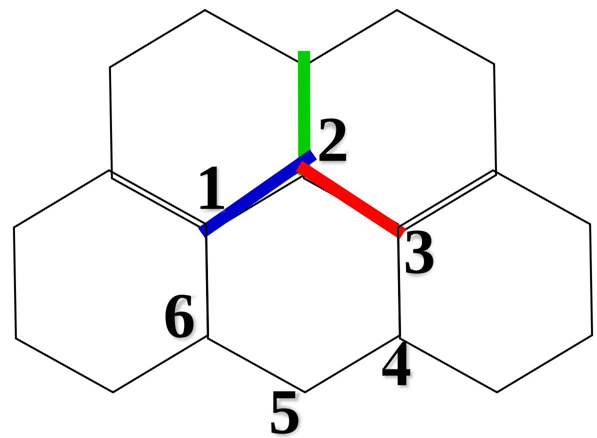
Geometrical Frustration of Isotropic Heisenberg spin systems

Birth of new family of models

Kitaev Model on a Honeycomb lattice

Kitaev 2001, 2003





Frustration in Spin Space

$$H = -J_x \sum_{\langle ij \rangle_x} \sigma_i^x \sigma_j^x - J_y \sum_{\langle ij \rangle_y} \sigma_i^y \sigma_j^y - J_z \sum_{\langle ij \rangle_z} \sigma_i^z \sigma_j^z$$

Flux Operator

$$B_p = \sigma_1^y \sigma_2^z \sigma_3^x \sigma_4^y \sigma_5^z \sigma_6^x$$

2N sites

$$[B_P, H] = 0 \qquad B_P^2 = 1$$

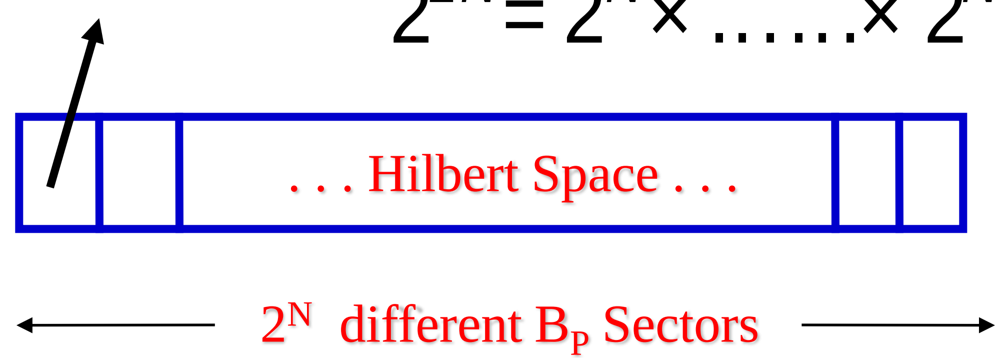
Eigen value B_P = ± 1

$$[B_P, B_{P'}] = 0$$

for every plaquette P

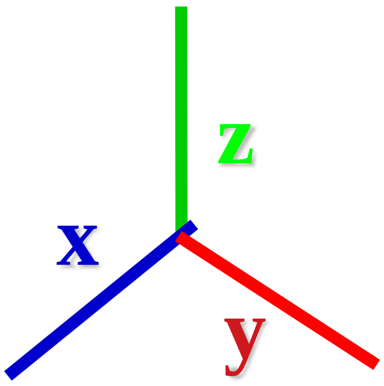
2^N states

$$2^{2N} = 2^N \times \ldots\ldots \times 2^N$$



2^N possible configurations of B_P

GB, S. Mandal and R. Shankar, Phys. Rev. Lett. 98 (2007)



It is a Gapless Spin Liquid

but spin correlation functions are ultra local

$\langle \sigma_i^a \sigma_j^b \rangle \neq 0$ only if i and j are nearest neighbors and $a = b$ is equal to the type of bond which joins i and j

Do the various features of the spin-1/2 Kitaev model survive for higher spins?

GB, D. Sen and R. Shankar, Phys. Rev. B 78 (2008) 115116

The conserved quantities on each hexagon survive:

$$W = e^{i\pi\,(S_1^y + S_2^z + S_3^x + S_4^y + S_5^z + S_6^x)}$$

$J_1 = J_2 = J_3$

Classical Ground States

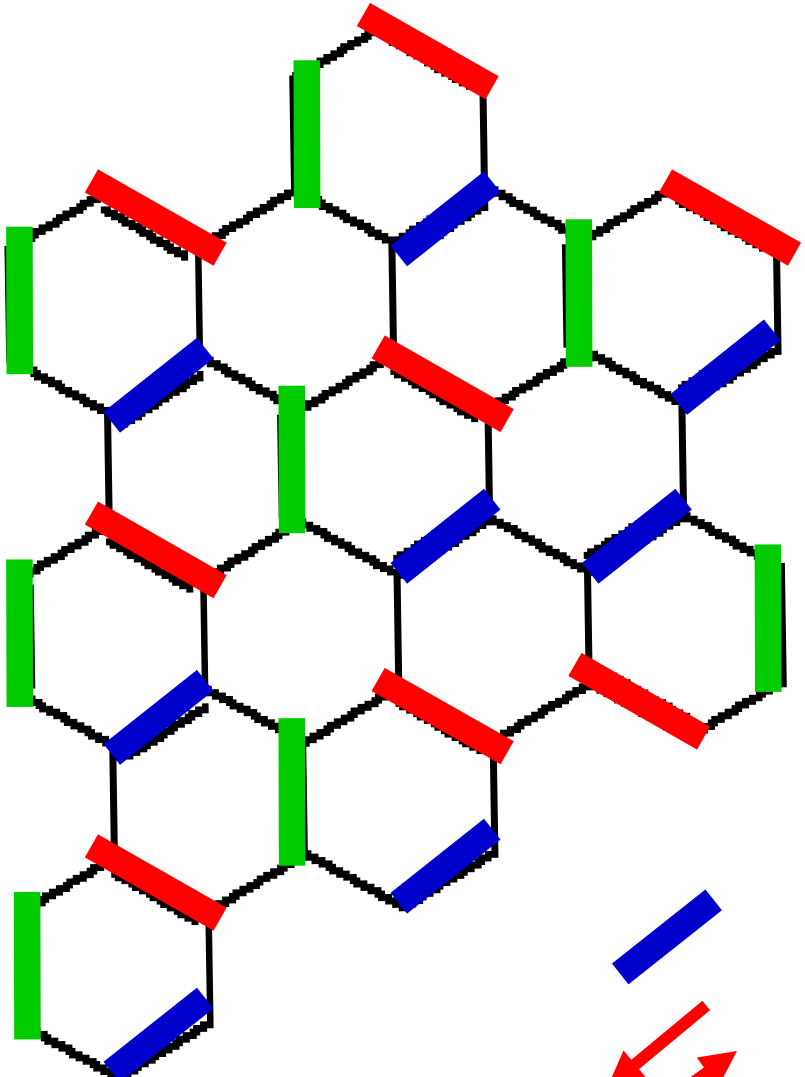
A sublattice pointing in some direction and all the spins In the B sublattice pointing in the opposite direction

States in which pairs of nearest neighbor spins on, say, a xx bond point along the $\pm \hat{x}$ direction

The number of such discrete states is equal to the number of dimer coverings of the honeycomb lattice which is 1.175^N times 1.414^N (due to the choice of $\pm$), which gives 1.662^N discrete classical ground states

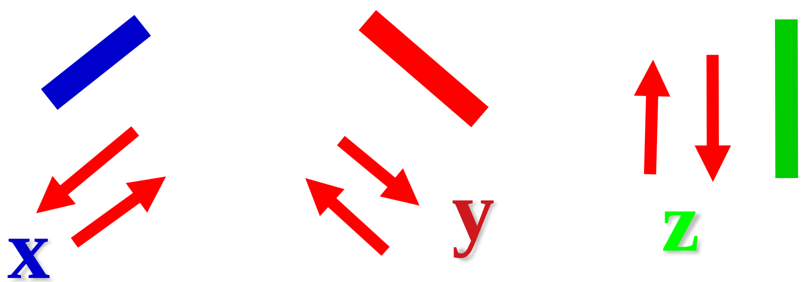
GB, D. Sen and R. Shankar, Phys. Rev. B 78 (2008) 115116

Cartesian Ground States



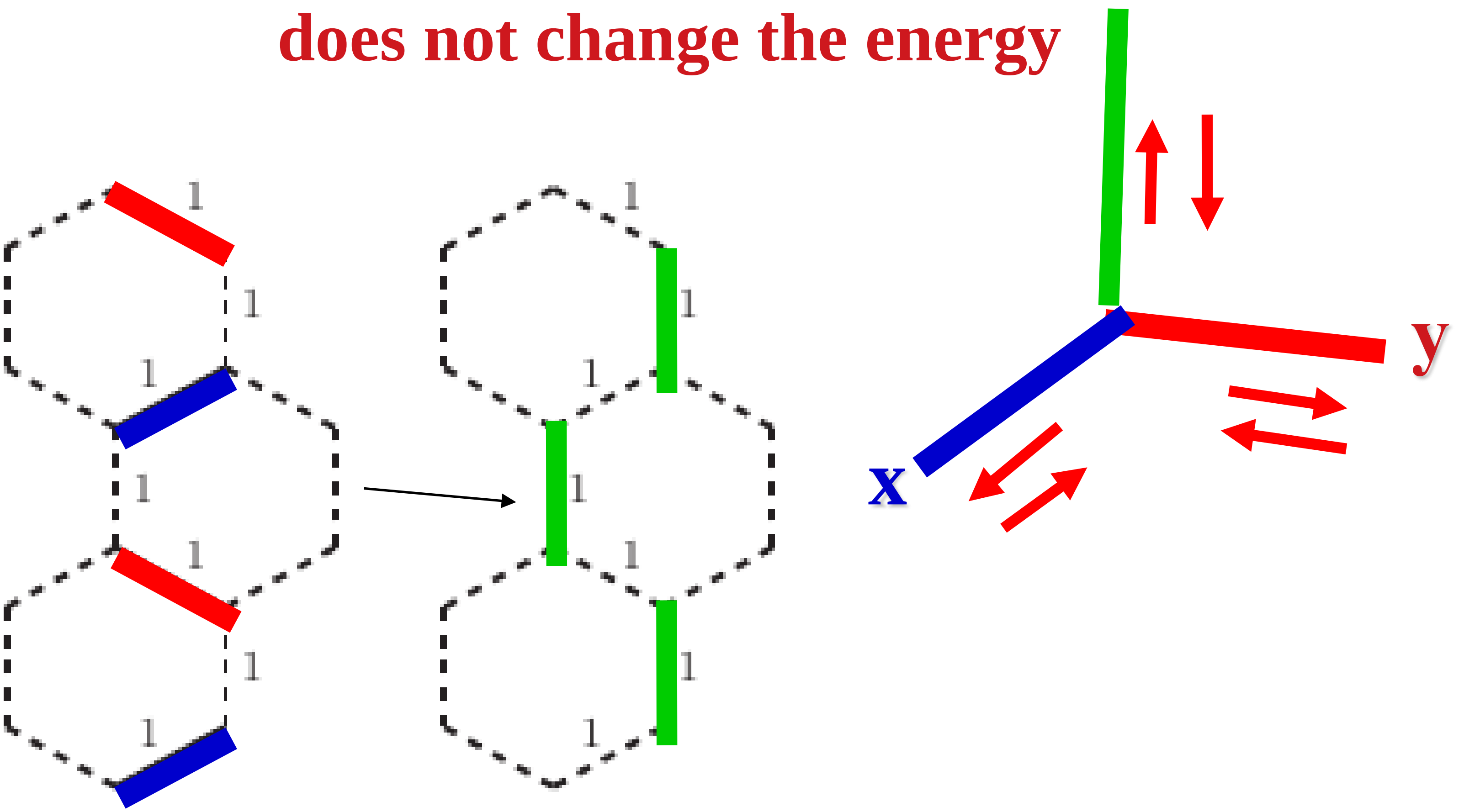
Cartesiana ground state
Degeneracy is related to
number of Dimer Coverings

$$(1.662)^N$$



GB, D. Sen and R. Shankar, Phys. Rev. B 78 (2008) 115116

Sliding: a continuous operation that does not change the energy



GB, D. Sen and R. Shankar, Phys. Rev. B 78 (2008) 115116

Ground state manifold contains atleast 1.662^N discrete set of points connected by Flat valleys

PHYSICAL REVIEW E **82**, 031113 (2010)

Classical Heisenberg spins on a hexagonal lattice with Kitaev couplings

Samarth Chandra,* Kabir Ramola,† and Deepak Dhar‡

GB, D. Sen and R. Shankar, Phys. Rev. B 78 (2008) 115116

PHYSICAL REVIEW E **82**, 031113 (2010)

Classical Heisenberg spins on a hexagonal lattice with Kitaev couplings

Samarth Chandra,[*] Kabir Ramola,[†] and Deepak Dhar[‡]

Chandra, Ramola and Dhar found the complete manifold of ground states of classical spin Kitaev model. Found a gauge structure.

Quantum spin liquid in the semiclassical regime

Ioannis Rousochatzakis[1], Yuriy Sizyuk[1] & Natalia B. Perkins[1]

Nature Communications | DOI: 10.1038/s41467-018-03934-1

A nice parametrization all classical ground states of Kitaev model

Find an effective Toric Code Hamiltonian interms of an emergent Pseudo spin variable on an emergent Kagame lattice

Shubhankar Khatua, R. Shankar and R. Ganesh PHYSICAL REVIEW B **97**, 054403 (2018)

Quantum spin quadrum

$$H_N = J\left[\sum_{j=1}^{N} \vec{S}_j\right]^2 .$$

N = 4

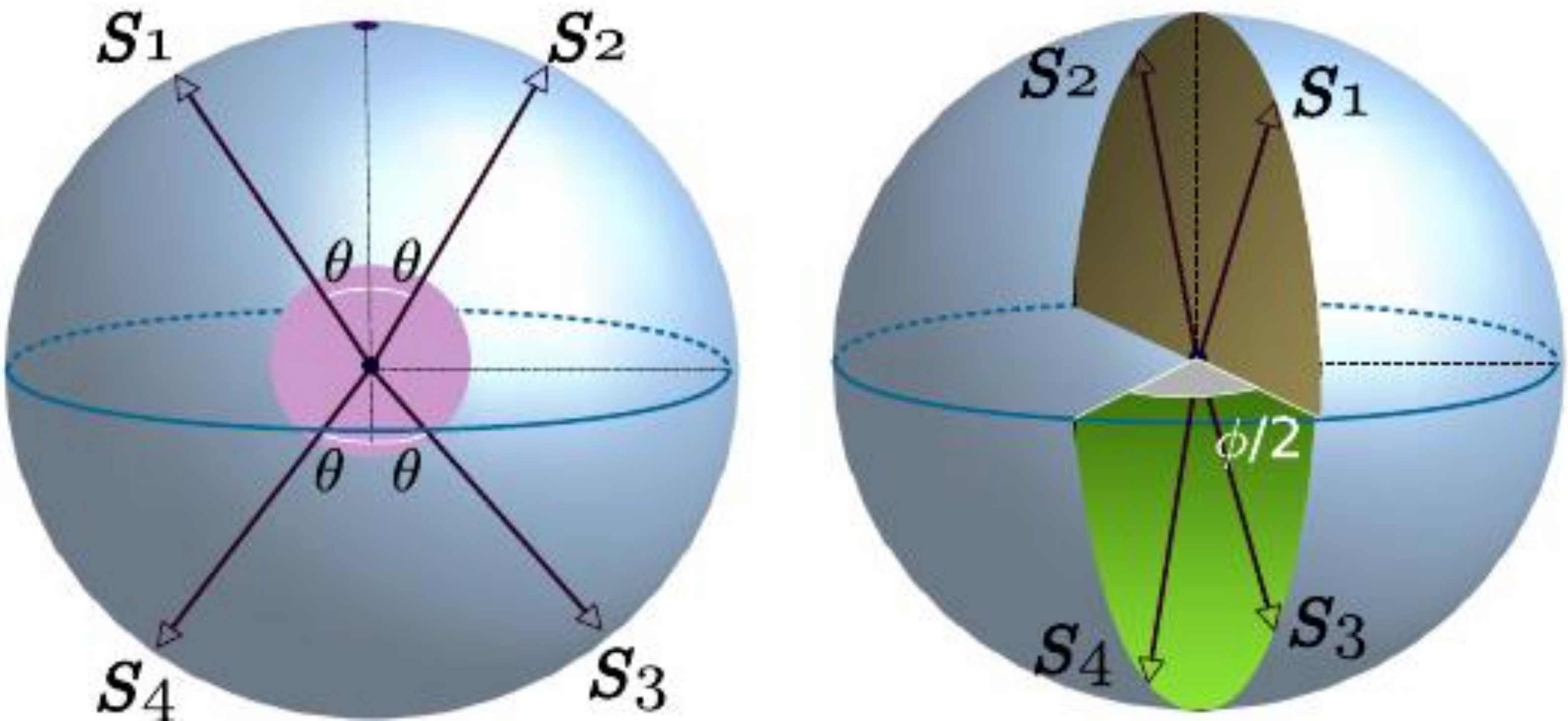
$$\hat{n}_1 = \sin\theta\left(\cos\frac{\phi}{4}\,\hat{x} + \sin\frac{\phi}{4}\,\hat{y}\right) + \cos\theta\,\hat{z},$$

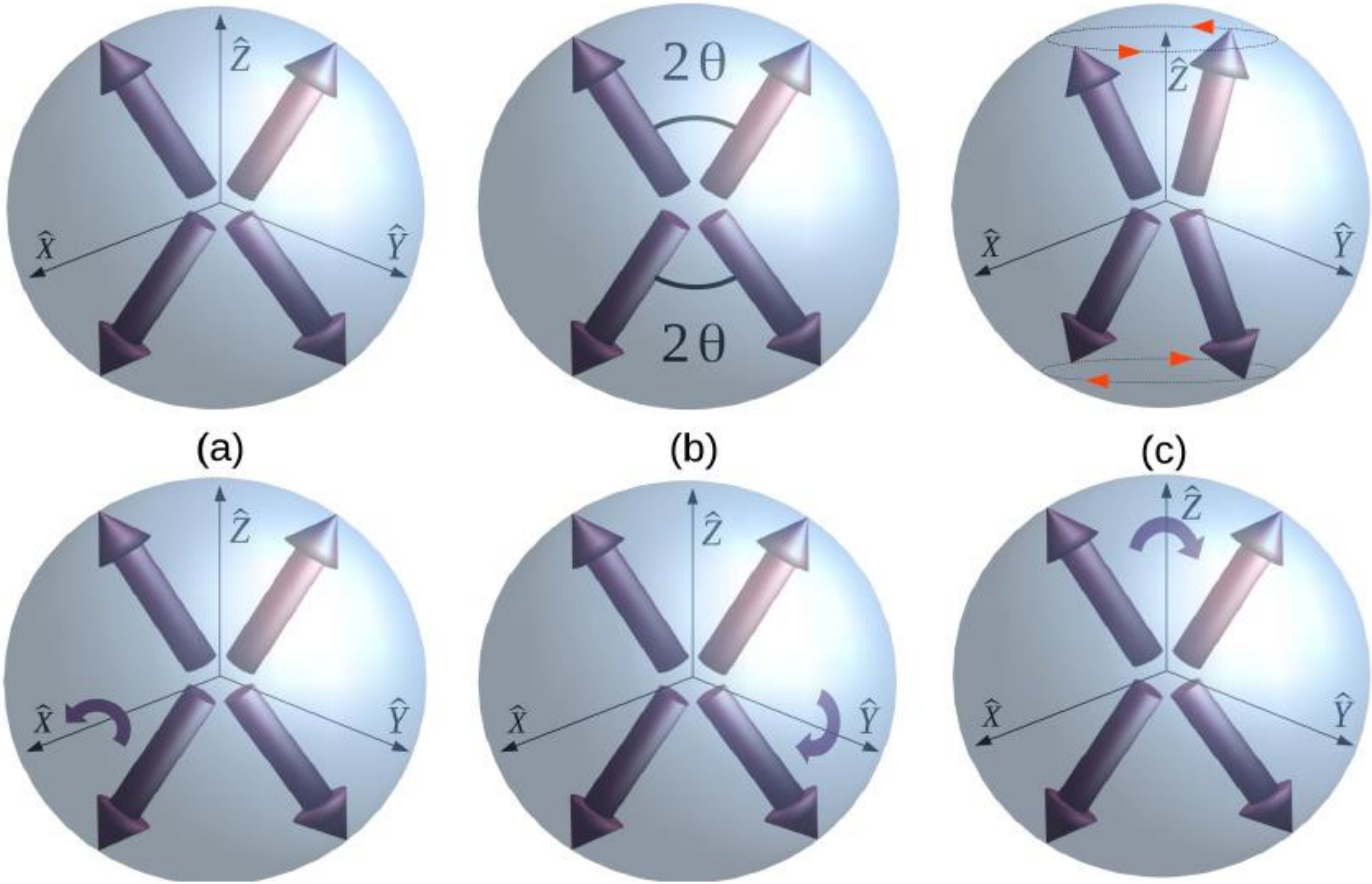
$$\hat{n}_2 = \sin\theta\left(-\cos\frac{\phi}{4}\,\hat{x} - \sin\frac{\phi}{4}\,\hat{y}\right) + \cos\theta\,\hat{z},$$

$$\hat{n}_3 = \sin\theta\left(-\cos\frac{\phi}{4}\,\hat{x} + \sin\frac{\phi}{4}\,\hat{y}\right) - \cos\theta\,\hat{z},$$

$$\hat{n}_4 = \sin\theta\left(\cos\frac{\phi}{4}\,\hat{x} - \sin\frac{\phi}{4}\,\hat{y}\right) - \cos\theta\,\hat{z}.$$

Rigid Rotor & Spin-S





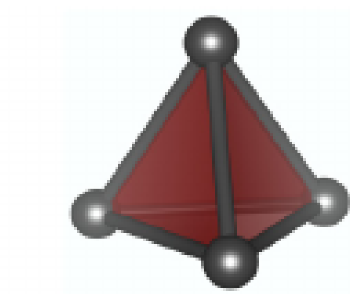
$\hat{Z}$
$\hat{X}$
$\hat{Y}$
2θ
2θ
(a)
(b)
(c)

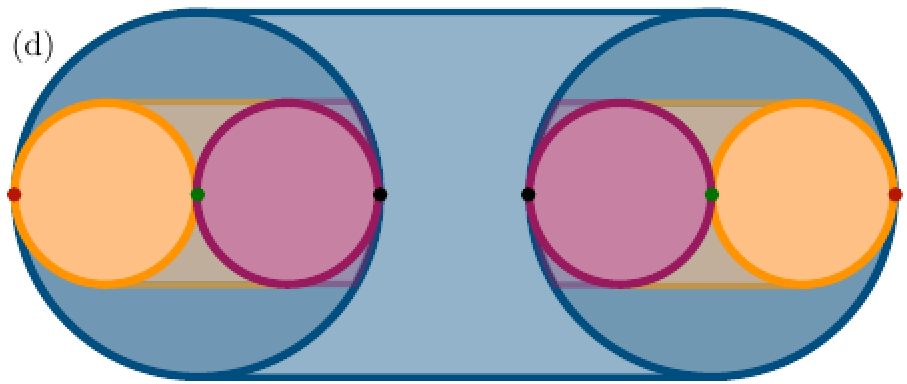
Effective theories for quantum spin clusters: Geometric phases and state selection by singularity

PHYSICAL REVIEW B **100**, 134411 (2019)

S. Khatua, D. Sen and R. Ganesh

XY Quadrumer



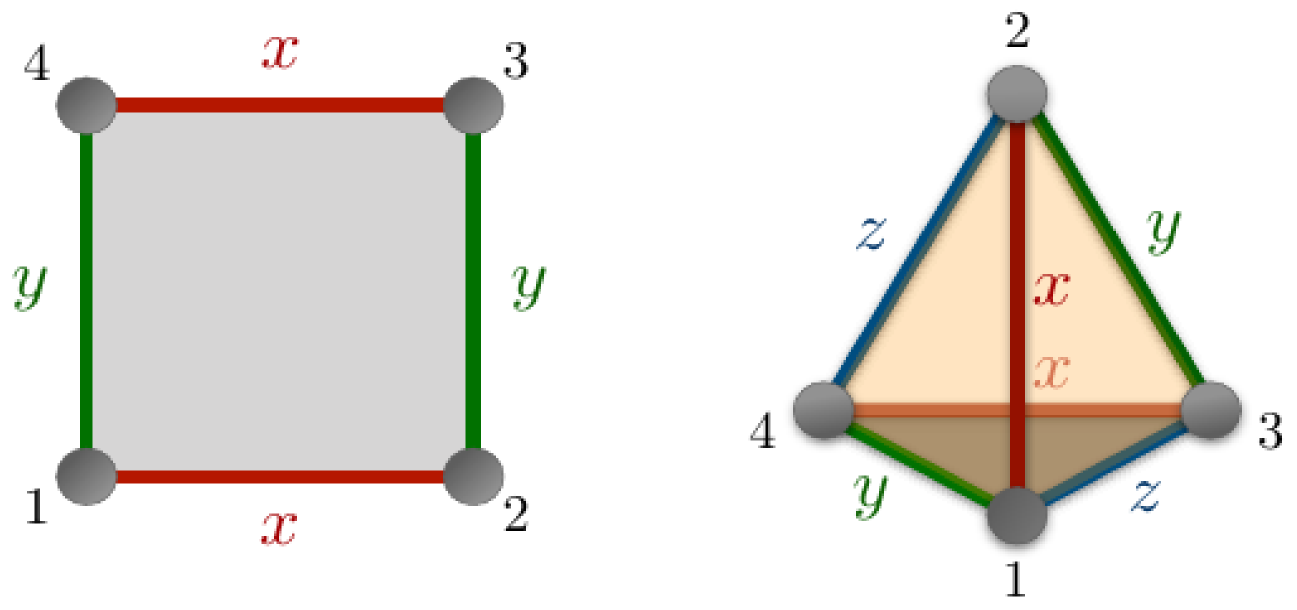


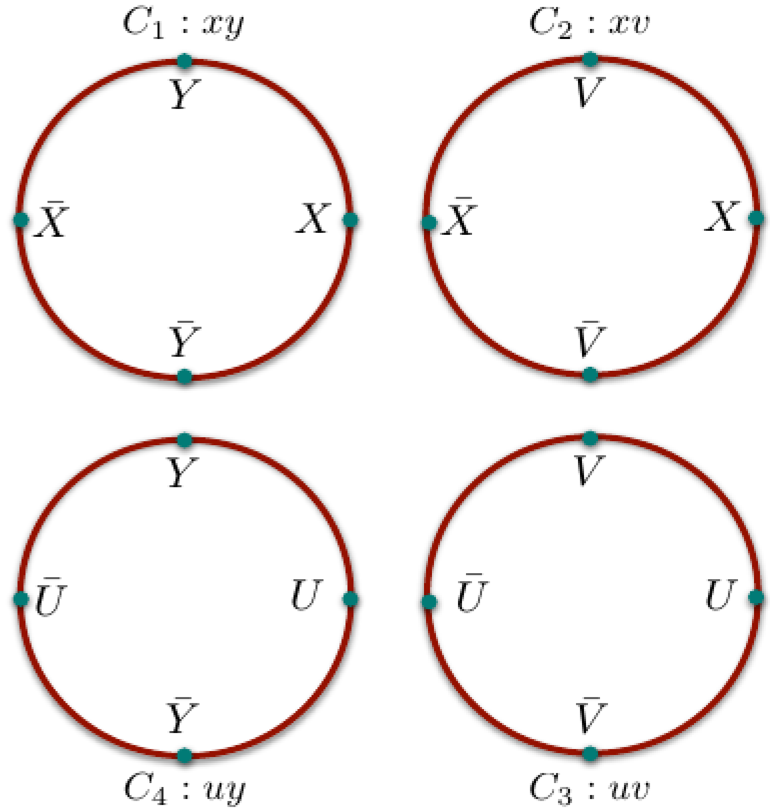
Cross-section view of the ground-state space.
We have three tori, with each pair of tori touching along a line.

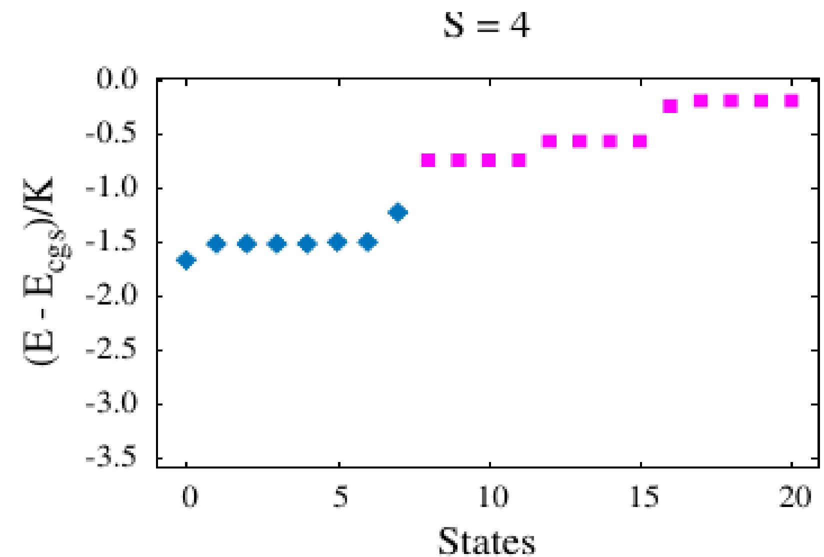
Order by singularity in Kitaev clusters

Sarvesh Srinivasan, Subhankar Khatua, G. Baskaran and R. Ganesh

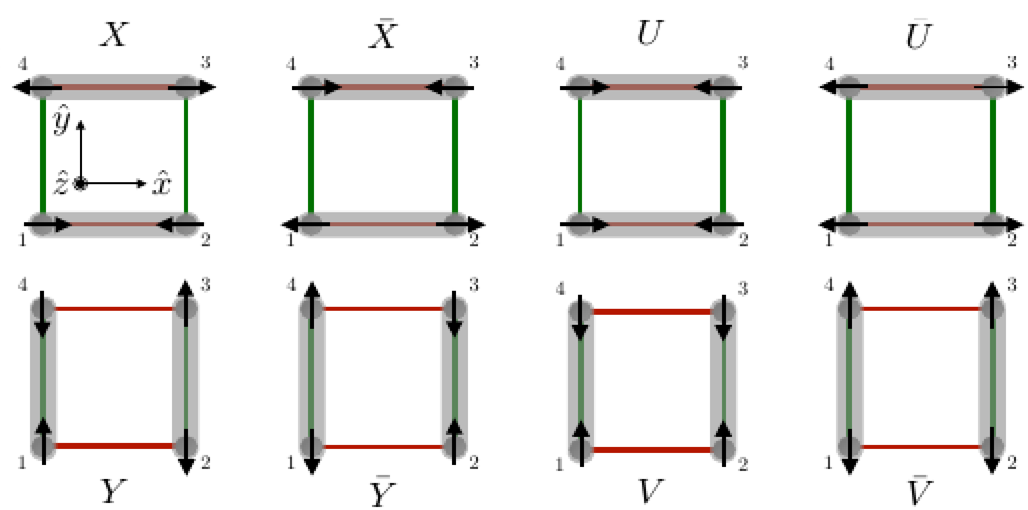
arXiv:1912.04341

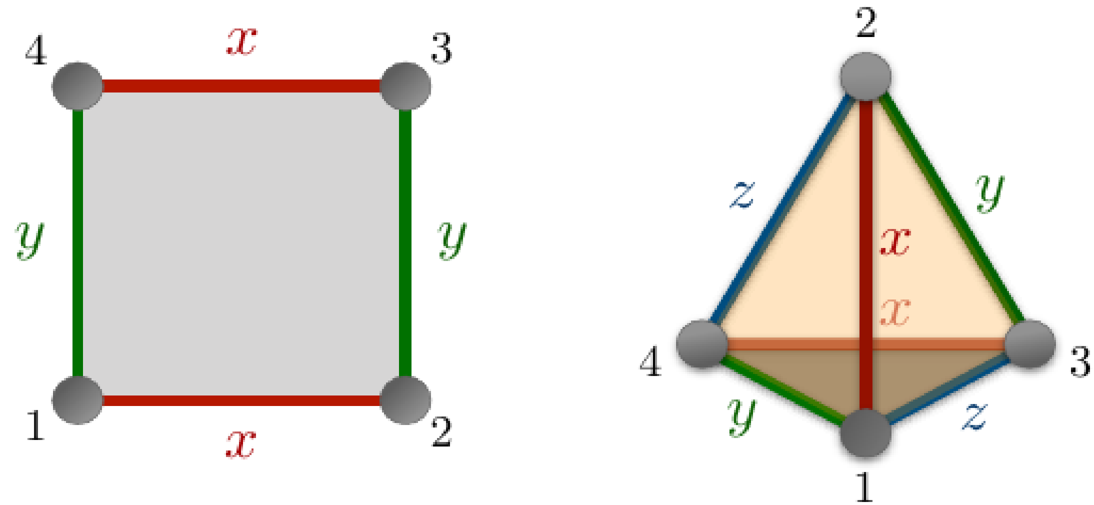






Four circles Embedded in Four Dimensions





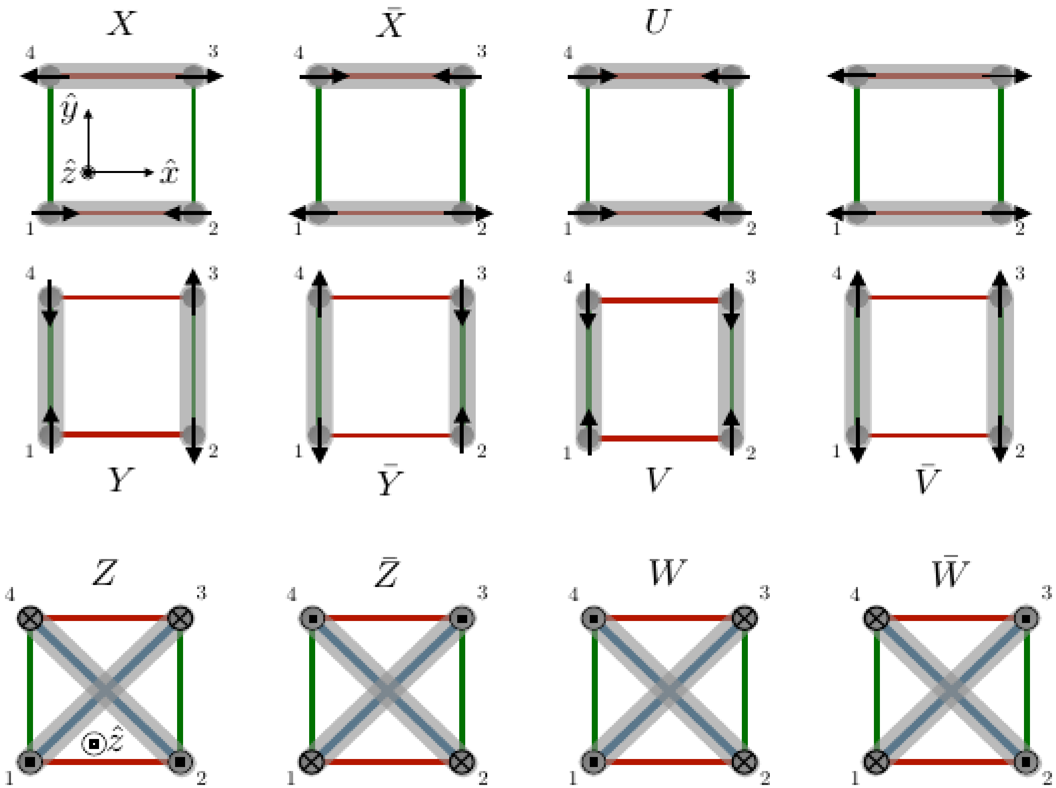
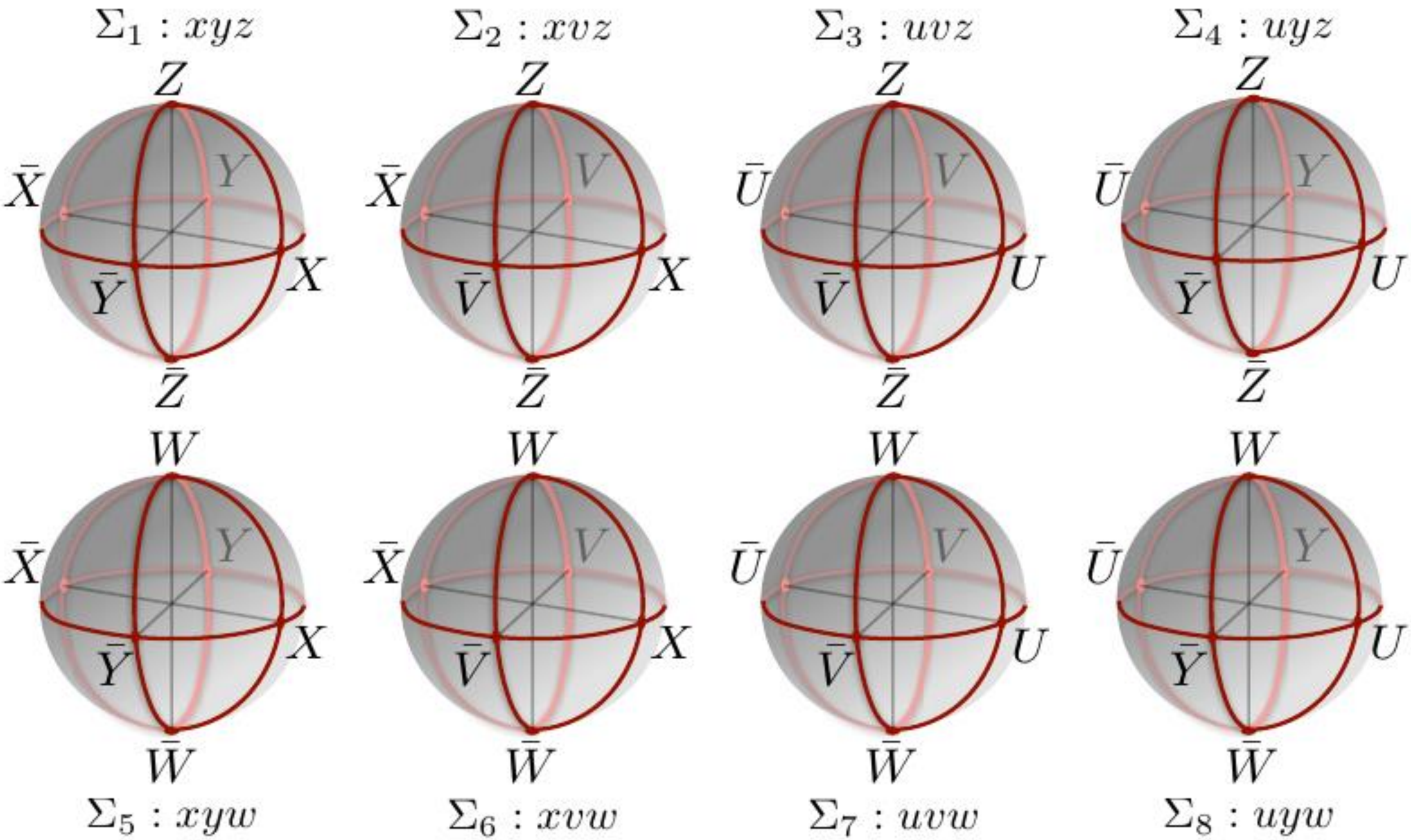
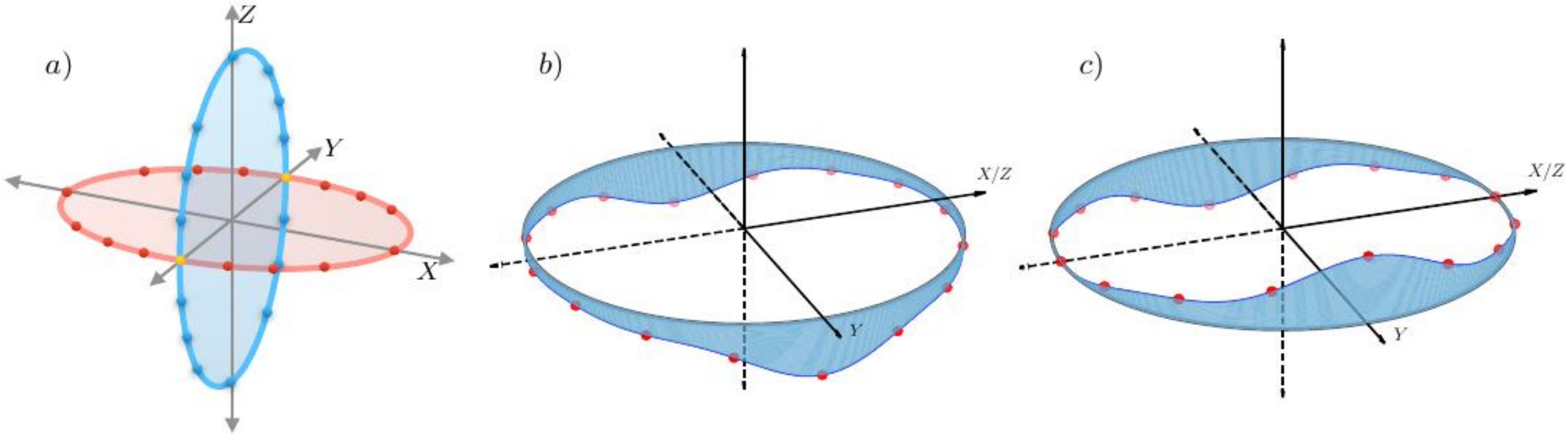


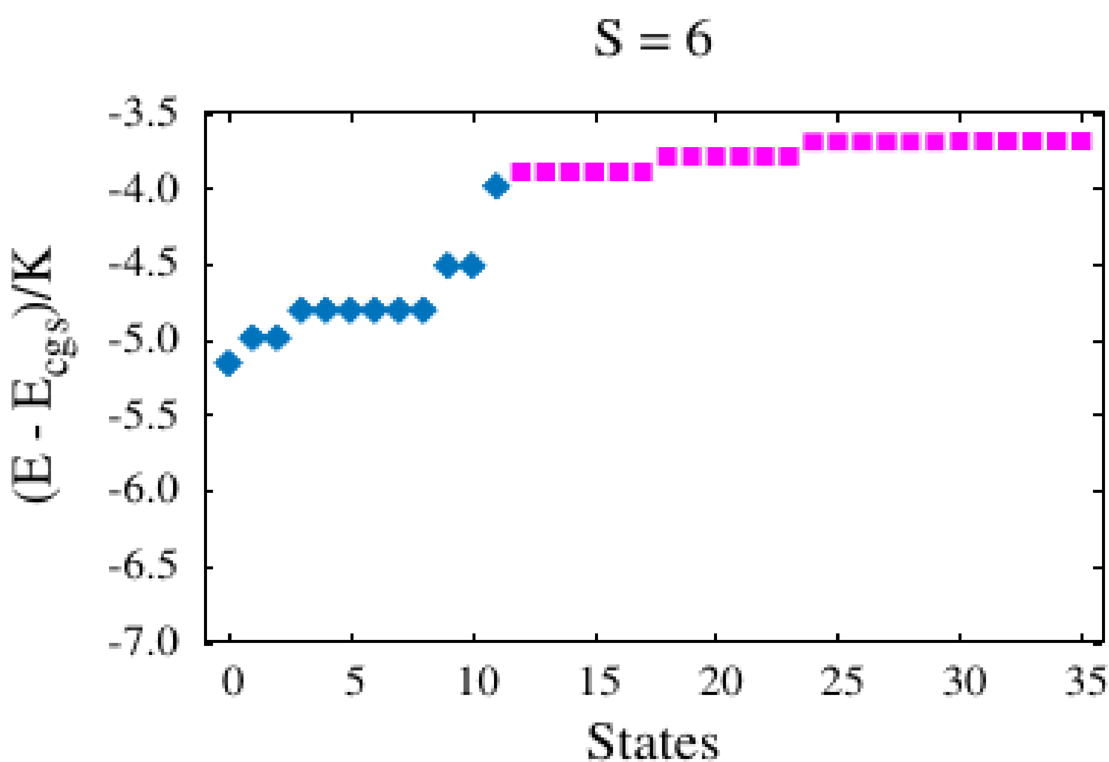
FIG. 11: Additional cartesian states that emerge in the Kitaev tetrahedron.



Eight spheres embedded in six dimensions.
Each sphere lies in a 3 dimensional subspace.

Example: Σ1 sphere lies in the space spanned by x, y and z coordinates.


a)
b)
c)
Z
Y
X
X/Z
Y
X/Z
Y


S = 6
$(E - E_{cgs})/K$
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-6.5
-7.0
0
5
10
15
20
25
30
35
States

Thank you for your attention

Kitaev's method of solution

2 Majorana fermions make one complex or Dirac fermion

$$\psi^{+}=\xi+i\zeta$$

$$2=\sqrt{2}\times\sqrt{2}$$

$$\{\psi,\psi^{+}\}=1$$

$$\{\xi,\zeta\}=0$$

$$\xi^{2}=\zeta^{2}=1$$

Introduce 4 Majorana fermions at each site:

$$c^{\alpha},\ \ \alpha=0,x,y,z \qquad \{c^{\alpha},c^{\beta}\}=2\delta_{\alpha\beta}$$

$$D_i|\Psi\rangle_{\rm phys} = |\Psi\rangle_{\rm phys}$$

$$D_i \equiv c_i\ c_i^x c_i^y c_i^z$$

Dimension of Physical Hilbert Space 2^{2N}

Dimension of Enlarged Hilbert Space 4^{2N}

$$\sigma_i^a = i c_i c_i^a, \qquad a=x,y,z$$

$$[\sigma_i^a,\sigma_j^b] = i\epsilon_{abc}\sigma^c\delta_{ij}$$

$$H = -\sum_{a=x,y,z} J_a \sum_{\langle ij \rangle_a} i c_i \hat{u}_{\langle ij \rangle_a} c_j$$

$$\hat{u}_{\langle ij \rangle_a} \equiv i c_i^a c_j^a$$

$$\boxed{[H, \hat{u}_{\langle ij \rangle_a}] = 0}$$

$\hat{u}_{\langle ij \rangle_a}$ **eigen value** $u_{\langle ij \rangle_a} = \pm 1$

$u_{\langle ij \rangle_a}$ (Ising) Z_2 gauge fields on the bonds

Local Z_2 gauge symmetry $u_{\langle ij \rangle_a} \longrightarrow \tau_i u_{\langle ij \rangle_a} \tau_j$

with $\tau_i \pm 1$

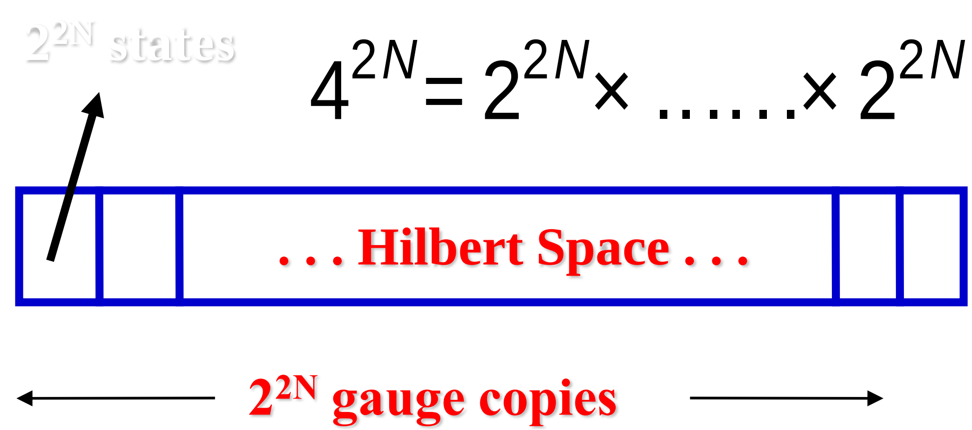
Dimension of
Physical Hilbert Space 2^{2N}

Dimension of
Enlarged Hilbert Space 4^{2N}

Spectrum is identical in all gauge copies

Unphysical Hilbert space has correct energy spectrucm !

$$4^{2N} = 2^{2N} \times \ldots\ldots \times 2^{2N}$$



On the nature of the wave function

Kitaev model does not have a continuous SU(2) symmetry

Physics is independent of sign of J

Proliferation of short range singlet and triplet bonds

It is a generalised triplet-singlet RVB (TS-RVB)

Spin of a Majorana fermion (?) - half integer (Shankar)

Exact results on large S Kitaev Model

GB, Diptiman Sen, Shankar
PRB 2008

Connection of classical ground states
to dimer covering on the hexagonal lattice

Topology of the classical ground state manifold
(cartesian states connected by flat lines in spin space)

Finite S:
Exact conserved plaquette operators for
Spin-S Kitaev model
Majorana fermion operators for half integer cases

An exactly solvable Kitaev type model for
arbitrary half integer spin

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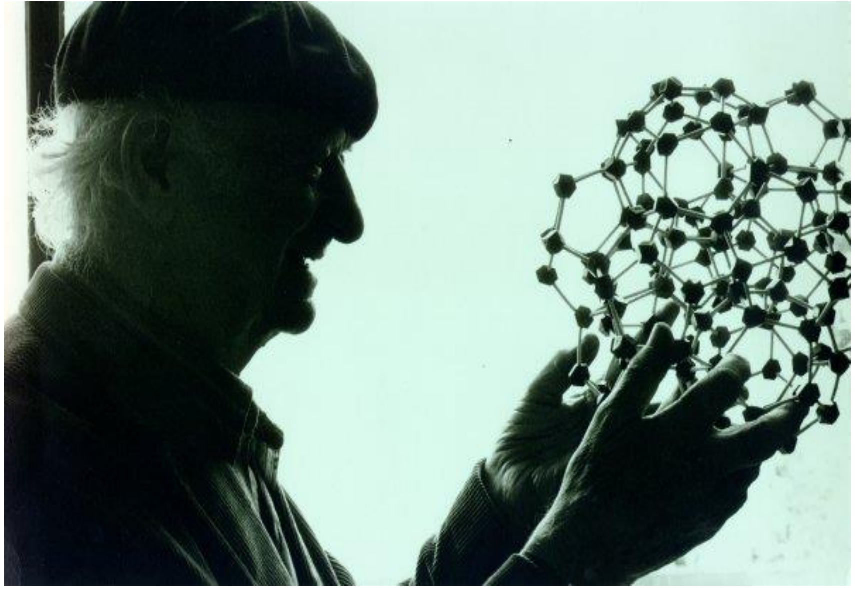
Perimeter Scholars International (PSI) Program

An example of respect by an individual
for theoretical physics as something
that helps transform society
(other examples – Kavli, Yuri Millner, Simons
Mahendralal Sarkar, Tata, Alagappa, Birla, Mehta, A C Muthiah
Infosys, Krish Gopalakrishnan, Wipro, HCL, SASTRA ...

Donor of
150 Million $
(Black Berry Chief)

A total of 400 Million $



Pauling was fond of Bonds

So is Anderson

But Anderson focussed on the spins in the bonds